AME CET Mathematics

Sample Paper – 4

Duration: 20 Minutes **Maximum Marks: 80**

Instructions

- This paper contains **20** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics section of **AME CET** entrance.
- Each correct answer carries **+4 marks**. Each wrong answer carries -1 **mark**. Unattempted questions carry **0 marks**.
- Only **one** option is correct per question. Choose carefully.
- Syllabus level: **Class 11 and 12 NCERT Mathematics** (Sets and Relations to Probability and Statistics).
- Use of mobile phones, calculators, or any electronic gadget is strictly prohibited.

Q1. The value of $\int_1^e \ln x \, dx$ is:

(A) $e - 1$

(B) 1

(C) e

(D) 0

Q2. The value of $\int_0^{\pi/2} \cos^2 x \, dx$ is:

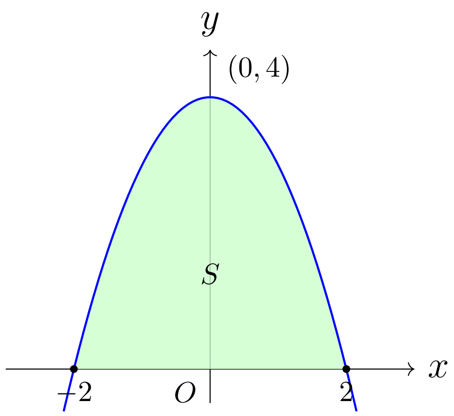
(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{3}$

(C) 1

(D) $\frac{\pi}{4}$

Q3. The area of the region bounded by the parabola $y = 4 - x^2$ and the x-axis, shown shaded below, is:



(A) $\frac{32}{3}$

(B) $\frac{8}{3}$

(C) $\frac{16}{3}$

(D) $\frac{64}{3}$

Q4. The distance between the points $(1,\ 2)$ and $(4,\ 6)$ is:

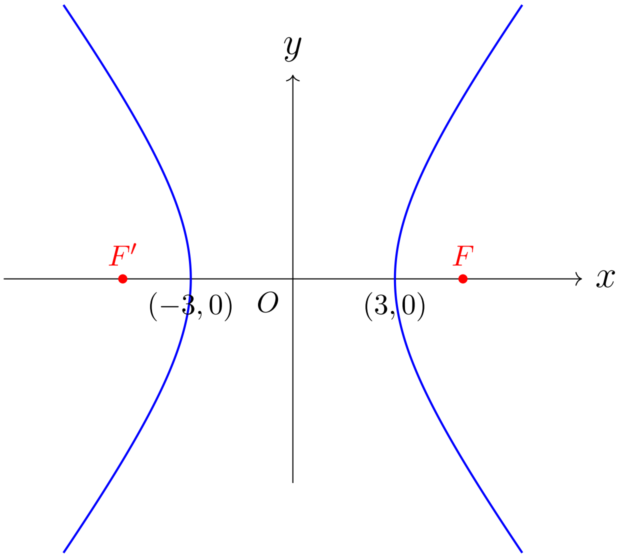
(A) 7

(B) $\sqrt{7}$

(C) 5

(D) $\sqrt{5}$

Q5. The eccentricity of the hyperbola $\frac{x^2}{9} - \frac{y^2}{16} = 1$, sketched below, is:



(A) $\frac{4}{3}$

(B) $\frac{5}{3}$

(C) $\frac{4}{5}$

(D) $\frac{3}{5}$

Q6. The coordinates of the focus of the parabola $x^2 = 12y$ are:

(A) $(0,\ 3)$

(B) $(3,\ 0)$

(C) $(0,\ -3)$

(D) $(-3,\ 0)$

Q7. The determinant of the diagonal matrix $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix}$ is:

(A) 9

(B) 14

(C) 24

(D) 0

Q8. If $A = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$, then A^{-1} equals:

(A) $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$

(B) $\begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}$

(C) $\begin{pmatrix} \frac{1}{3} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$

(D) $\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{3} \end{pmatrix}$

Q9. $\lim_{x \to 2} \dfrac{x^2 - 4}{x - 2}$ equals:

(A) 0

(B) 2

(C) 4

(D) ∞

Q10. If $y = e^{x^2}$, then $\dfrac{dy}{dx}$ is:

(A) e^{x^2}

(B) $2x\, e^{x^2}$

(C) $x^2 e^{x^2 - 1}$

(D) $2x\, e^{2x}$

Q11. If $\sin\theta = \dfrac{3}{5}$ and θ is acute, then $\tan\theta$ equals:

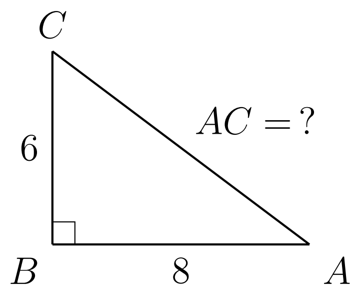
(A) $\dfrac{4}{3}$

(B) $\dfrac{3}{5}$

(C) $\dfrac{5}{4}$

(D) $\frac{3}{4}$

Q12. In the right-angled triangle below, the two legs are 6 and 8. The length of the hypotenuse AC is:



(A) 10

(B) $\sqrt{14}$

(C) 14

(D) 48

Q13. A fair coin is tossed 3 times. The probability of getting **exactly** two heads is:

(A) $\frac{1}{2}$

(B) $\frac{1}{4}$

(C) $\frac{1}{8}$

(D) $\frac{3}{8}$

Q14. A box contains 3 defective and 7 good items. Two items are drawn simultaneously at random. The probability that **both** are good is:

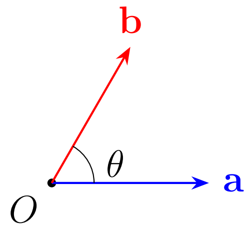
(A) $\frac{7}{10}$

(B) $\frac{49}{100}$

(C) $\frac{7}{15}$

(D) $\frac{1}{15}$

Q15. Two vectors $\mathbf{a}$ and $\mathbf{b}$ satisfy $|\mathbf{a}| = 5$, $|\mathbf{b}| = 4$ and $\mathbf{a} \cdot \mathbf{b} = 10$. The angle θ between them is:



(A) $60°$

(B) $30°$

(C) $45°$

(D) $90°$

Q16. The area of the parallelogram whose adjacent sides have magnitudes $|\mathbf{a}| = 3$ and $|\mathbf{b}| = 6$ with an included angle of $90°$ is:

(A) 9

(B) 18

(C) $\frac{18}{2}$

(D) 0

Q17. The value of $\binom{8}{2}$ is:

(A) 28

(B) 56

(C) 64

(D) 16

Q18. The sum of the first 8 terms of the geometric progression $1, 2, 4, 8, \ldots$ is:

(A) 256

(B) 255

(C) 511

(D) 128

Q19. The distance of the point $(1, 2, 2)$ from the origin is:

(A) 9

(B) 5

(C) $\sqrt{5}$

(D) 3

Q20. The general solution of the differential equation $\frac{dy}{dx} = 2y$ is:

(A) $y = 2x + C$

(B) $y = Cx^2$

(C) $y = Ce^{2x}$

(D) $y = Ce^{x}$

Detailed Solutions

Q1.

Solution

Concept – Integration by Parts:
$\int u\,dv = uv - \int v\,du$. Write $\ln x = 1 \cdot \ln x$ and choose $u = \ln x$, $dv = dx$.

Step 1 – Assign u and dv:

$$u = \ln x \;\Rightarrow\; du = \frac{1}{x}\,dx, \qquad dv = dx \;\Rightarrow\; v = x$$

Step 2 – Apply the formula:

$$\int_1^e \ln x\,dx = \Big[x\ln x\Big]_1^e - \int_1^e x\cdot\frac{1}{x}\,dx$$

Step 3 – Simplify the remaining integral:

$$\int_1^e x\cdot\frac{1}{x}\,dx = \int_1^e 1\,dx = \Big[x\Big]_1^e = e - 1$$

Step 4 – Evaluate the boundary term:

$$\Big[x\ln x\Big]_1^e = e\ln e - 1\cdot\ln 1 = e\cdot 1 - 0 = e$$

Step 5 – Combine:

$$\int_1^e \ln x\,dx = e - (e-1) = 1$$

Why other options are wrong:

- (A) $e-1$: stopping at the value of $\int_1^e 1\,dx$ alone.
- (C) e: keeping only the boundary term $[x\ln x]_1^e$.
- (D) 0: cancelling the two terms with the wrong sign.

