# AIIMS Paramedical Mathematics Sample Paper – 9

**Duration: 30 Minutes** **Maximum Marks: 30**

## Instructions

- This paper contains **30** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics portion of **AIIMS Paramedical** entrance.
- Each correct answer carries **+1 mark**. An incorrect answer attracts **a penalty of** $-\frac{1}{3}$ **mark**. An unattempted question carries **0 marks**.
- Only **one** option is correct. Choose carefully.
- Syllabus level: **Class 11 & 12 NCERT Mathematics**
- Use of mobile phones, calculators, or electronic gadgets is strictly prohibited.

**Q1.** In a survey of $120$ students, $65$ read English newspapers, $50$ read Hindi newspapers and $25$ read both. The number of students who read neither newspaper is

(A) $20$

(B) $30$

(C) $40$

(D) $35$

**Q2.** The function $f(x) = x^3 \sin x$ defined on $\mathbb{R}$ is

(A) odd and periodic

(B) odd but not even

(C) even but not periodic

(D) neither even nor odd

**Q3.** The maximum value of the expression $3\sin x + 4\cos x$ for real $x$ is

(A) $5$

(B) $7$

(C) $1$

(D) $25$

**Q4.** The principal solutions of the equation $\sin x = \frac{1}{2}$ in the interval $[0, 2\pi)$ are

(A) $\frac{\pi}{6}$ and $\frac{2\pi}{3}$

(B) $\frac{\pi}{3}$ and $\frac{2\pi}{3}$

(C) $\frac{\pi}{6}$ and $\frac{7\pi}{6}$

(D) $\frac{\pi}{6}$ and $\frac{5\pi}{6}$

**Q5.** The equation $|z - (2 + 3i)| = 5$ represents, in the Argand plane, the locus of the point $z$ which is

(A) a straight line through $(2, 3)$

(B) a circle of radius $5$ centred at $(2, 3)$

(C) a circle of radius $25$ centred at $(2, 3)$

(D) an ellipse with foci at $(2, 3)$

**Q6.** A committee of $3$ is to be formed by choosing $1$ teacher from $4$ teachers and $2$ students from $5$ students. The total number of ways of forming the committee is

(A) $20$

(B) $30$

(C) $40$

(D) $60$

**Q7.** Using the binomial theorem, the approximate value of $(1.02)^4$, correct to two terms of the expansion, is

(A) $1.08$

(B) $1.04$

(C) $1.16$

(D) $1.02$

**Q8.** The sum of the infinite series $1+\frac{2}{3}+\frac{3}{3^2}+\frac{4}{3^3}+\cdots$ (an arithmetic–geometric series) is

