AME CET Mathematics

Sample Paper – 7

Duration: 20 Minutes **Maximum Marks: 80**

Instructions

- This paper contains **20** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics section of **AME CET** entrance.
- Each correct answer carries **+4 marks**. Each wrong answer carries -1 **mark**. Unattempted questions carry **0 marks**.
- Only **one** option is correct per question. Choose carefully.
- Syllabus level: **Class 11 and 12 NCERT Mathematics** (Sets and Relations to Probability and Statistics).
- Use of mobile phones, calculators, or any electronic gadget is strictly prohibited.

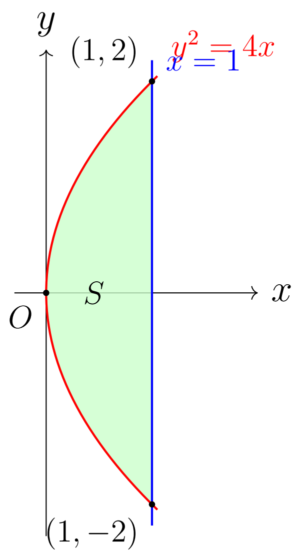
Q1. The value of $\int_0^1 \frac{1}{1+x^2}\,dx$ is:

(A) $\frac{\pi}{2}$
(B) $\frac{\pi}{4}$
(C) 1
(D) $\frac{\pi}{3}$

Q2. The value of $\int_0^{\pi/4} \sec^2 x\,dx$ is:

(A) 1
(B) $\frac{\pi}{4}$
(C) $\sqrt{2}$
(D) 2

Q3. The area of the region enclosed by the parabola $y^2 = 4x$ and the line $x = 1$, shown shaded below, is:



(A) $\frac{4}{3}$

(B) 2

(C) $\frac{8}{3}$

(D) $\frac{16}{3}$

Q4. The equation of the circle with centre at the origin that passes through the point $(3,\ 4)$ is:

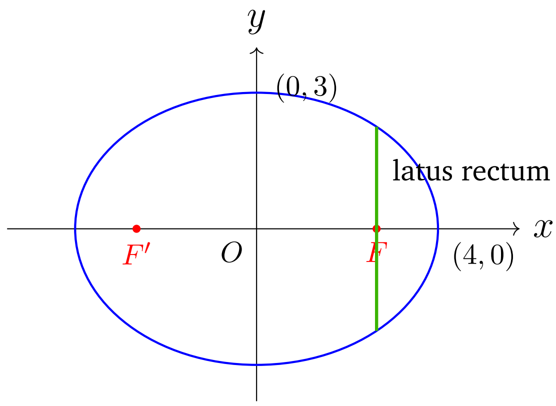
(A) $x^2 + y^2 = 5$

(B) $x^2 + y^2 = 7$

(C) $x^2 + y^2 = 49$

(D) $x^2 + y^2 = 25$

Q5. The length of the latus rectum of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$, sketched below, is:



(A) $\frac{9}{2}$

(B) $\frac{3}{2}$

(C) $\frac{9}{4}$

(D) 9

Q6. The coordinates of the focus of the parabola $y^2 = -8x$ are:

(A) $(2,\ 0)$

(B) $(-2,\ 0)$

(C) $(0,\ -2)$

(D) $(0,\ 2)$

Q7. If $\det\begin{pmatrix} x & 2 \\ 3 & 4 \end{pmatrix} = 10$, then the value of x is:

(A) 1

(B) 2

(C) 4

(D) 8

Q8. If $A = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$, then A^{-1} equals:

(A) $\begin{pmatrix} 3 & -2 \\ -1 & 1 \end{pmatrix}$

(B) $\begin{pmatrix} 3 & 2 \\ 1 & 1 \end{pmatrix}$

(C) $\begin{pmatrix} 1 & -2 \\ -1 & 3 \end{pmatrix}$

(D) $\begin{pmatrix} -3 & 2 \\ 1 & -1 \end{pmatrix}$

Q9. $\lim_{x\to 0} \frac{\sin 5x}{x}$ equals:

(A) 1

(B) $\frac{1}{5}$

(C) 0

(D) 5

Q10. If $y = \ln(\cos x)$, then $\frac{dy}{dx}$ is:

(A) $\tan x$

(B) $-\tan x$

(C) $\cot x$

(D) $-\cot x$

Q11. The value of θ in the interval $0 \le \theta \le \pi$ for which $\cos\theta = \frac{1}{2}$ is:

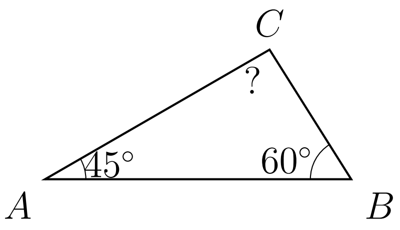
(A) 30°

(B) 45°

(C) 60°

(D) 90°

Q12. In triangle ABC, $\angle A = 45^\circ$ and $\angle B = 60^\circ$. The measure of the third angle $\angle C$ is:



(A) 60°

(B) 90°

(C) 105°

(D) 75°

Q13. A box contains 5 red and 3 blue balls. Two balls are drawn one after another **without** replacement. The probability that both balls are blue is:

(A) $\frac{3}{28}$

(B) $\frac{9}{64}$

(C) $\frac{1}{4}$

(D) $\frac{3}{8}$

Q14. For two events A and B, $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cup B) = 0.7$. The value of $P(A \cap B)$ is:

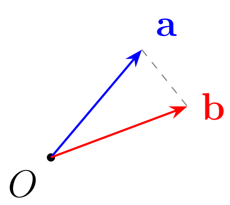
(A) 0.1

(B) 0.2

(C) 0.3

(D) 0.9

Q15. If $\mathbf{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\mathbf{b} = 4\hat{i} + 5\hat{j} + 6\hat{k}$, the value of $\mathbf{a} \cdot \mathbf{b}$ is:



(A) 21

(B) 26

(C) 28

(D) 32

Q16. If $|\mathbf{a}| = 2$, $|\mathbf{b}| = 2$ and the angle between $\mathbf{a}$ and $\mathbf{b}$ is $45°$, then $|\mathbf{a} \times \mathbf{b}|$ equals:

(A) 4

(B) 2

(C) $2\sqrt{2}$

(D) $\sqrt{2}$

Q17. The number of diagonals of a regular octagon (an eight-sided polygon) is:

(A) 20

(B) 28

(C) 16

(D) 40

Q18. The fifth term of the geometric progression $3,\ 6,\ 12,\ \ldots$ is:

(A) 24

(B) 48

(C) 96

(D) 36

Q19. The distance of the point $(3,\ 4,\ 12)$ from the origin is:

(A) 7

(B) 19

(C) $\sqrt{19}$

(D) 13

Q20. The general solution of the differential equation $\frac{dy}{dx} = x^2$ is:

(A) $y = 2x + C$

(B) $y = x^3 + C$

(C) $y = \frac{x^3}{3} + C$

(D) $y = \frac{x^2}{2} + C$

Detailed Solutions

Q1.

Solution

Concept – Standard integral:

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1} x + C$$

Step 1 – Write the antiderivative:

$$\int_0^1 \frac{1}{1+x^2}\,dx = \left[\tan^{-1} x\right]_0^1$$

Step 2 – Substitute the upper limit:

$$\tan^{-1}(1) = \frac{\pi}{4}$$

Step 3 – Substitute the lower limit:

$$\tan^{-1}(0) = 0$$

Step 4 – Subtract:

$$\frac{\pi}{4} - 0 = \frac{\pi}{4}$$

Why other options are wrong:

- (A) $\frac{\pi}{2}$: value of $\tan^{-1} x$ as $x \to \infty$, not at $x = 1$.
- (C) 1: confusing this with $\int_0^1 1\,dx$.
- (D) $\frac{\pi}{3}$: using $\tan^{-1}(\sqrt{3})$ instead of $\tan^{-1}(1)$.

Answer: (B) ← Go Back to Q1

Q2.