Answer: (B) ← Go Back to Q1

Q2.

Solution

Concept – Half-angle identity:

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

Step 1 – Rewrite the integrand:

$$\int_0^{\pi/2} \cos^2 x\,dx = \frac{1}{2}\int_0^{\pi/2} (1+\cos 2x)\,dx$$

Step 2 – Integrate term by term:

$$= \frac{1}{2}\left[x + \frac{\sin 2x}{2}\right]_0^{\pi/2}$$

Step 3 – Substitute the upper limit $x = \frac{\pi}{2}$:

$$\frac{\pi}{2} + \frac{\sin\pi}{2} = \frac{\pi}{2} + 0 = \frac{\pi}{2}$$

Step 4 – Substitute the lower limit $x = 0$:

$$0 + \frac{\sin 0}{2} = 0$$

Step 5 – Combine:

$$= \frac{1}{2}\left(\frac{\pi}{2} - 0\right) = \frac{\pi}{4}$$

Why other options are wrong:

- (A) $\frac{\pi}{2}$: forgetting the outer $\frac{1}{2}$ factor.
- (B) $\frac{\pi}{3}$: an arithmetic slip with the limits.
- (C) 1: integrating $\cos x$ rather than $\cos^2 x$.

Answer: (D) ← Go Back to Q2

Q3.

Solution

Concept – Area between a curve and the x-axis:

$$A = \int_a^b y\,dx \quad \text{between the points where } y = 0$$

Step 1 – Find where the curve meets the x-axis (set $y = 0$):

$$4 - x^2 = 0 \ \Rightarrow\ x^2 = 4 \ \Rightarrow\ x = -2,\ x = 2$$

Step 2 – Set up the integral over $[-2, 2]$:

$$A = \int_{-2}^{2} (4 - x^2)\,dx$$

Step 3 – Use symmetry (the integrand is even):

$$A = 2\int_0^2 (4 - x^2)\,dx$$

Step 4 – Integrate:

$$= 2\left[4x - \frac{x^3}{3}\right]_0^2$$

Step 5 – Evaluate:

$$= 2\left(8 - \frac{8}{3}\right) = 2 \cdot \frac{24 - 8}{3} = 2 \cdot \frac{16}{3} = \frac{32}{3}$$

Why other options are wrong:

- (C) $\frac{16}{3}$: integrating only over $[0, 2]$ (half the region).
- (B) $\frac{8}{3}$: integrating only $\int_0^2 x^2\,dx$.
- (D) $\frac{64}{3}$: doubling the correct area.

Answer: (A) ← Go Back to Q3

Q4.

Solution

Concept – Distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Step 1 – Identify the coordinates:

$$(x_1, y_1) = (1, 2), \qquad (x_2, y_2) = (4, 6)$$

Step 2 – Compute the differences:

$$x_2 - x_1 = 4 - 1 = 3, \qquad y_2 - y_1 = 6 - 2 = 4$$

Step 3 – Substitute:

$$d = \sqrt{3^2 + 4^2} = \sqrt{9 + 16}$$

Step 4 – Simplify:

$$d = \sqrt{25} = 5$$

Why other options are wrong:

- (A) 7: adding the differences $(3 + 4)$ instead of using the formula.
- (B) $\sqrt{7}$: adding the differences before squaring.
- (D) $\sqrt{5}$: forgetting to square the differences.

Answer: (C) ← Go Back to Q4

Q5.

Solution

Concept – Eccentricity of hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$:

$$c^2 = a^2 + b^2, \qquad e = \frac{c}{a}$$

Step 1 – Read off a^2 and b^2:

$$a^2 = 9, \qquad b^2 = 16$$

Step 2 – Compute c^2:

$$c^2 = 9 + 16 = 25$$

Step 3 – Compute c and a:

$$c = 5, \qquad a = 3$$

Step 4 – Compute the eccentricity:

$$e = \frac{c}{a} = \frac{5}{3}$$

Why other options are wrong:

- (A) $\frac{4}{3}$: using b/a instead of c/a.
- (D) $\frac{3}{5}$: inverting c/a.
- (C) $\frac{4}{5}$: using the ellipse formula $c^2 = a^2 - b^2$ with the wrong a.

Answer: (B) ← Go Back to Q5

Q6.

Solution

Concept – Standard parabola $x^2 = 4ay$:
This parabola opens upward with axis along the y-axis; its focus is at $(0, a)$.

