AIIMS Paramedical Mathematics Sample Paper – 7

Duration: 30 Minutes **Maximum Marks: 30**

Instructions

- This paper contains **30** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics portion of **AIIMS Paramedical** entrance.
- Each correct answer carries **+1 mark**. Each incorrect answer attracts **a penalty of $-\frac{1}{3}$ mark**. Unattempted questions carry **0** marks.
- Only **one** option is correct. Choose carefully.
- Syllabus level: **Class 11 & 12 NCERT Mathematics**
- Use of mobile phones, calculators, or electronic gadgets is strictly prohibited.

Q1. If a set A has 4 elements and a set B has 3 elements, then the total number of relations that can be defined from A to B is

(A) 2^{12}

(B) 2^7

(C) 12

(D) 4^3

Q2. The range of the function $f(x) = \dfrac{x-2}{x-3}$, $x \neq 3$, is

(A) $\mathbb{R}$

(B) $\mathbb{R} - \{0\}$

(C) $\mathbb{R} - \{1\}$

(D) $\mathbb{R} - \{3\}$

Q3. Using a product-to-sum transformation, the value of $2\sin 75^\circ \cos 15^\circ$ is

(A) $\frac{1}{2}$

(B) $\frac{2+\sqrt{3}}{2}$

(C) $\frac{\sqrt{3}}{2}$

(D) $1+\sqrt{3}$

Q4. The principal value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)+\cos^{-1}\left(-\frac{1}{2}\right)$ is

(A) $\frac{\pi}{2}$

(B) $\frac{2\pi}{3}$

(C) $\frac{\pi}{6}$

(D) $\frac{\pi}{3}$

Q5. One of the square roots of the complex number $3+4i$ is

(A) $2+i$

(B) $1+2i$

(C) $2-i$

(D) $\sqrt{3}+2i$

Q6. The number of ways in which the letters of the word **DELHI** can be arranged so that the two vowels E and I are never together is

(A) 48

(B) 96

(C) 72

(D) 120

Q7. The 4th term from the end in the expansion of $(x+1)^9$ is

(A) $126\,x^4$

(B) $84\,x^3$

(C) $126\,x^3$

(D) $84\,x^4$

Q8. The sum of three numbers in arithmetic progression is 24 and the sum of their squares is 200. The numbers (in increasing order) are

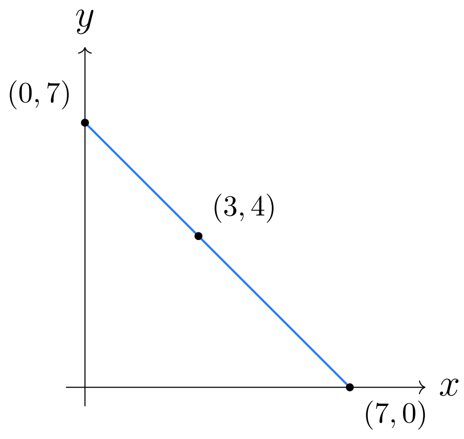
(A) $4, 8, 12$

(B) $5, 8, 11$

(C) $7, 8, 9$

(D) $6, 8, 10$

Q9. A straight line passes through the point $(3, 4)$ and cuts off equal positive intercepts on the coordinate axes. Its equation is



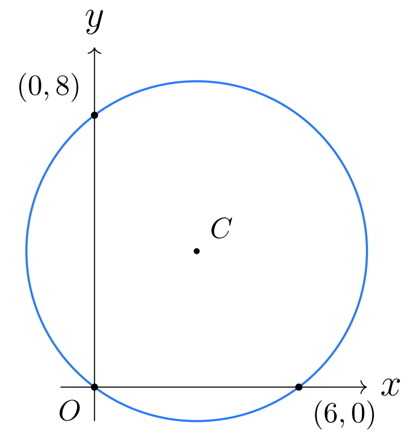
(A) $x + y = 7$

(B) $x - y = 7$

(C) $x + y = 12$

(D) $4x + 3y = 24$

Q10. The equation of the circle passing through the points $(0, 0)$, $(6, 0)$ and $(0, 8)$ is



(A) $x^2 + y^2 - 6x - 8y + 24 = 0$

(B) $x^2 + y^2 + 6x + 8y = 0$

(C) $x^2 + y^2 - 6x - 8y = 0$

(D) $x^2 + y^2 - 3x - 4y = 0$

Q11. The value of $\lim\limits_{x \to 0} \dfrac{e^{2x} - 1}{\sin 3x}$ is

(A) 1

(B) $\dfrac{2}{3}$

(C) $\dfrac{3}{2}$

(D) 0

Q12. Using the first-principle definition of the derivative, $\dfrac{d}{dx}(\sqrt{x})$ equals

(A) $\dfrac{1}{x}$

(B) $2\sqrt{x}$

(C) $\sqrt{x}$

(D) $\dfrac{1}{2\sqrt{x}}$

Q13. If $A = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$, then $A^2 - 5A$ equals

