AIIMS Paramedical Mathematics Sample Paper – 8

Duration: 30 Minutes **Maximum Marks: 30**

Instructions

- This paper contains **30** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics portion of **AIIMS Paramedical** entrance.
- Each correct answer carries **+1 mark**; each incorrect answer carries a penalty of $-\frac{1}{3}$ mark; an unattempted question carries **0** mark.
- Only **one** option is correct. Choose carefully.
- Syllabus level: **Class 11 & 12 NCERT Mathematics**
- Use of mobile phones, calculators, or electronic gadgets is strictly prohibited.

Q1. If $A = \{1, 2, 3\}$, then the number of elements in the power set $P(A)$, and the number of proper subsets of A, are respectively

(A) 8 and 7

(B) 6 and 5

(C) 8 and 8

(D) 7 and 6

Q2. The function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2$ is

(A) one-one and onto

(B) neither one-one nor onto

(C) one-one but not onto

(D) onto but not one-one

Q3. In a triangle ABC where $A+B+C = \pi$, the value of $\tan A+\tan B+\tan C$ is equal to

(A) 0

(B) 1

(C) $\tan A \tan B \tan C$

(D) $\cot A + \cot B + \cot C$

Q4. The range of the principal value branch of $\cos^{-1} x$ is

(A) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(B) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(C) $(0, \pi)$

(D) $[0, \pi]$

Q5. If z_1 and z_2 are two complex numbers with $|z_1| = 3$ and $|z_2| = 4$, then the maximum possible value of $|z_1 + z_2|$ is

(A) 7

(B) 5

(C) 1

(D) 12

Q6. The number of ways of distributing 10 identical chocolates among 3 children so that each child gets at least one chocolate is

(A) 30

(B) 36

(C) 45

(D) 28

Q7. In the binomial expansion of $(1 + x)^n$, the ratio of the coefficient of the 5th term to the coefficient of the 4th term equals

(A) $\frac{n-3}{5}$

(B) $\dfrac{n-4}{5}$

(C) $\dfrac{n-3}{4}$

(D) $\dfrac{n-4}{4}$

Q8. The number of bacteria in a culture doubles every hour. If there are 500 bacteria initially, the number present at the end of 5 hours is

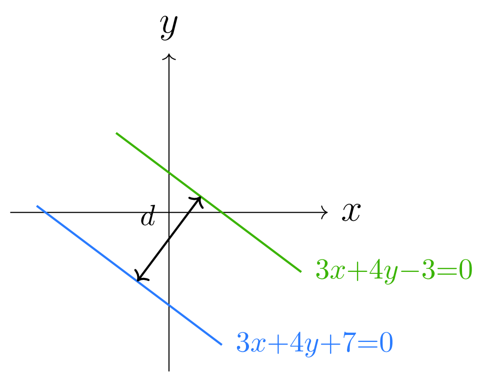
(A) 8000

(B) 32000

(C) 4000

(D) 16000

Q9. The distance between the two parallel lines $3x+4y+7=0$ and $3x+4y-3=0$ is



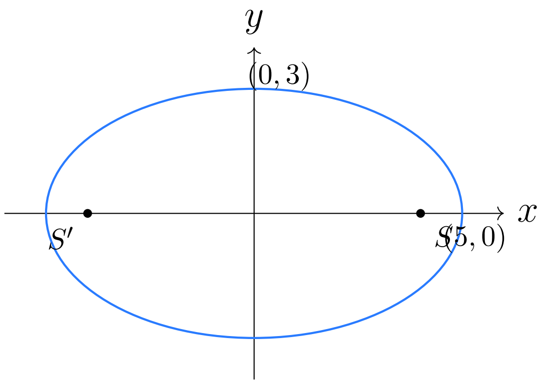
(A) 2

(B) 1

(C) 10

(D) $\dfrac{4}{5}$

Q10. For the ellipse $\dfrac{x^2}{25}+\dfrac{y^2}{9}=1$, the coordinates of the foci are



(A) $(\pm 3, 0)$

(B) $(\pm 4, 0)$

(C) $(0, \pm 4)$

(D) $(\pm 5, 0)$

Q11. The value of $\lim_{x \to 2} \frac{x^2 - 4}{x^2 - 3x + 2}$ is

(A) 2

(B) 1

(C) 4

(D) $\frac{1}{2}$

Q12. Which of the following statements about a function f is correct?

(A) If f is continuous at a point, it must be differentiable there.

(B) Differentiability does not imply continuity.

(C) $f(x) = |x|$ is differentiable at $x = 0$.

(D) If f is differentiable at a point, then it is continuous there.

