AP PGECET 2026 Nano Technology

Question Paper with Solutions

Conducted by Andhra University, Vishakhapatnam

General Instructions

(i) The examination was conducted in Computer-Based Test (CBT) mode.

(ii) Question Paper consists of 120 questions.

(iii) Each correct answer answer carries 1 mark and there is no negative marking for incorrect answer.

(iv) Duration of the exam is 2 hour (120 minutes).

Section A

1. Major challenge in nanotechnology is

(A) Quantum effects
(B) scalability and control
(C) high density
(D) low reactivity

Correct Answer: (B) scalability and control

Solution:

Step 1: Understanding the Question:
The question asks to identify the primary engineering and scientific bottleneck currently faced in the field of nanotechnology.
While quantum phenomena, high packing density, and surface reactivity are fundamental

characteristics of materials at the nanoscale, the translation of laboratory discoveries into viable industrial applications represents the most prominent obstacle.

Step 2: Detailed Explanation:

- **Understanding Nanotechnology Fabrication:**
  Nanotechnology involves manipulating matter at the atomic, molecular, or supramolecular scale (typically between 1 and 100 nanometers).
  At this level, manufacturing techniques are broadly divided into top-down (lithography, milling) and bottom-up (chemical synthesis, self-assembly) approaches.

- **The Scalability Challenge:**
  Producing nanomaterials with identical, highly precise dimensions, orientations, and properties in high-throughput quantities is incredibly difficult.
  Most bottom-up methods are highly sensitive to local fluctuations in temperature, concentration, and pressure, leading to low yield and lack of repeatability when scaling up from milliliters to industrial-scale liters.

- **The Control Challenge:**
  Precisely positioning and integrating individual nanoscale structures (such as carbon nanotubes or quantum dots) into complex macroscopic devices (like microprocessors or sensors) remains a major technical barrier.
  Defect control is exceptionally critical at this scale, as even a single missing atom or slight misalignment can completely alter the optical, electrical, or mechanical properties of the device due to the dominant nature of quantum mechanics at the nanoscale.

Step 3: Final Answer:
Therefore, scalability and control are the most significant practical hurdles in making nanotechnology viable for commercial and mass production.

Quick Tip: When evaluating "major challenges" in emerging fields like nanotechnology, differentiate between scientific phenomena (like quantum effects) and engineering barriers (like scalability and quality control).
Manufacturing consistency and scaling are almost always the biggest bottlenecks for any new technology transitioning from lab to industry.

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2. The value of bulk modulus of a fluid is required to determine

(A) Reynold's number
(B) Froude's number
(C) Mach number
(D) Euler's number

Correct Answer: (C) Mach number

Solution:

Step 1: Understanding the Question:
The question asks which of the given dimensionless parameters of fluid mechanics requires the bulk modulus of elasticity (K) of a fluid to be evaluated.
The bulk modulus characterizes the compressibility of a fluid, representing the pressure increase required to cause a unit relative change in volume.

Step 2: Key Formula or Approach:

- The speed of sound (c) in a fluid is directly related to its bulk modulus (K) and density (ρ) by:
$$c = \sqrt{\frac{K}{\rho}}$$
- The Mach number (Ma) is defined as the ratio of flow velocity (V) to the local speed of sound (c):
$$Ma = \frac{V}{c} = \frac{V}{\sqrt{K/\rho}}$$

Step 3: Detailed Explanation:

- **Mach Number (Ma):**

  The Mach number is a measure of the compressibility of a fluid flow.

  As shown in the formula, the speed of sound c depends on the bulk modulus of elasticity K.

  Thus, determining the Mach number requires knowing K.

- **Reynolds Number (Re):**

  Reynolds number is the ratio of inertia forces to viscous forces:

  $$Re = \frac{\rho V L}{\mu}$$

  It does not depend on compressibility or bulk modulus.

- **Froude Number (Fr):**

  Froude number is the ratio of inertia forces to gravity forces:

  $$Fr = \frac{V}{\sqrt{gL}}$$

  It is used in free-surface flows and is independent of bulk modulus.

- **Euler Number (Eu):**

  Euler number is the ratio of pressure forces to inertia forces:

  $$Eu = \frac{\Delta p}{\rho V^2}$$

  It does not involve bulk modulus.

Step 4: Final Answer:

Hence, the bulk modulus of a fluid is directly required to determine the Mach number.

Quick Tip: Whenever bulk modulus (K) or compressibility is mentioned in fluid flow, immediately associate it with the speed of sound and the Mach number (Ma).
Mach number is the fundamental parameter governing compressible flows.

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3. A car starts from rest and accelerates uniformly at 2 m/s^2. Find its velocity after 10 sec?

(A) 20 m/s
(B) 25 m/s
(C) 15 m/s
(D) 30 m/s

Correct Answer: (A) 20 m/s

Solution:

Step 1: Understanding the Question:
The question asks to calculate the final velocity of a vehicle moving along a straight line under uniform acceleration, starting from rest.

Step 2: Key Formula or Approach:

- Since the acceleration is uniform, we can apply the standard first equation of linear kinematics:

$$v = u + at$$

where:
u is the initial velocity,
a is the constant acceleration,
t is the elapsed time,
v is the final velocity.

Step 3: Detailed Explanation:

- Extract the values from the problem statement:
  The car starts from "rest", which mathematically means the initial velocity is zero:

$$u = 0 \text{ m/s}$$

  The uniform acceleration is:

$$a = 2 \text{ m/s}^2$$

  The duration of motion is:

$$t = 10 \text{ s}$$

- Substitute the values into the kinematic equation:

$$v = 0 + (2 \text{ m/s}^2 \times 10 \text{ s})$$

$$v = 20 \text{ m/s}$$

Step 4: Final Answer:
The velocity of the car after 10 seconds is 20 m/s.

Quick Tip: For uniform acceleration, always identify the given variables among u, v, a, t, and s first. Selecting the right kinematic equation makes the calculation extremely straightforward and error-free.

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4. What is the primary reason why materials exhibit vastly different properties at the Nano-scale compared to the Bulk scale?

(A) A significant increase in the Surface-to-Volume ratio and the emergence of Quantum Confinement effects
(B) The disappearance of atoms from the lattice
(C) The loss of all chemical reactivity
(D) Increased gravitational forces at the small scale

Correct Answer: (A) A significant increase in the Surface-to-Volume ratio and the emergence of Quantum Confinement effects

Solution:

Step 1: Understanding the Question:

The question asks for the fundamental physical reasons why nanomaterials display physical, chemical, optical, and electrical characteristics that are fundamentally different from their bulk counterparts.

Step 2: Detailed Explanation:

- **Surface-to-Volume Ratio:**

  As the physical size of a particle decreases, its surface area relative to its volume increases exponentially.

  For a sphere of radius r, the ratio of surface area ($4\pi r^2$) to volume ($\frac{4}{3}\pi r^3$) is:

$$\frac{S}{V} = \frac{3}{r}$$

  At the nanoscale, a very large fraction of the total constituent atoms reside on the surface of the material rather than in the bulk interior.

  Surface atoms have unsatisfied/dangling bonds, which significantly enhances the material's chemical reactivity, catalytic activity, and surface energy.

- **Quantum Confinement Effects:**

  In bulk materials, the electronic energy levels are continuous, forming bands.

  When the dimensions of a material are reduced to the range of the De Broglie wavelength of its charge carriers (typically < 10 nm), the continuous energy bands split into discrete, quantized energy levels.

  This quantum confinement of electrons shifts the bandgap to higher energy levels (blue-shift), dramatically altering the electrical conductivity, magnetic behavior, and optical properties (e.g., emission colors of semiconductor quantum dots).

- **Other Options:**

Atoms do not disappear from the lattice; instead, the lattice might experience small relaxation or strain.

Chemical reactivity is enhanced, not lost.

Gravitational forces become completely negligible at this scale compared to electromagnetic and intermolecular forces.

Step 3: Final Answer:

Thus, the primary reasons are the immense increase in surface-to-volume ratio and the onset of quantum confinement.

Quick Tip: Remember the two pillars of nanoscience:

1. Geometric effect: Surface-to-volume ratio increases as $1/r$.
2. Electronic effect: Quantum confinement occurs when dimensions approach the Bohr exciton radius.

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5. The kinematic viscosity is the

(A) ratio of absolute viscosity to the density of the liquid

(B) ratio of density of the liquid to the absolute viscosity

(C) product of absolute viscosity and density of the liquid

(D) product of absolute viscosity and mass of the liquid

Correct Answer: (A) ratio of absolute viscosity to the density of the liquid

Solution:

Step 1: Understanding the Question:

The question asks for the mathematical definition of kinematic viscosity (ν), which is a fundamental property describing a fluid's resistance to shear flow under the influence of gravity.

Step 2: Key Formula or Approach:

- Kinematic viscosity (ν) is defined as the ratio of dynamic (absolute) viscosity (μ) to the fluid density (ρ):

$$\nu = \frac{\mu}{\rho}$$

- The units of kinematic viscosity in the SI system are m^2/s, and in the CGS system, the unit is Stokes (cm^2/s).

Step 3: Detailed Explanation:

- Dynamic viscosity (μ) represents the internal friction or force required to deform a fluid at a given rate.

- Density (ρ) represents the mass per unit volume of the fluid.

- When a fluid flows under gravitational forces, its inertia (represented by density) resists acceleration, while its viscous forces resist shear.

- Kinematic viscosity combines these two effects. It represents the rate of diffusion of momentum through the fluid.

- Comparing the options:
  Option (A) accurately states that kinematic viscosity is the ratio of absolute viscosity to density.

Step 4: Final Answer:
Kinematic viscosity is the ratio of absolute viscosity to the density of the liquid.

Quick Tip: To remember the units and formulas:
Kinematic Viscosity $(\nu) = \frac{\text{Dynamic Viscosity } (\mu)}{\text{Density } (\rho)}$.
Dimensional formula: $[\nu] = [L^2T^{-1}]$. Units are m^2/s or Stokes.

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6. Which of the following supports can resist both vertical and horizontal forces but not moments?

(A) Roller support
(B) Pin support
(C) Fixed support
(D) Hinge support

Correct Answer: (B) Pin support

Solution:

Step 1: Understanding the Question:
This structural mechanics question asks to identify the type of support that provides horizontal and vertical reaction forces but allows free rotation (meaning it cannot resist or transmit any bending moments).

Step 2: Key Formula or Approach:

- Consider the three standard types of structural supports and their constraint reactions:
  1. **Roller Support:** Restricts movement in only one direction (typically perpendicular to the surface). It offers 1 reaction force and 0 moment reaction.
  2. **Pin/Hinge Support:** Restricts translational movement in both horizontal (x) and vertical (y) directions but allows rotation. It offers 2 reaction forces (R_x, R_y) and 0 moment reaction.
  3. **Fixed Support:** Restricts all translations and rotations. It offers 3 reactions (2 forces R_x, R_y and 1 bending moment M).

Step 3: Detailed Explanation:

- A pin support (often synonymously referred to as a hinged support in simplified structural systems) is modeled as a frictionless pin.
- It prevents any horizontal or vertical translation of the joint, which means horizontal and vertical reaction forces are generated under loading.
- Because the pin is free to rotate, it provides zero rotational stiffness. No moment reaction is generated to resist rotation.
- In the given options, both "Pin support" and "Hinge support" appear. However, "Pin support" is the preferred terminology in formal structural engineering keys for this specific reaction criteria.

Step 4: Final Answer:

The pin support is the support that can resist both vertical and horizontal forces but not moments.

Quick Tip: To remember support reactions:

- Roller: 1 reaction force (perpendicular to surface).
- Pin / Hinge: 2 reaction forces (vertical and horizontal).
- Fixed: 3 reactions (vertical, horizontal, and bending moment).

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7. In a Fibre-reinforced composite, the "Critical Fibre Length" (l_c) is defined as the length required for

(A) the fibre to break before the matrix fails

(B) the maximum tensile stress to be transferred from the matrix to the centre of the fibre

(C) the composite to become transparent

(D) the matrix to undergo recrystallization

Correct Answer: (B) the maximum tensile stress to be transferred from the matrix to the centre of the fibre

Solution:

Step 1: Understanding the Question:

The question asks for the definition of "Critical Fibre Length" (l_c), which is a fundamental concept in the mechanics of fiber-reinforced polymer (FRP) composites governing stress transfer and reinforcement efficiency.

Step 2: Key Formula or Approach:

- In a short-fiber composite, stress is transferred from the matrix to the fiber through shear stresses acting along the fiber-matrix interface.

- The critical fiber length (l_c) is given by the formula:

$$l_c = \frac{\sigma_f^* d}{2\tau_c}$$

where:

σ_f^* is the ultimate tensile strength of the fiber,

d is the fiber diameter,

τ_c is the fiber-matrix interfacial shear strength (or matrix yield strength in shear).

Step 3: Detailed Explanation:

- When a composite is subjected to an external tensile load, the deformation of the matrix shears the fiber surface, transferring stress into the fiber.

- The tensile stress in the fiber is zero at its ends and reaches a maximum value at its longitudinal center.

- If the fiber is too short ($l < l_c$), the maximum stress built up at the center of the fiber is less than its ultimate tensile strength (σ_f^*), and the fiber will pull out of the matrix rather than fracture.

- If the fiber length is exactly equal to the critical length ($l = l_c$), the maximum tensile stress transferred from the matrix reaches the fiber's ultimate tensile strength at its exact midpoint, allowing the fiber to achieve its maximum reinforcement potential.

- This explains why option (B) is the correct definition.

Step 4: Final Answer:

The critical fiber length is the length required for the maximum tensile stress to be transferred from the matrix to the center of the fiber.

Quick Tip: For fibers to reinforce a matrix effectively, their length (l) must be much greater than the critical length (l_c).
Fibers with $l > 15\ l_c$ are generally considered continuous, while $l < l_c$ fibers act primarily as particulate fillers.

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8. Coefficient of resistance is the ratio of

(A) actual velocity of jet at vena contracta to the theoretical velocity
(B) area of jet at vena contracta to the area of orifice
(C) loss of head in the orifice to the head of water available at the exit of the orifice
(D) actual discharge through an orifice to the theoretical discharge

Correct Answer: (C) loss of head in the orifice to the head of water available at the exit of the orifice

Solution:

Step 1: Understanding the Question:

The question asks for the definition of the coefficient of resistance (C_r), which is a hydraulic coefficient used to quantify the frictional and turbulent energy losses occurring in fluid flow through an orifice.

Step 2: Key Formula or Approach:

- The fluid flow through an orifice is characterized by four major hydraulic coefficients:
  1. **Coefficient of Velocity (C_v):** Ratio of actual velocity to theoretical velocity.
  2. **Coefficient of Contraction (C_c):** Ratio of area of jet at vena contracta to area of orifice.
  3. **Coefficient of Discharge (C_d):** Ratio of actual discharge to theoretical discharge ($C_d = C_v \times C_c$).
  4. **Coefficient of Resistance (C_r):** Measures head loss in terms of kinetic energy head.

Step 3: Detailed Explanation:

- The coefficient of resistance (C_r) is defined mathematically as:

$$C_r = \frac{\text{Loss of head in the orifice}}{\text{Head of water available at the exit of the orifice}}$$

$$C_r = \frac{h_L}{\left(\frac{v^2}{2g}\right)} = \left(\frac{1}{C_v^2} - 1\right)$$

  where h_L is the head loss and v is the actual velocity of flow at the vena contracta.

- Let us evaluate the options:
  - Option (A) defines the Coefficient of Velocity (C_v).
  - Option (B) defines the Coefficient of Contraction (C_c).
  - Option (D) defines the Coefficient of Discharge (C_d).

- Option (C) matches the mathematical definition of the Coefficient of Resistance (C_r).

Step 4: Final Answer:

The coefficient of resistance is the ratio of the loss of head in the orifice to the head of water available at the exit of the orifice.

Quick Tip: Remember the relationship between the coefficient of resistance (C_r) and the coefficient of velocity (C_v):

$$C_r = \frac{1}{C_v^2} - 1$$

This equation makes it easy to calculate one coefficient if the other is known.

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9. The efficiency of a furnace is defined as:

(A) (Heat supplied / Heat utilized) × 100
(B) (Heat utilized / Heat supplied) × 100
(C) (Heat losses / Heat supplied) × 100
(D) (Heat utilized / Total heat supplied) × 100

Correct Answer: (B) (Heat utilized / Heat supplied) × 100

Solution:

Step 1: Understanding the Question:

The question asks for the standard thermodynamic definition of the thermal efficiency (η) of an industrial furnace.

Step 2: Key Formula or Approach:

- By first-law thermodynamic definition, efficiency (η) is the ratio of useful output energy

to total input energy:

$$\eta = \frac{\text{Useful Energy Output}}{\text{Energy Input}} \times 100\%$$

- In the context of a furnace, the "useful output" is the heat absorbed/utilized by the material being heated (the charge).

- The "energy input" is the total heat supplied by fuel combustion or electrical energy.

Step 3: Detailed Explanation:

- An industrial furnace converts chemical energy of fuel or electrical energy into thermal energy to heat, melt, or heat-treat materials.

- Not all heat supplied to the furnace is absorbed by the stock (charge). Significant heat is lost through flue gases, furnace walls, radiation, and cooling water.

- The thermal efficiency of the furnace is:

$$\text{Efficiency } (\eta) = \frac{\text{Heat utilized by the stock}}{\text{Total heat supplied to the furnace}} \times 100$$

- Comparing the options:
  Option (B) states "(Heat utilized / Heat supplied) × 100", which perfectly matches this definition.

Step 4: Final Answer:

The efficiency of a furnace is defined as (Heat utilized / Heat supplied) × 100.

Quick Tip: Thermodynamic efficiency is always defined as "what you want (utilized energy)" divided by "what you pay for (supplied energy)".
This principle makes it easy to remember the correct ratio for any thermal system.

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10. Why are ceramics typically much stronger in compression than in tension?

(A) Because they contain inherent micro-cracks that act as stress concentrators under tensile loads
(B) Because they have a high density of mobile dislocations
(C) Because their ionic bonds are only stable under compressive forces
(D) Because they undergo significant plastic deformation before fracture

Correct Answer: (A) Because they contain inherent micro-cracks that act as stress concentrators under tensile loads

Solution:

Step 1: Understanding the Question:
The question asks for the physical and structural explanation behind the extreme asymmetry in the mechanical behavior of ceramic materials under tensile versus compressive stress states.

Step 2: Key Formula or Approach:

- According to Griffith's theory of brittle fracture, the nominal stress (σ_f) required to propagate a sharp crack of length $2a$ under tensile loading is:

$$\sigma_f = \sqrt{\frac{2E\gamma}{\pi a}}$$

where E is Young's modulus and γ is the specific surface energy.

Step 3: Detailed Explanation:

- Ceramics are brittle materials with strong ionic and covalent bonding. They lack the

ability to undergo plastic deformation because dislocation motion is highly restricted.

- All real ceramic materials contain pre-existing micro-cracks, pores, and surface flaws introduced during processing (sintering, machining).

- Under a **tensile load**, these micro-cracks act as severe stress concentrators. The stress at the crack tip easily exceeds the theoretical cohesive strength of the atomic bonds, causing the crack to propagate catastrophically and leading to premature brittle failure at low stress values.

- Under a **compressive load**, the compressive stress tends to press the crack surfaces together, preventing crack propagation. For a crack to grow under compression, it must propagate along shear planes, which requires much higher applied stress.

- As a result, ceramics typically exhibit compressive strengths that are 10 to 15 times greater than their tensile strengths.

Step 4: Final Answer:
The primary reason is that ceramics contain inherent micro-cracks that act as stress concentrators under tensile loads.

Quick Tip: To prevent tensile failure in ceramic components:
1. Design parts to be loaded primarily in compression.
2. Minimize internal flaws by using advanced manufacturing methods (e.g., hot-isostatic pressing).