(A) $-7I$

(B) $7I$

(C) $5I$

(D) $-5I$

Q14. For the determinant $\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$, the cofactor of the element $a_{23} = 6$ is

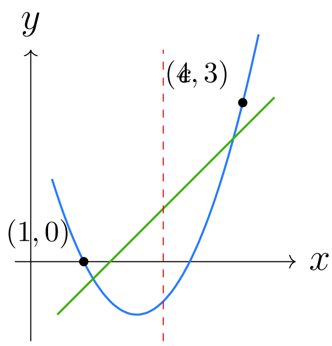
(A) -6

(B) 0

(C) 6

(D) -3

Q15. For the function $f(x) = x^2 - 4x + 3$ on the interval $[1, 4]$, the value of c guaranteed by Lagrange's mean value theorem is



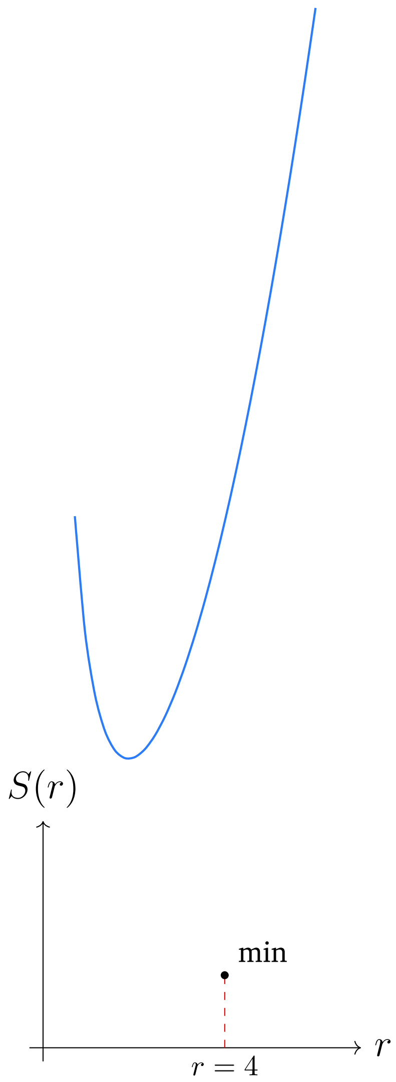
(A) 2

(B) $\frac{5}{2}$

(C) 3

(D) $\frac{3}{2}$

Q16. A manufacturer wants a closed right circular cylinder of volume 128π cm^3 to be made using the least possible sheet metal. The radius r (in cm) that minimises the total surface area is



(A) 8

(B) 2

(C) 6

(D) 4

Q17. The value of the integral $\int e^x \left(\frac{1}{x} + \log x\right) dx$ is (where C is an arbitrary constant)

(A) $e^x \log x + C$

(B) $\frac{e^x}{x} + C$

(C) $e^x \left(\log x - \frac{1}{x}\right) + C$

(D) $e^x + \log x + C$

Q18. Using a suitable substitution, $\int \frac{2x}{1+x^2}\,dx$ equals (where C is an arbitrary constant)

(A) $\frac{1}{1+x^2} + C$

(B) $\tan^{-1} x + C$

(C) $\log(1+x^2) + C$

(D) $2\log(1+x^2) + C$

Q19. The value of the definite integral $\int_0^{\pi/2} \cos x\,dx$ is

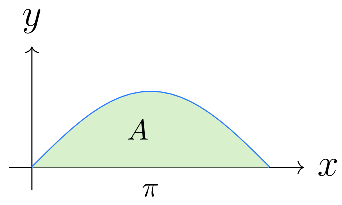
(A) 0

(B) 1

(C) $\frac{\pi}{2}$

(D) -1

Q20. The area bounded by the curve $y = \sin x$, the x-axis and the ordinates $x = 0$ and $x = \pi$ is



(A) 0

(B) π

(C) 1

(D) 2

Q21. The integrating factor of the linear differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$ is

(A) x

(B) $\frac{1}{x}$

(C) x^2

(D) e^x

Q22. The particular solution of $\frac{dy}{dx} = 2x$ satisfying $y(1) = 3$ is

(A) $y = x^2 + 3$

(B) $y = 2x^2 + 1$

(C) $y = x^2 + 2$

(D) $y = x^2 - 2$

Q23. The magnitude of the vector $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ is

(A) 5

(B) 7

(C) $\sqrt{11}$

(D) 11

Q24. The scalar projection (component) of the vector $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ along $\vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}$ is

(A) $\frac{4}{3}$

(B) 3

(C) $\frac{10}{3}$

(D) $\frac{8}{3}$

Q25. The points $A(1, 2, 3)$, $B(2, 4, 5)$ and $C(4, 8, 9)$ are collinear if the value of the z-coordinate replaced by λ at $C(4, 8, \lambda)$ equals

(A) 9

(B) 8

(C) 7

(D) 11

Q26. The plane $2x + 3y + 4z = 12$ cuts intercepts on the coordinate axes of lengths