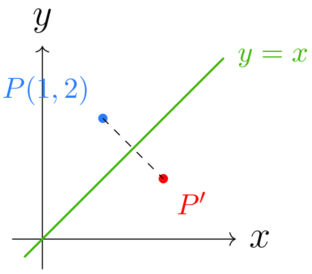
(A) $\frac{3}{2}$

(B) $3$

(C) $2$

(D) $\frac{9}{4}$

**Q9.** The image of the point $P(1,2)$ in the line $y=x$ is the point



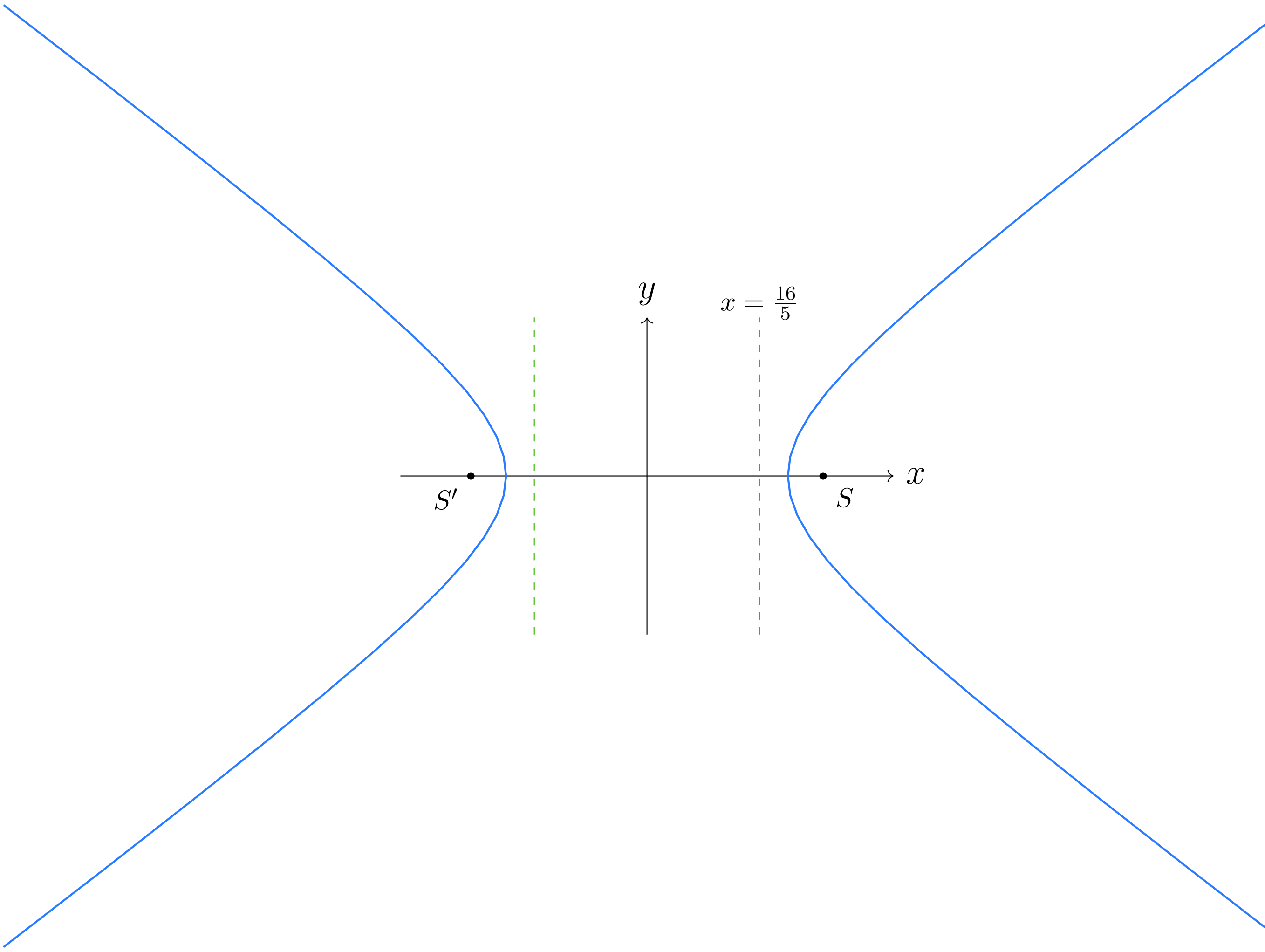

(A) $(-1,-2)$

(B) $(2,1)$

(C) $(-2,-1)$

(D) $(1,2)$

**Q10.** For the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$, the eccentricity $e$ and the equations of the directrices are




(A) $e = \frac{4}{5},\ x = \pm\frac{16}{5}$

(B) $e = \frac{3}{4},\ x = \pm\frac{16}{3}$

(C) $e = \frac{5}{4},\ x = \pm\frac{16}{5}$

(D) $e = \frac{5}{4},\ x = \pm\frac{16}{3}$

**Q11.** For the function $f(x) = \frac{|x-3|}{x-3}$ $(x \neq 3)$, the left-hand and right-hand limits at $x = 3$ tell us that

(A) $\lim_{x\to 3} f(x)$ does not exist

(B) $\lim_{x\to 3} f(x) = 1$

(C) $\lim_{x\to 3} f(x) = -1$

(D) $\lim_{x\to 3} f(x) = 0$

**Q12.** If $y = \sin^{-1}(2x\sqrt{1-x^2})$, $-\frac{1}{\sqrt{2}} < x < \frac{1}{\sqrt{2}}$, then $\frac{dy}{dx}$ equals

(A) $\frac{1}{\sqrt{1-x^2}}$

(B) $\frac{-2}{\sqrt{1-x^2}}$

(C) $\frac{1}{2\sqrt{1-x^2}}$

(D) $\frac{2}{\sqrt{1-x^2}}$

**Q13.** If $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$, then which of the following is correct?

(A) $AB = BA$

(B) $AB \neq BA$

(C) $AB = I$

(D) $AB = -BA$

**Q14.** If $A$ is a non-singular square matrix of order $3$ with $|A| = 4$, then the value of $|\operatorname{adj} A|$ is

(A) $4$

(B) $64$

(C) $16$

(D) $12$

**Q15.** The function $f(x) = x^3 - 3x^2 + 1$ has its critical points (where $f'(x) = 0$) at

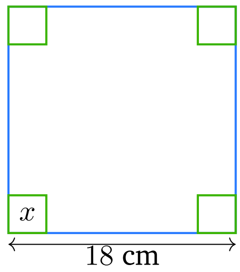
(A) $x = 0$ and $x = 2$

(B) $x = 1$ and $x = 2$

(C) $x = 0$ and $x = 3$

(D) $x = -1$ and $x = 2$

**Q16.** An open box is made from a square sheet of side 18 cm by cutting equal squares of side $x$ from each corner and folding up the sides. The value of $x$ that gives the maximum volume is




(A) 6

(B) 4.5

(C) 9

(D) 3

**Q17.** The value of the integral $\int \frac{dx}{x^2+9}$ is

(A) $\frac{1}{9}\tan^{-1}\frac{x}{3}+C$

(B) $\frac{1}{3}\tan^{-1}\frac{x}{3}+C$

(C) $3\tan^{-1}\frac{x}{3}+C$

(D) $\tan^{-1}\frac{x}{3}+C$

**Q18.** The value of the integral $\int x^2e^x\,dx$ is

(A) $e^x(x^2+2x+2)+C$

(B) $e^x(x^2+2x-2)+C$

(C) $e^x(x^2-2x+2)+C$

(D) $e^x(x^2-2x-2)+C$

**Q19.** Using the symmetry property of definite integrals, the value of $\int_{-2}^{2}(x^3+x\cos x+1$ is

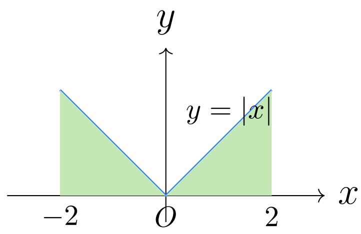
(A) 4

(B) 0

(C) 8

(D) 2

**Q20.** The area of the region bounded by the curve $y = |x|$, the $x$-axis and the lines $x = -2$ and $x = 2$ is




(A) 2

(B) 8

(C) 6

(D) 4

**Q21.** The general solution of the differential equation $\dfrac{dy}{dx} = \dfrac{\cos x}{\sin y}$ is

(A) $\cos y + \sin x = C$

(B) $\cos y = \sin x + C$

(C) $\sin y + \cos x = C$

(D) $\sin y = \cos x + C$

**Q22.** Treating $x$ as the dependent variable, the integrating factor of the linear differential equation $\dfrac{dx}{dy} + x = y$ is