Q13. If A is a square matrix such that $A^T A = I$ (an orthogonal matrix), then the value of $|A|$ (its determinant) is

(A) ± 1

(B) 0

(C) 2

(D) only 1

Q14. The system of equations $x + y = 3$ and $2x + 2y = 6$ is

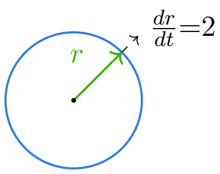
(A) inconsistent with no solution

(B) consistent with infinitely many solutions

(C) consistent with a unique solution

(D) inconsistent with a unique solution

Q15. A spherical balloon is being inflated so that its radius increases at a rate of 2 cm/s. The rate at which its volume increases when the radius is 5 cm is



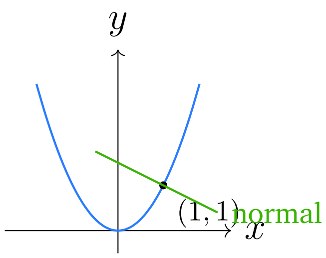
(A) 100π

(B) 50π

(C) 200π

(D) 400π

Q16. The equation of the normal to the curve $y = x^2$ at the point $(1, 1)$ is



(A) $2x - y - 1 = 0$

(B) $x - 2y + 1 = 0$

(C) $2x + y - 3 = 0$

(D) $x + 2y = 3$

Q17. The value of $\int \frac{dx}{\sqrt{9-x^2}}$ is (where C is the constant of integration)

(A) $\sin^{-1}\frac{x}{3}+C$

(B) $\frac{1}{3}\sin^{-1}\frac{x}{3}+C$

(C) $\sin^{-1}\frac{x}{9}+C$

(D) $\tan^{-1}\frac{x}{3}+C$

Q18. Using integration by parts, $\int \tan^{-1}x\,dx$ equals (where C is the constant of integration)

(A) $x\tan^{-1}x+\frac{1}{2}\log(1+x^2)+C$

(B) $x\tan^{-1}x-\log(1+x^2)+C$

(C) $\tan^{-1}x-\frac{1}{2}\log(1+x^2)+C$

(D) $x\tan^{-1}x-\frac{1}{2}\log(1+x^2)+C$

Q19. Using a symmetry property, the value of $\int_{-\pi/2}^{\pi/2}\sin^3 x\cos^2 x\,dx$ is

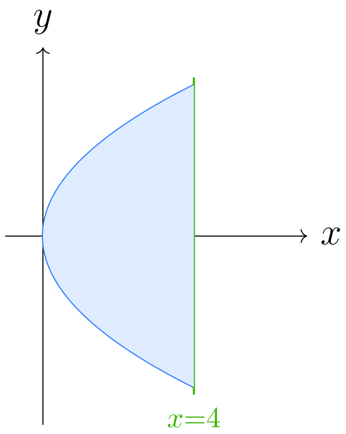
(A) $\frac{2}{15}$

(B) $\frac{4}{15}$

(C) 0

(D) $\frac{\pi}{2}$

Q20. The area bounded by the parabola $y^2=4x$ and the line $x=4$ (above the x-axis and below it) is



(A) $\frac{32}{3}$
(B) 16
(C) $\frac{16}{3}$
(D) $\frac{64}{3}$

Q21. The differential equation representing the family of curves $y = mx$, where m is an arbitrary constant, is

(A) $x\frac{dy}{dx} = y$
(B) $\frac{dy}{dx} = x$
(C) $y\frac{dy}{dx} = x$
(D) $\frac{dy}{dx} = xy$

Q22. The solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is (where C is an arbitrary constant)

(A) $y = x + C$
(B) $y = Cx$
(C) $y = Cx^2$
(D) $xy = C$

Q23. If $\vec{a}$ and $\vec{b}$ are two non-zero vectors such that $\vec{a} \cdot \vec{b} = 0$, then the vectors are

(A) parallel

(B) equal

(C) perpendicular

(D) anti-parallel

Q24. For the unit vectors $\hat{\imath}$ and $\hat{\jmath}$ along the x- and y-axes, the direction of $\hat{\imath} \times \hat{\jmath}$, by the right-hand rule, is along

(A) $-\hat{k}$

(B) $\hat{\imath}$

(C) $\hat{\jmath}$

(D) $\hat{k}$

Q25. The lines $\dfrac{x-1}{2} = \dfrac{y}{3} = \dfrac{z+1}{4}$ and $\dfrac{x}{4} = \dfrac{y-2}{6} = \dfrac{z}{8}$ are

(A) parallel

(B) perpendicular

(C) coincident

(D) intersecting at right angles

Q26. The distance between the two parallel planes $2x - 2y + z + 3 = 0$ and $2x - 2y + z - 6 = 0$ is

(A) 9

(B) 3

(C) 1

(D) $\dfrac{9}{2}$

Q27. If the probability that it rains on a given day is 0.3, then the probability that it does not rain on that day is