(A) $2, 3, 4$

(B) $12, 12, 12$

(C) $6, 4, 3$

(D) $3, 4, 6$

Q27. The probability that a student passes in Mathematics is 0.7 and in English is 0.6, with the two events independent. The probability that the student passes in at least one subject is

(A) 0.42

(B) 0.76

(C) 0.30

(D) 0.88

Q28. A fair coin is tossed 5 times. The probability of getting exactly 3 heads is

(A) $\frac{3}{16}$

(B) $\frac{5}{16}$

(C) $\frac{1}{2}$

(D) $\frac{10}{32}$

Q29. For a data set the mean is 40 and the standard deviation is 8. The coefficient of variation is

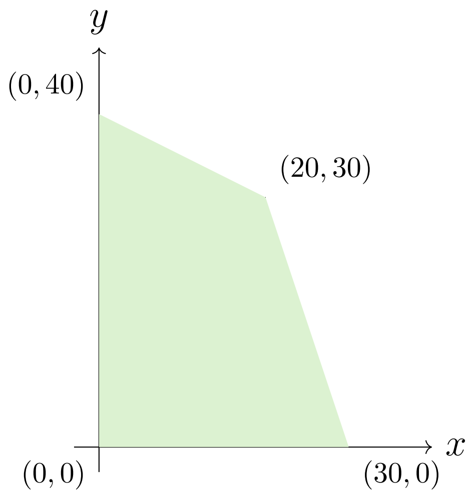
(A) 20%

(B) 32%

(C) 5%

(D) 48%

Q30. A factory makes two products X and Y. Each unit of X gives a profit of Rs. 40 and each unit of Y gives Rs. 50. The feasible region of the LPP has corner points $(0,0)$, $(30,0)$, $(20,30)$ and $(0,40)$. The maximum profit (in rupees) is



(A) 1200

(B) 2000

(C) 2300

(D) 2700

Detailed Solutions

Q1.

Solution

Concept — Number of relations: A relation from A to B is any subset of $A \times B$. If $A \times B$ has m ordered pairs, the number of subsets (relations) is 2^m.

Step 1 — Count ordered pairs: $n(A) = 4$ and $n(B) = 3$, so $n(A \times B) = 4 \times 3 = 12$.

Step 2 — Count subsets: number of relations $= 2^{n(A\times B)} = 2^{12}$.

Why other options are wrong:

- 2^7: would need 7 ordered pairs, not 12.
- 12: that is only the count of ordered pairs, not relations.
- 4^3: that is the number of *functions* from A to B, not relations.

Final Answer: The number of relations is $\boxed{2^{12}}$.

Answer: (A) Go Back to Q 1

Q2.

Solution

Concept — Range of a rational function: Set $y = f(x)$, solve for x in terms of y, and exclude any y for which no real x exists.

Step 1 — Put y equal to the function: $y = \dfrac{x-2}{x-3}$.

Step 2 — Cross-multiply: $y(x-3) = x-2$, so $yx - 3y = x - 2$.

Step 3 — Collect x terms: $yx - x = 3y - 2$, so $x(y-1) = 3y - 2$.

Step 4 — Solve for x: $x = \dfrac{3y-2}{y-1}$.

Step 5 — Identify excluded value: x is undefined when $y - 1 = 0$, i.e. $y = 1$. So $y = 1$ is not attained.

Why other options are wrong:

- $\mathbb{R}$: misses the excluded value $y = 1$.
- $\mathbb{R} - \{0\}$ and $\mathbb{R} - \{3\}$: those are not the values blocked by the algebra.

Final Answer: Range $= \boxed{\mathbb{R} - \{1\}}$.

Answer: (C) Go Back to Q 2

Q3.

Solution

Concept — Product to sum: $2\sin A\cos B = \sin(A+B)+\sin(A-B)$.

Step 1 — Apply the formula: with $A = 75°$, $B = 15°$,

$$2\sin 75°\cos 15° = \sin(75°+15°)+\sin(75°-15°).$$

Step 2 — Simplify the angles: $= \sin 90° + \sin 60°$.

Step 3 — Substitute known values: $\sin 90° = 1$ and $\sin 60° = \frac{\sqrt{3}}{2}$.

Step 4 — Add: $= 1+\frac{\sqrt{3}}{2} = \frac{2+\sqrt{3}}{2}$.

Why other options are wrong:

- $\frac{\sqrt{3}}{2}$ and $\frac{1}{2}$: drop one of the two sine terms.
- $1+\sqrt{3}$: doubles the second term incorrectly.

Final Answer: $\boxed{\frac{2+\sqrt{3}}{2}}$.

Answer: (B) Go Back to Q 3

Q4.

Solution

Concept — Principal values: $\sin^{-1}$ has range $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ and $\cos^{-1}$ has range $[0,\pi]$.

Step 1 — First term: $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$, since $\sin\left(-\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$ and $-\frac{\pi}{3}$ lies in the range.

Step 2 — Second term: $\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$, since $\cos\frac{2\pi}{3} = -\frac{1}{2}$ and $\frac{2\pi}{3} \in [0,\pi]$.

