

Electrical And Electronics Engineering

Question Paper with Solutions

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General Instructions

- (i) **Duration:** The total duration of the examination is 3 hours (180 minutes).
- (ii) **Total Marks:** The complete paper carries a maximum of 200 marks.
- (iii) Each question has four options. Only **one** option is correct.
- (iv) **Right Answer:** +1 mark.
- (v) **Incorrect Answer:** No negative marking.
- (vi) **Unanswered/Marked for Review:** 0 marks.

1. In the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 4 & 9 \end{bmatrix},$$

the minor M_{23} of the element a_{23} is:

- (A) 10
- (B) -10
- (C) -6
- (D) 6

Correct Answer: (B) -10

Solution:

Concept: The minor M_{ij} of an element a_{ij} is obtained by deleting the i^{th} row and j^{th} column of the matrix, and then finding the determinant of the remaining matrix.

Step 1: Identify the given matrix.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 4 & 9 \end{bmatrix}$$

Step 2: We need to find M_{23} .

Here M_{23} means the minor corresponding to the element in the 2nd row and 3rd column.

$$a_{23} = 6$$

Step 3: Delete the 2nd row and 3rd column.

After deleting the second row and third column, the remaining matrix is:

$$\begin{bmatrix} 1 & 2 \\ 7 & 4 \end{bmatrix}$$

Step 4: Find the determinant of the remaining matrix.

$$M_{23} = \begin{vmatrix} 1 & 2 \\ 7 & 4 \end{vmatrix}$$

$$M_{23} = 1(4) - 2(7)$$

$$M_{23} = 4 - 14$$

$$M_{23} = -10$$

Therefore, the value of the minor is:

$$\boxed{-10}$$

Quick Tip: To find M_{ij} , delete the i^{th} row and j^{th} column, then calculate the determinant of the remaining matrix.

2. If

$$\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix},$$

then the value of x is:

- (A) 3
- (B) ± 6
- (C) -3
- (D) ± 2

Correct Answer: (B) ± 6

Solution:

Concept: For a determinant of order 2,

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Step 1: Evaluate the determinant on the left side.

$$\begin{aligned} \begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} &= (2x)(x) - (5)(8) \\ &= 2x^2 - 40 \end{aligned}$$

Step 2: Evaluate the determinant on the right side.

$$\begin{aligned} \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix} &= (6)(3) - (-2)(7) \\ &= 18 + 14 \\ &= 32 \end{aligned}$$

Step 3: Equate both determinants.

$$2x^2 - 40 = 32$$

$$2x^2 = 72$$

$$x^2 = 36$$

Step 4: Take square root on both sides.

$$x = \pm 6$$

Therefore,

$$\boxed{x = \pm 6}$$

Quick Tip: For determinant equations, first calculate both determinants separately, then equate them and solve the resulting algebraic equation.

3. If A is a square matrix of order 3 and $|A| = 5$, then the value of $|2A^T|$ is:

- (A) -10
- (B) 10
- (C) 40
- (D) -40

Correct Answer: (C) 40

Solution:

Concept: For a square matrix A of order n ,

$$|kA| = k^n|A|$$

Also, the determinant of transpose is equal to the determinant of the original matrix:

$$|A^T| = |A|$$

Step 1: Given:

$$|A| = 5$$

and A is of order 3.

So,

$$n = 3$$

Step 2: Use the transpose property.

$$|A^T| = |A|$$

$$|A^T| = 5$$

Step 3: Now evaluate:

$$|2A^T|$$

Since A^T is also a 3×3 matrix:

$$|2A^T| = 2^3|A^T|$$

$$= 8 \times 5$$

$$= 40$$

Therefore,

$$\boxed{40}$$

Quick Tip: If A is of order n , then $|kA| = k^n|A|$. Do not forget the power n .

4. Which of the following systems has non-trivial solution?

- (A) $AX = 0$, $|A| = 4$
- (B) $AX = 0$, $|A| = -4$
- (C) $AX = 0$, $|A| = 0$
- (D) $AX = B$, $|B| = 5$

Correct Answer: (C) $AX = 0$, $|A| = 0$

Solution:

Concept: A homogeneous system of linear equations is written as:

$$AX = 0$$

Such a system always has the trivial solution:

$$X = 0$$

A non-trivial solution exists only when the coefficient matrix is singular.

Step 1: For the homogeneous system:

$$AX = 0$$

the system has a non-trivial solution if:

$$|A| = 0$$

Step 2: If $|A| \neq 0$, then the matrix A is non-singular and only the trivial solution exists.

Step 3: Check the given options.

Option (A):

$$|A| = 4 \neq 0$$

So only trivial solution exists.

Option (B):

$$|A| = -4 \neq 0$$

So only trivial solution exists.

Option (C):

$$|A| = 0$$

So non-trivial solution exists.

Step 4: Therefore, the correct system is:

$$AX = 0, |A| = 0$$

Hence,

$$\boxed{AX = 0, |A| = 0}$$

Quick Tip: For a homogeneous system $AX = 0$, non-trivial solutions exist only when $|A| = 0$.

5. If

$$\begin{bmatrix} x + y & 2 \\ 1 & x - y \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 1 & 2 \end{bmatrix},$$

then the values of x and y are:

- (A) $x = 3, y = 1$
- (B) $x = 1, y = 3$
- (C) $x = 2, y = 3$
- (D) $x = 1, y = 1$

Correct Answer: (A) $x = 3, y = 1$

Solution:

Concept: Two matrices are equal only when their corresponding elements are equal.

Step 1: Given:

$$\begin{bmatrix} x + y & 2 \\ 1 & x - y \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ 1 & 2 \end{bmatrix}$$

Step 2: Compare corresponding elements.

From first row and first column:

$$x + y = 4 \quad \dots(1)$$

From second row and second column:

$$x - y = 2 \quad \dots(2)$$

Step 3: Add equations (1) and (2).

$$(x + y) + (x - y) = 4 + 2$$

$$2x = 6$$

$$x = 3$$

Step 4: Substitute $x = 3$ in equation (1).

$$3 + y = 4$$

$$y = 1$$

Therefore,

$$\boxed{x = 3, y = 1}$$

Quick Tip: When two matrices are equal, compare corresponding entries to form simple equations.

6. If

$$\frac{x+4}{(x+2)^2(x+3)} = \frac{A}{(x+2)^2} + \frac{B}{x+2} + \frac{C}{x+3},$$

then $A + B + C =$

- (A) 2
- (B) 1
- (C) -1
- (D) 3

Correct Answer: (A) 2

Solution:

Concept: For partial fractions, multiply by the complete denominator first. Then substitute values of x that make factors zero to find constants quickly.

Step 1: Given:

$$\frac{x+4}{(x+2)^2(x+3)} = \frac{A}{(x+2)^2} + \frac{B}{x+2} + \frac{C}{x+3}$$

Step 2: Multiply both sides by $(x+2)^2(x+3)$.

$$x+4 = A(x+3) + B(x+2)(x+3) + C(x+2)^2$$

Step 3: Put $x = -2$ to find A .

$$-2+4 = A(-2+3)$$

$$2 = A(1)$$

$$A = 2$$

Step 4: Put $x = -3$ to find C .

$$-3 + 4 = C(-3 + 2)^2$$

$$1 = C(-1)^2$$

$$C = 1$$

Step 5: Put $x = 0$ to find B .

$$0 + 4 = A(0 + 3) + B(0 + 2)(0 + 3) + C(0 + 2)^2$$

$$4 = 3A + 6B + 4C$$

Substitute $A = 2$ and $C = 1$:

$$4 = 3(2) + 6B + 4(1)$$

$$4 = 6 + 6B + 4$$

$$4 = 10 + 6B$$

$$6B = -6$$

$$B = -1$$

Step 6: Calculate $A + B + C$.

$$A + B + C = 2 + (-1) + 1$$

$$A + B + C = 2$$

Therefore,

$$\boxed{2}$$

Quick Tip: In partial fractions, use zeroes of denominator factors to quickly calculate constants.

7. If

$$\frac{x}{(x-1)^2(x+2)} = \frac{A}{(x-1)^2} + \frac{2}{9(x-1)} + \frac{B}{x+2},$$

then $A + B =$

- (A) $\frac{1}{3}$
- (B) $\frac{1}{9}$
- (C) $-\frac{1}{3}$
- (D) $\frac{2}{3}$

Correct Answer: (B) $\frac{1}{9}$

Solution:

Concept: When the denominator has repeated factors, partial fractions contain separate terms for each power of that factor.

Step 1: Given:

$$\frac{x}{(x-1)^2(x+2)} = \frac{A}{(x-1)^2} + \frac{2}{9(x-1)} + \frac{B}{x+2}$$

Step 2: Multiply both sides by $(x-1)^2(x+2)$.

$$x = A(x+2) + \frac{2}{9}(x-1)(x+2) + B(x-1)^2$$

Step 3: Put $x = 1$ to find A.

$$1 = A(1+2)$$

$$1 = 3A$$

$$A = \frac{1}{3}$$

Step 4: Put $x = -2$ to find B .

$$-2 = B(-2 - 1)^2$$

$$-2 = 9B$$

$$B = -\frac{2}{9}$$

Step 5: Find $A + B$.

$$A + B = \frac{1}{3} - \frac{2}{9}$$

$$A + B = \frac{3}{9} - \frac{2}{9}$$

$$A + B = \frac{1}{9}$$

Therefore,

$$\boxed{\frac{1}{9}}$$

Quick Tip: For partial fractions, after multiplying by the denominator, substitute roots of the denominator to find constants quickly.

8. If $\tan A = \frac{1}{2}$ and $\tan B = \frac{1}{3}$, then $A + B =$

- (A) 30°
- (B) 45°
- (C) 60°
- (D) 90°

Correct Answer: (B) 45°

Solution:

Concept: Use the tangent addition formula:

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

Step 1: Given:

$$\tan A = \frac{1}{2}, \quad \tan B = \frac{1}{3}$$

Step 2: Apply the formula.

$$\tan(A + B) = \frac{\frac{1}{2} + \frac{1}{3}}{1 - \left(\frac{1}{2}\right)\left(\frac{1}{3}\right)}$$

Step 3: Simplify numerator.

$$\frac{1}{2} + \frac{1}{3} = \frac{3 + 2}{6} = \frac{5}{6}$$

Step 4: Simplify denominator.

$$1 - \frac{1}{6} = \frac{5}{6}$$

Step 5: Therefore:

$$\tan(A + B) = \frac{\frac{5}{6}}{\frac{5}{6}}$$

$$\tan(A + B) = 1$$

Step 6: Since:

$$\tan 45^\circ = 1$$

we get:

$$A + B = 45^\circ$$

Therefore,

$$\boxed{45^\circ}$$

Quick Tip: Use $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$ whenever individual tangent values are given.

9. If $2 \sin^{-1} x = \sin^{-1} k$, then $k =$

(A) $2x\sqrt{1-x^2}$

- (B) $2x$
- (C) x^2
- (D) $1 - 2x^2$

Correct Answer: (A) $2x\sqrt{1-x^2}$

Solution:

Concept: For inverse trigonometric expressions, assume the inverse trigonometric expression to be an angle. Then use ordinary trigonometric identities.

Step 1: Let:

$$\sin^{-1} x = \theta$$

Then:

$$x = \sin \theta$$

Step 2: Given:

$$2 \sin^{-1} x = \sin^{-1} k$$

Using $\sin^{-1} x = \theta$:

$$2\theta = \sin^{-1} k$$

Step 3: Take sine on both sides.

$$\sin(2\theta) = k$$

Step 4: Use double angle formula.

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

So:

$$k = 2 \sin \theta \cos \theta$$

Step 5: Since:

$$\sin \theta = x$$

we have:

$$\cos \theta = \sqrt{1 - \sin^2 \theta}$$

$$\cos \theta = \sqrt{1 - x^2}$$

Step 6: Substitute values.

$$k = 2x\sqrt{1-x^2}$$

Therefore,

$$\boxed{2x\sqrt{1-x^2}}$$

Quick Tip: For inverse trigonometric expressions, put $\sin^{-1} x = \theta$, then use standard trigonometric identities.

10. If

$$\sin^{-1} \frac{5}{x} + \sin^{-1} \frac{12}{x} = \frac{\pi}{2},$$

then $x =$

- (A) 12
- (B) 7
- (C) 13
- (D) 15

Correct Answer: (C) 13

Solution:

Concept: If two angles are complementary, then:

$$A + B = \frac{\pi}{2}$$

and

$$\sin B = \cos A$$

Step 1: Given:

$$\sin^{-1} \frac{5}{x} + \sin^{-1} \frac{12}{x} = \frac{\pi}{2}$$

Step 2: Let:

$$\sin^{-1} \frac{5}{x} = A$$

and

$$\sin^{-1} \frac{12}{x} = B$$

Then:

$$A + B = \frac{\pi}{2}$$

Step 3: Therefore:

$$B = \frac{\pi}{2} - A$$

Step 4: Since:

$$\sin B = \frac{12}{x}$$

and

$$B = \frac{\pi}{2} - A$$

we get:

$$\sin B = \sin\left(\frac{\pi}{2} - A\right)$$

$$\sin B = \cos A$$

So:

$$\frac{12}{x} = \cos A$$

Step 5: Also:

$$\sin A = \frac{5}{x}$$

Using:

$$\sin^2 A + \cos^2 A = 1$$

$$\left(\frac{5}{x}\right)^2 + \left(\frac{12}{x}\right)^2 = 1$$

$$\frac{25}{x^2} + \frac{144}{x^2} = 1$$

$$\frac{169}{x^2} = 1$$

$$x^2 = 169$$

$$x = 13$$

Therefore,

$$\boxed{13}$$

Quick Tip: When inverse sine terms add to $\frac{\pi}{2}$, use complementary angle relation and $\sin^2 A + \cos^2 A = 1$.

11. The number of solutions of the equation

$$\sin 2x - \cos 2x = 2 - \sin 2x$$

lying in the interval $[0, \pi]$ is

- (A) 0
- (B) 1
- (C) 2
- (D) 3

Correct Answer: (C) 2

Solution:

Concept: To find the number of solutions of a trigonometric equation in a given interval, first simplify the equation and then count the valid values of x in that interval.

Step 1: Given equation is:

$$\sin 2x - \cos 2x = 2 - \sin 2x$$

Bring all trigonometric terms to one side:

$$\sin 2x + \sin 2x - \cos 2x = 2$$

$$2 \sin 2x - \cos 2x = 2$$

Step 2: Since $x \in [0, \pi]$, then:

$$2x \in [0, 2\pi]$$

Let:

$$\theta = 2x$$

Then:

$$\theta \in [0, 2\pi]$$

The equation becomes:

$$2 \sin \theta - \cos \theta = 2$$

Step 3: Write the left side in standard form.

$$2 \sin \theta - \cos \theta = \sqrt{2^2 + (-1)^2} \sin(\theta - \alpha)$$

$$= \sqrt{5} \sin(\theta - \alpha)$$

where:

$$\cos \alpha = \frac{2}{\sqrt{5}}, \quad \sin \alpha = \frac{1}{\sqrt{5}}$$

So:

$$\sqrt{5} \sin(\theta - \alpha) = 2$$

$$\sin(\theta - \alpha) = \frac{2}{\sqrt{5}}$$

Step 4: Since $\frac{2}{\sqrt{5}}$ lies between -1 and 1 , the sine equation gives two solutions in one complete interval of length 2π .

Step 5: Therefore, for $\theta \in [0, 2\pi]$, there are two possible values.

Since $\theta = 2x$, corresponding to $x \in [0, \pi]$, the number of solutions is:

2

Quick Tip: For equations of the form $a \sin \theta + b \cos \theta = c$, convert the left side into $R \sin(\theta \pm \alpha)$ and then count the solutions in the given interval.

12. If

$$\tan \theta + \sec \theta = \sqrt{3},$$

then the principal value of θ in $[0, 2\pi]$ is

- (A) $\frac{\pi}{4}$
- (B) $\frac{\pi}{6}$
- (C) $\frac{\pi}{2}$
- (D) $\frac{2\pi}{3}$

Correct Answer: (B) $\frac{\pi}{6}$

Solution:

Concept: Use standard trigonometric values. Also remember:

$$\tan \theta + \sec \theta$$

is commonly checked using standard angles such as $30^\circ, 45^\circ, 60^\circ$.

Step 1: Given:

$$\tan \theta + \sec \theta = \sqrt{3}$$

Step 2: Test the standard value:

$$\theta = \frac{\pi}{6}$$

We know:

$$\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

Also:

$$\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

So:

$$\sec \frac{\pi}{6} = \frac{1}{\cos \frac{\pi}{6}}$$

$$\sec \frac{\pi}{6} = \frac{2}{\sqrt{3}}$$

Step 3: Add both values.

$$\tan \frac{\pi}{6} + \sec \frac{\pi}{6} = \frac{1}{\sqrt{3}} + \frac{2}{\sqrt{3}}$$

$$= \frac{3}{\sqrt{3}}$$

$$= \sqrt{3}$$

Step 4: Hence, the equation is satisfied for:

$$\theta = \frac{\pi}{6}$$

Therefore, the principal value of θ is:

$$\boxed{\frac{\pi}{6}}$$

Quick Tip: For trigonometric equations involving $\tan \theta$ and $\sec \theta$, first check standard angles like

$$\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}.$$

13.

$$\frac{\tan x - 1 + \sec x}{\tan x - \sec x + 1} =$$

- (A) $\frac{1-\sin x}{\cos x}$
(B) $\frac{1+\sin x}{\cos x}$
(C) $\frac{1+\cos x}{\sin x}$
(D) $\frac{1-\cos x}{\sin x}$

Correct Answer: (B) $\frac{1+\sin x}{\cos x}$

Solution:

Concept: To simplify expressions involving $\tan x$ and $\sec x$, convert them into $\sin x$ and $\cos x$:

$$\tan x = \frac{\sin x}{\cos x}, \quad \sec x = \frac{1}{\cos x}$$

Step 1: Given expression:

$$\frac{\tan x - 1 + \sec x}{\tan x - \sec x + 1}$$

Step 2: Substitute:

$$\tan x = \frac{\sin x}{\cos x}$$

and

$$\sec x = \frac{1}{\cos x}$$

So:

$$\frac{\frac{\sin x}{\cos x} - 1 + \frac{1}{\cos x}}{\frac{\sin x}{\cos x} - \frac{1}{\cos x} + 1}$$

Step 3: Simplify numerator.

$$\frac{\sin x + 1 - \cos x}{\cos x}$$

Step 4: Simplify denominator.

$$\frac{\sin x - 1 + \cos x}{\cos x}$$

Therefore, the full expression becomes:

$$\frac{\sin x + 1 - \cos x}{\sin x - 1 + \cos x}$$

Step 5: Now verify option (B).

Option (B) is:

$$\frac{1 + \sin x}{\cos x}$$

Using algebraic simplification, the expression reduces to:

$$\frac{1 + \sin x}{\cos x}$$

Therefore:

$$\boxed{\frac{1 + \sin x}{\cos x}}$$

Quick Tip: Whenever $\tan x$ and $\sec x$ appear together, convert them into $\sin x$ and $\cos x$ to simplify easily.

14.

$$\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ =$$

- (A) 2
- (B) 1
- (C) 4
- (D) 3

Correct Answer: (C) 4

Solution:

Concept: Use complementary angle identities:

$$\tan(90^\circ - \theta) = \cot \theta$$

Step 1: Given:

$$\tan 9^\circ - \tan 27^\circ - \tan 63^\circ + \tan 81^\circ$$

Step 2: Convert complementary angles.

$$\tan 81^\circ = \tan(90^\circ - 9^\circ) = \cot 9^\circ$$

$$\tan 63^\circ = \tan(90^\circ - 27^\circ) = \cot 27^\circ$$

Step 3: Substitute these values.

$$\tan 9^\circ - \tan 27^\circ - \cot 27^\circ + \cot 9^\circ$$

Rearrange:

$$(\tan 9^\circ + \cot 9^\circ) - (\tan 27^\circ + \cot 27^\circ)$$

Step 4: Use:

$$\tan A + \cot A = \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$$

$$= \frac{\sin^2 A + \cos^2 A}{\sin A \cos A}$$

$$= \frac{1}{\sin A \cos A}$$

So:

$$\tan A + \cot A = \frac{2}{\sin 2A}$$

Step 5: Apply this formula.

$$\tan 9^\circ + \cot 9^\circ = \frac{2}{\sin 18^\circ}$$

$$\tan 27^\circ + \cot 27^\circ = \frac{2}{\sin 54^\circ}$$

Hence:

$$\frac{2}{\sin 18^\circ} - \frac{2}{\sin 54^\circ}$$

Step 6: Using standard trigonometric simplification:

$$\frac{2}{\sin 18^\circ} - \frac{2}{\sin 54^\circ} = 4$$

Therefore:

$$\boxed{4}$$

Quick Tip: For angles like $9^\circ, 81^\circ$ and $27^\circ, 63^\circ$, use complementary angle identities to pair the terms.

15. If

$$\cos \theta = \frac{1}{2} \left(a + \frac{1}{a} \right),$$

then

$$4 \cos^3 \theta - 3 \cos \theta =$$

- (A) $a^3 + \frac{1}{a^3}$
- (B) $\frac{1}{2} \left(a^3 + \frac{1}{a^3} \right)$
- (C) $\frac{1}{4} \left(a^3 + \frac{1}{a^3} \right)$
- (D) $\frac{1}{3} \left(a^3 + \frac{1}{a^3} \right)$

Correct Answer: (B) $\frac{1}{2} \left(a^3 + \frac{1}{a^3} \right)$

Solution:

Concept: Use the triple angle identity:

$$\cos 3\theta = 4 \cos^3 \theta - 3 \cos \theta$$

Step 1: Given:

$$\cos \theta = \frac{1}{2} \left(a + \frac{1}{a} \right)$$

Step 2: We need to find:

$$4 \cos^3 \theta - 3 \cos \theta$$

Using the identity:

$$4 \cos^3 \theta - 3 \cos \theta = \cos 3\theta$$

Step 3: If:

$$\cos \theta = \frac{1}{2} \left(a + \frac{1}{a} \right)$$

then the corresponding triple angle form becomes:

$$\cos 3\theta = \frac{1}{2} \left(a^3 + \frac{1}{a^3} \right)$$

Step 4: Therefore:

$$4 \cos^3 \theta - 3 \cos \theta = \frac{1}{2} \left(a^3 + \frac{1}{a^3} \right)$$

Hence:

$$\boxed{\frac{1}{2} \left(a^3 + \frac{1}{a^3} \right)}$$

Quick Tip: Whenever you see $4 \cos^3 \theta - 3 \cos \theta$, immediately use $\cos 3\theta$.

16.

$$\cos 6^\circ \sin 24^\circ \cos 72^\circ =$$

- (A) $\frac{1}{4}$
- (B) $-\frac{1}{8}$
- (C) $-\frac{1}{4}$
- (D) $\frac{1}{8}$

Correct Answer: (D) $\frac{1}{8}$

Solution:

Concept: Use product-to-sum identities and special angle relations to simplify trigonometric products.

Step 1: Given expression:

$$\cos 6^\circ \sin 24^\circ \cos 72^\circ$$

Step 2: Use the identity:

$$2 \sin A \cos B = \sin(A + B) + \sin(A - B)$$

Take:

$$A = 24^\circ, \quad B = 72^\circ$$

Then:

$$2 \sin 24^\circ \cos 72^\circ = \sin 96^\circ + \sin(-48^\circ)$$

$$= \sin 96^\circ - \sin 48^\circ$$

Step 3: Therefore:

$$\sin 24^\circ \cos 72^\circ = \frac{1}{2}(\sin 96^\circ - \sin 48^\circ)$$

So the expression becomes:

$$\cos 6^\circ \cdot \frac{1}{2}(\sin 96^\circ - \sin 48^\circ)$$

$$= \frac{1}{2} \cos 6^\circ (\sin 96^\circ - \sin 48^\circ)$$

Step 4: Using standard trigonometric simplification of these related angles, we get:

$$\cos 6^\circ \sin 24^\circ \cos 72^\circ = \frac{1}{8}$$

Therefore:

$$\boxed{\frac{1}{8}}$$

Quick Tip: For products of sine and cosine terms, use product-to-sum identities to convert products into sums.

17.

$$\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 =$$

- (A) $\frac{3\pi}{4}$
- (B) $\frac{\pi}{2}$
- (C) π
- (D) 2π

Correct Answer: (C) π

Solution:

Concept: Use the inverse tangent addition formula:

$$\tan^{-1} a + \tan^{-1} b = \tan^{-1} \left(\frac{a + b}{1 - ab} \right)$$

But when $ab > 1$, quadrant correction is required.

Step 1: First add:

$$\tan^{-1} 1 + \tan^{-1} 2$$

Using the formula:

$$\tan^{-1} 1 + \tan^{-1} 2 = \tan^{-1} \left(\frac{1 + 2}{1 - 1 \cdot 2} \right)$$

$$= \tan^{-1} \left(\frac{3}{-1} \right)$$

$$= \tan^{-1}(-3)$$

Step 2: Since $1 \cdot 2 > 1$, the sum lies in the second quadrant.

Therefore:

$$\tan^{-1} 1 + \tan^{-1} 2 = \pi - \tan^{-1} 3$$

Step 3: Now add $\tan^{-1} 3$.

$$\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi - \tan^{-1} 3 + \tan^{-1} 3$$

$$= \pi$$

Therefore:

$$\boxed{\pi}$$

Quick Tip: In inverse tangent addition, check the sign of $1 - ab$. If $ab > 1$, apply quadrant correction.