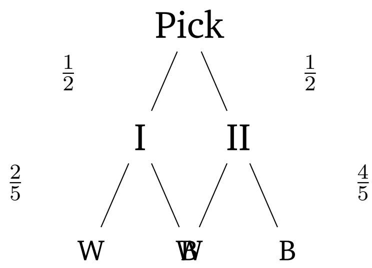
(A) 0.3

(B) 0.5

(C) 0.7

(D) 1.3

Q28. Bag I has 2 white and 3 black balls; Bag II has 4 white and 1 black ball. A bag is chosen at random and a white ball is drawn. By Bayes' theorem, the probability that it came from Bag II is



(A) $\frac{1}{3}$

(B) $\frac{1}{2}$

(C) $\frac{2}{5}$

(D) $\frac{2}{3}$

Q29. A group of 20 students has mean marks 60 and another group of 30 students has mean marks 50. The combined mean of all 50 students is

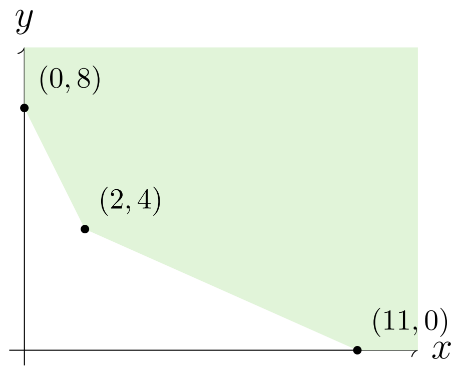
(A) 54

(B) 55

(C) 56

(D) 52

Q30. A diet problem requires minimising the cost $Z = 5x + 7y$ over a feasible region whose corner points are $(0, 8)$, $(2, 4)$ and $(11, 0)$. The minimum value of Z occurs and equals



(A) 40

(B) 38

(C) 56

(D) 55

Detailed Solutions

Q1.

Solution

Concept — Power set and subsets: For a set with n elements, the power set has 2^n elements and the number of proper subsets is $2^n - 1$.

Step 1 — Count the elements: $A = \{1, 2, 3\}$ has $n = 3$ elements.

Step 2 — Size of the power set: $|P(A)| = 2^3 = 8$.

Step 3 — Proper subsets: every subset except A itself is proper, giving $2^3 - 1 = 7$.

Why other options are wrong:

- 6 and 5: uses $2n$ instead of 2^n.
- 8 and 8: forgets to remove A itself.
- 7 and 6: subtracts one from the power-set count by mistake.

Final Answer: 8 and $7 \Rightarrow \boxed{\text{A}}$

Answer: (A) **Go Back to Q1**

Q2.

Solution

Concept — One-one and onto: A function is one-one if distinct inputs give distinct outputs, and onto if every element of the codomain is attained.

Step 1 — Test one-one: $f(2) = 4$ and $f(-2) = 4$, so two distinct inputs share an output; f is not one-one.

Step 2 — Test onto: the outputs of x^2 are never negative, so values like $-1 \in \mathbb{R}$ are not attained; f is not onto.

Step 3 — Classify: f is neither one-one nor onto.

Why other options are wrong:

- one-one and onto: fails both tests.
- one-one but not onto: fails the one-one test.
- onto but not one-one: fails the onto test.

Final Answer: neither one-one nor onto $\Rightarrow \boxed{\text{B}}$

Answer: (B) **Go Back to Q2**

Q3.

Solution

Concept — Conditional identity: When $A + B + C = \pi$, the tangents satisfy $\tan A + \tan B + \tan C = \tan A\ \tan B\ \tan C$.

Step 1 — Use the angle sum: $A + B = \pi - C$.

Step 2 — Take tangents: $\tan(A + B) = \tan(\pi - C) = -\tan C$.

Step 3 — Expand the left side: $\dfrac{\tan A + \tan B}{1 - \tan A \tan B} = -\tan C$.

Step 4 — Cross-multiply: $\tan A + \tan B = -\tan C(1 - \tan A \tan B)$.

Step 5 — Rearrange: $\tan A + \tan B + \tan C = \tan A\ \tan B\ \tan C$.

Why other options are wrong:

- 0: holds only for the cotangents in special cases, not here.
- 1: not a general identity.
- $\cot A + \cot B + \cot C$: a different and unrelated expression.

Final Answer: $\tan A\ \tan B\ \tan C \Rightarrow \boxed{\text{C}}$

Answer: (C) **Go Back to Q3**

Q4.

Solution

Concept — Principal value branches: Each inverse trig function is restricted to a fixed range to make it single-valued.

Step 1 — Recall the rule for $\cos^{-1}$: the principal branch of $\cos^{-1} x$ takes values in $[0, \pi]$.