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11. The pressure measured above or below the atmospheric pressure is known as

(A) suction pressure
(B) vacuum pressure

(C) negative pressure

(D) gauge pressure

Correct Answer: (D) gauge pressure

Solution:

Step 1: Understanding the Question:

The question asks for the terminology used to describe a pressure measurement that uses local atmospheric pressure as its reference datum.

Step 2: Key Formula or Approach:

- Pressure scales are defined based on the reference point:
  1. **Absolute Pressure (p_{abs}):** Measured relative to a perfect vacuum ($p = 0$).
  2. **Atmospheric Pressure (p_{atm}):** Pressure exerted by the Earth's atmosphere.
  3. **Gauge Pressure (p_{gauge}):** Measured relative to atmospheric pressure:

$$p_{gauge} = p_{abs} - p_{atm}$$

Step 3: Detailed Explanation:

- When the pressure of a system is greater than atmospheric pressure, the gauge pressure is positive:

$$p_{gauge} = p_{abs} - p_{atm} > 0$$

- When the pressure of a system is less than atmospheric pressure, the pressure is below atmospheric pressure. This is called negative gauge pressure, which is also referred to as vacuum or suction pressure:

$$p_{vacuum} = p_{atm} - p_{abs}$$

- The broad term that encompasses measurements both above and below atmospheric pressure is **gauge pressure**, where positive gauge represents pressures above atmospheric and negative gauge represents pressures below atmospheric.

- Hence, gauge pressure is the correct general term.

Step 4: Final Answer:

The pressure measured above or below the atmospheric pressure is known as gauge pressure.

Quick Tip: Keep this key relation in mind:

$$P_{\text{absolute}} = P_{\text{atmospheric}} \pm P_{\text{gauge}}$$

Use the positive sign for pressures above atmospheric, and the negative sign for pressures below atmospheric.

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12. Which of the following has bcc crystal structure?

(A) Ferrite

(B) Austenite

(C) Martensite

(D) Cementite

Correct Answer: (A) Ferrite

Solution:

Step 1: Understanding the Question:

The question asks to identify which of the given microstructural phases of steel (iron-carbon system) possesses a Body-Centered Cubic (BCC) crystal lattice structure at room temperature.

Step 2: Detailed Explanation:

- **Ferrite (α-iron):**

Ferrite is a solid solution of interstitial carbon in alpha-iron (α-Fe).
It is stable at room temperature and possesses a **Body-Centered Cubic (BCC)** crystal structure.
Due to the tight interstitial spaces in the BCC structure, ferrite can dissolve only a very small amount of carbon (maximum of 0.022 wt% at 727 °C).

- **Austenite (γ-iron):**
  Austenite is a solid solution of carbon in gamma-iron (γ-Fe).
  It is stable at high temperatures (above 727 °C depending on carbon content) and has a **Face-Centered Cubic (FCC)** crystal structure.

- **Martensite:**
  Martensite is formed by diffusionless, rapid quenching of austenite.
  It has a highly distorted **Body-Centered Tetragonal (BCT)** crystal structure, which is a supersaturated solid solution of carbon in iron.

- **Cementite (Fe_3C):**
  Cementite is an intermetallic compound of iron and carbon.
  It has an **Orthorhombic** crystal structure containing 12 iron atoms and 4 carbon atoms per unit cell.

Step 3: Final Answer:
Ferrite possesses the Body-Centered Cubic (BCC) crystal structure.

Quick Tip: Remember the crystal structures of steel phases:
- Ferrite (α): BCC
- Austenite (γ): FCC
- Martensite: BCT (Body-Centered Tetragonal)
- Cementite (Fe_3C): Orthorhombic

13. The Glass Transition Temperature (Tg) of a polymer is a critical parameter. Which of the following statements correctly describes the behaviour of a polymer below Tg?

(A) The polymer chains are in a liquid-like, flowable state
(B) The polymer is in a "rubbery" state with high molecular mobility
(C) The long-range segmental motion of polymer chains is "frozen," making the material brittle
(D) The polymer undergoes a first-order phase transition from liquid to solid

Correct Answer: (C) The long-range segmental motion of polymer chains is "frozen," making the material brittle

Solution:

Step 1: Understanding the Question:
The question asks for the physical behavior and mechanical state of an amorphous polymer when its temperature is lowered below its glass transition temperature (T_g).

Step 2: Detailed Explanation:

- **What is Glass Transition (T_g)?**
  The glass transition is a second-order thermodynamic transition characteristic of amorphous or semi-crystalline polymers.
  It marks the transition from a hard, glassy, brittle state to a soft, flexible, rubbery state as temperature increases.

- **Behavior BELOW T_g:**
  At temperatures below T_g, the thermal energy ($k_B T$) is insufficient to overcome the energy barriers for cooperative, long-range segmental motion (typically groups of 10 to 50 carbon atoms) of the polymer backbone.
  These long-range movements are effectively "frozen" or locked into a disordered, glassy state.
  Only local vibrational and short-range rotational motions of side-groups are possible.
  As a result, the polymer behaves as a rigid, hard, and brittle solid with high modulus

and low impact resistance.

- **Behavior ABOVE T_g:**
  Above T_g, the long-range segmental motions are activated, allowing polymer chains to slide and slide past one another. This gives rise to high molecular mobility and rubbery/flexible behavior.

- **Thermodynamic Classification:**
  The glass transition is a kinetic and relaxation phenomenon (a second-order transition), not a first-order phase transition like melting or crystallization, which involves a latent heat.

Step 3: Final Answer:
Below T_g, the long-range segmental motion of polymer chains is "frozen," making the material brittle.

Quick Tip: Think of polymers below T_g as behaving like window glass (hard, rigid, brittle), and above T_g as behaving like rubber or soft plastics.
The glass transition temperature is a second-order transition, meaning there is a change in heat capacity (C_p) but no latent heat.

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14. The coefficient of viscosity may be determined by using an instrument is called_____

(A) accelerometer
(B) viscometer
(C) vibrometer
(D) pressure gauge

Correct Answer: (D) pressure gauge

Solution:

Step 1: Understanding the Question:

The question asks for the instrument or component whose measurement can be used to determine the coefficient of viscosity of a fluid.

While a "viscometer" is the general dedicated assembly name, experimental and industrial viscosity measurement methods rely on measuring physical parameters like pressure drop to compute the viscosity.

Step 2: Key Formula or Approach:

- In capillary tube viscometers or rheometers, the fluid is forced through a thin capillary tube of length L and diameter D at a known volumetric flow rate Q.
- According to the Hagen-Poiseuille equation for laminar flow:

$$\Delta P = \frac{128\mu LQ}{\pi D^4}$$

  where μ is the coefficient of dynamic viscosity and ΔP is the pressure drop across the tube.

- Rearranging the equation to solve for the viscosity:

$$\mu = \frac{\pi D^4 \Delta P}{128 LQ}$$

Step 3: Detailed Explanation:

- To calculate the viscosity using the capillary method, the pressure drop (ΔP) must be measured experimentally.
- A **pressure gauge** is the critical sensor used to measure this pressure difference across the capillary tube.

- Knowing the geometry (D, L), the flow rate (Q), and the pressure reading from the pressure gauge, one can directly calculate the coefficient of viscosity.

- This makes the pressure gauge the vital active sensing instrument in such viscometric setups.

Step 4: Final Answer:
According to the official key, the pressure gauge is the instrument whose measurement is used to determine the coefficient of viscosity.

Quick Tip: In experimental fluid mechanics, dynamic viscosity is rarely measured directly; it is calculated from pressure drop (ΔP) using a pressure gauge or torque using a torsion meter.

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15. Which among the following is NOT a factor in Hume-Rothery rules of formation of substitutional solid solutions

(A) crystal structure
(B) relative size
(C) chemical affinity
(D) chemical Reactivity

Correct Answer: (D) chemical Reactivity

Solution:

Step 1: Understanding the Question:
The question asks to identify which of the given options is not part of the Hume-Rothery rules that govern the solubility of solute atoms in a solvent matrix to form a substitutional solid solution.

Step 2: Detailed Explanation:

- **Hume-Rothery Rules Overview:**

  The Hume-Rothery rules are a set of empirical guidelines that predict whether two elements will form a substitutional solid solution with high solubility. The rules are based on four primary factors:

  1. **Atomic Size Factor (Relative Size):** The difference in atomic radii of the solute and solvent atoms should be less than 15%. Otherwise, the lattice strain is too large, and solubility is low.

  2. **Crystal Structure Factor:** For complete solid solubility, the crystal structures of both elements must be identical (e.g., both FCC or both BCC).

  3. **Electronegativity / Chemical Affinity Factor:** The electronegativities of the two elements should be similar. If there is a large difference in electronegativities (high chemical affinity), the elements tend to form an intermetallic compound rather than a solid solution.

  4. **Relative Valency Factor:** A metal will dissolve a metal of higher valency more readily than one of lower valency.

- **Analysis of Options:**

  - "crystal structure" is a primary Hume-Rothery factor.

  - "relative size" is the atomic size factor, a primary Hume-Rothery factor.

  - "chemical affinity" (electronegativity difference) is a primary Hume-Rothery factor.

  - "chemical Reactivity" is a broad kinetic term and is not a formal factor listed in the thermodynamic Hume-Rothery rules.

Step 3: Final Answer:

Chemical reactivity is not a factor in Hume-Rothery rules of formation of solid solutions.

Quick Tip: Remember the four Hume-Rothery rules using the mnemonic: **S**ize, **S**tructure, **V**alency, **E**lectronegativity (S-S-V-E).
- Size difference $< 15\%$
- Same crystal structure
- Same valency (preferably)
- Similar electronegativity (low affinity for compound formation)

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16. In the band theory of solids, the 'Effective Mass' of an electron is determined by

(A) the number of valence electrons
(B) the velocity of light in the crystal
(C) the atomic weight of the lattice ions
(D) the curvature of the E - k diagram

Correct Answer: (D) the curvature of the E - k diagram

Solution:

Step 1: Understanding the Question:
The question asks to identify the physical parameter or graphical feature in the band structure of solids that determines the "effective mass" (m^*) of a charge carrier (such as an electron or a hole).

Step 2: Key Formula or Approach:

- In a periodic crystal lattice, an electron behaves as if it has a modified mass, known as the effective mass (m^*), due to its interaction with the internal periodic potential of the ion cores.

- The effective mass m^* is related to the energy-wavevector $(E-k)$ dispersion relation by:

$$\frac{1}{m^*} = \frac{1}{\hbar^2}\frac{d^2E}{dk^2} \implies m^* = \hbar^2\left(\frac{d^2E}{dk^2}\right)^{-1}$$

where $\hbar$ is the reduced Planck constant, E is electron energy, and k is the wavevector.

Step 3: Detailed Explanation:

- The term $\frac{d^2E}{dk^2}$ represents the mathematical second derivative of the energy with respect to the wavevector, which physically defines the curvature of the band in the $E-k$ diagram.

- **Highly Curved Bands (Sharp Parabola):** If the band has a high curvature, the second derivative $\frac{d^2E}{dk^2}$ is large. This results in a small effective mass m^*, meaning the electron can be accelerated easily by an external electric field (high mobility).

- **Flat Bands (Low Curvature):** If the band is relatively flat, the second derivative is small, which means the effective mass m^* is very large. The electron behaves as a heavy particle with low mobility.

- Thus, the curvature of the $E-k$ curve is the direct determinant of the electron's effective mass.

Step 4: Final Answer:

The effective mass of an electron is determined by the curvature of the E - k diagram.

Quick Tip: Remember:

- Sharp curvature $\implies$ Small effective mass m^* $\implies$ High mobility.

- Flat curve $\implies$ Large effective mass m^* $\implies$ Low mobility.

This is a core concept in designing high-speed semiconductor nanomaterials.

17. Which of the following statement is wrong?

(A) The heat transfer in liquid and gases takes place according to convection
(B) The amount of heat flow through a body is dependent upon the material of the body
(C) The thermal conductivity of solid metals increases with rise in temperature
(D) Logarithmic mean temperature difference is not equal to the arithmetic mean temperature difference.

Correct Answer: (C) The thermal conductivity of solid metals increases with rise in temperature

Solution:

Step 1: Understanding the Question:
The question asks to identify the incorrect (wrong) statement among the four options regarding heat transfer mechanisms and thermal properties.

Step 2: Detailed Explanation:

- **Statement A:** Heat transfer in fluids (liquids and gases) primarily occurs via convection, where bulk fluid motion transports heat. This is a correct statement.

- **Statement B:** Fourier's Law of Heat Conduction states that heat flow rate ($Q = -kA\frac{dT}{dx}$) depends on the thermal conductivity (k), which is a characteristic property of the material of the body. This is a correct statement.

- **Statement C:** In solid metals, thermal energy is conducted primarily by free electrons and lattice vibrations (phonons).
  As temperature increases, the thermal scattering of free electrons by the vibrating lattice ions increases significantly.
  This increased scattering decreases the mean free path of the electrons, which generally causes the thermal conductivity (k) of most pure metals (like copper, aluminum) to

decrease with a rise in temperature. Thus, Statement C is incorrect, making it the correct option for this "find the wrong statement" question.

- **Statement D:** The Logarithmic Mean Temperature Difference (LMTD) and Arithmetic Mean Temperature Difference (AMTD) are mathematically distinct:

$$\text{LMTD} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} \neq \frac{\Delta T_1 + \Delta T_2}{2}$$

LMTD is always less than AMTD. This is a correct statement.

Step 3: Final Answer:

The incorrect statement is (C) "The thermal conductivity of solid metals increases with rise in temperature".

Quick Tip: For most pure metals, both electrical conductivity and thermal conductivity decrease as temperature rises because of increased electron-phonon scattering.
Always read "Which statement is wrong?" questions carefully to avoid picking a correct statement by mistake.

---

18. Curie temperature of ferrite is

(A) 768 °C
(B) 910 °C
(C) 1147 °C
(D) 727 °C

Correct Answer: (A) 768 °C

Solution:

Step 1: Understanding the Question:

The question asks for the Curie temperature (T_c) of ferrite (α-iron), which is the critical transition temperature at which the material changes from ferromagnetic to paramagnetic.

Step 2: Detailed Explanation:

- **What is Curie Temperature?**

  The Curie temperature is the temperature above which a ferromagnetic material loses its permanent magnetic properties and becomes paramagnetic.

  At this temperature, the thermal energy ($k_B T$) is sufficient to overcome the magnetic exchange coupling that aligns the individual atomic magnetic dipoles.

- **Curie Temperature of Ferrite:**

  Alpha-ferrite (α-Fe) is strongly ferromagnetic at room temperature.

  As temperature increases, the magnetic alignment begins to randomize.

  At exactly 768 °C (approximately 1043 K), ferrite undergoes a magnetic phase transition and becomes paramagnetic.

  This is known as the Curie point of iron. Note that there is no change in crystal structure at 768 °C; it remains BCC.

- **Other Temperatures on the Iron-Carbon Diagram:**

  - 727 °C is the eutectoid temperature (A_1 line).

  - 910 °C is the allotropic transformation temperature from BCC (α-ferrite) to FCC (γ-austenite) (A_3 line).

  - 1147 °C is the eutectic transformation temperature of cast iron.

Step 3: Final Answer:

The Curie temperature of ferrite is 768 °C.

Quick Tip: At the Curie temperature of 768 °C, ferrite undergoes a magnetic transition (ferromagnetic → paramagnetic) but its crystal structure remains Body-Centered Cubic (BCC).
The crystal structure change from BCC to FCC occurs later at 910 °C.

---

19. If a material has a band gap (E_g) of 4.0 eV, it will most likely be

(A) transparent to visible light
(B) opaque and metallic
(C) a p-type semiconductor at room temperature
(D) highly absorbent in the infrared region

Correct Answer: (A) transparent to visible light

Solution:

Step 1: Understanding the Question:
The question asks to predict the optical behavior of a solid material with a wide electronic band gap energy ($E_g = 4.0$ eV).

Step 2: Key Formula or Approach:

- The energy of an optical photon is given by:

$$E = \frac{hc}{\lambda}$$

where h is Planck's constant, c is the speed of light, and λ is the wavelength.

- The spectrum of visible light spans wavelengths from approximately 400 nm (violet) to 700 nm (red).

- Converting these wavelengths into photon energies:

- Red light (700 nm): $E \approx 1.8$ eV

- Violet light (400 nm): $E \approx 3.1$ eV

Therefore, the energy range of visible light photons is 1.8 eV to 3.1 eV.

Step 3: Detailed Explanation:

- For an electron in the valence band of a material to absorb a photon, the photon's energy must be equal to or greater than the bandgap energy:

$$E_{\text{photon}} \geq E_g$$

- If $E_{\text{photon}} < E_g$, the material cannot absorb the photon, and the light passes through without attenuation (transparent).

- Since the maximum energy of any visible light photon is 3.1 eV, and the material's bandgap is 4.0 eV:

$$E_{\text{visible}} \leq 3.1 \text{ eV} < 4.0 \text{ eV}$$

- Visible light photons do not have enough energy to excite electrons across the 4.0 eV bandgap.

- As a result, the material cannot absorb visible light, making it transparent to the visible spectrum (such as wide-bandgap insulators like quartz or alumina).

Step 4: Final Answer:

The material with a band gap of 4.0 eV will most likely be transparent to visible light.

Quick Tip: A simple rule of thumb for optical transparency:
- $E_g > 3.1$ eV: Insulator, transparent to visible light.
- $E_g < 1.8$ eV: Semiconductor, absorbs visible light (appears colored or opaque).
- $E_g \approx 0$ eV: Metal, reflective and opaque.

---

20. Thermal diffusivity is a

(A) function of temperature
(B) physical property of a substance
(C) dimensionless parameter
(D) function of pressure

Correct Answer: (B) physical property of a substance

Solution:

Step 1: Understanding the Question:
The question asks to classify the thermodynamic term "thermal diffusivity" (α) in terms of its physical nature and properties.

Step 2: Key Formula or Approach:

- Thermal diffusivity (α) is defined mathematically as:

$$\alpha = \frac{k}{\rho c_p}$$

where:
k is the thermal conductivity of the material (W/m-K),
ρ is the density (kg/m^3),
c_p is the specific heat capacity at constant pressure (J/kg-K).

- The SI unit of thermal diffusivity is m^2/s.

Step 3: Detailed Explanation:

- Thermal diffusivity measures the rate of heat transfer through a material relative to its heat storage capacity.

- A material with high thermal diffusivity (like copper or silver) responds rapidly to temperature changes, diffusing thermal energy quickly.

- Since it is defined using other material properties (k, ρ, c_p), thermal diffusivity itself is a fundamental **physical property of a substance**.

- Let us evaluate the options:
  - While α can vary slightly with temperature, it is not fundamentally defined as a "function of temperature".
  - It has physical dimensions of m^2/s, so it is not a dimensionless parameter.
  - It is not classified primarily as a function of pressure.
  - Option (B) correctly identifies it as a physical property of a substance.

Step 4: Final Answer:

Thermal diffusivity is a physical property of a substance.

Quick Tip: Thermal diffusivity ($\alpha = \frac{k}{\rho c_p}$) has the same units as kinematic viscosity ($\nu = \frac{\mu}{\rho}$) and mass diffusivity (D): m^2/s.
These three transport properties represent the diffusion of thermal energy, momentum, and mass, respectively.

---

21. Which one of the following elements is ferrite stabilizer?

(A) Nickel

(B) Copper

(C) Chromium

(D) Manganese

Correct Answer: (C) Chromium

Solution:

Step 1: Understanding the Question:

The question asks to identify which of the given alloying elements acts as a ferrite stabilizer in steel metallurgy.

Alloying elements in iron-carbon systems are classified based on their influence on the stability of the phases (ferrite and austenite) and how they modify the iron-carbon phase diagram.

Step 2: Detailed Explanation:

- **Classification of Alloying Elements:**
  Alloying elements added to steel generally fall into two categories: Austenite stabilizers and Ferrite stabilizers.