Step 3 — Add: $-\frac{\pi}{3}+\frac{2\pi}{3} = \frac{\pi}{3}$.

Why other options are wrong:

- $\frac{2\pi}{3}$: forgets the negative first term.
- $\frac{\pi}{2}$ and $\frac{\pi}{6}$: arise from wrong principal values.

Final Answer: $\boxed{\frac{\pi}{3}}$.

Answer: (D) Go Back to Q 4

Q5.

Solution

Concept — Square root of a complex number: Let $\sqrt{3+4i} = a + bi$. Then $a^2 - b^2 = 3$ and $2ab = 4$.

Step 1 — Set up equations: $a^2 - b^2 = 3$ and $ab = 2$.

Step 2 — Use the modulus: $a^2 + b^2 = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$.

Step 3 — Solve the pair: adding $a^2 - b^2 = 3$ and $a^2 + b^2 = 5$ gives $2a^2 = 8$, so $a^2 = 4$, $a = 2$. Then $b^2 = 1$, $b = 1$.

Step 4 — Check the sign: $ab = 2 > 0$, so a and b have the same sign; take $a = 2, b = 1$. Hence $\sqrt{3+4i} = 2 + i$.

Verification: $(2+i)^2 = 4 + 4i + i^2 = 4 + 4i - 1 = 3 + 4i$. ✓

Why other options are wrong:

- $2 - i$: squares to $3 - 4i$.
- $1 + 2i$: squares to $-3 + 4i$.
- $\sqrt{3} + 2i$: does not satisfy the equations.

Final Answer: $\boxed{2+i}$.

Answer: (A) Go Back to Q 5

Q6.

Solution

Concept — Never-together count: (Never together) $=$ (Total arrangements) $-$ (Arrangements with the two together).

Step 1 — Total arrangements: the word DELHI has 5 distinct letters, so total $= 5! = 120$.

Step 2 — Vowels together: treat E and I as one block. Then there are 4 units to arrange: $4! = 24$. The two vowels inside the block swap in $2! = 2$ ways. So together $= 24 \times 2 = 48$.

Step 3 — Subtract: never together $= 120 - 48 = 72$.

Why other options are wrong:

- 48: that is the count *with* the vowels together.
- 120: that is the total without any restriction.
- 96: double-counts the block swap.

Final Answer: $\boxed{72}$.

Answer: (C) Go Back to Q 6

Q7.

Solution

Concept — rth term from the end: In the expansion of $(a+b)^n$ (which has $n+1$ terms), the rth term from the end is the $(n-r+2)$th term from the beginning.

Step 1 — Identify the position: here $n = 9$ and $r = 4$. Term number from the beginning $= n - r + 2 = 9 - 4 + 2 = 7$. So we need the 7th term.

Step 2 — General term: for $(x+1)^9$, $T_{k+1} = \binom{9}{k} x^{9-k}(1)^k = \binom{9}{k} x^{9-k}$.

Step 3 — Use $k = 6$ for the 7th term: $T_7 = \binom{9}{6} x^{9-6} = \binom{9}{6} x^3$.

Step 4 — Evaluate the coefficient: $\binom{9}{6} = \binom{9}{3} = \dfrac{9 \cdot 8 \cdot 7}{3 \cdot 2 \cdot 1} = 84$. Hence $T_7 = 84\,x^3$.

Why other options are wrong:

- $126\,x^4$ and $126\,x^3$: use $\binom{9}{4} = 126$, the wrong term index.
- $84\,x^4$: wrong power of x.

Final Answer: $\boxed{84\,x^3}$.

Answer: (B) Go Back to Q 7

Q8.

Solution

Concept — Three numbers in AP: Take them as $a-d,\ a,\ a+d$ to use symmetry.

Step 1 — Use the sum: $(a-d)+a+(a+d)=3a=24$, so $a=8$.

Step 2 — Use the sum of squares: $(a-d)^2+a^2+(a+d)^2=3a^2+2d^2=200$.

Step 3 — Substitute $a=8$**:** $3(64)+2d^2=200$, so $192+2d^2=200$.

Step 4 — Solve for d**:** $2d^2=8$, $d^2=4$, $d=2$.

Step 5 — Write the numbers: $a-d,\ a,\ a+d=6,8,10$.

Why other options are wrong:

- $4,8,12$: sum of squares $=224\neq 200$.
- $5,8,11$: sum of squares $=210\neq 200$.
- $7,8,9$: sum of squares $=194\neq 200$.

Final Answer: $\boxed{6,8,10}$.

Answer: (D) Go Back to Q 8

Q9.

Solution

Concept — Equal intercepts: A line with equal intercepts a on both axes is $\frac{x}{a}+\frac{y}{a}=1$, i.e. $x+y=a$.

Step 1 — Write the family: $x+y=a$.

Step 2 — Pass through $(3,4)$**:** $3+4=a$, so $a=7$.

Step 3 — State the equation: $x+y=7$, with positive intercepts 7 and 7.

