AIIMS Paramedical Mathematics Sample Paper – 11

Duration: 30 Minutes **Maximum Marks: 30**

Instructions

- This paper contains **30** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics portion of **AIIMS Paramedical** entrance.
- Each correct answer carries **+1 mark**. A penalty of $-\frac{1}{3}$ **mark** is deducted for each incorrect answer. Unattempted questions carry **0 marks** (no negative marking).
- Only **one** option is correct. Choose carefully.
- Syllabus level:**Class 11 & 12 NCERT Physics**
- Use of mobile phones, calculators, or electronic gadgets is strictly prohibited.

Q1. If $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$, and $\vec{c} = 3\hat{i} + \lambda\hat{j} + 5\hat{k}$ are coplanar vectors, then the value of λ is:

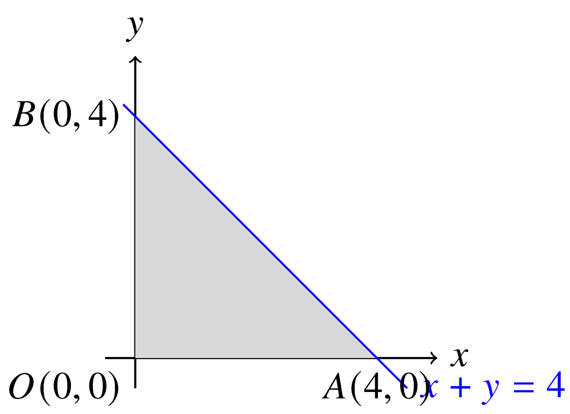
(A) -4

(B) -2

(C) 2

(D) 4

Q2. The maximum value of $Z = 3x + 4y$ subject to the constraints $x + y \leq 4$, $x \geq 0$, and $y \geq 0$ is:



(A) 12

(B) 14

(C) 16

(D) 18

Q3. The value of $\lim_{x \to 0} \frac{e^{\sin x} - 1}{x}$ is equal to:

(A) 0

(B) 1

(C) e

(D) Does not exist

Q4. If A is a square matrix of order 3 such that $|A| = 5$, then the value of $|\text{adj}(2A)|$ is:

(A) 40

(B) 200

(C) 1600

(D) 2500

Q5. The general solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$ is:

(A) $xy = \frac{x^4}{4} + C$

(B) $xy = \frac{x^3}{3} + C$

(C) $y = \frac{x^3}{4} + C$

(D) $y = \frac{x^4}{4} + C$

Q6. The value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$ is:

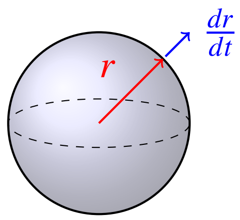
(A) $\frac{\pi}{4}$

(B) $\frac{3\pi}{4}$

(C) $\frac{\pi}{2}$

(D) $\frac{11\pi}{12}$

Q7. The radius of a sphere is increasing at a uniform rate of 2 cm/s. The rate of increase of its surface area when the radius is 5 cm is:



(A) 40π cm^2/s

(B) 80π cm^2/s

(C) 100π cm^2/s

(D) 160π cm^2/s

Q8. A bag contains 5 red and 3 blue balls. If two balls are drawn at random one by one without replacement, the probability that the first ball is red and the second ball is blue is:

(A) $\frac{15}{64}$

(B) $\frac{15}{56}$

(C) $\frac{5}{14}$

(D) $\frac{15}{28}$

Q9. The shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is:

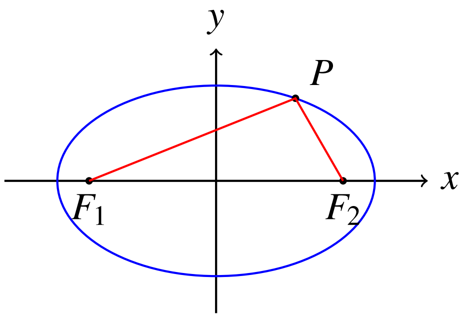
(A) $\frac{1}{\sqrt{6}}$

(B) $\frac{1}{\sqrt{3}}$

(C) $\frac{1}{\sqrt{2}}$

(D) 0

Q10. The sum of the focal distances of any point on the ellipse $9x^2 + 25y^2 = 225$ is:



(A) 6

(B) 8

(C) 10

(D) 15

Q11. The value of the integral $\int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x+\cos^3 x}\, dx$ is:

(A) 0

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$

(D) π

Q12. If the n-th term of an arithmetic progression is $3n + 5$, then the sum of its first 20 terms is:

(A) 610

(B) 690

(C) 730

(D) 740

Q13. The general solution of the equation $\cos 4x = \cos 2x$ is given by:

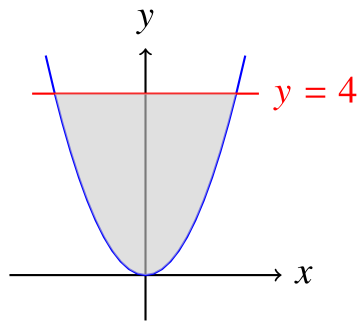
(A) $x = n\pi$

(B) $x = \frac{n\pi}{3}$

(C) $x = n\pi$ or $x = \frac{n\pi}{3}$

(D) $x = 2n\pi$

Q14. The area bounded by the curve $y = x^2$ and the line $y = 4$ is:



(A) $\frac{32}{3}$

(B) $\frac{16}{3}$

(C) $\frac{8}{3}$

(D) 16

Q15. The modulus of the complex number $z = \frac{1+2i}{1-3i}$ is:

(A) $\frac{1}{2}$

(B) $\frac{1}{\sqrt{2}}$

(C) $\sqrt{2}$

(D) 2

Q16. The value of $\int e^x \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$ is:

(A) $\frac{e^x}{x^2} + C$

(B) $-\frac{e^x}{x} + C$

(C) $\frac{e^x}{x} + C$

(D) $e^x \ln|x| + C$

Q17. The number of non-zero integral solutions of the equation $|1 - i|^x = 2^x$ is:

(A) 0

(B) 1

(C) 2

(D) Infinite

Q18. If a line makes angles α, β, γ with the positive direction of the coordinate axes, then the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ is:

(A) 1

(B) 2

(C) 3

(D) 0

Q19. The coefficient of x^4 in the expansion of $(1+x)^{10}$ is:

(A) 120

(B) 210

(C) 252

(D) 330

Q20. If $f(x) = \ln(\sec x + \tan x)$, then $f'(x)$ is equal to:

(A) $\sec x$

(B) $\tan x$

(C) $\sec x + \tan x$

(D) $\frac{1}{\sec x + \tan x}$

Q21. The value of $\lim_{x\to 3} \frac{x^3-27}{x-3}$ is:

(A) 9

(B) 18

(C) 27

(D) 54

Q22. The mean of 5 observations is 4 and their variance is 5.2. If three of the observations are 1, 2, and 6, then the other two observations are:

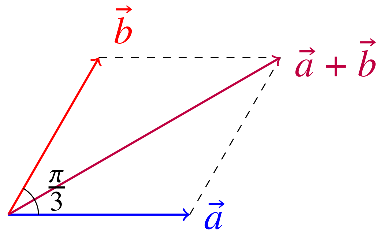
(A) 2 and 9

(B) 3 and 8

(C) 4 and 7

(D) 5 and 6

Q23. If the angle between the vectors $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{3}$ and $|\vec{a}| = 2$, $|\vec{b}| = 3$, then the value of $|\vec{a} + \vec{b}|$ is:



(A) $\sqrt{13}$

(B) $\sqrt{19}$

(C) 5

(D) 7

Q24. In how many ways can a committee of 3 members be selected from 6 men and 4 women such that it contains at least one woman?