Step 2 — Check endpoints: $\cos^{-1} 1 = 0$ and $\cos^{-1}(-1) = \pi$, so both endpoints are included.

Step 3 — State the range: the range is $[0, \pi]$.

Why other options are wrong:

- $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$: this is the range of $\sin^{-1} x$.
- $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$: this is the range of $\tan^{-1} x$.
- $(0, \pi)$: wrongly excludes the endpoints.

Final Answer: $[0, \pi] \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q4

Q5.

Solution

Concept — Triangle inequality: For any two complex numbers, $|z_1 + z_2| \le |z_1| + |z_2|$, with equality when they point in the same direction.

Step 1 — Apply the inequality: $|z_1 + z_2| \le |z_1| + |z_2|$.

Step 2 — Substitute the magnitudes: $|z_1 + z_2| \le 3 + 4$.

Step 3 — Add: $|z_1 + z_2| \le 7$.

Step 4 — Identify the maximum: the largest value, attained when z_1 and z_2 are co-directional, is 7.

Why other options are wrong:

- 5: comes from $\sqrt{3^2 + 4^2}$, the perpendicular case.
- 1: this is the minimum value $|z_1| - |z_2|$.
- 12: comes from multiplying the magnitudes.

Final Answer: maximum $= 7 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q5

Q6.

Solution

Concept — Identical objects, each gets at least one: The number of ways to distribute n identical objects among r groups with each getting at least one is $\binom{n-1}{r-1}$.

Step 1 — Identify the values: $n = 10$ chocolates, $r = 3$ children.

Step 2 — Apply the formula: number $= \binom{10-1}{3-1} = \binom{9}{2}$.

Step 3 — Evaluate: $\binom{9}{2} = \dfrac{9 \times 8}{2} = 36$.

Why other options are wrong:

- $45 = \binom{10}{2}$: allows a child to get zero.
- $30, 28$: arithmetic slips in the binomial coefficient.

Final Answer: 36 ways $\Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q6

Q7.

Solution

Concept — Coefficients in $(1+x)^n$**:** The coefficient of the $(r+1)$th term is $\binom{n}{r}$, and $\dfrac{\binom{n}{r}}{\binom{n}{r-1}} = \dfrac{n-r+1}{r}$.

Step 1 — Identify the terms: the 5th term has coefficient $\binom{n}{4}$ and the 4th term has coefficient $\binom{n}{3}$.

Step 2 — Form the ratio: $\dfrac{\binom{n}{4}}{\binom{n}{3}} = \dfrac{n-4+1}{4} = \dfrac{n-3}{4}$.

Step 3 — State the answer: the ratio is $\dfrac{n-3}{4}$.

Why other options are wrong:

- $\dfrac{n-3}{5}, \dfrac{n-4}{5}$: divide by the wrong index 5.
- $\dfrac{n-4}{4}$: uses $n-r$ instead of $n-r+1$.

Final Answer: $\dfrac{n-3}{4} \Rightarrow \boxed{C}$

Answer: (C) Go Back to Q7

Q8.

Solution

Concept — Geometric progression: Repeated doubling gives a GP with $a_n = a\,r^n$, where $r = 2$.

Step 1 — Set up the GP: initial count $a = 500$, common ratio $r = 2$, number of doublings $= 5$.

Step 2 — Write the count after 5 **hours:** 500×2^5.

Step 3 — Evaluate the power: $2^5 = 32$.

Step 4 — Multiply: $500 \times 32 = 16000$.

Why other options are wrong:

- $8000 = 500 \times 16$: uses 2^4 (only 4 doublings).

- $32000 = 500 \times 64$: uses 2^6.
- $4000 = 500 \times 8$: uses 2^3.

Final Answer: 16000 bacteria $\Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q8

Q9.

Solution

Concept — Distance between parallel lines: For $ax+by+c_1 = 0$ and $ax+by+c_2 = 0$, the distance is $\dfrac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$.

Step 1 — Identify the constants: $a = 3$, $b = 4$, $c_1 = 7$, $c_2 = -3$.

Step 2 — Numerator: $|c_1 - c_2| = |7 - (-3)| = 10$.

Step 3 — Denominator: $\sqrt{3^2 + 4^2} = \sqrt{25} = 5$.

Step 4 — Divide: $\dfrac{10}{5} = 2$.

Why other options are wrong:

- 1: halves the numerator.
- 10: forgets to divide by $\sqrt{a^2 + b^2}$.
- $\dfrac{4}{5}$: subtracts the constants wrongly.

Final Answer: distance $= 2 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q9

Q10.

Solution

Concept — Foci of an ellipse: For $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ with $a > b$, the foci are $(\pm c, 0)$ where $c^2 = a^2 - b^2$.