(A) $e^{-y}$

(B) $e^{x}$

(C) $e^{y}$

(D) $e^{-x}$

**Q23.** The position vector of the point that divides the line segment joining the points with position vectors $\vec{a} = 2\hat{i} + 3\hat{j}$ and $\vec{b} = 4\hat{i} - \hat{j}$ in the ratio $1 : 1$ (the midpoint) is

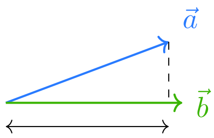
(A) $3\hat{i} + \hat{j}$

(B) $6\hat{i} + 2\hat{j}$

(C) $\hat{i} + 2\hat{j}$

(D) $3\hat{i} + 2\hat{j}$

**Q24.** The scalar component (projection) of the vector $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$ along the vector $\vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}$ is




(A) $\frac{4}{3}$

(B) $\frac{6}{3}$

(C) $\frac{10}{9}$

(D) $\frac{8}{3}$

**Q25.** The image of the point $P(1, 2, 3)$ in the plane $x + y + z = 0$ is the point

(A) $(-3, -2, -1)$

(B) $(-5, -4, -3)$

(C) $(-1, -2, -3)$

(D) $(3, 2, 1)$

**Q26.** The Cartesian equation of the line passing through the point $(1, 2, 3)$ with direction ratios $2, 3, 4$ is

(A) $\frac{x+1}{2} = \frac{y+2}{3} = \frac{z+3}{4}$

(B) $\dfrac{x-2}{1} = \dfrac{y-3}{2} = \dfrac{z-4}{3}$

(C) $\dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z-3}{4}$

(D) $\dfrac{x-1}{1} = \dfrac{y-2}{2} = \dfrac{z-3}{3}$

**Q27.** For two mutually exclusive events $A$ and $B$ with $P(A) = 0.3$ and $P(B) = 0.45$, the value of $P(A \cup B)$ is

(A) $0.75$

(B) $0.135$

(C) $0.15$

(D) $1$

**Q28.** A random variable $X$ takes the values $0, 1, 2$ with probabilities $0.2,\ 0.5,\ 0.3$ respectively. The expected value $E(X)$ is

(A) $0.5$

(B) $1.5$

(C) $0.9$

(D) $1.1$

**Q29.** The variance of the observations $2, 4, 6, 8, 10$ is

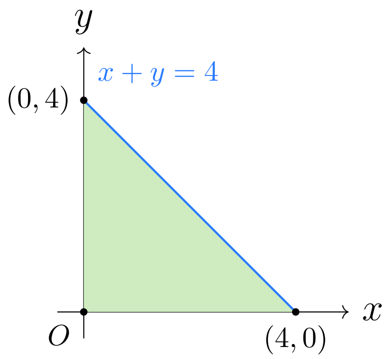
(A) $4$

(B) $8$

(C) $6$

(D) $10$

**Q30.** For the linear programming problem: maximise and minimise $Z = 3x + 2y$ subject to $x \geq 0,\ y \geq 0,\ x+y \leq 4$, the maximum and minimum values of $Z$ over the feasible region are




(A) max $= 8$, min $= 0$

(B) max $= 8$, min $= 2$

(C) max $= 12$, min $= 0$

(D) max $= 12$, min $= 8$

# Detailed Solutions

## Q1.

**Solution**

**Concept — Sets (inclusion–exclusion):** For two sets, $n(E \cup H) = n(E) + n(H) - n(E \cap H)$, and "neither" is the complement within the universal set.

**Step 1 — Students reading at least one:** $n(E \cup H) = 65 + 50 - 25$.

**Step 2 — Add and subtract:** $65 + 50 = 115$, then $115 - 25 = 90$.

**Step 3 — Students reading neither:** Total minus at least one $= 120 - 90 = 30$.

**Why other options are wrong:**

- Option (A) $20$: forgets to subtract the overlap.
- Option (C) $40$: subtracts both individual totals wrongly.
- Option (D) $35$: arithmetic slip on the overlap.

**Final Answer:** $30$ students read neither $\Rightarrow$ $\boxed{\text{B}}$

Answer: (B) Go Back to Q1

## Q2.

**Solution**

**Concept — Even/odd and periodic functions:** $f$ is odd if $f(-x) = -f(x)$, even if $f(-x) = f(x)$; periodic if $f(x + T) = f(x)$ for some fixed $T > 0$.

**Step 1 — Replace $x$ by $-x$:** $f(-x) = (-x)^3 \sin(-x) = (-x^3)(-\sin x) = x^3 \sin x$.

**Step 2 — Compare:** $f(-x) = x^3 \sin x = f(x)$, so $f$ is *even*.

**Step 3 — Periodicity:** The factor $x^3$ grows without bound, so no fixed period $T$ exists; $f$ is not periodic.

**Why other options are wrong:**

- Options (A),(B): claim the function is odd, but it is even.
- Option (D): it is even, so "neither" is false.

