AME CET Mathematics

Sample Paper – 5

Duration: 20 Minutes **Maximum Marks: 80**

Instructions

- This paper contains **20** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics section of **AME CET** entrance.
- Each correct answer carries **+4 marks**. Each wrong answer carries -1 **mark**. Unattempted questions carry **0 marks**.
- Only **one** option is correct per question. Choose carefully.
- Syllabus level: **Class 11 and 12 NCERT Mathematics** (Sets and Relations to Probability and Statistics).
- Use of mobile phones, calculators, or any electronic gadget is strictly prohibited.

Q1. The value of $\int_0^{\pi/2} \sin x \, \cos x \, dx$ is:

(A) 1

(B) $\frac{1}{4}$

(C) $\frac{1}{2}$

(D) 0

Q2. The value of $\int_0^{1} e^{2x} \, dx$ is:

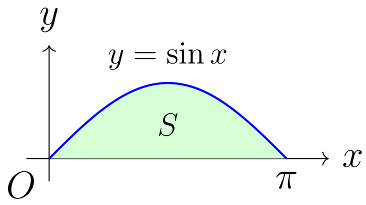
(A) $\frac{e^2 - 1}{2}$

(B) $e^2 - 1$

(C) $2(e^2 - 1)$

(D) e^2

Q3. The area of the region bounded by $y = \sin x$ and the x-axis from $x = 0$ to $x = \pi$, shaded below, is:



(A) 1

(B) 2

(C) π

(D) 0

Q4. The slope of the line $3x + 4y = 12$ is:

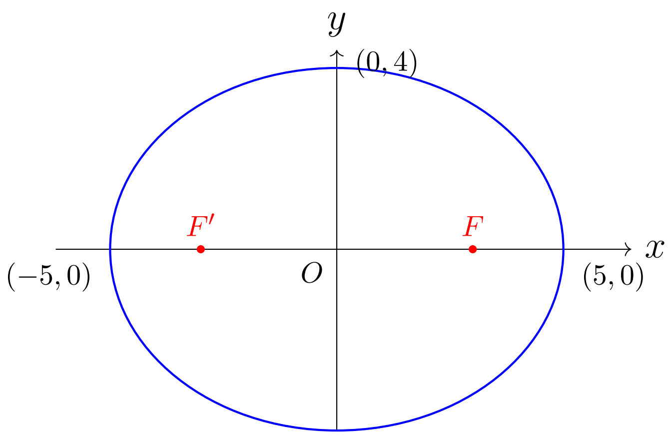
(A) $\frac{3}{4}$

(B) $\frac{4}{3}$

(C) $-\frac{4}{3}$

(D) $-\frac{3}{4}$

Q5. The coordinates of the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$, sketched below, are:



(A) $(\pm 4,\ 0)$

(B) $(0, \pm 3)$

(C) $(\pm 3, 0)$

(D) $(\pm 5, 0)$

Q6. The length of the latus rectum of the parabola $y^2 = 16x$ is:

(A) 4

(B) 16

(C) 32

(D) 8

Q7. The value of the determinant $\begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{vmatrix}$ is:

(A) 6

(B) 0

(C) 24

(D) 1

Q8. If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, then $2A$ equals:

(A) $\begin{pmatrix} 2 & 4 \\ 6 & 8 \end{pmatrix}$

(B) $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

(C) $\begin{pmatrix} 3 & 4 \\ 5 & 6 \end{pmatrix}$

(D) $\begin{pmatrix} 2 & 2 \\ 3 & 8 \end{pmatrix}$

Q9. $\lim_{x \to 0} \frac{e^{2x} - 1}{x}$ equals:

(A) 1

(B) $\frac{1}{2}$

(C) 0

(D) 2

Q10. If $y = \frac{x}{x+1}$, then $\frac{dy}{dx}$ is:

(A) $\frac{1}{x+1}$

(B) $-\frac{1}{(x+1)^2}$

(C) $\frac{1}{(x+1)^2}$

(D) $\frac{x}{(x+1)^2}$

Q11. The value of $\tan 15°$ is:

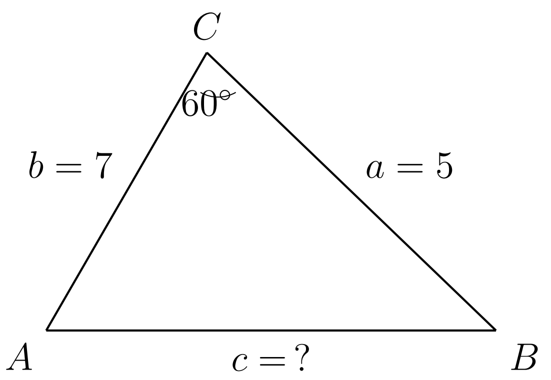
(A) $2-\sqrt{3}$

(B) $2+\sqrt{3}$

(C) $\sqrt{3}-1$

(D) $\frac{1}{\sqrt{3}}$

Q12. In triangle ABC, $a = 5$, $b = 7$ and the included angle $C = 60°$. The length of side c is:



(A) $\sqrt{109}$

(B) $\sqrt{39}$

(C) $\sqrt{74}$

(D) $\sqrt{29}$

Q13. A bag contains 4 white and 6 black balls. One ball is drawn at random. The probability that it is white is:

(A) $\frac{2}{5}$

(B) $\frac{3}{5}$

(C) $\frac{4}{6}$

(D) $\frac{1}{2}$

Q14. An urn contains 6 red and 4 green balls. Two balls are drawn simultaneously at random. The probability of getting **one red and one green** ball is:

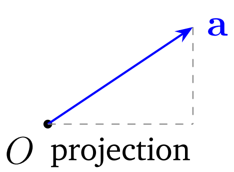
(A) $\frac{1}{3}$

(B) $\frac{2}{15}$

(C) $\frac{12}{25}$

(D) $\frac{8}{15}$

Q15. The magnitude of the vector $\mathbf{a} = 2\hat{i} + 2\hat{j} + \hat{k}$ is:



(A) $\sqrt{5}$

(B) 3

(C) 5

(D) $\sqrt{7}$

Q16. If $|\mathbf{a}| = 4$, $|\mathbf{b}| = 3$ and $\mathbf{a}$ is perpendicular to $\mathbf{b}$, then $|\mathbf{a} \times \mathbf{b}|$ equals:

(A) 0

(B) 6

(C) 12

(D) 7

Q17. The number of ways to arrange 4 distinct books on a shelf is:

(A) 16

(B) 24

(C) 12

(D) 256

Q18. The sum of the first 20 terms of the arithmetic progression $2, 5, 8, \dots$ is:

(A) 590

(B) 305

(C) 610

(D) 1220

Q19. A line makes angles 60° and 60° with the positive x- and y-axes respectively. The angle it makes with the positive z-axis is:

(A) 45°

(B) 30°

(C) 60°

(D) 90°

Q20. The general solution of the differential equation $\dfrac{dy}{dx} = -\dfrac{y}{x}$ is:

(A) $y = Cx$

(B) $y = Ce^{-x}$

(C) $y = -x + C$

(D) $y = \dfrac{C}{x}$

Detailed Solutions

Q1.

Solution

Concept – Double-angle identity:

$$\sin x \cos x = \frac{1}{2}\sin 2x$$

Step 1 – Rewrite the integrand:

$$\int_0^{\pi/2} \sin x \cos x \, dx = \frac{1}{2}\int_0^{\pi/2} \sin 2x \, dx$$

Step 2 – Integrate $\sin 2x$**:**

$$\int \sin 2x \, dx = -\frac{\cos 2x}{2}$$

Step 3 – Apply the limits:

$$= \frac{1}{2}\left[-\frac{\cos 2x}{2}\right]_0^{\pi/2} = -\frac{1}{4}\Big[\cos 2x\Big]_0^{\pi/2}$$

Step 4 – Substitute the limits:

$$= -\frac{1}{4}\big(\cos\pi - \cos 0\big) = -\frac{1}{4}\big(-1-1\big)$$

Step 5 – Simplify:

$$= -\frac{1}{4}\times(-2) = \frac{1}{2}$$

Why other options are wrong:

- (A) 1: forgetting the $\frac{1}{2}$ factor from the identity.
- (B) $\frac{1}{4}$: an extra halving of the correct value.
- (D) 0: wrongly assuming the integrand is odd over the interval.

Answer: (C) ← Go Back to Q1

Q2.