Step 1 — Read off a^2 and b^2: $a^2 = 25$, $b^2 = 9$, and since $25 > 9$ the major axis is along the x-axis.

Step 2 — Compute c^2: $c^2 = 25 - 9 = 16$.

Step 3 — Take the square root: $c = 4$.

Step 4 — Write the foci: $(\pm 4, 0)$.

Why other options are wrong:

- $(\pm 3, 0)$: uses b instead of c.
- $(0, \pm 4)$: places foci on the wrong axis.
- $(\pm 5, 0)$: uses a instead of c.

Final Answer: foci $(\pm 4, 0) \Rightarrow \boxed{\text{B}}$

Answer: (B) **Go Back to Q10**

Q11.

Solution

Concept — Limit of a rational function: If direct substitution gives $\frac{0}{0}$, factor and cancel the common term.

Step 1 — Factor the numerator: $x^2 - 4 = (x - 2)(x + 2)$.

Step 2 — Factor the denominator: $x^2 - 3x + 2 = (x - 1)(x - 2)$.

Step 3 — Cancel $(x - 2)$: the expression becomes $\frac{x+2}{x-1}$.

Step 4 — Substitute $x = 2$: $\frac{2+2}{2-1} = \frac{4}{1} = 4$.

Why other options are wrong:

- $2, 1, \frac{1}{2}$: result from cancelling incorrectly or wrong factoring.

Final Answer: limit $= 4 \Rightarrow \boxed{\text{C}}$

Answer: (C) **Go Back to Q11**

Q12.

Solution

Concept — Continuity vs differentiability: Differentiability at a point implies continuity there, but continuity does not imply differentiability.

Step 1 — Check the valid direction: if $f'(a)$ exists, then f must be continuous at a; this is a standard theorem.

Step 2 — Counterexample for the converse: $f(x) = |x|$ is continuous at 0 but has no derivative there (corner point), so continuity does not give differentiability.

Step 3 — Pick the correct statement: "If f is differentiable at a point, then it is continuous there" is the true statement.

Why other options are wrong:

- "continuous $\Rightarrow$ differentiable": false, by $|x|$.
- "differentiability does not imply continuity": false; it does.
- "$|x|$ is differentiable at 0": false; it has a corner.

Final Answer: differentiable $\Rightarrow$ continuous $\Rightarrow$ $\boxed{\text{D}}$

Answer: (D) Go Back to Q12

Q13.

Solution

Concept — Orthogonal matrix: If $A^T A = I$, take determinants on both sides and use $|A^T| = |A|$.

Step 1 — Take determinants: $|A^T A| = |I| = 1$.

Step 2 — Split the product: $|A^T|\,|A| = 1$.

Step 3 — Use $|A^T| = |A|$: $|A|^2 = 1$.

Step 4 — Solve: $|A| = \pm 1$.

Why other options are wrong:

- 0: an orthogonal matrix is invertible, so the determinant cannot be 0.
- 2: $|A|^2 = 1$ forbids this.
- only 1: misses the -1 possibility (e.g. a reflection).

Final Answer: $|A| = \pm 1 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q13

Q14.

Solution

Concept — Consistency of linear equations: Two lines that are identical give infinitely many solutions (consistent and dependent).

Step 1 — Compare the equations: $x + y = 3$ and $2x + 2y = 6$.

Step 2 — Simplify the second: dividing $2x + 2y = 6$ by 2 gives $x + y = 3$.

Step 3 — Observe: both equations are the same line, so every point on it is a solution.

Step 4 — Conclude: the system is consistent with infinitely many solutions.

Why other options are wrong:

- inconsistent / no solution: would need parallel distinct lines.
- unique solution: would need intersecting non-identical lines.
- "inconsistent with a unique solution": self-contradictory.

Final Answer: infinitely many solutions $\Rightarrow$ $\boxed{\text{B}}$

Answer: (B) **Go Back to Q14**

Q15.

Solution

Concept — Related rates: For a sphere $V = \frac{4}{3}\pi r^3$, so $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$.

Step 1 — Differentiate the volume: $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$.

Step 2 — Substitute $r = 5$ and $\frac{dr}{dt} = 2$: $\frac{dV}{dt} = 4\pi(5)^2(2)$.

Step 3 — Evaluate the square: $5^2 = 25$.

Step 4 — Multiply: $4\pi \times 25 \times 2 = 200\pi$.

Why other options are wrong:

- 100π: forgets the factor $\frac{dr}{dt} = 2$.
- 50π: uses $2\pi r$ instead of $4\pi r^2$.
- 400π: doubles the correct value.

Final Answer: $\frac{dV}{dt} = 200\pi$ cm^3/s $\Rightarrow$ $\boxed{\text{C}}$

Answer: (C) Go Back to Q15

Q16.