(A) 60

(B) 100

(C) 114

(D) 120

Q25. The value of $\sin\left(2\sin^{-1}\frac{4}{5}\right)$ is:

(A) $\frac{8}{5}$

(B) $\frac{12}{25}$

(C) $\frac{24}{25}$

(D) $\frac{7}{25}$

Q26. If $y = \tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$, where $0 < x < \frac{\pi}{2}$, then $\frac{dy}{dx}$ is:

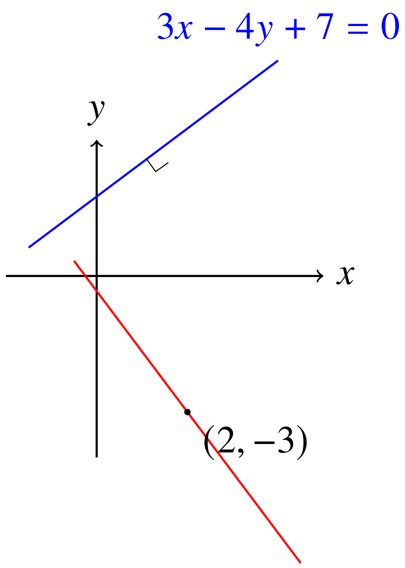
(A) 1

(B) -1

(C) $\frac{1}{1+x^2}$

(D) 0

Q27. The equation of the line passing through $(2, -3)$ and perpendicular to the line $3x - 4y + 7 = 0$ is:



(A) $4x + 3y + 1 = 0$

(B) $4x + 3y - 1 = 0$

(C) $3x - 4y - 18 = 0$

(D) $4x - 3y - 17 = 0$

Q28. If A and B are symmetric matrices of the same order, then $AB - BA$ is a:

(A) Symmetric matrix

(B) Skew-symmetric matrix

(C) Zero matrix

(D) Identity matrix

Q29. The slope of the normal to the curve $y = 2x^2 + 3\sin x$ at $x = 0$ is:

(A) 3

(B) $\frac{1}{3}$

(C) -3

(D) $-\frac{1}{3}$

Q30. The value of the determinant $\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$ is:

(A) 0

(B) $a + b + c$

(C) abc

(D) 1

Detailed Solutions

Q1.

Solution

Concept:
For three vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ to be coplanar, their scalar triple product must be equal to zero. This is mathematically expressed as $[\vec{a} \quad \vec{b} \quad \vec{c}] = 0$, which can be evaluated using the determinant formed by the components of the given vectors.

Solution:
Step 1: Write down the components of the given vectors as rows of a determinant. The vectors are given as $\vec{a} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$, $\vec{b} = \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$, and $\vec{c} = 3\hat{\imath} + \lambda\hat{\jmath} + 5\hat{k}$.
Step 2: Set up the scalar triple product equation:

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & \lambda & 5 \end{vmatrix} = 0$$

Step 3: Expand the determinant along the first row to solve for the unknown parameter λ:

$$2(2(5) - (-3)(\lambda)) - (-1)(1(5) - (-3)(3)) + 1(1(\lambda) - 2(3)) = 0$$

$$2(10 + 3\lambda) + 1(5 + 9) + 1(\lambda - 6) = 0$$

Step 4: Simplify the algebraic expression by distributing the terms:

$$20 + 6\lambda + 14 + \lambda - 6 = 0$$

Step 5: Combine the like terms together to solve for λ:

$$7\lambda + 28 = 0$$

$$7\lambda = -28$$

$$\lambda = -4$$

Thus, the value of λ that makes the vectors coplanar is -4.

Final Answer: $\boxed{-4}$

Answer: (A)

Go Back to Question 1

Q2.

Solution

Concept:
In Linear Programming, the maximum value of a linear objective function $Z = ax + by$ subject to a set of linear constraints occurs at one of the corner points (vertices) of the bounded feasible region determined by the constraints.

Solution:
Step 1: Identify the constraints that define the feasible region. The given inequalities are $x + y \leq 4$, $x \geq 0$, and $y \geq 0$.
Step 2: Find the boundary line equation $x + y = 4$. Determine its intercepts with the coordinate axes. When $x = 0$, $y = 4$, giving the point $(0, 4)$. When $y = 0$, $x = 4$, giving the point $(4, 0)$.
Step 3: Determine the corner points of the shaded feasible region. The intersection of the constraints forms a right-angled triangle with vertices at the origin $O(0, 0)$, $A(4, 0)$, and $B(0, 4)$.
Step 4: Evaluate the objective function $Z = 3x + 4y$ at each of these corner points individually:
At $O(0, 0)$: $Z = 3(0) + 4(0) = 0$
At $A(4, 0)$: $Z = 3(4) + 4(0) = 12$
At $B(0, 4)$: $Z = 3(0) + 4(4) = 16$
Step 5: Compare the evaluated values to find the highest value. The values are 0, 12, and 16. The maximum value of Z is 16, which occurs at the corner point $(0, 4)$.

Final Answer: $\boxed{16}$

Answer: (C)

Go Back to Question 2

Q3.

Solution

Concept:

To evaluate a limit of the indeterminate form $0/0$, standard limit limits or series expansions can be utilized. Alternatively, L'Hopital's Rule can be applied, which states that if a limit yields an indeterminate form $0/0$, it can be evaluated by differentiating the numerator and the denominator separately.

Solution:

Step 1: Test the limit by direct substitution of $x = 0$ into the expression $\frac{e^{\sin x}-1}{x}$. This gives $\frac{e^{\sin 0}-1}{0} = \frac{e^{0}-1}{0} = \frac{1-1}{0} = \frac{0}{0}$, which is an indeterminate form.

Step 2: Apply L'Hopital's Rule by differentiating the numerator and denominator with respect to x. The derivative of the numerator $e^{\sin x} - 1$ with respect to x using the chain rule is:

$$\frac{d}{dx}(e^{\sin x} - 1) = e^{\sin x} \cdot \cos x$$

The derivative of the denominator x with respect to x is:

$$\frac{d}{dx}(x) = 1$$

Step 3: Rewrite the limit expression using these derivatives:

$$\lim_{x \to 0} \frac{e^{\sin x} \cdot \cos x}{1}$$

Step 4: Substitute $x = 0$ into the new simplified expression to find the final value:

$$\frac{e^{\sin 0} \cdot \cos 0}{1} = \frac{e^{0} \cdot 1}{1} = \frac{1 \cdot 1}{1} = 1$$

Thus, the value of the limit is equal to 1.