Solution

Concept – Standard integral:

$$\int \sec^2 x\,dx = \tan x + C$$

Step 1 – Write the antiderivative:

$$\int_0^{\pi/4} \sec^2 x\,dx = \left[\tan x\right]_0^{\pi/4}$$

Step 2 – Substitute the upper limit:

$$\tan\frac{\pi}{4} = 1$$

Step 3 – Substitute the lower limit:

$$\tan 0 = 0$$

Step 4 – Subtract:

$$1 - 0 = 1$$

Why other options are wrong:

- (B) $\frac{\pi}{4}$: treating the integral of $\sec^2 x$ as the integral of a constant over $[0, \pi/4]$.
- (C) $\sqrt{2}$: using $\sec\frac{\pi}{4} = \sqrt{2}$ as the answer instead of $\tan\frac{\pi}{4}$.
- (D) 2: doubling the correct value.

Answer: (A) ← Go Back to Q2

Q3.

Solution

Concept – Area by horizontal strips / symmetry:
The parabola $y^2 = 4x$ is symmetric about the x-axis, so the area between it and $x = 1$ is twice the area above the x-axis.

Step 1 – Express y on the upper branch:

$$y^2 = 4x \;\Rightarrow\; y = 2\sqrt{x} \quad (y \geq 0)$$

Step 2 – Use symmetry to set up the integral:

$$A = 2\int_0^1 2\sqrt{x}\,dx$$

Step 3 – Integrate $\sqrt{x} = x^{1/2}$:

$$\int x^{1/2}\,dx = \frac{x^{3/2}}{3/2} = \frac{2}{3}x^{3/2}$$

Step 4 – Apply the limits:

$$A = 4\left[\frac{2}{3}x^{3/2}\right]_0^1 = 4\cdot\frac{2}{3}(1-0)$$

Step 5 – Simplify:

$$A = \frac{8}{3}$$

Why other options are wrong:

- (A) $\frac{4}{3}$: computing only the upper-half area and forgetting the factor 2.
- (B) 2: integrating $2\sqrt{x}$ once without doubling for symmetry.
- (D) $\frac{16}{3}$: doubling the correct answer one time too many.

Answer: (C) ← Go Back to Q3

Q4.

Solution

Concept – Circle centred at the origin:

$$x^2 + y^2 = r^2$$

The radius equals the distance from the origin to any point on the circle.

Step 1 – Find r as the distance from $(0, 0)$ to $(3, 4)$:

$$r = \sqrt{3^2 + 4^2}$$

Step 2 – Compute:

$$r = \sqrt{9 + 16} = \sqrt{25} = 5$$

Step 3 – Substitute $r^2 = 25$:

$$x^2 + y^2 = 25$$

Why other options are wrong:

- (A) $x^2 + y^2 = 5$: using $r = 5$ as r^2 by mistake.
- (B) $x^2 + y^2 = 7$: adding $3 + 4$ instead of using the distance formula.
- (C) $x^2 + y^2 = 49$: using $r = 7$ (the sum $3 + 4$) so that $r^2 = 49$.

Answer: (D) ← Go Back to Q4

Q5.

Solution

Concept – Latus rectum of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with $a > b$:

$$\ell = \frac{2b^2}{a}$$

Step 1 – Read off a^2 and b^2:

$$a^2 = 16, \quad b^2 = 9$$

Step 2 – Find a:

$$a = \sqrt{16} = 4$$

Step 3 – Substitute into the formula:

$$\ell = \frac{2(9)}{4} = \frac{18}{4}$$

Step 4 – Simplify:

$$\ell = \frac{9}{2}$$

Why other options are wrong:

- (B) $\frac{3}{2}$: using $\frac{2b}{a}$ instead of $\frac{2b^2}{a}$ (i.e. b not b^2).
- (C) $\frac{9}{4}$: forgetting the factor 2 in the numerator.
- (D) 9: computing $\frac{2b^2}{a}$ with $a = 2$ instead of $a = 4$.

Answer: (A) ← Go Back to Q5

Q6.

Solution

Concept – Standard parabola $y^2 = -4ax$ (opens left):
Focus is at $(-a, 0)$ and the axis is the x-axis.

Step 1 – Match $y^2 = -8x$ to $y^2 = -4ax$:

$$4a = 8 \;\Rightarrow\; a = 2$$

Step 2 – State the focus:

$$\text{Focus} = (-a, 0) = (-2, 0)$$

Why other options are wrong:

- (A) $(2,0)$: focus of the rightward parabola $y^2 = 8x$, the wrong direction.
- (C) $(0,-2)$: confusing this with a vertical parabola $x^2 = -8y$.
- (D) $(0,2)$: incorrect axis and sign.

