AP EAPCET 2026 May 12 Shift 1

Question Paper (Memory-Based) with Solutions

Conducted by JNTU, KAKINADA on behalf of APSCHE

General Instructions

(i) The test is of 3 hours duration.

(ii) This test paper consists of 160 questions. The maximum marks are 160.

(iii) Physics and Chemistry contains 40 questions each and Mathematics contains 80 questions.

(iv) Each question carries +1 marks for correct answer and there is no negative marking for wrong answer.

1. Let $f(x)=\int \frac{\sqrt{x}}{(1+x)^2}dx(x\geq 0)$. Then $f(3)-f(1)$ is equal to:

(A) $-\frac{\pi}{12}+\frac{1}{2}+\frac{\sqrt{3}}{4}$

(B) $\frac{\pi}{12}+\frac{1}{2}-\frac{\sqrt{3}}{4}$

(C) $-\frac{\pi}{6}+\frac{1}{2}+\frac{\sqrt{3}}{4}$

(D) $\frac{\pi}{6}+\frac{1}{2}-\frac{\sqrt{3}}{4}$

Correct Answer: (B) $\frac{\pi}{12}+\frac{1}{2}-\frac{\sqrt{3}}{4}$

Solution: **Concept:** The expression $f(3)-f(1)$ represents the definite integral of the function from 1 to 3. Specifically, by the Fundamental Theorem of Calculus:

$$f(3)-f(1)=\int_1^3 \frac{\sqrt{x}}{(1+x)^2}dx$$

To solve this, we use a trigonometric substitution to simplify the radical and the denominator. Since we have a term like $(1+x)$, substituting $x=\tan^2\theta$ is effective because $1+\tan^2\theta=\sec^2\theta$.

Step 1: Substitution and changing limits. Let $x=\tan^2\theta$. Then:

$$dx=2\tan\theta\sec^2\theta\,d\theta$$

Now, let's find the new limits for θ:

- When $x = 1$, $\tan^2\theta = 1 \Rightarrow \tan\theta = 1 \Rightarrow \theta = \frac{\pi}{4}$.
- When $x = 3$, $\tan^2\theta = 3 \Rightarrow \tan\theta = \sqrt{3} \Rightarrow \theta = \frac{\pi}{3}$.

Step 2: Transforming the integral. Substitute the values into the integral:

$$I = \int_{\pi/4}^{\pi/3} \frac{\sqrt{\tan^2\theta}}{(1+\tan^2\theta)^2} \cdot (2\tan\theta\sec^2\theta)\, d\theta$$

Using the identity $1 + \tan^2\theta = \sec^2\theta$:

$$I = \int_{\pi/4}^{\pi/3} \frac{\tan\theta}{(\sec^2\theta)^2} \cdot 2\tan\theta\sec^2\theta\, d\theta$$

$$I = \int_{\pi/4}^{\pi/3} \frac{2\tan^2\theta\sec^2\theta}{\sec^4\theta}\, d\theta = 2\int_{\pi/4}^{\pi/3} \frac{\tan^2\theta}{\sec^2\theta}\, d\theta$$

Convert to sine and cosine:

$$I = 2\int_{\pi/4}^{\pi/3} \frac{\sin^2\theta/\cos^2\theta}{1/\cos^2\theta}\, d\theta = 2\int_{\pi/4}^{\pi/3} \sin^2\theta\, d\theta$$

Step 3: Integrating and applying limits. Use the power-reduction identity $\sin^2\theta = \frac{1-\cos 2\theta}{2}$:

$$I = 2\int_{\pi/4}^{\pi/3} \frac{1-\cos 2\theta}{2}\, d\theta = \int_{\pi/4}^{\pi/3} (1-\cos 2\theta)\, d\theta$$

$$I = \left[\theta - \frac{\sin 2\theta}{2}\right]_{\pi/4}^{\pi/3}$$

Evaluate at the upper limit ($\pi/3$): $\frac{\pi}{3} - \frac{\sin(2\pi/3)}{2} = \frac{\pi}{3} - \frac{\sqrt{3}/2}{2} = \frac{\pi}{3} - \frac{\sqrt{3}}{4}$.

Evaluate at the lower limit ($\pi/4$): $\frac{\pi}{4} - \frac{\sin(2\pi/4)}{2} = \frac{\pi}{4} - \frac{\sin(\pi/2)}{2} = \frac{\pi}{4} - \frac{1}{2}$.