Why other options are wrong:

- $x-y=7$: intercepts 7 and -7, not equal positive.
- $x+y=12$: does not pass through $(3,4)$.
- $4x+3y=24$: has unequal intercepts 6 and 8.

Final Answer: $\boxed{x+y=7}$.

Answer: (A) Go Back to Q 9

Q10.

Solution

Concept — Circle through three points: Use the general form $x^2 + y^2 + 2gx + 2fy + c = 0$ and substitute each point.

Step 1 — Use $(0,0)$**:** $0 + 0 + 0 + 0 + c = 0 \Rightarrow c = 0$.

Step 2 — Use $(6,0)$**:** $36 + 12g + c = 0 \Rightarrow 36 + 12g = 0 \Rightarrow g = -3$.

Step 3 — Use $(0,8)$**:** $64 + 16f + c = 0 \Rightarrow 64 + 16f = 0 \Rightarrow f = -4$.

Step 4 — Write the circle: $x^2+y^2+2(-3)x+2(-4)y+0 = 0$, i.e. $x^2+y^2-6x-8y = 0$.

Why other options are wrong:

- $x^2 + y^2 - 6x - 8y + 24 = 0$: nonzero c, so it misses the origin.
- $x^2 + y^2 + 6x + 8y = 0$: wrong signs for g, f.
- $x^2 + y^2 - 3x - 4y = 0$: halves the coefficients incorrectly.

Final Answer: $\boxed{x^2 + y^2 - 6x - 8y = 0}$.

Answer: (C) Go Back to Q 10

Q11.

Solution

Concept — Standard limits: $\lim_{x\to 0} \frac{e^{kx} - 1}{kx} = 1$ and $\lim_{x\to 0} \frac{\sin kx}{kx} = 1$.

Step 1 — Rewrite with standard forms:

$$\frac{e^{2x} - 1}{\sin 3x} = \frac{e^{2x} - 1}{2x} \cdot \frac{3x}{\sin 3x} \cdot \frac{2x}{3x}.$$

Step 2 — Take limits factor by factor: $\frac{e^{2x} - 1}{2x} \to 1$, $\frac{3x}{\sin 3x} \to 1$, and $\frac{2x}{3x} = \frac{2}{3}$.

Step 3 — Multiply: limit $= 1 \cdot 1 \cdot \frac{2}{3} = \frac{2}{3}$.

Cross-check (L'Hopital): derivative of top $= 2e^{2x} \to 2$; derivative of bottom $= 3\cos 3x \to 3$; ratio $= \frac{2}{3}$. ✓

Why other options are wrong:

- 1: ignores the coefficient ratio.

- $\frac{3}{2}$: inverts the ratio.
- 0: ignores that both top and bottom vanish.

Final Answer: $\boxed{\frac{2}{3}}$.

Answer: (B) Go Back to Q 11

Q12.

Solution

Concept — First principle: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$.

Step 1 — Write the quotient: with $f(x) = \sqrt{x}$,

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}.$$

Step 2 — Rationalise: multiply by $\frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$:

$$= \lim_{h \to 0} \frac{(x+h) - x}{h\left(\sqrt{x+h} + \sqrt{x}\right)}.$$

Step 3 — Simplify the numerator: $(x+h) - x = h$, so

$$= \lim_{h \to 0} \frac{h}{h\left(\sqrt{x+h} + \sqrt{x}\right)} = \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}}.$$

Step 4 — Take the limit: $= \frac{1}{\sqrt{x} + \sqrt{x}} = \frac{1}{2\sqrt{x}}$.

Why other options are wrong:

- $2\sqrt{x}$: that is an antiderivative, not the derivative.
- $\frac{1}{x}$ and $\sqrt{x}$: wrong power rule.

Final Answer: $\boxed{\frac{1}{2\sqrt{x}}}$.

Answer: (D) Go Back to Q 12

Q13.

Solution

Concept — Matrix polynomial: Compute A^2, then subtract $5A$, and compare with a scalar multiple of I.

Step 1 — Compute A^2: with $A = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$,

$$A^2 = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ 5 & 8 \end{bmatrix}.$$

Step 2 — Compute $5A$: $5A = \begin{bmatrix} 10 & -5 \\ 5 & 15 \end{bmatrix}$.

Step 3 — Subtract: $A^2 - 5A = \begin{bmatrix} 3-10 & -5+5 \\ 5-5 & 8-15 \end{bmatrix} = \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix}$.

Step 4 — Recognise the result: $\begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} = -7I$.

Why other options are wrong:

- $7I$: sign error.
- $5I$ and $-5I$: wrong scalar.

Final Answer: $\boxed{-7I}$.

Answer: (A) Go Back to Q 13

Q14.

Solution

Concept — Cofactor: $C_{ij} = (-1)^{i+j} M_{ij}$, where M_{ij} is the minor obtained by deleting row i and column j.

Step 1 — Find the minor M_{23}: delete row 2 and column 3 from $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$, leaving $\begin{vmatrix} 1 & 2 \\ 7 & 8 \end{vmatrix}$.

Step 2 — Evaluate the minor: $M_{23} = (1)(8) - (2)(7) = 8 - 14 = -6$.