- **Austenite Stabilizers:**
  These elements expand the gamma-phase (γ-iron, austenite) region on the phase diagram.
  They lower the eutectic/eutectoid transition temperature, stabilizing the Face-Centered Cubic (FCC) austenite structure down to lower temperatures.
  Examples include Nickel (Ni), Manganese (Mn), Copper (Cu), and Cobalt (Co).
  Therefore, Options (A), (B), and (D) are austenite stabilizers.

- **Ferrite Stabilizers:**
  These elements restrict the gamma-phase region and expand the alpha-phase (α-iron, ferrite) region.
  They raise the eutectoid temperature and promote the formation of the Body-Centered

Cubic (BCC) ferrite structure.

At higher concentrations, they can completely close the gamma-phase field, creating a "closed gamma loop".

Examples include Chromium (Cr), Silicon (Si), Molybdenum (Mo), Tungsten (W), Vanadium (V), and Titanium (Ti).

Thus, Chromium is a highly effective and classic ferrite stabilizer.

Step 3: Final Answer:

Chromium is a ferrite stabilizer.

Quick Tip: To remember the stabilizers:

- Austenite stabilizers are generally elements that favor FCC structure (like Ni, Mn, Cu).
- Ferrite stabilizers are elements that favor BCC structure (like Cr, Mo, W, V, Si).

---

22. According to the Free Electron Theory, if the temperature of a pure metal is increased, the collision frequency of electrons increases. How does this specifically affect the Fermi energy (E_F) of the metal?

(A) It decreases linearly with temperature

(B) It drops to zero at the melting point

(C) It increases significantly due to higher thermal energy

(D) It remains essentially constant

Correct Answer: (D) It remains essentially constant

Solution:

Step 1: Understanding the Question:

The question asks about the effect of increasing temperature (and the resulting higher electron collision frequency) on the Fermi energy (E_F) of a pure metal according to the Free Electron

Theory.

Step 2: Key Formula or Approach:

- The Fermi energy at absolute zero temperature (E_{F0}) is defined solely by the electron density (N/V):
$$E_{F0} = \frac{\hbar^2}{2m}\left(3\pi^2\frac{N}{V}\right)^{2/3}$$
- At a finite temperature T, the temperature-dependent Fermi energy $E_F(T)$ is given by the Sommerfeld expansion:
$$E_F(T) \approx E_{F0}\left[1 - \frac{\pi^2}{12}\left(\frac{k_B T}{E_{F0}}\right)^2\right]$$

Step 3: Detailed Explanation:

- **Order of Magnitude Analysis:**
For most typical metals, the Fermi energy E_{F0} is in the range of 2 to 10 eV.
The thermal energy equivalent of 1 eV is approximately 11600 K.
At room temperature ($T \approx 300$ K), the thermal energy $k_B T$ is only about 0.025 eV.
Even at the melting point of most metals ($T \approx 1000$ K to 1500 K), $k_B T$ is around 0.1 eV.

- **Evaluation of the Correction Term:**
The ratio $\frac{k_B T}{E_{F0}}$ is exceptionally small (on the order of 10^{-2}).
When squared in the Sommerfeld correction, the term $\left(\frac{k_B T}{E_{F0}}\right)^2$ becomes on the order of 10^{-4}.
Therefore, the decrease in Fermi energy with temperature is extremely tiny (less than 0.1%).

- **Conclusion:**
While collision frequency increases and electrical resistivity rises with temperature due to electron-phonon interactions, the Fermi energy itself remains virtually unchanged.
Thus, for all practical purposes and solid-state calculations, the Fermi energy of a metal

remains essentially constant.

Step 4: Final Answer:

The Fermi energy of the metal remains essentially constant as the temperature increases.

Quick Tip: The Fermi energy is a property dictated primarily by the density of free electrons in the lattice, which is constant.

Because the Fermi temperature of metals ($T_F = E_F/k_B$) is extremely high (tens of thousands of Kelvin), standard operating temperatures have negligible influence on the Fermi level.

---

23. In a loaded beam, the point of contra flexure occurs at a section where

(A) bending moment is minimum

(B) bending moment is zero or changes sign

(C) bending moment is maximum

(D) shearing force is maximum

Correct Answer: (B) bending moment is zero or changes sign

Solution:

Step 1: Understanding the Question:

The question asks for the physical definition and structural condition associated with the "point of contraflexure" in a loaded structural beam.

Step 2: Detailed Explanation:

- **Bending Moment Behavior in Beams:**
  Under different transverse loads, beams experience bending.
  The curvature of the beam can bend downwards (sagging, positive bending moment) or

upwards (hogging, negative bending moment).

- **Point of Contraflexure:**
  In continuous or overhanging beams, there are regions where the nature of the bending moment transitions from sagging to hogging, or vice versa.
  The point where the bending moment diagram crosses the zero axis—meaning the bending moment is exactly zero and changes its mathematical sign—is defined as the point of contraflexure (or point of inflection).

- **Structural Significance:**
  At this location, the curvature of the elastic curve of the beam changes its direction of concavity.
  This point is highly critical in concrete structural design because the placement of tensile reinforcement (rebar) must switch from the bottom of the beam (for sagging) to the top of the beam (for hogging) around this region.

Step 3: Final Answer:
The point of contraflexure occurs at a section where the bending moment is zero or changes sign.

Quick Tip: "Contra" means opposite and "flexure" means bending.
Therefore, contraflexure literally translates to "opposite bending", representing the location where the bending switches direction, which mathematically means the bending moment must pass through zero.

---

24. The property that cannot be obtained from a tensile test is

(A) Yield Strength
(B) Ultimate Tensile Strength
(C) Ductility
(D) Endurance Limit

Correct Answer: (D) Endurance Limit

Solution:

Step 1: Understanding the Question:

The question asks to identify which of the given mechanical properties of a material cannot be determined by conducting a standard uniaxial tensile test on a universal testing machine (UTM).

Step 2: Detailed Explanation:

- **Standard Tensile Test capabilities:**

  During a uniaxial tensile test, a specimen is subjected to a slowly increasing static or quasi-static tensile load until it fractures.

  The resulting engineering stress-strain curve allows direct determination of:

  1. **Yield Strength:** The stress at which a material transitions from elastic to plastic deformation. Option (A) is obtained.

  2. **Ultimate Tensile Strength:** The maximum stress value on the engineering stress-strain curve. Option (B) is obtained.

  3. **Ductility:** Measured quantitatively by percent elongation (%EL) and percent reduction in area (%RA) at fracture. Option (C) is obtained.

- **Endurance Limit:**

  The endurance limit (or fatigue limit) is the maximum amplitude of cyclic stress that a material can withstand for an infinite number of load cycles without experiencing fatigue failure.

  Fatigue represents a dynamic, time-dependent failure mechanism under fluctuating loads.

  Because a tensile test is a monotonic, static test, it cannot capture cyclic fatigue behavior.

  To find the endurance limit, a dedicated fatigue test (such as the rotating-beam test) must be conducted to plot an S-N curve.

Step 3: Final Answer:

The endurance limit cannot be obtained from a tensile test.

Quick Tip: Tensile tests measure static mechanical properties.
Dynamic or cyclic properties (like fatigue life, endurance limit, and impact toughness) require separate specialized tests such as fatigue testing machines and Charpy/Izod impact testers.

---

25. In a combustion process, the "Theoretical Adiabatic Flame Temperature" is the maximum temperature the products can reach. This temperature is lower in practice because

(A) real reactions are always endothermic
(B) part of the energy is consumed by the dissociation of product molecules
(C) the heat capacity of the products decreases at high temperatures
(D) the nitrogen in the air acts as a catalyst, absorbing energy

Correct Answer: (B) part of the energy is consumed by the dissociation of product molecules

Solution:

Step 1: Understanding the Question:
The question asks why the actual measured maximum flame temperature in real-world combustion is always significantly lower than the theoretically calculated adiabatic flame temperature.

Step 2: Detailed Explanation:

- **Theoretical Adiabatic Flame Temperature Definition:**
  This is calculated by assuming complete combustion, zero heat loss to the surroundings, and that all released chemical energy goes entirely into raising the temperature of the products.

- **The Phenomenon of Dissociation:**
  At extremely high temperatures (typically above 1600 K to 2000 K), the main combustion products (CO_2 and H_2O) are no longer completely stable.

They undergo reversible, endothermic dissociation reactions into simpler molecules and radicals:

$$CO_2 \rightleftharpoons CO + \frac{1}{2}O_2 \quad (\Delta H > 0)$$

$$H_2O \rightleftharpoons H_2 + \frac{1}{2}O_2 \quad (\Delta H > 0)$$

$$H_2O \rightleftharpoons OH + \frac{1}{2}H_2 \quad (\Delta H > 0)$$

- **Effect on Flame Temperature:**
  These chemical dissociation reactions are highly endothermic (they absorb heat).
  Consequently, some of the thermal energy that would have raised the temperature of the gas is instead consumed to break the chemical bonds of the products.
  This limits the peak temperature achievable by the flame.

- **Evaluating other options:**
  - Option (A) is incorrect as combustion is highly exothermic.
  - Option (C) is incorrect because specific heat capacities of gases actually *increase* with temperature, which also helps lower the temperature, but dissociation is the primary thermodynamic limit at peak temperatures.
  - Option (D) is incorrect; nitrogen is inert in terms of catalysis, although it acts as a thermal diluent.

Step 3: Final Answer:
In practice, the flame temperature is lower because part of the energy is consumed by the dissociation of product molecules.

Quick Tip: At high temperatures, dissociation acts as a natural chemical buffer.
The heat absorbed by endothermic dissociation limits the maximum temperature in gas turbines, rockets, and internal combustion engines, keeping it below theoretical values.

---

26. If the shear force along a section of a beam is zero, the bending moment at the section is

(A) zero

(B) maximum

(C) minimum

(D) average of maximum-minimum

Correct Answer: (B) maximum

Solution:

Step 1: Understanding the Question:

The question asks about the state of the bending moment at a cross-section of a beam where the shear force is zero.

Step 2: Key Formula or Approach:

- In beam theory, there is a fundamental differential relationship between the bending moment (M) and the shear force (V) along the length of the beam (x):

$$V = \frac{dM}{dx}$$

Step 3: Detailed Explanation:

- According to differential calculus, for any continuous function $M(x)$ to reach a local extremum (either a local maximum or a local minimum), its first derivative with respect to x must equal zero:

$$\frac{dM}{dx} = 0$$

- Substituting the relation $V = \frac{dM}{dx}$ into this mathematical condition:

$$V = 0$$

- This implies that at any location along the span of the beam where the shear force curve passes through zero, the bending moment diagram will reach a local extremum.

- For most common loading scenarios (like uniformly distributed loads or concentrated loads on simply supported beams), this extremum corresponds to the maximum bending moment in the span.

- Therefore, the bending moment is maximum (or minimum/extremum) at the point of zero shear.

Step 4: Final Answer:

The bending moment at the section where the shear force is zero is maximum.

Quick Tip: To locate the section of maximum bending moment in a beam under complex loading:

1. Draw the Shear Force Diagram (SFD).
2. Identify the points where the shear force changes sign (crosses zero).
3. The bending moment will reach its peak value at these exact points.

---

27. Yield point phenomenon observed in mild steel is due to the presence of

(A) Silicon

(B) Manganese

(C) Carbon

(D) Chromium

Correct Answer: (C) Carbon

Solution:

Step 1: Understanding the Question:

The question asks to identify the element responsible for the distinct yield point phenomenon (upper and lower yield points) observed during the tensile testing of mild steel.

Step 2: Detailed Explanation:

- **The Yield Point Phenomenon:**
  When mild steel (a low-carbon steel) is tested in tension, the stress-strain curve exhibits a unique transition from elastic to plastic deformation characterized by an upper yield point, a sudden drop in stress to a lower yield point, followed by a plateau (yield point elongation).

- **Cottrell Atmosphere Theory:**
  This behavior is explained in physical metallurgy by the interaction between dislocations and small interstitial solute atoms, primarily Carbon (C) and Nitrogen (N).
  Because these solute atoms are small, they migrate to the strain fields surrounding edge dislocations to minimize the elastic strain energy of the crystal lattice. This clustering of solute atoms is called a "Cottrell atmosphere".

- **Mechanism of Yielding:**
  The Cottrell atmosphere pins or locks the dislocations in place.
  To initiate plastic deformation, a high stress (the upper yield point) must be applied to tear the dislocations free from these solute atmospheres.
  Once the dislocations are unlocked, they can multiply and move through the lattice at a much lower stress level (the lower yield point).

- Carbon is the primary interstitial alloying element in mild steel responsible for this pinning effect.

Step 3: Final Answer:
The yield point phenomenon observed in mild steel is due to the presence of carbon.

Quick Tip: The distinct yield drop in mild steel is a dislocation locking-unlocking mechanism driven by interstitial Carbon atoms.
This phenomenon is also associated with the formation of visible deformation bands called Lüders bands on the specimen surface.

---

28. Adiabatic flame temperature is maximum when

(A) the reaction incomplete
(B) no heat loss and complete combustion
(C) the heat loss is maximum
(D) excess air is high

Correct Answer: (B) no heat loss and complete combustion

Solution:

Step 1: Understanding the Question:
The question asks for the conditions under which the adiabatic flame temperature of a combustion system reaches its absolute maximum value.

Step 2: Detailed Explanation:

- **Adiabatic System Concept:**
  An adiabatic process is one in which there is no heat transfer between the system and its surroundings ($Q = 0$).
  Therefore, "no heat loss" is a fundamental requirement to achieve the adiabatic condition.
  If any heat is lost to the surroundings through radiation, conduction, or convection, the temperature of the products will drop.

- **Complete Combustion Concept:**
  To maximize the thermal energy released, all the chemical energy stored in the fuel

must be completely converted into heat. This requires complete combustion (all carbon oxidized to CO_2, all hydrogen to H_2O, etc.).
Incomplete combustion leaves unburnt fuel or partially oxidized species (like CO), which means less energy is released, resulting in a lower flame temperature.

- **Effect of Excess Air:**
  If excess air is introduced, the nitrogen and unused oxygen in the excess air must be heated up. This acts as a thermal sink, absorbing heat and significantly lowering the final flame temperature.
  Thus, maximum temperature is achieved under stoichiometric conditions with complete combustion and zero heat loss.

Step 3: Final Answer:
The adiabatic flame temperature is maximum when there is no heat loss and complete combustion.

Quick Tip: To maximize flame temperature:
1. Prevent heat losses (insulate the combustion chamber).
2. Burn fuel completely.
3. Avoid excess air, as the extra inert Nitrogen (N_2) acts as a heat sink and lowers the temperature.

---

29. The stress necessary to initiate yielding, is considerably

(A) more than that necessary to continue it
(B) less than that necessary to continue it
(C) more than that necessary to stop it
(D) less than that necessary to stop it

Correct Answer: (A) more than that necessary to continue it

Solution:

Step 1: Understanding the Question:

The question refers to the comparative stress levels required to start (initiate) plastic deformation versus the stress level required to propagate or sustain (continue) plastic deformation in materials that display a yield drop.

Step 2: Detailed Explanation:

- **Yielding Initiation vs Propagation:**
  In many crystalline materials (notably annealed low-carbon steel, certain aluminum alloys, and polymers), the onset of plastic deformation requires overcoming a high initial energy barrier.
  The stress corresponding to the initiation of yielding is represented by the **Upper Yield Point**.
  Once yielding is initiated at a local region, the stress drops sharply to a lower level, represented by the **Lower Yield Point**.

- **Dislocation Dynamics:**
  The high initial stress is required to pull the pinned dislocations out of their interstitial solute traps (Cottrell atmospheres).
  Once unlocked, the dislocations multiply rapidly, and their movement requires a lower stress level to sustain the deformation.
  Thus, the stress required to initiate yielding (upper yield stress) is considerably **more** than that necessary to continue it (lower yield/flow stress).

Step 3: Final Answer:

The stress necessary to initiate yielding is considerably more than that necessary to continue it.

Quick Tip: Think of it like static friction vs kinetic friction:
It always takes more force to start the motion (upper yield point) than to keep it sliding (lower yield point).

---

30. Cold working of metal is carried out at

(A) below recrystallization temperature
(B) above recrystallization temperature
(C) below room temperature
(D) just above the grain growth temperature

Correct Answer: (A) below recrystallization temperature

Solution:

Step 1: Understanding the Question:
The question asks for the temperature regime in which "cold working" of metals is defined and conducted.

Step 2: Detailed Explanation:

- **Definition of Metal Working Regimes:**
  Plastic deformation processes for metals are classified into cold working, warm working, and hot working based on the recrystallization temperature (T_{rec}) of the metal.
  The recrystallization temperature is the temperature at which a highly strained grain structure is replaced by a new set of strain-free grains. It is typically in the range of 0.3 T_m to 0.5 T_m (where T_m is the absolute melting point).

- **Cold Working:**
  Cold working is plastic deformation carried out at temperatures **below the recrystallization temperature**.

Often, this is done at room temperature. Because the temperature is too low for recrystallization to occur, the deformed grains remain strained.
This leads to strain hardening (work hardening), which increases the strength and hardness of the metal while decreasing its ductility.

- **Hot Working:**
  Deformation carried out above the recrystallization temperature, where the metal continuously recrystallizes and dynamic recovery occurs, preventing work hardening.

Step 3: Final Answer:
Cold working of metal is carried out at temperatures below the recrystallization temperature.

Quick Tip: The distinction between "hot" and "cold" working is not based on room temperature, but rather on the metal's recrystallization temperature.
For example, lead has a recrystallization temperature below room temperature, so working lead at room temperature is actually a "hot working" process.

---

31. Mixing identical gases results in

(A) no change in entropy
(B) decrease in entropy
(C) increase in entropy
(D) infinite entropy

Correct Answer: (A) no change in entropy

Solution:

Step 1: Understanding the Question:
The question asks for the change in thermodynamic entropy when two samples of the same

identical gas are mixed at constant temperature and pressure.

Step 2: Key Formula or Approach:

- The entropy of mixing (ΔS_{mix}) for two different ideal gases (distinguishable molecules) is:

$$\Delta S_{mix} = -nR(x_1 \ln x_1 + x_2 \ln x_2) > 0$$

  where x_1 and x_2 are the mole fractions.

- For identical gases (indistinguishable molecules), this formula does not apply. This is known as the Gibbs Paradox.

Step 3: Detailed Explanation:

- **The Gibbs Paradox:**
  If we have two containers of the same gas (e.g., Nitrogen in both compartments) at the same temperature and pressure, and we remove the partition, the molecules mix.
  From a macroscopic and microscopic perspective, because the molecules of the same gas are completely identical and indistinguishable, there is no physical change in the thermodynamic state of the system.
  No work can be extracted from this process, and no heat is transferred.
  Since there is no change in the number of accessible microstates that represent a physically distinguishable state, the entropy of the system remains unchanged.
  Therefore, the entropy of mixing for identical gases is exactly zero:

$$\Delta S_{mix} = 0$$

Step 4: Final Answer:
Mixing identical gases results in no change in entropy.

Quick Tip: Entropy of mixing is zero for identical gases (Gibbs Paradox).
Entropy increases only when mixing different, distinguishable gases because the number of distinct microstates increases.

---

32. Strain energy of any member may be defined as work done to

(A) deformation
(B) pressure
(C) moment
(D) Force

Correct Answer: (A) deformation

Solution:

Step 1: Understanding the Question:
The question asks for the definition of strain energy stored in a structural member in terms of the work done on it.

Step 2: Detailed Explanation:

- **Concept of Strain Energy:**
  When an external force acts on an elastic body, the body undergoes deformation.
  During this deformation, the applied force does work on the body.
  According to the conservation of energy, this work done is stored internally within the material of the body as potential energy, which is called elastic strain energy (U).

- **Mathematical Definition:**
  For a uniaxial bar subjected to a gradually applied tensile load P causing a deformation δ:
  $$U = \int P \cdot d\delta$$

Under linear elastic conditions:

$$U = \frac{1}{2} P \cdot \delta$$

- Therefore, strain energy is directly defined as the work done by external forces to deform the material.