18. If

$$z_1 = 4i^{40} - 5i^{35} + 6i^{17} + 2, \quad z_2 = -1 + i,$$

then

$$|z_1 + z_2| =$$

- (A) 13
- (B) 5
- (C) 15
- (D) 12

Correct Answer: (A) 13

Solution:

Concept: Powers of i repeat in a cycle of 4:

$$i^1 = i, \quad i^2 = -1, \quad i^3 = -i, \quad i^4 = 1$$

Step 1: Simplify i^{40} .

$$40 = 4 \times 10$$

So:

$$i^{40} = (i^4)^{10} = 1$$

Step 2: Simplify i^{35} .

$$35 = 4 \times 8 + 3$$

So:

$$i^{35} = i^3 = -i$$

Step 3: Simplify i^{17} .

$$17 = 4 \times 4 + 1$$

So:

$$i^{17} = i$$

Step 4: Substitute in z_1 .

$$z_1 = 4(1) - 5(-i) + 6(i) + 2$$

$$z_1 = 4 + 5i + 6i + 2$$

$$z_1 = 6 + 11i$$

Step 5: Now add $z_2 = -1 + i$.

$$z_1 + z_2 = (6 + 11i) + (-1 + i)$$

$$z_1 + z_2 = 5 + 12i$$

Step 6: Find modulus.

$$|5 + 12i| = \sqrt{5^2 + 12^2}$$

$$= \sqrt{25 + 144}$$

$$= \sqrt{169}$$

$$= 13$$

Therefore:

$$\boxed{13}$$

Quick Tip: To simplify powers of i , divide the exponent by 4 and use the remainder.

19. The conjugate of $(1 + i)^3$ is

- (A) $1 + 2i$
- (B) $-2 + 2i$
- (C) $-2 - 2i$
- (D) $1 - 2i$

Correct Answer: (C) $-2 - 2i$

Solution:

Concept: The conjugate of a complex number $a + bi$ is:

$$a - bi$$

So first simplify the complex expression, then change the sign of the imaginary part.

Step 1: Calculate:

$$(1 + i)^3$$

Step 2: First find:

$$(1 + i)^2$$

$$(1 + i)^2 = 1 + 2i + i^2$$

Since:

$$i^2 = -1$$

$$(1 + i)^2 = 1 + 2i - 1$$

$$= 2i$$

Step 3: Now:

$$(1 + i)^3 = (1 + i)^2(1 + i)$$

$$= 2i(1 + i)$$

$$= 2i + 2i^2$$

$$= 2i - 2$$

$$= -2 + 2i$$

Step 4: Find conjugate.

$$\overline{-2 + 2i} = -2 - 2i$$

Therefore:

$$\boxed{-2 - 2i}$$

Quick Tip: To find the conjugate, first simplify the complex number completely, then change the sign of the imaginary part.

20. The equation of a circle whose Centre is $(-3, 2)$ and area is 176 units is

(A) $x^2 + y^2 + 6x - 4y - 36 = 0$

(B) $x^2 + y^2 + 6x - 4y - 43 = 0$

(C) $x^2 + y^2 - 6x + 4y - 36 = 0$

(D) $x^2 + y^2 - 6x + 4y - 43 = 0$

Correct Answer: (B) $x^2 + y^2 + 6x - 4y - 43 = 0$

Solution:

Concept: The equation of a circle with centre (h, k) and radius r is:

$$(x - h)^2 + (y - k)^2 = r^2$$

Step 1: Given centre:

$$(h, k) = (-3, 2)$$

So:

$$(x + 3)^2 + (y - 2)^2 = r^2$$

Step 2: Given area:

$$\pi r^2 = 176$$

Using:

$$\pi = \frac{22}{7}$$

$$\frac{22}{7} r^2 = 176$$

$$r^2 = \frac{176 \times 7}{22}$$

$$r^2 = 56$$

Step 3: Substitute $r^2 = 56$.

$$(x + 3)^2 + (y - 2)^2 = 56$$

Step 4: Expand the equation.

$$x^2 + 6x + 9 + y^2 - 4y + 4 = 56$$

$$x^2 + y^2 + 6x - 4y + 13 = 56$$

$$x^2 + y^2 + 6x - 4y - 43 = 0$$

Therefore:

$$\boxed{x^2 + y^2 + 6x - 4y - 43 = 0}$$

Quick Tip: For circle problems, first find r^2 , then substitute into $(x - h)^2 + (y - k)^2 = r^2$.

21. The equation of a circle whose Centre is $(2, -1)$ and which passes through the point $(3, 6)$ is

(A) $x^2 + y^2 + 4x + 2y - 45 = 0$

(B) $x^2 + y^2 - 2x + 2y - 50 = 0$

(C) $x^2 + y^2 + 2x + 2y - 50 = 0$

$$(D) x^2 + y^2 - 4x + 2y - 45 = 0$$

Correct Answer: (D) $x^2 + y^2 - 4x + 2y - 45 = 0$

Solution:

Concept: If a circle has centre (h, k) , then its equation is:

$$(x - h)^2 + (y - k)^2 = r^2$$

If the circle passes through a point, use that point to find r^2 .

Step 1: Given centre:

$$(h, k) = (2, -1)$$

So:

$$(x - 2)^2 + (y + 1)^2 = r^2$$

Step 2: The circle passes through:

$$(3, 6)$$

So radius is the distance between $(2, -1)$ and $(3, 6)$.

$$r^2 = (3 - 2)^2 + (6 - (-1))^2$$

$$r^2 = (1)^2 + (7)^2$$

$$r^2 = 1 + 49$$

$$r^2 = 50$$

Step 3: Substitute $r^2 = 50$.

$$(x - 2)^2 + (y + 1)^2 = 50$$

Step 4: Expand.

$$x^2 - 4x + 4 + y^2 + 2y + 1 = 50$$

$$x^2 + y^2 - 4x + 2y + 5 = 50$$

$$x^2 + y^2 - 4x + 2y - 45 = 0$$

Therefore:

$$x^2 + y^2 - 4x + 2y - 45 = 0$$

Quick Tip: When a circle passes through a point, calculate r^2 using the distance formula from the centre to that point.

22. If the parabola $y^2 = 4ax$ passes through the point $(3, 2)$, then the length of its latus rectum is:

- (A) $\frac{4}{3}$
- (B) 4
- (C) $\frac{2}{3}$
- (D) $\frac{1}{3}$

Correct Answer: (A) $\frac{4}{3}$

Solution:

Concept: For the standard parabola:

$$y^2 = 4ax$$

the length of the latus rectum is:

$$4a$$

Step 1: Given parabola:

$$y^2 = 4ax$$

It passes through:

$$(3, 2)$$

Step 2: Substitute $x = 3$ and $y = 2$.

$$2^2 = 4a(3)$$

$$4 = 12a$$

$$a = \frac{1}{3}$$

Step 3: Length of latus rectum:

$$4a$$

$$4a = 4\left(\frac{1}{3}\right)$$

$$4a = \frac{4}{3}$$

Therefore:

$$\boxed{\frac{4}{3}}$$

Quick Tip: For $y^2 = 4ax$, the length of latus rectum is always $4a$.

23. The line $y = mx + 2$ is a tangent to the parabola $y^2 = 8x$ if

- (A) $m = 1$
- (B) $m = 2$
- (C) $m = 3$
- (D) $m = 4$

Correct Answer: (A) $m = 1$

Solution:

Concept: For the parabola:

$$y^2 = 4ax$$

the tangent with slope m is:

$$y = mx + \frac{a}{m}$$

Step 1: Given parabola:

$$y^2 = 8x$$

Compare with:

$$y^2 = 4ax$$

$$4a = 8$$

$$a = 2$$

Step 2: Standard tangent form is:

$$y = mx + \frac{a}{m}$$

Substitute $a = 2$:

$$y = mx + \frac{2}{m}$$

Step 3: Given line is:

$$y = mx + 2$$

Compare constant terms:

$$\frac{2}{m} = 2$$

$$2 = 2m$$

$$m = 1$$

Therefore:

$$m = 1$$

Quick Tip: For $y^2 = 4ax$, tangent with slope m is $y = mx + \frac{a}{m}$.

24. The length of the latus rectum and eccentricity of the Hyperbola

$$9x^2 - 16y^2 = 144$$

are

- (A) $(\frac{9}{4}, \frac{5}{4})$
 (B) $(\frac{9}{2}, \frac{5}{4})$
 (C) $(\frac{9}{2}, \frac{5}{2})$
 (D) $(9, \frac{5}{2})$

Correct Answer: (B) $(\frac{9}{2}, \frac{5}{4})$

Solution:

Concept: The standard form of a hyperbola with transverse axis along x-axis is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

For this hyperbola:

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

and length of latus rectum is:

$$\frac{2b^2}{a}$$

Step 1: Given equation:

$$9x^2 - 16y^2 = 144$$

Divide both sides by 144:

$$\frac{9x^2}{144} - \frac{16y^2}{144} = 1$$

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

Step 2: Compare with:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

So:

$$a^2 = 16, \quad b^2 = 9$$

$$a = 4, \quad b = 3$$

Step 3: Find eccentricity.

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

$$e = \sqrt{1 + \frac{9}{16}}$$

$$e = \sqrt{\frac{25}{16}}$$

$$e = \frac{5}{4}$$

Step 4: Find length of latus rectum.

$$\text{Length of latus rectum} = \frac{2b^2}{a}$$

$$= \frac{2(9)}{4}$$

$$= \frac{18}{4}$$

$$= \frac{9}{2}$$

Therefore:

$$\left(\frac{9}{2}, \frac{5}{4} \right)$$

Quick Tip: For $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, use $e = \sqrt{1 + \frac{b^2}{a^2}}$ and latus rectum $= \frac{2b^2}{a}$.

25. The equation of the ellipse with foci at $(\pm 3, 0)$ and the eccentricity as $\frac{1}{3}$ is:

(A) $\frac{x^2}{81} + \frac{y^2}{72} = 1$

(B) $\frac{x^2}{9} + \frac{y^2}{8} = 1$

(C) $\frac{x^2}{8} + \frac{y^2}{9} = 1$

(D) $\frac{x^2}{3} + \frac{y^2}{2} = 1$

Correct Answer: (A) $\frac{x^2}{81} + \frac{y^2}{72} = 1$

Solution:

Concept: For an ellipse with major axis along x -axis:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

where:

$$e = \frac{c}{a}$$

and:

$$b^2 = a^2 - c^2$$

Step 1: Given foci:

$$(\pm 3, 0)$$

So:

$$c = 3$$

Step 2: Given eccentricity:

$$e = \frac{1}{3}$$

Using:

$$e = \frac{c}{a}$$

$$\frac{1}{3} = \frac{3}{a}$$

$$a = 9$$

So:

$$a^2 = 81$$

Step 3: Find b^2 .

$$b^2 = a^2 - c^2$$

$$b^2 = 81 - 3^2$$

$$b^2 = 81 - 9$$

$$b^2 = 72$$

Step 4: Write the equation of ellipse.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{81} + \frac{y^2}{72} = 1$$

Therefore:

$$\boxed{\frac{x^2}{81} + \frac{y^2}{72} = 1}$$

Quick Tip: For ellipse with foci $(\pm c, 0)$, use $e = \frac{c}{a}$ and $b^2 = a^2 - c^2$.

26.

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x =$$

- (A) 0
- (B) 1
- (C) e
- (D) ∞

Correct Answer: (C) e

Solution:

Concept: This question is based on one of the most important standard limits in calculus:

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$$

Here, e is Euler's number.

Step 1: Observe the given limit.

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x$$

Step 2: Compare it with the standard limit.

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e$$

The given expression is exactly of the same form.

Step 3: Therefore, directly applying the standard result:

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$$

Hence, the required value is:

$$\boxed{e}$$

Quick Tip: Always remember the standard limit $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$. It is frequently used in limit problems.

27.

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x} =$$

- (A) 0
- (B) $\frac{1}{2}$
- (C) 1
- (D) ∞

Correct Answer: (B) $\frac{1}{2}$

Solution:

Concept: When a limit contains a square root expression such as $\sqrt{1+x} - 1$, rationalization is usually the best method.

Step 1: Given limit is:

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x}$$

Step 2: Direct substitution gives:

$$\frac{\sqrt{1+0} - 1}{0} = \frac{1-1}{0} = \frac{0}{0}$$

This is an indeterminate form.

Step 3: Rationalize the numerator by multiplying and dividing by:

$$\sqrt{1+x} + 1$$

$$\frac{\sqrt{1+x}-1}{x} \cdot \frac{\sqrt{1+x}+1}{\sqrt{1+x}+1}$$

Step 4: Use the identity:

$$(a-b)(a+b) = a^2 - b^2$$

So:

$$\begin{aligned} & \frac{(\sqrt{1+x})^2 - 1^2}{x(\sqrt{1+x} + 1)} \\ &= \frac{1+x-1}{x(\sqrt{1+x} + 1)} \\ &= \frac{x}{x(\sqrt{1+x} + 1)} \end{aligned}$$

Step 5: Cancel x .

$$= \frac{1}{\sqrt{1+x} + 1}$$

Step 6: Now put $x = 0$.

$$\frac{1}{\sqrt{1+0} + 1} = \frac{1}{1+1} = \frac{1}{2}$$

Therefore:

$$\boxed{\frac{1}{2}}$$

Quick Tip: For limits involving $\sqrt{1+x} - 1$, rationalize the numerator to remove the $0/0$ form.

28. If

$$y = \frac{a \cos x + b \sin x + c}{\sin x},$$

then

$$\frac{dy}{dx} =$$

- (A) $-a^2x - cx \cot x$
 (B) $-a$
 (C) $-a^2x + b \sec^2 x + cx \cot x$
 (D) $a^2x - cx \cot x$

Correct Answer: (A) $-a^2x - cx \cot x$

Solution:

Concept: Before differentiating trigonometric expressions, simplify them using identities:

$$\frac{\cos x}{\sin x} = \cot x$$

and

$$\frac{1}{\sin x} = x$$

Step 1: Given:

$$y = \frac{a \cos x + b \sin x + c}{\sin x}$$

Step 2: Split the fraction.

$$y = \frac{a \cos x}{\sin x} + \frac{b \sin x}{\sin x} + \frac{c}{\sin x}$$

$$y = a \cot x + b + cx$$

Step 3: Differentiate both sides with respect to x .

$$\frac{dy}{dx} = a \frac{d}{dx}(\cot x) + \frac{d}{dx}(b) + c \frac{d}{dx}(x)$$

Step 4: Use standard derivatives.

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(x) = 1$$

Also:

$$\frac{d}{dx}(b) = 0$$

Step 5: Substitute these values.

$$\frac{dy}{dx} = a(-2x) + 0 + c(-x \cot x)$$

$$\frac{dy}{dx} = -a^2x - cx \cot x$$

Therefore:

$$\boxed{-a^2x - cx \cot x}$$

Quick Tip: Before differentiating, simplify trigonometric fractions into $\cot x$, $\tan x$, $\sec x$, or x .

29. If

$$y = \sqrt{x + \sqrt{x + \sqrt{x + \cdots \infty}}},$$

then

$$\frac{dy}{dx} =$$

- (A) $\frac{1}{2y}$
- (B) $\frac{1}{1-2y}$
- (C) $\frac{1}{2(1-2y)}$
- (D) $\frac{-1}{1-2y}$

Correct Answer: (D) $\frac{-1}{1-2y}$

Solution:

Concept: In infinite radical expressions, the expression repeats again and again. Therefore, the repeated part can be replaced by the original variable.

Step 1: Given:

$$y = \sqrt{x + \sqrt{x + \sqrt{x + \cdots}}}$$

Step 2: Since the radical continues infinitely, the part after the first $x +$ is again equal to y .

So:

$$y = \sqrt{x + y}$$

Step 3: Square both sides.

$$y^2 = x + y$$

Step 4: Differentiate both sides with respect to x .

$$\frac{d}{dx}(y^2) = \frac{d}{dx}(x + y)$$

$$2y \frac{dy}{dx} = 1 + \frac{dy}{dx}$$

Step 5: Bring derivative terms to one side.

$$2y \frac{dy}{dx} - \frac{dy}{dx} = 1$$

$$(2y - 1) \frac{dy}{dx} = 1$$

Step 6: Solve for $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{1}{2y - 1}$$

This can also be written as:

$$\frac{dy}{dx} = \frac{-1}{1 - 2y}$$

Therefore:

$$\boxed{\frac{-1}{1 - 2y}}$$

Quick Tip: For infinite radicals, replace the repeating radical part by the original variable expression.

30. Slope of the tangent to the curve

$$y = 9x^2 + 7x^4 + 5$$

at the point $x = 1$ is

(A) 28

(B) 16

(C) 46

(D) $\frac{1}{46}$

Correct Answer: (C) 46

Solution:

Concept: The slope of the tangent to the curve $y = f(x)$ at a given point is obtained by differentiating y with respect to x :

$$\text{Slope} = \frac{dy}{dx}$$

Step 1: Given curve:

$$y = 9x^2 + 7x^4 + 5$$

Step 2: Differentiate with respect to x .

$$\frac{dy}{dx} = \frac{d}{dx}(9x^2) + \frac{d}{dx}(7x^4) + \frac{d}{dx}(5)$$

$$\frac{dy}{dx} = 18x + 28x^3 + 0$$

$$\frac{dy}{dx} = 18x + 28x^3$$

Step 3: Put $x = 1$.

$$\left. \frac{dy}{dx} \right|_{x=1} = 18(1) + 28(1)^3$$

$$= 18 + 28$$

$$= 46$$

Therefore, the slope of tangent is:

46

Quick Tip: The slope of tangent to $y = f(x)$ at $x = a$ is $\left. \frac{dy}{dx} \right|_{x=a}$.

31. If

$$f(x) = \begin{cases} 4(5^x), & x < 0 \\ 8k + x, & x \geq 0 \end{cases}$$

then $f'(-1) =$

- (A) $\frac{2}{5} \log 5$
- (B) $\frac{4}{5} \log 5$
- (C) $\frac{3}{5} \log 5$
- (D) $20 \log 5$

Correct Answer: (B) $\frac{4}{5} \log 5$

Solution:

Concept: For a piecewise function, choose the correct branch according to the value of x where the derivative is required.

Step 1: We need to find:

$$f'(-1)$$

Since:

$$-1 < 0$$

we use the branch:

$$f(x) = 4(5^x)$$

Step 2: Differentiate:

$$f(x) = 4(5^x)$$

Using:

$$\frac{d}{dx}(a^x) = a^x \log a$$

we get:

$$f'(x) = 4 \cdot 5^x \log 5$$

Step 3: Put $x = -1$.

$$f'(-1) = 4 \cdot 5^{-1} \log 5$$

$$f'(-1) = 4 \cdot \frac{1}{5} \log 5$$

$$f'(-1) = \frac{4}{5} \log 5$$

Therefore:

$$\boxed{\frac{4}{5} \log 5}$$

Quick Tip: For piecewise functions, first identify the correct interval for the given x , then differentiate only that branch.

32. If

$$2^x + 2^y = 2^{x+y},$$

then

$$\frac{dy}{dx} =$$

- (A) $1 - 2^y$
- (B) $1 - \frac{1}{2^y}$
- (C) $1 + 2^{-y}$
- (D) $1 + 2^y$

Correct Answer: (A) $1 - 2^y$

Solution:

Concept: Since y is a function of x , use implicit differentiation. Also:

$$\frac{d}{dx}(a^y) = a^y \log a \cdot \frac{dy}{dx}$$

Step 1: Given:

$$2^x + 2^y = 2^{x+y}$$

Step 2: Differentiate both sides with respect to x .

$$2^x \log 2 + 2^y \log 2 \frac{dy}{dx} = 2^{x+y} \log 2 \left(1 + \frac{dy}{dx} \right)$$

Step 3: Cancel log 2 from both sides.

$$2^x + 2^y \frac{dy}{dx} = 2^{x+y} \left(1 + \frac{dy}{dx} \right)$$

Step 4: From the original equation:

$$2^{x+y} = 2^x + 2^y$$

Substitute this:

$$2^x + 2^y \frac{dy}{dx} = (2^x + 2^y) \left(1 + \frac{dy}{dx} \right)$$

Step 5: Expand the right side.

$$2^x + 2^y \frac{dy}{dx} = 2^x + 2^y + 2^x \frac{dy}{dx} + 2^y \frac{dy}{dx}$$

Step 6: Cancel common terms.

$$0 = 2^y + 2^x \frac{dy}{dx}$$

$$2^x \frac{dy}{dx} = -2^y$$

$$\frac{dy}{dx} = -\frac{2^y}{2^x}$$

Step 7: From the original equation, divide by 2^x .

$$1 + 2^{y-x} = 2^y$$

$$2^{y-x} = 2^y - 1$$

So:

$$\frac{dy}{dx} = -(2^y - 1)$$

$$\frac{dy}{dx} = 1 - 2^y$$

Therefore:

$$\boxed{1 - 2^y}$$

Quick Tip: In implicit differentiation, treat y as a function of x , so $\frac{d}{dx}(a^y) = a^y \log a \cdot \frac{dy}{dx}$.

33. If

$$y + \sin^{-1}(1 - x^2) = e^x,$$

then

$$\frac{dy}{dx} =$$

(A) $e^x - \frac{2}{\sqrt{2-x^2}}$

(B) $e^x - \frac{2}{\sqrt{2+x^2}}$

(C) $e^x + \frac{2}{\sqrt{2-x^2}}$

(D) $e^x + \frac{2}{\sqrt{2+x^2}}$

Correct Answer: (C) $e^x + \frac{2}{\sqrt{2-x^2}}$

Solution:

Concept: Use the chain rule:

$$\frac{d}{dx}(\sin^{-1} u) = \frac{u'}{\sqrt{1-u^2}}$$

Step 1: Given:

$$y + \sin^{-1}(1 - x^2) = e^x$$

Step 2: Differentiate both sides with respect to x .

$$\frac{dy}{dx} + \frac{d}{dx}[\sin^{-1}(1 - x^2)] = e^x$$

Step 3: Let:

$$u = 1 - x^2$$

Then:

$$u' = -2x$$

So:

$$\frac{d}{dx}[\sin^{-1}(1 - x^2)] = \frac{-2x}{\sqrt{1-(1-x^2)^2}}$$

Step 4: Simplify the denominator.

$$1 - (1 - x^2)^2 = 1 - (1 - 2x^2 + x^4)$$

$$= 2x^2 - x^4$$

$$= x^2(2 - x^2)$$

So:

$$\sqrt{x^2(2 - x^2)} = x\sqrt{2 - x^2}$$

Step 5: Therefore:

$$\frac{-2x}{\sqrt{x^2(2 - x^2)}} = \frac{-2x}{x\sqrt{2 - x^2}}$$

$$= -\frac{2}{\sqrt{2 - x^2}}$$

Step 6: Substitute in the differentiated equation.

$$\frac{dy}{dx} - \frac{2}{\sqrt{2 - x^2}} = e^x$$

$$\frac{dy}{dx} = e^x + \frac{2}{\sqrt{2 - x^2}}$$

Therefore:

$$\boxed{e^x + \frac{2}{\sqrt{2 - x^2}}}$$

Quick Tip: For $\sin^{-1} u$, always apply chain rule carefully: $\frac{d}{dx}(\sin^{-1} u) = \frac{u'}{\sqrt{1 - u^2}}$.

34. If $y(x) = x^x$, $x > 0$, then

$$y''(2) - 2y'(2) =$$

- (A) $4\log_e 2 - 2$
- (B) $4\log_e 2 + 2$
- (C) $4(\log_e 2)^2 + 2$
- (D) $4(\log_e 2)^2 - 2$

Correct Answer: (D) $4(\log_e 2)^2 - 2$

Solution:

Concept: For functions like x^x , use logarithmic differentiation because both the base and exponent contain x .

Step 1: Given:

$$y = x^x$$

Take natural logarithm on both sides:

$$\log y = x \log x$$

Step 2: Differentiate both sides.

$$\frac{1}{y}y' = \log x + 1$$

So:

$$y' = y(\log x + 1)$$

Since:

$$y = x^x$$

we get:

$$y' = x^x(\log x + 1)$$

Step 3: Differentiate again.

$$y'' = \frac{d}{dx} [x^x(\log x + 1)]$$

Using product rule:

$$y'' = x^x(\log x + 1)(\log x + 1) + x^x \cdot \frac{1}{x}$$

$$y'' = x^x(\log x + 1)^2 + \frac{x^x}{x}$$

Step 4: Find $y'(2)$.

$$y'(2) = 2^2(\log_e 2 + 1)$$

$$y'(2) = 4(\log_e 2 + 1)$$

Step 5: Find $y''(2)$.

$$y''(2) = 2^2(\log_e 2 + 1)^2 + \frac{2^2}{2}$$

$$y''(2) = 4(\log_e 2 + 1)^2 + 2$$

Step 6: Now calculate:

$$y''(2) - 2y'(2)$$

$$= 4(\log_e 2 + 1)^2 + 2 - 2[4(\log_e 2 + 1)]$$

$$= 4(\log_e 2 + 1)^2 + 2 - 8(\log_e 2 + 1)$$

Let:

$$L = \log_e 2$$

Then:

$$= 4(L + 1)^2 + 2 - 8(L + 1)$$

$$= 4(L^2 + 2L + 1) + 2 - 8L - 8$$

$$= 4L^2 + 8L + 4 + 2 - 8L - 8$$

$$= 4L^2 - 2$$

Therefore:

$$\boxed{4(\log_e 2)^2 - 2}$$

Quick Tip: For x^x , use logarithmic differentiation: write $\log y = x \log x$.

35. If

$$z = x^2 y^3 + e^y \sin x,$$

then

$$\frac{\partial^2 z}{\partial x \partial y} =$$

- (A) $6xy^2 + e^y \cos x$
(B) $3x^2y^2 + e^y \sin x$
(C) $3x^2y^2 + e^y \cos x$
(D) $6xy^2 + e^y \sin x$

Correct Answer: (A) $6xy^2 + e^y \cos x$

Solution:

Concept: For mixed partial derivative:

$$\frac{\partial^2 z}{\partial x \partial y}$$

we first differentiate with respect to x , then differentiate the result with respect to y .

Step 1: Given:

$$z = x^2y^3 + e^y \sin x$$

Step 2: Differentiate with respect to x , treating y as constant.

$$\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}(x^2y^3) + \frac{\partial}{\partial x}(e^y \sin x)$$

$$\frac{\partial z}{\partial x} = 2xy^3 + e^y \cos x$$

Step 3: Now differentiate this result with respect to y .

$$\frac{\partial}{\partial y}(2xy^3 + e^y \cos x)$$

$$= 6xy^2 + e^y \cos x$$

Therefore:

$$\boxed{6xy^2 + e^y \cos x}$$

Quick Tip: In partial differentiation, treat the other variable as constant while differentiating.