**Final Answer:** even but not periodic $\Rightarrow$ $\boxed{\text{C}}$

Answer: (C) Go Back to Q2

## Q3.

**Solution**

**Concept — Maximum of $a\sin x + b\cos x$:** The expression ranges over $[-\sqrt{a^2+b^2},\ \sqrt{a^2+b^2}]$.

**Step 1 — Identify constants:** Here $a = 3$, $b = 4$.

**Step 2 — Compute amplitude:** $\sqrt{a^2+b^2} = \sqrt{3^2+4^2} = \sqrt{9+16} = \sqrt{25} = 5$.

**Step 3 — State maximum:** The greatest value is $+\sqrt{25} = 5$.

**Why other options are wrong:**

- Option (B) $7$: adds $3+4$ directly, which is wrong.
- Option (C) $1$: this is the minimum-style slip $|4-3|$.
- Option (D) $25$: forgets the square root.

**Final Answer:** maximum $= 5 \Rightarrow \boxed{\text{A}}$

**Answer: (A)** Go Back to Q3

## Q4.

**Solution**

**Concept — Principal solutions:** The solutions of $\sin x = k$ lying in $[0, 2\pi)$ are the principal solutions.

**Step 1 — Reference angle:** $\sin x = \frac{1}{2}$ gives reference angle $\frac{\pi}{6}$.

**Step 2 — Quadrants where $\sin$ is positive:** First and second quadrants.

**Step 3 — Write the two solutions:** First quadrant $x = \frac{\pi}{6}$; second quadrant $x = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$.

**Why other options are wrong:**

- Options (A),(B): use $\frac{2\pi}{3}$, whose sine is $\frac{\sqrt{3}}{2}$, not $\frac{1}{2}$.
- Option (C) $\frac{7\pi}{6}$: lies in the third quadrant where sine is negative.

**Final Answer:** $\frac{\pi}{6}$ and $\frac{5\pi}{6} \Rightarrow \boxed{\text{D}}$

**Answer: (D)** Go Back to Q4

## Q5.

**Solution**

**Concept — Locus from modulus:** $|z - z_0| = r$ is the set of points at fixed distance $r$ from $z_0$, i.e. a circle of radius $r$ centred at $z_0$.

**Step 1 — Identify the centre:** $z_0 = 2 + 3i$ corresponds to the point $(2, 3)$.

**Step 2 — Identify the radius:** The right-hand side is $5$, so $r = 5$.

**Step 3 — Conclusion:** The locus is a circle of radius $5$ centred at $(2, 3)$.

**Why other options are wrong:**

- Option (A): a modulus equation is not a line.
- Option (C): the radius is $5$, not $25$.
- Option (D): an ellipse needs a sum of two distances.

**Final Answer:** circle, radius $5$, centre $(2, 3) \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q5

## Q6.

**Solution**

**Concept — Selection from different groups:** Independent choices are multiplied; each choice uses ${}^nC_r$.

**Step 1 — Choose the teacher:** ${}^4C_1 = 4$ ways.

**Step 2 — Choose the students:** ${}^5C_2 = \dfrac{5 \times 4}{2 \times 1} = 10$ ways.

**Step 3 — Multiply:** Total $= 4 \times 10 = 40$ ways.

**Why other options are wrong:**

- Option (A) $20$: uses ${}^5C_1$ instead of ${}^5C_2$.
- Option (B) $30$: incorrect combination value.
- Option (D) $60$: over-counts the students.

**Final Answer:** $40$ ways $\Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q6

## Q7.

**Solution**

**Concept — Binomial approximation:** For small $h$, $(1+h)^n \approx 1+nh$ keeping the first two terms.

**Step 1 — Write in binomial form:** $(1.02)^4 = (1+0.02)^4$ with $n=4$, $h=0.02$.

**Step 2 — Apply the two-term approximation:** $1+nh = 1+4(0.02)$.

**Step 3 — Evaluate:** $4 \times 0.02 = 0.08$, so the value is $1+0.08 = 1.08$.

**Why other options are wrong:**

- Option (B) $1.04$: uses $n=2$ by mistake.
- Option (C) $1.16$: doubles the correction term.
- Option (D) $1.02$: drops the factor $n=4$.

**Final Answer:** $(1.02)^4 \approx 1.08 \Rightarrow \boxed{\text{A}}$

**Answer: (A)** Go Back to Q7

## Q8.

**Solution**

**Concept — Arithmetic–geometric series:** $S = \sum_{n=1}^{\infty} n\,r^{n-1} = \dfrac{1}{(1-r)^2}$ for $|r|<1$.

**Step 1 — Identify the form:** The series $1 + \dfrac{2}{3} + \dfrac{3}{3^2} + \cdots$ has $n$-th term $n\,r^{n-1}$ with $r = \dfrac{1}{3}$.

**Step 2 — Apply the formula:** $S = \dfrac{1}{\left(1-\frac{1}{3}\right)^2} = \dfrac{1}{\left(\frac{2}{3}\right)^2}$.

**Step 3 — Simplify:** $\left(\frac{2}{3}\right)^2 = \dfrac{4}{9}$, so $S = \dfrac{1}{4/9} = \dfrac{9}{4}$.

**Why other options are wrong:**

- Option (A) $\dfrac{3}{2}$: uses $\dfrac{1}{1-r}$ instead of its square.
- Options (B),(C): ignore the linear factor $n$.