Answer: (B) ← Go Back to Q6

Q7.

Solution

Concept – 2×2 determinant:

$$\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

Step 1 – Expand the determinant:

$$\det\begin{pmatrix} x & 2 \\ 3 & 4 \end{pmatrix} = 4x - 6$$

Step 2 – Set equal to 10:

$$4x - 6 = 10$$

Step 3 – Add 6 to both sides:

$$4x = 16$$

Step 4 – Divide by 4:

$$x = 4$$

Why other options are wrong:

- (A) 1: solving $4x - 6 = -2$ or a similar arithmetic slip.
- (B) 2: forgetting the -6 and solving $4x = 8$.
- (D) 8: solving $4x - 6 = 10$ as $4x = 8$ then misreading, or using $2x$.

Answer: (C) ← Go Back to Q7

Q8.

Solution

Concept – 2×2 matrix inverse:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

Step 1 – Compute $\det(A)$:

$$\det(A) = 1 \times 3 - 2 \times 1 = 3 - 2 = 1$$

Step 2 – Form the adjugate (swap main diagonal, negate off-diagonal):

$$\text{adj}(A) = \begin{pmatrix} 3 & -2 \\ -1 & 1 \end{pmatrix}$$

Step 3 – Divide by $\det(A) = 1$:

$$A^{-1} = \begin{pmatrix} 3 & -2 \\ -1 & 1 \end{pmatrix}$$

Step 4 – Verify $A \cdot A^{-1} = I$:

$$\begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 3 & -2 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 3-2 & -2+2 \\ 3-3 & -2+3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \checkmark$$

Why other options are wrong:

- (B): no sign change on off-diagonal entries.
- (C): main diagonal not swapped.
- (D): all signs negated incorrectly.

Answer: (A) ← Go Back to Q8

Q9.

Solution

Concept – Standard trigonometric limit:

$$\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1$$

Step 1 – Introduce $5x$ in the denominator:

$$\lim_{x\to 0}\frac{\sin 5x}{x}=\lim_{x\to 0}\frac{\sin 5x}{5x}\cdot 5$$

Step 2 – Apply the standard limit (with $\theta = 5x \to 0$):

$$\lim_{x\to 0}\frac{\sin 5x}{5x}=1$$

Step 3 – Multiply by the factor 5:

$$1\times 5=5$$

Why other options are wrong:

- (A) 1: applying the standard limit but dropping the coefficient 5.
- (B) $\frac{1}{5}$: inverting the coefficient.
- (C) 0: wrongly substituting $x = 0$ directly into the original form.

Answer: (D) ← Go Back to Q9

Q10.

Solution

Concept – Chain rule:

$$\frac{d}{dx}\left[\ln f(x)\right]=\frac{f'(x)}{f(x)}$$

Step 1 – Identify $f(x) = \cos x$:

$$y=\ln(\cos x)$$

Step 2 – Differentiate $f(x)$:

$$\frac{d}{dx}(\cos x)=-\sin x$$

Step 3 – Apply the chain rule:

$$\frac{dy}{dx} = \frac{-\sin x}{\cos x}$$

Step 4 – Simplify:

$$= -\tan x$$

Why other options are wrong:

- (A) $\tan x$: missing the negative sign from differentiating $\cos x$.
- (C) $\cot x$: confusing $\frac{\sin}{\cos}$ with $\frac{\cos}{\sin}$.
- (D) $-\cot x$: inverting the ratio and keeping the sign.

Answer: (B) ← Go Back to Q10

Q11.

Solution

Concept – Reference angle for cosine:
On $[0, \pi]$, cosine is a one-to-one decreasing function, so $\cos\theta = \frac{1}{2}$ has exactly one solution.