Solution

Concept – Integral of e^{ax}:

$$\int e^{ax}\,dx = \frac{e^{ax}}{a}$$

Step 1 – Integrate with $a = 2$:

$$\int_0^1 e^{2x}\,dx = \left[\frac{e^{2x}}{2}\right]_0^1$$

Step 2 – Substitute the upper limit $x = 1$:

$$\frac{e^2}{2}$$

Step 3 – Substitute the lower limit $x = 0$:

$$\frac{e^0}{2} = \frac{1}{2}$$

Step 4 – Subtract:

$$\frac{e^2}{2} - \frac{1}{2} = \frac{e^2 - 1}{2}$$

Why other options are wrong:

- (B) $e^2 - 1$: forgetting to divide by $a = 2$.
- (C) $2(e^2 - 1)$: multiplying by a instead of dividing.
- (D) e^2: ignoring the lower-limit contribution.

Answer: (A) ← Go Back to Q2

Q3.

Solution

Concept – Area under a curve above the x-axis:

$$A = \int_a^b y\,dx, \quad \text{where } y \geq 0 \text{ on } [a, b]$$

Step 1 – Note $\sin x \geq 0$ on $[0, \pi]$, so the area is:

$$A = \int_0^{\pi} \sin x\,dx$$

Step 2 – Find the antiderivative:

$$\int \sin x\, dx = -\cos x$$

Step 3 – Apply the limits:

$$A = \Big[-\cos x\Big]_0^{\pi}$$

Step 4 – Substitute the limits:

$$= -\cos\pi - (-\cos 0) = -(-1) - (-1)$$

Step 5 – Simplify:

$$= 1 + 1 = 2$$

Why other options are wrong:

- (A) 1: integrating over only half the interval, $[0, \frac{\pi}{2}]$.
- (C) π: confusing the area with the width of the interval.
- (D) 0: this is $\int_0^{2\pi} \sin x\, dx$, where the symmetric halves cancel.

Answer: (B) ← Go Back to Q3

Q4.

Solution

Concept – Slope of $Ax + By = C$:
Rewrite in the form $y = mx + c$; the coefficient m is the slope.

Step 1 – Start from the given line:

$$3x + 4y = 12$$

Step 2 – Isolate the y-term:

$$4y = -3x + 12$$

Step 3 – Divide by 4:

$$y = -\frac{3}{4}x + 3$$

Step 4 – Read off the slope:

$$m = -\frac{3}{4}$$

Why other options are wrong:

- (A) $\frac{3}{4}$: dropping the negative sign.

- (B) $\frac{4}{3}$: inverting the ratio (using B/A).
- (C) $-\frac{4}{3}$: inverting the ratio with the sign.

Answer: (D) ← Go Back to Q4

Q5.

Solution

Concept – Foci of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with $a > b$:

$$c^2 = a^2 - b^2, \qquad \text{foci at } (\pm c,\ 0)$$

Step 1 – Read off a^2 and b^2:

$$a^2 = 25, \quad b^2 = 16$$

Step 2 – Compute c^2:

$$c^2 = 25 - 16 = 9$$

Step 3 – Compute c:

$$c = 3$$

Step 4 – State the foci (major axis along x):

$$(\pm 3,\ 0)$$

Why other options are wrong:

- (A) $(\pm 4, 0)$: using $c = \sqrt{a^2 + b^2}$ or a wrong subtraction.
- (B) $(0, \pm 3)$: placing foci on the minor axis.
- (D) $(\pm 5, 0)$: confusing the foci with the vertices $(\pm a, 0)$.

Answer: (C) ← Go Back to Q5

Q6.

Solution

Concept – Latus rectum of $y^2 = 4ax$:
The length of the latus rectum equals $4a$.

Step 1 – Match $y^2 = 16x$ to $y^2 = 4ax$:

$$4a = 16 \;\Rightarrow\; a = 4$$

Step 2 – State the length of the latus rectum:

$$\text{Length} = 4a = 16$$

Why other options are wrong:

- (A) 4: reporting the value of a instead of $4a$.
- (D) 8: reporting $2a$ (the semi-latus rectum doubled incorrectly).
- (C) 32: doubling the correct $4a$.

Answer: (B) ← Go Back to Q6

Q7.

Solution

Concept – Determinant of a triangular matrix:
For an upper- (or lower-) triangular matrix, the determinant is the product of the diagonal entries.

Step 1 – Confirm the matrix is upper-triangular:

$$\begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{vmatrix}$$

All entries below the main diagonal are zero.