**Final Answer:** $S = \dfrac{9}{4} \Rightarrow \boxed{\text{D}}$

**Answer: (D)** Go Back to Q8

## Q9.

**Solution**

**Concept — Reflection in $y = x$:** The image of $(a, b)$ in the line $y = x$ is $(b, a)$; the coordinates are simply interchanged.

**Step 1 — Identify the point:** $P = (1, 2)$, so $a = 1$, $b = 2$.

**Step 2 — Interchange coordinates:** Image $= (b, a) = (2, 1)$.

**Step 3 — Check:** The midpoint of $P(1, 2)$ and $P'(2, 1)$ is $\left(\frac{3}{2}, \frac{3}{2}\right)$, which lies on $y = x$, as required.

**Why other options are wrong:**

- Options (A),(C): negate coordinates, which is reflection through the origin, not $y = x$.
- Option (D): is the original point, unchanged.

**Final Answer:** image $= (2, 1) \Rightarrow \boxed{\text{B}}$

**Answer: (B)** Go Back to Q9

## Q10.

**Solution**

**Concept — Hyperbola elements:** For $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$, $b^2 = a^2(e^2 - 1)$ and the directrices are $x = \pm\dfrac{a}{e}$.

**Step 1 — Read $a$ and $b$:** $a^2 = 16 \Rightarrow a = 4$; $b^2 = 9 \Rightarrow b = 3$.

**Step 2 — Find $e$:** $b^2 = a^2(e^2 - 1) \Rightarrow 9 = 16(e^2 - 1) \Rightarrow e^2 - 1 = \dfrac{9}{16} \Rightarrow e^2 = \dfrac{25}{16} \Rightarrow e = \dfrac{5}{4}$.

**Step 3 — Directrices:** $x = \pm\dfrac{a}{e} = \pm\dfrac{4}{5/4} = \pm\dfrac{16}{5}$.

**Why other options are wrong:**

- Options (A),(B): give $e < 1$, impossible for a hyperbola.
- Option (D): uses $\dfrac{16}{3}$ instead of $\dfrac{a}{e} = \dfrac{16}{5}$.

**Final Answer:** $e = \dfrac{5}{4}$, $x = \pm\dfrac{16}{5} \Rightarrow \boxed{\text{C}}$

**Answer: (C)** Go Back to Q10

## Q11.

**Solution**

**Concept — Existence of a limit:** $\lim_{x \to a} f(x)$ exists only if the left-hand and right-hand limits are equal.

**Step 1 — Right-hand limit ($x > 3$):** Then $x - 3 > 0$, so $|x - 3| = x - 3$ and $f(x) = \frac{x-3}{x-3} = 1$. Hence RHL $= 1$.

**Step 2 — Left-hand limit ($x < 3$):** Then $x - 3 < 0$, so $|x - 3| = -(x - 3)$ and $f(x) = \frac{-(x-3)}{x-3} = -1$. Hence LHL $= -1$.

**Step 3 — Compare:** LHL $= -1 \neq 1 =$ RHL, so the limit does not exist.

**Why other options are wrong:**

- Options (B),(C): report only one of the one-sided limits.
- Option (D) $0$: averages the two sides, which is not how limits work.

**Final Answer:** the limit does not exist $\Rightarrow \boxed{A}$

Answer: (A) Go Back to Q11

## Q12.

**Solution**

**Concept — Inverse trig substitution:** With $x = \sin\theta$, $2x\sqrt{1 - x^2} = \sin 2\theta$, simplifying the inverse sine.

**Step 1 — Substitute:** Let $x = \sin\theta$, so $\theta = \sin^{-1} x$ and $\sqrt{1 - x^2} = \cos\theta$.

**Step 2 — Simplify the argument:** $2x\sqrt{1 - x^2} = 2\sin\theta\cos\theta = \sin 2\theta$, so $y = \sin^{-1}(\sin 2\theta) = 2\theta = 2\sin^{-1} x$ on the given interval.

**Step 3 — Differentiate:** $\frac{dy}{dx} = 2 \cdot \frac{1}{\sqrt{1 - x^2}} = \frac{2}{\sqrt{1 - x^2}}$.

**Why other options are wrong:**

- Option (A): forgets the factor $2$.
- Option (B): wrong sign.
- Option (C): halves instead of doubles.

**Final Answer:** $\frac{dy}{dx} = \frac{2}{\sqrt{1 - x^2}} \Rightarrow \boxed{D}$

Answer: (D) Go Back to Q12

## Q13.

### Solution

**Concept — Matrix multiplication is not commutative:** In general $AB \neq BA$; we test by direct computation.

**Step 1 — Compute $AB$:** $AB = \begin{pmatrix}1 & 2\\0 & 1\end{pmatrix}\begin{pmatrix}1 & 0\\3 & 1\end{pmatrix} = \begin{pmatrix}1+6 & 0+2\\0+3 & 0+1\end{pmatrix} = \begin{pmatrix}7 & 2\\3 & 1\end{pmatrix}$.