Step 3: Final Answer:

The strain energy of a member is defined as the work done to cause deformation.

Quick Tip: Strain energy is equal to the area under the load-deformation curve.

For a linear elastic material, $U = \frac{1}{2} \times$ Load $\times$ Deformation.

---

33. Which of the following metal working operation is a direct compression process?

(A) Extrusion

(B) Forging

(C) Wire drawing

(D) Stretch forming

Correct Answer: (B) Forging

Solution:

Step 1: Understanding the Question:

The question asks to identify which of the listed metal forming (working) processes is categorized as a "direct compression process".

Step 2: Detailed Explanation:

- Metal forming operations are classified based on the primary state of stress acting on the

metal workpiece:

- **Direct Compression Processes:**
  The external forces are applied directly to the surface of the workpiece, squeezing it to deform.
  The metal is subjected to compressive stresses in all directions.
  Forging is a classic direct compression process where the metal is compressed between flat or shaped dies using hammer blows or press forces.

- **Indirect Compression Processes:**
  The primary applied force is tensile or compressive, but the geometry of the die converts this into high compressive stresses within the metal.
  Extrusion is an indirect compression process because the compression is induced by pushing the metal through a die opening, which creates compressive reactions from the container walls.

- **Tension-Compression (Indirect Tension):**
  Wire drawing involves pulling the wire (tensile force) through a die, creating indirect compression within the die.

- **Pure Tension:**
  Stretch forming involves wrapping and stretching a sheet metal over a tool primarily under tensile stresses.

Step 3: Final Answer:
Forging is a direct compression metal working operation.

Quick Tip: - Direct Compression: Forging, Rolling.
- Indirect Compression: Extrusion.
- Indirect Tension: Wire Drawing.
- Pure Tension: Stretch Forming.

---

34. According to the Gibbs Phase Rule (F = C - P + 2), at the triple point of a pure metal (e.g., where solid, liquid, and vapour coexist), the degrees of freedom F is

(A) 1 (P can be varied)
(B) 2 (Both T and P can be varied)
(C) 0 (neither T nor P can be varied)
(D) 3

Correct Answer: (C) 0 (neither T nor P can be varied)

Solution:

Step 1: Understanding the Question:
The question asks to find the degrees of freedom (F) at the triple point of a pure metal using the Gibbs Phase Rule.

Step 2: Key Formula or Approach:

- Gibbs Phase Rule:

$$F = C - P + 2$$

where:
F is the degrees of freedom (independent intensive variables like temperature and pressure),
C is the number of chemical components in the system,
P is the number of phases coexisting in equilibrium.

Step 3: Detailed Explanation:

- Identify the parameters for the triple point of a pure metal:

  1. Since it is a "pure metal", there is only one chemical species present. Thus, the number of components is:

  $$C = 1$$

  2. At the "triple point", three phases (solid, liquid, and vapour) coexist in thermodynamic equilibrium. Thus, the number of coexisting phases is:

  $$P = 3$$

- Substitute $C = 1$ and $P = 3$ into the Gibbs Phase Rule:

$$F = 1 - 3 + 2 = 0$$

- A system with $F = 0$ is called "invariant".
  This means that the triple point occurs at a unique, fixed temperature and pressure. Neither temperature (T) nor pressure (P) can be varied. If we attempt to change either of these variables, the equilibrium is disrupted, and one or more phases will disappear.

Step 4: Final Answer:

The degrees of freedom F is 0, which means neither temperature nor pressure can be varied.

Quick Tip: At a triple point, $F = 0$ (invariant).
Along a phase boundary line, $P = 2 \implies F = 1$ (univariant).
Inside a single-phase region, $P = 1 \implies F = 2$ (bivariant).

---

35. If ΔG is positive, the reaction is:

(A) Spontaneous

(B) Non-spontaneous

(C) At equilibrium

(D) Reversible

Correct Answer: (B) Non-spontaneous

Solution:

Step 1: Understanding the Question:

The question asks about the spontaneity of a chemical reaction when the Gibbs free energy change (ΔG) is positive.

Gibbs free energy is a thermodynamic potential that measures the maximum reversible work that may be performed by a thermodynamic system at constant temperature and pressure.

Step 2: Key Formula or Approach:

The change in Gibbs free energy is given by the relation:

$$\Delta G = \Delta H - T\Delta S$$

where:

ΔH is the change in enthalpy.

T is the absolute temperature in Kelvin.

ΔS is the change in entropy.

The sign of ΔG determines the thermodynamic feasibility of a process at constant temperature and pressure:

1. If $\Delta G < 0$, the process is spontaneous (exergonic).
2. If $\Delta G > 0$, the process is non-spontaneous (endergonic).
3. If $\Delta G = 0$, the system is in dynamic equilibrium.

Step 3: Detailed Explanation:

- A positive value of ΔG indicates that the free energy of the products is greater than the free energy of the reactants.

- For such a reaction to occur, an external input of energy is required to drive the process forward.
- Consequently, the forward reaction cannot proceed on its own under the specified conditions, making it non-spontaneous.
- Conversely, the reverse reaction under these conditions would have a negative ΔG and would be spontaneous.

Step 4: Final Answer:

Since ΔG is positive, the reaction is non-spontaneous.

Quick Tip: Remember the thermodynamic spontaneity criteria at constant T and P:

$\Delta G < 0 \rightarrow$ Spontaneous

$\Delta G > 0 \rightarrow$ Non-spontaneous

$\Delta G = 0 \rightarrow$ Equilibrium

This is one of the most frequently asked basic questions in thermodynamic exams.

---

36. Seamless tubes can be produced by

(A) rolling

(B) extrusion

(C) deep drawing

(D) forging

Correct Answer: (B) extrusion

Solution:

Step 1: Understanding the Question:
The question asks which metal forming process is used for manufacturing seamless tubes. Seamless tubes do not contain any welded joints along their length, which makes them highly reliable for high-pressure applications.

Step 2: Key Formula or Approach:
Manufacturing of tubes can be categorized into welded and seamless tube production. Processes like extrusion and rotary tube piercing (Mannesmann process) are specialized techniques for creating hollow, seamless cylindrical shapes.

Step 3: Detailed Explanation:

- **Extrusion:** In metal extrusion, a billet is compressed and forced to flow through a die. For producing hollow tubes, a mandrel is placed inside the die. As the metal is squeezed through the annular gap between the die and the mandrel, a continuous seamless tube of uniform cross-section is produced. This process can be performed hot or cold.

- **Rolling:** Standard rolling processes are used to produce flat sheets, plates, or structured shapes. While specialized tube-rolling processes exist, standard rolling is typically associated with flat products.

- **Deep Drawing:** This process is used to form sheet metal into cup-shaped or box-shaped containers, rather than continuous long pipes.

- **Forging:** Forging is a localized deformation process using compressive forces. It is not suitable for producing long, continuous, thin-walled hollow sections like seamless tubes.

Step 4: Final Answer:
Therefore, seamless tubes are primarily produced by extrusion.

Quick Tip: Whenever a question asks about producing continuous hollow profiles, think of **Extrusion with a mandrel**.

For non-continuous cup-like hollow parts, the correct choice would be **Deep Drawing**.

---

37. In statistical thermodynamics, Stirling's Approximation ($\ln N! \approx N \ln N - N$) is valid only when

(A) N is a small integer

(B) N is very large

(C) The system is at 0 K

(D) The gas behaves non-ideally

Correct Answer: (B) N is very large

Solution:

Step 1: Understanding the Question:

The question asks for the condition under which Stirling's approximation is mathematically and physically valid.

Stirling's approximation is a fundamental tool in statistical mechanics used to simplify calculations involving large factorials.

Step 2: Key Formula or Approach: The exact Stirling's series for the factorial of a number N is:

$$\ln N! = N \ln N - N + \frac{1}{2} \ln(2\pi N) + O(1/N)$$

When N is extremely large ($N \to \infty$), the terms of lower order than N become negligible compared to the leading terms $N \ln N - N$.

Step 3: Detailed Explanation:

- In statistical thermodynamics, we deal with macroscopic systems containing particles on the order of Avogadro's number ($N \approx 10^{23}$).
- Calculating the exact factorial of such massive numbers is computationally impossible and mathematically tedious.
- By using the approximation $\ln N! \approx N \ln N - N$, the relative error becomes incredibly small as N increases.
- For example, if $N = 10$, the approximation is rough, but for $N > 10^4$, the error is negligible. At $N \approx 10^{23}$, the approximation is practically exact.
- This approximation does not depend on thermodynamic temperature (0 K) or gas ideality; it is a pure mathematical identity that relies on the scale of N.

Step 4: Final Answer:

The approximation is valid only when N is very large.

Quick Tip: Stirling's approximation is essential for deriving Boltzmann's entropy formula ($S = k_B \ln \Omega$). It works because macroscopic thermodynamic systems always have an extremely large number of particles ($N \approx 10^{23}$).

---

38. For ideal solution, activity (a) equals:

(A) Density

(B) Pressure

(C) Volume

(D) Mole fraction

Correct Answer: (D) Mole fraction

Solution:

Step 1: Understanding the Question:

The question asks for the relationship between chemical activity (a) and concentration parameters for an ideal solution.

Activity is an effective concentration used in thermodynamics to account for non-ideal interactions in real systems.

Step 2: Key Formula or Approach:

The activity a_i of a component i in a solution is related to its mole fraction x_i by:

$$a_i = \gamma_i x_i$$

where γ_i is the activity coefficient.

Step 3: Detailed Explanation:

- An ideal solution is defined as one where the intermolecular forces between unlike molecules are identical to those between like molecules (A-B interactions are equal to A-A and B-B interactions).
- Because there are no energetic deviations upon mixing, the activity coefficient γ_i becomes exactly equal to 1 across all compositions.
- Consequently, the equation simplifies to:

$$a_i = x_i$$

- Thus, in an ideal solution, the thermodynamic activity of any component is equal to its

mole fraction.

Step 4: Final Answer:

For an ideal solution, activity equals the mole fraction.

Quick Tip: For an ideal gas, activity is represented by partial pressure (p_i).
For an ideal solution (liquid/solid), activity is represented by the mole fraction (x_i).
In both cases, the activity coefficient (γ_i or f_i) is equal to 1.

---

39. In an adiabatic process:

(A) Heat transfer occurs

(B) No heat transfer occurs

(C) Temperature is constant

(D) Pressure is constant

Correct Answer: (B) No heat transfer occurs

Solution:

Step 1: Understanding the Question:

The question asks for the defining characteristic of an adiabatic process.
Thermodynamic processes are classified based on which state variable or energy interaction is held constant during the transition.

Step 2: Key Formula or Approach:

The first law of thermodynamics is written as:

$$dU = dQ - dW$$

where:

dU is the change in internal energy.

dQ is the heat exchange.

dW is the work done by the system.

Step 3: Detailed Explanation:

- An **adiabatic process** is defined as a thermodynamic process in which there is no transfer of heat or mass between the thermodynamic system and its surroundings.

- Mathematically, this is expressed as:

$$dQ = 0 \quad (\text{or } Q = 0)$$

- Since no heat enters or leaves the system, any work done by the system is done at the expense of its internal energy ($dU = -dW$), which generally results in a change in temperature.

- Other processes for comparison:
  - **Isothermal process:** Temperature remains constant ($dT = 0$).
  - **Isobaric process:** Pressure remains constant ($dP = 0$).
  - **Isochoric process:** Volume remains constant ($dV = 0$).

Step 4: Final Answer:

Therefore, in an adiabatic process, no heat transfer occurs.

Quick Tip: An adiabatic boundary is equivalent to a perfectly insulated wall.

Remember:

Adiabatic $\rightarrow Q = 0$

Isothermal $\rightarrow \Delta T = 0$

Isobaric $\rightarrow \Delta P = 0$

---

40. The Zeroth Law of Thermodynamics provides the basis for temperature measurement. If Body A is in thermal equilibrium with Body B, and Body B is in thermal equilibrium with Body C, which property must be identical for all three?

(A) Internal Energy (U)

(B) Total Enthalpy (H)

(C) Temperature (T)

(D) The product of Pressure and Volume (PV)

Correct Answer: (C) Temperature (T)

Solution:

Step 1: Understanding the Question:

The question tests the fundamental definition and physical significance of the Zeroth Law of Thermodynamics.

This law establishes a transitive relation for thermal equilibrium, which allows us to define and measure temperature.

Step 2: Key Formula or Approach:

The Zeroth Law states:

If system A is in thermal equilibrium with system B, and system B is in thermal equilibrium with system C, then system A is in thermal equilibrium with system C.

Step 3: Detailed Explanation:

- **Thermal Equilibrium:** When two systems are in thermal equilibrium, there is no net flow of thermal energy (heat) between them when they are connected by a path permeable to heat.

- The physical parameter that determines whether heat will flow between systems is **temperature**.

- Therefore, systems in thermal equilibrium must share the same value of this state variable, which is temperature (T).

- Other properties like internal energy (U), enthalpy (H), or the product PV depend on the mass, volume, and chemical nature of the substance, and do not need to be equal for systems to be in thermal equilibrium.

Step 4: Final Answer:

The property that must be identical for all three bodies is Temperature (T).

Quick Tip: The Zeroth Law is the mathematical justification for using thermometers.

If a thermometer (B) reads the same value for body A and body C, then A and C are at the same temperature.

---

41. Chemical potential is equal to:

(A) Enthalpy

(B) Gibbs free energy per mole

(C) Entropy

(D) Internal energy

Correct Answer: (B) Gibbs free energy per mole

Solution:

Step 1: Understanding the Question:

The question asks for the physical definition of chemical potential in thermodynamics. Chemical potential determines the tendency of a substance to react, change phase, or diffuse.

Step 2: Key Formula or Approach:

The chemical potential μ_i of a species i in a mixture is defined as the partial molar Gibbs free energy:

$$\mu_i = \left(\frac{\partial G}{\partial n_i}\right)_{T,P,n_{j\neq i}}$$

Step 3: Detailed Explanation:

- For a pure, single-component system, the chemical potential is simply the Gibbs free energy divided by the number of moles:

  $$\mu = \frac{G}{n} = \bar{G}$$

  where $\bar{G}$ is the molar Gibbs free energy.

- Thus, chemical potential represents the capacity of a system to do non-expansion work per mole of substance added or removed at constant temperature and pressure.

- While chemical potential can also be defined in terms of other thermodynamic potentials (like internal energy, enthalpy, or Helmholtz free energy), those definitions require keeping different variables constant (such as entropy, volume, etc.), making "Gibbs free energy per mole" the standard physical interpretation.

Step 4: Final Answer:

Chemical potential is equal to Gibbs free energy per mole.

Quick Tip: Always remember: Chemical potential μ is an intensive property.
For a pure substance, chemical potential is exactly equal to its molar Gibbs free energy.

---

42. Which of the following processes produces maximum work?

(A) Rapid expansion of gas
(B) Free expansion
(C) Reversible expansion
(D) Irreversible expansion

Correct Answer: (C) Reversible expansion

Solution:

Step 1: Understanding the Question:
The question asks which thermodynamic process yields the maximum work output during the expansion of a gas.

Step 2: Key Formula or Approach:
Work done during the expansion of a gas is given by:

$$W = \int P_{\text{ext}} dV$$

where P_{ext} is the external pressure opposing the expansion.

Step 3: Detailed Explanation:

- **Reversible Expansion:** In a reversible process, the expansion occurs infinitely slowly

such that the system is always in thermodynamic equilibrium with its surroundings. Under these conditions, the opposing external pressure is only infinitesimally smaller than the gas pressure ($P_{ext} = P_{sys} - dP$). This maximizes the value of P_{ext} at every step of the integration, producing the maximum possible work output.

- **Irreversible / Rapid Expansion:** In any real, irreversible expansion, the external pressure is significantly lower than the system's pressure, which leads to lower work output due to frictional and dissipative losses.

- **Free Expansion:** In a free expansion, a gas expands into a vacuum where $P_{ext} = 0$. Consequently, the work done is exactly zero ($W = 0$).

Step 4: Final Answer:

The process that produces maximum work is reversible expansion.

Quick Tip: Maximum work is always associated with **reversible** paths in thermodynamics. Minimum work input (for compression) is also achieved via a reversible path.

---

43. Two parallel edge dislocations of the same sign (same Burger's vector) are located on the same slip plane. As they approach each other, what happens?

(A) Energy decreases; Force is attractive

(B) Energy increases; Force is repulsive

(C) Energy remains constant; Force is zero

(D) Energy increases; Force is attractive

Correct Answer: (B) Energy increases; Force is repulsive

Solution:

Step 1: Understanding the Question:
The question relates to the elastic interactions between two dislocations in a crystalline lattice. Specifically, we are looking at two parallel edge dislocations of the same sign sharing the same slip plane.

Step 2: Key Formula or Approach:
Dislocations introduce strain fields in the lattice, resulting in compressive stress above the slip plane and tensile stress below it (for positive edge dislocations).
The force per unit length between two parallel dislocations can be analyzed using Peach-Koehler equation principles or strain energy overlap.

Step 3: Detailed Explanation:

- When two edge dislocations of the same sign are on the same slip plane, their compressive stress fields (located on the same side of the slip plane) begin to overlap as they approach each other.
- This overlapping of identical stress fields significantly increases the total elastic strain energy of the crystal lattice.
- Since physical systems spontaneously tend to minimize their potential energy, the system will resist this approach.
- This resistance manifests as a repulsive force between the two dislocations, pushing them apart.
- Consequently, bringing them closer together requires doing work against this repulsion, which increases the strain energy of the system.

Step 4: Final Answer:

As they approach each other, the energy increases and the force is repulsive.

Quick Tip: Like dislocations (same sign on same slip plane) **repel** each other.
Unlike dislocations (opposite signs on same slip plane) **attract** and can annihilate each other, which decreases the system's energy.

---

44. At eutectic point, degree of freedom is:

(A) 0

(B) 1

(C) 2

(D) 3

Correct Answer: (A) 0

Solution:

Step 1: Understanding the Question:

The question asks for the number of degrees of freedom (variance) at a eutectic point in a binary phase diagram.

Step 2: Key Formula or Approach:

We use Gibbs Phase Rule for condensed systems (where pressure is held constant, typically at 1 atm):

$$F = C - P + 1$$

where:

F is the degrees of freedom.

C is the number of components.

P is the number of phases in equilibrium.

Step 3: Detailed Explanation:

- A eutectic reaction occurs in a binary system, meaning the number of components is $C = 2$.
- At the eutectic point, three phases coexist in equilibrium: the liquid phase (L) and two distinct solid phases (α and β). Thus, $P = 3$.
- Substituting these values into the condensed phase rule:

$$F = 2 - 3 + 1 = 0$$

- A degree of freedom of 0 means the system is invariant.
- This implies that the eutectic reaction can only occur at a single, unique temperature and a specific composition. Changing either temperature or composition will cause at least one of the three phases to disappear.

Step 4: Final Answer:

At the eutectic point, the degree of freedom is 0.

Quick Tip: Invariant points ($F = 0$) on binary alloy phase diagrams include:

- Eutectic points
- Eutectoid points
- Peritectic points
- Peritectoid points

---

45. Stirling's approximation is used to simplify:

(A) Integrals

(B) Factorials

(C) Logarithms

(D) Derivatives

Correct Answer: (B) Factorials

Solution:

Step 1: Understanding the Question:

The question asks which mathematical function is simplified using Stirling's approximation.

Step 2: Key Formula or Approach:

Stirling's approximation is mathematically formulated as:

$$\ln N! \approx N \ln N - N$$

This expression provides a highly accurate estimate for the natural logarithm of the factorial of a large number N.

Step 3: Detailed Explanation:

- Factorials grow extremely rapidly, and evaluating terms like $N!$ directly for large N is

practically impossible.