Step 1 – Match $x^2 = 12y$ to $x^2 = 4ay$:

$$4a = 12 \;\Rightarrow\; a = 3$$

Step 2 – State the focus:

$$\text{Focus} = (0, a) = (0, 3)$$

Why other options are wrong:

- (B) $(3, 0)$: focus of the rightward parabola $y^2 = 12x$.
- (C) $(0, -3)$: focus of the downward parabola $x^2 = -12y$.
- (D) $(-3, 0)$: focus of a leftward parabola, wrong axis.

Answer: (A) ← Go Back to Q6

Q7.

Solution

Concept – Determinant of a diagonal (or triangular) matrix:
It equals the product of the diagonal entries.

Step 1 – Identify the diagonal entries:

$$d_1 = 2, \quad d_2 = 3, \quad d_3 = 4$$

Step 2 – Multiply them:

$$\det(A) = 2 \times 3 \times 4$$

Step 3 – Compute:

$$= 6 \times 4 = 24$$

Why other options are wrong:

- (A) 9: multiplying only 3×3 or mis-reading entries.
- (D) 0: assuming the off-diagonal zeros force a zero determinant.
- (B) 14: adding the entries $(2 + 3 + 4 + 5)$ instead of multiplying.

Answer: (C) ← Go Back to Q7

Q8.

Solution

Concept – Inverse of a diagonal matrix:
For a diagonal matrix, invert each non-zero diagonal entry; off-diagonal entries stay 0.

Step 1 – Compute $\det(A)$**:**

$$\det(A) = 2 \times 3 - 0 \times 0 = 6 \neq 0$$

Step 2 – Invert each diagonal entry:

$$\frac{1}{2} \text{ and } \frac{1}{3}$$

Step 3 – Write the inverse:

$$A^{-1} = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{3} \end{pmatrix}$$

Step 4 – Verify $A\,A^{-1} = I$:

$$\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{3} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \checkmark$$

Why other options are wrong:

- (A): this is A itself, not its inverse.
- (B): swapping the diagonal entries does not invert them.
- (C): the reciprocals are placed in the wrong positions.

Answer: (D) ← Go Back to Q8

Q9.

Solution

Concept – Factor and cancel the $\frac{0}{0}$ form:
Direct substitution gives $\frac{0}{0}$, so factor the numerator.

Step 1 – Factor the numerator using $a^2 - b^2 = (a-b)(a+b)$:

$$x^2 - 4 = (x-2)(x+2)$$

Step 2 – Cancel the common factor $(x-2)$:

$$\frac{x^2-4}{x-2} = \frac{(x-2)(x+2)}{x-2} = x+2 \quad (x \neq 2)$$

Step 3 – Substitute $x = 2$ into the simplified form:

$$\lim_{x \to 2}(x+2) = 2+2 = 4$$

Why other options are wrong:

- (A) 0: assuming the numerator's zero dominates.
- (B) 2: substituting only the cancelled factor.
- (D) ∞: forgetting that the $(x-2)$ factors cancel.

Answer: (C) ← Go Back to Q9

Q10.

Solution

Concept – Chain rule:

$$\frac{d}{dx}e^{f(x)} = e^{f(x)} \cdot f'(x)$$

Step 1 – Identify the inner function:

$$f(x) = x^2$$

Step 2 – Differentiate the inner function:

$$f'(x) = 2x$$

Step 3 – Apply the chain rule:

$$\frac{dy}{dx} = e^{x^2} \cdot 2x = 2x\, e^{x^2}$$

Why other options are wrong:

- (A) e^{x^2}: treating e^{x^2} like e^x and skipping the chain factor.
- (C) $x^2 e^{x^2-1}$: applying the power rule, which does not apply to exponentials.
- (D) $2x\, e^{2x}$: differentiating the exponent inside the wrong base.

Answer: (B) ← Go Back to Q10

Q11.

Solution

Concept – Right-triangle ratios:

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$, $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$, $\tan\theta = \frac{\sin\theta}{\cos\theta}$.

Step 1 – Use $\sin\theta = \frac{3}{5}$ (opposite $= 3$, hypotenuse $= 5$).