Step 3 — Apply the sign: $C_{23} = (-1)^{2+3} M_{23} = (-1)^5(-6) = -(-6) = 6$.

Why other options are wrong:

- -6: that is the minor, before applying the sign.
- 0: the determinant of the whole matrix is 0, but that is a different quantity.
- -3: arithmetic slip.

Final Answer: $\boxed{6}$.

Answer: (C) Go Back to Q 14

Q15.

Solution

Concept — Lagrange's MVT: If f is continuous on $[a, b]$ and differentiable on (a, b), there is $c \in (a, b)$ with $f'(c) = \dfrac{f(b) - f(a)}{b - a}$.

Step 1 — Endpoint values: $f(1) = 1 - 4 + 3 = 0$ and $f(4) = 16 - 16 + 3 = 3$.

Step 2 — Average slope: $\dfrac{f(4) - f(1)}{4 - 1} = \dfrac{3 - 0}{3} = 1$.

Step 3 — Derivative: $f'(x) = 2x - 4$.

Step 4 — Solve $f'(c) = 1$: $2c - 4 = 1$, so $2c = 5$, $c = \dfrac{5}{2}$.

Step 5 — Check the interval: $\dfrac{5}{2} = 2.5 \in (1, 4)$. ✓

Why other options are wrong:

- 2 and 3: do not satisfy $2c - 4 = 1$.
- $\dfrac{3}{2}$: gives $f'(c) = -1$, the wrong slope.

Final Answer: $\boxed{c = \dfrac{5}{2}}$.

Answer: (B) Go Back to Q 15

Q16.

Solution

Concept — Minimum-surface cylinder: For fixed volume $V = \pi r^2 h$, minimise total surface $S = 2\pi r^2 + 2\pi r h$ by differentiating in r.

Step 1 — Express h: $V = 128\pi = \pi r^2 h \Rightarrow h = \dfrac{128}{r^2}$.

Step 2 — Surface in one variable: $S = 2\pi r^2 + 2\pi r \cdot \dfrac{128}{r^2} = 2\pi r^2 + \dfrac{256\pi}{r}$.

Step 3 — Differentiate: $\dfrac{dS}{dr} = 4\pi r - \dfrac{256\pi}{r^2}$.

Step 4 — Set to zero: $4\pi r = \dfrac{256\pi}{r^2} \Rightarrow r^3 = \dfrac{256}{4} = 64 \Rightarrow r = 4$.

Step 5 — Confirm minimum: $\dfrac{d^2S}{dr^2} = 4\pi + \dfrac{512\pi}{r^3} > 0$, so $r = 4$ gives a minimum.

Why other options are wrong:

- $8, 2, 6$: do not satisfy $r^3 = 64$.

Final Answer: $\boxed{r = 4 \text{ cm}}$.

Answer: (D) Go Back to Q 16

Q17.

Solution

Concept — $\int e^x(f + f')\,dx$: $\displaystyle\int e^x\big(f(x) + f'(x)\big)dx = e^x f(x) + C$.

Step 1 — Identify f and f': take $f(x) = \log x$. Then $f'(x) = \dfrac{1}{x}$.

Step 2 — Match the integrand: $\dfrac{1}{x} + \log x = f'(x) + f(x)$, which is exactly the required form.

Step 3 — Apply the rule: $\displaystyle\int e^x\left(\frac{1}{x} + \log x\right)dx = e^x \log x + C$.

Why other options are wrong:

- $\dfrac{e^x}{x} + C$: that is $\int e^x\left(\dfrac{1}{x} - \dfrac{1}{x^2}\right)dx$, a different integrand.
- $e^x\left(\log x - \dfrac{1}{x}\right) + C$: wrong sign pairing.
- $e^x + \log x + C$: not an antiderivative of the product.

Final Answer: $\boxed{e^x \log x + C}$.

Answer: (A) Go Back to Q 17

Q18.

Solution

Concept — Substitution: When the numerator is the derivative of the denominator, $\int \frac{g'(x)}{g(x)} dx = \log|g(x)| + C$.

Step 1 — Choose substitution: let $t = 1 + x^2$. Then $dt = 2x\,dx$.

Step 2 — Rewrite the integral: $\int \frac{2x}{1+x^2} dx = \int \frac{dt}{t}$.

Step 3 — Integrate: $= \log|t| + C$.

Step 4 — Back-substitute: $= \log(1 + x^2) + C$ (positive, so modulus dropped).

Why other options are wrong:

- $\tan^{-1} x + C$: that is $\int \frac{1}{1+x^2} dx$, missing the $2x$.
- $\frac{1}{1+x^2} + C$: differentiates wrongly.
- $2\log(1 + x^2) + C$: extra factor of 2.

Final Answer: $\boxed{\log(1 + x^2) + C}$.

Answer: (C) Go Back to Q 18

Q19.

Solution

Concept — Fundamental theorem: $\int_a^b f(x)\,dx = F(b) - F(a)$, where $F' = f$.

Step 1 — Antiderivative: $\int \cos x\,dx = \sin x$.