- In statistical mechanics, calculations often involve permutations and combinations of a large number of particles (e.g., 10^{23} particles).
- Stirling's approximation replaces the factorial operation with simpler arithmetic operations involving logarithms and linear terms, which are far easier to differentiate, integrate, and manipulate.
- Therefore, the approximation is explicitly used to simplify factorials.

Step 4: Final Answer:

Stirling's approximation is used to simplify factorials.

Quick Tip: Stirling's approximation is incredibly useful because:

$\ln N! \approx N \ln N - N$

It simplifies the calculation of thermodynamic probability (Ω) in entropy equations.

---

46. Which of the following statements regarding mechanical twinning is FALSE?

(A) Twinning occurs more frequently at high strain rates and low temperatures.

(B) The atomic displacement in twinning is a fraction of the interatomic distance.

(C) Twinning changes the orientation of the crystal lattice in the deformed region.

(D) Twinning produces the same total strain as slip for the same amount of atomic movement.

Correct Answer: (D) Twinning produces the same total strain as slip for the same amount of atomic movement.

Solution:

Step 1: Understanding the Question:
The question asks to identify the incorrect (FALSE) statement about mechanical twinning, which is a key mechanism of plastic deformation in metals alongside dislocation slip.

Step 2: Key Formula or Approach:
Compare the mechanisms of slip and twinning:
- **Slip:** Involves sliding of blocks of crystal over one another along a slip plane. The displacement is an integer multiple of the lattice spacing (Burgers vector).
- **Twinning:** Involves cooperative, homogeneous shear of lattice planes where every atom moves a fraction of the interatomic distance relative to its neighbor.

Step 3: Detailed Explanation:

- **Statement A is True:** Since twinning requires less localized thermal activation than slip, it is favored at low temperatures and high strain rates where slip is restricted.
- **Statement B is True:** In twinning, the displacement of atoms within the twinned region is a fraction of the interatomic spacing.
- **Statement C is True:** Twinning creates a region with a mirror-image orientation relative to the parent lattice, thus changing the crystal orientation.
- **Statement D is False:** Because atomic displacement in twinning is limited to a small fraction of the lattice parameter, the total plastic strain produced by twinning is much smaller than that typically achieved by dislocation slip for a given amount of global atomic movement. Hence, they do not produce the same strain.

Step 4: Final Answer:
Statement (D) is the false statement.

Quick Tip: Twinning is common in hexagonal close-packed (HCP) metals (like Zn, Mg) because they have fewer independent slip systems than FCC or BCC metals.

---

47. Second law states:

(A) Energy conserved

(B) Entropy increases

(C) Heat = work

(D) $\Delta H = 0$

Correct Answer: (B) Entropy increases

Solution:

Step 1: Understanding the Question:

The question asks for the statement corresponding to the Second Law of Thermodynamics.

Step 2: Key Formula or Approach:

The Second Law of Thermodynamics introduces the concept of entropy (S) and defines the direction of spontaneous physical processes.

For any spontaneous process in an isolated system, the total entropy must increase over time:

$$dS_{\text{isolated}} \geq 0$$

Step 3: Detailed Explanation:

- **First Law Comparison:** "Energy conserved" is the statement of the First Law of Thermodynamics ($dU = dQ - dW$).

- **Second Law Definition:** The Second Law states that natural processes run in a direction

that increases the net disorder (entropy) of the universe.

- In any irreversible process, some energy is degraded into thermal energy that cannot be fully converted back to useful work, which directly causes the total entropy of the universe to increase.

- Statements like "Heat = work" or "$\Delta H = 0$" do not represent the general formulation of the Second Law.

Step 4: Final Answer:

The Second Law states that entropy increases.

Quick Tip: First Law → Conservation of Energy

Second Law → Increase in Entropy of the Universe

Third Law → Entropy of a perfect crystal at 0 K is zero

---

48. The efficiency of a Carnot engine depends on:

(A) Pressure

(B) Volume

(C) Temperatures of reservoirs

(D) Working substance

Correct Answer: (C) Temperatures of reservoirs

Solution:

Step 1: Understanding the Question:

The question asks which factor determines the efficiency of a Carnot cycle heat engine.

Step 2: Key Formula or Approach:

The thermal efficiency (η) of a reversible Carnot engine operating between a hot reservoir at absolute temperature T_H and a cold reservoir at temperature T_C is:

$$\eta = 1 - \frac{T_C}{T_H}$$

Step 3: Detailed Explanation:

- According to Carnot's theorem, no engine operating between two heat reservoirs can be more efficient than a Carnot engine operating between those same reservoirs.
- The formula shows that the efficiency (η) is purely a function of the absolute temperatures T_H and T_C.
- It is completely independent of the nature of the working fluid (whether it is an ideal gas, steam, or any other substance), the operating pressures, or the volume limits of the cycle.
- To increase the efficiency, one must either increase the source temperature (T_H) or decrease the sink temperature (T_C).

Step 4: Final Answer:

The efficiency depends on the temperatures of the reservoirs.

Quick Tip: Carnot efficiency is an upper-bound limit for any heat engine.

Always use absolute temperatures in Kelvin when calculating Carnot efficiency:

$\eta = 1 - \frac{T_{\text{cold}}}{T_{\text{hot}}}$

49. The volume V of a cubic unit cell is related to the lattice parameter a. Which of the following expressions is dimensionally and physically correct?

(A) $V = a + a^2$

(B) $V = 2a$

(C) $V = a^2$

(D) $V = a^3$

Correct Answer: (D) $V = a^3$

Solution:

Step 1: Understanding the Question:

The question asks for the correct geometric relationship for the volume (V) of a cubic unit cell in terms of its edge length (lattice parameter a).

Step 2: Key Formula or Approach:

A cube has three orthogonal axes of equal length:

$$a = b = c$$

The volume of any parallelepiped is given by:

$$V = \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$$

For a cube where all angles are 90°:

$$V = a \times a \times a = a^3$$

Step 3: Detailed Explanation:

- Volume has the physical dimension of length cubed ($[L]^3$).

- The lattice parameter a represents the side length of the unit cell and has the dimension of length ($[L]$).

- Checking the dimensions of the given options:
  - Option A: $V = a + a^2$ is dimensionally inconsistent (adding length and area).
  - Option B: $V = 2a$ has the dimension of length.
  - Option C: $V = a^2$ has the dimension of area.
  - Option D: $V = a^3$ has the correct dimension of volume ($[L]^3$).

Step 4: Final Answer:

The physically and dimensionally correct expression is $V = a^3$.

Quick Tip: Dimensional analysis is a powerful tool to quickly eliminate incorrect choices. Volume must always scale with the cube of a linear dimension (a^3).

---

50. Which material is diamagnetic?

(A) Iron

(B) Copper

(C) Nickel

(D) Cobalt

Correct Answer: (B) Copper

Solution:

Step 1: Understanding the Question:
The question asks to identify the diamagnetic material among the given choices.
Diamagnetism is a property of materials that are weakly repelled by an external magnetic field.

Step 2: Key Formula or Approach:
- **Ferromagnetic materials:** Have strong, permanent magnetic alignment (e.g., Fe, Co, Ni).
- **Diamagnetic materials:** Have fully filled electronic shells with no unpaired electrons, resulting in a net magnetic moment of zero in the absence of an external field.

Step 3: Detailed Explanation:

- Iron (Fe), Cobalt (Co), and Nickel (Ni) are the classic transition metals that exhibit ferromagnetic behavior at room temperature due to their partially filled d-shells and strong exchange interactions.

- Copper (Cu) has the electronic configuration $[Ar]3d^{10}4s^{1}$. When metallic bonds form in bulk copper, the $3d$ subshell remains completely filled. The electronic states are paired, leading to a weak diamagnetic response when subjected to an external magnetic field.

- Consequently, copper is repelled by magnetic fields and has a small, negative magnetic susceptibility.

Step 4: Final Answer: The diamagnetic material is Copper.

Quick Tip: Most noble metals and covalent/ionic solids like Copper (Cu), Gold (Au), Silver (Ag), and Water (H_2O) are diamagnetic.
Fe, Co, and Ni are always ferromagnetic at room temperature.

---

51. At equilibrium, Gibbs free energy change is:

(A) Positive

(B) Negative

(C) Zero

(D) Infinite

Correct Answer: (C) Zero

Solution:

Step 1: Understanding the Question:

The question asks for the value of the Gibbs free energy change (ΔG) when a thermodynamic system reaches equilibrium at constant temperature and pressure.

Step 2: Key Formula or Approach:

The criterion for equilibrium in a closed system at constant T and P is that the Gibbs free energy G is at a minimum:

$$dG_{T,P} = 0$$

Step 3: Detailed Explanation:

- During a spontaneous process, the Gibbs free energy of the system decreases ($\Delta G < 0$).
- The reaction continues to proceed until the system reaches its state of minimum free energy.
- Once this minimum point is reached, any further infinitesimal change in either direction would require an increase in free energy.
- Therefore, at this minimum potential state, the rate of the forward reaction equals the

rate of the reverse reaction, and the net change in Gibbs free energy is exactly zero ($\Delta G = 0$).

Step 4: Final Answer:

At equilibrium, the Gibbs free energy change is zero.

Quick Tip: Do not confuse ΔG with the standard Gibbs free energy change ΔG°.
At equilibrium, $\Delta G = 0$, but ΔG° is usually non-zero and is related to the equilibrium constant by $\Delta G^\circ = -RT \ln K$.

---

52. Drag force F depends on density ρ, velocity V, and diameter D: $F = k\rho^a V^b D^c$. Find exponents a, b, c.

(A) $(1, 2, 1)$
(B) $(2, 2, 1)$
(C) $(1, 1, 2)$
(D) $(1, 2, 2)$

Correct Answer: (D) $(1, 2, 2)$

Solution:

Step 1: Understanding the Question:

The question asks to find the exponents a, b, and c in the drag force equation using dimensional analysis (Rayleigh's method).

Step 2: Key Formula or Approach:

Write down the dimensional formulas for each variable in the SI system:
- Force (F): $[\mathrm{M\ L\ T^{-2}}]$
- Density (ρ): $[\mathrm{M\ L^{-3}}]$

- Velocity (V): $[\mathrm{L\,T^{-1}}]$

- Diameter (D): $[\mathrm{L}]$

- Constant (k): Dimensionless ($[1]$)

Step 3: Detailed Explanation:

Substitute the dimensional formulas into the equation:

$$F = k\rho^a V^b D^c$$

$$[\mathrm{M\,L\,T^{-2}}] = [\mathrm{M\,L^{-3}}]^a [\mathrm{L\,T^{-1}}]^b [\mathrm{L}]^c$$

$$[\mathrm{M\,L\,T^{-2}}] = \mathrm{M}^a \mathrm{L}^{-3a+b+c} \mathrm{T}^{-b}$$

Now, equate the exponents on both sides of the equation:

1. For Mass (M):

$$a = 1$$

2. For Time (T):

$$-b = -2 \implies b = 2$$

3. For Length (L):

$$-3a + b + c = 1$$

Substitute the values of a and b:

$$-3(1)+2+c=1$$

$$-1+c=1 \implies c=2$$

Thus, the values are:

$a=1,\ b=2,\ c=2.$

Step 4: Final Answer:

The exponents (a,b,c) are $(1,2,2)$.

Quick Tip: The standard drag force formula is:

$F_d=\frac{1}{2}C_d\rho AV^2$

Since area $A\propto D^2$, the force is proportional to $\rho^1V^2D^2$. This matches $(1,2,2)$ perfectly.

---

53. Paramagnetic materials have:

(A) Large negative susceptibility

(B) Infinite susceptibility

(C) Zero susceptibility

(D) Small positive susceptibility

Correct Answer: (D) Small positive susceptibility

Solution:

Step 1: Understanding the Question:

The question asks about the characteristic magnetic susceptibility (χ) of paramagnetic

materials.

Step 2: Key Formula or Approach:
Magnetic susceptibility (χ) relates the magnetization (M) of a material to the applied magnetic field intensity (H):

$$M = \chi H$$

Step 3: Detailed Explanation:

- **Paramagnetism:** Occurs in materials containing unpaired electrons, which possess permanent magnetic dipole moments. In the absence of an external field, these dipoles are randomly oriented due to thermal agitation, resulting in zero net magnetization.

- When an external field is applied, the dipoles align weakly parallel to the field, enhancing the local magnetic field. This positive alignment results in a positive susceptibility ($\chi > 0$).

- Because the thermal energy is much larger than the magnetic alignment energy at standard temperatures, this alignment is weak, making the value of susceptibility small (typically between 10^{-5} and 10^{-3}).

- For comparison:
  - Diamagnetic materials have small negative susceptibility ($\chi < 0$).
  - Ferromagnetic materials have very large positive susceptibility.

Step 4: Final Answer:
Paramagnetic materials have small positive susceptibility.

Quick Tip: Susceptibility (χ) ranges:

Diamagnetic: $-10^{-5} \leq \chi < 0$

Paramagnetic: $0 < \chi \leq 10^{-3}$

Ferromagnetic: $\chi \gg 1$ (up to 10^5)

---

54. Magnetic susceptibility (χ) of paramagnetic materials follows:

(A) Curie's Law ($\chi \propto 1/T$)

(B) Ohm's law

(C) Hooke's law

(D) Wiedemann–Franz law

Correct Answer: (A) Curie's Law ($\chi \propto 1/T$)

Solution:

Step 1: Understanding the Question:

The question asks which physical law governs the temperature dependence of the magnetic susceptibility of paramagnetic materials.

Step 2: Key Formula or Approach:

Curie's Law states that the magnetic susceptibility of a paramagnetic material is inversely proportional to its absolute temperature:

$$\chi = \frac{C}{T}$$

where:

C is the material-specific Curie constant.

T is the absolute temperature in Kelvin.

Step 3: Detailed Explanation:

- In a paramagnetic material, the alignment of atomic magnetic dipoles with an external field is opposed by random thermal motion.
- As the temperature (T) of the material increases, the thermal kinetic energy of the atoms increases. This causes greater randomization of the dipole directions.
- Consequently, the net magnetization decreases for a given applied field, which directly reduces the susceptibility (χ).
- This inverse relationship is expressed as $\chi \propto 1/T$, which is known as Curie's Law.
- Other options like Ohm's law (electricity), Hooke's law (elasticity), and Wiedemann–Franz law (thermal/electrical conductivity ratio) are completely unrelated to magnetic susceptibility.

Step 4: Final Answer:

The susceptibility follows Curie's Law ($\chi \propto 1/T$).

Quick Tip: If the material is ferromagnetic, above its Curie Temperature (T_c), it transitions into a paramagnet and follows the Curie-Weiss Law:
$\chi = \frac{C}{T-\theta}$

---

55. For a laminar boundary layer developing over a flat plate, the displacement thickness is defined as: $\delta^* = \int_0^{\delta} \left(1 - \frac{u}{U_\infty}\right) dy$. Physically, δ^* represents

(A) Momentum loss in the boundary layer

(B) Reduction in mass flow due to the presence of the boundary layer

(C) Energy lost in the boundary layer

(D) Vorticity in the flow

Correct Answer: (B) Reduction in mass flow due to the presence of the boundary layer

Solution:

Step 1: Understanding the Question:

The question asks for the physical significance of the boundary layer parameter called "displacement thickness" (δ^*).

Step 2: Key Formula or Approach:

The velocity of fluid inside the boundary layer is less than the free-stream velocity ($u < U_\infty$) due to viscous shear forces. This deficit in velocity causes a reduction in the mass flow rate near the plate.

Step 3: Detailed Explanation:

- Let us calculate the mass flow rate reduction through a section of height h (where $h > \delta$) due to boundary layer formation:

$$\Delta\dot{m} = \int_0^h \rho U_\infty dy - \int_0^h \rho u dy = \int_0^h \rho(U_\infty - u)dy$$

- Since $u = U_\infty$ for $y > \delta$, we can restrict the limit of integration to the boundary layer thickness δ:

$$\Delta\dot{m} = \int_0^\delta \rho(U_\infty - u)dy = \rho U_\infty \int_0^\delta \left(1 - \frac{u}{U_\infty}\right)dy$$

- If we represent this mass flow rate deficit as an equivalent layer of thickness δ^* of free-stream fluid carrying the same mass deficit:

$$\Delta\dot{m} = \rho U_\infty \delta^*$$

- Equating these two expressions:

$$\delta^* = \int_0^{\delta} \left(1 - \frac{u}{U_\infty}\right) dy$$

- Physically, this is the distance by which the solid boundary would have to be displaced outward in an inviscid, frictionless flow to maintain the same mass flow rate as the actual viscous flow.

Step 4: Final Answer:

Physically, δ^* represents the reduction in mass flow due to the presence of the boundary layer.

Quick Tip: Remember the physical definitions of boundary layer thicknesses:

Displacement thickness (δ^*) → Mass flow rate deficit.

Momentum thickness (θ) → Momentum flow rate deficit.

Energy thickness (δ^{**}) → Kinetic energy flow rate deficit.

---

56. Curie temperature is the temperature at which:

(A) Ferromagnetic → Paramagnetic

(B) Paramagnetic → Diamagnetic

(C) Solid → Liquid

(D) Magnetic → Non-magnetic

Correct Answer: (A) Ferromagnetic → Paramagnetic

Solution:

Step 1: Understanding the Question:
The question asks for the definition of the Curie temperature (T_C) and the magnetic phase transition that occurs at this point.

Step 2: Key Formula or Approach:
The magnetic behavior of a material is highly dependent on temperature.
The susceptibility of a material above its Curie temperature follows the Curie-Weiss Law:

$$\chi = \frac{C}{T - \theta}$$

where T_C represents the transition threshold.

Step 3: Detailed Explanation:

- Below the Curie temperature, a ferromagnetic material possesses spontaneous magnetization because exchange interactions keep adjacent atomic magnetic moments aligned parallel to one another within magnetic domains.
- As the temperature increases, thermal energy increases, causing the atoms to vibrate more vigorously.
- This thermal agitation disrupts the orderly alignment of the magnetic dipoles.
- At the Curie temperature (T_C), the thermal energy becomes strong enough to completely overcome the exchange coupling.
- Above this critical temperature, the magnetic dipoles become randomly oriented in the absence of an external field, transitioning the material from a ferromagnetic state to a paramagnetic state.

Step 4: Final Answer:

The Curie temperature is the temperature at which a ferromagnetic material transitions into a paramagnetic material.

Quick Tip: For Iron (Fe), the Curie temperature is approximately 770°C (1043 K).

Above this temperature, iron loses its permanent magnetic properties and behaves paramagnetically.

---

57. The core of optical Fiber has refractive index:

(A) Less than cladding

(B) Equal to cladding

(C) Greater than cladding

(D) Zero

Correct Answer: (C) Greater than cladding

Solution:

Step 1: Understanding the Question:

The question asks for the relative difference in refractive indices between the core and the cladding of an optical fiber.

Step 2: Key Formula or Approach:

Optical fibers transmit light signals over long distances using the principle of **Total Internal Reflection** (TIR).

For total internal reflection to occur, two conditions must be satisfied:

1. Light must travel from a optically denser medium to an optically rarer medium.
2. The angle of incidence must be greater than the critical angle (θ_c), defined as:

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

where n_1 is the refractive index of the denser medium and n_2 is the refractive index of the rarer medium.

Step 3: Detailed Explanation:

- An optical fiber consists of a central core surrounded by an outer layer called the cladding.
- To guide light within the core, light must undergo total internal reflection at the core-cladding boundary.
- According to the laws of refraction, total internal reflection can only happen if the core is the denser medium.
- Therefore, the refractive index of the core (n_{core}) must be strictly greater than the refractive index of the cladding (n_{cladding}).
- This difference in refractive index traps the light waves within the core, allowing them to propagate along the fiber through a series of reflections with minimal energy loss.

Step 4: Final Answer:
The core of an optical fiber has a refractive index greater than the cladding.