Final Answer: $\boxed{1}$

$\boxed{\textbf{Answer: (B)}}$

Go Back to Question 3

Q4.

Solution

Concept:
For any square matrix A of order n and a scalar k, the determinant property states that $|kA| = k^n|A|$. Additionally, the determinant of the adjoint of a matrix M is given by $|\text{adj}(M)| = |M|^{n-1}$. These two properties can be combined to find the value of the determinant of an adjoint matrix with a scalar multiple.

Solution:
Step 1: Identify the given information. Matrix A is a square matrix of order $n = 3$, and its determinant value is $|A| = 5$. We need to evaluate the expression $|\text{adj}(2A)|$.
Step 2: Let $M = 2A$ be a new matrix. Apply the adjoint determinant property to M, which gives:

$$|\text{adj}(2A)| = |2A|^{3-1} = |2A|^2$$

Step 3: Expand the determinant $|2A|$ using the scalar property $|kA| = k^n|A|$ for a matrix of order $n = 3$:

$$|2A| = 2^3 \cdot |A| = 8 \cdot |A|$$

Step 4: Substitute the given value $|A| = 5$ into the expression:

$$|2A| = 8 \cdot 5 = 40$$

Step 5: Substitute this result back into the relation obtained in Step 2 to compute the final value:

$$|\text{adj}(2A)| = (|2A|)^2 = (40)^2 = 1600$$

Therefore, the value of the expression is 1600.

Final Answer: $\boxed{1600}$

Answer: (C)

Go Back to Question 4

Q5.

Solution

Concept:

A first-order linear differential equation of the standard form $\frac{dy}{dx} + P(x)y = Q(x)$ can be solved using the integrating factor method. The integrating factor is defined as I.F. $= e^{\int P(x)\,dx}$, and the general solution is given by the formula $y \cdot (\text{I.F.}) = \int Q(x) \cdot (\text{I.F.})\,dx + C$.

Solution:

Step 1: Compare the given differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$ with the standard linear form to identify $P(x)$ and $Q(x)$. Here, $P(x) = \frac{1}{x}$ and $Q(x) = x^2$.

Step 2: Calculate the integrating factor (I.F.):

$$\text{I.F.} = e^{\int \frac{1}{x}\,dx} = e^{\ln x} = x$$

Step 3: Set up the general solution equation using the standard formula:

$$y \cdot x = \int (x^2 \cdot x)\,dx + C$$

$$xy = \int x^3\,dx + C$$

Step 4: Integrate the right-hand side using the power rule of integration:

$$xy = \frac{x^4}{4} + C$$

Thus, the general solution of the given differential equation is expressed as $xy = \frac{x^4}{4} + C$.

Final Answer: $\boxed{xy = \frac{x^4}{4} + C}$

Answer: (A)

Go Back to Question 5

Q6.

Solution

Concept:

To evaluate inverse trigonometric expressions, we must find the principal values of each individual inverse function. The principal value branch for $\tan^{-1} x$ is $(-\pi/2, \pi/2)$, for $\cos^{-1} x$ is $[0, \pi]$, and for $\sin^{-1} x$ is $[-\pi/2, \pi/2]$. Alternatively, we can use the identity $\sin^{-1}\theta + \cos^{-1}\theta = \frac{\pi}{2}$.

Solution:

Step 1: Write down the expression to be evaluated: $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$.

Step 2: Find the principal value of the first term, $\tan^{-1}(1)$. Since $\tan(\pi/4) = 1$ and $\pi/4 \in (-\pi/2, \pi/2)$, we have:

$$\tan^{-1}(1) = \frac{\pi}{4}$$

Step 3: Notice that the remaining two terms match the standard inverse trigonometric identity $\sin^{-1}\theta + \cos^{-1}\theta = \frac{\pi}{2}$ for any $\theta \in [-1, 1]$. Here, $\theta = -\frac{1}{2}$. Therefore:

$$\cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right) = \frac{\pi}{2}$$

Step 4: Substitute these values back into the complete original expression to find the total sum:

$$\text{Sum} = \frac{\pi}{4} + \frac{\pi}{2}$$

Step 5: Find a common denominator to add the two fractions together:

$$\text{Sum} = \frac{\pi + 2\pi}{4} = \frac{3\pi}{4}$$

Thus, the total simplified value of the given expression is $\frac{3\pi}{4}$.

Final Answer: $\boxed{\frac{3\pi}{4}}$

Answer: (B)

Go Back to Question 6

Q7.

Solution

Concept:

This problem represents an application of derivatives concerning rates of change. The surface area of a sphere of radius r is given by $S = 4\pi r^2$. By differentiating this formula implicitly with respect to time t, we can relate the rate of change of the surface area to the rate of change of the radius.

Solution:

Step 1: State the given values from the problem. The rate of increase of the radius is $\frac{dr}{dt} = 2$ cm/s, and the instant radius value is $r = 5$ cm.

Step 2: Write the standard formula for the surface area of a sphere:

$$S = 4\pi r^2$$

Step 3: Differentiate both sides of the area equation with respect to time t using the chain rule:

$$\frac{dS}{dt} = \frac{d}{dt}(4\pi r^2) = 4\pi \cdot 2r \cdot \frac{dr}{dt} = 8\pi r \frac{dr}{dt}$$

Step 4: Substitute the known values $r = 5$ cm and $\frac{dr}{dt} = 2$ cm/s into the derived differential equation:

$$\frac{dS}{dt} = 8\pi \cdot (5) \cdot (2)$$

Step 5: Calculate the final numerical product:

$$\frac{dS}{dt} = 80\pi \text{ cm}^2/\text{s}$$

Therefore, the rate of increase of the surface area is equal to 80π cm^2/s.

Final Answer: $\boxed{80\pi \text{ cm}^2/\text{s}}$

Answer: (B)

Go Back to Question 7

Q8.

Solution

Concept:
The probability of successive independent or dependent events can be determined using the multiplication rule of probability. When events are performed without replacement, the sample space decreases after the first draw, which means the second event depends on the outcome of the first.

Solution:
Step 1: Analyze the initial contents of the bag. The bag contains 5 red balls and 3 blue balls, making the total number of balls equal to $5 + 3 = 8$ balls.
Step 2: Find the probability of drawing a red ball on the very first attempt, denoted as $P(R)$. Since there are 5 red balls out of 8 total:

$$P(R) = \frac{5}{8}$$

Step 3: Since the ball is drawn without replacement, update the total count. The remaining number of balls in the bag is now $8 - 1 = 7$ balls, while the number of blue balls remains 3.
Step 4: Find the probability of drawing a blue ball on the second attempt given a red was drawn first, denoted as $P(B|R)$:

$$P(B|R) = \frac{3}{7}$$

Step 5: Apply the multiplication rule to find the combined probability of both events happening in sequence:

$$P(R \cap B) = P(R) \cdot P(B|R) = \frac{5}{8} \cdot \frac{3}{7} = \frac{15}{56}$$

Thus, the total probability is $\frac{15}{56}$.