Step 2 – Multiply the diagonal entries:

$$\det = 1 \times 1 \times 1 = 1$$

Step 3 – Verify by expansion along the first column:

$$1 \cdot \begin{vmatrix} 1 & 4 \\ 0 & 1 \end{vmatrix} - 0 + 0 = 1\,(1 \times 1 - 4 \times 0) = 1$$

Why other options are wrong:

- (A) 6: multiplying $1 \times 2 \times 3$ (using the top row, not the diagonal).
- (B) 0: wrongly assuming a triangular matrix is singular.
- (C) 24: product of all off-diagonal-ish entries, an invalid rule.

Answer: (D) ← Go Back to Q7

Q8.

Solution

Concept – Scalar multiple of a matrix:
Multiplying a matrix by a scalar multiplies *every* entry by that scalar.

Step 1 – Write the matrix A:

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

Step 2 – Multiply each entry by 2:

$$2 \times 1 = 2, \quad 2 \times 2 = 4, \quad 2 \times 3 = 6, \quad 2 \times 4 = 8$$

Step 3 – Assemble the result:

$$2A = \begin{pmatrix} 2 & 4 \\ 6 & 8 \end{pmatrix}$$

Why other options are wrong:

- (B): the matrix A unchanged (forgot to multiply).
- (C): adding 2 to each entry instead of multiplying.
- (D): only some entries scaled, others left or wrongly handled.

Answer: (A) ← Go Back to Q8

Q9.

Solution

Concept – Standard exponential limit:

$$\lim_{u \to 0} \frac{e^u - 1}{u} = 1$$

Step 1 – Introduce $2x$ in numerator and denominator:

$$\lim_{x\to 0}\frac{e^{2x}-1}{x}=\lim_{x\to 0}\frac{e^{2x}-1}{2x}\times 2$$

Step 2 – Let $u=2x$ (so $u\to 0$ as $x\to 0$):

$$=2\lim_{u\to 0}\frac{e^{u}-1}{u}$$

Step 3 – Apply the standard limit:

$$=2\times 1=2$$

Why other options are wrong:

- (A) 1: applying the standard limit without the factor 2.
- (B) $\frac{1}{2}$: dividing by 2 instead of multiplying.
- (C) 0: wrongly evaluating $\frac{0}{0}$ directly as 0.

Answer: (D) ← Go Back to Q9

Q10.

Solution

Concept – Quotient rule:

$$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{u'v-uv'}{v^2}$$

Step 1 – Identify u and v:

$$u=x\ \Rightarrow\ u'=1,\qquad v=x+1\ \Rightarrow\ v'=1$$

Step 2 – Substitute into the rule:

$$\frac{dy}{dx}=\frac{(1)(x+1)-(x)(1)}{(x+1)^2}$$

Step 3 – Simplify the numerator:

$$(x+1)-x=1$$

Step 4 – Write the final derivative:

$$\frac{dy}{dx}=\frac{1}{(x+1)^2}$$

Why other options are wrong:

- (A) $\frac{1}{x+1}$: forgetting to square the denominator.
- (B) $-\frac{1}{(x+1)^2}$: a sign slip in the numerator.
- (D) $\frac{x}{(x+1)^2}$: using uv' in the numerator only.

Answer: (C) ← Go Back to Q10

Q11.

Solution

Concept – Tangent of a difference:

$$\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B}$$

Step 1 – Write $15^\circ = 45^\circ - 30^\circ$**:**

$$\tan 15^\circ = \frac{\tan 45^\circ - \tan 30^\circ}{1+\tan 45^\circ \tan 30^\circ}$$

Step 2 – Substitute $\tan 45^\circ = 1$, $\tan 30^\circ = \frac{1}{\sqrt{3}}$**:**

$$= \frac{1-\dfrac{1}{\sqrt{3}}}{1+\dfrac{1}{\sqrt{3}}}$$

Step 3 – Multiply numerator and denominator by $\sqrt{3}$**:**

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

Step 4 – Rationalise by $(\sqrt{3}-1)$**:**

$$= \frac{(\sqrt{3}-1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)} = \frac{3-2\sqrt{3}+1}{3-1}$$

Step 5 – Simplify:

$$= \frac{4-2\sqrt{3}}{2} = 2-\sqrt{3}$$

Why other options are wrong:

- (B) $2+\sqrt{3}$: this is $\tan 75^\circ$, the complement.
- (C) $\sqrt{3}-1$: the un-rationalised numerator only.