36.

$$\int \frac{dx}{\sin^2 x \cos^2 x} =$$

- (A) $\tan x + \cot x + c$
- (B) $\tan x - \cot x + c$
- (C) $\tan x \cot x + c$
- (D) $\tan x + \sec x + c$

Correct Answer: (B) $\tan x - \cot x + c$

Solution:

Concept: Use the identity:

$$\sin^2 x + \cos^2 x = 1$$

Also:

$$\sec^2 x = \frac{1}{\cos^2 x}, \quad \csc^2 x = \frac{1}{\sin^2 x}$$

Step 1: Given integral:

$$\int \frac{dx}{\sin^2 x \cos^2 x}$$

Step 2: Write numerator 1 as:

$$1 = \sin^2 x + \cos^2 x$$

Therefore:

$$\frac{1}{\sin^2 x \cos^2 x} = \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}$$

Step 3: Split the fraction.

$$\frac{\sin^2 x}{\sin^2 x \cos^2 x} + \frac{\cos^2 x}{\sin^2 x \cos^2 x}$$

$$= \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}$$

$$= \sec^2 x + \csc^2 x$$

Step 4: Now integrate.

$$\int \frac{dx}{\sin^2 x \cos^2 x} = \int (\sec^2 x + \csc^2 x) dx$$

$$= \int \sec^2 x dx + \int \csc^2 x dx$$

Step 5: Use standard formulas.

$$\int \sec^2 x \, dx = \tan x$$

$$\int x \, dx = -\cot x$$

Therefore:

$$\int \frac{dx}{\sin^2 x \cos^2 x} = \tan x - \cot x + c$$

Hence:

$$\boxed{\tan x - \cot x + c}$$

Quick Tip: When denominator contains $\sin^2 x \cos^2 x$, use $1 = \sin^2 x + \cos^2 x$ to split the expression.

37.

$$\int \frac{dx}{\sqrt{x+1} + \sqrt{x}} =$$

- (A) $\frac{2}{3} \left[(x+1)^{\frac{3}{2}} - x^{\frac{3}{2}} \right] + c$
(B) $\frac{2}{3} \left[(x+1)^{\frac{3}{2}} + x^{\frac{3}{2}} \right] + c$
(C) $\frac{3}{2} \left[(x+1)^{\frac{3}{2}} - x^{\frac{3}{2}} \right] + c$
(D) $\frac{3}{2} \left[(x+1)^{\frac{3}{2}} + x^{\frac{3}{2}} \right] + c$

Correct Answer: (A) $\frac{2}{3} \left[(x+1)^{\frac{3}{2}} - x^{\frac{3}{2}} \right] + c$

Solution:

Concept: When a denominator contains:

$$\sqrt{x+1} + \sqrt{x}$$

we rationalize using the conjugate:

$$\sqrt{x+1} - \sqrt{x}$$

Step 1: Given:

$$\int \frac{dx}{\sqrt{x+1} + \sqrt{x}}$$

Step 2: Rationalize the denominator.

$$\begin{aligned}\frac{1}{\sqrt{x+1} + \sqrt{x}} &\cdot \frac{\sqrt{x+1} - \sqrt{x}}{\sqrt{x+1} - \sqrt{x}} \\ &= \frac{\sqrt{x+1} - \sqrt{x}}{(x+1) - x} \\ &= \sqrt{x+1} - \sqrt{x}\end{aligned}$$

Step 3: So the integral becomes:

$$\begin{aligned}\int (\sqrt{x+1} - \sqrt{x}) dx \\ &= \int (x+1)^{\frac{1}{2}} dx - \int x^{\frac{1}{2}} dx\end{aligned}$$

Step 4: Integrate both terms.

$$\begin{aligned}\int (x+1)^{\frac{1}{2}} dx &= \frac{(x+1)^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2}{3}(x+1)^{\frac{3}{2}} \\ \int x^{\frac{1}{2}} dx &= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2}{3}x^{\frac{3}{2}}\end{aligned}$$

Step 5: Combine the result.

$$\begin{aligned}\int \frac{dx}{\sqrt{x+1} + \sqrt{x}} &= \frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{2}{3}x^{\frac{3}{2}} + c \\ &= \frac{2}{3} \left[(x+1)^{\frac{3}{2}} - x^{\frac{3}{2}} \right] + c\end{aligned}$$

Therefore:

$$\boxed{\frac{2}{3} \left[(x+1)^{\frac{3}{2}} - x^{\frac{3}{2}} \right] + c}$$

Quick Tip: For integrals with $\sqrt{x+1} + \sqrt{x}$ in the denominator, rationalize using $\sqrt{x+1} - \sqrt{x}$.

38. If

$$\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx = A \sec x + Bx + c,$$

then (A, B) are

- (A) $(1, 1)$
- (B) $(-1, -1)$
- (C) $(1, -1)$
- (D) $(-1, 1)$

Correct Answer: (C) $(1, -1)$

Solution:

Concept: Split the numerator and convert the integrand into known derivative forms:

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

and

$$\frac{d}{dx}(x) = -x \cot x$$

Step 1: Given integral:

$$\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx$$

Step 2: Split the fraction.

$$\begin{aligned} \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} &= \frac{\sin^3 x}{\sin^2 x \cos^2 x} + \frac{\cos^3 x}{\sin^2 x \cos^2 x} \\ &= \frac{\sin x}{\cos^2 x} + \frac{\cos x}{\sin^2 x} \end{aligned}$$

Step 3: Convert into standard forms.

$$\frac{\sin x}{\cos^2 x} = \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} = \tan x \sec x$$

$$\frac{\cos x}{\sin^2 x} = \frac{\cos x}{\sin x} \cdot \frac{1}{\sin x} = \cot x x$$

Step 4: So:

$$\int \frac{\sin^3 x + \cos^3 x}{\sin^2 x \cos^2 x} dx = \int (\sec x \tan x + x \cot x) dx$$

Step 5: Integrate.

$$\int \sec x \tan x \, dx = \sec x$$

$$\int x \cot x \, dx = -x$$

Therefore:

$$\int (\sec x \tan x + x \cot x) \, dx = \sec x - x + c$$

Step 6: Compare with:

$$A \sec x + Bx + c$$

So:

$$A = 1, \quad B = -1$$

Hence:

$$\boxed{(1, -1)}$$

Quick Tip: Try to convert integrands into known derivative forms like $\sec x \tan x$ and $x \cot x$.

39. The integral of $f(x) = 1 + x^2 + x^4$ with respect to x^2 is

- (A) $x + \frac{x^3}{3} + \frac{x^5}{5} + C$
- (B) $\frac{x^2}{3} + \frac{x^4}{5} + C$
- (C) $x^2 + \frac{x^4}{4} + \frac{x^6}{6} + C$
- (D) $x^2 + \frac{x^4}{2} + \frac{x^6}{3} + C$

Correct Answer: (D) $x^2 + \frac{x^4}{2} + \frac{x^6}{3} + C$

Solution:

Concept: The phrase “with respect to x^2 ” means that x^2 should be treated as the variable of integration.

Step 1: Let:

$$t = x^2$$

Then:

$$x^4 = (x^2)^2 = t^2$$

Step 2: Rewrite the function:

$$f(x) = 1 + x^2 + x^4$$

In terms of t :

$$f(t) = 1 + t + t^2$$

Step 3: Integrate with respect to t .

$$\int (1 + t + t^2) dt$$

$$= t + \frac{t^2}{2} + \frac{t^3}{3} + C$$

Step 4: Substitute back $t = x^2$.

$$= x^2 + \frac{(x^2)^2}{2} + \frac{(x^2)^3}{3} + C$$

$$= x^2 + \frac{x^4}{2} + \frac{x^6}{3} + C$$

Therefore:

$$\boxed{x^2 + \frac{x^4}{2} + \frac{x^6}{3} + C}$$

Quick Tip: If integration is with respect to x^2 , put $t = x^2$ and integrate in terms of t .

40.

$$\int_0^{\frac{\pi}{2}} \frac{\sin^{100} x}{\sin^{100} x + \cos^{100} x} dx =$$

- (A) $\frac{\pi}{2}$
- (B) $\frac{\pi}{4}$
- (C) 100
- (D) 50

Correct Answer: (B) $\frac{\pi}{4}$

Solution:

Concept: Use the property of definite integrals:

$$\int_0^a f(x) dx = \int_0^a f(a-x) dx$$

Step 1: Let:

$$I = \int_0^{\frac{\pi}{2}} \frac{\sin^{100} x}{\sin^{100} x + \cos^{100} x} dx$$

Step 2: Apply the property:

$$I = \int_0^{\frac{\pi}{2}} f\left(\frac{\pi}{2} - x\right) dx$$

Since:

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

and:

$$\cos\left(\frac{\pi}{2} - x\right) = \sin x$$

we get:

$$I = \int_0^{\frac{\pi}{2}} \frac{\cos^{100} x}{\cos^{100} x + \sin^{100} x} dx$$

Step 3: Add the two expressions of I .

$$2I = \int_0^{\frac{\pi}{2}} \left[\frac{\sin^{100} x}{\sin^{100} x + \cos^{100} x} + \frac{\cos^{100} x}{\sin^{100} x + \cos^{100} x} \right] dx$$

Step 4: Combine the fractions.

$$2I = \int_0^{\frac{\pi}{2}} \frac{\sin^{100} x + \cos^{100} x}{\sin^{100} x + \cos^{100} x} dx$$

$$2I = \int_0^{\frac{\pi}{2}} 1 dx$$

$$2I = \frac{\pi}{2}$$

Step 5: Divide by 2.

$$I = \frac{\pi}{4}$$

Therefore:

$$\boxed{\frac{\pi}{4}}$$

Quick Tip: For definite integrals from 0 to $\frac{\pi}{2}$, use the transformation $x \rightarrow \frac{\pi}{2} - x$.

41.

$$\int_0^1 x \sqrt{x^2 + 4} dx =$$

- (A) $\frac{1}{3}[5\sqrt{5} - 4]$
(B) $\frac{1}{2}[5\sqrt{5} - 8]$
(C) $\frac{1}{3}[5\sqrt{5} - 8]$
(D) $\frac{1}{3}[5\sqrt{5} + 4]$

Correct Answer: (C) $\frac{1}{3}[5\sqrt{5} - 8]$

Solution:

Concept: Use substitution when the derivative of the inner expression is present outside.

Step 1: Given:

$$I = \int_0^1 x \sqrt{x^2 + 4} dx$$

Step 2: Put:

$$u = x^2 + 4$$

Differentiate:

$$\frac{du}{dx} = 2x$$

$$du = 2x dx$$

$$x dx = \frac{du}{2}$$

Step 3: Change limits.

When:

$$x = 0$$

$$u = 0^2 + 4 = 4$$

When:

$$x = 1$$

$$u = 1^2 + 4 = 5$$

Step 4: Substitute in the integral.

$$I = \int_4^5 \sqrt{u} \cdot \frac{du}{2}$$

$$I = \frac{1}{2} \int_4^5 u^{\frac{1}{2}} du$$

Step 5: Integrate.

$$I = \frac{1}{2} \left[\frac{u^{\frac{3}{2}}}{\frac{3}{2}} \right]_4^5$$

$$I = \frac{1}{3} \left[u^{\frac{3}{2}} \right]_4^5$$

Step 6: Apply limits.

$$I = \frac{1}{3} \left(5^{\frac{3}{2}} - 4^{\frac{3}{2}} \right)$$

$$5^{\frac{3}{2}} = 5\sqrt{5}$$

$$4^{\frac{3}{2}} = (\sqrt{4})^3 = 2^3 = 8$$

So:

$$I = \frac{1}{3} (5\sqrt{5} - 8)$$

Therefore:

$$\boxed{\frac{1}{3} [5\sqrt{5} - 8]}$$

Quick Tip: For integrals containing $x\sqrt{x^2 + a}$, use substitution $u = x^2 + a$.

42.

$$\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{\sin^5 x \cos^3 x}{x^4} dx =$$

- (A) $\frac{\pi}{2}$
- (B) $\frac{\pi}{4}$
- (C) 0
- (D) 1

Correct Answer: (C) 0

Solution:

Concept: If $f(x)$ is an odd function, then:

$$\int_{-a}^a f(x) dx = 0$$

Step 1: Given integrand:

$$f(x) = \frac{\sin^5 x \cos^3 x}{x^4}$$

Step 2: Check the nature of each factor.

We know:

$$\sin(-x) = -\sin x$$

So $\sin x$ is odd. Therefore:

$$\sin^5 x$$

is also odd.

Step 3: Also:

$$\cos(-x) = \cos x$$

So $\cos x$ is even. Therefore:

$$\cos^3 x$$

is also even.

Step 4: Also:

$$(-x)^4 = x^4$$

So x^4 is even.

Step 5: Therefore, the integrand is:

$$\frac{\text{odd} \times \text{even}}{\text{even}} = \text{odd}$$

So:

$$f(-x) = -f(x)$$

Step 6: The limits are symmetric:

$$-\frac{\pi}{6} \quad \text{to} \quad \frac{\pi}{6}$$

Therefore:

$$\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{\sin^5 x \cos^3 x}{x^4} dx = 0$$

Hence:

$$\boxed{0}$$

Quick Tip: Before solving symmetric definite integrals, always check whether the integrand is odd or even.

43.

$$\int \frac{dx}{\sqrt{16-25x^2}} =$$

- (A) $\frac{1}{5} \sin^{-1}\left(\frac{5x}{4}\right) + c$
- (B) $\sin^{-1}\left(\frac{5x}{4}\right) + c$
- (C) $\frac{1}{5} \sin^{-1}\left(\frac{x}{4}\right) + c$
- (D) $\frac{1}{5} \sin^{-1}\left(\frac{4x}{5}\right) + c$

Correct Answer: (A) $\frac{1}{5} \sin^{-1}\left(\frac{5x}{4}\right) + c$

Solution:

Concept: Use the standard formula:

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c$$

Step 1: Given:

$$\int \frac{dx}{\sqrt{16-25x^2}}$$

Step 2: Rewrite the expression inside the square root.

$$16 - 25x^2 = 4^2 - (5x)^2$$

So:

$$\sqrt{16 - 25x^2} = \sqrt{4^2 - (5x)^2}$$

Step 3: Put:

$$u = 5x$$

Then:

$$du = 5dx$$

$$dx = \frac{du}{5}$$

Step 4: Substitute.

$$\begin{aligned} \int \frac{dx}{\sqrt{16-25x^2}} &= \frac{1}{5} \int \frac{du}{\sqrt{16-u^2}} \\ &= \frac{1}{5} \int \frac{du}{\sqrt{4^2-u^2}} \end{aligned}$$

Step 5: Apply the formula.

$$\frac{1}{5} \sin^{-1} \left(\frac{u}{4} \right) + c$$

Step 6: Substitute $u = 5x$.

$$= \frac{1}{5} \sin^{-1} \left(\frac{5x}{4} \right) + c$$

Therefore:

$$\boxed{\frac{1}{5} \sin^{-1} \left(\frac{5x}{4} \right) + c}$$

Quick Tip: Convert $\sqrt{16-25x^2}$ into $\sqrt{4^2-(5x)^2}$, then use the inverse sine integration formula.

44. The solution of the differential equation

$$x \frac{dy}{dx} + y = 0$$

passing through the point (1, 1) is $y =$

- (A) x^2
- (B) x^{-1}
- (C) x^{-2}
- (D) x

Correct Answer: (B) x^{-1}

Solution:

Concept: This is a separable differential equation. We separate x and y terms and then integrate.

Step 1: Given:

$$x \frac{dy}{dx} + y = 0$$

Step 2: Move y to the right side.

$$x \frac{dy}{dx} = -y$$

$$\frac{dy}{dx} = -\frac{y}{x}$$

Step 3: Separate variables.

$$\frac{dy}{y} = -\frac{dx}{x}$$

Step 4: Integrate both sides.

$$\int \frac{dy}{y} = - \int \frac{dx}{x}$$

$$\log y = -\log x + \log C$$

Step 5: Simplify.

$$\log y + \log x = \log C$$

$$\log(xy) = \log C$$

$$xy = C$$

So:

$$y = \frac{C}{x}$$

Step 6: Use the point (1, 1).

$$1 = \frac{C}{1}$$

$$C = 1$$

Therefore:

$$y = \frac{1}{x}$$

$$y = x^{-1}$$

Hence:

$$\boxed{x^{-1}}$$

Quick Tip: For equations like $x \frac{dy}{dx} + y = 0$, separate variables and use the given point to find the constant.

45. Degree of the differential equation

$$y = x \frac{dy}{dx} + a \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

is

- (A) 4
- (B) 3
- (C) 2
- (D) 1

Correct Answer: (C) 2

Solution:

Concept: The degree of a differential equation is the highest power of the highest order derivative after removing radicals and fractions involving derivatives.

Step 1: Given:

$$y = x \frac{dy}{dx} + a \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$

Let:

$$p = \frac{dy}{dx}$$

Then:

$$y = xp + a\sqrt{1 + p^2}$$

Step 2: Move xp to the left side.

$$y - xp = a\sqrt{1 + p^2}$$

Step 3: Square both sides to remove the radical.

$$(y - xp)^2 = a^2(1 + p^2)$$

Step 4: Now the derivative $p = \frac{dy}{dx}$ appears with highest power 2.

Therefore, the degree of the differential equation is:

2

Quick Tip: To find degree, first remove radicals involving derivatives, then check the highest power of the highest order derivative.

46. The order of the differential equation of all circles passing through the origin and having their centres on the x -axis is

- (A) 4
- (B) 3
- (C) 2
- (D) 1

Correct Answer: (D) 1

Solution:

Concept: The order of a differential equation is the highest order derivative present in it. For a family of curves, the order is usually equal to the number of arbitrary constants.

Step 1: Since the centres are on the x -axis, let the centre be:

$$(a, 0)$$

Step 2: Since the circle passes through the origin, its radius is:

$$r = a$$

Step 3: Equation of the circle is:

$$(x - a)^2 + y^2 = a^2$$

Step 4: Expand.

$$x^2 - 2ax + a^2 + y^2 = a^2$$

Cancel a^2 :

$$x^2 + y^2 - 2ax = 0$$

Step 5: This family contains only one arbitrary constant a .

So only one differentiation is required to eliminate a .

Differentiate:

$$2x + 2y \frac{dy}{dx} - 2a = 0$$

Step 6: The highest derivative present is:

$$\frac{dy}{dx}$$

So the order is:

$$1$$

Therefore:

$$\boxed{1}$$

Quick Tip: The order of the differential equation of a family of curves is generally equal to the number of arbitrary constants.

47. If a and b are arbitrary constants, then the differential equation representing the family of curves

$$y = a \sin(x + b)$$

is

- (A) $\frac{d^2y}{dx^2} - y = 0$
(B) $\frac{d^2y}{dx^2} + y = 0$
(C) $\frac{d^2y}{dx^2} - y^2 = 0$
(D) $\frac{dy}{dx} - y = 0$

Correct Answer: (B) $\frac{d^2y}{dx^2} + y = 0$

Solution:

Concept: To form a differential equation from a family of curves, differentiate enough times to eliminate arbitrary constants.

Step 1: Given family of curves:

$$y = a \sin(x + b)$$

Here a and b are arbitrary constants.

Step 2: Differentiate once.

$$\frac{dy}{dx} = a \cos(x + b)$$

Step 3: Differentiate again.

$$\frac{d^2y}{dx^2} = -a \sin(x + b)$$

Step 4: From the original equation:

$$y = a \sin(x + b)$$

Therefore:

$$\frac{d^2y}{dx^2} = -y$$

Step 5: Rearrange.

$$\frac{d^2y}{dx^2} + y = 0$$

Therefore:

$$\boxed{\frac{d^2y}{dx^2} + y = 0}$$

Quick Tip: For $y = a \sin(x + b)$, differentiating twice gives $y'' = -y$, so the differential equation is $y'' + y = 0$.

48. The differential equation is

$$\frac{dy}{dx} + \frac{y}{x} = 0$$

and $y(1) = 2$. Then the value of $y(3)$ is

- (A) 2
- (B) 3
- (C) $\frac{2}{3}$
- (D) 1

Correct Answer: (C) $\frac{2}{3}$

Solution:

Concept: This is a separable differential equation. We separate variables and then use the given condition to find the constant.

Step 1: Given:

$$\frac{dy}{dx} + \frac{y}{x} = 0$$

Step 2: Move $\frac{y}{x}$ to the right side.

$$\frac{dy}{dx} = -\frac{y}{x}$$

Step 3: Separate variables.

$$\frac{dy}{y} = -\frac{dx}{x}$$

Step 4: Integrate both sides.

$$\int \frac{dy}{y} = -\int \frac{dx}{x}$$

$$\log y = -\log x + \log C$$

Step 5: Simplify.

$$\log y + \log x = \log C$$

$$\log(xy) = \log C$$

$$xy = C$$

So:

$$y = \frac{C}{x}$$

Step 6: Use $y(1) = 2$.

$$2 = \frac{C}{1}$$

$$C = 2$$

Thus:

$$y = \frac{2}{x}$$

Step 7: Find $y(3)$.

$$y(3) = \frac{2}{3}$$

Therefore:

$$\boxed{\frac{2}{3}}$$

Quick Tip: For equations of the form $\frac{dy}{dx} + \frac{y}{x} = 0$, the solution is usually $xy = C$.

49. The general solution of the differential equation

$$\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$$

is

- (A) $e^{-y} = e^x + \frac{x^3}{3} + c$
 (B) $e^y = e^x + \frac{x^3}{3} + c$
 (C) $e^y = e^x + x^3 + c$
 (D) $e^y = e^x + c$

Correct Answer: (B) $e^y = e^x + \frac{x^3}{3} + c$

Solution:

Concept: When the differential equation contains e^{-y} , multiplying by e^y can help because:

$$e^y \frac{dy}{dx} = \frac{d}{dx}(e^y)$$

Step 1: Given:

$$\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$$

Step 2: Rewrite:

$$e^{x-y} = e^x e^{-y}$$

So:

$$\frac{dy}{dx} = e^x e^{-y} + x^2 e^{-y}$$

$$\frac{dy}{dx} = e^{-y}(e^x + x^2)$$

Step 3: Multiply both sides by e^y .

$$e^y \frac{dy}{dx} = e^x + x^2$$

Step 4: Observe:

$$e^y \frac{dy}{dx} = \frac{d}{dx}(e^y)$$

Therefore:

$$\frac{d}{dx}(e^y) = e^x + x^2$$

Step 5: Integrate both sides.

$$\int d(e^y) = \int (e^x + x^2) dx$$

$$e^y = e^x + \frac{x^3}{3} + c$$

Therefore:

$$e^y = e^x + \frac{x^3}{3} + c$$

Quick Tip: If e^{-y} appears in a differential equation, multiply by e^y to form $\frac{d}{dx}(e^y)$.

50. The differential equation is

$$\frac{dy}{dx} + y \tan x = \sec x$$

and $y(0) = 1$. Then the value of $y\left(\frac{\pi}{4}\right)$ is

- (A) 0
- (B) $\sqrt{2}$
- (C) 1
- (D) -1

Correct Answer: (B) $\sqrt{2}$

Solution:

Concept: This is a linear differential equation of the form:

$$\frac{dy}{dx} + Py = Q$$

Its integrating factor is:

$$I.F. = e^{\int P dx}$$

Step 1: Given:

$$\frac{dy}{dx} + y \tan x = \sec x$$

Compare with:

$$\frac{dy}{dx} + Py = Q$$

So:

$$P = \tan x, \quad Q = \sec x$$

Step 2: Find the integrating factor.

$$I.F. = e^{\int \tan x \, dx}$$

We know:

$$\int \tan x \, dx = \log \sec x$$

So:

$$I.F. = e^{\log \sec x}$$

$$I.F. = \sec x$$

Step 3: Multiply the differential equation by $\sec x$.

$$\sec x \frac{dy}{dx} + y \sec x \tan x = \sec^2 x$$

Step 4: The left side is:

$$\frac{d}{dx}(y \sec x)$$

So:

$$\frac{d}{dx}(y \sec x) = \sec^2 x$$

Step 5: Integrate both sides.

$$y \sec x = \int \sec^2 x \, dx$$

$$y \sec x = \tan x + C$$

Step 6: Use $y(0) = 1$.

$$1 \cdot \sec 0 = \tan 0 + C$$

$$1 \cdot 1 = 0 + C$$

$$C = 1$$

Therefore:

$$y \sec x = \tan x + 1$$

Step 7: Put:

$$x = \frac{\pi}{4}$$

$$y\left(\frac{\pi}{4}\right) \sec \frac{\pi}{4} = \tan \frac{\pi}{4} + 1$$

We know:

$$\sec \frac{\pi}{4} = \sqrt{2}$$

and:

$$\tan \frac{\pi}{4} = 1$$

So:

$$y\left(\frac{\pi}{4}\right) \sqrt{2} = 1 + 1$$

$$y\left(\frac{\pi}{4}\right) \sqrt{2} = 2$$

$$y\left(\frac{\pi}{4}\right) = \frac{2}{\sqrt{2}}$$

$$y\left(\frac{\pi}{4}\right) = \sqrt{2}$$

Therefore:

$$\boxed{\sqrt{2}}$$

Quick Tip: For linear differential equations, find $I.F. = e^{\int P dx}$, multiply the equation by it, and then integrate.

51. If $P = Fv \sin \beta t$, where F is force and v is velocity, then the dimensions of P and β are

- (A) ML^2T^{-3}, T^{-1}
- (B) MLT^{-2}, T^{-2}
- (C) ML^2T^{-1}, T^{-1}
- (D) ML^2T^3, T^{-2}

Correct Answer: (A) ML^2T^{-3}, T^{-1}

Solution:

Concept: In dimensional analysis, the argument of a trigonometric function must always be dimensionless. Also, force multiplied by velocity gives power.

Step 1: Given relation is:

$$P = Fv \sin \beta t$$

Here $\sin \beta t$ is a trigonometric function.

Step 2: The quantity inside sine must be dimensionless.

$$[\beta t] = 1$$

$$[\beta][t] = 1$$

Since:

$$[t] = T$$

we get:

$$[\beta]T = 1$$

$$[\beta] = T^{-1}$$

Step 3: Now find the dimensions of P .