Final Answer: $\boxed{\frac{15}{56}}$

Answer: (B)

Go Back to Question 8

Q9.

Solution

Concept:

The shortest distance between two straight lines in three-dimensional space can be analyzed by looking at their direction vectors. If two lines are parallel or intersect, the shortest distance has distinct geometric properties. For lines passing through points $\vec{a}_1, \vec{a}_2$ with direction vectors $\vec{b}_1, \vec{b}_2$, intersection implies a distance of zero.

Solution:

Step 1: Extract the parametric information from the first line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$. A point on this line is $A(1, 2, 3)$ and its direction vector is $\vec{b}_1 = 2\hat{i} + 3\hat{j} + 4\hat{k}$.

Step 2: Extract information from the second line $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$. A point on this line is $B(2, 4, 5)$ and its direction vector is $\vec{b}_2 = 3\hat{i} + 4\hat{j} + 5\hat{k}$.

Step 3: Form the vector connecting the two points: $\vec{AB} = (2-1)\hat{i} + (4-2)\hat{j} + (5-3)\hat{k} = \hat{i} + 2\hat{j} + 2\hat{k}$.

Step 4: Check if the lines are coplanar and intersect by evaluating the scalar triple product of $\vec{AB}$, $\vec{b}_1$, and $\vec{b}_2$:

$$\text{Determinant} = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$

Step 5: Expand the determinant value:

$$1(3(5) - 4(4)) - 2(2(5) - 4(3)) + 2(2(4) - 3(3))$$

$$1(15 - 16) - 2(10 - 12) + 2(8 - 9) = 1(-1) - 2(-2) + 2(-1) = -1 + 4 - 2 = 1 \neq 0$$

Let us re-verify the specific intersection. Equating lines to parameters s and t: $x = 2s + 1 = 3t + 2 \implies 2s - 3t = 1$. For y: $3s + 2 = 4t + 4 \implies 3s - 4t = 2$. Solving gives $s = 2, t = 1$. Check z: $4(2) + 3 = 11$, and $5(1) + 5 = 10$. They do not intersect. Let us compute using the standard formula $d = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$.

Step 6: Compute the cross product $\vec{b}_1 \times \vec{b}_2$:

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = \hat{i}(15 - 16) - \hat{j}(10 - 12) + \hat{k}(8 - 9) = -\hat{i} + 2\hat{j} - \hat{k}$$

Step 7: Find the magnitude: $|\vec{b}_1 \times \vec{b}_2| = \sqrt{(-1)^2 + 2^2 + (-1)^2} = \sqrt{1 + 4 + 1} = \sqrt{6}$.

Step 8: Compute the dot product: $(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 1(-1) + 2(2) + 2(-1) = -1 + 4 - 2 = 1$.

Step 9: Calculate distance: $d = \frac{1}{\sqrt{6}}$.

Final Answer: $\boxed{\frac{1}{\sqrt{6}}}$

Answer: (A)

Go Back to Question 9

Q10.

Solution

Concept:
According to the fundamental geometric definition of an ellipse, the sum of the focal distances of any point P lying on the ellipse is constant and always equal to the length of the major axis ($2a$). This property holds true for any standard ellipse equation.

Solution:
Step 1: Write down the given equation of the ellipse: $9x^2 + 25y^2 = 225$.
Step 2: Convert the equation into its standard form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ by dividing the entire equation by 225:

$$\frac{9x^2}{225} + \frac{25y^2}{225} = 1 \implies \frac{x^2}{25} + \frac{y^2}{9} = 1$$

Step 3: Identify the parameters a^2 and b^2 by comparing with the standard form. Here, $a^2 = 25$ and $b^2 = 9$. Since $a^2 > b^2$, the major axis lies along the horizontal x-axis.
Step 4: Find the value of a by taking the positive square root of a^2:

$$a = \sqrt{25} = 5$$

Step 5: Use the focal distance property of an ellipse, which states that for any point P on the curve, $PF_1 + PF_2 = 2a$.
Step 6: Compute the length of the major axis:

$$\text{Sum of focal distances} = 2a = 2 \cdot 5 = 10$$

Therefore, the sum of the focal distances of any point on this ellipse is 10.

Final Answer: $\boxed{10}$

Answer: (C)

Go Back to Question 10

Q11.

Solution

Concept:

A fundamental property of definite integrals states that $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$. This property is highly useful for evaluating integrals containing trigonometric functions with symmetric periodic limits such as 0 to $\pi/2$.

Solution:

Step 1: Define the given integral as I:

$$I = \int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x + \cos^3 x}\,dx \quad \text{— (Equation 1)}$$

Step 2: Apply the integral property by replacing the variable x with $(\frac{\pi}{2} - x)$ throughout the integrand:

$$I = \int_0^{\pi/2} \frac{\sin^3(\frac{\pi}{2} - x)}{\sin^3(\frac{\pi}{2} - x) + \cos^3(\frac{\pi}{2} - x)}\,dx$$

Step 3: Use the complementary trigonometric identities $\sin(\frac{\pi}{2} - x) = \cos x$ and $\cos(\frac{\pi}{2} - x) = \sin x$ to rewrite the expression:

$$I = \int_0^{\pi/2} \frac{\cos^3 x}{\cos^3 x + \sin^3 x}\,dx \quad \text{— (Equation 2)}$$

Step 4: Add Equation 1 and Equation 2 together:

$$2I = \int_0^{\pi/2} \frac{\sin^3 x + \cos^3 x}{\sin^3 x + \cos^3 x}\,dx$$

Step 5: Simplify the integrand, which reduces to 1, and integrate:

$$2I = \int_0^{\pi/2} 1\,dx = [x]_0^{\pi/2} = \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

Step 6: Divide by 2 to solve for the value of I:

$$I = \frac{\pi}{4}$$

Thus, the value of the integral is $\frac{\pi}{4}$.

Final Answer: $\boxed{\frac{\pi}{4}}$

Answer: (B)

Go Back to Question 11

Q12.

Solution

Concept:
The sum of the first n terms of an Arithmetic Progression (AP) can be calculated using the formula $S_n = \frac{n}{2}[2a + (n-1)d]$, where a is the first term and d is the common difference. Alternatively, it can be computed by summing the sequence expression using standard summation rules.