Subtracting the two:

$$I = \left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right) - \left(\frac{\pi}{4} - \frac{1}{2}\right)$$

$$I = \frac{\pi}{3} - \frac{\pi}{4} + \frac{1}{2} - \frac{\sqrt{3}}{4} = \frac{\pi}{12} + \frac{1}{2} - \frac{\sqrt{3}}{4}$$

Quick Tip: When you see $\sqrt{x}$ and $(1+x)$ in the same integral, the substitution $x = \tan^2\theta$ is usually the fastest way to collapse the denominator using Pythagorean identities.

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2. Gas is being pumped into a spherical balloon at the rate of $30\,\text{ft}^3/\text{min}$. Then the rate at which the radius increases when it reaches the value $15\,\text{ft}$ is:

(A) $\frac{1}{30\pi}$ ft/min
(B) $\frac{1}{15\pi}$ ft/min
(C) $\frac{1}{20\pi}$ ft/min
(D) $\frac{1}{25\pi}$ ft/min

Correct Answer: (A) $\frac{1}{30\pi}$ ft/min

Solution:

Concept: This is a related rates problem. We are given the rate of change of volume (dV/dt) and need to find the rate of change of the radius (dr/dt). The volume V of a sphere is given by:

$$V = \frac{4}{3}\pi r^3$$

Step 1: Differentiating the volume formula. We differentiate both sides of the volume equation with respect to time t using the chain rule:

$$\frac{dV}{dt} = \frac{4}{3}\pi \cdot \frac{d}{dt}(r^3)$$

$$\frac{dV}{dt} = \frac{4}{3}\pi \cdot 3r^2\frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$$

Step 2: Plugging in given values.

We are given:

- $\frac{dV}{dt} = 30\,\text{ft}^3/\text{min}$
- $r = 15\,\text{ft}$

Substitute these into our differentiated equation:

$$30 = 4\pi(15)^2 \frac{dr}{dt}$$

$$30 = 4\pi(225)\frac{dr}{dt}$$

$$30 = 900\pi \frac{dr}{dt}$$

Step 3: Solving for dr/dt.

$$\frac{dr}{dt} = \frac{30}{900\pi}$$

Simplify the fraction by dividing both numerator and denominator by 30:

$$\frac{dr}{dt} = \frac{1}{30\pi} \text{ ft/min}$$

Quick Tip: In related rates, always identify your "knowns" and your "unknowns" before differentiating. Remember that $\frac{dV}{dt} = (\text{Surface Area}) \times \frac{dr}{dt}$ for a sphere.

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3. Let the function $f(x)$ be defined as $f(x) = \frac{x-|x|}{x}$, then:

(A) the function is continuous everywhere

(B) the function is not continuous

(C) the function is continuous when $x < 0$

(D) the function is continuous for all x except zero

Correct Answer: (D) the function is continuous for all x except zero

Solution:

Concept: A function is continuous where it is defined and the limit equals the function value. First, we must redefine $f(x)$ by removing the absolute value sign based on the domain of x.

Step 1: Redefining the function for different intervals. Recall the definition of $|x|$:

$$|x| = \begin{cases} x & \text{if } x > 0 \\ -x & \text{if } x < 0 \end{cases}$$

Note that at $x = 0$, the denominator of $f(x)$ is zero, so the function is undefined at $x = 0$.

- For $x > 0$: $f(x) = \frac{x-x}{x} = \frac{0}{x} = 0$.
- For $x < 0$: $f(x) = \frac{x-(-x)}{x} = \frac{x+x}{x} = \frac{2x}{x} = 2$.

Step 2: Analyzing continuity.

The function is a constant 0 for all $x \in (0, \infty)$, which is continuous. The function is a constant 2 for all $x \in (-\infty, 0)$, which is also continuous. However, at $x = 0$, the function is not defined. Furthermore, the left-hand limit (2) is not equal to the right-hand limit (0), showing a jump discontinuity.

Step 3: Conclusion.

Since the function is defined and constant in the intervals $(-\infty, 0)$ and $(0, \infty)$, it is continuous on those intervals. The only point of concern is $x = 0$, where the function fails to exist. Thus, the function is continuous for all x except zero.

Quick Tip: Always check the domain first. If a function's denominator is zero at a point, it cannot be continuous there, regardless of whether the limit exists.