Solution

Concept — Equation of the normal: The normal at a point has slope $-\frac{1}{(dy/dx)}$ and passes through that point.

Step 1 — Find the tangent slope: $\frac{dy}{dx} = 2x$, so at $(1, 1)$ the slope is 2.

Step 2 — Normal slope: $-\frac{1}{2}$.

Step 3 — Point-slope form: $y - 1 = -\frac{1}{2}(x - 1)$.

Step 4 — Clear the fraction: $2(y - 1) = -(x - 1)$, i.e. $2y - 2 = -x + 1$.

Step 5 — Rearrange: $x + 2y - 3 = 0$, i.e. $x + 2y = 3$.

Why other options are wrong:

- $2x - y - 1 = 0$: this is the tangent line, not the normal.
- $x - 2y + 1 = 0$: has slope $\frac{1}{2}$, the wrong sign for the normal.
- $2x + y - 3 = 0$: wrong slope -2.

Final Answer: normal: $x + 2y = 3 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q16

Q17.

Solution

Concept — Standard form: $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\frac{x}{a} + C$.

Step 1 — Identify a: here $a^2 = 9$, so $a = 3$.

Step 2 — Apply the formula: $\int \frac{dx}{\sqrt{9 - x^2}} = \sin^{-1}\frac{x}{3} + C$.

Why other options are wrong:

- $\frac{1}{3}\sin^{-1}\frac{x}{3}$: inserts a stray factor $\frac{1}{3}$.
- $\sin^{-1}\frac{x}{9}$: uses a^2 instead of a inside.
- $\tan^{-1}\frac{x}{3}$: that is for $\frac{1}{a^2 + x^2}$.

Final Answer: $\sin^{-1}\frac{x}{3} + C \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q17

Q18.

Solution

Concept — Integration by parts: $\int u\,dv = uv - \int v\,du$; take $u = \tan^{-1}x$ and $dv = dx$.

Step 1 — Choose parts: $u = \tan^{-1}x \Rightarrow du = \frac{dx}{1+x^2}$, and $dv = dx \Rightarrow v = x$.

Step 2 — Apply the formula: $\int \tan^{-1}x\,dx = x\tan^{-1}x - \int \frac{x}{1+x^2}\,dx$.

Step 3 — Evaluate the remaining integral: $\int \frac{x}{1+x^2}\,dx = \frac{1}{2}\log(1+x^2)$.

Step 4 — Combine: $x\tan^{-1}x - \frac{1}{2}\log(1+x^2) + C$.

Why other options are wrong:

- $+\frac{1}{2}\log$: wrong sign on the log term.
- $-\log$: missing the factor $\frac{1}{2}$.
- $\tan^{-1}x - \frac{1}{2}\log$: drops the factor x from uv.

Final Answer: $x\tan^{-1}x - \frac{1}{2}\log(1+x^2) + C \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q18

Q19.

Solution

Concept — Symmetric limits: $\int_{-a}^{a} f(x)\,dx = 0$ if f is odd, and $= 2\int_0^a f(x)\,dx$ if f is even.

Step 1 — Test the parity: let $g(x) = \sin^3 x\cos^2 x$.

Step 2 — Replace x by $-x$: $\sin^3(-x)\cos^2(-x) = (-\sin x)^3(\cos x)^2 = -\sin^3 x\cos^2 x$.

Step 3 — Conclude oddness: $g(-x) = -g(x)$, so g is odd.

Step 4 — Apply the property: the integral of an odd function over $[-a, a]$ is 0.

Why other options are wrong:

- $\frac{2}{15}, \frac{4}{15}$: these would arise only if the integrand were even.
- $\frac{\pi}{2}$: unrelated to this odd integrand.

Final Answer: integral $= 0 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q19

Q20.

Solution

Concept — Area by integration: For $y^2 = 4x$ bounded by $x = 4$, use symmetry about the x-axis and integrate $y = 2\sqrt{x}$.

Step 1 — Use symmetry: total area $= 2\int_0^4 2\sqrt{x}\,dx$.

Step 2 — Simplify the integrand: $= 4\int_0^4 x^{1/2}\,dx$.

Step 3 — Integrate: $\int x^{1/2}\,dx = \frac{2}{3}x^{3/2}$, so area $= 4 \cdot \frac{2}{3}\left[x^{3/2}\right]_0^4$.

Step 4 — Apply limits: $4^{3/2} = 8$, so $4 \cdot \frac{2}{3} \cdot 8 = \frac{64}{3}$.

Why other options are wrong:

- $\frac{32}{3}$: the area of only the upper half.
- 16: drops the factor $\frac{2}{3}$.
- $\frac{16}{3}$: arithmetic slip.

Final Answer: area $= \frac{64}{3}$ square units $\Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q20

Q21.