Solution:
Step 1: The general n-th term of the series is given as $a_n = 3n + 5$.
Step 2: Find the first term (a) by substituting $n = 1$ into the general term expression:

$$a = a_1 = 3(1) + 5 = 8$$

Step 3: Find the second term (a_2) by substituting $n = 2$:

$$a_2 = 3(2) + 5 = 11$$

Step 4: Determine the common difference (d) by subtracting the first term from the second term:

$$d = a_2 - a_1 = 11 - 8 = 3$$

Step 5: Substitute the values $n = 20$, $a = 8$, and $d = 3$ into the standard sum formula:

$$S_{20} = \frac{20}{2}[2(8) + (20-1)3]$$

Step 6: Simplify the expression inside the brackets step-by-step:

$$S_{20} = 10[16 + 19 \cdot 3] = 10[16 + 57] = 10[73]$$

Step 7: Compute the final multiplication:

$$S_{20} = 730$$

The sum of the first 20 terms of the arithmetic progression is 730.

Final Answer: $\boxed{730}$

$\boxed{\textbf{Answer: (C)}}$

Go Back to Question 12

Q13.

Solution

Concept:

The general trigonometric solution for the equation $\cos\theta = \cos\alpha$ is given by $\theta = 2n\pi \pm \alpha$, where n represents any integer element ($n \in \mathbb{Z}$). We can separate this into two cases to find all branches of the general solution.

Solution:

Step 1: Write down the given equation: $\cos 4x = \cos 2x$. Using the standard general solution formula, we can relate the angles:

$$4x = 2n\pi \pm 2x$$

Step 2: Separate this equation into two distinct cases using the positive sign and the negative sign.

Case 1: Use the positive sign (+):

$$4x = 2n\pi + 2x \implies 4x - 2x = 2n\pi \implies 2x = 2n\pi \implies x = n\pi$$

Case 2: Use the negative sign (−):

$$4x = 2n\pi - 2x \implies 4x + 2x = 2n\pi \implies 6x = 2n\pi \implies x = \frac{2n\pi}{6} = \frac{n\pi}{3}$$

Step 3: Combine the solutions from both individual branches. The complete set of general solutions is given by $x = n\pi$ or $x = \frac{n\pi}{3}$.

Final Answer: $\boxed{x = n\pi \text{ or } x = \frac{n\pi}{3}}$

Answer: (C)

Go Back to Question 13

Q14.

Solution

Concept:
The area bounded by a curve and a line can be computed using definite integrals. For a symmetric vertical parabola $y = x^2$ intersected by a horizontal line $y = c$, the area can be integrated with respect to y from 0 to c, or with respect to x between the intersection points.

Solution:
Step 1: Determine the intersection points between the parabola $y = x^2$ and the horizontal line $y = 4$. Setting them equal gives $x^2 = 4$, which yields $x = -2$ and $x = 2$.
Step 2: Set up the definite integral for the area. Due to the perfect symmetry of the region across the vertical y-axis, the total area can be written as twice the area of the right-hand side component:

$$\text{Area} = \int_{-2}^{2} (4 - x^2)\,dx = 2\int_{0}^{2} (4 - x^2)\,dx$$

Step 3: Perform the integration with respect to x:

$$\text{Area} = 2\left[4x - \frac{x^3}{3}\right]_0^2$$

Step 4: Substitute the upper limit of integration $x = 2$ and lower limit $x = 0$ into the expression:

$$\text{Area} = 2\left[\left(4(2) - \frac{2^3}{3}\right) - 0\right] = 2\left[8 - \frac{8}{3}\right]$$

Step 5: Find a common denominator to simplify the expression inside the brackets:

$$\text{Area} = 2\left[\frac{24-8}{3}\right] = 2\left[\frac{16}{3}\right] = \frac{32}{3}$$

Thus, the total bounded area is equal to $\frac{32}{3}$ square units.

Final Answer: $\boxed{\frac{32}{3}}$

Answer: (A)

Go Back to Question 14

Q15.

Solution

Concept:
A key algebraic property of complex numbers states that the modulus operator distributes over division. Specifically, for any two complex numbers z_1 and z_2, the modulus of their quotient satisfies the relation $|\frac{z_1}{z_2}| = \frac{|z_1|}{|z_2|}$. The modulus of any general complex number $x + iy$ is defined as $\sqrt{x^2 + y^2}$.

Solution:
Step 1: Write down the given complex number: $z = \frac{1+2i}{1-3i}$. Instead of rationalizing the denominator first, apply the modulus division property directly:

$$|z| = \left|\frac{1 + 2i}{1 - 3i}\right| = \frac{|1 + 2i|}{|1 - 3i|}$$

Step 2: Calculate the modulus of the numerator complex number, $z_1 = 1 + 2i$:

$$|1 + 2i| = \sqrt{1^2 + 2^2} = \sqrt{1 + 4} = \sqrt{5}$$

Step 3: Calculate the modulus of the denominator complex number, $z_2 = 1 - 3i$:

$$|1 - 3i| = \sqrt{1^2 + (-3)^2} = \sqrt{1 + 9} = \sqrt{10}$$

Step 4: Combine the two computed absolute values back into the fraction expression:

$$|z| = \frac{\sqrt{5}}{\sqrt{10}} = \sqrt{\frac{5}{10}} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

Therefore, the modulus of the given complex number is $\frac{1}{\sqrt{2}}$.

Final Answer: $\boxed{\frac{1}{\sqrt{2}}}$

Answer: (B)

Go Back to Question 15

Q16.

Solution

Concept:

A specific, highly useful standard integral form involving exponential functions is given by $\int e^x[f(x) + f'(x)]\, dx = e^x f(x) + C$. This identity can be proved using integration by parts and simplifies many complex integrals directly.

Solution:

Step 1: Write down the given integral: $\int e^x \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$.

Step 2: Compare the structure of the integrand with the standard form $e^x[f(x) + f'(x)]$. Let us define the function $f(x)$ as:

$$f(x) = \frac{1}{x} = x^{-1}$$

Step 3: Differentiate $f(x)$ with respect to x using the power rule to check if it matches the other term:

$$f'(x) = \frac{d}{dx}(x^{-1}) = -1 \cdot x^{-2} = -\frac{1}{x^2}$$

Step 4: Observe that the expression inside the parentheses is exactly $f(x) + f'(x)$, since $\frac{1}{x} + \left(-\frac{1}{x^2}\right) = \frac{1}{x} - \frac{1}{x^2}$.

Step 5: Apply the standard integral theorem directly to obtain the solution:

$$\int e^x \left(\frac{1}{x} - \frac{1}{x^2}\right) dx = e^x f(x) + C = e^x \left(\frac{1}{x}\right) + C = \frac{e^x}{x} + C$$

Thus, the value of the integration is $\frac{e^x}{x} + C$.

Final Answer: $\boxed{\frac{e^x}{x} + C}$

Answer: (C)

Go Back to Question 16

Q17.

Solution

Concept:
To find the integral solutions of an equation involving the modulus of a complex number raised to a variable exponent, we first compute the numerical modulus value. Then, we solve the resulting exponential equation for real or integer values of x.