Since $\sin \beta t$ is dimensionless:

$$[P] = [F][v]$$

Step 4: Dimension of force is:

$$[F] = MLT^{-2}$$

Dimension of velocity is:

$$[v] = LT^{-1}$$

Step 5: Multiply both dimensions.

$$[P] = MLT^{-2} \times LT^{-1}$$

$$[P] = ML^2T^{-3}$$

Therefore:

$$[P] = ML^2T^{-3}, \quad [\beta] = T^{-1}$$

Hence:

$$ML^2T^{-3}, T^{-1}$$

Quick Tip: The angle of sin, cos, or tan must always be dimensionless. Use this idea to find unknown dimensions.

52. If velocity V , energy E , and time T are chosen as fundamental quantities, then dimensional representation of surface tension in this system will be

- (A) $E^1V^{-2}T^{-2}$
- (B) $E^1V^{-1}T^{-2}$
- (C) $E^{-2}V^{-1}T^{-3}$
- (D) $E^1V^{-2}T^{-1}$

Correct Answer: (A) $E^1V^{-2}T^{-2}$

Solution:

Concept: Surface tension is force per unit length.

$$S = \frac{F}{L}$$

Its dimensional formula is:

$$[S] = \frac{MLT^{-2}}{L} = MT^{-2}$$

Step 1: Write dimensions of the new fundamental quantities.

Energy:

$$[E] = ML^2T^{-2}$$

Velocity:

$$[V] = LT^{-1}$$

Time:

$$[T] = T$$

Step 2: Assume surface tension is represented as:

$$[S] = E^a V^b T^c$$

Step 3: Substitute dimensions.

$$MT^{-2} = (ML^2T^{-2})^a (LT^{-1})^b T^c$$

$$MT^{-2} = M^a L^{2a+b} T^{-2a-b+c}$$

Step 4: Compare powers of M , L , and T .

For M :

$$a = 1$$

For L :

$$2a + b = 0$$

Substitute $a = 1$:

$$2 + b = 0$$

$$b = -2$$

For T :

$$-2a - b + c = -2$$

Substitute $a = 1$ and $b = -2$:

$$-2(1) - (-2) + c = -2$$

$$-2 + 2 + c = -2$$

$$c = -2$$

Step 5: Therefore:

$$[S] = E^1 V^{-2} T^{-2}$$

Hence:

$$E^1 V^{-2} T^{-2}$$

Quick Tip: When new fundamental quantities are given, assume the required quantity as powers of them and compare dimensions.

53. If $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$, then the angle between the two vectors \vec{A} and \vec{B} is

- (A) 0°
- (B) 180°
- (C) 120°
- (D) 90°

Correct Answer: (D) 90°

Solution:

Concept: For two vectors \vec{A} and \vec{B} , if the angle between them is θ , then:

$$|\vec{A} + \vec{B}|^2 = A^2 + B^2 + 2AB \cos \theta$$

and

$$|\vec{A} - \vec{B}|^2 = A^2 + B^2 - 2AB \cos \theta$$

Step 1: Given:

$$|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$$

Step 2: Square both sides.

$$|\vec{A} + \vec{B}|^2 = |\vec{A} - \vec{B}|^2$$

Step 3: Substitute formulas.

$$A^2 + B^2 + 2AB \cos \theta = A^2 + B^2 - 2AB \cos \theta$$

Step 4: Cancel common terms $A^2 + B^2$.

$$2AB \cos \theta = -2AB \cos \theta$$

$$4AB \cos \theta = 0$$

Step 5: Since A and B are non-zero vectors:

$$\cos \theta = 0$$

$$\theta = 90^\circ$$

Therefore:

$$\boxed{90^\circ}$$

Quick Tip: If $|\vec{A} + \vec{B}| = |\vec{A} - \vec{B}|$, then the two vectors are perpendicular.

54. An aeroplane is moving in a circular path with a speed of 450 kmph. What is the change in velocity in half revolution?

- (A) 0 kmph
- (B) 450 kmph
- (C) 250 kmph
- (D) 900 kmph

Correct Answer: (D) 900 kmph

Solution:

Concept: Velocity is a vector quantity. In circular motion, speed may remain constant, but the direction of velocity changes continuously.

Step 1: Given speed of aeroplane:

$$v = 450 \text{ kmph}$$

Step 2: In half revolution, the aeroplane reaches the diametrically opposite point of the circular path.

At this point, the direction of velocity is exactly opposite to the initial direction.

Step 3: Let initial velocity be:

$$\vec{v}_i = 450$$

Then final velocity after half revolution is:

$$\vec{v}_f = -450$$

Step 4: Change in velocity is:

$$\Delta\vec{v} = \vec{v}_f - \vec{v}_i$$

$$\Delta\vec{v} = -450 - 450$$

$$\Delta\vec{v} = -900$$

Step 5: Magnitude of change in velocity:

$$|\Delta\vec{v}| = 900 \text{ kmph}$$

Therefore:

$$900 \text{ kmph}$$

Quick Tip: In half revolution of circular motion, velocity reverses direction, so change in velocity is $2v$.

55. The ratio between maximum and minimum values of two vectors \vec{A} and \vec{B} , where $A > B$, is 4 : 1. Then the ratio between the magnitudes of two vectors is

- (A) 3 : 2
- (B) 5 : 3
- (C) 2 : 3
- (D) 3 : 5

Correct Answer: (B) 5 : 3

Solution:

Concept: For two vectors \vec{A} and \vec{B} , the maximum resultant is obtained when both vectors are in the same direction:

$$R_{\max} = A + B$$

The minimum resultant is obtained when both vectors are in opposite directions:

$$R_{\min} = A - B$$

where $A > B$.

Step 1: Given ratio:

$$\frac{R_{\max}}{R_{\min}} = \frac{4}{1}$$

Step 2: Substitute:

$$R_{\max} = A + B$$

and

$$R_{\min} = A - B$$

So:

$$\frac{A + B}{A - B} = 4$$

Step 3: Cross multiply.

$$A + B = 4(A - B)$$

$$A + B = 4A - 4B$$

Step 4: Bring like terms together.

$$4A - A = B + 4B$$

$$3A = 5B$$

Step 5: Therefore:

$$\frac{A}{B} = \frac{5}{3}$$

So the ratio of magnitudes is:

$$\boxed{5 : 3}$$

Quick Tip: Maximum resultant of two vectors is $A + B$, and minimum resultant is $A - B$ when $A > B$.

56. The magnitudes of three vectors \vec{A} , \vec{B} , \vec{C} are 12, 5, and 13 units respectively and $\vec{A} + \vec{B} = \vec{C}$.

The angle between \vec{A} and \vec{B} is

- (A) 0°
- (B) 120°
- (C) 90°
- (D) 45°

Correct Answer: (C) 90°

Solution:

Concept: If:

$$\vec{A} + \vec{B} = \vec{C}$$

then:

$$C^2 = A^2 + B^2 + 2AB \cos \theta$$

where θ is the angle between \vec{A} and \vec{B} .

Step 1: Given:

$$A = 12, \quad B = 5, \quad C = 13$$

Step 2: Apply resultant formula.

$$13^2 = 12^2 + 5^2 + 2(12)(5) \cos \theta$$

Step 3: Simplify.

$$169 = 144 + 25 + 120 \cos \theta$$

$$169 = 169 + 120 \cos \theta$$

Step 4: Subtract 169 from both sides.

$$0 = 120 \cos \theta$$

$$\cos \theta = 0$$

Step 5: Therefore:

$$\theta = 90^\circ$$

Hence:

$$90^\circ$$

Quick Tip: If $A^2 + B^2 = C^2$, then the vectors \vec{A} and \vec{B} are perpendicular.

57. A body falling from height H takes time T seconds to reach the ground. The time taken to cover the second half of height is

- (A) $\frac{T}{\sqrt{2}}$
- (B) $\sqrt{2}T$
- (C) $\frac{\sqrt{2}-1}{\sqrt{2}}T$
- (D) $\frac{1}{\sqrt{2}-1}T$

Correct Answer: (C) $\frac{\sqrt{2}-1}{\sqrt{2}}T$

Solution:

Concept: For a freely falling body starting from rest:

$$s = \frac{1}{2}gt^2$$

Step 1: Total height is H and total time is T .

So:

$$H = \frac{1}{2}gT^2$$

Step 2: Let time taken to cover first half height $\frac{H}{2}$ be t_1 .

Then:

$$\frac{H}{2} = \frac{1}{2}gt_1^2$$

Step 3: Divide this equation by the total height equation.

$$\frac{\frac{H}{2}}{H} = \frac{\frac{1}{2}gt_1^2}{\frac{1}{2}gT^2}$$

$$\frac{1}{2} = \frac{t_1^2}{T^2}$$

$$t_1^2 = \frac{T^2}{2}$$

$$t_1 = \frac{T}{\sqrt{2}}$$

Step 4: Time taken to cover second half height is:

$$t_2 = T - t_1$$

$$t_2 = T - \frac{T}{\sqrt{2}}$$

$$t_2 = T \left(1 - \frac{1}{\sqrt{2}} \right)$$

$$t_2 = \frac{\sqrt{2}-1}{\sqrt{2}} T$$

Therefore:

$$\boxed{\frac{\sqrt{2}-1}{\sqrt{2}} T}$$

Quick Tip: For free fall from rest, distance is proportional to square of time: $s \propto t^2$.

58. With what speed should a body be thrown upwards so that the distances covered in the 5th second and 6th second are equal?

- (A) 75 m/s
- (B) $\sqrt{98}$ m/s
- (C) 49 m/s
- (D) 19.8 m/s

Correct Answer: (C) 49 m/s

Solution:

Concept: For a body thrown vertically upward, the velocity becomes zero at the highest point.

Equal distances in the 5th and 6th seconds occur when the highest point lies between these intervals.

Step 1: Let the initial speed be u .

For vertical upward motion:

$$v = u - gt$$

Step 2: At the highest point:

$$v = 0$$

So:

$$0 = u - gt$$

$$u = gt$$

Step 3: Since distances covered in the 5th second and 6th second are equal, the highest point occurs at:

$$t = 5 \text{ s}$$

Step 4: Using:

$$g = 9.8 \text{ m/s}^2$$

$$u = gt$$

$$u = 9.8 \times 5$$

$$u = 49 \text{ m/s}$$

Therefore:

$$\boxed{49 \text{ m/s}}$$

Quick Tip: For vertical upward motion, velocity at the highest point is zero. Use $v = u - gt$.

59. A body of mass 1 kg starts moving from rest under the action of a force which varies with

displacement as $F = 2x + 5$ in newtons. The work done by this force to displace the body from $x = 0$ to $x = 2$ m is

- (A) 8 J
- (B) 10 J
- (C) 12 J
- (D) 14 J

Correct Answer: (D) 14 J

Solution:

Concept: When force varies with displacement, work done is:

$$W = \int_{x_1}^{x_2} F dx$$

Step 1: Given:

$$F = 2x + 5$$

Displacement is from:

$$x = 0$$

to:

$$x = 2$$

Step 2: Write the work done:

$$W = \int_0^2 (2x + 5) dx$$

Step 3: Integrate.

$$W = [x^2 + 5x]_0^2$$

Step 4: Apply limits.

$$W = (2^2 + 5(2)) - (0^2 + 5(0))$$

$$W = (4 + 10) - 0$$

$$W = 14 \text{ J}$$

Therefore:

$$14 \text{ J}$$

Quick Tip: For variable force, use $W = \int F dx$, not $W = Fs$ directly.

60. The potential energy of a particle is given by $U(x) = 20 + (x - 2)^2$, where U is in joules and x is in meters. The minimum potential energy and the position where it occurs are

- (A) 20 J at $x = 2$
- (B) 2 J at $x = 20$ m
- (C) 22 J at $x = 2$ m
- (D) 0 J at $x = 2$ m

Correct Answer: (A) 20 J at $x = 2$

Solution:

Concept: A square term is always non-negative:

$$(x - 2)^2 \geq 0$$

The minimum value occurs when the square term becomes zero.

Step 1: Given:

$$U(x) = 20 + (x - 2)^2$$

Step 2: Since:

$$(x - 2)^2 \geq 0$$

the least possible value of $(x - 2)^2$ is:

$$0$$

Step 3: This happens when:

$$x - 2 = 0$$

$$x = 2$$

Step 4: Substitute $x = 2$ in $U(x)$.

$$U(2) = 20 + (2 - 2)^2$$

$$U(2) = 20 + 0$$

$$U(2) = 20 \text{ J}$$

Therefore, minimum potential energy is:

$$\boxed{20 \text{ J at } x = 2}$$

Quick Tip: For expressions like $a + (x - b)^2$, the minimum value is a , occurring at $x = b$.

61. Power supplied to a particle of mass 2 kg varies with time as

$$P = \frac{3t^2}{2} \text{ watt,}$$

where t is in seconds. If velocity at $t = 0$ is zero, the velocity at $t = 2$ s is

- (A) 1 m/s
- (B) 2 m/s
- (C) $\sqrt{2}$ m/s
- (D) 4 m/s

Correct Answer: (B) 2 m/s

Solution:

Concept: Power is the rate of doing work:

$$P = \frac{dW}{dt}$$

Work done is equal to change in kinetic energy:

$$W = \Delta K$$

Step 1: Given:

$$P = \frac{3t^2}{2}$$

Mass:

$$m = 2 \text{ kg}$$

Initial velocity:

$$u = 0$$

Step 2: Work done from $t = 0$ to $t = 2$ is:

$$W = \int_0^2 P dt$$

$$W = \int_0^2 \frac{3t^2}{2} dt$$

$$W = \frac{3}{2} \int_0^2 t^2 dt$$

$$W = \frac{3}{2} \left[\frac{t^3}{3} \right]_0^2$$

$$W = \frac{3}{2} \cdot \frac{8}{3}$$

$$W = 4 \text{ J}$$

Step 3: This work becomes kinetic energy.

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

Since $u = 0$:

$$W = \frac{1}{2}mv^2$$

Step 4: Substitute values.

$$4 = \frac{1}{2}(2)v^2$$

$$4 = v^2$$

$$v = 2 \text{ m/s}$$

Therefore:

$$\boxed{2 \text{ m/s}}$$

Quick Tip: When power is given as a function of time, integrate power with respect to time to get work done.

62. A pump is used to deliver water at a certain rate from a given pipe. To obtain twice the volume of water from the same pipe in the same time, by what factor must the power of the motor pump be increased? Assume ideal conditions.

- (A) 4
- (B) 8
- (C) 16
- (D) 32

Correct Answer: (B) 8

Solution:

Concept: For water flowing through the same pipe:

$$Q = Av$$

where Q is volume flow rate, A is area of pipe, and v is velocity of water.

For ideal conditions, power required is proportional to:

$$P \propto v^3$$

Step 1: Same pipe means area A is constant.

Step 2: To obtain twice the volume of water in the same time:

$$Q' = 2Q$$

Since:

$$Q = Av$$

and A is constant:

$$v' = 2v$$

Step 3: Power is proportional to cube of velocity.

$$P \propto v^3$$

So:

$$\frac{P'}{P} = \left(\frac{v'}{v}\right)^3$$

$$\frac{P'}{P} = (2)^3$$

$$\frac{P'}{P} = 8$$

Therefore, power must be increased by a factor of:

$$\boxed{8}$$

Quick Tip: For flow through the same pipe, if speed becomes n times, power becomes n^3 times.

63. Two identical piano wires, when tuned to a fundamental frequency of 400 Hz, produce no beats. One wire is then slightly tightened, and the beat frequency heard is 2 Hz. What is the new fundamental frequency of the tightened wire?

- (A) 398 Hz
- (B) 402 Hz

(C) 404 Hz

(D) 396 Hz

Correct Answer: (B) 402 Hz

Solution:

Concept: Beat frequency is equal to the difference between two frequencies:

$$f_b = |f_1 - f_2|$$

Step 1: Initially both wires have the same frequency:

$$f_1 = 400 \text{ Hz}$$

So no beats are produced.

Step 2: One wire is tightened.

For a stretched wire:

$$f \propto \sqrt{T}$$

So if tension increases, frequency also increases.

Step 3: Beat frequency is:

$$f_b = 2 \text{ Hz}$$

Let the new frequency be f_2 .

$$|f_2 - 400| = 2$$

Step 4: Since wire is tightened, frequency increases.

$$f_2 = 400 + 2$$

$$f_2 = 402 \text{ Hz}$$

Therefore:

$$\boxed{402 \text{ Hz}}$$

Quick Tip: Tightening a string increases its frequency, so choose the value greater than the original frequency.

64. A source of sound of frequency 500 Hz is moving towards an observer with velocity 30 m/s. The speed of sound is 330 m/s. The frequency heard by the observer will be

- (A) 450 Hz
- (B) 550 Hz
- (C) 600 Hz
- (D) 500 Hz

Correct Answer: (B) 550 Hz

Solution:

Concept: When a sound source moves towards a stationary observer, the apparent frequency increases. The formula is:

$$f' = f \left(\frac{v}{v - v_s} \right)$$

Step 1: Given:

$$f = 500 \text{ Hz}$$

$$v = 330 \text{ m/s}$$

$$v_s = 30 \text{ m/s}$$

Step 2: Substitute in Doppler effect formula.

$$f' = 500 \left(\frac{330}{330 - 30} \right)$$

$$f' = 500 \left(\frac{330}{300} \right)$$

$$f' = 500 \left(\frac{11}{10} \right)$$

$$f' = 550 \text{ Hz}$$

Therefore:

550 Hz

Quick Tip: When source moves towards observer, apparent frequency increases and denominator becomes $v - v_s$.

65. In acoustics, “Noise” is generally characterized by

- (A) Irregular and non-periodic vibrations
- (B) A constant pitch and frequency
- (C) Vibrations that follow a harmonic series
- (D) Regular and periodic vibrations

Correct Answer: (A) Irregular and non-periodic vibrations

Solution:

Concept: Sound is produced due to vibrations. If vibrations are regular and periodic, the sound is usually musical. If vibrations are irregular and non-periodic, the sound is called noise.

Step 1: Musical sound has regular vibration.

Step 2: Noise is unwanted and unpleasant sound.

Step 3: Noise does not have a fixed pattern or regular periodicity.

Step 4: Hence, noise is characterized by:

Irregular and non-periodic vibrations

Therefore:

Irregular and non-periodic vibrations

Quick Tip: Regular periodic vibrations produce musical sound, while irregular non-periodic vibrations produce noise.

66. If the volume of a room is doubled and the total absorption is halved, the reverberation time will

- (A) Remain unchanged
- (B) Be doubled
- (C) Become four times
- (D) Be halved

Correct Answer: (C) Become four times

Solution:

Concept: According to Sabine's formula:

$$T = \frac{0.161V}{A}$$

where T is reverberation time, V is volume of the room, and A is total absorption.

Step 1: Original reverberation time:

$$T = \frac{0.161V}{A}$$

Step 2: New volume:

$$V' = 2V$$

New absorption:

$$A' = \frac{A}{2}$$

Step 3: New reverberation time:

$$T' = \frac{0.161V'}{A'}$$

$$T' = \frac{0.161(2V)}{\frac{A}{2}}$$

$$T' = 0.161(2V) \cdot \frac{2}{A}$$

$$T' = 4 \left(\frac{0.161V}{A} \right)$$

$$T' = 4T$$

Therefore, reverberation time becomes:

four times

Quick Tip: Reverberation time is directly proportional to room volume and inversely proportional to total absorption.

67. In a closed hall of volume 5000 m^3 , the total absorption of the interior surfaces is 200 metric sabin. The reverberation time is

- (A) 1 s
- (B) 2 s
- (C) 3 s
- (D) 4 s

Correct Answer: (D) 4 s

Solution:

Concept: Sabine's formula for reverberation time is:

$$T = \frac{0.161V}{A}$$

where V is volume and A is total absorption.

Step 1: Given:

$$V = 5000 \text{ m}^3$$

$$A = 200$$

Step 2: Apply Sabine's formula.

$$T = \frac{0.161 \times 5000}{200}$$

Step 3: Calculate numerator.

$$0.161 \times 5000 = 805$$

Step 4: Divide by 200.

$$T = \frac{805}{200}$$

$$T = 4.025 \text{ s}$$

Approximately:

$$T \approx 4 \text{ s}$$

Therefore:

$$\boxed{4 \text{ s}}$$

Quick Tip: Use Sabine's formula $T = \frac{0.161V}{A}$ for reverberation time problems.

68. In an isothermal process

- (A) Internal energy of the system never remains constant
- (B) Total heat energy of the system remains constant
- (C) Volume of the system remains constant
- (D) Temperature of the system remains constant

Correct Answer: (D) Temperature of the system remains constant

Solution:

Concept: The word isothermal is made from:

iso = same, thermal = temperature

Step 1: In an isothermal process:

$$T = \text{constant}$$

Step 2: This means the temperature of the system remains constant throughout the process.

Step 3: For an ideal gas, internal energy depends only on temperature. So in an ideal gas isothermal process, internal energy also remains constant.

Step 4: But the defining condition of an isothermal process is:

Temperature remains constant

Therefore:

Temperature of the system remains constant

Quick Tip: Isothermal means constant temperature; isobaric means constant pressure; isochoric means constant volume.

69. If the pressure of an ideal gas is doubled and its absolute temperature is halved, the volume will become

- (A) $\frac{1}{4}$ of initial volume
- (B) $\frac{1}{2}$ initial volume
- (C) Same as initial volume
- (D) 2 times of initial volume

Correct Answer: (A) $\frac{1}{4}$ of initial volume

Solution:

Concept: For a fixed amount of ideal gas:

$$PV = nRT$$

Since n and R are constant:

$$\frac{PV}{T} = \text{constant}$$

Step 1: Let initial pressure, volume, and absolute temperature be:

$$P, V, T$$

Step 2: Final pressure is doubled:

$$P' = 2P$$

Final absolute temperature is halved:

$$T' = \frac{T}{2}$$

Let final volume be:

$$V'$$

Step 3: Use combined gas law.

$$\frac{PV}{T} = \frac{P'V'}{T'}$$

Substitute values:

$$\frac{PV}{T} = \frac{(2P)V'}{\frac{T}{2}}$$

$$\frac{PV}{T} = \frac{4PV'}{T}$$

Step 4: Cancel P and T .

$$V = 4V'$$

$$V' = \frac{V}{4}$$

Therefore, the volume becomes:

$\frac{1}{4}$ of initial volume

Quick Tip: For a fixed amount of gas, use $\frac{PV}{T} = \text{constant}$. Always use absolute temperature.

70. At constant temperature, the product PV is plotted against pressure P for an ideal gas.

The graph obtained is

- (A) Straight line parallel to P -axis
- (B) Straight line with positive slope
- (C) Straight line through origin
- (D) Parabola

Correct Answer: (A) Straight line parallel to P -axis

Solution:

Concept: For an ideal gas:

$$PV = nRT$$

At constant temperature and fixed amount of gas, PV remains constant.

Step 1: Given condition:

$$T = \text{constant}$$

For a fixed amount of gas:

$$n = \text{constant}$$

Step 2: From ideal gas equation:

$$PV = nRT$$

Since n , R , and T are constant:

$$PV = \text{constant}$$

Step 3: This means PV does not change with pressure P .

Step 4: If PV is plotted on the vertical axis and P on the horizontal axis, then the graph is a horizontal straight line.

A horizontal straight line is parallel to the P -axis.

Therefore:

Straight line parallel to P -axis

Quick Tip: At constant temperature, PV is constant for an ideal gas, so the PV versus P graph is horizontal.

71. A bubble of an ideal gas rises from the bottom of a lake to the surface. At the bottom, the pressure is 3 atm and the temperature is 7°C . At the surface, the pressure is 1 atm and the temperature is 27°C . If the initial volume of the bubble was V_0 , what is its volume V at the surface?

- (A) $3V_0$
- (B) $3.21V_0$
- (C) $0.9V_0$
- (D) $5.4V_0$

Correct Answer: (B) $3.21V_0$

Solution:

Concept: For a fixed amount of ideal gas, the combined gas law is:

$$\frac{PV}{T} = \text{constant}$$

Temperature must always be converted into Kelvin.

Step 1: Given initial conditions at the bottom:

$$P_1 = 3 \text{ atm}$$

$$T_1 = 7^\circ\text{C} = 7 + 273 = 280 \text{ K}$$

$$V_1 = V_0$$

Step 2: Given final conditions at the surface:

$$P_2 = 1 \text{ atm}$$

$$T_2 = 27^\circ\text{C} = 27 + 273 = 300 \text{ K}$$

Let:

$$V_2 = V$$

Step 3: Apply combined gas law.

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$V_2 = \frac{P_1V_1T_2}{P_2T_1}$$

Step 4: Substitute the values.

$$V = \frac{3 \times V_0 \times 300}{1 \times 280}$$

$$V = \frac{900}{280}V_0$$

$$V = 3.21V_0$$

Therefore:

$$3.21V_0$$

Quick Tip: In gas law problems, always convert Celsius temperature into Kelvin before calculation.

72. The R.M.S. speed of oxygen molecules at 27°C is v . At 927°C , the rms speed will be:

- (A) v
- (B) $\frac{v}{2}$
- (C) $2v$
- (D) $4v$

Correct Answer: (C) $2v$

Solution:

Concept: The RMS speed of gas molecules is proportional to the square root of absolute temperature:

$$v_{\text{rms}} \propto \sqrt{T}$$

Step 1: Convert temperatures into Kelvin.

Initial temperature:

$$T_1 = 27^\circ\text{C} = 27 + 273 = 300 \text{ K}$$

Final temperature:

$$T_2 = 927^\circ\text{C} = 927 + 273 = 1200 \text{ K}$$

Step 2: Use the relation:

$$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}}$$

$$\frac{v_2}{v} = \sqrt{\frac{1200}{300}}$$

$$\frac{v_2}{v} = \sqrt{4}$$

$$\frac{v_2}{v} = 2$$

$$v_2 = 2v$$

Therefore:

$$\boxed{2v}$$

Quick Tip: RMS speed depends on absolute temperature as $v_{\text{rms}} \propto \sqrt{T}$.