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4. If the direction ratios of two lines are given by $l + m + n = 0$ and $mn - 2ln + lm = 0$, then the angle between the lines is:

(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{2}$

(D) 0

Correct Answer: (C) $\frac{\pi}{2}$

Solution:

Concept: The angle θ between two lines with direction ratios (l_1, m_1, n_1) and (l_2, m_2, n_2) is found using:

$$\cos\theta = \frac{|l_1 l_2 + m_1 m_2 + n_1 n_2|}{\sqrt{l_1^2 + m_1^2 + n_1^2}\sqrt{l_2^2 + m_2^2 + n_2^2}}$$

If the dot product $l_1 l_2 + m_1 m_2 + n_1 n_2 = 0$, the lines are perpendicular ($\theta = \pi/2$).

Step 1: Substituting one variable.

From $l + m + n = 0$, we have $n = -(l + m)$. Substitute this into the second equation $mn - 2ln + lm = 0$:

$$m(-(l+m)) - 2l(-(l+m)) + lm = 0$$

$$-ml - m^2 + 2l^2 + 2lm + lm = 0$$

$$2l^2 + 2lm - m^2 = 0$$

Step 2: Forming a quadratic in (l/m).

Divide the equation by m^2:

$$2\left(\frac{l}{m}\right)^2 + 2\left(\frac{l}{m}\right) - 1 = 0$$

Let the two roots (direction ratios of the two lines) be $\frac{l_1}{m_1}$ and $\frac{l_2}{m_2}$.

From the properties of quadratic equations:

Product of roots: $\frac{l_1 l_2}{m_1 m_2} = \frac{c}{a} = \frac{-1}{2} \Rightarrow 2l_1 l_2 = -m_1 m_2 \Rightarrow 2l_1 l_2 + m_1 m_2 = 0$.

Step 3: Finding the final condition.

We also have $n_1 = -(l_1 + m_1)$ and $n_2 = -(l_2 + m_2)$.

Product $n_1 n_2 = (l_1 + m_1)(l_2 + m_2) = l_1 l_2 + l_1 m_2 + l_2 m_1 + m_1 m_2$.

Using the sum of roots $\frac{l_1}{m_1} + \frac{l_2}{m_2} = -\frac{b}{a} = -1$, we get $l_1 m_2 + l_2 m_1 = -m_1 m_2$.

Substitute this back: $n_1 n_2 = l_1 l_2 - m_1 m_2 + m_1 m_2 = l_1 l_2$.

Now calculate $l_1 l_2 + m_1 m_2 + n_1 n_2$:

Using $m_1 m_2 = -2l_1 l_2$:

$$l_1 l_2 + (-2l_1 l_2) + l_1 l_2 = 0$$

Since the sum of the products of the direction ratios is zero, the lines are perpendicular.

Quick Tip: In problems involving homogeneous equations in l, m, n, always aim to reduce the system to a quadratic in terms of a ratio like l/m or m/n.

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5. The equation of the normal to the curve $y = \log_e x$ at the point $P(1,0)$ is ____.

(A) $2x + y = 2$

(B) $x - 2y = 1$

(C) $x - y = 1$

(D) $x + y = 1$

Correct Answer: (D) $x + y = 1$

Solution:

Concept: The slope of the tangent to a curve $y = f(x)$ at point (x_1, y_1) is $m_t = f'(x_1)$. The slope of the normal is the negative reciprocal: $m_n = -1/m_t$. The equation of the line is then $y - y_1 = m_n(x - x_1)$.

Step 1: Finding the slope of the tangent.

The given curve is $y = \ln x$.

Differentiating with respect to x:

$$\frac{dy}{dx} = \frac{1}{x}$$

At the point $P(1, 0)$, the slope of the tangent m_t is:

$$m_t = \left.\frac{dy}{dx}\right|_{x=1} = \frac{1}{1} = 1$$

Step 2: Finding the slope of the normal.

Since the normal is perpendicular to the tangent:

$$m_n = -\frac{1}{m_t} = -\frac{1}{1} = -1$$

Step 3: Equation of the normal line.

Using the point-slope form with point $(1, 0)$ and slope $m_n = -1$:

$$y - 0 = -1(x - 1)$$

$$y = -x + 1$$

$$x + y = 1$$

Quick Tip: For $y = \ln x$, the tangent at $(1, 0)$ is always $y = x - 1$. Swapping the coefficients and changing the sign of one (perpendicular line property) gives the normal $x + y = k$.

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6. The radius in the hydrogen atom in the ground state is 0.53 Å. The radius of Li^{+2} ion (atomic number $= 3$) in a similar state is .....

(a) 0.53Å

(b) 1.06Å

(c) 0.17Å

(d) 0.265Å

Correct Answer: (c) 0.17Å

Solution:

Concept: According to Bohr's model of the atom, the radius of the n^{th} orbit for a hydrogen-like species (single-electron systems like H, He^+, Li^{2+}, etc.) is given by the formula:

$$r_n = a_0 \frac{n^2}{Z}$$

Where:

- r_n is the radius of the n^{th} orbit.