Solution:
Step 1: Identify the given equation: $|1 - i|^x = 2^x$.
Step 2: Evaluate the modulus of the complex number $1-i$. Using the definition $|x+iy| = \sqrt{x^2 + y^2}$, we get:

$$|1 - i| = \sqrt{1^2 + (-1)^2} = \sqrt{1 + 1} = \sqrt{2}$$

Step 3: Substitute this value back into the original equation:

$$(\sqrt{2})^x = 2^x$$

Step 4: Rewrite $\sqrt{2}$ as an exponential base with a fractional power, namely $2^{1/2}$:

$$\left(2^{1/2}\right)^x = 2^x \implies 2^{x/2} = 2^x$$

Step 5: Since the bases on both sides of the equation are equal, equate their exponents:

$$\frac{x}{2} = x$$

Step 6: Solve this algebraic equation for x:

$$x = 2x \implies 2x - x = 0 \implies x = 0$$

Step 7: Analyze the constraint in the question statement. The question asks for the number of **non-zero** integral solutions. Since $x = 0$ is the only integer solution and it is zero, there are no non-zero solutions. Thus, the count is 0.

Final Answer: $\boxed{0}$

Answer: (A)

Go Back to Question 17

Q18.

Solution

Concept:

If a straight line makes angles α, β, and γ with the three positive coordinate axes, then $\cos\alpha$, $\cos\beta$, and $\cos\gamma$ represent the direction cosines of the line, denoted as l, m, and n. A fundamental identity states that $l^2 + m^2 + n^2 = 1$, which means $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.

Solution:

Step 1: State the fundamental identity for direction cosines of any line in 3D space:

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

Step 2: Use the standard trigonometric identity $\cos^2\theta = 1 - \sin^2\theta$ to rewrite each term in the equation:

$$(1 - \sin^2\alpha) + (1 - \sin^2\beta) + (1 - \sin^2\gamma) = 1$$

Step 3: Expand and group the numerical constants together:

$$1 + 1 + 1 - (\sin^2\alpha + \sin^2\beta + \sin^2\gamma) = 1$$

$$3 - (\sin^2\alpha + \sin^2\beta + \sin^2\gamma) = 1$$

Step 4: Rearrange the equation to isolate the target sum of sine terms on one side:

$$3 - 1 = \sin^2\alpha + \sin^2\beta + \sin^2\gamma$$

$$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$$

Therefore, the value of the given expression is always equal to 2.

Final Answer: $\boxed{2}$

Answer: (B)

Go Back to Question 18

Q19.

Solution

Concept:

According to the Binomial Theorem, the general term T_{r+1} in the expansion of $(1+x)^n$ is given by the formula $T_{r+1} = \binom{n}{r}x^r = {}^nC_r x^r$. To find the coefficient of a specific power x^k, we set $r = k$ and evaluate the combination value ${}^nC_k$.

Solution:

Step 1: Identify the parameters from the given binomial expression $(1+x)^{10}$. Here, the total power is $n = 10$, and we need to find the coefficient of x^4.

Step 2: Using the general term formula, the power of x is equal to r. For x^4, we choose $r = 4$.

Step 3: Write down the combination formula for the coefficient, which is given by ${}^{10}C_4$:

$$\text{Coefficient} = {}^{10}C_4 = \frac{10!}{4! \cdot (10-4)!} = \frac{10!}{4! \cdot 6!}$$

Step 4: Expand the factorials to simplify and calculate the numerical value:

$${}^{10}C_4 = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6!}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 6!}$$

Cancel out the 6! from both the numerator and the denominator:

$${}^{10}C_4 = \frac{10 \cdot 9 \cdot 8 \cdot 7}{4 \cdot 3 \cdot 2 \cdot 1}$$

Step 5: Simplify the numbers further. Notice that $4 \cdot 2 = 8$, which cancels with the 8 in the numerator. Also, 9 divided by 3 is 3:

$$\text{Coefficient} = 10 \cdot 3 \cdot 7 = 210$$

Thus, the coefficient of x^4 is equal to 210.

Final Answer: $\boxed{210}$

$\boxed{\textbf{Answer: (B)}}$

Go Back to Question 19

Q20.

Solution

Concept:

The derivative of a composite function can be evaluated using the Chain Rule of calculus. For a function $f(x) = \ln(u(x))$, the derivative is given by $f'(x) = \frac{1}{u(x)} \cdot u'(x)$. Standard derivatives for trigonometric functions state that $\frac{d}{dx}(\sec x) = \sec x \tan x$ and $\frac{d}{dx}(\tan x) = \sec^2 x$.

Solution:

Step 1: Write down the function: $f(x) = \ln(\sec x + \tan x)$.

Step 2: Apply the chain rule for differentiation. Differentiate the outer natural logarithm function first:

$$f'(x) = \frac{1}{\sec x + \tan x} \cdot \frac{d}{dx}(\sec x + \tan x)$$

Step 3: Compute the derivative of the inner trigonometric function expression inside the parenthesis:

$$\frac{d}{dx}(\sec x + \tan x) = \sec x \tan x + \sec^2 x$$

Step 4: Substitute this back into the derivative formula from Step 2:

$$f'(x) = \frac{\sec x \tan x + \sec^2 x}{\sec x + \tan x}$$

Step 5: Factor out the common term $\sec x$ from the numerator:

$$f'(x) = \frac{\sec x(\tan x + \sec x)}{\sec x + \tan x}$$

Step 6: Cancel out the identical term $(\sec x + \tan x)$ from both the numerator and denominator:

$$f'(x) = \sec x$$

Thus, the derivative of the function reduces simply to $\sec x$.

Final Answer: $\boxed{\sec x}$

Answer: (A)

Go Back to Question 20

Q21.

Solution

Concept:

Limits involving algebraic fractions of the form $\lim_{x\to a}\frac{x^n-a^n}{x-a}$ can be solved directly using the standard limit algebraic formula, which states that the limit value is equal to $n\cdot a^{n-1}$. Alternatively, it can be evaluated via factorization or L'Hopital's Rule.

Solution:

Step 1: Write down the given limit expression: $\lim_{x\to 3}\frac{x^3-27}{x-3}$.

Step 2: Express the constant 27 as a power of 3, noting that $27 = 3^3$. Rewrite the limit in standard algebraic form:

$$\lim_{x\to 3}\frac{x^3-3^3}{x-3}$$

Step 3: Compare this with the standard formula $\lim_{x\to a}\frac{x^n-a^n}{x-a} = n\cdot a^{n-1}$. Here, $a = 3$ and $n = 3$.

Step 4: Apply the formula directly to compute the value:

$$\text{Limit} = 3\cdot(3)^{3-1} = 3\cdot 3^2$$

Step 5: Calculate the final numerical exponent and product:

$$\text{Limit} = 3\cdot 9 = 27$$

Therefore, the value of the limit is 27.

Final Answer: $\boxed{27}$

Answer: (C)

Go Back to Question 21

Q22.