**Step 2 — Compute $BA$:** $BA = \begin{pmatrix}1 & 0\\3 & 1\end{pmatrix}\begin{pmatrix}1 & 2\\0 & 1\end{pmatrix} = \begin{pmatrix}1 & 2\\3 & 7\end{pmatrix}$.

**Step 3 — Compare:** $AB = \begin{pmatrix}7 & 2\\3 & 1\end{pmatrix} \neq \begin{pmatrix}1 & 2\\3 & 7\end{pmatrix} = BA$.

**Why other options are wrong:**

- Option (A): false, the products differ.
- Option (C): $AB$ is not the identity.
- Option (D): $AB \neq -BA$.

**Final Answer:** $AB \neq BA \Rightarrow \boxed{\text{B}}$

**Answer: (B)** Go Back to Q13

## Q14.

### Solution

**Concept — Determinant of the adjoint:** For an $n \times n$ matrix, $|\operatorname{adj} A| = |A|^{n-1}$.

**Step 1 — Identify $n$:** The matrix has order 3, so $n = 3$ and $n - 1 = 2$.

**Step 2 — Apply the formula:** $|\operatorname{adj} A| = |A|^2$.

**Step 3 — Substitute $|A| = 4$:** $|\operatorname{adj} A| = 4^2 = 16$.

**Why other options are wrong:**

- Option (A) 4: uses power 1 instead of $n - 1 = 2$.
- Option (B) 64: uses power 3.
- Option (D) 12: multiplies $4 \times 3$ by mistake.

**Final Answer:** $|\operatorname{adj} A| = 16 \Rightarrow \boxed{\text{C}}$

**Answer: (C)** Go Back to Q14

## Q15.

**Solution**

**Concept — Critical points:** These occur where $f'(x) = 0$ (or is undefined).

**Step 1 — Differentiate:** $f(x) = x^3 - 3x^2 + 1 \Rightarrow f'(x) = 3x^2 - 6x$.

**Step 2 — Set derivative to zero:** $3x^2 - 6x = 0 \Rightarrow 3x(x - 2) = 0$.

**Step 3 — Solve:** $x = 0$ or $x = 2$.

**Why other options are wrong:**

- Option (B): misses the root $x = 0$.
- Option (C): wrong second root.
- Option (D): introduces a spurious root $x = -1$.

**Final Answer:** $x = 0$ and $x = 2 \Rightarrow \boxed{\text{A}}$

**Answer: (A)** Go Back to Q15

## Q16.

**Solution**

**Concept — Maximising volume of an open box:** Express $V$ in terms of $x$, then set $\frac{dV}{dx} = 0$.

**Step 1 — Volume function:** Base side $= 18 - 2x$, height $= x$, so $V = x(18 - 2x)^2$.

**Step 2 — Differentiate:** $V = x(18 - 2x)^2$, $\frac{dV}{dx} = (18 - 2x)^2 + x \cdot 2(18 - 2x)(-2) = (18 - 2x)\big[(18 - 2x) - 4x\big] = (18 - 2x)(18 - 6x)$.

**Step 3 — Solve** $\frac{dV}{dx} = 0$**:** $(18 - 2x)(18 - 6x) = 0 \Rightarrow x = 9$ or $x = 3$. Since $x = 9$ gives zero base, the valid maximiser is $x = 3$.

**Why other options are wrong:**

- Options (A),(B): do not satisfy $\frac{dV}{dx} = 0$.
- Option (C) 9: makes the base length zero, giving zero volume.

**Final Answer:** $x = 3$ cm $\Rightarrow \boxed{\text{D}}$

**Answer: (D)** Go Back to Q16

## Q17.

**Solution**

**Concept — Standard arctangent integral:** $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$.

**Step 1 — Identify $a$:** Here $a^2 = 9$, so $a = 3$.

**Step 2 — Apply the formula:** $\int \frac{dx}{x^2+9} = \frac{1}{3}\tan^{-1}\frac{x}{3} + C$.

**Why other options are wrong:**

- Option (A): uses $\frac{1}{a^2} = \frac{1}{9}$ instead of $\frac{1}{a}$.
- Option (C): multiplies by $a$ instead of dividing.
- Option (D): omits the factor $\frac{1}{a}$.

**Final Answer:** $\frac{1}{3}\tan^{-1}\frac{x}{3} + C \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q17

## Q18.

**Solution**

**Concept — Repeated integration by parts:** Apply $\int u\,dv = uv - \int v\,du$ twice, with the polynomial as $u$.

**Step 1 — First pass:** $u = x^2,\ dv = e^x dx \Rightarrow \int x^2 e^x dx = x^2 e^x - \int 2x\,e^x dx$.

**Step 2 — Second pass:** $\int 2x\,e^x dx = 2x\,e^x - \int 2e^x dx = 2xe^x - 2e^x$.

**Step 3 — Combine:** $\int x^2 e^x dx = x^2 e^x - (2xe^x - 2e^x) = e^x(x^2 - 2x + 2) + C$.

**Why other options are wrong:**

- Option (A): wrong sign on the linear term.
- Options (B),(D): wrong sign on the constant term.

**Final Answer:** $e^x(x^2 - 2x + 2) + C \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q18

## Q19.

**Solution**

**Concept — Odd/even symmetry:** $\int_{-a}^{a} (\text{odd})\, dx = 0$ and $\int_{-a}^{a} (\text{even})\, dx = 2\int_{0}^{a} (\text{even})\, dx$.