Solution

Concept — Forming a DE: Differentiate the family and eliminate the arbitrary constant.

Step 1 — Differentiate $y = mx$: $\frac{dy}{dx} = m$.

Step 2 — Express m from the original: from $y = mx$, $m = \frac{y}{x}$.

Step 3 — Eliminate m: $\frac{dy}{dx} = \frac{y}{x}$.

Step 4 — Write without fractions: $x\frac{dy}{dx} = y$.

Why other options are wrong:

- $\frac{dy}{dx} = x$: ignores the constant m.
- $y\frac{dy}{dx} = x$: wrong placement of x and y.
- $\frac{dy}{dx} = xy$: not obtained from this family.

Final Answer: $x\frac{dy}{dx} = y \Rightarrow \boxed{\text{A}}$

Answer: (A) **Go Back to Q21**

Q22.

Solution

Concept — Separable variables: Separate y and x to opposite sides and integrate.

Step 1 — Separate: $\frac{dy}{y} = \frac{dx}{x}$.

Step 2 — Integrate both sides: $\log|y| = \log|x| + \log C$.

Step 3 — Combine the logs: $\log|y| = \log|Cx|$.

Step 4 — Remove the logarithm: $y = Cx$.

Why other options are wrong:

- $y = x + C$: comes from $\frac{dy}{dx} = 1$, not $\frac{y}{x}$.
- $y = Cx^2$: wrong power of x.
- $xy = C$: solves $\frac{dy}{dx} = -\frac{y}{x}$.

Final Answer: $y = Cx \Rightarrow \boxed{\text{B}}$

Answer: (B) **Go Back to Q22**

Q23.

Solution

Concept — Dot product and angle: $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$, so the sign of the dot product reflects the angle.

Step 1 — Set the dot product to zero: $|\vec{a}||\vec{b}| \cos\theta = 0$.

Step 2 — Use non-zero magnitudes: since $|\vec{a}| \neq 0$ and $|\vec{b}| \neq 0$, we need $\cos\theta = 0$.

Step 3 — Solve for θ: $\theta = \frac{\pi}{2}$, so the vectors are perpendicular.

Why other options are wrong:

- parallel: would give $\cos\theta = 1$, dot product positive.
- equal: a special parallel case.
- anti-parallel: gives $\cos\theta = -1$, dot product negative.

Final Answer: perpendicular $\Rightarrow \boxed{\text{C}}$

Answer: (C) **Go Back to Q23**

Q24.

Solution

Concept — Right-hand rule: The cross product $\hat{i} \times \hat{j}$ points in the direction given by curling the right hand from $\hat{i}$ to $\hat{j}$.

Step 1 — Recall the cyclic rule: $\hat{i} \times \hat{j} = \hat{k}$, $\hat{j} \times \hat{k} = \hat{i}$, $\hat{k} \times \hat{i} = \hat{j}$.

Step 2 — Apply directly: $\hat{i} \times \hat{j} = \hat{k}$.

Step 3 — Confirm by right-hand rule: fingers from x to y make the thumb point along $+z$.

Why other options are wrong:

- $-\hat{k}$: that is $\hat{j} \times \hat{i}$, the reverse order.
- $\hat{i}, \hat{j}$: not perpendicular to both $\hat{i}$ and $\hat{j}$.

Final Answer: $\hat{i} \times \hat{j} = \hat{k} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q24

Q25.

Solution

Concept — Parallel lines in 3D: Two lines are parallel when their direction ratios are proportional.

Step 1 — Read the directions: first line has direction $\langle 2, 3, 4 \rangle$, second has $\langle 4, 6, 8 \rangle$.

Step 2 — Check proportionality: $\frac{4}{2} = \frac{6}{3} = \frac{8}{4} = 2$, so the directions are proportional.

Step 3 — Test for coincidence: the point $(1, 0, -1)$ of the first line does not satisfy the second, so they are not coincident.

Step 4 — Conclude: the lines are parallel (and distinct).

Why other options are wrong:

- perpendicular: would need the dot product of directions to be 0, but $2(4) + 3(6) + 4(8) = 58 \neq 0$.
- coincident: the points differ.
- intersecting at right angles: not perpendicular and not intersecting.

Final Answer: the lines are parallel $\Rightarrow$ $\boxed{\text{A}}$

Answer: (A) Go Back to Q25

Q26.

Solution

Concept — Distance between parallel planes: For $ax + by + cz + d_1 = 0$ and $ax + by + cz + d_2 = 0$, the distance is $\frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$.

Step 1 — Identify the constants: $a = 2$, $b = -2$, $c = 1$, $d_1 = 3$, $d_2 = -6$.

Step 2 — Numerator: $|d_1 - d_2| = |3 - (-6)| = 9$.