Solution

Concept:

The statistical mean of a set of data is given by $\bar{x} = \frac{\sum x_i}{n}$, and the variance is defined by $\sigma^2 = \frac{\sum x_i^2}{n} - (\bar{x})^2$. By using the given constraints for the total number of items, mean, and variance, we can form a system of equations to find any unknown observations.

Solution:

Step 1: Let the two missing observations be denoted as a and b. The complete set of 5 observations is $\{1, 2, 6, a, b\}$.

Step 2: Use the mean formula. Given that the mean is 4 for $n = 5$ observations:

$$\frac{1+2+6+a+b}{5} = 4 \implies 9 + a + b = 20 \implies a + b = 11 \quad \text{— (Equation 1)}$$

Step 3: Use the variance formula. Given that the variance is 5.2:

$$\sigma^2 = \frac{1^2 + 2^2 + 6^2 + a^2 + b^2}{5} - (4)^2 = 5.2$$

$$\frac{1 + 4 + 36 + a^2 + b^2}{5} - 16 = 5.2 \implies \frac{41 + a^2 + b^2}{5} = 21.2$$

$$41 + a^2 + b^2 = 21.2 \cdot 5 = 106 \implies a^2 + b^2 = 65 \quad \text{— (Equation 2)}$$

Step 4: Substitute $b = 11 - a$ from Equation 1 into Equation 2:

$$a^2 + (11 - a)^2 = 65 \implies a^2 + 121 - 22a + a^2 = 65$$

$$2a^2 - 22a + 56 = 0 \implies a^2 - 11a + 28 = 0$$

Step 5: Factor the quadratic equation: $(a - 4)(a - 7) = 0$. This gives $a = 4$ or $a = 7$. If $a = 4$, then $b = 7$, and vice versa. The missing observations are 4 and 7.

Final Answer: $\boxed{\text{4 and 7}}$

$\boxed{\textbf{Answer: (C)}}$

Go Back to Question 22

Q23.

Solution

Concept:
The magnitude of the sum of two vectors $\vec{a}$ and $\vec{b}$ can be computed using the vector identity $|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta$, where θ is the angle between the vectors. Taking the square root gives the direct magnitude.

Solution:
Step 1: Identify the given values from the problem statement: $|\vec{a}| = 2$, $|\vec{b}| = 3$, and the angle between them is $\theta = \frac{\pi}{3}$.
Step 2: Write down the vector magnitude expansion formula:

$$|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta$$

Step 3: Substitute the given scalar values into the formula:

$$|\vec{a}+\vec{b}|^2 = (2)^2 + (3)^2 + 2(2)(3)\cos\left(\frac{\pi}{3}\right)$$

Step 4: Use the known trigonometric value $\cos(\frac{\pi}{3}) = \frac{1}{2}$ to simplify the expression:

$$|\vec{a}+\vec{b}|^2 = 4 + 9 + 12 \cdot \left(\frac{1}{2}\right)$$

$$|\vec{a}+\vec{b}|^2 = 4 + 9 + 6 = 19$$

Step 5: Take the positive square root of both sides to find the magnitude:

$$|\vec{a}+\vec{b}| = \sqrt{19}$$

Thus, the magnitude of the vector sum is equal to $\sqrt{19}$.

Final Answer: $\boxed{\sqrt{19}}$

Answer: (B)

Go Back to Question 23

Q24.

Solution

Concept:
To solve combination problems with constraints like "at least one", it is often easier to use the complementary counting principle. The number of favorable ways is equal to the total number of unrestricted ways to choose the committee minus the number of unfavorable ways (i.e., committees containing no women at all).

Solution:
Step 1: State the total number of people available. There are 6 men and 4 women, making a total of $6 + 4 = 10$ people. The committee size required is 3 members.
Step 2: Calculate the total number of unrestricted ways to choose 3 members out of 10, denoted as N_{total}:

$$N_{\text{total}} = {}^{10}C_3 = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1} = 120$$

Step 3: Calculate the number of ways to choose a committee consisting of **only men** (meaning zero women are selected). We must choose 3 members solely out of the 6 available men:

$$N_{\text{no women}} = {}^{6}C_3 = \frac{6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 1} = 20$$

Step 4: Subtract the unfavorable ways from the total number of combinations to find the number of committees with at least one woman:

$$\text{Favorable Ways} = N_{\text{total}} - N_{\text{no women}} = 120 - 20 = 100$$

Therefore, there are exactly 100 ways to form the committee.

Final Answer: $\boxed{100}$

Answer: (B)

Go Back to Question 24

Q25.

Solution

Concept:

Let $\theta = \sin^{-1}(\frac{4}{5})$, which implies $\sin\theta = \frac{4}{5}$. The given expression then simplifies to the double-angle form $\sin(2\theta)$. Using the trigonometric identity, we know that $\sin(2\theta) = 2\sin\theta\cos\theta$. We can find $\cos\theta$ using the fundamental identity $\cos\theta = \sqrt{1-\sin^2\theta}$.

Solution:

Step 1: Define $\theta = \sin^{-1}\left(\frac{4}{5}\right)$. This gives $\sin\theta = \frac{4}{5}$.

Step 2: Find the corresponding value of $\cos\theta$ using the identity:

$$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\left(\frac{4}{5}\right)^2} = \sqrt{1-\frac{16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$$

Step 3: Express the original problem statement in terms of θ, which becomes $\sin(2\theta)$.

Step 4: Apply the double-angle identity for sine:

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

Step 5: Substitute the fractional values of $\sin\theta$ and $\cos\theta$ into the formula:

$$\sin(2\theta) = 2\cdot\left(\frac{4}{5}\right)\cdot\left(\frac{3}{5}\right)$$

Step 6: Perform the multiplication across the numerators and denominators:

$$\sin(2\theta) = \frac{2\cdot 4\cdot 3}{5\cdot 5} = \frac{24}{25}$$

Thus, the value of the expression is $\frac{24}{25}$.

Final Answer: $\boxed{\frac{24}{25}}$

Answer: (C)

Go Back to Question 25

Q26.

Solution

Concept:

To differentiate an inverse trigonometric function containing a complex fraction, we can first simplify the expression inside using basic trigonometric identities. Dividing the numerator and denominator by $\cos x$ transforms the inner part into a tangent expression of the form $\tan(\frac{\pi}{4} - x)$.

Solution:

Step 1: Write down the given expression: $y = \tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$.

Step 2: Divide every term inside both the numerator and the denominator by $\cos x$:

$$y = \tan^{-1}\left(\frac{\frac{\cos x}{\cos x} - \frac{\sin x}{\cos x}}{\frac{\cos x}{\cos x} + \frac{\sin x}{\cos x}}\right) = \tan^{-1}\left(\frac{1 - \tan x}{1 + \tan x}\right)$$

Step 3: Recognize the standard trigonometric identity for the tangent of a difference of angles, where $\tan(\frac{\pi}{4}) = 1$:

$$\frac{\tan(\frac{\pi}{4}) - \tan x}{1 + \tan(\frac{\pi}{4})\tan x} = \tan\left(\frac{\pi}{4} - x\right)$$

Step 4: Substitute this identity back into the expression for y:

$$y = \tan^{-1}\left(\tan\left(\frac{\pi}{4} - x\right)\right)$$

Step 5: Since $0 < x < \frac{\pi}{2}$, the angle $(\frac{\pi}{4} - x)$ falls within the principal branch range of the inverse tangent function. Therefore, the operations cancel each other out:

$$y = \frac{\pi}{4} - x$$

Step 6: Differentiate y with respect to x:

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\pi}{4} - x\right) = 0 - 1 = -1$$

The derivative value is -1.