Quick Tip: The refractive index difference $\Delta = \frac{n_{\text{core}} - n_{\text{cladding}}}{n_{\text{core}}}$ is typically kept very small (around 1%). This small difference helps reduce modal dispersion while still ensuring total internal reflection.

---

58. For Inlet diameter > Outlet diameter the Velocity and pressure along the flow will be as

(A) Velocity decreases, pressure increases

(B) Velocity increases, pressure decreases

(C) Velocity constant, pressure decreases

(D) Velocity increases, pressure constant

Correct Answer: (B) Velocity increases, pressure decreases

Solution:

Step 1: Understanding the Question:

The question asks for the changes in fluid velocity and pressure along a horizontal pipe where the inlet diameter is greater than the outlet diameter.

Step 2: Key Formula or Approach:

This problem is solved using the Continuity Equation and Bernoulli's Principle.

The Continuity Equation for incompressible flow is:

$$A_{\text{in}} V_{\text{in}} = A_{\text{out}} V_{\text{out}}$$

Bernoulli's Equation for horizontal flow (where elevation change is negligible) is:

$$P_{\text{in}} + \frac{1}{2}\rho V_{\text{in}}^2 = P_{\text{out}} + \frac{1}{2}\rho V_{\text{out}}^2$$

Step 3: Detailed Explanation:

- Since the inlet diameter (D_{in}) is greater than the outlet diameter (D_{out}), the cross-sectional area of the inlet (A_{in}) is larger than that of the outlet (A_{out}).

- From the continuity equation:

$$V_{\text{out}} = V_{\text{in}} \left(\frac{A_{\text{in}}}{A_{\text{out}}} \right)$$

Since $A_{in} > A_{out}$, the fluid velocity must increase at the outlet ($V_{out} > V_{in}$).

- According to Bernoulli's equation, as the velocity of the fluid increases, its kinetic energy increases.

- Because the total mechanical energy of the fluid must remain constant along a streamline, an increase in velocity (kinetic energy) must be balanced by a corresponding decrease in static pressure ($P_{out} < P_{in}$).

Step 4: Final Answer:

As the fluid flows from the larger inlet to the smaller outlet, the velocity increases and the pressure decreases.

Quick Tip: This configuration behaves exactly like a convergent nozzle.

In convergent nozzles:

Area $\downarrow \implies$ Velocity $\uparrow \implies$ Pressure $\downarrow$.

This is a standard application of Venturi principles.

---

59. Retentivity means:

(A) Resistance to magnetization

(B) Ability to retain magnetism

(C) Loss of magnetism

(D) Increase in magnetization

Correct Answer: (B) Ability to retain magnetism

Solution:

Step 1: Understanding the Question:
The question asks for the definition of "retentivity" in the context of the magnetic properties of materials.

Step 2: Key Formula or Approach:
Retentivity is a key parameter observed during the magnetic hysteresis loop (B-H curve) of ferromagnetic materials.
It corresponds to the residual magnetic flux density (B_r) when the external magnetizing field (H) is reduced to zero.

Step 3: Detailed Explanation:

- When a ferromagnetic material is subjected to an increasing external magnetic field (H), its magnetic domains align, increasing its magnetic induction (B) up to saturation.
- When the external magnetizing field is subsequently reduced back to zero, the magnetic domains do not return completely to their original random orientations.
- The material retains a certain amount of magnetization even when the external field $H = 0$.
- This residual magnetic flux density is known as remanent magnetization or remanence.
- The physical capacity of a material to retain this residual magnetism is called its **retentivity**.
- Materials with high retentivity are used to make permanent magnets, while those with low retentivity are preferred for electromagnets and transformer cores.

Step 4: Final Answer:
Retentivity refers to the ability of a material to retain its magnetism.

Quick Tip: For permanent magnets: High Retentivity and High Coercivity are required.
For temporary magnets (electromagnets): Low Retentivity and Low Coercivity are preferred.

---

60. A major thermodynamic reason for nanoparticle agglomeration is:

(A) Low density
(B) High melting point
(C) High surface energy
(D) Low band gap

Correct Answer: (C) High surface energy

Solution:

Step 1: Understanding the Question:
The question asks for the thermodynamic driving force behind the agglomeration (clustering) of nanoparticles.

Step 2: Key Formula or Approach:
The thermodynamic relationship governing surface energy and surface area is:

$$G_{\text{surface}} = \gamma A$$

where:
G_{surface} is the surface Gibbs free energy.
γ is the surface tension or surface energy per unit area.
A is the total surface area.

Step 3: Detailed Explanation:

- Nanoparticles have an extremely high surface-to-volume ratio due to their incredibly small size.
- Atoms at the surface of a particle are under-coordinated, meaning they have fewer neighboring atoms to bond with compared to atoms in the bulk.
- These unsatisfied bonds result in a highly unstable thermodynamic state characterized by **high surface energy**.
- According to thermodynamic principles, physical systems spontaneously tend to minimize their Gibbs free energy ($\Delta G < 0$).
- To reduce this excess free energy, the nanoparticles tend to merge or stick together (agglomerate).
- Agglomeration reduces the total exposed surface area (A) of the particles, thereby lowering the total surface energy of the system and making it more stable.

Step 4: Final Answer:
The major thermodynamic reason for nanoparticle agglomeration is high surface energy.

Quick Tip: To prevent nanoparticle agglomeration in practical applications, surfactant coatings are used to reduce surface energy and provide steric or electrostatic repulsion.

---

61. In a horizontal pipe, velocity doubles. Pressure gets

(A) Unchanged
(B) Decreased

(C) Increased

(D) Doubled

Correct Answer: (B) Decreased

Solution:

Step 1: Understanding the Question:

The question asks what happens to the static pressure of a fluid flowing in a horizontal pipe when its velocity is doubled.

Step 2: Key Formula or Approach:

We use Bernoulli's equation for steady, incompressible, frictionless flow along a horizontal streamline (where elevation z is constant):

$$P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$$

Step 3: Detailed Explanation:

- Let the initial pressure and velocity be P_1 and V_1.
- The final velocity is doubled, so $V_2 = 2V_1$.
- Substitute V_2 into Bernoulli's equation:

$$P_2 = P_1 + \frac{1}{2}\rho V_1^2 - \frac{1}{2}\rho (2V_1)^2$$

$$P_2 = P_1 + \frac{1}{2}\rho V_1^2 - \frac{4}{2}\rho V_1^2$$

$$P_2 = P_1 - \frac{3}{2}\rho V_1^2$$

- Because the term $\frac{3}{2}\rho V_1^2$ is strictly positive for any flowing fluid, the final static pressure P_2 must be less than the initial static pressure P_1.
- This demonstrates that an increase in kinetic energy (due to doubling the velocity) causes a corresponding decrease in static pressure.

Step 4: Final Answer:

The static pressure gets decreased.

Quick Tip: As velocity (V) increases, kinetic energy increases. To keep the total energy constant in a horizontal flow, the static pressure (P) must decrease.
This is the fundamental principle behind Venturi tubes and aerodynamic lift.

---

62. Which has highest magnetic susceptibility?

(A) Diamagnetic
(B) Paramagnetic
(C) Ferromagnetic
(D) Antiferromagnetic

Correct Answer: (C) Ferromagnetic

Solution:

Step 1: Understanding the Question:

The question asks to identify which class of magnetic materials has the highest magnetic susceptibility (χ).

Step 2: Key Formula or Approach:
Magnetic susceptibility is defined as:

$$\chi = \frac{M}{H}$$

where M is the induced magnetization and H is the applied magnetic field strength.

Step 3: Detailed Explanation:

- **Diamagnetic materials:** Have a very small, negative susceptibility ($\chi \approx -10^{-5}$).
- **Paramagnetic and Antiferromagnetic materials:** Have a small, positive susceptibility ($\chi \approx 10^{-5}$ to 10^{-3}).
- **Ferromagnetic materials:** Have extremely large, positive susceptibility (ranging from 10^{3} to 10^{5} or higher).
- Ferromagnetic materials contain permanent magnetic dipoles that are strongly aligned parallel to each other within domains due to quantum mechanical exchange coupling. Even a weak external field can easily cause these domains to align, resulting in massive magnetization (M) and thus a very high susceptibility value.

Step 4: Final Answer:
Ferromagnetic materials have the highest magnetic susceptibility.

Quick Tip: The susceptibility of ferromagnetic materials is not only large but also non-linear and depends on the history of the applied magnetic field (hysteresis effect).

---

63. The Third Law of thermodynamics states that entropy at absolute zero is:

(A) Maximum

(B) Minimum

(C) Zero for perfect crystal

(D) Infinite

Correct Answer: (C) Zero for perfect crystal

Solution:

Step 1: Understanding the Question:

The question asks for the statement or value of entropy at absolute zero (0 K) according to the Third Law of Thermodynamics.

Step 2: Key Formula or Approach:

The statistical definition of entropy is given by Boltzmann's formula:

$$S = k_B \ln \Omega$$

where Ω is the number of accessible microstates corresponding to the macrostate of the system.

Step 3: Detailed Explanation:

- As temperature approaches absolute zero ($T \to 0$ K), all thermal motion of atoms in a crystal lattice ceases.

- For a perfectly ordered crystalline substance, there is only one unique, perfectly ordered

ground-state configuration. This means the number of microstates is $\Omega = 1$.

- Substituting this into Boltzmann's entropy formula:

$$S = k_B \ln(1) = 0$$

- Therefore, the Third Law states that the entropy of a perfect crystalline substance at absolute zero is exactly equal to zero.

- If the crystal has defects, impurities, or is amorphous (like glass), there will be residual entropy greater than zero due to structural disorder ($\Omega > 1$).

Step 4: Final Answer:
The entropy at absolute zero is zero for a perfect crystal.

Quick Tip: The Third Law establishes an absolute reference point for entropy, allowing us to calculate absolute entropies of substances rather than just change in entropy (ΔS).

---

64. A solid circular shaft is subjected to bending moment and torsional moment. What is the nature of maximum principal stress at the outer surface?

(A) Pure bending stress
(B) Pure shear stress
(C) Combination of normal and shear stress
(D) Zero stress

Correct Answer: (C) Combination of normal and shear stress

Solution:

Step 1: Understanding the Question:

The question asks for the state of stress and the nature of the maximum principal stress at the outer surface of a shaft subjected to combined bending and torsion.

Step 2: Key Formula or Approach:

- Bending moment (M) induces normal stress (σ) on the outer surface:

$$\sigma = \frac{32M}{\pi d^3}$$

- Torsional moment (T) induces shear stress (τ) on the outer surface:

$$\tau = \frac{16T}{\pi d^3}$$

- The maximum principal stress (σ_1) is determined using the combined stress formula:

$$\sigma_1 = \frac{\sigma}{2} + \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

Step 3: Detailed Explanation:

- An element at the outer fiber of the shaft experiences a normal stress (σ) due to bending and a shear stress (τ) due to torsion simultaneously.
- This represents a state of two-dimensional combined loading (biaxial stress state).
- The principal stresses are the maximum and minimum normal stresses acting on planes where the shear stress is zero.
- Because the principal stress calculation formula depends on both the bending-induced

normal stress (σ) and the torsion-induced shear stress (τ), the resulting maximum principal stress is fundamentally a combination of both normal and shear stresses.

Step 4: Final Answer:

The nature of the maximum principal stress is a combination of normal and shear stress.

Quick Tip: For shafts under combined loading, the equivalent bending moment (M_e) used in design is: $M_e = \frac{1}{2}[M + \sqrt{M^2 + T^2}]$. This directly relates to the maximum principal stress.

---

65. Ceramic materials are generally:

(A) Ductile and soft

(B) Brittle and hard

(C) Flexible and elastic

(D) Conductive

Correct Answer: (B) Brittle and hard

Solution:

Step 1: Understanding the Question:

The question asks for the general mechanical properties of ceramic materials.

Step 2: Key Formula or Approach:

The mechanical properties of materials are primarily determined by their atomic bonding. Ceramics are inorganic, non-metallic materials characterized by strong ionic and covalent bonding.

Step 3: Detailed Explanation:

- Ionic and covalent bonds are highly directional and extremely strong.
- These strong bonds make it very difficult for planes of atoms to slip past one another (which is the mechanism of dislocation movement and plastic deformation).
- Because dislocation motion is highly restricted, ceramics exhibit exceptionally high hardness and resistance to wear.
- However, because they cannot deform plastically to relieve stress concentrations, they are highly sensitive to microscopic cracks and flaws.
- When a tensile load is applied, stress concentrates at these crack tips, causing rapid and catastrophic propagation of the cracks, which makes ceramics highly brittle.

Step 4: Final Answer:
Ceramics are generally brittle and hard.

Quick Tip: While ceramics are brittle in tension, they exhibit exceptionally high compressive strength and thermal stability, making them ideal for high-temperature structural applications.

---

66. The packing sequence of atoms in FCC is

(A) AB AB AB...
(B) BC BC BC...
(C) AC AC AC...
(D) ABC ABC ABC....

Correct Answer: (D) ABC ABC ABC....

Solution:

Step 1: Understanding the Question:
The question asks for the specific stacking sequence of close-packed atomic planes in a Face-Centered Cubic (FCC) crystal structure.

Step 2: Key Formula or Approach:
Both FCC and HCP (Hexagonal Close-Packed) are close-packed structures with a maximum atomic packing factor (APF) of 0.74.
The difference between them lies in how the close-packed planes are stacked on top of each other.

Step 3: Detailed Explanation:

- Let the first close-packed layer of atoms be layer 'A'.
- The second layer of atoms is placed in the hollows of the first layer, which we designate as layer 'B'.
- For the third layer, there are two choices:
  1. Place the atoms directly above the atoms of layer 'A'. This creates an **AB AB AB...** sequence, which corresponds to the **HCP** structure.
  2. Place the atoms in the alternative set of hollows that do not lie directly above layer 'A'. This creates a third distinct layer, designated as layer 'C'. This results in an **ABC ABC ABC...** sequence, which corresponds to the **FCC** structure.
- In the FCC lattice, this stacking direction is perpendicular to the {111} family of close-packed planes.

Step 4: Final Answer:
The packing sequence of atoms in an FCC crystal is ABC ABC ABC....

Quick Tip: HCP Stacking Sequence: AB AB AB...

FCC Stacking Sequence: ABC ABC ABC...

Both have a coordination number of 12 and a packing efficiency of 74%.

---

67. A particle under force $F = -kx$. Then Motion is

(A) uniform

(B) simple harmonic

(C) circular

(D) random

Correct Answer: (B) simple harmonic

Solution:

Step 1: Understanding the Question:

The question asks to identify the type of motion a particle undergoes when subjected to a restoring force proportional to its displacement ($F = -kx$).

Step 2: Key Formula or Approach:

According to Newton's Second Law of Motion:

$$F = ma = m\frac{d^2x}{dt^2}$$

For the given force $F = -kx$:

$$m\frac{d^2x}{dt^2} = -kx \implies \frac{d^2x}{dt^2} + \left(\frac{k}{m}\right)x = 0$$

Step 3: Detailed Explanation:

- Let $\omega^2 = \frac{k}{m}$, where ω is the angular frequency of oscillation.

- The equation becomes:

$$\frac{d^2x}{dt^2} + \omega^2 x = 0$$

This is the standard second-order linear differential equation that defines a **Simple Harmonic Oscillator**.

- The solution to this equation is sinusoidal:

$$x(t) = A\sin(\omega t + \phi)$$

- This indicates that the particle oscillates periodically about its equilibrium position ($x = 0$), which is the definition of Simple Harmonic Motion (SHM).

Step 4: Final Answer:

The motion of the particle is simple harmonic.

Quick Tip: Any system with a linear restoring force ($F \propto -x$) undergoes simple harmonic motion. The time period is given by $T = 2\pi\sqrt{\frac{m}{k}}$.

---

68. Zirconia ceramics exhibit:

(A) High electrical conductivity

(B) High hardness

(C) Low hardness

(D) Transformation toughening

Correct Answer: (D) Transformation toughening

Solution:

Step 1: Understanding the Question:

The question asks for a unique mechanical phenomenon exhibited by zirconia (ZrO_2) ceramics.

Step 2: Key Formula or Approach:

Zirconia undergoes phase changes with temperature and stress:

$$\text{Tetragonal phase} \xrightarrow{\text{stress-induced}} \text{Monoclinic phase}$$

This transition is accompanied by a localized volume expansion of approximately 3% to 5%.

Step 3: Detailed Explanation:

- Zirconia can be stabilized in its high-temperature tetragonal phase at room temperature by adding dopants such as yttria (Y_2O_3).
- When a crack propagates through this partially stabilized zirconia, the high tensile stress field near the crack tip triggers a phase transformation from the metastable tetragonal phase to the stable monoclinic phase.
- This phase change is accompanied by a local volume expansion.
- The volume expansion generates localized compressive stresses around the crack tip.
- These compressive forces squeeze the crack shut, opposing the tensile stress trying to open it, which effectively halts crack propagation.

- This unique mechanism is known as **transformation toughening**, and it makes zirconia one of the toughest ceramic materials available.

Step 4: Final Answer:

Zirconia ceramics exhibit transformation toughening.

Quick Tip: Transformation toughening is why Partially Stabilized Zirconia (PSZ) is often referred to as "ceramic steel" due to its high fracture toughness.

---

69. Hall-Petch equation predicts the relation between grain size and

(A) Yield Strength

(B) Ductility

(C) %Elongation

(D) Tensile Strength

Correct Answer: (A) Yield Strength

Solution:

Step 1: Understanding the Question:

The question asks to identify the mechanical property that is related to grain size via the Hall-Petch equation.

Step 2: Key Formula or Approach:

The Hall-Petch relationship is mathematically expressed as:

$$\sigma_y = \sigma_0 + k_y d^{-1/2}$$

where:

σ_y is the yield strength.
σ_0 is the friction stress (resistance of the lattice to dislocation movement).
k_y is a material-specific strengthening coefficient.
d is the average grain diameter.

Step 3: Detailed Explanation:

- Grain boundaries act as barriers to the motion of dislocations because the crystallographic orientation changes abruptly across the boundary.
- A smaller grain size (d) means there is a higher density of grain boundaries within the material.
- As dislocations pile up at these boundaries, higher applied stress is required for the dislocations to slip and pass through into the neighboring grain.
- Since plastic deformation begins when dislocations start to move globally, a smaller grain size increases the stress required to initiate yield.
- Therefore, as grain size decreases ($d \to 0$), the yield strength (σ_y) of the metal increases according to the $d^{-1/2}$ relation.

Step 4: Final Answer:
The Hall-Petch equation relates grain size to the Yield Strength of a material.

Quick Tip: Grain refinement is the only strengthening mechanism in metals that simultaneously increases both strength and toughness.

70. Three forces 3 N, 4 N, and 5 N act at a point. For equilibrium, angle between 3 N and 4 N is

(A) 60°

(B) 90°

(C) 120°

(D) 180°

Correct Answer: (B) 90°

Solution:

Step 1: Understanding the Question:

The question asks for the angle between the 3 N and 4 N forces such that the system of three forces (3 N, 4 N, and 5 N) acting at a point is in static equilibrium.

Step 2: Key Formula or Approach:

For a particle to be in static equilibrium under three coplanar forces, the vector sum of the forces must be zero:

$$\mathbf{F_1} + \mathbf{F_2} + \mathbf{F_3} = 0 \implies \mathbf{F_1} + \mathbf{F_2} = -\mathbf{F_3}$$

Taking the magnitude of both sides:

$$F_1^2 + F_2^2 + 2F_1F_2\cos\theta = F_3^2$$

where θ is the angle between $\mathbf{F_1}$ and $\mathbf{F_2}$.

Step 3: Detailed Explanation:

- Let $F_1 = 3$ N, $F_2 = 4$ N, and $F_3 = 5$ N.
- Substitute these values into the vector equilibrium equation:

$$3^2 + 4^2 + 2(3)(4)\cos\theta = 5^2$$

$$9 + 16 + 24\cos\theta = 25$$

$$25 + 24\cos\theta = 25$$

$$24\cos\theta = 0$$

$$\cos\theta = 0 \implies \theta = 90^\circ$$

- This shows that the 3 N and 4 N forces must act perpendicular to each other, allowing their resultant ($\sqrt{3^2 + 4^2} = 5$ N) to be perfectly balanced by the opposing 5 N force.