Step 3 — Denominator: $\sqrt{2^2 + (-2)^2 + 1^2} = \sqrt{4 + 4 + 1} = \sqrt{9} = 3$.

Step 4 — Divide: $\frac{9}{3} = 3$.

Why other options are wrong:

- 9: forgets to divide by $\sqrt{a^2+b^2+c^2}$.
- 1: divides by 9 instead of 3.
- $\frac{9}{2}$: uses a wrong denominator.

Final Answer: distance $= 3 \Rightarrow \boxed{\text{B}}$

Answer: (B) **Go Back to Q26**

Q27.

Solution

Concept — Complementary events: $P(\text{not } E) = 1 - P(E)$.

Step 1 — Identify $P(E)$: probability of rain $= 0.3$.

Step 2 — Apply the complement rule: $P(\text{no rain}) = 1 - 0.3$.

Step 3 — Subtract: $1 - 0.3 = 0.7$.

Why other options are wrong:

- 0.3: that is the probability of rain itself.
- 0.5: not the complement of 0.3.
- 1.3: a probability cannot exceed 1.

Final Answer: $P(\text{no rain}) = 0.7 \Rightarrow \boxed{\text{C}}$

Answer: (C) **Go Back to Q27**

Q28.

Solution

Concept — Bayes' theorem: $P(\text{II} \mid W) = \dfrac{P(\text{II})P(W \mid \text{II})}{P(\text{I})P(W \mid \text{I}) + P(\text{II})P(W \mid \text{II})}$.

Step 1 — Prior probabilities: $P(\text{I}) = P(\text{II}) = \frac{1}{2}$.

Step 2 — Conditional white probabilities: $P(W \mid \text{I}) = \frac{2}{5}$, $P(W \mid \text{II}) = \frac{4}{5}$.

Step 3 — Numerator: $\frac{1}{2} \cdot \frac{4}{5} = \frac{4}{10} = \frac{2}{5}$.

Step 4 — Denominator: $\frac{1}{2} \cdot \frac{2}{5} + \frac{1}{2} \cdot \frac{4}{5} = \frac{1}{5} + \frac{2}{5} = \frac{3}{5}$.

Step 5 — Divide: $\dfrac{2/5}{3/5} = \dfrac{2}{3}$.

Why other options are wrong:

- $\frac{1}{3}$: this is $P(\text{I} \mid W)$, the complement.
- $\frac{1}{2}$: ignores the differing white-ball counts.
- $\frac{2}{5}$: stops at the numerator.

Final Answer: $P(\text{II} \mid W) = \dfrac{2}{3} \Rightarrow \boxed{\text{D}}$

Answer: (D) **Go Back to Q28**

Q29.

Solution

Concept — Combined mean: The combined mean of two groups is $\dfrac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$.

Step 1 — List the data: $n_1 = 20$, $\bar{x}_1 = 60$, $n_2 = 30$, $\bar{x}_2 = 50$.

Step 2 — Total of the first group: $20 \times 60 = 1200$.

Step 3 — Total of the second group: $30 \times 50 = 1500$.

Step 4 — Combined total and count: sum $= 1200 + 1500 = 2700$, count $= 20 + 30 = 50$.

Step 5 — Divide: $\dfrac{2700}{50} = 54$.

Why other options are wrong:

- 55: the simple average of 60 and 50, ignoring group sizes.
- $56, 52$: arithmetic slips in the weighting.

Final Answer: combined mean $= 54 \Rightarrow \boxed{\text{A}}$

Answer: (A) **Go Back to Q29**

Q30.

Solution

Concept — LPP corner-point method: The optimum of a linear objective over a feasible region occurs at a corner point; evaluate Z at each.

Step 1 — Evaluate at $(0, 8)$: $Z = 5(0) + 7(8) = 0 + 56 = 56$.

Step 2 — Evaluate at $(2, 4)$: $Z = 5(2) + 7(4) = 10 + 28 = 38$.

Step 3 — Evaluate at $(11, 0)$: $Z = 5(11) + 7(0) = 55 + 0 = 55$.

Step 4 — Compare the values: the corner values are 56, 38 and 55.

Step 5 — Pick the smallest: the minimum is 38, attained at $(2, 4)$.

Why other options are wrong:

- 56: value at $(0, 8)$, the largest.
- 55: value at $(11, 0)$.
- 40: does not match any corner evaluation.

Final Answer: minimum cost $Z = 38 \Rightarrow \boxed{B}$

Answer: (B) Go Back to Q30

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	B	3	C	4	D	5	A
6	B	7	C	8	D	9	A	10	B
11	C	12	D	13	A	14	B	15	C
16	D	17	A	18	D	19	C	20	D
21	A	22	B	23	C	24	D	25	A
26	B	27	C	28	D	29	A	30	B