Step 1 – Recall the standard value:

$$\cos 60^\circ = \frac{1}{2}$$

Step 2 – Check the interval:

$$60^\circ \in [0^\circ, 180^\circ] \quad \checkmark$$

Step 3 – State the value:

$$\theta = 60^\circ = \frac{\pi}{3}$$

Why other options are wrong:

- (A) 30°: this gives $\cos 30^\circ = \frac{\sqrt{3}}{2}$, not $\frac{1}{2}$.
- (B) 45°: this gives $\cos 45^\circ = \frac{1}{\sqrt{2}}$.
- (D) 90°: this gives $\cos 90^\circ = 0$.

Answer: (C) ← Go Back to Q11

Q12.

Solution

Concept – Angle sum of a triangle:

$$\angle A + \angle B + \angle C = 180^\circ$$

Step 1 – Substitute the known angles:

$$45^\circ + 60^\circ + \angle C = 180^\circ$$

Step 2 – Add the two known angles:

$$105^\circ + \angle C = 180^\circ$$

Step 3 – Solve for $\angle C$:

$$\angle C = 180^\circ - 105^\circ = 75^\circ$$

Why other options are wrong:

- (A) 60°: assuming an equilateral-type split rather than using the angle sum.
- (B) 90°: subtracting only one of the given angles from 180° incorrectly.
- (C) 105°: stating the sum of the two known angles instead of the remaining angle.

Answer: (D) ← Go Back to Q12

Q13.

Solution

Concept – Multiplication rule for dependent events:

$$P(A \cap B) = P(A) \cdot P(B \mid A)$$

Step 1 – Total balls: $5 + 3 = 8$.

Step 2 – Probability first ball is blue:

$$P(\text{1st blue}) = \frac{3}{8}$$

Step 3 – After removing one blue, the box has 5 red and 2 blue (7 total).

Probability second ball is blue:

$$P(\text{2nd blue} \mid \text{1st blue}) = \frac{2}{7}$$

Step 4 – Combined probability:

$$P(\text{both blue}) = \frac{3}{8} \times \frac{2}{7} = \frac{6}{56} = \frac{3}{28}$$

Why other options are wrong:

- (B) $\frac{9}{64}$: treating the draws as *with* replacement: $\left(\frac{3}{8}\right)^2$.
- (C) $\frac{1}{4}$: rough estimate of $\frac{2}{8}$ for the second draw alone.
- (D) $\frac{3}{8}$: only the probability of the first blue ball.

Answer: (A) ← Go Back to Q13

Q14.

Solution

Concept – Addition rule of probability:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Step 1 – Substitute the known values:

$$0.7 = 0.5 + 0.4 - P(A \cap B)$$

Step 2 – Add the individual probabilities:

$$0.7 = 0.9 - P(A \cap B)$$

Step 3 – Rearrange for $P(A \cap B)$:

$$P(A \cap B) = 0.9 - 0.7$$

Step 4 – Compute:

$$P(A \cap B) = 0.2$$

Why other options are wrong:

- (A) 0.1: arithmetic slip in $0.9 - 0.7$.
- (C) 0.3: subtracting 0.4 from 0.7 instead of using the full rule.
- (D) 0.9: stopping at $P(A) + P(B)$ without subtracting $P(A \cup B)$.

Answer: (B) ← Go Back to Q14

Q15.

Solution

Concept – Dot product:

$$\mathbf{a}\cdot\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$$

Step 1 – Write components:

$$\mathbf{a} = (1,\,2,\,3), \quad \mathbf{b} = (4,\,5,\,6)$$

Step 2 – Multiply component pairs:

$$1\times 4 = 4$$

$$2\times 5 = 10$$

$$3\times 6 = 18$$

Step 3 – Sum:

$$\mathbf{a}\cdot\mathbf{b} = 4 + 10 + 18 = 32$$

Why other options are wrong:

- (A) 21: omitting one product, e.g. $4 + 10 +$ (missing 18) plus a slip.
- (B) 26: dropping a term, e.g. $4+?\,+18$ with a wrong middle product.
- (C) 28: arithmetic slip, e.g. taking $2\times 5 = 8$.

Answer: (D) ← Go Back to Q15

Q16.