- a_0 is the Bohr radius (radius of H-atom in ground state, $\approx 0.529\text{Å}$ or 0.53Å).
- n is the principal quantum number (orbit number).
- Z is the atomic number of the element.

Step 1: Identify the given values for Hydrogen and Lithium. For the Hydrogen atom in the ground state:

- Principal quantum number $(n) = 1$ (since it is the ground state).
- Atomic number $(Z_H) = 1$.
- Radius $(r_H) = 0.53\text{Å}$.

For the Li^{+2} ion in a similar state:

- Principal quantum number $(n) = 1$ (as it is in a "similar state" i.e., ground state).
- Atomic number for Lithium $(Z_{Li}) = 3$.

Step 2: Establishing the relationship between radii. Using the formula $r \propto \frac{n^2}{Z}$, and since n is constant $(n = 1)$ for both cases, we can state that:

$$r \propto \frac{1}{Z}$$

Therefore, the ratio of the radius of the Li^{+2} ion to the radius of the Hydrogen atom is:

$$\frac{r_{Li^{+2}}}{r_H} = \frac{Z_H}{Z_{Li}}$$

Step 3: Calculating the final radius.

Substitute the known values into the ratio:

$$\frac{r_{Li^{+2}}}{0.53} = \frac{1}{3}$$

Now, solve for $r_{Li^{+2}}$:

$$r_{Li^{+2}} = \frac{0.53}{3}$$

$$r_{Li^{+2}} = 0.17666...\text{Å}$$

Rounding to two decimal places as per the options provided:

$$r_{Li^{+2}} \approx 0.17\text{Å}$$

Quick Tip: To quickly solve Bohr radius problems, remember that as the nuclear charge (Z) increases, the nucleus pulls the electron closer, making the radius smaller. Since Lithium has 3 protons ($Z = 3$) compared to Hydrogen's 1, its ground state radius must be exactly one-third of Hydrogen's.

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7. The energy of the second bohr orbit of the hydrogen atom is $-328\,\text{kJ mol}^{-1}$; hence the energy of the fourth bohr orbit would be:

(a) $-41\,\text{kJ/mol}$
(b) $-82\,\text{kJ/mol}$
(c) $-164\,\text{kJ/mol}$
(d) $-1312\,\text{kJ/mol}$

Correct Answer: (b) $-82\,\text{kJ/mol}$

Solution:

Concept: According to Bohr's theory of the hydrogen atom, the energy of an electron in the n^{th} orbit (E_n) is inversely proportional to the square of the principal quantum number (n). The mathematical expression for the energy of an orbit is:

$$E_n = -\frac{k \cdot Z^2}{n^2}$$

Where:

- E_n is the energy of the n^{th} orbit.
- Z is the atomic number (for Hydrogen, $Z = 1$).
- n is the principal quantum number (orbit number).
- k is a constant representing the energy of the ground state.

From this formula, we can derive a proportionality relationship for a specific element:

$$E_n \propto \frac{1}{n^2}$$

Step 1: Establishing the ratio between two orbits. Let E_2 be the energy of the second orbit ($n_1 = 2$) and E_4 be the energy of the fourth orbit ($n_2 = 4$). Based on the inverse-square relationship:

$$\frac{E_4}{E_2} = \frac{n_1^2}{n_2^2}$$

Substituting the orbit numbers:

$$\frac{E_4}{E_2} = \frac{2^2}{4^2} = \frac{4}{16} = \frac{1}{4}$$

Step 2: Substituting the given value. We are given that the energy of the second Bohr orbit (E_2) is $-328\,\text{kJ mol}^{-1}$. Using the relationship derived in Step 1:

$$E_4 = E_2 \times \frac{1}{4}$$

$$E_4 = -328 \times \frac{1}{4}$$

Step 3: Final Calculation. Perform the division:

$$E_4 = -\frac{328}{4}$$

$$E_4 = -82\,\text{kJ mol}^{-1}$$

This calculation shows that as the electron moves further from the nucleus (to a higher orbit), the energy level increases (becomes less negative).

Quick Tip: To solve these quickly: if the orbit number doubles (from 2 to 4), the energy becomes $\frac{1}{2^2} = \frac{1}{4}$ of its previous value. Conversely, if the orbit number triples, the energy becomes $\frac{1}{9}$ of the original value. Always remember that energy levels get closer together as n increases.