Step 2 — Apply limits: $\int_0^{\pi/2} \cos x\,dx = [\sin x]_0^{\pi/2} = \sin\frac{\pi}{2} - \sin 0$.

Step 3 — Evaluate: $= 1 - 0 = 1$.

Why other options are wrong:

- 0: would be the value of $\int_0^{\pi} \cos x\,dx$.

- $\frac{\pi}{2}$: confuses the integral of a constant.
- -1: wrong sign.

Final Answer: $\boxed{1}$.

Answer: (B) Go Back to Q 19

Q20.

Solution

Concept — Area under a curve: Since $\sin x \geq 0$ on $[0, \pi]$, area $= \displaystyle\int_0^{\pi} \sin x \, dx$.

Step 1 — Antiderivative: $\displaystyle\int \sin x \, dx = -\cos x$.

Step 2 — Apply limits: $\left[-\cos x\right]_0^{\pi} = -\cos \pi - (-\cos 0)$.

Step 3 — Evaluate: $= -(-1) - (-(1)) = 1 + 1 = 2$.

Why other options are wrong:

- 0: that is $\int_0^{2\pi} \sin x \, dx$ where positive and negative areas cancel.
- π: confuses area with interval length.
- 1: only half the region.

Final Answer: $\boxed{2}$ square units.

Answer: (D) Go Back to Q 20

Q21.

Solution

Concept — Integrating factor: For $\frac{dy}{dx} + P(x)y = Q(x)$, the integrating factor is $\mu = e^{\int P \, dx}$.

Step 1 — Identify P: here $P(x) = \frac{1}{x}$.

Step 2 — Integrate P: $\displaystyle\int \frac{1}{x} \, dx = \log x$.

Step 3 — Form the IF: $\mu = e^{\log x} = x$.

Why other options are wrong:

- $\frac{1}{x} = e^{-\log x}$: uses $P = -\frac{1}{x}$.

- x^2: uses $P = \frac{2}{x}$.
- e^x: uses $P = 1$.

Final Answer: $\boxed{x}$.

Answer: (A) Go Back to Q 21

Q22.

Solution

Concept — Particular solution: Integrate the differential equation, then use the initial condition to fix the constant.

Step 1 — Integrate: $\int dy = \int 2x\,dx \Rightarrow y = x^2 + C$.

Step 2 — Apply $y(1) = 3$**:** $3 = 1^2 + C = 1 + C$, so $C = 2$.

Step 3 — Write the solution: $y = x^2 + 2$.

Why other options are wrong:

- $y = x^2 + 3$: uses $C = 3$, fails at $x = 1$.
- $y = 2x^2 + 1$: wrong integration.
- $y = x^2 - 2$: wrong sign of constant.

Final Answer: $\boxed{y = x^2 + 2}$.

Answer: (C) Go Back to Q 22

Q23.

Solution

Concept — Magnitude of a vector: $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

Step 1 — Square the components: $2^2 = 4$, $(-3)^2 = 9$, $6^2 = 36$.

Step 2 — Add: $4 + 9 + 36 = 49$.

Step 3 — Take the square root: $|\vec{a}| = \sqrt{49} = 7$.

Why other options are wrong:

- 5: drops the $\hat{k}$ component.
- 11: adds magnitudes without squaring.
- $\sqrt{11}$: misadds the squares.

Final Answer: $\boxed{7}$.

Answer: (B) Go Back to Q 23

Q24.

Solution

Concept — Scalar projection: The component of $\vec{a}$ along $\vec{b}$ is $\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$.

Step 1 — Dot product: $\vec{a}\cdot\vec{b} = (2)(1) + (1)(2) + (2)(2) = 2 + 2 + 4 = 8.$

Step 2 — Magnitude of $\vec{b}$: $|\vec{b}| = \sqrt{1^2 + 2^2 + 2^2} = \sqrt{1 + 4 + 4} = \sqrt{9} = 3.$

Step 3 — Divide: component $= \dfrac{8}{3}$.

Why other options are wrong:

- $\dfrac{4}{3}$: halves the dot product.
- 3: gives the magnitude of $\vec{b}$, not the projection.
- $\dfrac{10}{3}$: arithmetic slip in the dot product.

Final Answer: $\boxed{\dfrac{8}{3}}$.

Answer: (D) Go Back to Q 24

Q25.

Solution

Concept — Collinearity in 3D: Three points are collinear if $\vec{AB}$ and $\vec{AC}$ are parallel, i.e. proportional component-wise.

Step 1 — Form $\vec{AB}$: $B - A = (2 - 1,\ 4 - 2,\ 5 - 3) = (1, 2, 2).$

Step 2 — Form $\vec{AC}$: with $C = (4, 8, \lambda)$, $C - A = (3, 6, \lambda - 3)$.

Step 3 — Impose proportionality: $\dfrac{3}{1} = \dfrac{6}{2} = \dfrac{\lambda - 3}{2}$. The first two ratios are 3, so $\dfrac{\lambda - 3}{2} = 3$.