Step 2 – Find the adjacent side by Pythagoras:

$$\text{adjacent} = \sqrt{5^2 - 3^2} = \sqrt{25-9} = \sqrt{16} = 4$$

Step 3 – Compute $\cos\theta$:

$$\cos\theta = \frac{4}{5} \quad (\theta \text{ acute, so positive})$$

Step 4 – Compute $\tan\theta$:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{3/5}{4/5} = \frac{3}{4}$$

Why other options are wrong:

- (A) $\frac{4}{3}$: this is $\cot\theta$, the reciprocal of $\tan\theta$.
- (B) $\frac{3}{5}$: this is $\sin\theta$, not $\tan\theta$.
- (C) $\frac{5}{4}$: this is $\sec\theta$.

Answer: (D) ← Go Back to Q11

Q12.

Solution

Concept – Pythagoras' theorem:
In a right triangle, $\text{hypotenuse}^2 = (\text{leg}_1)^2 + (\text{leg}_2)^2$.

Step 1 – Identify the two legs:

$$\text{leg}_1 = 6, \qquad \text{leg}_2 = 8$$

Step 2 – Square and add:

$$AC^2 = 6^2 + 8^2 = 36 + 64 = 100$$

Step 3 – Take the positive square root:

$$AC = \sqrt{100} = 10$$

Why other options are wrong:

- (C) 14: adding the legs $(6 + 8)$ instead of using Pythagoras.
- (B) $\sqrt{14}$: adding the legs before squaring.
- (D) 48: multiplying the legs (6×8).

Answer: (A) ← Go Back to Q12

Q13.

Solution

Concept – Binomial probability:

$$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$$

Here $n = 3$, $p = \frac{1}{2}$, $k = 2$.

Step 1 – Count the favourable arrangements $\binom{3}{2}$:

$$\binom{3}{2} = 3 \quad \text{(HHT, HTH, THH)}$$

Step 2 – Each outcome has probability:

$$\left(\frac{1}{2}\right)^3 = \frac{1}{8}$$

Step 3 – Multiply:

$$P(\text{exactly 2 heads}) = 3 \times \frac{1}{8} = \frac{3}{8}$$

Why other options are wrong:

- (A) $\frac{1}{2}$: a guess ignoring the count of arrangements.
- (B) $\frac{1}{4}$: using $n = 2$ tosses instead of 3.
- (C) $\frac{1}{8}$: counting only one arrangement (HHT) and dropping the factor 3.

Answer: (D) ← Go Back to Q13

Q14.

Solution

Concept – Classical probability using combinations:

$$P = \frac{\text{favourable outcomes}}{\text{total outcomes}}$$

Step 1 – Total ways to choose 2 from 10 items:

$$\binom{10}{2} = \frac{10 \times 9}{2} = 45$$

Step 2 – Ways to choose 2 good items from the 7 good ones:

$$\binom{7}{2} = \frac{7 \times 6}{2} = 21$$

Step 3 – Form the probability:

$$P = \frac{21}{45}$$

Step 4 – Simplify (divide by 3):

$$P = \frac{7}{15}$$

Why other options are wrong:

- (A) $\frac{7}{10}$: the probability that the *first* single draw is good, not both.
- (B) $\frac{49}{100}$: using $\left(\frac{7}{10}\right)^2$, i.e. drawing *with* replacement.
- (D) $\frac{1}{15}$: using $\binom{3}{2}/\binom{10}{2}$, the probability that both are defective.

Answer: (C) ← Go Back to Q14

Q15.

Solution

Concept – Dot product and angle:

$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}|\,|\mathbf{b}| \cos\theta \;\Rightarrow\; \cos\theta = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|\,|\mathbf{b}|}$$

Step 1 – Substitute the given values:

$$\cos\theta = \frac{10}{5 \times 4}$$

Step 2 – Simplify:

$$\cos\theta = \frac{10}{20} = \frac{1}{2}$$

Step 3 – Find θ:

$$\theta = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$$

Why other options are wrong:

- (B) 30°: using $\cos 30^\circ = \frac{\sqrt{3}}{2}$, which would need $\mathbf{a} \cdot \mathbf{b} = 10\sqrt{3}$.
- (C) 45°: corresponds to $\cos\theta = \frac{1}{\sqrt{2}}$, not $\frac{1}{2}$.
- (D) 90°: would require $\mathbf{a} \cdot \mathbf{b} = 0$.

Answer: (A) ← Go Back to Q15

Q16.