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8. The IUPAC name of an element with atomic number 119 is:

(a) Ununennium

(b) Unnilennium

(c) Unununium

(d) ununoctium

Correct Answer: (a) Ununennium

Solution:

Concept: For elements with atomic numbers greater than 100, IUPAC established a systematic nomenclature based directly on the digits of the atomic number.

This ensures a temporary name is available before a permanent name (usually honoring a scientist or place) is officially adopted.

The names are derived using numerical roots for each digit:

The name is constructed by putting the roots together in order of the digits and adding the suffix "-ium" at the end.

Step 1: Breaking down the atomic number digits. The given atomic number is 119. We identify the roots for each individual digit:

- First digit: 1 → root is "un"
- Second digit: 1 → root is "un"
- Third digit: 9 → root is "enn"

Step 2: Assembling the roots. Combine the roots in the order of the digits:

$$\text{un} + \text{un} + \text{enn}$$

This gives us the base string: "ununenn".

Step 3: Applying the suffix and final naming. Add the mandatory IUPAC suffix "-ium" to the end of the root string:

$$\text{ununenn} + \text{ium} = \text{Ununennium}$$

The symbol for the element is derived from the first letter of each root (capitalizing the first one): Uue.

Note on spelling: If a root ends in 'enn' and the suffix is 'ium', we keep all three 'n's (enn-ium). If a root ends in 'bi' or 'tri', we drop one 'i' (e.g., bi-ium becomes bium).

Quick Tip: To never forget the roots: 0 is Nil (nothing), 1 is Un (uni-), 2 is Bi (bi-), and 9 is Enn (think of 'enneagon' for a 9-sided shape). Just string them together and add "-ium"!

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9. Which of the following orders is not correct for the given property?

(a) $Li < Na < K$ – metallic radius

(b) $Br < F < Cl$ – electron gain enthalpy

(c) $C < N < O$ – first ionisation enthalpy

(d) $Mg^{+2} < Na^{+} < F^{-}$ – ionic radius

Correct Answer: (c) $C < N < O$ – first ionisation enthalpy

Solution: **Concept:** Periodic properties follow specific trends across periods and down groups. However, there are significant exceptions caused by electronic configurations (half-filled and fully-filled stability) and electron-electron repulsions.

- **Atomic/Metallic Radius:** Increases down a group due to the addition of new shells.
- **Electron Gain Enthalpy:** Generally becomes more negative across a period. For halogens, Cl $>$ F due to inter-electronic repulsion in the small F atom.
- **Ionisation Enthalpy:** Generally increases across a period but decreases if an atom has a stable configuration.
- **Ionic Radius:** In isoelectronic species, radius decreases as nuclear charge (Z) increases.

Step 1: Evaluating Option (a): Metallic Radius Li, Na, and K are Alkali metals (Group 1). As we move down the group from Lithium to Potassium, the number of energy shells increases ($n = 2$ for Li, $n = 3$ for Na, $n = 4$ for K). This outweighs the increase in nuclear charge, resulting in an increase in metallic radius. Order: Li $<$ Na $<$ K. (Correct)

Step 2: Evaluating Option (b): Electron Gain Enthalpy Usually, Electron Gain Enthalpy decreases down a group.

However, Fluorine (F) has a very small size. When an electron is added to its $2p$ orbital, it faces significant inter-electronic repulsion. Chlorine (Cl) is larger, allowing the electron to be added more easily with a higher release of energy. Order of magnitude: Br $<$ F $<$ Cl. (Correct)

Step 3: Evaluating Option (c): First Ionisation Enthalpy Across the second period (C, N, O), ionisation enthalpy generally increases. Let's look at the electronic configurations:

- $\mathrm{C}(Z = 6) : 1s^2 2s^2 2p^2$
- $\mathrm{N}(Z = 7) : 1s^2 2s^2 2p^3$ (Exactly half-filled p-subshell)
- $\mathrm{O}(Z = 8) : 1s^2 2s^2 2p^4$

Nitrogen has a half-filled $2p$ subshell, which is exceptionally stable. Removing an electron from Nitrogen requires more energy than removing one from Oxygen, where electron-electron repulsion in the paired $2p$ orbital makes removal easier. Correct Order: C $<$ O $<$ N. The given order C $<$ N $<$ O is Incorrect.

Step 4: Evaluating Option (d): Ionic Radius Mg^{+2}, Na^{+}, and F^{-} all have 10 electrons (isoelectronic).