- (D) $\frac{1}{\sqrt{3}}$: this is $\tan 30°$.

Answer: (A) ← Go Back to Q11

Q12.

Solution

Concept – Cosine Rule:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Step 1 – Substitute $a = 5$, $b = 7$, $C = 60°$:

$$c^2 = 5^2 + 7^2 - 2(5)(7)\cos 60°$$

Step 2 – Compute each part:

$$c^2 = 25 + 49 - 70 \times \frac{1}{2}$$

Step 3 – Simplify:

$$c^2 = 74 - 35 = 39$$

Step 4 – Take the positive square root:

$$c = \sqrt{39}$$

Why other options are wrong:

- (A) $\sqrt{109}$: using $+2ab\cos C$ (wrong sign in the rule).
- (C) $\sqrt{74}$: dropping the $-2ab\cos C$ term entirely.
- (D) $\sqrt{29}$: using 70 instead of 35 for $2ab\cos 60°$.

Answer: (B) ← Go Back to Q12

Q13.

Solution

Concept – Classical probability:

$$P(E) = \frac{\text{favourable outcomes}}{\text{total outcomes}}$$

Step 1 – Total number of balls:

$$4 + 6 = 10$$

Step 2 – Number of white balls (favourable):

$$4$$

Step 3 – Compute the probability:

$$P(\text{white}) = \frac{4}{10}$$

Step 4 – Reduce the fraction:

$$= \frac{2}{5}$$

Why other options are wrong:

- (B) $\frac{3}{5}$: this is $P(\text{black}) = \frac{6}{10}$.
- (C) $\frac{4}{6}$: dividing white by black instead of by the total.
- (D) $\frac{1}{2}$: assuming equal numbers of each colour.

Answer: (A) ← Go Back to Q13

Q14.

Solution

Concept – Classical probability using combinations:

$$P = \frac{\text{favourable outcomes}}{\text{total outcomes}}$$

Step 1 – Total ways to choose 2 balls from 10:

$$\binom{10}{2} = \frac{10 \times 9}{2} = 45$$

Step 2 – Ways to choose 1 red from 6 AND 1 green from 4:

$$\binom{6}{1} \times \binom{4}{1} = 6 \times 4 = 24$$

Step 3 – Form the probability:

$$P = \frac{24}{45}$$

Step 4 – Reduce the fraction:

$$= \frac{8}{15}$$

Why other options are wrong:

- (A) $\frac{1}{3}$: equals $\binom{6}{2}/\binom{10}{2} = 15/45$, the probability both are red.
- (B) $\frac{2}{15}$: equals $\binom{4}{2}/\binom{10}{2} = 6/45$, the probability both are green.
- (C) $\frac{12}{25}$: using $\frac{6}{10} \times \frac{4}{10} \times 2$ with replacement, not simultaneous draw.

Answer: (D) ← Go Back to Q14

Q15.

Solution

Concept – Magnitude of a vector:

$$|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

Step 1 – Read off the components of a $= 2\hat{i} + 2\hat{j} + \hat{k}$**:**

$$a_1 = 2, \quad a_2 = 2, \quad a_3 = 1$$

Step 2 – Square each component:

$$2^2 = 4, \quad 2^2 = 4, \quad 1^2 = 1$$

Step 3 – Add the squares:

$$4 + 4 + 1 = 9$$

Step 4 – Take the square root:

$$|\mathbf{a}| = \sqrt{9} = 3$$

Why other options are wrong:

- (A) $\sqrt{5}$: using only two of the three components.
- (C) 5: adding the components $(2 + 2 + 1)$ instead of their squares.
- (D) $\sqrt{7}$: a squaring error in one component.

Answer: (B) ← Go Back to Q15

Q16.

Solution

Concept – Magnitude of cross product:

$$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin\theta$$

Step 1 – State the given values:

$$|\mathbf{a}| = 4, \quad |\mathbf{b}| = 3, \quad \theta = 90^\circ$$

Step 2 – Evaluate $\sin 90^\circ$:

$$\sin 90^\circ = 1$$

Step 3 – Substitute:

$$|\mathbf{a} \times \mathbf{b}| = 4 \times 3 \times 1 = 12$$

Why other options are wrong:

- (A) 0: this is the *dot* product of perpendicular vectors, not the cross product.
- (B) 6: using $\sin 90^\circ = \frac{1}{2}$ by mistake.
- (D) 7: adding the magnitudes $(4 + 3)$ instead of multiplying.