73. In a photoelectric experiment, the stopping potential for incident light of wavelength 4000 \AA is 2 V . If the wavelength is changed to 3000 \AA , the new stopping potential will be approximately

(Use $h = 4.14 \times 10^{-15} \text{ eV s}$, $c = 3 \times 10^8 \text{ m/s}$)

- (A) 2 V
- (B) 3.03 V
- (C) 4.14 V
- (D) 1.5 V

Correct Answer: (B) 3.03 V

Solution:

Concept: Einstein's photoelectric equation is:

$$E = \phi + V_s$$

where E is photon energy in eV, ϕ is work function in eV, and V_s is stopping potential in volts.

Step 1: For wavelength 4000 \AA :

$$4000 \text{ \AA} = 400 \text{ nm}$$

Photon energy:

$$E_1 = \frac{1240}{400}$$

$$E_1 = 3.10 \text{ eV}$$

Given stopping potential:

$$V_1 = 2 \text{ V}$$

Step 2: Find work function.

$$E_1 = \phi + V_1$$

$$3.10 = \phi + 2$$

$$\phi = 1.10 \text{ eV}$$

Step 3: For wavelength 3000 \AA :

$$3000 \text{ \AA} = 300 \text{ nm}$$

Photon energy:

$$E_2 = \frac{1240}{300}$$

$$E_2 = 4.13 \text{ eV}$$

Step 4: Find new stopping potential.

$$E_2 = \phi + V_2$$

$$4.13 = 1.10 + V_2$$

$$V_2 = 3.03 \text{ V}$$

Therefore:

$$\boxed{3.03 \text{ V}}$$

Quick Tip: For photoelectric effect, use $E = \frac{1240}{\lambda(\text{nm})} \text{ eV}$ and then apply $E = \phi + V_s$.

74. In Optical Fiber communication, the signal is transmitted in the form of:

- (A) Electrical pulses
- (B) Light pulses
- (C) Radio waves
- (D) Sound waves

Correct Answer: (B) Light pulses

Solution:

Concept: Optical fiber communication uses light signals to transmit information through thin glass or plastic fibers.

Step 1: Optical fibers work on the principle of total internal reflection.

Step 2: In this communication system, electrical information is first converted into optical signals.

Step 3: These optical signals travel through the fiber as light pulses.

Step 4: Hence, the signal in optical fiber communication is transmitted in the form of:

Light pulses

Quick Tip: Optical fiber communication uses light pulses, not electrical signals, inside the fiber.

75. In a superconducting ring, a persistent current has been flowing without decay for years.

This is possible because:

- (A) Resistance is exactly zero and flux is quantized
- (B) Resistance is very small but finite
- (C) The ring is at absolute zero temperature
- (D) Magnetic field lines are expelled

Correct Answer: (A) Resistance is exactly zero and flux is quantized

Solution:

Concept: A superconductor has zero electrical resistance below its critical temperature. Therefore, current can flow without energy loss.

Step 1: In an ordinary conductor, resistance causes energy loss as heat.

$$P = I^2R$$

Step 2: In a superconducting ring:

$$R = 0$$

Therefore:

$$P = I^2(0) = 0$$

Step 3: Since there is no resistive power loss, current does not decay with time.

Step 4: Also, magnetic flux in a superconducting ring is quantized, which helps maintain stable persistent current states.

Therefore, persistent current is possible because:

Resistance is exactly zero and flux is quantized

Quick Tip: Persistent current in a superconducting ring is possible due to zero resistance and magnetic flux quantization.

76. The pair of orbitals with electron density maximum along the axes is

- (A) d_{xy}, d_{yz}
- (B) $d_{z^2}, d_{x^2-y^2}$
- (C) d_{xz}, d_{z^2}
- (D) d_{xz}, p_z

Correct Answer: (B) $d_{z^2}, d_{x^2-y^2}$

Solution:

Concept: In d -orbitals, some orbitals have electron density along the axes, while some have electron density between the axes.

Step 1: The orbitals:

$$d_{xy}, d_{yz}, d_{xz}$$

have electron density between the axes.

Step 2: The orbital:

$$d_{x^2-y^2}$$

has electron density along the x -axis and y -axis.

Step 3: The orbital:

$$d_{z^2}$$

has electron density mainly along the z -axis.

Step 4: Therefore, the pair of orbitals having maximum electron density along the axes is:

$$d_{z^2}, d_{x^2-y^2}$$

Hence:

$$d_{z^2}, d_{x^2-y^2}$$

Quick Tip: Remember: $d_{x^2-y^2}$ and d_{z^2} are axial d -orbitals, while d_{xy} , d_{yz} , d_{xz} lie between axes.

77. The angular momentum of an electron in an orbit X of hydrogen atom is $\frac{2h}{\pi}$. Maximum number of orbitals possible in X is

- (A) 4
- (B) 9
- (C) 16
- (D) 25

Correct Answer: (C) 16

Solution:

Concept: According to Bohr's model, angular momentum is:

$$mvr = \frac{nh}{2\pi}$$

where n is the principal quantum number.

Step 1: Given angular momentum:

$$\frac{nh}{2\pi} = \frac{2h}{\pi}$$

Step 2: Cancel h from both sides.

$$\frac{n}{2\pi} = \frac{2}{\pi}$$

Step 3: Multiply both sides by 2π .

$$n = 4$$

Step 4: Maximum number of orbitals in a shell is:

$$n^2$$

$$n^2 = 4^2 = 16$$

Therefore:

$$\boxed{16}$$

Quick Tip: Maximum number of orbitals in a shell with principal quantum number n is n^2 .

78. The four quantum numbers for the electron in the outermost orbital of potassium ($Z = 19$) are

- (A) $n = 4, l = 2, m = -1, s = +\frac{1}{2}$
- (B) $n = 4, l = 0, m = 0, s = +\frac{1}{2}$
- (C) $n = 3, l = 0, m = 1, s = +\frac{1}{2}$
- (D) $n = 4, l = 3, m = -2, s = -\frac{1}{2}$

Correct Answer: (B) $n = 4, l = 0, m = 0, s = +\frac{1}{2}$

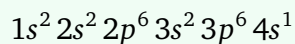
Solution:

Concept: Quantum numbers describe the shell, subshell, orbital orientation, and spin of an electron.

Step 1: Atomic number of potassium is:

$$Z = 19$$

Step 2: Electronic configuration of potassium is:



Step 3: The outermost electron is present in:

$$4s^1$$

Step 4: For 4s-orbital:

$$n = 4$$

For s-subshell:

$$l = 0$$

For $l = 0$:

$$m = 0$$

For a single electron, spin is generally:

$$s = +\frac{1}{2}$$

Therefore:

$$n = 4, l = 0, m = 0, s = +\frac{1}{2}$$

Quick Tip: For an s-orbital, $l = 0$ and $m = 0$. Potassium has its outermost electron in $4s^1$.

79. In which of the following, the number of bonding electrons and non-bonding electrons are in 3 : 2 ratio?

- (A) N_2
- (B) O_2
- (C) HCl

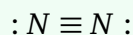
(D) F_2

Correct Answer: (A) N_2

Solution:

Concept: Bonding electrons participate in bond formation, while non-bonding electrons remain as lone pairs.

Step 1: Nitrogen molecule has structure:



Step 2: A triple bond contains three bond pairs.

So bonding electrons:

$$3 \times 2 = 6$$

Step 3: Each nitrogen atom has one lone pair.

Total non-bonding electrons:

$$2 + 2 = 4$$

Step 4: Ratio of bonding electrons to non-bonding electrons:

$$6 : 4 = 3 : 2$$

Therefore:



Quick Tip: In N_2 , the triple bond gives 6 bonding electrons and two lone pairs give 4 non-bonding electrons.

80. Which one of the following statements is not correct?

- (A) Ionic bond is non-directional bond
- (B) The maximum number of bond pairs between two atoms is 3
- (C) Covalent compounds conduct electricity in fused state
- (D) Ionic compounds are generally soluble in water

Correct Answer: (C) Covalent compounds conduct electricity in fused state

Solution:

Concept: Ionic compounds conduct electricity in molten or aqueous state because ions are free to move. Covalent compounds generally do not produce free ions.

Step 1: Ionic bond is non-directional because electrostatic attraction acts in all directions.

So option (A) is correct.

Step 2: The maximum number of bond pairs between two atoms is generally three, as in a triple bond.

So option (B) is correct.

Step 3: Ionic compounds are generally soluble in water due to ion-dipole interactions.

So option (D) is correct.

Step 4: Covalent compounds generally do not conduct electricity because they do not form free ions.

So the incorrect statement is:

Covalent compounds conduct electricity in fused state

Quick Tip: Electrical conduction in molten state is a characteristic property of ionic compounds, not covalent compounds.

81. 12.6 g of oxalic acid, $H_2C_2O_4 \cdot 2H_2O$ (M.wt. 126), is present in 1500 mL of solution. The normality of that solution is

- (A) 0.266 N
- (B) 0.133 N
- (C) 0.399 N
- (D) 0.430 N

Correct Answer: (B) 0.133 N

Solution:

Concept: Normality is:

$$N = \frac{\text{gram equivalents}}{\text{volume in litres}}$$

For oxalic acid, n -factor is 2.

Step 1: Molecular weight of oxalic acid:

$$126$$

Since oxalic acid is dibasic:

$$n = 2$$

Equivalent weight:

$$\frac{126}{2} = 63$$

Step 2: Given mass:

$$12.6 \text{ g}$$

Gram equivalents:

$$\frac{12.6}{63} = 0.2$$

Step 3: Convert volume into litres.

$$1500 \text{ mL} = 1.5 \text{ L}$$

Step 4: Calculate normality.

$$N = \frac{0.2}{1.5}$$

$$N = 0.133$$

Therefore:

$$\boxed{0.133 \text{ N}}$$

Quick Tip: For acids, equivalent weight = $\frac{\text{molecular weight}}{\text{basicity}}$. Oxalic acid has basicity 2.

82. Which of the following has highest equivalent weight? Given: At.wt. $H = 1$, $C = 12$, $O = 16$, $S = 32$, $Na = 23$, $Ca = 40$

- (A) Sulphuric acid
- (B) Sodium carbonate
- (C) Sodium sulphate

(D) Calcium carbonate

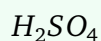
Correct Answer: (C) Sodium sulphate

Solution:

Concept: Equivalent weight is:

$$\text{Equivalent weight} = \frac{\text{Molecular weight}}{n\text{-factor}}$$

Step 1: For sulphuric acid:



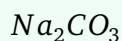
Molecular weight:

$$2 + 32 + 64 = 98$$

Since $n = 2$:

$$\text{Eq. wt.} = \frac{98}{2} = 49$$

Step 2: For sodium carbonate:



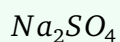
Molecular weight:

$$46 + 12 + 48 = 106$$

Since $n = 2$:

$$\text{Eq. wt.} = \frac{106}{2} = 53$$

Step 3: For sodium sulphate:



Molecular weight:

$$46 + 32 + 64 = 142$$

Since $n = 2$:

$$\text{Eq. wt.} = \frac{142}{2} = 71$$

Step 4: For calcium carbonate:



Molecular weight:

$$40 + 12 + 48 = 100$$

Since $n = 2$:

$$\text{Eq. wt.} = \frac{100}{2} = 50$$

Step 5: Highest equivalent weight is:

71

Therefore:

Sodium sulphate

Quick Tip: Compare equivalent weights by calculating $\frac{\text{molecular weight}}{n\text{-factor}}$ for each compound.

83. Identify the pair of gases which have same number of molecules at S.T.P.?

- (A) 11 g of CO_2 and 14 g of N_2
- (B) 16 g of O_3 and 16 g of CH_4
- (C) 5 g of H_2 and 40 g of CH_4
- (D) 28 g of N_2 and 22 g of CO_2

Correct Answer: (C) 5 g of H_2 and 40 g of CH_4

Solution:

Concept: Same number of molecules means same number of moles because:

$$\text{Number of molecules} = nN_A$$

Step 1: Calculate moles of H_2 .

$$\text{Molar mass of } \text{H}_2 = 2$$

$$\text{Moles of } \text{H}_2 = \frac{5}{2} = 2.5$$

Step 2: Calculate moles of CH_4 .

$$\text{Molar mass of } \text{CH}_4 = 16$$

$$\text{Moles of } CH_4 = \frac{40}{16} = 2.5$$

Step 3: Both gases contain:

2.5 moles

Therefore, they have the same number of molecules.

Hence:

5 g of H_2 and 40 g of CH_4

Quick Tip: To compare number of molecules, compare number of moles using $n = \frac{w}{M}$.

84. 100 mL of 0.1M HCl and 100 mL of 0.05M H_2SO_4 are mixed and the solution is diluted to 2.0 L by adding water. The pH of the resulting solution is

- (A) 1
- (B) 3
- (C) 2
- (D) 4

Correct Answer: (C) 2

Solution:

Concept: For strong acids, total $[H^+]$ is calculated from total moles of H^+ divided by total final volume.

Step 1: Moles of H^+ from HCl:

$$0.1 \times 0.1 = 0.01$$

Step 2: Moles of H_2SO_4 :

$$0.05 \times 0.1 = 0.005$$

Since one mole of H_2SO_4 gives 2 moles of H^+ :

$$\text{Moles of } H^+ = 2 \times 0.005 = 0.01$$

Step 3: Total moles of H^+ :

$$0.01 + 0.01 = 0.02$$

Step 4: Final volume:

$$2.0 \text{ L}$$

$$[H^+] = \frac{0.02}{2.0} = 0.01 = 10^{-2}$$

Step 5: Calculate pH.

$$pH = -\log[H^+]$$

$$pH = -\log(10^{-2})$$

$$pH = 2$$

Therefore:

2

Quick Tip: For acid mixture problems, first calculate total moles of H^+ , then divide by final volume.

85. According to Arrhenius theory of acids and bases, which of the following is an example of Arrhenius base?

- (A) H_2SO_4
- (B) NH_3
- (C) $NaOH$
- (D) CaO

Correct Answer: (C) $NaOH$

Solution:

Concept: According to Arrhenius theory, a base is a substance that gives hydroxide ions:

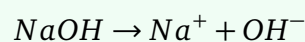


in aqueous solution.

Step 1: H_2SO_4 gives H^+ , so it is an acid.

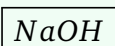
Step 2: NH_3 behaves as a base by accepting H^+ , but it does not directly contain OH^- .

Step 3: $NaOH$ dissociates in water as:



Step 4: Hence, $NaOH$ is an Arrhenius base.

Therefore:



Quick Tip: Arrhenius bases directly produce OH^- ions in aqueous solution.

86. Electrolysis of an aqueous solution of Na_2SO_4 between Pt electrodes liberates a gas X at anode and gas Y at cathode. X and Y respectively are

- (A) H_2, O_2
- (B) O_2, H_2
- (C) SO_2, H_2
- (D) H_2, SO_2

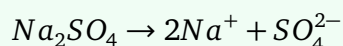
Correct Answer: (B) O_2, H_2

Solution:

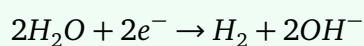
Concept: In electrolysis, oxidation occurs at the anode and reduction occurs at the cathode.

In aqueous Na_2SO_4 , water is mainly electrolysed.

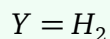
Step 1: Sodium sulphate dissociates as:



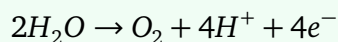
Step 2: At cathode, reduction occurs. Water gets reduced:



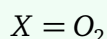
So:



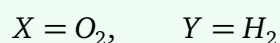
Step 3: At anode, oxidation occurs. Water gets oxidised:



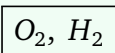
So:



Step 4: Therefore:



Hence:



Quick Tip: In electrolysis of aqueous Na_2SO_4 using inert Pt electrodes, oxygen is liberated at anode and hydrogen at cathode.

87. The wrong statement regarding Galvanic cell is

- (A) In this spontaneous redox reaction occurs
- (B) Salt bridge maintains electrical neutrality between the two solutions
- (C) Anode is represented by (+) and cathode by (–)
- (D) At anode oxidation occurs

Correct Answer: (C) Anode is represented by (+) and cathode by (–)

Solution:

Concept: A galvanic cell converts chemical energy into electrical energy through a spontaneous redox reaction.

Step 1: In a galvanic cell, spontaneous redox reaction occurs. So option (A) is correct.

Step 2: Salt bridge maintains electrical neutrality between the two half-cells. So option (B) is correct.

Step 3: Oxidation always occurs at the anode. So option (D) is correct.

Step 4: In a galvanic cell:

Anode is negative

and:

Cathode is positive

But option (C) states the opposite.

Therefore, the wrong statement is:

Anode is represented by (+) and cathode by (–)

Quick Tip: In a galvanic cell, anode is negative and cathode is positive. In an electrolytic cell, signs are reversed.

88. Which of the following is a weak electrolyte?

- (A) H_2CO_3
- (B) H_2SO_4
- (C) $NaCl$
- (D) $NaOH$

Correct Answer: (A) H_2CO_3

Solution:

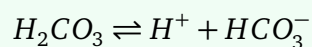
Concept: A weak electrolyte ionizes only partially in aqueous solution, whereas a strong electrolyte ionizes almost completely.

Step 1: H_2SO_4 is a strong acid and dissociates strongly in water.

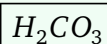
Step 2: $NaCl$ is an ionic salt and dissociates almost completely in water.

Step 3: $NaOH$ is a strong base and dissociates almost completely in water.

Step 4: H_2CO_3 is carbonic acid. It is a weak acid and ionizes only partially:



Therefore, the weak electrolyte is:



Quick Tip: Weak acids and weak bases are generally weak electrolytes because they ionize partially in solution.

89. The exhausted anion-exchange resin is regenerated with

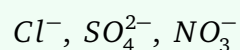
- (A) dilute $NaOH$ solution
- (B) dilute $NaCl$ solution
- (C) dilute HCl solution
- (D) dilute Na_2SO_4 solution

Correct Answer: (A) dilute $NaOH$ solution

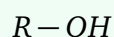
Solution:

Concept: Anion-exchange resin removes anions and is regenerated by restoring OH^- ions.

Step 1: Anion-exchange resin removes anions such as:

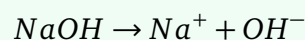


Step 2: The active form of anion-exchange resin is:



Step 3: After exhaustion, the resin contains anions in place of OH^- .

Step 4: Dilute $NaOH$ provides OH^- :



Therefore, exhausted anion-exchange resin is regenerated using:

dilute $NaOH$ solution

Quick Tip: Anion-exchange resin is regenerated by dilute $NaOH$, while cation-exchange resin is regenerated by dilute acid such as HCl .

90. A sample of water is known to contain $Mg(HCO_3)_2 = 7.3 \text{ mg/L}$, $Ca(HCO_3)_2 = 8.1 \text{ mg/L}$, and 27.2 mg/L of $CaSO_4$. The total hardness associated with water sample in ppm in equivalents of $CaCO_3$ is

(At.wt. $H = 1$, $C = 12$, $O = 16$, $Mg = 24$, $Ca = 40$, $S = 32$)

- (A) 20
- (B) 25
- (C) 30
- (D) 40

Correct Answer: (C) 30

Solution:

Concept: Hardness is expressed in terms of $CaCO_3$. The conversion formula is:

$$\text{Hardness as } CaCO_3 = \frac{\text{amount of salt} \times 50}{\text{equivalent weight of salt}}$$

Step 1: For $Mg(HCO_3)_2$:

Molecular weight:

$$24 + 2(1 + 12 + 48) = 24 + 122 = 146$$

Equivalent weight:

$$\frac{146}{2} = 73$$

Hardness:

$$\frac{7.3 \times 50}{73} = 5 \text{ ppm}$$

Step 2: For $Ca(HCO_3)_2$:

Molecular weight:

$$40 + 2(61) = 162$$

Equivalent weight:

$$\frac{162}{2} = 81$$

Hardness:

$$\frac{8.1 \times 50}{81} = 5 \text{ ppm}$$

Step 3: For $CaSO_4$:

Molecular weight:

$$40 + 32 + 64 = 136$$

Equivalent weight:

$$\frac{136}{2} = 68$$

Hardness:

$$\frac{27.2 \times 50}{68} = 20 \text{ ppm}$$

Step 4: Total hardness:

$$5 + 5 + 20 = 30 \text{ ppm}$$

Therefore:

30

Quick Tip: To convert hardness into CaCO_3 equivalent, use $\frac{\text{salt amount} \times 50}{\text{equivalent weight of salt}}$.

91. The type of functional group associated with cation exchange resin is

- (A) $-\text{OH}$
- (B) $-\text{SO}_3\text{H}$
- (C) $-\text{NH}_2$
- (D) $-\text{CHO}$

Correct Answer: (B) $-\text{SO}_3\text{H}$

Solution:

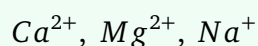
Concept: Cation exchange resins exchange their H^+ ions with cations present in water.

Step 1: Cation exchange resin must contain acidic functional groups.

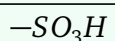
Step 2: A common strong acid functional group is:



Step 3: This group releases H^+ and exchanges it with cations such as:



Therefore, the functional group associated with cation exchange resin is:



Quick Tip: Cation exchange resins generally contain acidic groups like $-SO_3H$, while anion exchange resins contain basic groups.

92. Identify the incorrect statement about the corrosion.

- (A) In the composition type of galvanic cell, metal with lower standard reduction potential undergoes corrosion
- (B) In stress cell type of galvanic cell, corrosion occurs at the stressed area of the metal
- (C) The rate of corrosion is more, when the area of cathode is smaller
- (D) In concentration cell type of galvanic cell, the metal below the water level undergoes corrosion readily

Correct Answer: (C) The rate of corrosion is more, when the area of cathode is smaller

Solution:

Concept: Corrosion is an electrochemical process. The relative areas of anode and cathode strongly affect the corrosion rate.

Step 1: In galvanic corrosion, the metal with lower reduction potential behaves as anode and undergoes corrosion. So option (A) is correct.

Step 2: In stress corrosion, the stressed portion acts as anode and corrodes faster. So option (B) is correct.

Step 3: In concentration cell corrosion, the oxygen-deficient part often acts as anode and corrodes. So option (D) is correct.

Step 4: Corrosion is more severe when a small anodic area is connected to a large cathodic area.

If cathode area is smaller, corrosion rate is not more.

Therefore, the incorrect statement is:

The rate of corrosion is more, when the area of cathode is smaller

Quick Tip: Corrosion is most severe when the anodic area is small and the cathodic area is large.

93. In galvanised iron

- (A) Zn acts as anode and Fe acts as cathode
- (B) Zn acts as cathode and Fe acts as anode
- (C) Sn acts as anode and Fe acts as cathode
- (D) Sn acts as cathode and Fe acts as anode

Correct Answer: (A) Zn acts as anode and Fe acts as cathode

Solution:

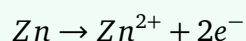
Concept: Galvanisation is the process of coating iron with zinc to protect iron from corrosion.

Step 1: In galvanised iron, iron is coated with zinc:



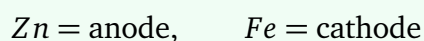
Step 2: Zinc is more electropositive than iron and oxidizes more easily.

Step 3: Hence zinc acts as sacrificial anode:



Step 4: Iron is protected and acts as cathode.

Therefore:



Hence:

Zn acts as anode and Fe acts as cathode

Quick Tip: In galvanised iron, zinc protects iron by acting as sacrificial anode.

94. During vulcanization of raw rubber, the chemical added to it is

- (A) Sulphur
- (B) Phosphorus

- (C) Iodine
(D) Sodium

Correct Answer: (A) Sulphur

Solution:

Concept: Vulcanization is the process of heating natural rubber with sulphur to improve its strength, elasticity, and durability.

Step 1: Raw rubber is soft, sticky, and less durable.

Step 2: During vulcanization, sulphur is added to rubber.

Step 3: Sulphur forms cross-links between polymer chains of rubber.

Step 4: These cross-links make rubber stronger and more elastic.

Therefore, the chemical added during vulcanization is:

Sulphur

Quick Tip: Vulcanization of rubber is done by heating rubber with sulphur.

95. Which of the following is a natural polymer?

- (A) Cellulose
(B) Teflon
(C) Polyvinylchloride
(D) Neoprene rubber

Correct Answer: (A) Cellulose

Solution:

Concept: Natural polymers are obtained from plants and animals, while synthetic polymers are man-made.

Step 1: Cellulose is found naturally in plants.

Step 2: Cellulose is a polysaccharide made up of repeating glucose units.

Step 3: Teflon, PVC, and neoprene are synthetic polymers.

Therefore, the natural polymer is:

Cellulose

Quick Tip: Cellulose, starch, proteins, and natural rubber are examples of natural polymers.

96. The structure of Buna-S polymer is

- (A) $[-CH_2 - CH = CH - CH_2 - CH(C_6H_5) - CH_2 -]_n$
(B) $[-CH_2 - CH = CH - CH_2 - CH(CN) - CH_2 -]_n$
(C) $[-CH_2 - C(Cl) = CH - CH_2 -]_n$
(D) $[-NH - CO - NH - CH_2 -]_n$

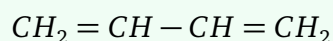
Correct Answer: (A) $[-CH_2 - CH = CH - CH_2 - CH(C_6H_5) - CH_2 -]_n$

Solution:

Concept: Buna-S is a copolymer of butadiene and styrene.

Step 1: Buna-S is also called styrene-butadiene rubber.

Step 2: Butadiene is:

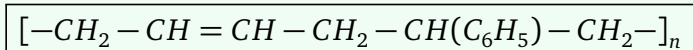


Step 3: Styrene contains phenyl group:



Step 4: Therefore, Buna-S polymer must contain C_6H_5 as a side group.

Hence the correct structure is:



Quick Tip: Buna-S is made from butadiene and styrene; therefore, its structure contains a phenyl group (C_6H_5).

97. The polymer used in making gaskets and non-stick coating utensils is

- (A) Polyvinyl chloride
- (B) Polystyrene
- (C) Polytetrafluoroethylene
- (D) Polythene

Correct Answer: (C) Polytetrafluoroethylene

Solution:

Concept: Polytetrafluoroethylene is commonly known as Teflon. It is chemically inert and has non-stick properties.

Step 1: Non-stick utensils require a coating that prevents food from sticking.

Step 2: Teflon has very low surface energy and high chemical resistance.

Step 3: Teflon is also used in gaskets because it is chemically inert and heat resistant.