- Mg^{+2}: $Z = 12$ (Strongest nuclear pull)

- Na^+: $Z = 11$
- F^-: $Z = 9$ (Weakest nuclear pull)

As Z increases, the radius decreases. Order: $Mg^{+2} < Na^+ < F^-$. (Correct)

Quick Tip: Always watch out for the "N vs O" and "Be vs B" traps in Ionisation Enthalpy! Half-filled and full-filled configurations jump ahead of the next element in the period.

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10. Identify the correct set of molecules with zero dipole moment:

(a) CO_2, NH_3, H_2O

(b) NH_3, NF_3, BF_3

(c) PF_3, NH_3, CH_4

(d) CH_4, BF_3, CO_2

Correct Answer: (d) CH_4, BF_3, CO_2

Solution:

Concept: The dipole moment (μ) of a molecule depends on both the polarity of the individual bonds and the geometric shape (symmetry) of the molecule. A molecule has a zero dipole moment if it is highly symmetrical, such that the individual bond dipoles cancel each other out as vectors.

- **Polar Molecules:** Contain lone pairs on the central atom or have asymmetrical surrounding atoms (e.g., NH_3, H_2O, NF_3).
- **Non-Polar Molecules:** Symmetrical shapes where bond dipoles cancel (e.g., Linear, Trigonal Planar, Tetrahedral with identical atoms).

Step 1: Analyzing molecules in the options. Let's examine the symmetry and dipole cancellation for each unique molecule mentioned:

- **CO_2:** It has a linear geometry (O=C=O). The two C=O bond dipoles are equal in magnitude but opposite in direction. They cancel out perfectly. $\mu = 0$.
- **BF_3:** It has a trigonal planar geometry. The three B-F bond dipoles are oriented at 120° to each other. Their vector sum is zero. $\mu = 0$.

- **CH_4:** It has a tetrahedral geometry. All four C-H bonds are identical and symmetrically arranged. The vector sum of these four dipoles is zero. $\mu = 0$.

Step 2: Identifying molecules with non-zero dipole moments. Now, let's see why the other sets are incorrect:

- **NH_3 and NF_3:** These have pyramidal geometry due to the presence of a lone pair on Nitrogen. The lone pair contributes to the dipole and prevents the bond dipoles from canceling. $\mu \neq 0$.
- **H_2O:** It has a bent (V-shape) geometry because of two lone pairs on Oxygen. The dipoles do not cancel. $\mu \neq 0$.
- **PF_3:** Similar to NH_3, it is pyramidal with a lone pair. $\mu \neq 0$.

Step 3: Final Selection.
Comparing our findings with the options:

- (a) Contains NH_3 and H_2O ($\mu \neq 0$).
- (b) Contains NH_3 and NF_3 ($\mu \neq 0$).
- (c) Contains PF_3 and NH_3 ($\mu \neq 0$).
- (d) Contains CH_4, BF_3, and CO_2. All three are symmetrical and have zero dipole moment.

Quick Tip: A quick shortcut: If the central atom has no lone pairs and all the surrounding atoms are the same, the molecule will almost always have a zero dipole moment due to its perfect symmetry!

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11. The potential energy of a simple harmonic oscillator when the particle is half way to its end point is (where E is the total energy):

(A) $\frac{2}{3}E$
(B) $\frac{1}{8}E$
(C) $\frac{1}{4}E$
(D) $\frac{1}{2}E$

Correct Answer: (C) $\frac{1}{4}E$

Solution:

Concept: In Simple Harmonic Motion (SHM), the total energy E of the oscillator is constant and is the sum of its kinetic energy and potential energy. The potential energy (U) at a displacement x is given by:

$$U = \frac{1}{2}kx^2$$

The total energy (E) occurs when the displacement is at its maximum (amplitude A), where kinetic energy is zero:

$$E = \frac{1}{2}kA^2$$

Step 1: Defining the displacement.

The problem states the particle is "half way to its end point." The end point of an oscillator is its amplitude A. Therefore, the displacement x is:

$$x = \frac{A}{2}$$

Step 2: Calculating Potential Energy at that point. Substitute $x = \frac{A}{2}$ into the potential energy formula:

$$U = \frac{1}{2}k\left(\frac{A}{2}\right)^2$$

$$U = \frac{1}{2}k\left(\frac{A^2}{4}\right)$$

$$U = \frac{1}{4}\left(\frac{1}{2}kA^2\right)$$

Step 3: Relating to Total Energy. Since we know that $E = \frac{1}{2}kA^2$, we can substitute E into our expression for U:

$$U = \frac{1}{4}E$$

Quick Tip: Remember that Potential Energy in SHM is proportional to the square of displacement ($U \propto x^2$). If the distance is halved, the energy becomes $(1/2)^2 = 1/4$ of the maximum energy.