Answer: (C) ← Go Back to Q16

Q17.

Solution

Concept – Arrangement of n distinct objects:
The number of arrangements (permutations) of n distinct objects in a row is $n!$.

Step 1 – Identify $n = 4$ distinct books:

Step 2 – Compute $4!$:

$$4! = 4 \times 3 \times 2 \times 1$$

Step 3 – Multiply:

$$= 24$$

Why other options are wrong:

- (A) 16: computing 4^2 instead of $4!$.
- (C) 12: computing 4×3 only $({}^4P_2)$.
- (D) 256: computing 4^4 (arrangements with repetition allowed).

Answer: (B) ← Go Back to Q17

Q18.

Solution

Concept – Sum of n terms of an AP:

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

Step 1 – Identify $a = 2$, $d = 5 - 2 = 3$, $n = 20$.

Step 2 – Substitute into the formula:

$$S_{20} = \frac{20}{2}\left[2(2) + (20-1)(3)\right]$$

Step 3 – Simplify inside the brackets:

$$= 10\left[4 + 19 \times 3\right] = 10\left[4 + 57\right]$$

Step 4 – Compute:

$$= 10 \times 61 = 610$$

Why other options are wrong:

- (A) 590: using $(n-1) = 18$ instead of 19.
- (B) 305: forgetting the factor $\frac{n}{2} = 10$ and using $\frac{n}{2}$ incorrectly as 5.
- (D) 1220: doubling the correct sum.

Answer: (C) ← Go Back to Q18

Q19.

Solution

Concept – Direction cosines identity:

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

where α, β, γ are the angles with the x-, y-, z-axes.

Step 1 – Substitute $\alpha = 60^\circ$, $\beta = 60^\circ$:

$$\cos^2 60^\circ + \cos^2 60^\circ + \cos^2\gamma = 1$$

Step 2 – Use $\cos 60^\circ = \frac{1}{2}$:

$$\frac{1}{4} + \frac{1}{4} + \cos^2\gamma = 1$$

Step 3 – Solve for $\cos^2\gamma$:

$$\cos^2\gamma = 1 - \frac{1}{2} = \frac{1}{2}$$

Step 4 – Take the square root:

$$\cos\gamma = \frac{1}{\sqrt{2}} \;\Rightarrow\; \gamma = 45^\circ$$

Why other options are wrong:

- (B) 30°: would give $\cos^2\gamma = \frac{3}{4}$, breaking the identity.
- (C) 60°: would give a total of $\frac{3}{4} \neq 1$.
- (D) 90°: would give $\cos\gamma = 0$, so the total is only $\frac{1}{2}$.

Answer: (A) ← Go Back to Q19

Q20.

Solution

Concept – Separation of variables:
Collect all y terms on one side and all x terms on the other, then integrate.

Step 1 – Separate the variables:

$$\frac{dy}{y} = -\frac{dx}{x}$$

Step 2 – Integrate both sides:

$$\int \frac{dy}{y} = -\int \frac{dx}{x}$$

$$\ln|y| = -\ln|x| + C_1$$

Step 3 – Combine the logarithms:

$$\ln|y| + \ln|x| = C_1 \;\Rightarrow\; \ln|xy| = C_1$$

Step 4 – Exponentiate:

$$|xy| = e^{C_1} \;\Rightarrow\; xy = C$$

Step 5 – Solve for y:

$$y = \frac{C}{x}$$

Why other options are wrong:

- (A) $y = Cx$: solution of $\frac{dy}{dx} = \frac{y}{x}$ (opposite sign).
- (B) $y = Ce^{-x}$: solution of $\frac{dy}{dx} = -y$, not $-\frac{y}{x}$.
- (C) $y = -x + C$: solution of $\frac{dy}{dx} = -1$.

Answer: (D) ← Go Back to Q20

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	C	2	A	3	B	4	D	5	C
6	B	7	D	8	A	9	D	10	C
11	A	12	B	13	A	14	D	15	B
16	C	17	B	18	C	19	A	20	D