Step 4 — Solve: $\lambda - 3 = 6 \Rightarrow \lambda = 9.$

Why other options are wrong:

- $8, 7, 11$: break the common ratio of 3.

Final Answer: $\boxed{\lambda = 9}$.

Answer: (A) Go Back to Q 25

Q26.

Solution

Concept — Intercept form of a plane: Writing $\frac{x}{a}+\frac{y}{b}+\frac{z}{c} = 1$ reads the intercepts a, b, c directly.

Step 1 — Divide by 12**:** $2x + 3y + 4z = 12 \Rightarrow \frac{2x}{12} + \frac{3y}{12} + \frac{4z}{12} = 1$.

Step 2 — Simplify each term: $\frac{x}{6} + \frac{y}{4} + \frac{z}{3} = 1$.

Step 3 — Read intercepts: $a = 6$, $b = 4$, $c = 3$.

Why other options are wrong:

- $2, 3, 4$: those are the coefficients, not the intercepts.
- $12, 12, 12$: forgets to divide by each coefficient.
- $3, 4, 6$: lists the intercepts in the wrong order for x, y, z.

Final Answer: Intercepts $\boxed{6, 4, 3}$ on the x, y, z axes.

Answer: (C) Go Back to Q 26

Q27.

Solution

Concept — At least one: $P(\text{at least one}) = 1 - P(\text{none})$. For independent events, $P(\text{none}) = P(\bar{M})P(\bar{E})$.

Step 1 — Failure probabilities: $P(\bar{M}) = 1 - 0.7 = 0.3$ and $P(\bar{E}) = 1 - 0.6 = 0.4$.

Step 2 — Probability of failing both: $P(\text{none}) = 0.3 \times 0.4 = 0.12$.

Step 3 — Complement: $P(\text{at least one}) = 1 - 0.12 = 0.88$.

Why other options are wrong:

- 0.42: that is $P(M) \cdot P(E)$, passing both.
- 0.76: misuses the addition rule without subtracting overlap.
- 0.30: only $P(\bar{M})$.

Final Answer: $\boxed{0.88}$.

Answer: (D) Go Back to Q 27

Q28.

Solution

Concept — Binomial probability: $P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$, with $n = 5$, $p = \frac{1}{2}$.

Step 1 — Count favourable arrangements: $\binom{5}{3} = 10$.

Step 2 — Probability of each outcome: $\left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$.

Step 3 — Multiply: $P(X = 3) = 10 \times \frac{1}{32} = \frac{10}{32} = \frac{5}{16}$.

Why other options are wrong:

- $\frac{3}{16}$: uses the wrong binomial coefficient.
- $\frac{1}{2}$: ignores the binomial structure.
- $\frac{10}{32}$: numerically equals $\frac{5}{16}$ but is left unreduced; the standard simplified form is $\frac{5}{16}$.

Final Answer: $\boxed{\frac{5}{16}}$.

Answer: (B) Go Back to Q 28

Q29.

Solution

Concept — Coefficient of variation: $\text{CV} = \frac{\sigma}{\bar{x}} \times 100\%$, where σ is the standard deviation and $\bar{x}$ the mean.

Step 1 — Substitute values: $\sigma = 8$, $\bar{x} = 40$.

Step 2 — Compute the ratio: $\frac{8}{40} = \frac{1}{5} = 0.2$.

Step 3 — Convert to percent: $0.2 \times 100\% = 20\%$.

Why other options are wrong:

- 32%: inverts the ratio or mis-divides.
- 5%: uses $\frac{2}{40}$ instead of $\frac{8}{40}$.

- 48%: unrelated arithmetic.

Final Answer: $\boxed{20\%}$.

Answer: (A) Go Back to Q 29

Q30.

Solution

Concept — LPP corner-point method: The maximum of a linear objective over a bounded feasible region occurs at a corner point. Evaluate Z at each corner.

Step 1 — Objective function: $Z = 40x + 50y$ (profit in rupees).

Step 2 — Evaluate at each corner:

- $(0, 0)$: $Z = 0$.
- $(30, 0)$: $Z = 40(30) + 50(0) = 1200$.
- $(20, 30)$: $Z = 40(20) + 50(30) = 800 + 1500 = 2300$.
- $(0, 40)$: $Z = 40(0) + 50(40) = 2000$.

Step 3 — Pick the largest: the maximum is $Z = 2300$ at $(20, 30)$.

Why other options are wrong:

- 1200: the value at $(30, 0)$, not the maximum.
- 2000: the value at $(0, 40)$, not the maximum.
- 2700: exceeds every corner value; not attainable in the region.

Final Answer: Maximum profit $=$ $\boxed{\text{Rs. } 2300}$ at $(20, 30)$.

Answer: (C) Go Back to Q 30

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	C	3	B	4	D	5	A
6	C	7	B	8	D	9	A	10	C
11	B	12	D	13	A	14	C	15	B
16	D	17	A	18	C	19	B	20	D
21	A	22	C	23	B	24	D	25	A
26	C	27	D	28	B	29	A	30	C