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12. A current loop of magnetic moment $0.5\,\mathrm{Am}^2$ is placed in a magnetic field of $0.4\,\mathrm{T}$. If the loop is rotated from an angle of 0° to 180°, what is the change in potential energy?

(A) $-0.4\,\mathrm{J}$

(B) $0.4\,\mathrm{J}$

(C) $-0.2\,\mathrm{J}$

(D) $0.2\,\mathrm{J}$

Correct Answer: (B) $0.4\,\mathrm{J}$

Solution:

Concept: The potential energy (U) of a magnetic dipole (current loop) in a uniform magnetic field B is given by the scalar product:

$$U = -\vec{M} \cdot \vec{B} = -MB\cos\theta$$

where M is the magnetic moment and θ is the angle between the magnetic moment and the magnetic field.

Step 1: Calculating Initial Potential Energy. Initially, the angle $\theta_1 = 0^\circ$.

$$U_{\text{initial}} = -MB\cos(0^\circ)$$

Since $\cos(0^\circ) = 1$:

$$U_{\text{initial}} = -MB$$

Substituting values: $U_{\text{initial}} = -(0.5)(0.4) = -0.2\,\mathrm{J}$.

Step 2: Calculating Final Potential Energy. Finally, the loop is rotated to $\theta_2 = 180^\circ$.

$$U_{\text{final}} = -MB\cos(180^\circ)$$

Since $\cos(180^\circ) = -1$:

$$U_{\text{final}} = -MB(-1) = +MB$$

Substituting values: $U_{\text{final}} = +(0.5)(0.4) = +0.2\,\text{J}$.

Step 3: Calculating the Change in Potential Energy. The change in potential energy (ΔU) is the difference between the final and initial states:

$$\Delta U = U_{\text{final}} - U_{\text{initial}}$$

$$\Delta U = 0.2\,\text{J} - (-0.2\,\text{J})$$

$$\Delta U = 0.2 + 0.2 = 0.4\,\text{J}$$

Quick Tip: The work done in rotating a dipole from θ_1 to θ_2 is always $\Delta U = MB(\cos\theta_1 - \cos\theta_2)$. For a 180° rotation from the stable equilibrium (0°), the work is always $2MB$.

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13. Twenty seven drops of same size are charged at 220 V each. They combine to form a bigger drop. Calculate the potential of the bigger drop.

(A) 1320 V

(B) 1520 V

(C) 1980 V

(D) 660 V

Correct Answer: (C) 1980 V

Solution:

Concept: When small charged droplets coalesce into a single large drop, two physical quantities remain conserved: Total Volume and Total Charge. The electric potential V of a spherical drop of radius r and charge q is given by:

$$V = \frac{kq}{r}$$

By finding the relationship between the radius and charge of the small drops versus the large drop, we can determine the new potential.

Step 1: Relation between radii using Volume Conservation. Let the radius of each small drop be r and the radius of the big drop be R. The volume of 27 small drops equals the volume of 1 big drop:

$$27 \times \left(\frac{4}{3}\pi r^3\right) = \frac{4}{3}\pi R^3$$

$$27r^3 = R^3$$

Taking the cube root on both sides:

$$R = 3r$$

Step 2: Relation between charges using Charge Conservation. Let the charge on each small drop be q and the charge on the big drop be Q.

$$Q = 27q$$

Step 3: Calculating the potential of the bigger drop. Potential of a small drop (v): $v = \frac{kq}{r} = 220$ V. Potential of the big drop (V):

$$V = \frac{kQ}{R}$$

Substitute $Q = 27q$ and $R = 3r$:

$$V = \frac{k(27q)}{3r} = 9\left(\frac{kq}{r}\right)$$

$$V = 9 \times v$$

$$V = 9 \times 220 = 1980 \text{ V}$$

Quick Tip: For n identical droplets combining, the potential of the big drop is always $V_{big} = n^{2/3} \times V_{small}$. Here, $27^{2/3} = (3^3)^{2/3} = 3^2 = 9$.

14. Two polaroids are oriented with their principal planes making an angle of 60°. Then the percentage of incident unpolarized light which passes through the system is:

(A) 100%

(B) 50%

(C) 12.5%

(D) 37.5%

Correct Answer: (C) 12.5%

Solution:

Concept: This problem involves two stages of light transmission:

1. Unpolarized light passing through the first polaroid (Polarizer): The intensity is always halved. 2. Polarized light passing through the second polaroid (Analyzer): The intensity follows Malus's Law: $I = I_0 \cos^2 \theta$.