Solution

Concept – Magnitude of cross product:

$$|\mathbf{a}\times\mathbf{b}| = |\mathbf{a}|\,|\mathbf{b}|\sin\theta$$

Step 1 – State given values:

$$|\mathbf{a}| = 2, \quad |\mathbf{b}| = 2, \quad \theta = 45^\circ$$

Step 2 – Recall $\sin 45°$:

$$\sin 45° = \frac{1}{\sqrt{2}}$$

Step 3 – Substitute:

$$|\mathbf{a} \times \mathbf{b}| = 2 \times 2 \times \frac{1}{\sqrt{2}} = \frac{4}{\sqrt{2}}$$

Step 4 – Rationalise:

$$\frac{4}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$$

Why other options are wrong:

- (A) 4: using $\sin 45° = 1$, i.e. treating the vectors as perpendicular.
- (B) 2: using $\sin 45° = \frac{1}{2}$ by mistake.
- (D) $\sqrt{2}$: dividing the correct answer by 2.

Answer: (C) ← Go Back to Q16

Q17.

Solution

Concept – Number of diagonals of an n-sided polygon:

$$D = \frac{n(n-3)}{2}$$

This counts all line segments joining pairs of vertices, $\binom{n}{2}$, minus the n sides.

Step 1 – Identify $n = 8$ (octagon).

Step 2 – Substitute into the formula:

$$D = \frac{8(8-3)}{2} = \frac{8 \times 5}{2}$$

Step 3 – Compute:

$$D = \frac{40}{2} = 20$$

Why other options are wrong:

- (B) 28: this is $\binom{8}{2}$, all segments *including* the 8 sides.
- (C) 16: subtracting $2n$ instead of n from $\binom{8}{2}$.
- (D) 40: forgetting to divide by 2.

Answer: (A) ← Go Back to Q17

Q18.

Solution

Concept – nth term of a GP:

$$a_n = a\,r^{\,n-1}$$

Step 1 – Identify the first term and common ratio:

$$a = 3, \qquad r = \frac{6}{3} = 2$$

Step 2 – Use $n = 5$:

$$a_5 = 3 \times 2^{\,5-1} = 3 \times 2^4$$

Step 3 – Evaluate 2^4:

$$2^4 = 16$$

Step 4 – Multiply:

$$a_5 = 3 \times 16 = 48$$

Why other options are wrong:

- (A) 24: computing the 4th term $3 \times 2^3 = 24$.
- (C) 96: computing the 6th term $3 \times 2^5 = 96$.
- (D) 36: using $a_5 = a + 4r$ as if it were an AP.

Answer: (B) ← Go Back to Q18

Q19.

Solution

Concept – Distance of a point from the origin in 3D:

$$d = \sqrt{x^2 + y^2 + z^2}$$

Step 1 – Substitute $(x, y, z) = (3, 4, 12)$:

$$d = \sqrt{3^2 + 4^2 + 12^2}$$

Step 2 – Square each coordinate:

$$3^2 = 9, \quad 4^2 = 16, \quad 12^2 = 144$$

Step 3 – Add:

$$9 + 16 + 144 = 169$$

Step 4 – Take the square root:

$$d = \sqrt{169} = 13$$

Why other options are wrong:

- (A) 7: adding $3 + 4$ and ignoring the z-coordinate.
- (B) 19: adding the coordinates $3 + 4 + 12$ directly.
- (C) $\sqrt{19}$: summing the coordinates before squaring incorrectly.

Answer: (D) ← Go Back to Q19

Q20.

Solution

Concept – Direct integration:
When $\dfrac{dy}{dx} = f(x)$, integrate both sides with respect to x.
Step 1 – Separate (already in directly integrable form):

$$dy = x^2\,dx$$

Step 2 – Integrate both sides:

$$\int dy = \int x^2\,dx$$

Step 3 – Apply the power rule on the right:

$$y = \frac{x^3}{3} + C$$

Why other options are wrong:

- (A) $y = 2x + C$: this is the solution of $\frac{dy}{dx} = 2$, not x^2.
- (B) $y = x^3 + C$: forgetting to divide by 3 from the power rule.
- (D) $y = \frac{x^2}{2} + C$: this is the solution of $\frac{dy}{dx} = x$, not x^2.

Answer: (C) ← Go Back to Q20

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	A	3	C	4	D	5	A
6	B	7	C	8	A	9	D	10	B
11	C	12	D	13	A	14	B	15	D
16	C	17	A	18	B	19	D	20	C