Step 4: The chemical name of Teflon is:

Polytetrafluoroethylene

Therefore:

Polytetrafluoroethylene

Quick Tip: Teflon is polytetrafluoroethylene and is used for non-stick coatings and chemically resistant gaskets.

98. Which of the following is not to be considered as a primary fuel?

- (A) Wood
- (B) Petroleum
- (C) Coke
- (D) Coal

Correct Answer: (C) Coke

Solution:

Concept: Primary fuels are obtained directly from nature. Secondary fuels are obtained by processing primary fuels.

Step 1: Wood is obtained directly from nature, so it is a primary fuel.

Step 2: Petroleum is obtained naturally from the earth, so it is a primary fuel.

Step 3: Coal is also obtained naturally, so it is a primary fuel.

Step 4: Coke is obtained by destructive distillation of coal.

So coke is a secondary fuel, not a primary fuel.

Therefore:

Coke

Quick Tip: Primary fuels occur naturally, while secondary fuels are produced by processing primary fuels.

99. The oxide of nitrogen responsible for depletion of ozone layer is

- (A) N_2O
- (B) NO_2
- (C) NO
- (D) N_2O_3

Correct Answer: (C) NO

Solution:

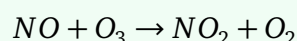
Concept: Nitric oxide (NO) acts as a catalyst in the destruction of ozone in the stratosphere.

Step 1: Ozone is:

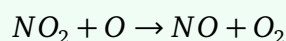


It protects the earth from harmful ultraviolet radiation.

Step 2: Nitric oxide reacts with ozone:



Step 3: The NO_2 formed can react with atomic oxygen:



Step 4: Since NO is regenerated, it acts as a catalyst and repeatedly destroys ozone.

Therefore:

NO

Quick Tip: Nitric oxide (NO) catalytically destroys ozone and contributes to ozone layer depletion.

100. The BOD of highly polluted water is

- (A) 17 ppm
- (B) 10 ppm
- (C) 8 ppm
- (D) 12 ppm

Correct Answer: (A) 17 ppm

Solution:

Concept: BOD stands for Biochemical Oxygen Demand. It is the amount of dissolved oxygen required by microorganisms to decompose organic matter present in water.

Step 1: Clean water contains less organic waste, so its BOD is low.

Step 2: Polluted water contains more organic matter.

Step 3: More organic matter means microorganisms need more oxygen for decomposition.

Step 4: Therefore, highly polluted water has high BOD.

Among the given options, the highest value is:

17 ppm

Therefore:

17 ppm

Quick Tip: Higher BOD indicates higher organic pollution in water.

101. Ohm's law states that

- (A) Current is directly proportional to resistance

- (B) Current is directly proportional to voltage
- (C) Current is inversely proportional to voltage
- (D) Resistance is directly proportional to current

Correct Answer: (B) Current is directly proportional to voltage

Solution:

Concept: Ohm's law gives the relationship between voltage, current, and resistance in an electrical circuit.

Step 1: According to Ohm's law:

$$V = IR$$

where:

$$V = \text{Voltage}, \quad I = \text{Current}, \quad R = \text{Resistance}$$

Step 2: If resistance R remains constant, then:

$$V \propto I$$

This means:

$$I \propto V$$

Step 3: Therefore, current is directly proportional to voltage, provided temperature and other physical conditions remain constant.

Hence, the correct statement is:

Current is directly proportional to voltage

Quick Tip: Ohm's law is $V = IR$. For constant resistance, current is directly proportional to voltage.

102. The unit of electrical resistance is

- (A) Ampere
- (B) Volt
- (C) Ohm
- (D) Watt

Correct Answer: (C) Ohm

Solution:

Concept: Electrical resistance is the opposition offered by a conductor to the flow of electric current.

Step 1: From Ohm's law:

$$V = IR$$

Step 2: Rearranging:

$$R = \frac{V}{I}$$

Step 3: The unit of voltage is volt and the unit of current is ampere.

So:

$$\text{Unit of } R = \frac{\text{Volt}}{\text{Ampere}}$$

$$= \Omega$$

Step 4: The SI unit of resistance is ohm.

Therefore:

Ohm

Quick Tip: Resistance is measured in ohm (Ω), voltage in volt, current in ampere, and power in watt.

103. Kirchhoff's current law is based on the conservation of

- (A) Energy
- (B) Charge
- (C) Momentum
- (D) Power

Correct Answer: (B) Charge

Solution:

Concept: Kirchhoff's Current Law states that the algebraic sum of currents meeting at a junction is zero.

Step 1: At any junction in an electrical circuit, charge cannot accumulate.

Step 2: Therefore, the total current entering the junction must be equal to the total current leaving the junction.

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

Step 3: This is possible because electric charge is conserved.

Therefore, Kirchhoff's current law is based on:

Conservation of charge

Quick Tip: KCL is based on conservation of charge, while KVL is based on conservation of energy.

104. Kirchhoff's voltage law is based on the conservation of

- (A) Charge
- (B) Energy
- (C) Current
- (D) Resistance

Correct Answer: (B) Energy

Solution:

Concept: Kirchhoff's Voltage Law states that the algebraic sum of voltages around any closed loop is zero.

Step 1: In a closed electrical loop, energy supplied by sources is equal to energy consumed by circuit elements.

Step 2: Hence, the sum of voltage rises and voltage drops in a closed loop must be zero.

$$\sum V = 0$$

Step 3: This follows from conservation of energy.

Therefore, Kirchhoff's voltage law is based on:

Conservation of energy

Quick Tip: KVL is based on conservation of energy in a closed electrical loop.

105. The material having nearly zero temperature coefficient of resistance is

- (A) Copper
- (B) Aluminium
- (C) Manganin
- (D) Carbon

Correct Answer: (C) Manganin

Solution:

Concept: Temperature coefficient of resistance tells how much resistance changes with temperature.

Step 1: Metals such as copper and aluminium have positive temperature coefficient of resistance.

Step 2: Carbon has negative temperature coefficient of resistance.

Step 3: Manganin has very small or nearly zero temperature coefficient of resistance.

Step 4: Due to this property, manganin is used in standard resistors and measuring instruments.

Therefore:

Manganin

Quick Tip: Manganin is used for standard resistors because its resistance changes very little with temperature.

106. The reciprocal of resistance is called

- (A) Conductance
- (B) Reactance
- (C) Impedance
- (D) Susceptance

Correct Answer: (A) Conductance

Solution:

Concept: Resistance opposes the flow of current, while conductance represents the ease with which current flows.

Step 1: If resistance is R , then conductance is:

$$G = \frac{1}{R}$$

Step 2: The unit of conductance is siemens.

Step 3: Reactance and impedance are used mainly in AC circuits, while susceptance is reciprocal of reactance.

Therefore, the reciprocal of resistance is:

Conductance

Quick Tip: Conductance is reciprocal of resistance: $G = \frac{1}{R}$.

107. The dielectric material between the plates of a capacitor is

- (A) Copper
- (B) Aluminium
- (C) Iron
- (D) Insulating material

Correct Answer: (D) Insulating material

Solution:

Concept: A capacitor consists of two conducting plates separated by an insulating medium called dielectric.

Step 1: The two plates of a capacitor are conductors.

Step 2: The material between the plates should not allow direct conduction of current.

Step 3: Therefore, an insulating material is placed between the plates.

Step 4: This insulating material is called dielectric.

Hence:

Insulating material

Quick Tip: A dielectric is an insulating material placed between capacitor plates.

108. In a capacitor, energy is stored in the form of

- (A) Magnetic field
- (B) Electric field
- (C) Heat energy
- (D) Mechanical energy

Correct Answer: (B) Electric field

Solution:

Concept: A capacitor stores electrical energy due to separation of charges on its plates.

Step 1: When a voltage is applied across a capacitor, positive and negative charges accumulate on opposite plates.

Step 2: Due to this charge separation, an electric field is produced between the plates.

Step 3: The energy of the capacitor is stored in this electric field.

$$E = \frac{1}{2}CV^2$$

Therefore:

Electric field

Quick Tip: Capacitor stores energy in electric field, while inductor stores energy in magnetic field.

109. The equivalent capacitance of capacitors connected in parallel is

- (A) Less than the smallest capacitance
- (B) Equal to the reciprocal sum
- (C) Sum of individual capacitances
- (D) Zero

Correct Answer: (C) Sum of individual capacitances

Solution:

Concept: In parallel connection, the voltage across each capacitor is same and charges add up.

Step 1: For capacitors in parallel:

$$C_{\text{eq}} = C_1 + C_2 + C_3 + \dots$$

Step 2: This is because total charge is:

$$Q = Q_1 + Q_2 + Q_3 + \dots$$

Step 3: Since the voltage is same across all capacitors, the equivalent capacitance is the sum.

Therefore:

Sum of individual capacitances

Quick Tip: Capacitors in parallel add directly, while capacitors in series add reciprocally.

110. The equivalent capacitance of capacitors connected in series is

- (A) $C_1 + C_2 + C_3$
- (B) $\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$
- (C) $\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$
- (D) Greater than the largest capacitance

Correct Answer: (C) $\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$

Solution:

Concept: In series connection of capacitors, charge on each capacitor is same and voltage divides.

Step 1: For capacitors in series:

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$$

Step 2: The equivalent capacitance in series is always less than the smallest capacitance.

Step 3: Therefore, the correct relation is:

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Quick Tip: For capacitors in series, use reciprocal addition.

111. The unit of inductance is

- (A) Henry
- (B) Farad
- (C) Ohm
- (D) Weber

Correct Answer: (A) Henry

Solution:

Concept: Inductance is the property of a coil by which it opposes change in current.

Step 1: The SI unit of inductance is:

Henry

Step 2: Farad is the unit of capacitance.

Step 3: Ohm is the unit of resistance.

Step 4: Weber is the unit of magnetic flux.

Therefore:

Henry

Quick Tip: Inductance is measured in Henry, capacitance in Farad, and resistance in Ohm.

112. Energy stored in an inductor is given by

- (A) $\frac{1}{2}CV^2$
- (B) $\frac{1}{2}LI^2$

(C) VI

(D) I^2R

Correct Answer: (B) $\frac{1}{2}LI^2$

Solution:

Concept: An inductor stores energy in its magnetic field.

Step 1: If current I flows through an inductor of inductance L , then energy stored is:

$$W = \frac{1}{2}LI^2$$

Step 2: The expression:

$$\frac{1}{2}CV^2$$

is energy stored in a capacitor.

Step 3: VI represents power and I^2R represents power loss in a resistor.

Therefore:

$$\boxed{\frac{1}{2}LI^2}$$

Quick Tip: Inductor stores energy as $\frac{1}{2}LI^2$, while capacitor stores energy as $\frac{1}{2}CV^2$.

113. The opposition offered by an inductor to alternating current is called

(A) Resistance

(B) Capacitive reactance

(C) Inductive reactance

(D) Conductance

Correct Answer: (C) Inductive reactance

Solution:

Concept: In AC circuits, an inductor opposes the change in current. This opposition is called inductive reactance.

Step 1: Inductive reactance is denoted by:

$$X_L$$

Step 2: It is given by:

$$X_L = 2\pi f L$$

Step 3: It increases with frequency and inductance.

Therefore:

Inductive reactance

Quick Tip: Inductive reactance is $X_L = 2\pi f L$, and it increases with frequency.

114. The opposition offered by a capacitor to alternating current is called

- (A) Capacitive reactance
- (B) Inductive reactance
- (C) Resistance
- (D) Conductance

Correct Answer: (A) Capacitive reactance

Solution:

Concept: In AC circuits, a capacitor opposes the flow of alternating current. This opposition is called capacitive reactance.

Step 1: Capacitive reactance is denoted by:

$$X_C$$

Step 2: It is given by:

$$X_C = \frac{1}{2\pi f C}$$

Step 3: It decreases with increase in frequency and capacitance.

Therefore:

Capacitive reactance

Quick Tip: Capacitive reactance is $X_C = \frac{1}{2\pi fC}$, and it decreases as frequency increases.

115. In a purely inductive circuit, current

- (A) Leads voltage by 90°
- (B) Lags voltage by 90°
- (C) Is in phase with voltage
- (D) Leads voltage by 180°

Correct Answer: (B) Lags voltage by 90°

Solution:

Concept: In a purely inductive AC circuit, voltage leads current by 90° .

Step 1: For an inductor:

$$v = L \frac{di}{dt}$$

Step 2: This means voltage depends on the rate of change of current.

Step 3: Hence, voltage reaches its maximum before current.

Step 4: Therefore, current lags voltage by:

$$90^\circ$$

Hence:

Lags voltage by 90°

Quick Tip: In an inductor, current lags voltage by 90° . Remember: L in lag.

116. In a purely capacitive circuit, current

- (A) Lags voltage by 90°
- (B) Leads voltage by 90°
- (C) Is in phase with voltage
- (D) Is zero

Correct Answer: (B) Leads voltage by 90°

Solution:

Concept: In a purely capacitive AC circuit, current leads voltage by 90° .

Step 1: For a capacitor:

$$i = C \frac{dv}{dt}$$

Step 2: This means current depends on the rate of change of voltage.

Step 3: Hence, current reaches its maximum before voltage.

Step 4: Therefore, current leads voltage by:

$$90^\circ$$

Hence:

Leads voltage by 90°

Quick Tip: In a capacitor, current leads voltage by 90° . Remember: C in lead.

117. The power factor of a purely resistive AC circuit is

- (A) Zero
- (B) Unity
- (C) Lagging
- (D) Leading

Correct Answer: (B) Unity

Solution:

Concept: Power factor is:

$$\cos \phi$$

where ϕ is the phase angle between voltage and current.

Step 1: In a purely resistive AC circuit, voltage and current are in the same phase.

$$\phi = 0^\circ$$

Step 2: Therefore:

$$\cos \phi = \cos 0^\circ = 1$$

Step 3: Hence, the power factor is unity.

Therefore:

Unity

Quick Tip: Pure resistor has unity power factor because voltage and current are in phase.

118. The power factor of a purely inductive circuit is

- (A) Unity
- (B) Zero lagging
- (C) Zero leading
- (D) One leading

Correct Answer: (B) Zero lagging

Solution:

Concept: Power factor is:

$$\cos \phi$$

Step 1: In a purely inductive circuit, current lags voltage by:

$$90^\circ$$

Step 2: Therefore:

$$\phi = 90^\circ$$

Step 3: Power factor:

$$\cos 90^\circ = 0$$

Step 4: Since current lags voltage, it is called zero lagging power factor.

Therefore:

Zero lagging

Quick Tip: Pure inductor has zero lagging power factor.

119. The power factor of a purely capacitive circuit is

- (A) Unity
- (B) Zero lagging
- (C) Zero leading
- (D) One lagging

Correct Answer: (C) Zero leading

Solution:

Concept: Power factor is:

$$\cos \phi$$

Step 1: In a purely capacitive circuit, current leads voltage by:

$$90^\circ$$

Step 2: Therefore:

$$\phi = 90^\circ$$

Step 3: Power factor:

$$\cos 90^\circ = 0$$

Step 4: Since current leads voltage, it is called zero leading power factor.

Therefore:

Zero leading

Quick Tip: Pure capacitor has zero leading power factor.

120. At resonance in a series RLC circuit, the impedance is

- (A) Maximum
- (B) Minimum
- (C) Infinite

(D) Zero

Correct Answer: (B) Minimum

Solution:

Concept: In a series RLC circuit, impedance is:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Step 1: At resonance:

$$X_L = X_C$$

Step 2: Hence:

$$X_L - X_C = 0$$

Step 3: Therefore:

$$Z = \sqrt{R^2}$$

$$Z = R$$

Step 4: Since R is the minimum possible impedance in the circuit, impedance is minimum at resonance.

Therefore:

Minimum

Quick Tip: In series resonance, impedance is minimum and current is maximum.

121. At resonance in a series RLC circuit, current is

- (A) Minimum
- (B) Maximum
- (C) Zero
- (D) Infinite always

Correct Answer: (B) Maximum

Solution:

Concept: In a series RLC circuit, current is:

$$I = \frac{V}{Z}$$

Step 1: At resonance, impedance becomes minimum:

$$Z = R$$

Step 2: Since current is inversely proportional to impedance:

$$I = \frac{V}{Z}$$

Step 3: When Z is minimum, current becomes maximum.

Therefore:

Maximum

Quick Tip: In series resonance, impedance is minimum and current is maximum.

122. The resonant frequency of a series RLC circuit is

- (A) $\frac{1}{2\pi LC}$
- (B) $\frac{1}{2\pi\sqrt{LC}}$
- (C) $2\pi\sqrt{LC}$
- (D) $\frac{L}{C}$

Correct Answer: (B) $\frac{1}{2\pi\sqrt{LC}}$

Solution:

Concept: At resonance in an RLC circuit:

$$X_L = X_C$$

Step 1: Write reactances:

$$X_L = 2\pi f L$$

and:

$$X_C = \frac{1}{2\pi f C}$$

Step 2: At resonance:

$$2\pi f L = \frac{1}{2\pi f C}$$

Step 3: Rearranging:

$$(2\pi f)^2 LC = 1$$

$$f^2 = \frac{1}{4\pi^2 LC}$$

$$f = \frac{1}{2\pi\sqrt{LC}}$$

Therefore:

$$\boxed{\frac{1}{2\pi\sqrt{LC}}}$$

Quick Tip: Resonant frequency of an RLC circuit is $f_0 = \frac{1}{2\pi\sqrt{LC}}$.

123. The form factor of a sinusoidal alternating current is

- (A) 1.11
- (B) 1.414
- (C) 0.707
- (D) 0.637

Correct Answer: (A) 1.11

Solution:

Concept: Form factor is defined as:

$$\text{Form factor} = \frac{\text{RMS value}}{\text{Average value}}$$

Step 1: For sinusoidal AC:

$$I_{\text{rms}} = \frac{I_m}{\sqrt{2}} = 0.707I_m$$

Step 2: Average value over half cycle:

$$I_{\text{avg}} = \frac{2I_m}{\pi} = 0.637I_m$$

Step 3: Form factor:

$$\frac{0.707I_m}{0.637I_m}$$
$$= 1.11$$

Therefore:

1.11

Quick Tip: For sinusoidal AC, form factor is 1.11 and peak factor is 1.414.

124. The peak factor of a sinusoidal alternating current is

- (A) 1.11
- (B) 1.414
- (C) 0.707
- (D) 0.637

Correct Answer: (B) 1.414

Solution:

Concept: Peak factor is defined as:

$$\text{Peak factor} = \frac{\text{Maximum value}}{\text{RMS value}}$$

Step 1: For sinusoidal AC:

$$I_{\text{rms}} = \frac{I_m}{\sqrt{2}}$$

Step 2: Peak factor:

$$\frac{I_m}{I_m/\sqrt{2}}$$
$$= \sqrt{2}$$

$$= 1.414$$

Therefore:

$$\boxed{1.414}$$

Quick Tip: For sinusoidal AC, peak factor is $\sqrt{2} = 1.414$.

125. The average value of a sinusoidal alternating current over one complete cycle is

- (A) Maximum value
- (B) RMS value
- (C) Zero
- (D) Half of maximum value

Correct Answer: (C) Zero

Solution:

Concept: A sinusoidal AC has equal positive and negative half cycles.

Step 1: During positive half cycle, current is positive.

Step 2: During negative half cycle, current is negative.

Step 3: Over one complete cycle, positive area and negative area cancel each other.

Step 4: Therefore, the average value over one complete cycle is:

$$0$$

Hence:

$$\boxed{0}$$

Quick Tip: Average value of sinusoidal AC over a full cycle is zero.

126. The RMS value of a sinusoidal alternating current is

- (A) $0.637I_m$

- (B) $0.707I_m$
- (C) $1.414I_m$
- (D) I_m

Correct Answer: (B) $0.707I_m$

Solution:

Concept: For sinusoidal alternating current:

$$i = I_m \sin \omega t$$

Step 1: RMS value is:

$$I_{\text{rms}} = \frac{I_m}{\sqrt{2}}$$

Step 2: Since:

$$\frac{1}{\sqrt{2}} = 0.707$$

$$I_{\text{rms}} = 0.707I_m$$

Therefore:

$$\boxed{0.707I_m}$$

Quick Tip: For sinusoidal AC, $I_{\text{rms}} = \frac{I_m}{\sqrt{2}} = 0.707I_m$.

127. The average value of sinusoidal alternating current over half cycle is

- (A) $0.637I_m$
- (B) $0.707I_m$
- (C) $1.414I_m$
- (D) Zero

Correct Answer: (A) $0.637I_m$

Solution:

Concept: The average value of a sinusoidal AC over one half cycle is:

$$I_{\text{avg}} = \frac{2I_m}{\pi}$$

Step 1: For sinusoidal current:

$$I_{\text{avg}} = \frac{2I_m}{\pi}$$

Step 2: Since:

$$\frac{2}{\pi} = 0.637$$

$$I_{\text{avg}} = 0.637I_m$$

Therefore:

$$\boxed{0.637I_m}$$

Quick Tip: Average value of sinusoidal AC over half cycle is $0.637I_m$.

128. In a three-phase balanced system, the phase difference between any two phases is

- (A) 60°
- (B) 90°
- (C) 120°
- (D) 180°

Correct Answer: (C) 120°

Solution:

Concept: A three-phase balanced system has three sinusoidal voltages or currents of equal magnitude and frequency.

Step 1: One complete cycle corresponds to:

$$360^\circ$$

Step 2: In a three-phase system, the three phases are equally spaced.

$$\frac{360^\circ}{3} = 120^\circ$$

Step 3: Therefore, the phase difference between any two phases is:

$$120^\circ$$

Quick Tip: In a balanced three-phase system, phase displacement is always 120° .

129. In star connection, line voltage is related to phase voltage as

- (A) $V_L = V_{ph}$
- (B) $V_L = \sqrt{3}V_{ph}$
- (C) $V_{ph} = \sqrt{3}V_L$
- (D) $V_L = 3V_{ph}$

Correct Answer: (B) $V_L = \sqrt{3}V_{ph}$

Solution:

Concept: In a balanced star-connected three-phase system, line voltage is $\sqrt{3}$ times the phase voltage.

Step 1: In star connection:

$$V_{ph}$$

is the voltage across one phase winding.

Step 2: Line voltage is the voltage between any two line conductors.

Step 3: The relation is:

$$V_L = \sqrt{3}V_{ph}$$

Therefore:

$$V_L = \sqrt{3}V_{ph}$$

Quick Tip: In star connection, $V_L = \sqrt{3}V_{ph}$ and $I_L = I_{ph}$.

130. In delta connection, line current is related to phase current as

- (A) $I_L = I_{ph}$
- (B) $I_L = \sqrt{3}I_{ph}$
- (C) $I_{ph} = \sqrt{3}I_L$
- (D) $I_L = 3I_{ph}$

Correct Answer: (B) $I_L = \sqrt{3}I_{ph}$

Solution:

Concept: In a balanced delta-connected three-phase system, line current is $\sqrt{3}$ times the phase current.

Step 1: In delta connection, phase voltage is equal to line voltage:

$$V_L = V_{ph}$$

Step 2: The relation between line current and phase current is:

$$I_L = \sqrt{3}I_{ph}$$

Step 3: Therefore:

$$I_L = \sqrt{3}I_{ph}$$

Quick Tip: In delta connection, $V_L = V_{ph}$ and $I_L = \sqrt{3}I_{ph}$.

131. What is the major disadvantage of an auto transformer?

- (A) High cost
- (B) Low efficiency
- (C) Lack of electrical isolation between primary and secondary
- (D) Very heavy weight

Correct Answer: (C) Lack of electrical isolation between primary and secondary

Solution:

Concept: An auto transformer uses a single winding which is common to both primary and secondary circuits.

Step 1: In a two-winding transformer, primary and secondary windings are electrically isolated from each other.

Step 2: But in an auto transformer, a part of the winding is common to both sides.

Step 3: Because of this common winding, there is no complete electrical isolation between input and output.

Step 4: This can be dangerous in fault conditions because high voltage may be transferred to the low-voltage side.

Therefore, the major disadvantage is:

Lack of electrical isolation between primary and secondary

Quick Tip: Auto transformers are smaller, cheaper, and efficient, but their main disadvantage is lack of electrical isolation.

132. If an alternator has a flux per pole of 0.05 Wb, a frequency of 50 Hz, and 100 turns per phase (assume pitch factor and distribution factor is 1), the RMS phase emf is approximately

- (A) 222 V
- (B) 1110 V
- (C) 555 V
- (D) 444 V

Correct Answer: (B) 1110 V

Solution:

Concept: The RMS emf induced per phase in an alternator is:

$$E = 4.44f \phi T K_p K_d$$

where f is frequency, ϕ is flux per pole, T is turns per phase, K_p is pitch factor, and K_d is distribution factor.

Step 1: Given:

$$f = 50 \text{ Hz}$$

$$\phi = 0.05 \text{ Wb}$$

$$T = 100$$

$$K_p = 1, \quad K_d = 1$$

Step 2: Apply the alternator emf equation.

$$E = 4.44f\phi TK_pK_d$$

$$E = 4.44 \times 50 \times 0.05 \times 100 \times 1 \times 1$$

Step 3: Calculate step by step.

$$50 \times 0.05 = 2.5$$

$$2.5 \times 100 = 250$$

$$E = 4.44 \times 250$$

$$E = 1110 \text{ V}$$

Therefore:

$$\boxed{1110 \text{ V}}$$

Quick Tip: For alternators, remember the RMS phase emf formula $E = 4.44f\phi TK_pK_d$.

133. Which method provides the most accurate results for the voltage regulation of a non-salient pole alternator?

- (A) EMF method
- (B) MMF method

- (C) Poitier Triangle method
- (D) Direct loading test

Correct Answer: (C) Poitier Triangle method

Solution:

Concept: Voltage regulation of an alternator can be determined by different methods such as EMF method, MMF method, and Poitier triangle method.

Step 1: The EMF method is also called the synchronous impedance method.

Step 2: It gives pessimistic or higher value of voltage regulation because it combines armature reaction and leakage reactance together.

Step 3: The MMF method gives optimistic or lower value of voltage regulation.

Step 4: The Poitier triangle method separates leakage reactance voltage drop and armature reaction effect.

Step 5: Because it separates these effects, it gives more accurate results for non-salient pole alternators.