**Step 1 — Classify each term:** $x^3$ is odd; $x \cos x$ is odd (odd $\times$ even); $1$ is even.

**Step 2 — Drop the odd parts:** $\int_{-2}^{2} (x^3 + x\cos x)\, dx = 0$.

**Step 3 — Integrate the even part:** $\int_{-2}^{2} 1\, dx = [x]_{-2}^{2} = 2 - (-2) = 4$.

**Why other options are wrong:**

- Option (B) $0$: forgets the constant term contributes.
- Option (C) $8$: uses an interval length of $8$.
- Option (D) $2$: integrates over half the interval only.

**Final Answer:** value $= 4 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q19

## Q20.

**Solution**

**Concept — Area under $y = |x|$:** Use symmetry; $|x|$ is even, so the area is twice the area from $0$ to $2$.

**Step 1 — Write the integral:** Area $= \int_{-2}^{2} |x|\, dx = 2\int_{0}^{2} x\, dx$.

**Step 2 — Integrate:** $\int_{0}^{2} x\, dx = \left[\frac{x^2}{2}\right]_0^2 = \frac{4}{2} = 2$.

**Step 3 — Double it:** Area $= 2 \times 2 = 4$.

**Why other options are wrong:**

- Option (A) $2$: gives only the right half.
- Option (B) $8$: doubles the answer once too often.
- Option (C) $6$: arithmetic slip.

**Final Answer:** area $= 4$ square units $\Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q20

## Q21.

**Solution**

**Concept — Separable differential equation:** Gather $y$ on one side and $x$ on the other, then integrate.

**Step 1 — Separate variables:** $\frac{dy}{dx} = \frac{\cos x}{\sin y} \Rightarrow \sin y\,dy = \cos x\,dx$.

**Step 2 — Integrate both sides:** $\int \sin y\,dy = \int \cos x\,dx \Rightarrow -\cos y = \sin x + C_1$.

**Step 3 — Tidy up:** Multiply by $-1$ and absorb the constant: $\cos y + \sin x = C$.

**Why other options are wrong:**

- Option (B): wrong sign linking the two terms.
- Options (C),(D): integrate $\sin y$ or $\cos x$ incorrectly.

**Final Answer:** $\cos y + \sin x = C \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q21

## Q22.

**Solution**

**Concept — Integrating factor (linear in $x$):** For $\frac{dx}{dy} + P(y)\,x = Q(y)$, the integrating factor is $e^{\int P\,dy}$.

**Step 1 — Identify $P(y)$:** The equation $\frac{dx}{dy} + x = y$ has $P(y) = 1$.

**Step 2 — Integrate $P$:** $\int 1\,dy = y$.

**Step 3 — Form the integrating factor:** I.F. $= e^{\int P\,dy} = e^y$.

**Why other options are wrong:**

- Option (A): wrong sign in the exponent.
- Options (B),(D): use $x$, but the independent variable here is $y$.

**Final Answer:** integrating factor $= e^y \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q22

## Q23.

### Solution

**Concept — Section formula (midpoint):** The midpoint of $\vec{a}$ and $\vec{b}$ has position vector $\frac{\vec{a}+\vec{b}}{2}$.

**Step 1 — Add the vectors:** $\vec{a}+\vec{b} = (2\hat{\imath}+3\hat{\jmath})+(4\hat{\imath}-\hat{\jmath}) = 6\hat{\imath}+2\hat{\jmath}$.

**Step 2 — Halve:** $\frac{6\hat{\imath}+2\hat{\jmath}}{2} = 3\hat{\imath}+\hat{\jmath}$.

**Why other options are wrong:**

- Option (B): forgets to divide by 2.
- Option (C): subtracts instead of adds.
- Option (D): error in the $\hat{\jmath}$ component.

**Final Answer:** midpoint $= 3\hat{\imath}+\hat{\jmath} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q23

## Q24.

### Solution

**Concept — Scalar projection:** The component of $\vec{a}$ along $\vec{b}$ is $\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$.

**Step 1 — Dot product:** $\vec{a}\cdot\vec{b} = (2)(1)+(1)(2)+(2)(2) = 2+2+4 = 8$.

**Step 2 — Magnitude of $\vec{b}$:** $|\vec{b}| = \sqrt{1^2+2^2+2^2} = \sqrt{1+4+4} = \sqrt{9} = 3$.

**Step 3 — Divide:** Projection $= \frac{8}{3}$.

**Why other options are wrong:**

- Option (A) $\frac{4}{3}$: uses a wrong dot product.
- Option (B): equals 2, dividing by the wrong magnitude.
- Option (C): divides by $|\vec{b}|^2$ instead of $|\vec{b}|$.

**Final Answer:** projection $= \frac{8}{3} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q24

## Q25.

**Solution**

**Concept — Image of a point in a plane:** The foot of perpendicular gives the reflection; here the plane $x + y + z = 0$ passes through the origin with normal $(1, 1, 1)$.

**Step 1 — Set up the line through $P$:** $\frac{x-1}{1} = \frac{y-2}{1} = \frac{z-3}{1} = t$, so the point is $(1+t,\ 2+t,\ 3+t)$.

**Step 2 — The image satisfies $t = -\frac{2(ax_0+by_0+cz_0+d)}{a^2+b^2+c^2}$:** Here $x_0 + y_0 + z_0 = 1 + 2 + 3 = 6$, $a^2 + b^2 + c^2 = 3$, so $t = -\frac{2(6)}{3} = -4$.