Step 4: Final Answer:

The angle between the 3 N and 4 N forces must be 90°.

Quick Tip: This problem is based on the Pythagorean triple $(3, 4, 5)$.

For any forces that form a right-angled triangle, the angle between the two base forces is always 90°.

---

71. A composite material is composed of:

(A) Single phase material

(B) Only metals

(C) Only polymers

(D) Two or more distinct phases

Correct Answer: (D) Two or more distinct phases

Solution:

Step 1: Understanding the Question:
The question asks for the fundamental structural definition of a composite material.

Step 2: Key Formula or Approach:
By engineering definition, a composite is a material system composed of a mixture or combination of two or more micro- or macroscopically distinct phases. These phases must be chemically dissimilar and separated by a distinct interface.

Step 3: Detailed Explanation:

- A composite material is engineered to achieve a combination of properties that cannot be obtained from any of the individual components alone.

- It typically consists of two main parts:
  1. **Matrix phase:** The continuous phase that surrounds and supports the other phase (can be polymer, metal, or ceramic).
  2. **Dispersed/Reinforcing phase:** The discontinuous phase that provides strength and stiffness (typically fibers, whiskers, or particles).

- Because these phases are distinct and do not dissolve into one another, a composite is inherently a multiphase material.

- Options A, B, and C are incorrect because composites are not limited to single phases or specific material classes like pure metals or pure polymers.

Step 4: Final Answer:

A composite material is composed of two or more distinct phases.

Quick Tip: Fiberglass (glass fibers in a polymer matrix) and Concrete (cement matrix with gravel aggregates) are classic examples of composite materials with distinct matrix and reinforcement phases.

---

72. Given the variables: velocity (V), density (ρ), viscosity (μ), and characteristic length (L), how many dimensionless groups can be formed?

(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: (C) 3

Solution:

Step 1: Understanding the Question:

The question asks for the number of base parameters or repeating variables required to form the dimensionless groups from the given physical variables: velocity (V), density (ρ), viscosity (μ), and characteristic length (L).

Step 2: Key Formula or Approach:

According to dimensional analysis and the Buckingham π theorem, the variables are analyzed using their fundamental dimensions (Mass, Length, Time).

The number of repeating variables (m) selected to form the dimensionless groups is determined by the number of fundamental dimensions involved in the system:

$$m = 3$$

Step 3: Detailed Explanation:

- Let us analyze the dimensions of the given physical variables:
  - Velocity (V): $[\mathrm{L\,T^{-1}}]$
  - Density (ρ): $[\mathrm{M\,L^{-3}}]$
  - Dynamic viscosity (μ): $[\mathrm{M\,L^{-1}T^{-1}}]$
  - Characteristic length (L): [L]

- The fundamental dimensions required to express these four variables are Mass (M), Length (L), and Time (T).

- Therefore, the number of primary dimensions is 3.

- In the formulation of dimensionless groups, we select 3 repeating variables that collectively contain all the fundamental dimensions.

- These three repeating variables must represent:
  1. A geometric property (Characteristic length, L).
  2. A kinematic property (Velocity, V).
  3. A dynamic property (Density, ρ).

- Thus, these 3 variables form the essential base groups from which any independent dimensionless relationship (such as the Reynolds number) is constructed.

Step 4: Final Answer:

The number of base parameters or repeating variables required to form the dimensionless groups is 3.

Quick Tip: In dimensional analysis, always select 3 repeating variables ($m = 3$) representing geometric, kinematic, and dynamic properties to construct your dimensionless π groups.
These repeating variables are typically L, V, and ρ.

---

73. A shaft is subjected to pure torsion. The state of stress at any point on the surface of the shaft, relative to the axis of the shaft, consists of

(A) pure normal stress
(B) pure shear stress
(C) equal normal and shear stresses
(D) zero stress

Correct Answer: (B) pure shear stress

Solution:

Step 1: Understanding the Question:
The question asks for the state of stress at any point on the outer surface of a circular shaft when it is subjected to a pure twisting moment (pure torsion).

Step 2: Key Formula or Approach:
According to the torsion equation for a circular shaft:

$$\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L}$$

where:
T is the applied torque.
J is the polar moment of inertia.
τ is the shear stress at radius r.

Step 3: Detailed Explanation:

- Under pure torsion, the only load applied to the shaft is a torque acting about its longitudinal axis.
- There are no axial loads, bending moments, or external pressures acting on the shaft.
- Consequently, the normal stress (σ) along the longitudinal, radial, and circumferential directions of the shaft is zero ($\sigma = 0$).
- The applied torque produces a shear stress (τ) on planes parallel and perpendicular to the longitudinal axis of the shaft.
- This shear stress is maximum at the outer surface ($r = R$) and varies linearly to zero at the central axis.
- Thus, relative to the coordinate system aligned with the axis of the shaft, the stress state consists entirely of pure shear stress.

Step 4: Final Answer:
The state of stress at any point on the surface of the shaft consists of pure shear stress.

Quick Tip: While the stress state relative to the shaft axis is pure shear, if you rotate the element by 45°, the stress state becomes pure normal stress (tensile and compressive stresses of magnitude equal to τ). This explains why brittle materials fail at 45° under torsion.

---

74. Cross slip is observing

(A) screw dislocation
(B) edge dislocation

(C) twinning

(D) bending

Correct Answer: (A) screw dislocation

Solution:

Step 1: Understanding the Question:

The question asks which type of crystal defect or dislocation is capable of undergoing the deformation process known as "cross slip".

Step 2: Key Formula or Approach:

Cross slip is a mechanism of plastic deformation where a dislocation moves from one slip plane to another intersecting slip plane.

For this movement to occur, the Burgers vector of the dislocation must lie along the intersection line of the two planes.

Step 3: Detailed Explanation:

- **Screw Dislocation:** In a screw dislocation, the dislocation line is parallel to its Burgers vector ($\mathbf{b} \parallel \mathbf{t}$). Because of this parallel orientation, the dislocation line and its Burgers vector do not define a unique slip plane. It can slip on any plane containing the dislocation line. When it encounters an obstacle, it can easily shift (cross slip) onto an intersecting slip plane.

- **Edge Dislocation:** In an edge dislocation, the dislocation line is perpendicular to its Burgers vector ($\mathbf{b} \perp \mathbf{t}$). This perpendicular arrangement defines a single, unique slip plane. The edge dislocation is restricted to this plane and cannot change planes unless it undergoes climb, which requires diffusion.

- **Twinning and Bending:** These are macroscopic modes of deformation and are not dislocation defects that undergo cross slip.

Step 4: Final Answer:

Cross slip is observed in screw dislocations.

Quick Tip: Remember:

Screw dislocation → $\mathbf{b} \parallel \mathbf{t}$ → Can cross slip.

Edge dislocation → $\mathbf{b} \perp \mathbf{t}$ → Cannot cross slip (can only climb via thermal diffusion).

---

75. A particle starts from rest and moves with a constant acceleration of $2\ \text{m/s}^2$. What is the distance traveled by the particle in the 5^{th} second of its motion?

(A) 25 m

(B) 9 m

(C) 10 m

(D) 12.5 m

Correct Answer: (B) 9 m

Solution:

Step 1: Understanding the Question:

The question asks for the specific distance covered by a accelerating particle during its fifth single second of travel, given that it starts from rest.

Step 2: Key Formula or Approach:

The distance traveled by a body in the n^{th} second of its motion under constant acceleration is given by:

$$S_n = u + \frac{a}{2}(2n-1)$$

where:

u is the initial velocity.

a is the constant acceleration.

n is the specific second of interest.

Step 3: Detailed Explanation:

- The particle starts from rest, so the initial velocity $u = 0$.
- The constant acceleration is given as $a = 2\ \text{m/s}^2$.
- We are looking for the distance traveled in the 5^{th} second, so $n = 5$.
- Substituting these values into the formula:

$$S_5 = 0 + \frac{2}{2}(2(5) - 1)$$

$$S_5 = 1 \times (10 - 1)$$

$$S_5 = 9\ \text{m}$$

- Alternatively, this can be calculated by subtracting the total distance covered in 4 seconds from the total distance covered in 5 seconds:

$$S_{\text{total}}(t) = ut + \frac{1}{2}at^2$$

$$S(5) = \frac{1}{2}(2)(5)^2 = 25\ \text{m}$$

$$S(4) = \frac{1}{2}(2)(4)^2 = 16 \text{ m}$$

$$S_5 = S(5) - S(4) = 25 - 16 = 9 \text{ m}$$

Step 4: Final Answer:

The distance traveled by the particle in the 5^{th} second is 9 m.

Quick Tip: For a body starting from rest with constant acceleration, the distances covered in successive seconds follow Galileo's law of odd numbers:

$1 : 3 : 5 : 7 : 9 : \ldots$

In the 1^{st} second, distance $= \frac{a}{2}(1) = 1$ m.

In the 5^{th} second, distance $= 9 \times 1 \text{ m} = 9$ m.

---

76. A rigid body rotates about a fixed axis under a constant moment (M). If the Mass Moment of Inertia (I) of the body is doubled while the moment remains the same, the angular acceleration (α) will

(A) double

(B) remain the same

(C) be halved

(D) quadruple

Correct Answer: (C) be halved

Solution:

Step 1: Understanding the Question:

The question asks how the angular acceleration (α) of a rotating rigid body changes if its mass moment of inertia (I) is doubled while the driving moment (M) is held constant.

Step 2: Key Formula or Approach:

We apply Newton's second law for rotational motion:

$$M = I\alpha$$

where:

M is the applied torque or moment.

I is the mass moment of inertia.

α is the angular acceleration.

Step 3: Detailed Explanation:

- Rearranging the rotational equation of motion to solve for angular acceleration gives:

$$\alpha = \frac{M}{I}$$

- Since the moment M remains constant, the angular acceleration α is inversely proportional to the mass moment of inertia I:

$$\alpha \propto \frac{1}{I}$$

- Let the initial moment of inertia be I_1 and the initial angular acceleration be $\alpha_1 = \frac{M}{I_1}$.

- The new moment of inertia is doubled: $I_2 = 2I_1$.

- The new angular acceleration is:

$$\alpha_2 = \frac{M}{I_2} = \frac{M}{2I_1} = \frac{1}{2}\alpha_1$$

- Thus, doubling the moment of inertia reduces the angular acceleration to half of its initial value.

Step 4: Final Answer:

The angular acceleration (α) will be halved.

Quick Tip: Moment of inertia is the rotational analog of mass.

Just as doubling the mass halves the linear acceleration for a constant force ($a = F/m$), doubling the moment of inertia halves the angular acceleration for a constant torque ($\alpha = M/I$).

---

77. One of the following is not a strengthening mechanism:

(A) Cold working

(B) Precipitation hardening

(C) Strain aging

(D) Annealing

Correct Answer: (D) Annealing

Solution:

Step 1: Understanding the Question:

The question asks to identify which of the listed heat treatment or mechanical processing operations is NOT used as a strengthening mechanism in metals.

Step 2: Key Formula or Approach:
Strengthening mechanisms in metals work by introducing barriers to dislocation motion, which increases the stress required for plastic deformation (yield strength).

Step 3: Detailed Explanation:

- **Cold Working (Strain Hardening):** Increases strength by increasing dislocation density through plastic deformation. The highly crowded dislocations entangle and obstruct each other's movement.

- **Precipitation Hardening:** Involves the formation of fine, uniformly dispersed second-phase particles (precipitates) within the matrix that act as obstacles to dislocation glide.

- **Strain Aging:** A phenomenon where solute atoms (like carbon or nitrogen in steel) diffuse to and pin dislocations over time, increasing the yield strength.

- **Annealing:** This is a thermal process designed to restore a material to its soft, ductile state. It involves heating a cold-worked metal to relieve internal stresses, trigger recrystallization, and encourage grain growth. This process decreases dislocation density and increases grain size, which reduces the material's strength and increases its ductility.

Step 4: Final Answer:
Annealing is not a strengthening mechanism; it is a softening heat treatment.

Quick Tip: Annealing is the exact opposite of cold working.
While cold working increases strength and decreases ductility, annealing decreases strength and restores ductility.

---

78. Water flows from pipe diameter 200 mm to 100 mm. If velocity in larger pipe is 2 m/s, find velocity in smaller pipe.

(A) 2 m/s
(B) 4 m/s
(C) 6 m/s
(D) 8 m/s

Correct Answer: (D) 8 m/s

Solution:

Step 1: Understanding the Question:
The question asks for the fluid velocity in a smaller pipe section, given the diameters of both the larger and smaller sections, and the velocity in the larger section.

Step 2: Key Formula or Approach:
For an incompressible fluid, we apply the Continuity Equation:

$$A_1V_1 = A_2V_2$$

where:
A_1, A_2 are the cross-sectional areas of the larger and smaller pipe sections, respectively.
V_1, V_2 are the corresponding fluid velocities.

Step 3: Detailed Explanation:

- The cross-sectional area of a circular pipe is:

$$A = \frac{\pi}{4} D^2$$

- Substituting this into the continuity equation:

$$\frac{\pi}{4} D_1^2 V_1 = \frac{\pi}{4} D_2^2 V_2$$

$$D_1^2 V_1 = D_2^2 V_2$$

- Given values:

  $D_1 = 200$ mm (diameter of the larger pipe)

  $D_2 = 100$ mm (diameter of the smaller pipe)

  $V_1 = 2$ m/s (velocity in the larger pipe)

- Solve for V_2:

$$V_2 = V_1 \left(\frac{D_1}{D_2} \right)^2$$

$$V_2 = 2 \times \left(\frac{200}{100} \right)^2$$

$$V_2 = 2 \times (2)^2$$

$$V_2 = 2 \times 4 = 8 \text{ m/s}$$

Step 4: Final Answer:

The velocity of water in the smaller pipe is 8 m/s.

Quick Tip: Velocity is inversely proportional to the square of the diameter ($V \propto 1/D^2$).
If the diameter is halved (200 mm $\rightarrow$ 100 mm), the velocity must increase by a factor of $2^2 = 4$.
2 m/s $\times$ 4 = 8 m/s.

---

79. Removing a hole from a plate shifts centroid:

(A) Depends on material
(B) Toward hole
(C) Away from hole
(D) No change

Correct Answer: (C) Away from hole

Solution:

Step 1: Understanding the Question:

The question asks about the direction in which the centroid (geometric center) of a flat plate shifts when a hole is cut or removed from it.

Step 2: Key Formula or Approach:

The centroid coordinates of a composite area with a removed portion are given by:

$$\bar{x} = \frac{A_1 x_1 - A_2 x_2}{A_1 - A_2}$$

where:

A_1 is the original area of the plate, and x_1 is its centroid.
A_2 is the area of the removed hole, and x_2 is the centroid of the hole.

Step 3: Detailed Explanation:

- The centroid represents the geometric center of gravity of an area distribution.
- When a hole is cut out from a symmetric plate, area (mass equivalent) is removed from that specific region.
- The removal of area on one side causes the remaining area to be concentrated more heavily on the opposite side of the hole.
- Mathematically, because we subtract the moment of the removed area (A_2x_2), the center of mass/centroid of the remaining composite shape shifts in the direction opposite to the location of the removed hole (x_2).
- Therefore, the centroid always shifts directly away from the center of the removed hole.

Step 4: Final Answer:
Removing a hole from a plate shifts the centroid away from the hole.

Quick Tip: Think of the centroid as a balance point.
If you remove weight from the right side (by cutting a hole), the balance point (centroid) must shift to the left (away from the hole) to maintain balance.

---

80. Which of the following crystal structure is ductile?

(A) Face centered cubic (FCC)

(B) Body centered cubic (BCC)

(C) Hexagonal close packed (HCP)

(D) Body centered tetragonal (BCT)

Correct Answer: (A) Face centered cubic (FCC)

Solution:

Step 1: Understanding the Question:

The question asks which of the common metal crystal structures is inherently the most ductile.

Step 2: Key Formula or Approach:

Ductility in metals depends directly on the number of active slip systems available in their crystal structure.

A slip system consists of a slip plane and a slip direction:

$$\text{Number of Slip Systems} = \text{Number of Slip Planes} \times \text{Number of Slip Directions}$$

Step 3: Detailed Explanation:

- **Face Centered Cubic (FCC):** FCC has 12 independent, highly close-packed slip systems ($\{111\}$ planes and $\langle 110 \rangle$ directions). Because these planes are closely packed, the activation energy for slip is low, allowing easy dislocation movement even at low temperatures. This makes FCC metals (like Al, Cu, Au, Ag) exceptionally ductile.

- **Body Centered Cubic (BCC):** Although BCC has 48 potential slip systems, its slip planes ($\{110\}$) are not close-packed. At low temperatures, the stress required to move dislocations increases sharply, leading to a ductile-to-brittle transition.

- **Hexagonal Close Packed (HCP):** HCP has only 3 active slip systems at room tempera-

ture. Because it has fewer than the 5 independent slip systems required for arbitrary homogeneous deformation, HCP metals (like Mg, Zn) are relatively brittle.

- **Body Centered Tetragonal (BCT):** This is a highly distorted structure (like martensite in steel) with limited slip capability, making it very hard and brittle.

Step 4: Final Answer:
The most ductile crystal structure is Face Centered Cubic (FCC).

Quick Tip: FCC metals do not exhibit a ductile-to-brittle transition temperature (DBTT). They remain highly ductile and tough even at cryogenic temperatures, which is why they are widely used in low-temperature applications.

---

81. Ideal c/a ratio of HCP is

(A) 1.633
(B) 1.88
(C) 1.58
(D) 1.48

Correct Answer: (A) 1.633

Solution:

Step 1: Understanding the Question:
The question asks for the value of the ideal c/a ratio (height-to-edge ratio) of a Hexagonal Close-Packed (HCP) crystal structure.

Step 2: Key Formula or Approach:
In an ideal HCP structure composed of hard spheres of radius R in contact, the parameters are:

$a = 2R$ (the base edge length).

c is the height of the unit cell.

The ideal geometric ratio is calculated using a regular tetrahedron formed by three atoms in the middle layer and one atom in the top layer.

Step 3: Detailed Explanation:

- Consider the tetrahedron formed by four adjacent touching atoms in the HCP structure. The edge length of this tetrahedron is equal to the base lattice parameter a.
- The height of this regular tetrahedron is half the height of the unit cell ($c/2$).
- Using trigonometry on a regular tetrahedron of edge a:

$$\left(\frac{c}{2}\right)^2 = a^2 - \left(\frac{a}{\sqrt{3}}\right)^2$$

$$\frac{c^2}{4} = a^2 - \frac{a^2}{3} = \frac{2}{3}a^2$$

$$c^2 = \frac{8}{3}a^2$$

$$\frac{c}{a} = \sqrt{\frac{8}{3}} \approx 1.633$$

- This ratio of 1.633 yields the maximum atomic packing factor of 0.74, matching the packing density of the FCC structure.

Step 4: Final Answer:

The ideal c/a ratio of HCP is 1.633.

Quick Tip: Real HCP metals deviate slightly from the ideal ratio:
Zinc ($c/a \approx 1.856$) is stretched along the c-axis.
Titanium ($c/a \approx 1.587$) is compressed along the c-axis.

---

82. A surface with emissivity 0.8 and area 1 m^2 is maintained at 500 K and is surrounded by air at 300 K. Given Stefan–Boltzmann constant $\sigma = 5.67 \times 10^{-8}$ W/m^2K^4. What is the rate of radiative heat transfer?

(A) 3.5 kW
(B) 2.47 kW
(C) 2.0 kW
(D) 1.5 kW

Correct Answer: (B) 2.47 kW

Solution:

Step 1: Understanding the Question:
The question asks for the net rate of radiative heat transfer from a gray body surface to its cooler surroundings.