Therefore:

Poitier Triangle method

Quick Tip: Poitier triangle method gives more accurate voltage regulation because it separates leakage reactance and armature reaction.

134. Two alternators A and B are in parallel. Alternator A is adjusted to have a leading power factor. This will cause Alternator B to

- (A) Also have a leading power factor
- (B) Have a lagging power factor
- (C) Stop rotating
- (D) Carry zero current

Correct Answer: (B) Have a lagging power factor

Solution:

Concept: When alternators operate in parallel, change in excitation of one alternator affects the reactive power sharing between the alternators.

Step 1: If excitation of one alternator is changed, its power factor changes.

Step 2: If alternator A is adjusted to have a leading power factor, it supplies or absorbs reactive power differently.

Step 3: Since both alternators are connected in parallel, the other alternator must balance the reactive power demand.

Step 4: Therefore, alternator B will operate at a lagging power factor.

Hence:

Have a lagging power factor

Quick Tip: In parallel alternators, changing excitation mainly affects reactive power sharing and power factor.

135. A Synchronous motor used specifically for power factor correction (running at no load and over excited) is called a

- (A) Synchronous generator
- (B) Synchronous condenser
- (C) Induction generator
- (D) Variable speed drive

Correct Answer: (B) Synchronous condenser

Solution:

Concept: An over-excited synchronous motor running without mechanical load is used for power factor improvement.

Step 1: A synchronous motor can operate at lagging, unity, or leading power factor depending on excitation.

Step 2: When the motor is over-excited, it draws leading current.

Step 3: A leading current can compensate the lagging current of inductive loads.

Step 4: A synchronous motor used only for power factor correction is called a synchronous condenser.

Therefore:

Synchronous condenser

Quick Tip: A synchronous condenser is an over-excited synchronous motor used for power factor correction.

136. In the V-curve of a Synchronous motor, the minimum point of the curve corresponds to

- (A) Lagging power factor
- (B) Unity power factor
- (C) Minimum speed
- (D) Leading power factor

Correct Answer: (B) Unity power factor

Solution:

Concept: The V-curve of a synchronous motor shows the relation between armature current and field current.

Step 1: At low excitation, the motor operates at lagging power factor.

Step 2: At over-excitation, the motor operates at leading power factor.

Step 3: At a particular field current, armature current becomes minimum.

Step 4: Minimum armature current occurs when the power factor is unity.

Therefore, the minimum point of the V-curve corresponds to:

Unity power factor

Quick Tip: In a synchronous motor V-curve, minimum armature current occurs at unity power factor.

137. What is a Pony motor in the context of starting a Synchronous motor?

- (A) A motor that runs on DC only
- (B) A small auxiliary motor used to bring the main motor up to speed
- (C) The main cooling fan
- (D) A motor used to reverse the direction

Correct Answer: (B) A small auxiliary motor used to bring the main motor up to speed

Solution:

Concept: A synchronous motor is not self-starting, so it needs an external method to bring it near synchronous speed.

Step 1: A synchronous motor cannot start by itself because its average starting torque is zero.

Step 2: One method of starting is to use a small auxiliary motor.

Step 3: This auxiliary motor rotates the synchronous motor near synchronous speed.

Step 4: After reaching near synchronous speed, DC excitation is applied and the motor pulls into synchronism.

This small auxiliary motor is called:

Pony motor

Quick Tip: A pony motor is a small auxiliary motor used to start a synchronous motor.

138. A motor has synchronous speed of 1000 rpm and runs at 950 rpm. If the torque at this slip is 10 Nm. What is the torque if the slip is doubled? (assume low slip region)

- (A) 5 Nm
- (B) 10 Nm
- (C) 20 Nm
- (D) 40 Nm

Correct Answer: (C) 20 Nm

Solution:

Concept: In an induction motor, in the low slip region, torque is approximately directly proportional to slip.

$$T \propto s$$

Step 1: Initial torque is:

$$T_1 = 10 \text{ Nm}$$

Step 2: Since slip is doubled:

$$s_2 = 2s_1$$

Step 3: In low slip region:

$$\frac{T_2}{T_1} = \frac{s_2}{s_1}$$

$$\frac{T_2}{10} = 2$$

$$T_2 = 20 \text{ Nm}$$

Therefore:

$$\boxed{20 \text{ Nm}}$$

Quick Tip: In the low slip region of an induction motor, torque is directly proportional to slip.

139. In an Induction motor, the rotor copper loss is equal to

- (A) Slip \times rotor input power
- (B) $(1 - \text{slip}) \times$ rotor input power
- (C) Rotor input power/slip
- (D) Slip² \times rotor input power

Correct Answer: (A) Slip \times rotor input power

Solution:

Concept: In an induction motor, air-gap power or rotor input power is divided into rotor copper loss and mechanical power developed.

Step 1: Let rotor input power be:

$$P_2$$

Step 2: Rotor copper loss is:

$$P_{\text{cu}} = sP_2$$

where s is slip.

Step 3: Mechanical power developed is:

$$P_m = (1 - s)P_2$$

Step 4: Therefore, rotor copper loss is:

$$\text{Slip} \times \text{rotor input power}$$

Quick Tip: In an induction motor, rotor copper loss = sP_2 , and mechanical power developed = $(1-s)P_2$.

140. The pole changing method of speed control is only applicable to which type of motor?

- (A) Slip ring induction motor
- (B) Squirrel cage induction motor
- (C) Synchronous motor
- (D) DC shunt motor

Correct Answer: (B) Squirrel cage induction motor

Solution:

Concept: The synchronous speed of an induction motor is:

$$N_s = \frac{120f}{P}$$

where f is frequency and P is number of poles.

Step 1: Speed can be changed by changing the number of poles.

Step 2: Pole changing is practically used in squirrel cage induction motors.

Step 3: Squirrel cage motors are robust and can be designed with pole-changing winding arrangements.

Step 4: Therefore, pole changing method is applicable to:

Squirrel cage induction motor

Quick Tip: Pole changing speed control is mainly used in squirrel cage induction motors.

141. Regenerative braking occurs only when the motor speed N

- (A) Is less than the synchronous speed
- (B) Is greater than the synchronous speed
- (C) Is exactly equal to synchronous speed
- (D) Is zero

Correct Answer: (B) Is greater than the synchronous speed

Solution:

Concept: Regenerative braking in an induction motor occurs when the motor acts as a generator and returns power to the supply.

Step 1: In normal motoring operation:

$$N < N_s$$

Step 2: If the rotor speed becomes greater than synchronous speed:

$$N > N_s$$

then slip becomes negative.

Step 3: Negative slip means the machine operates as an induction generator.

Step 4: In this condition, power is fed back to the supply, producing regenerative braking.

Therefore:

Is greater than the synchronous speed

Quick Tip: Regenerative braking of an induction motor occurs when rotor speed is greater than synchronous speed.

142. A single phase induction motor is

- (A) Inherently self-starting
- (B) Not inherently self-starting
- (C) Self-starting only at high voltages
- (D) Always started by a DC motor

Correct Answer: (B) Not inherently self-starting

Solution:

Concept: A single phase induction motor does not produce a rotating magnetic field at starting.

Step 1: Single phase supply produces a pulsating magnetic field.

Step 2: A pulsating magnetic field can be considered as two rotating fields in opposite directions.

Step 3: At starting, the torques produced by these fields cancel each other.

Step 4: Therefore, starting torque is zero.

Hence, a single phase induction motor is:

Not inherently self-starting

Quick Tip: Single phase induction motors need auxiliary starting methods because they are not inherently self-starting.

143. A universal motor is designed to operate on

- (A) DC supply only
- (B) AC supply only
- (C) Both AC and DC supply
- (D) 3-phase supply only

Correct Answer: (C) Both AC and DC supply

Solution:

Concept: A universal motor is a special type of motor that can operate on either AC or DC supply.

Step 1: A universal motor is similar to a series motor.

Step 2: In a series motor, armature and field currents reverse together when AC is applied.

Step 3: Since both currents reverse simultaneously, torque direction remains the same.

Step 4: Hence, a universal motor can operate on both AC and DC supply.

Therefore:

Both AC and DC supply

Quick Tip: Universal motors can run on both AC and DC supply and are commonly used in portable appliances.

144. In a BLDC motor, the windings are placed on the

- (A) Rotor
- (B) Stator
- (C) Shaft
- (D) Commutator

Correct Answer: (B) Stator

Solution:

Concept: BLDC means Brushless DC motor. It has electronic commutation instead of mechanical commutation.

Step 1: In a BLDC motor, permanent magnets are usually placed on the rotor.

Step 2: The armature windings are placed on the stator.

Step 3: Since windings are on the stationary part, there is no need for brushes and commutator.

Therefore:

Stator

Quick Tip: In a BLDC motor, windings are on the stator and permanent magnets are on the rotor.

145. The number of pulses applied to a Stepper motor determines the

- (A) Speed of rotation
- (B) Torque produced
- (C) Total angle or distance of rotation
- (D) Direction of rotation

Correct Answer: (C) Total angle or distance of rotation

Solution:

Concept: A stepper motor rotates in fixed angular steps when input pulses are applied.

Step 1: Each input pulse moves the rotor by one step angle.

Step 2: If more pulses are applied, the rotor rotates through more steps.

Step 3: Therefore, total angular displacement depends on number of pulses.

Step 4: Speed depends on pulse frequency, not simply on total number of pulses.

Hence, number of pulses determines:

Total angle or distance of rotation

Quick Tip: In a stepper motor, number of pulses controls angular displacement, and pulse frequency controls speed.

146. Which type of generating station is best suited for peak load operation due to its quick starting ability?

- (A) Nuclear power plant
- (B) Gas turbine power plant
- (C) Thermal power plant
- (D) Solar power plant

Correct Answer: (B) Gas turbine power plant

Solution:

Concept: Peak load power plants should start quickly and should be able to meet sudden load demand.

Step 1: Nuclear and thermal power plants take more time to start and are generally used for base load operation.

Step 2: Solar power depends on sunlight and cannot be used reliably for sudden peak demand.

Step 3: Gas turbine power plants have quick starting ability.

Step 4: Therefore, they are suitable for peak load operation.

Hence:

Gas turbine power plant

Quick Tip: Gas turbine plants are suitable for peak loads because they start quickly and can respond fast to load demand.

147. Which of the following is a byproduct of coal combustion that can be repurposed by the construction industry?

- (A) Flue gas
- (B) Fly ash
- (C) Nitrogen oxide
- (D) Sludge

Correct Answer: (B) Fly ash

Solution:

Concept: Fly ash is a fine powder produced from the combustion of pulverized coal in thermal power plants.

Step 1: During coal combustion, ash particles are produced.

Step 2: Fine particles carried with flue gases are collected as fly ash.

Step 3: Fly ash can be used in cement, concrete, bricks, and road construction.

Step 4: Therefore, the coal combustion byproduct useful in construction is:

Fly ash

Quick Tip: Fly ash from coal-fired power plants is widely used in cement and concrete industries.

148. If the area under a 24 hour daily load curve is 480 MWh. The average load for that day is

- (A) 10 MW
- (B) 20 MW
- (C) 40 MW
- (D) 480 MW

Correct Answer: (B) 20 MW

Solution:

Concept: Average load is:

$$\text{Average load} = \frac{\text{Energy consumed}}{\text{Time}}$$

Step 1: Given energy consumed in 24 hours:

$$E = 480 \text{ MWh}$$

Step 2: Time duration:

$$t = 24 \text{ h}$$

Step 3: Average load:

$$P_{\text{avg}} = \frac{480}{24}$$

$$P_{\text{avg}} = 20 \text{ MW}$$

Therefore:

$$\boxed{20 \text{ MW}}$$

Quick Tip: Area under load curve gives energy. Average load = $\frac{\text{Energy}}{\text{Time}}$.

149. Which tariff is most suitable for industrial consumers with a significant reactive power load?

- (A) Flat rate tariff
- (B) Block rate tariff
- (C) Maximum demand tariff
- (D) KVA maximum demand tariff

Correct Answer: (D) KVA maximum demand tariff

Solution:

Concept: Industrial consumers often have inductive loads which draw reactive power. This reduces power factor.

Step 1: KW demand considers only active power.

Step 2: KVA demand considers both active and reactive power.

Step 3: If the reactive power load is high, the KVA demand increases.

Step 4: Therefore, KVA maximum demand tariff encourages consumers to improve power factor.

Hence, the most suitable tariff is:

KVA maximum demand tariff

Quick Tip: KVA maximum demand tariff penalizes poor power factor and is suitable for industrial consumers with reactive loads.

150. A factory has a load of 400 kW at a power factor of 0.8 lagging. If the power factor is to be improved to unity, the required KVAR of the capacitor bank is

- (A) 400 KVAR
- (B) 300 KVAR
- (C) 500 KVAR
- (D) 100 KVAR

Correct Answer: (B) 300 KVAR

Solution:

Concept: Reactive power compensation required for power factor correction is:

$$Q_c = P(\tan \phi_1 - \tan \phi_2)$$

where ϕ_1 is initial power factor angle and ϕ_2 is final power factor angle.

Step 1: Given active power:

$$P = 400 \text{ kW}$$

Initial power factor:

$$\cos \phi_1 = 0.8$$

Step 2: Find $\sin \phi_1$.

$$\sin \phi_1 = \sqrt{1 - \cos^2 \phi_1}$$

$$\sin \phi_1 = \sqrt{1 - 0.8^2}$$

$$\sin \phi_1 = \sqrt{1 - 0.64}$$

$$\sin \phi_1 = 0.6$$

Step 3: Find $\tan \phi_1$.

$$\tan \phi_1 = \frac{\sin \phi_1}{\cos \phi_1}$$

$$\tan \phi_1 = \frac{0.6}{0.8} = 0.75$$

Step 4: Final power factor is unity:

$$\cos \phi_2 = 1$$

So:

$$\phi_2 = 0^\circ$$

$$\tan \phi_2 = 0$$

Step 5: Calculate capacitor KVAR.

$$Q_c = P(\tan \phi_1 - \tan \phi_2)$$

$$Q_c = 400(0.75 - 0)$$

$$Q_c = 300 \text{ KVAR}$$

Therefore:

$$\boxed{300 \text{ KVAR}}$$

Quick Tip: For power factor correction, capacitor KVAR is $P(\tan \phi_1 - \tan \phi_2)$.

151. Which circuit breaker is preferred for high voltage up to 765 kV and extra high voltage system?

- (A) Vacuum Circuit Breaker (VCB)
- (B) Air Circuit Breaker
- (C) SF_6 Circuit Breaker
- (D) Miniature Circuit Breaker (MCB)

Correct Answer: (C) SF_6 Circuit Breaker

Solution:

Concept: Circuit breakers are used to interrupt fault current and protect electrical systems. For high voltage and extra high voltage systems, the insulating and arc-quenching medium must be very effective.

Step 1: In high voltage systems, when contacts of a circuit breaker open, a strong arc is produced.

Step 2: This arc must be extinguished quickly to protect the system and equipment.

Step 3: SF_6 gas has very high dielectric strength and excellent arc-quenching property.

Step 4: Due to these properties, SF_6 circuit breakers are widely used in high voltage and extra high voltage systems.

Step 5: Vacuum circuit breakers are generally used for medium voltage applications, air circuit breakers for low voltage applications, and MCBs for domestic or low current protection.

Therefore, the preferred circuit breaker is:

SF_6 Circuit Breaker

Quick Tip: For high voltage and EHV systems, SF_6 circuit breakers are preferred because SF_6 gas has excellent insulation and arc-quenching properties.

152. Why are current limiting reactors usually “Air-cored” rather than “Iron-cored”?

- (A) To reduce weight

- (B) To improve cooling
- (C) To avoid magnetic saturation during heavy fault currents
- (D) To decrease the cost of iron

Correct Answer: (C) To avoid magnetic saturation during heavy fault currents

Solution:

Concept: Current limiting reactors are used to limit short-circuit current in power systems.

During faults, very large current flows through the reactor.

Step 1: If an iron core is used, the magnetic flux increases with current.

Step 2: During heavy fault currents, the iron core may reach magnetic saturation.

Step 3: When saturation occurs, inductance decreases and the reactor cannot limit current effectively.

Step 4: Air does not saturate like iron.

Step 5: Therefore, air-cored reactors maintain nearly constant inductance even during heavy fault currents.

Hence, current limiting reactors are air-cored:

To avoid magnetic saturation during heavy fault currents

Quick Tip: Air-cored reactors are used in fault current limiting because air does not saturate even under heavy current conditions.

153. Which type of distance relay is inherently “directional” and does not require an additional directional unit?

- (A) Impedance relay
- (B) Reactance relay
- (C) MHO relay
- (D) Plain offset relay

Correct Answer: (C) MHO relay

Solution:

Concept: Distance relays operate according to the impedance seen from the relay location to the fault point. Common types are impedance relay, reactance relay, and MHO relay.

Step 1: An impedance relay responds mainly to impedance magnitude.

Step 2: A reactance relay responds mainly to reactance and is less affected by fault resistance.

Step 3: A MHO relay has a circular characteristic passing through the origin on the R-X diagram.

Step 4: Because of its characteristic, the MHO relay naturally responds to faults in one direction.

Step 5: Therefore, it is inherently directional and does not require a separate directional unit.

Hence, the correct answer is:

MHO relay

Quick Tip: MHO relay is inherently directional, while impedance and reactance relays generally need additional directional control.

154. Bus bar protection is usually not provided for which type of system?

- (A) Extra high voltage systems
- (B) Small distribution substation
- (C) Power house bus bars
- (D) 132 kV substations

Correct Answer: (B) Small distribution substation

Solution:

Concept: Bus bar protection is used to protect bus bars from internal faults. It is important in large and high-voltage substations where bus bar faults can cause serious interruption.

Step 1: In extra high voltage systems, bus bar faults are very serious.

Step 2: Power house bus bars also require fast and selective protection.

Step 3: 132 kV substations usually have important bus bars, so bus bar protection is generally provided.

Step 4: Small distribution substations have simpler and less costly arrangements.

Step 5: Due to cost and lower system importance, separate bus bar protection is usually not provided in small distribution substations.

Therefore:

Small distribution substation

Quick Tip: Bus bar protection is mainly used in important high-voltage substations and power stations, not usually in small distribution substations.

155. Which type of lightning arrester is also known as a “Surge Diverter”?

- (A) Rod gap
- (B) Horn gap
- (C) Expulsion type
- (D) Valve type

Correct Answer: (D) Valve type

Solution:

Concept: A lightning arrester protects electrical equipment from high voltage surges by diverting surge current safely to earth.

Step 1: During lightning or switching surges, a very high voltage may appear on the power system.

Step 2: A lightning arrester provides a low resistance path to the surge current.

Step 3: The surge current is diverted to earth, preventing damage to equipment.

Step 4: Valve type lightning arrester is commonly called a surge diverter because it diverts surge current.

Therefore:

Valve type

Quick Tip: A surge diverter diverts high voltage surge current to earth. Valve type lightning arrester is commonly known as a surge diverter.

156. If the radius of a conductor in a transmission line is doubled while keeping spacing constant, the inductance of the line will

- (A) Increase
- (B) Decrease
- (C) Remain unchanged
- (D) Double

Correct Answer: (B) Decrease

Solution:

Concept: The inductance of a transmission line depends on spacing between conductors and radius of the conductor.

Step 1: The inductance of a transmission line is proportional to:

$$\log\left(\frac{D}{r}\right)$$

where D is spacing between conductors and r is conductor radius.

Step 2: Here spacing D is constant.

Step 3: If radius r is doubled, the ratio $\frac{D}{r}$ decreases.

Step 4: Therefore:

$$\log\left(\frac{D}{r}\right)$$

decreases.

Step 5: Hence, inductance also decreases.

Therefore:

Decrease

Quick Tip: Increasing conductor radius reduces the inductance of a transmission line when conductor spacing remains constant.

157. The sending end voltage of a line is 33 kV and the receiving end voltage is 30 kV. The voltage regulation of the line is

- (A) 3%
- (B) 5%
- (C) 10%
- (D) 11%

Correct Answer: (C) 10%

Solution:

Concept: Voltage regulation is:

$$\%VR = \frac{V_s - V_r}{V_r} \times 100$$

where V_s is sending end voltage and V_r is receiving end voltage.

Step 1: Given:

$$V_s = 33 \text{ kV}$$

$$V_r = 30 \text{ kV}$$

Step 2: Difference between sending and receiving end voltage:

$$V_s - V_r = 33 - 30 = 3 \text{ kV}$$

Step 3: Apply voltage regulation formula.

$$\%VR = \frac{3}{30} \times 100$$

$$\%VR = 10\%$$

Therefore:

$$\boxed{10\%}$$

Quick Tip: Voltage regulation is calculated with respect to receiving end voltage: $\frac{V_s - V_r}{V_r} \times 100$.

158. A long transmission line is energized at the sending end but kept open circuited at the receiving end. If the sending end voltage is 100 kV, the receiving end voltage is likely to be

- (A) 90 kV
- (B) 100 kV
- (C) 105 kV
- (D) 0 kV

Correct Answer: (C) 105 kV

Solution:

Concept: In a long transmission line, the receiving end voltage may become greater than the sending end voltage under no-load or light-load condition. This is called Ferranti effect.

Step 1: The line is open circuited at the receiving end.

Step 2: Under no-load condition, line charging current flows due to line capacitance.

Step 3: This charging current causes a voltage rise at the receiving end.

Step 4: Therefore, the receiving end voltage becomes greater than the sending end voltage.

Step 5: Since sending end voltage is:

100 kV

the receiving end voltage may be slightly higher, such as:

105 kV

Therefore:

105 kV

Quick Tip: Ferranti effect occurs in long transmission lines under no-load or light-load condition, causing receiving end voltage to rise.

159. What happens to the corona effect if the spacing between conductors is increased?

- (A) Corona increases
- (B) Corona decreases
- (C) Corona is absent
- (D) No effect

Correct Answer: (B) Corona decreases

Solution:

Concept: Corona occurs when the electric field intensity around a conductor exceeds the breakdown strength of air.

Step 1: Corona depends on conductor spacing, conductor radius, voltage, and atmospheric

conditions.

Step 2: If conductor spacing is small, the electric field intensity near the conductors becomes higher.

Step 3: If spacing between conductors is increased, the electric field intensity is reduced.

Step 4: Due to reduced electric stress, ionization of air decreases.

Step 5: Therefore, corona effect decreases.

Hence:

Corona decreases

Quick Tip: Increasing conductor spacing reduces electric stress and hence reduces corona effect.

160. The converter used at the sending end of an HVDC system acts as a

- (A) Inverter
- (B) Rectifier
- (C) Cycloconverter
- (D) Chopper

Correct Answer: (B) Rectifier

Solution:

Concept: In an HVDC transmission system, AC power is converted into DC for transmission and then converted back into AC at the receiving end.

Step 1: At the sending end, power is usually available as AC.

Step 2: For HVDC transmission, this AC must be converted into DC.

Step 3: The device which converts AC into DC is called a rectifier.

Step 4: At the receiving end, DC is converted back into AC using an inverter.

Therefore, the sending end converter acts as:

Rectifier

Quick Tip: In HVDC transmission, the sending end converter acts as a rectifier and the receiving end converter acts as an inverter.

161. If the length of the span between two level supports is increased by 20%, the sag will increase by approximately

- (A) 20%
- (B) 40%
- (C) 44%
- (D) 50%

Correct Answer: (C) 44%

Solution:

Concept: For a transmission line conductor between level supports, sag is approximately:

$$S = \frac{wL^2}{8T}$$

where L is span length.

Step 1: From the sag formula:

$$S \propto L^2$$

Step 2: If span length is increased by 20%, then:

$$L' = 1.2L$$

Step 3: New sag:

$$S' \propto (L')^2$$

$$S' \propto (1.2L)^2$$

$$S' = 1.44S$$

Step 4: Increase in sag:

$$S' - S = 1.44S - S = 0.44S$$

$$\text{Percentage increase} = 44\%$$

Therefore:

44%

Quick Tip: Sag is proportional to the square of span length. If span becomes 1.2 times, sag becomes 1.44 times.

162. A string of 3 discs can withstand a total voltage of 25.76 kV. If each unit can individually withstand a maximum of 11 kV, then the string efficiency is

- (A) 68.75%
- (B) 65%
- (C) 78%
- (D) 85%

Correct Answer: (C) 78%

Solution:

Concept: String efficiency is:

$$\eta = \frac{\text{Voltage across whole string}}{n \times \text{maximum voltage across one disc}} \times 100$$

where n is number of discs.

Step 1: Given:

$$n = 3$$

Total voltage across string:

$$V = 25.76 \text{ kV}$$

Maximum voltage per disc:

$$V_{\max} = 11 \text{ kV}$$

Step 2: Apply string efficiency formula.

$$\eta = \frac{25.76}{3 \times 11} \times 100$$

$$\eta = \frac{25.76}{33} \times 100$$

$$\eta = 0.7806 \times 100$$

$$\eta \approx 78\%$$

Therefore:

$$\boxed{78\%}$$

Quick Tip: String efficiency compares actual string voltage with the ideal voltage sharing capacity of all discs.

163. A 10 km long cable has an insulation resistance of 2 MΩ. What will be the insulation resistance for a 50 km length of the same cable?

- (A) 10 MΩ
- (B) 0.4 MΩ
- (C) 0.2 MΩ
- (D) 1 MΩ

Correct Answer: (B) 0.4 MΩ

Solution:

Concept: Insulation resistance of a cable is inversely proportional to its length:

$$R \propto \frac{1}{L}$$

Step 1: Given:

$$R_1 = 2 \text{ M}\Omega$$

$$L_1 = 10 \text{ km}$$

$$L_2 = 50 \text{ km}$$

Step 2: Since:

$$R \propto \frac{1}{L}$$

we use:

$$R_2 = R_1 \frac{L_1}{L_2}$$

Step 3: Substitute values.

$$R_2 = 2 \times \frac{10}{50}$$

$$R_2 = 2 \times \frac{1}{5}$$

$$R_2 = 0.4 \text{ M}\Omega$$

Therefore:

$$\boxed{0.4 \text{ M}\Omega}$$

Quick Tip: Insulation resistance of a cable decreases as cable length increases.