Step 1: Transmission through the first polaroid. Let the incident unpolarized light intensity be I_{in}. After passing through the first polaroid, the light becomes plane-polarized, and its intensity I_1 is:

$$I_1 = \frac{I_{in}}{2}$$

Step 2: Transmission through the second polaroid (Malus's Law). The polarized light I_1 now hits the second polaroid at an angle $\theta = 60^\circ$. The resulting intensity I_2 is:

$$I_2 = I_1 \cos^2(60^\circ)$$

Substitute the value of $\cos(60^\circ) = \frac{1}{2}$:

$$I_2 = I_1 \left(\frac{1}{2}\right)^2 = \frac{I_1}{4}$$

Step 3: Calculating final percentage. Substitute I_1 from Step 1:

$$I_2 = \left(\frac{I_{in}}{2}\right) \times \frac{1}{4} = \frac{I_{in}}{8}$$

To find the percentage:

$$\text{Percentage} = \frac{I_2}{I_{in}} \times 100 = \frac{1}{8} \times 100 = 12.5\%$$

Quick Tip: Always remember: First polaroid → divide by 2. Second polaroid → multiply by $\cos^2\theta$. For 60°, $\cos^2(60^\circ)$ is 0.25 or 1/4. Half of 1/4 is 1/8, which is 12.5%.

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15. The equation of path of a projectile moving in x-y plane is given by $y = x - \frac{x^2}{25}$, then the initial speed and maximum height of particle will be (take $g = 10\,\text{m/s}^2$):

(A) $4\sqrt{10}\,\text{m/s}$ and $\frac{29}{4}\,\text{m}$

(B) $25\sqrt{10}\,\text{m/s}$ and $\frac{21}{4}\,\text{m}$

(C) $2\sqrt{10}\,\text{m/s}$ and $\frac{31}{4}\,\text{m}$

(D) $5\sqrt{10}\,\text{m/s}$ and $\frac{25}{4}\,\text{m}$

Correct Answer: (D) $5\sqrt{10}\,\text{m/s}$ and $\frac{25}{4}\,\text{m}$

Solution:

Concept: The standard trajectory equation for a projectile launched with initial velocity u at an angle θ with the horizontal is:

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

By comparing the given equation $y = x - \frac{x^2}{25}$ with this standard form, we can extract the values for the launch angle and the initial velocity. Once these are found, the maximum height H_{max} can be calculated using the formula:

$$H_{max} = \frac{u^2\sin^2\theta}{2g}$$

Step 1: Comparing the equations to find the launch angle. Comparing $y = x - \frac{x^2}{25}$ with $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$: The coefficient of x gives:

$$\tan\theta = 1 \implies \theta = 45^\circ$$

At $\theta = 45^\circ$, we have:

$$\sin 45^\circ = \frac{1}{\sqrt{2}}, \quad \cos 45^\circ = \frac{1}{\sqrt{2}}, \quad \text{and} \quad \cos^2 45^\circ = \frac{1}{2}$$

Step 2: Finding the initial speed (u). The coefficient of x^2 gives:

$$\frac{g}{2u^2 \cos^2 \theta} = \frac{1}{25}$$

Substitute $g = 10$ and $\cos^2 \theta = 1/2$:

$$\frac{10}{2u^2(1/2)} = \frac{1}{25} \implies \frac{10}{u^2} = \frac{1}{25}$$

$$u^2 = 250 \implies u = \sqrt{25 \times 10} = 5\sqrt{10}\,\text{m/s}$$

Step 3: Calculating the maximum height (H_{max}). Now, substitute $u^2 = 250$ and $\sin^2 \theta = 1/2$ into the maximum height formula:

$$H_{max} = \frac{u^2 \sin^2 \theta}{2g} = \frac{250 \times (1/2)}{2 \times 10}$$

$$H_{max} = \frac{125}{20}$$

Divide both by 5:

$$H_{max} = \frac{25}{4}\,\text{m}$$

Thus, the initial speed is $5\sqrt{10}\,\text{m/s}$ and the maximum height is $\frac{25}{4}$ m.

Quick Tip: To find the maximum height quickly from a trajectory equation $y = ax - bx^2$, you can use calculus. The maximum height occurs at the vertex where $dy/dx = 0$. Here, $1 - (2x/25) = 0 \implies x = 12.5$. Plugging $x = 12.5$ into the equation gives $y = 12.5 - (12.5)^2/25 = 12.5 - 6.25 = 6.25$, which is exactly $25/4$.