Solution

Concept – Area of a parallelogram from adjacent sides:

$$\text{Area} = |\mathbf{a} \times \mathbf{b}| = |\mathbf{a}|\,|\mathbf{b}|\sin\theta$$

Step 1 – State the given values:

$$|\mathbf{a}| = 3, \quad |\mathbf{b}| = 6, \quad \theta = 90^\circ$$

Step 2 – Evaluate $\sin 90^\circ$:

$$\sin 90^\circ = 1$$

Step 3 – Substitute:

$$\text{Area} = 3 \times 6 \times 1 = 18$$

Why other options are wrong:

- (A) 9: taking $\frac{1}{2}|\mathbf{a}||\mathbf{b}|$, the triangle area formula.
- (C) $\frac{18}{2} = 9$: again using the triangle (half) area.
- (D) 0: using $\sin 0^\circ$ instead of $\sin 90^\circ$.

Answer: (B) ← Go Back to Q16

Q17.

Solution

Concept – Combination (order does not matter):

$$\binom{n}{r} = \frac{n!}{r!\,(n-r)!}$$

Step 1 – Identify $n = 8$, $r = 2$:

Step 2 – Expand (cancel $6!$):

$$\binom{8}{2} = \frac{8 \times 7}{2 \times 1}$$

Step 3 – Compute:

$$= \frac{56}{2} = 28$$

Why other options are wrong:

- (D) 16: computing 8×2.
- (B) 56: using $P(8, 2) = 8 \times 7$ (permutations, not combinations).
- (C) 64: computing 8^2.

Answer: (A) ← Go Back to Q17

Q18.

Solution

Concept – Sum of n terms of a GP ($r \neq 1$):

$$S_n = a \cdot \frac{r^n - 1}{r - 1}$$

Step 1 – Identify $a = 1$, $r = 2$, $n = 8$.

Step 2 – Compute $r^n = 2^8$**:**

$$2^8 = 256$$

Step 3 – Substitute:

$$S_8 = 1 \cdot \frac{256 - 1}{2 - 1} = \frac{255}{1}$$

Step 4 – Final value:

$$S_8 = 255$$

Why other options are wrong:

- (A) 256: using 2^8 itself, forgetting the -1.
- (C) 511: using $2^9 - 1$, i.e. 9 terms.
- (D) 128: using only the 8th term 2^7.

Answer: (B) ← Go Back to Q18

Q19.

Solution

Concept – Distance from the origin in 3D:

$$d = \sqrt{x^2 + y^2 + z^2}$$

Step 1 – Identify the coordinates:

$$(x, y, z) = (1, 2, 2)$$

Step 2 – Square each coordinate:

$$1^2 = 1, \quad 2^2 = 4, \quad 2^2 = 4$$

Step 3 – Add them:

$$1 + 4 + 4 = 9$$

Step 4 – Take the square root:

$$d = \sqrt{9} = 3$$

Why other options are wrong:

- (A) 9: stopping at the sum of squares without taking the root.
- (B) 5: adding the coordinates $(1 + 2 + 2)$.
- (C) $\sqrt{5}$: dropping one of the coordinates from the sum.

Answer: (D) ← Go Back to Q19

Q20.

Solution

Concept – Separation of variables:
Collect all y terms on one side and x terms on the other, then integrate.

Step 1 – Separate variables:

$$\frac{dy}{y} = 2\,dx$$

Step 2 – Integrate both sides:

$$\int \frac{dy}{y} = \int 2\,dx$$

$$\ln|y| = 2x + C_1$$

Step 3 – Exponentiate:

$$|y| = e^{2x+C_1} = e^{C_1} \cdot e^{2x}$$

Step 4 – Let $C = \pm e^{C_1}$ (arbitrary constant):

$$y = Ce^{2x}$$

Why other options are wrong:

- (A) $y = 2x + C$: solution of $\frac{dy}{dx} = 2$, not $2y$.

- (B) $y = Cx^2$: solution of a different (homogeneous-type) equation.
- (D) $y = Ce^x$: solution of $\frac{dy}{dx} = y$, missing the factor 2 in the exponent.

Answer: (C) ← Go Back to Q20

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	D	3	A	4	C	5	B
6	A	7	C	8	D	9	C	10	B
11	D	12	A	13	D	14	C	15	A
16	B	17	A	18	B	19	D	20	C