**Step 3 — Substitute $t = -4$:** Image $= (1-4,\ 2-4,\ 3-4) = (-3, -2, -1)$.

**Why other options are wrong:**

- Option (B): overshoots by using $t = -6$ instead of $-4$.
- Option (C): merely negates each coordinate, ignoring the plane.
- Option (D): is the foot/translation, not the reflection.

**Final Answer:** image $= (-3, -2, -1) \Rightarrow \boxed{A}$

**Answer: (A)** Go Back to Q25

## Q26.

**Solution**

**Concept — Cartesian form of a line:** A line through $(x_1, y_1, z_1)$ with direction ratios $a, b, c$ is $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$.

**Step 1 — Insert the point:** $(x_1, y_1, z_1) = (1, 2, 3)$.

**Step 2 — Insert the direction ratios:** $a = 2$, $b = 3$, $c = 4$.

**Step 3 — Write the equation:** $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$.

**Why other options are wrong:**

- Option (A): uses $+$ signs, i.e. the point $(-1, -2, -3)$.
- Option (B): mismatches point and direction ratios.
- Option (D): wrong direction ratios.

**Final Answer:** $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \Rightarrow \boxed{C}$

Answer: (C) Go Back to Q26

## Q27.

**Solution**

**Concept — Mutually exclusive events:** If $A$ and $B$ cannot occur together, $P(A \cap B) = 0$, so $P(A \cup B) = P(A) + P(B)$.

**Step 1 — Apply the rule:** $P(A \cup B) = P(A) + P(B) = 0.3 + 0.45$.

**Step 2 — Add:** $0.3 + 0.45 = 0.75$.

**Why other options are wrong:**

- Option (B) $0.135$: multiplies the probabilities (treats events as independent).
- Option (C) $0.15$: subtracts the probabilities.
- Option (D) $1$: assumes the events are exhaustive.

**Final Answer:** $P(A \cup B) = 0.75 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q27

## Q28.

**Solution**

**Concept — Expected value:** $E(X) = \sum x_i\, p_i$, the probability-weighted average of the values.

**Step 1 — Form each product:** $0 \times 0.2 = 0;\ \ 1 \times 0.5 = 0.5;\ \ 2 \times 0.3 = 0.6$.

**Step 2 — Sum the products:** $E(X) = 0 + 0.5 + 0.6 = 1.1$.

**Why other options are wrong:**

- Option (A) $0.5$: counts only the middle term.
- Option (B) $1.5$: takes a plain average of $0, 1, 2$.
- Option (C) $0.9$: arithmetic slip on the last product.

**Final Answer:** $E(X) = 1.1 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q28

## Q29.

**Solution**

**Concept — Variance:** $\sigma^2 = \frac{1}{n}\sum(x_i - \bar{x})^2$, where $\bar{x}$ is the mean.

**Step 1 — Mean:** $\bar{x} = \frac{2+4+6+8+10}{5} = \frac{30}{5} = 6$.

**Step 2 — Squared deviations:** $(2-6)^2 = 16$, $(4-6)^2 = 4$, $(6-6)^2 = 0$, $(8-6)^2 = 4$, $(10-6)^2 = 16$; sum $= 40$.

**Step 3 — Divide by $n$:** $\sigma^2 = \frac{40}{5} = 8$.

**Why other options are wrong:**

- Option (A) $4$: halves the variance.
- Option (C) $6$: reports the mean instead.
- Option (D) $10$: arithmetic error in the sum.

**Final Answer:** variance $= 8 \Rightarrow \boxed{\text{B}}$

**Answer: (B)** Go Back to Q29

## Q30.

**Solution**

**Concept — Graphical LPP:** The optimum of a linear objective over a polygon occurs at a corner (vertex) of the feasible region.

**Step 1 — Identify the corners:** The feasible region for $x \geq 0,\ y \geq 0,\ x + y \leq 4$ is the triangle with vertices $(0, 0)$, $(4, 0)$, $(0, 4)$.

**Step 2 — Evaluate $Z = 3x + 2y$ at each corner:** $Z(0, 0) = 0$; $Z(4, 0) = 3(4) + 2(0) = 12$; $Z(0, 4) = 3(0) + 2(4) = 8$.

**Step 3 — Pick max and min:** Maximum $= 12$ at $(4, 0)$; minimum $= 0$ at $(0, 0)$.

**Why other options are wrong:**

- Option (A): uses $8$ as the maximum, but $12$ is larger.
- Option (B): wrong on both values.
- Option (D): omits the origin where $Z = 0$.

**Final Answer:** max $= 12$, min $= 0 \Rightarrow \boxed{\text{C}}$

**Answer: (C)** Go Back to Q30

## Answer Key

| Q | Ans | Q | Ans | Q | Ans | Q | Ans | Q | Ans |
|---|---|---|---|---|---|---|---|---|---|
| 1 | B | 2 | C | 3 | A | 4 | D | 5 | B |
| 6 | C | 7 | A | 8 | D | 9 | B | 10 | C |
| 11 | A | 12 | D | 13 | B | 14 | C | 15 | A |
| 16 | D | 17 | B | 18 | C | 19 | A | 20 | D |
| 21 | B | 22 | C | 23 | A | 24 | D | 25 | A |
| 26 | C | 27 | A | 28 | D | 29 | B | 30 | C |