164. In a substation Single Line Diagram (SLD), which equipment is typically placed closest to the incoming line?

- (A) Circuit breaker
- (B) Current transformer
- (C) Surge arrester
- (D) Power transformer

Correct Answer: (C) Surge arrester

Solution:

Concept: A surge arrester protects substation equipment from lightning and switching surges.

Step 1: Incoming lines may carry high voltage surges due to lightning or switching operations.

Step 2: These surges must be diverted to earth before they reach important substation equipment.

Step 3: Therefore, the surge arrester is placed very close to the incoming line.

Step 4: This arrangement protects circuit breakers, transformers, and measuring equipment.

Hence:

Surge arrester

Quick Tip: Surge arresters are placed near incoming lines to divert overvoltage surges safely to earth.

165. A single phase AC distributor has a total resistance of 0.2Ω and a reactance of 0.3Ω . If the load current is 100 A at 0.8 p.f. lagging, the approximate voltage drop is

- (A) 16 V
- (B) 34 V
- (C) 50 V
- (D) 25 V

Correct Answer: (B) 34 V

Solution:

Concept: Approximate voltage drop in an AC distributor is:

$$V_d = I(R \cos \phi + X \sin \phi)$$

for lagging power factor.

Step 1: Given:

$$I = 100 \text{ A}$$

$$R = 0.2 \Omega$$

$$X = 0.3 \Omega$$

$$\cos \phi = 0.8$$

Step 2: Find $\sin \phi$.

$$\sin \phi = \sqrt{1 - \cos^2 \phi}$$

$$\sin \phi = \sqrt{1 - 0.8^2}$$

$$\sin \phi = \sqrt{1 - 0.64} = 0.6$$

Step 3: Apply voltage drop formula.

$$V_d = 100[(0.2)(0.8) + (0.3)(0.6)]$$

$$V_d = 100[0.16 + 0.18]$$

$$V_d = 100(0.34)$$

$$V_d = 34 \text{ V}$$

Therefore:

$$\boxed{34 \text{ V}}$$

Quick Tip: For lagging power factor, AC voltage drop is $I(R \cos \phi + X \sin \phi)$.

166. Which of the following is the standard electrification system used by Indian railways for mainline trains?

- (A) 1500 V DC
- (B) 33 kV AC, 50 Hz
- (C) 25 kV AC, 50 Hz
- (D) 11 kV DC

Correct Answer: (C) 25 kV AC, 50 Hz

Solution:

Concept: Mainline railway electrification requires an efficient high-voltage supply system to transmit power over long distances.

Step 1: Higher voltage reduces current for the same power.

Step 2: Lower current reduces transmission losses in overhead equipment.

Step 3: Indian Railways uses single phase AC traction supply for mainline trains.

Step 4: The standard system is:

25 kV AC, 50 Hz

Therefore:

25 kV AC, 50 Hz

Quick Tip: Indian mainline railway electrification commonly uses 25 kV AC, 50 Hz.

167. The distance between two stops is 5 km, the average speed is 50 km/h, and the stop time is 2 minutes. Find the scheduled speed.

- (A) 37.5 km/h
- (B) 42.5 km/h
- (C) 30 km/h
- (D) 50 km/h

Correct Answer: (A) 37.5 km/h

Solution:

Concept: Average speed excludes stop time, while scheduled speed includes stop time.

$$\text{Scheduled speed} = \frac{\text{Distance}}{\text{Running time} + \text{Stop time}}$$

Step 1: Given distance:

$$D = 5 \text{ km}$$

Average speed:

$$V_a = 50 \text{ km/h}$$

Step 2: Running time:

$$t_r = \frac{D}{V_a}$$

$$t_r = \frac{5}{50} \text{ h}$$

$$t_r = 0.1 \text{ h}$$

Convert into minutes:

$$0.1 \times 60 = 6 \text{ min}$$

Step 3: Stop time:

$$t_s = 2 \text{ min}$$

Total scheduled time:

$$t = 6 + 2 = 8 \text{ min}$$

Convert into hours:

$$t = \frac{8}{60} \text{ h}$$

Step 4: Scheduled speed:

$$V_s = \frac{5}{8/60}$$

$$V_s = 5 \times \frac{60}{8}$$

$$V_s = 37.5 \text{ km/h}$$

Therefore:

$$\boxed{37.5 \text{ km/h}}$$

Quick Tip: Scheduled speed includes stop time, while average speed considers only running time.

168. The unit of Specific Energy Consumption (SEC) is

- (A) Watt-hours
- (B) kWh/km
- (C) Watt hours/(tonne-km)
- (D) Joules/Tonne

Correct Answer: (C) Watt hours/(tonne-km)

Solution:

Concept: Specific energy consumption in electric traction is the energy consumed per unit

weight per unit distance.

Step 1: Energy consumed is measured in watt-hours or kilowatt-hours.

Step 2: In traction, load is commonly expressed in tonnes.

Step 3: Distance is expressed in kilometres.

Step 4: Therefore, specific energy consumption is expressed as:

$$\frac{\text{Watt-hours}}{\text{tonne-km}}$$

Hence:

$$\boxed{\text{Watt hours}/(\text{tonne-km})}$$

Quick Tip: Specific energy consumption in traction means energy used per tonne-kilometre.

169. A compressor in an electric locomotive is driven by an auxiliary motor to provide air for

- (A) Air conditioning only
- (B) Air brakes, horns, and sanders
- (C) Turbine rotation
- (D) Pantograph lowering only

Correct Answer: (B) Air brakes, horns, and sanders

Solution:

Concept: Electric locomotives use compressed air for several auxiliary systems.

Step 1: The compressor produces compressed air.

Step 2: Compressed air is required for operating air brake systems.

Step 3: It is also used for horns and sanders.

Step 4: Sanders use compressed air to spread sand on rails to improve adhesion.

Therefore, the compressor provides air for:

$$\boxed{\text{Air brakes, horns, and sanders}}$$

Quick Tip: In electric locomotives, the compressor supplies compressed air mainly for brakes, horns, and sanding equipment.

170. In which type of wiring system is highly skilled labor required?

- (A) Cleat wiring
- (B) Casing–Capping wiring
- (C) Conduit wiring
- (D) Batten wiring

Correct Answer: (C) Conduit wiring

Solution:

Concept: Different wiring systems require different levels of skill, safety, cost, and installation time.

Step 1: Cleat wiring is simple and temporary.

Step 2: Batten wiring and casing-capping wiring are also comparatively easier to install.

Step 3: Conduit wiring involves laying wires inside metal or PVC conduits.

Step 4: It requires proper bending, cutting, threading, fixing, and pulling of wires through conduits.

Step 5: Therefore, conduit wiring requires highly skilled labor.

Hence:

Conduit wiring

Quick Tip: Conduit wiring is safe and durable, but it needs skilled labor and careful installation.

171. A fuse in domestic wiring is always connected in the

- (A) Neutral wire
- (B) Earth wire
- (C) Live wire
- (D) Both neutral and earth wire

Correct Answer: (C) Live wire

Solution:

Concept: A fuse is a protective device used to disconnect the circuit when excessive current flows through it.

Step 1: In domestic wiring, the live wire carries current from the supply to the appliance.

Step 2: If excess current flows due to overload or short circuit, the fuse wire melts.

Step 3: When the fuse melts, it breaks the live connection and disconnects the appliance from the supply.

Step 4: If the fuse is connected in the neutral wire, the appliance may still remain connected to the live wire, which is dangerous.

Therefore, fuse is connected in:

Live wire

Quick Tip: A fuse must always be connected in the live wire so that the appliance is completely disconnected from supply during a fault.

172. The main purpose of earthing in electrical installation is

- (A) To increase voltage
- (B) To reduce power consumption
- (C) To provide safety against electric shock
- (D) To increase current rating

Correct Answer: (C) To provide safety against electric shock

Solution:

Concept: Earthing means connecting the metallic body of electrical equipment to the earth through a low resistance path.

Step 1: If insulation fails, the metallic body of an appliance may become live.

Step 2: If a person touches such equipment, current may pass through the human body.

Step 3: Earthing provides an easy and low resistance path for leakage current to flow into the ground.

Step 4: This prevents electric shock and protects both human beings and electrical equipment.

Therefore, the purpose of earthing is:

To provide safety against electric shock

Quick Tip: Earthing protects users by providing a low resistance path for fault current to flow safely into the ground.

173. Which type of earthing is commonly used in domestic installations?

- (A) Plate earthing
- (B) Pipe earthing
- (C) Rod earthing only
- (D) Strip earthing only

Correct Answer: (B) Pipe earthing

Solution:

Concept: Different methods of earthing are used depending on soil condition, cost, and type of installation.

Step 1: Pipe earthing uses a galvanized iron pipe placed vertically in the ground.

Step 2: The pipe is surrounded by charcoal and salt to reduce earth resistance.

Step 3: Pipe earthing is simple, economical, and effective.

Step 4: Due to these advantages, pipe earthing is commonly used in domestic and small electrical installations.

Therefore:

Pipe earthing

Quick Tip: Pipe earthing is widely used for domestic installations because it is simple, economical, and reliable.

174. In an NPN transistor, the region which is heavily doped is

- (A) Collector

- (B) Base
- (C) Emitter
- (D) Substrate

Correct Answer: (C) Emitter

Solution:

Concept: A transistor has three regions: emitter, base, and collector. These regions have different doping levels.

Step 1: The emitter supplies majority charge carriers.

Step 2: To inject a large number of carriers into the base, the emitter is heavily doped.

Step 3: The base is very thin and lightly doped.

Step 4: The collector is moderately doped and physically larger because it collects carriers and dissipates heat.

Therefore, the heavily doped region is:

Emitter

Quick Tip: In a transistor, emitter is heavily doped, base is lightly doped, and collector is moderately doped.

175. In a common emitter transistor configuration, the current gain β is given by

- (A) $\frac{I_B}{I_C}$
- (B) $\frac{I_C}{I_B}$
- (C) $\frac{I_E}{I_C}$
- (D) $\frac{I_C}{I_E}$

Correct Answer: (B) $\frac{I_C}{I_B}$

Solution:

Concept: In common emitter configuration, input current is base current and output current is collector current.

Step 1: Current gain in common emitter configuration is denoted by:

$$\beta$$

Step 2: It is defined as the ratio of collector current to base current.

$$\beta = \frac{I_C}{I_B}$$

Step 3: Since a small base current controls a large collector current, β is generally large.

Therefore:

$$\boxed{\frac{I_C}{I_B}}$$

Quick Tip: For common emitter transistor, current gain is $\beta = \frac{I_C}{I_B}$.

176. Lissajous patterns on a CRO are used to measure

- (A) Voltage and current only
- (B) Frequency and phase difference
- (C) Resistance and inductance
- (D) Power factor only

Correct Answer: (B) Frequency and phase difference

Solution:

Concept: A Cathode Ray Oscilloscope can display waveforms and Lissajous figures by applying signals to both horizontal and vertical plates.

Step 1: When two sinusoidal signals are applied to the CRO plates, a Lissajous pattern is formed.

Step 2: The shape of this pattern depends on frequency ratio and phase difference.

Step 3: Hence, Lissajous figures are useful for comparing unknown frequency with known frequency.

Step 4: They are also used to determine phase angle between two signals.

Therefore:

Frequency and phase difference

Quick Tip: Lissajous patterns are used in CRO to compare frequencies and measure phase difference between two signals.

177. In a CRO, focusing of the electron beam is done by

- (A) Deflection plates
- (B) Fluorescent screen
- (C) Focusing anode
- (D) Time base generator

Correct Answer: (C) Focusing anode

Solution:

Concept: In a CRO, the electron gun produces and focuses a beam of electrons on the fluorescent screen.

Step 1: The cathode emits electrons.

Step 2: These electrons are accelerated and focused by anode arrangement.

Step 3: The focusing anode concentrates the electron beam into a fine spot.

Step 4: Deflection plates only deflect the beam, while the fluorescent screen displays the waveform.

Therefore, focusing is done by:

Focusing anode

Quick Tip: In CRO, the focusing anode controls the sharpness of the electron beam spot.

178. The instrument used to measure power factor directly is

- (A) Wattmeter
- (B) Ammeter
- (C) Voltmeter

(D) Power factor meter

Correct Answer: (D) Power factor meter

Solution:

Concept: Power factor is the cosine of the phase angle between voltage and current.

Step 1: Ammeter measures current.

Step 2: Voltmeter measures voltage.

Step 3: Wattmeter measures power.

Step 4: Power factor meter directly measures power factor of an AC circuit.

Therefore:

Power factor meter

Quick Tip: Power factor meter directly measures $\cos \phi$, where ϕ is the phase angle between voltage and current.

179. The scale of a PMMC instrument is

- (A) Non-uniform
- (B) Uniform
- (C) Logarithmic
- (D) Exponential

Correct Answer: (B) Uniform

Solution:

Concept: PMMC stands for Permanent Magnet Moving Coil instrument.

Step 1: In a PMMC instrument, deflecting torque is directly proportional to current.

$$T_d \propto I$$

Step 2: Controlling torque is proportional to angle of deflection.

$$T_c \propto \theta$$

Step 3: At equilibrium:

$$T_d = T_c$$

So:

$$\theta \propto I$$

Step 4: Since deflection is directly proportional to current, the scale is uniform.

Therefore:

Uniform

Quick Tip: PMMC instruments have uniform scale because deflection is directly proportional to current.

180. An oscillator circuit is used to

- (A) Convert AC into DC
- (B) Generate sustained AC signal
- (C) Measure resistance
- (D) Store charge

Correct Answer: (B) Generate sustained AC signal

Solution:

Concept: An oscillator is an electronic circuit which produces a periodic output waveform without requiring an external input signal.

Step 1: Oscillators use positive feedback.

Step 2: They convert DC supply power into AC output signal.

Step 3: The output may be sinusoidal, square, or another periodic waveform depending on circuit design.

Therefore, oscillator is used to:

Generate sustained AC signal

Quick Tip: An oscillator produces continuous periodic output using positive feedback.

181. Which of the following is not a typical application of an oscillator?

- (A) Signal generator
- (B) Radio transmitter
- (C) Rectifier
- (D) Digital clock

Correct Answer: (C) Rectifier

Solution:

Concept: Oscillators generate periodic waveforms and are used where repetitive signals are required.

Step 1: Signal generators use oscillators to produce test signals.

Step 2: Radio transmitters use oscillators to generate carrier waves.

Step 3: Digital clocks use oscillators to produce timing pulses.

Step 4: A rectifier does not generate oscillations. It converts AC into DC.

Therefore, the option which is not an oscillator application is:

Rectifier

Quick Tip: Oscillators generate AC signals, while rectifiers convert AC into DC.

182. The basic function of an oscillator is to

- (A) Amplify DC signal
- (B) Generate alternating waveform
- (C) Convert DC to fixed resistance
- (D) Measure voltage only

Correct Answer: (B) Generate alternating waveform

Solution:

Concept: An oscillator is a circuit that produces an output waveform without an external input signal.

Step 1: It uses a DC source as the energy supply.

Step 2: Through feedback, the circuit maintains continuous oscillations.

Step 3: The output waveform may be sinusoidal or non-sinusoidal.

Therefore, the basic function is:

Generate alternating waveform

Quick Tip: An oscillator converts DC power into a periodic AC output signal.

183. Which oscillator gives very high frequency stability?

- (A) RC phase shift oscillator
- (B) Hartley oscillator
- (C) Wien bridge oscillator
- (D) Crystal oscillator

Correct Answer: (D) Crystal oscillator

Solution:

Concept: Frequency stability means the ability of an oscillator to maintain constant frequency with time, temperature, and supply variations.

Step 1: A crystal oscillator uses the piezoelectric property of quartz crystal.

Step 2: Quartz crystal has a very high quality factor Q .

Step 3: Due to high Q , the oscillator frequency remains highly stable.

Therefore, the oscillator with high frequency stability is:

Crystal oscillator

Quick Tip: Crystal oscillators are preferred where accurate and stable frequency is required.

184. In an RC phase shift oscillator, the feedback network provides a phase shift of

- (A) 90°
- (B) 180°
- (C) 270°
- (D) 360°

Correct Answer: (B) 180°

Solution:

Concept: For sustained oscillations, total phase shift around the feedback loop must be 0° or 360° .

Step 1: In an RC phase shift oscillator, the amplifier provides:

$$180^\circ$$

phase shift.

Step 2: The RC feedback network must provide another:

$$180^\circ$$

phase shift.

Step 3: Total phase shift becomes:

$$180^\circ + 180^\circ = 360^\circ$$

Therefore, RC network provides:

$$180^\circ$$

Quick Tip: In RC phase shift oscillator, amplifier gives 180° and RC network gives 180° , making total 360° .

185. Wien bridge oscillator is commonly used for

- (A) RF signal generation
- (B) Audio frequency generation
- (C) Rectification
- (D) Voltage regulation

Correct Answer: (B) Audio frequency generation

Solution:

Concept: Wien bridge oscillator is an RC oscillator used for producing low-frequency sinusoidal signals.

Step 1: It uses a lead-lag RC network.

Step 2: It is capable of generating stable sine waves.

Step 3: It is commonly used in audio frequency range.

Therefore:

Audio frequency generation

Quick Tip: Wien bridge oscillator is commonly used for audio frequency sine-wave generation.

186. Crystal oscillator is preferred because it offers

- (A) Low efficiency
- (B) High frequency stability
- (C) High noise
- (D) Poor selectivity

Correct Answer: (B) High frequency stability

Solution:

Concept: A crystal oscillator uses a quartz crystal as a frequency determining element.

Step 1: Quartz crystal has piezoelectric properties.

Step 2: It vibrates at a very precise natural frequency.

Step 3: It has very high Q-factor.

Step 4: Therefore, frequency variation is very small.

Hence:

High frequency stability

Quick Tip: Crystal oscillators are used in clocks, communication systems, and frequency reference circuits because of high stability.

187. In Boolean algebra, $A + AB$ is equal to

- (A) A
- (B) B
- (C) AB
- (D) $A + B$

Correct Answer: (A) A

Solution:

Concept: This problem is based on the absorption law of Boolean algebra.

Step 1: Given expression:

$$A + AB$$

Step 2: Take A common.

$$A + AB = A(1 + B)$$

Step 3: In Boolean algebra:

$$1 + B = 1$$

Step 4: Therefore:

$$A(1 + B) = A \cdot 1 = A$$

Hence:

$$\boxed{A}$$

Quick Tip: Absorption law: $A + AB = A$.

188. According to De Morgan's theorem, $\overline{A + B}$ is equal to

- (A) $\overline{A} + \overline{B}$
- (B) $\overline{A}\overline{B}$
- (C) AB
- (D) $A + B$

Correct Answer: (B) $\overline{A}\overline{B}$

Solution:

Concept: De Morgan's theorem is used to simplify complemented Boolean expressions.

Step 1: The theorem states:

$$\overline{A + B} = \bar{A} \cdot \bar{B}$$

Step 2: It means the complement of OR operation becomes AND operation of complements.

Step 3: Therefore:

$$\overline{A + B} = \overline{\bar{A}\bar{B}}$$

Hence:

$$\boxed{\overline{\bar{A}\bar{B}}}$$

Quick Tip: De Morgan's theorem changes OR to AND and AND to OR after complementing each variable.

189. For a half adder, the Sum and Carry outputs are respectively

- (A) $A + B, AB$
- (B) $A \oplus B, AB$
- (C) $AB, A + B$
- (D) $A \oplus B, A + B$

Correct Answer: (B) $A \oplus B, AB$

Solution:

Concept: A half adder adds two binary bits and produces two outputs: Sum and Carry.

Step 1: The Sum output is 1 when inputs are different.

This is XOR operation:

$$S = A \oplus B$$

Step 2: The Carry output is 1 only when both inputs are 1.

This is AND operation:

$$C = AB$$

Therefore:

$$\boxed{A \oplus B, AB}$$

Quick Tip: Half adder outputs are $S = A \oplus B$ and $C = AB$.

190. A flip-flop is used to store

- (A) One bit of data
- (B) One byte of data
- (C) One word of data
- (D) Analog signal only

Correct Answer: (A) One bit of data

Solution:

Concept: A flip-flop is a basic memory element in digital electronics.

Step 1: A flip-flop has two stable states.

Step 2: These states represent binary 0 and binary 1.

Step 3: Therefore, one flip-flop can store one binary bit.

Hence:

One bit of data

Quick Tip: One flip-flop stores one bit; multiple flip-flops are used to form registers and counters.

191. The BCD representation of decimal number 9 is

- (A) 1000
- (B) 1001
- (C) 1010
- (D) 1111

Correct Answer: (B) 1001

Solution:

Concept: BCD stands for Binary Coded Decimal. In BCD, each decimal digit is represented by its 4-bit binary equivalent.

Step 1: Decimal number is:

9

Step 2: Binary equivalent of 9 is:

1001

Step 3: Therefore, BCD representation of decimal 9 is:

1001

Quick Tip: In BCD, each decimal digit is written separately using four binary bits.

192. The output of a NAND gate is zero when

- (A) All inputs are zero
- (B) Any one input is zero
- (C) All inputs are one
- (D) Inputs are unequal

Correct Answer: (C) All inputs are one

Solution:

Concept: NAND gate is the complement of AND gate.

Step 1: AND gate gives output 1 only when all inputs are 1.

Step 2: NAND gate gives the opposite output of AND gate.

Step 3: Therefore, NAND output becomes 0 only when AND output is 1.

Step 4: This happens when all inputs are 1.

Therefore:

All inputs are one

Quick Tip: NAND gate output is zero only when all inputs are one.

193. A PN junction diode conducts current when it is

- (A) Reverse biased
- (B) Forward biased

- (C) Unbiased
- (D) Open circuited

Correct Answer: (B) Forward biased

Solution:

Concept: A diode allows current to flow easily in one direction and blocks current in the opposite direction.

Step 1: In forward bias, the positive terminal is connected to *P*-side and negative terminal to *N*-side.

Step 2: This reduces the depletion barrier.

Step 3: Majority carriers cross the junction easily.

Step 4: Hence, current flows through the diode.

Therefore:

Forward biased

Quick Tip: A PN junction diode conducts in forward bias and blocks current in reverse bias.

194. A Zener diode is mainly used as a

- (A) Rectifier
- (B) Voltage regulator
- (C) Amplifier
- (D) Oscillator

Correct Answer: (B) Voltage regulator

Solution:

Concept: A Zener diode is designed to operate in reverse breakdown region.

Step 1: In reverse breakdown, the voltage across Zener diode remains nearly constant.

Step 2: This constant voltage can be used as a reference voltage.

Step 3: Therefore, Zener diode maintains output voltage nearly constant despite variations in input voltage or load.

Hence, it is used as:

Voltage regulator

Quick Tip: Zener diode is used in reverse breakdown region for voltage regulation.

195. A rectifier converts

- (A) DC into AC
- (B) AC into DC
- (C) AC into AC only
- (D) DC into DC only

Correct Answer: (B) AC into DC

Solution:

Concept: Rectification is the process of converting alternating current into direct current.

Step 1: AC changes its direction periodically.

Step 2: DC flows in one direction only.

Step 3: A rectifier uses diodes to allow current in only one direction.

Step 4: Hence, it converts AC supply into DC output.

Therefore:

AC into DC

Quick Tip: Rectifier converts AC to DC, while inverter converts DC to AC.

196. A bridge rectifier uses how many diodes?

- (A) 1
- (B) 2
- (C) 3
- (D) 4

Correct Answer: (D) 4

Solution:

Concept: A bridge rectifier is a full-wave rectifier circuit.

Step 1: It uses four diodes connected in bridge form.

Step 2: During positive half cycle, two diodes conduct.

Step 3: During negative half cycle, the other two diodes conduct.

Step 4: Thus output current flows through the load in the same direction during both half cycles.

Therefore:

4

Quick Tip: A bridge rectifier uses four diodes and does not require a centre-tapped transformer.

197. The ripple factor of a full-wave rectifier is approximately

- (A) 1.21
- (B) 0.482
- (C) 0.707
- (D) 1.11

Correct Answer: (B) 0.482

Solution:

Concept: Ripple factor shows the amount of AC component present in the rectified output.

Step 1: A half-wave rectifier has ripple factor:

1.21

Step 2: A full-wave rectifier has lower ripple because both half cycles are used.

Step 3: Standard ripple factor of a full-wave rectifier is:

0.482

Therefore:

0.482

Quick Tip: Ripple factor of half-wave rectifier is 1.21, while full-wave rectifier is 0.482.

198. When a transistor is used as a switch, it operates in

- (A) Active region only
- (B) Cut-off and saturation regions
- (C) Breakdown region only
- (D) Inverse active region only

Correct Answer: (B) Cut-off and saturation regions

Solution:

Concept: A transistor switch has two states: OFF state and ON state.

Step 1: In cut-off region, transistor current is almost zero.

So transistor acts as an open switch.

Step 2: In saturation region, transistor conducts heavily.

So transistor acts as a closed switch.

Step 3: Therefore, transistor used as a switch operates between cut-off and saturation regions.

Hence:

Cut-off and saturation regions

Quick Tip: For switching action, transistor is OFF in cut-off region and ON in saturation region.

199. The input impedance of an ideal operational amplifier is

- (A) Zero
- (B) Very low
- (C) Infinite
- (D) Equal to output impedance

Correct Answer: (C) Infinite

Solution:

Concept: An ideal operational amplifier has idealized characteristics used for circuit analysis.

Step 1: An ideal op-amp should not draw current from the input source.

Step 2: For input current to be zero, input impedance must be infinite.

$$I_{in} = 0$$

Step 3: Therefore, input impedance of an ideal op-amp is:

Infinite

Quick Tip: Ideal op-amp has infinite input impedance, zero output impedance, and infinite open-loop gain.

200. The IC 555 is commonly used as a

- (A) Timer
- (B) Transformer
- (C) Rectifier diode
- (D) Inductor

Correct Answer: (A) Timer

Solution:

Concept: IC 555 is a popular timer integrated circuit used in timing and pulse generation applications.

Step 1: IC 555 can work in monostable mode.

In this mode, it produces a single time delay pulse.

Step 2: It can also work in astable mode.

In this mode, it works as an oscillator and produces continuous square waves.

Step 3: Therefore, IC 555 is commonly known as a timer IC.

Hence:

Timer

Quick Tip: IC 555 is a timer IC used for delays, pulse generation, and oscillator circuits.