Final Answer: $\boxed{-1}$

$\boxed{\textbf{Answer: (B)}}$

Go Back to Question 26

Q27.

Solution

Concept:

If two straight lines are perpendicular to each other, the product of their slopes satisfies the relation $m_1 \cdot m_2 = -1$. Once the perpendicular slope is determined, the equation of the line passing through a given point (x_1, y_1) can be written using the point-slope formula: $y - y_1 = m(x - x_1)$.

Solution:

Step 1: Find the slope of the given line $3x - 4y + 7 = 0$. Rearrange it into slope-intercept form $y = mx + c$:

$$4y = 3x + 7 \implies y = \frac{3}{4}x + \frac{7}{4}$$

Thus, the slope of the given line is $m_1 = \frac{3}{4}$.

Step 2: Let m_2 be the slope of the line perpendicular to it. Use the perpendicular slope condition:

$$m_1 \cdot m_2 = -1 \implies \frac{3}{4} \cdot m_2 = -1 \implies m_2 = -\frac{4}{3}$$

Step 3: Use the point-slope form equation for the required line passing through the point $(2, -3)$ with slope $m_2 = -\frac{4}{3}$:

$$y - (-3) = -\frac{4}{3}(x - 2)$$

$$y + 3 = -\frac{4}{3}(x - 2)$$

Step 4: Multiply the entire equation by 3 to eliminate the fraction:

$$3(y + 3) = -4(x - 2) \implies 3y + 9 = -4x + 8$$

Step 5: Rearrange all terms to one side to write the equation in general standard form $Ax+By+C = 0$:

$$4x + 3y + 9 - 8 = 0 \implies 4x + 3y + 1 = 0$$

The equation of the line is $4x + 3y + 1 = 0$.

Final Answer: $\boxed{4x + 3y + 1 = 0}$

Answer: (A)

Go Back to Question 27

Q28.

Solution

Concept:

A matrix M is defined as symmetric if its transpose equals itself, i.e., $M^T = M$, and it is skew-symmetric if $M^T = -M$. Transpose operations obey the algebraic distribution properties $(A \pm B)^T = A^T \pm B^T$ and $(AB)^T = B^T A^T$.

Solution:

Step 1: Identify the given mathematical properties of matrices A and B. Since both are symmetric matrices, we can write:

$$A^T = A \quad \text{and} \quad B^T = B$$

Step 2: Let us define a new matrix X representing the expression to be analyzed: $X = AB - BA$.

Step 3: To determine the nature of matrix X, take the transpose of both sides of the expression:

$$X^T = (AB - BA)^T$$

Step 4: Apply the subtraction property of transposes to separate the terms:

$$X^T = (AB)^T - (BA)^T$$

Step 5: Apply the reversal rule for the transpose of a product of matrices, which states $(AB)^T = B^T A^T$:

$$X^T = B^T A^T - A^T B^T$$

Step 6: Substitute the symmetric properties $A^T = A$ and $B^T = B$ established in Step 1 into the expression:

$$X^T = BA - AB$$

Step 7: Factor out a negative sign from the right-hand side to compare it with the original matrix X:

$$X^T = -(AB - BA) = -X$$

Since $X^T = -X$, by definition, the matrix $AB - BA$ is a skew-symmetric matrix.

Final Answer: $\boxed{\text{Skew-symmetric matrix}}$

$\boxed{\textbf{Answer: (B)}}$

Go Back to Question 28

Q29.

Solution

Concept:

The slope of the tangent line to a curve $y = f(x)$ at a given point is equal to the value of the first derivative $\frac{dy}{dx}$ evaluated at that specific point. Since the normal line is perpendicular to the tangent line, the slope of the normal is given by the negative reciprocal of the tangent slope, $m_{\text{normal}} = -\frac{1}{m_{\text{tangent}}}$.

Solution:

Step 1: Write down the equation of the curve: $y = 2x^2 + 3\sin x$.

Step 2: Differentiate y with respect to x to find the general formula for the slope of the tangent:

$$\frac{dy}{dx} = \frac{d}{dx}(2x^2) + \frac{d}{dx}(3\sin x) = 4x + 3\cos x$$

Step 3: Evaluate this derivative at the given point $x = 0$ to calculate the slope of the tangent line (m_t):

$$m_t = \left.\frac{dy}{dx}\right|_{x=0} = 4(0) + 3\cos(0) = 0 + 3(1) = 3$$

Step 4: Use the perpendicular relation to calculate the slope of the normal line (m_n):

$$m_n = -\frac{1}{m_t} = -\frac{1}{3}$$

Therefore, the slope of the normal to the curve at $x = 0$ is equal to $-\frac{1}{3}$.

Final Answer: $\boxed{-\frac{1}{3}}$

Answer: (D)

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Q30.

Solution

Concept:
Determinants can be simplified efficiently using elementary row or column operations before expansion. If any two rows or columns of a determinant become identical or proportional as a result of these operations, the total value of the determinant is automatically equal to zero.

Solution:
Step 1: Write down the given determinant expression:

$$\Delta = \begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$$

Step 2: Apply the column operation $C_3 \to C_3 + C_2$. This replaces the third column with the sum of the elements in the second and third columns:

$$\Delta = \begin{vmatrix} 1 & a & a+b+c \\ 1 & b & a+b+c \\ 1 & c & a+b+c \end{vmatrix}$$

Step 3: Take out the common scalar factor $(a + b + c)$ from the third column (C_3) using the linearity property of determinants:

$$\Delta = (a+b+c)\begin{vmatrix} 1 & a & 1 \\ 1 & b & 1 \\ 1 & c & 1 \end{vmatrix}$$

Step 4: Observe the columns of the remaining determinant. Column 1 (C_1) and Column 3 (C_3) contain identical entries, as both are composed entirely of ones.
Step 5: According to standard determinant properties, since $C_1 = C_3$, the value of the determinant is zero:

$$\Delta = (a+b+c) \cdot 0 = 0$$

Thus, the final simplified value of the determinant is 0.

Final Answer: $\boxed{0}$

Answer: (A)

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Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	C	3	B	4	C	5	A
6	B	7	B	8	B	9	A	10	C
11	B	12	C	13	C	14	A	15	B
16	C	17	A	18	B	19	B	20	A
21	C	22	C	23	B	24	B	25	C
26	B	27	A	28	B	29	D	30	A