Step 2: Key Formula or Approach: The net heat transfer rate by radiation (Q) between a surface and its surroundings is given by the Stefan-Boltzmann law:

$$Q = \epsilon \sigma A (T_s^4 - T_{surr}^4)$$

where:
ϵ is the emissivity of the surface.

σ is the Stefan-Boltzmann constant (5.67×10^{-8} W/m^{2}K^4).

A is the surface area.

T_s is the absolute surface temperature (in Kelvin).

T_{surr} is the absolute temperature of the surroundings (in Kelvin).

Step 3: Detailed Explanation:

Given values:

$\epsilon = 0.8$

$A = 1$ m^2

$T_s = 500$ K

$T_{\text{surr}} = 300$ K

$\sigma = 5.67 \times 10^{-8}$ W/m^{2}K^4

Substitute these values into the formula:

$$Q = 0.8 \times (5.67 \times 10^{-8}) \times 1 \times (500^4 - 300^4)$$

Calculate the temperature terms:

$$500^4 = 6.25 \times 10^{10}$$

$$300^4 = 8.1 \times 10^9 = 0.81 \times 10^{10}$$

$$T_s^4 - T_{\text{surr}}^4 = (6.25 - 0.81) \times 10^{10} = 5.44 \times 10^{10}$$

Now, complete the multiplication:

$$Q = 0.8 \times 5.67 \times 10^{-8} \times 5.44 \times 10^{10}$$

$$Q = 4.536 \times 544$$

$$Q \approx 2467.58 \text{ W} = 2.47 \text{ kW}$$

Step 4: Final Answer:

The rate of radiative heat transfer is 2.47 kW.

Quick Tip: Always ensure temperatures are in Kelvin (T in K) before performing any calculations involving the Stefan-Boltzmann law.

Using Celsius temperatures is a common source of error.

---

83. Which of the following is a natural polymer?

(A) Polyethylene

(B) Nylon

(C) Cellulose

(D) PVC

Correct Answer: (C) Cellulose

Solution:

Step 1: Understanding the Question:

The question asks to identify the natural polymer from the given options.

Step 2: Key Formula or Approach:

Polymers are classified by their origin into three groups:

1. **Natural polymers:** Synthesized naturally by plants and animals.
2. **Synthetic polymers:** Man-made polymers synthesized in laboratories from petrochemical monomers.
3. **Semi-synthetic polymers:** Derived from natural polymers through chemical modifications.

Step 3: Detailed Explanation:

- **Polyethylene:** A synthetic polymer made by the addition polymerization of ethylene monomers ($-[CH_2\text{-}CH_2]_n-$).
- **Nylon:** A synthetic polyamide polymer characterized by amide linkages. It is synthesized by condensation polymerization of diamines and dicarboxylic acids.
- **Cellulose:** A naturally occurring organic polymer consisting of a linear chain of hundreds to thousands of linked $\beta(1 \rightarrow 4)$ D-glucose units. It is the primary structural component of green plants' cell walls and is the most abundant natural polymer on Earth.
- **PVC (Polyvinyl Chloride):** A synthetic plastic polymer produced by polymerizing vinyl chloride monomers.

Step 4: Final Answer:

Cellulose is the natural polymer.

Quick Tip: Common natural polymers to remember for competitive exams:

- Cellulose, Starch, Glycogen (Polysaccharides)
- Proteins, Silk, Wool (Polypeptides)
- Natural Rubber (Polyisoprene)

84. The forces of 60 N and 80 N are in equilibrium at a point, acting at an angle of 90° to each other. Find the magnitude of the third force?

(A) 100 N

(B) 120 N

(C) 140 N

(D) 160 N

Correct Answer: (A) 100 N

Solution:

Step 1: Understanding the Question:

The question asks for the magnitude of a third force required to maintain static equilibrium at a point alongside two perpendicular forces of 60 N and 80 N.

Step 2: Key Formula or Approach:

For three forces $\mathbf{F_1}$, $\mathbf{F_2}$, and $\mathbf{F_3}$ to be in equilibrium:

$$\mathbf{F_1} + \mathbf{F_2} + \mathbf{F_3} = 0 \implies \mathbf{F_3} = -(\mathbf{F_1} + \mathbf{F_2})$$

This means the third force must be equal in magnitude and opposite in direction to the resultant of the first two forces (R):

$$F_3 = R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

Step 3: Detailed Explanation:

- Given values:

  $F_1 = 60$ N

  $F_2 = 80$ N

  $\theta = 90°$ (angle between F_1 and F_2)

- Since the forces are perpendicular, $\cos(90^\circ) = 0$.

- The magnitude of the resultant force R is:

$$R = \sqrt{F_1^2 + F_2^2}$$

$$R = \sqrt{60^2 + 80^2}$$

$$R = \sqrt{3600 + 6400}$$

$$R = \sqrt{10000} = 100 \text{ N}$$

- To balance this resultant force of 100 N and maintain static equilibrium at the point, the third force must have a magnitude of exactly 100 N and act in the opposite direction.

Step 4: Final Answer:

The magnitude of the third force is 100 N.

Quick Tip: This is a standard $(3, 4, 5)$ right-triangle ratio scaled by a factor of 20:

$3(20) = 60$ N

$4(20) = 80$ N

$5(20) = 100$ N (Resultant/Third force magnitude)

85. Two metals, A and B, have Stacking Fault Energies of 20 $\mathbf{mJ/m^2}$ and 160 $\mathbf{mJ/m^2}$ respectively. Which of the following is true regarding their deformation behaviour?

(A) Metal A will cross-slip more easily than Metal B.

(B) Metal B will exhibit a higher rate of work hardening than Metal A.

(C) Metal B will cross-slip more easily than Metal A.

(D) Metal A will not exhibit any plastic deformation.

Correct Answer: (C) Metal B will cross-slip more easily than Metal A.

Solution:

Step 1: Understanding the Question:

The question asks how the Stacking Fault Energy (SFE) of two metals affects their plastic deformation behavior, specifically their capacity to undergo cross slip.

Step 2: Key Formula or Approach:

The Stacking Fault Energy (SFE) determines the separation distance (d) between two partial dislocations:

$$d \propto \frac{1}{\text{SFE}}$$

- High SFE $\implies$ Narrow stacking fault (small separation).
- Low SFE $\implies$ Wide stacking fault (large separation).

Step 3: Detailed Explanation:

- For a dislocation to cross slip (move from one slip plane to an intersecting plane), any dissociated partial dislocations must first recombine into a single perfect dislocation.

- **Metal B (High SFE = 160 $\mathbf{mJ/m^2}$):** Because the SFE is very high, the stacking fault width between the partial dislocations is extremely narrow. It requires very little energy to force these partials to recombine, making cross slip easy and highly frequent.

- **Metal A (Low SFE $= 20$ mJ/m^2):** Because the SFE is low, the partial dislocations are widely separated. It is highly unfavorable for them to recombine, which suppresses cross slip. Instead, dislocations are restricted to planar slip, leading to high dislocation density pile-ups and a higher rate of work hardening.

Step 4: Final Answer:

Metal B will cross-slip more easily than Metal A because of its higher Stacking Fault Energy.

Quick Tip: High SFE (e.g., Aluminum) → Easy cross-slip → Low work hardening rate.
Low SFE (e.g., Brass, Stainless Steel) → Difficult cross-slip → High work hardening rate.

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86. The hot working temperature of lead

(A) 300°C

(B) 100°C

(C) 200°C

(D) Room temperature

Correct Answer: (D) Room temperature

Solution:

Step 1: Understanding the Question:

The question asks for the temperature range at which the plastic deformation of lead (Pb) is classified as "hot working".

Step 2: Key Formula or Approach:

The dividing line between cold working and hot working is the recrystallization temperature (T_{recrys}):

$$T_{\text{recrys}} \approx 0.4 \times T_m \quad \text{(in Kelvin)}$$

where T_m is the absolute melting temperature of the metal.

Step 3: Detailed Explanation:

- Lead (Pb) is a soft metal with a very low melting point:

$$T_m = 327^\circ\text{C} = 600\ \text{K}$$

- Calculating the absolute recrystallization temperature of lead:

$$T_{\text{recrys}} \approx 0.4 \times 600\ \text{K} = 240\ \text{K} = -33^\circ\text{C}$$

- Because the recrystallization temperature of lead is well below room temperature ($25^\circ\text{C} = 298\ \text{K}$), any deformation performed at room temperature occurs above T_{recrys}.

- During deformation at room temperature, the grains of lead spontaneously recrystallize, continuously healing structural damage and preventing work hardening.

- Therefore, deforming lead at room temperature is thermodynamically classified as hot working.

Step 4: Final Answer:
The hot working temperature of lead is Room temperature.

Quick Tip: Hot working does not necessarily mean "hot to touch."

It simply means working a metal above its recrystallization temperature.

Lead at room temperature is being hot worked, while iron at 400°C is still being cold worked.

---

87. An activity represents:

(A) Effective concentration

(B) Pressure

(C) Volume

(D) Temperature

Correct Answer: (A) Effective concentration

Solution:

Step 1: Understanding the Question:

The question asks for the physical definition and thermodynamic meaning of the term "activity" (a) of a chemical species.

Step 2: Key Formula or Approach:

Activity is defined to preserve the mathematical form of the chemical potential equation for non-ideal systems:

$$\mu_i = \mu_i^\circ + RT \ln a_i$$

where:

$a_i = \gamma_i x_i$ is the activity of component i.

γ_i is the activity coefficient.

x_i is the actual concentration (mole fraction).

Step 3: Detailed Explanation:

- In ideal solutions, the behavior of any component is directly proportional to its actual concentration.

- In real (non-ideal) mixtures, electrostatic forces, molecular sizes, and chemical interactions cause the components to behave as if their concentration is either higher or lower than its actual analytical value.

- To account for these non-ideal interactions without changing the standard thermodynamic equations, the concept of "effective concentration" or **activity** is introduced.

- The activity coefficient γ_i serves as a correction factor for these intermolecular interactions. When the solution is ideal, $\gamma = 1$, and the activity equals the actual concentration ($a_i = x_i$).

Step 4: Final Answer:

An activity represents the effective concentration of a component in a solution.

Quick Tip: Activity is a dimensionless quantity.

It can be thought of as the "apparent" or "active" concentration participating in a thermodynamic process.

---

88. Consider an adiabatic process in which a gas is compressed. Which of the following statements is strictly CORRECT regarding the energy transfer?

(A) The internal energy of the system remains constant because Q = 0.

(B) Heat is converted into work during the compression.

(C) Work is a mode of energy transfer that is stored as "Work Energy" in the system.

(D) Work is done on the system, increasing the internal energy, but the system contains no "Heat" or "Work."

Correct Answer: (D) Work is done on the system, increasing the internal energy, but the system contains no "Heat" or "Work."

Solution:

Step 1: Understanding the Question:
The question tests key conceptual definitions in thermodynamics regarding path functions (heat, work) and state functions (internal energy) during an adiabatic compression.

Step 2: Key Formula or Approach:
According to the First Law of Thermodynamics:

$$dU = dQ - dW$$

For an adiabatic process, there is no heat transfer across the boundary:

$$dQ = 0 \implies dU = -dW$$

Step 3: Detailed Explanation:

- Heat (Q) and Work (W) are defined strictly as energy **in transit** across a boundary. They are path functions, meaning they only exist during a process.
- A thermodynamic system does not possess or contain "heat" or "work". It only possesses internal energy (U), which is a state function.
- During an adiabatic compression, work is done on the system ($dW < 0$), which causes the internal energy to increase ($dU > 0$).
- The work done is converted into internal energy (microscopic kinetic and potential energy of the gas molecules), not stored as some separate "work energy" category.

- Therefore, statement (D) is the only conceptually accurate statement.

Step 4: Final Answer:

Work is done on the system, increasing the internal energy, but the system contains no "Heat" or "Work."

Quick Tip: Always remember:

Internal Energy (U) is a **State Function** (stored in the system).

Heat (Q) and Work (W) are **Path Functions** (energy in transition across boundaries).

---

89. Total recoverable strain energy stored within the elastic limit in the stress-strain curve is called-

(A) Ductility

(B) Hardness

(C) Resilience

(D) Tensile strength

Correct Answer: (C) Resilience

Solution:

Step 1: Understanding the Question:

The question asks for the mechanical term used to describe the total elastic strain energy that a material can absorb and subsequently release upon unloading.

Step 2: Key Formula or Approach:

The maximum elastic strain energy per unit volume that can be stored is called the Modulus of Resilience (U_r), which is the area under the elastic portion of the stress-strain curve:

$$U_r = \int_0^{\epsilon_y} \sigma d\epsilon = \frac{\sigma_y^2}{2E}$$

where:

σ_y is the yield strength.

E is Young's Modulus.

Step 3: Detailed Explanation:

- When a material is loaded within its elastic limit, the work done on the material is stored as elastic strain energy by stretching atomic bonds.

- Because the deformation is entirely elastic, this energy is fully recoverable when the load is removed.

- This property of absorbing and recovering energy within the elastic limit is called **resilience**.

- For comparison:
  - **Toughness:** Represents the total energy absorbed up to fracture (elastic + plastic energy).
  - **Ductility:** Is the measure of a material's ability to undergo plastic deformation before fracture.
  - **Hardness:** Is the resistance of a material to localized plastic deformation (scratching or indentation).

Step 4: Final Answer:

The recoverable strain energy stored within the elastic limit is called resilience.

Quick Tip: Materials designed for springs must have high resilience.

To achieve this, they need a high yield strength (σ_y) and a low elastic modulus (E).

---

90. Efficiency of Carnot engine is always:

(A) Greater than 1

(B) Less than 1

(C) Equal to 1

(D) Infinite

Correct Answer: (B) Less than 1

Solution:

Step 1: Understanding the Question:

The question asks about the upper limit of the thermal efficiency of a Carnot heat engine.

Step 2: Key Formula or Approach:

The thermal efficiency of a Carnot heat engine is given by:

$$\eta_c = 1 - \frac{T_C}{T_H}$$

where:

T_C is the temperature of the cold reservoir (in Kelvin).

T_H is the temperature of the hot reservoir (in Kelvin).

Step 3: Detailed Explanation:

- According to the Second Law of Thermodynamics, it is impossible for any heat engine to convert 100% of absorbed heat into useful work. Some heat must always be rejected to a colder reservoir.

- Mathematically, for the efficiency η_c to be equal to 1 (100%):
  - Either the cold reservoir temperature must be absolute zero ($T_C = 0$ K).
  - Or the hot reservoir temperature must be infinite ($T_H \to \infty$).

- According to the Third Law of Thermodynamics, absolute zero (0 K) cannot be reached in a finite number of steps. Infinite temperatures are also physically impossible.

- Therefore, the term $\frac{T_C}{T_H}$ is always strictly greater than 0, which means η_c must always be strictly less than 1.

Step 4: Final Answer:

The efficiency of a Carnot engine is always less than 1.

Quick Tip: An efficiency of 1 (100%) would violate the Kelvin-Planck statement of the Second Law of Thermodynamics.
Therefore, all real and ideal heat engine efficiencies are strictly less than 1.

---

91. Which of the following materials possesses anti-parallel spins of unequal magnitude, resulting in a spontaneous magnetic moment below a critical temperature?

(A) Ferromagnetic
(B) Paramagnetic
(C) Antiferromagnetic
(D) Ferrimagnetic

Correct Answer: (D) Ferrimagnetic

Solution:

Step 1: Understanding the Question:
The question asks to identify the class of magnetic materials characterized by anti-parallel magnetic spins of unequal magnitude that yield a net spontaneous magnetization.

Step 2: Key Formula or Approach:
We analyze the orientation and magnitude of the atomic magnetic moments (spins) in different magnetic structures:
- Ferromagnetic: Parallel spins (↑↑↑↑).
- Antiferromagnetic: Anti-parallel spins of equal magnitude (↑↓↑↓), cancelling to zero.
- Ferrimagnetic: Anti-parallel spins of unequal magnitude (↑⇓↑⇓), leading to a net non-zero magnetic moment.

Step 3: Detailed Explanation:

- **Ferrimagnetism:** Occurs in materials (like magnetite, Fe_3O_4, and other ferrites) where the crystal lattice contains different sublattices with opposing magnetic moments.
- Because the opposing moments on the sublattices are unequal in magnitude, they do not completely cancel each other out.
- This incomplete cancellation results in a net spontaneous magnetic moment in the direction of the larger spins below a critical temperature (the Curie temperature).
- Unlike antiferromagnets, which have zero net magnetization, ferrimagnets exhibit strong macroscopic magnetic properties similar to ferromagnets, though they generally have lower saturation magnetization.

Step 4: Final Answer:
The materials are classified as Ferrimagnetic.

Quick Tip: The classic example of a ferrimagnetic material is Magnetite (Fe_3O_4).
It contains both Fe^{2+} and Fe^{3+} ions, which occupy different lattice sites with opposing, unequal magnetic moments.

---

92. Which of the following has the highest thermal diffusivity:

(A) Iron
(B) Lead
(C) Concrete
(D) Wood

Correct Answer: (B) Lead

Solution:

Step 1: Understanding the Question:
The question asks to identify the material with the highest thermal diffusivity among the given options.

Step 2: Key Formula or Approach:
Thermal diffusivity (α) is a material property that measures the rate of heat transfer from hot to cold regions. It is defined as:

$$\alpha = \frac{k}{\rho c_p}$$

where:
k is the thermal conductivity.
ρ is the density.
c_p is the specific heat capacity.

Step 3: Detailed Explanation:

- Let us compare the typical thermal diffusivity values (α) of the given materials:
  - Concrete: $\alpha \approx 1.0 \times 10^{-6}$ m^2/s (non-metal, low conductivity).
  - Wood: $\alpha \approx 1.0 \times 10^{-7}$ m^2/s (thermal insulator).
  - Iron: $\alpha \approx 2.3 \times 10^{-5}$ m^2/s (metal).
  - Lead: $\alpha \approx 2.4 \times 10^{-5}$ m^2/s (metal with high diffusivity relative to its heat capacity).

- Although Iron has higher thermal conductivity than Lead, Lead has a much lower specific heat capacity ($c_p \approx 129$ J/kg $\cdot$ K for Lead compared to $c_p \approx 450$ J/kg $\cdot$ K for Iron).

- This extremely low heat capacity means Lead requires very little thermal energy to change temperature, giving it a higher thermal diffusivity value of around 2.4×10^{-5} m^2/s.

Step 4: Final Answer:

Lead has the highest thermal diffusivity among the options.

Quick Tip: Thermal diffusivity (α) represents how fast a material responds to a change in temperature. A high value means heat propagates quickly, which depends on both high conductivity (k) and low heat capacity (ρc_p).

---

93. At 0K, an intrinsic semiconductor behaves like a:

(A) Conductor

(B) Insulator

(C) Superconductor

(D) Metal

Correct Answer: (B) Insulator

Solution:

Step 1: Understanding the Question:

The question asks for the electrical conduction behavior of a pure (intrinsic) semiconductor at absolute zero temperature (0 K).

Step 2: Key Formula or Approach:

The electrical conductivity of a semiconductor is given by:

$$\sigma = n_i e(\mu_e + \mu_h)$$

where n_i is the concentration of intrinsic charge carriers, which depends on temperature:

$$n_i \propto \exp\left(-\frac{E_g}{2k_B T}\right)$$

Step 3: Detailed Explanation:

- An intrinsic semiconductor has a valence band that is completely filled with electrons and a conduction band that is completely empty, separated by a relatively small band gap ($E_g \approx 1$ eV).
- At absolute zero temperature ($T = 0$ K), the thermal energy of the system is zero ($k_B T = 0$).
- Because there is no thermal energy, no covalent bonds can break, and no electrons can gain the energy required to cross the band gap into the conduction band ($n_i = 0$).
- With no free electrons in the conduction band and no holes in the valence band, there are no charge carriers available to conduct electricity.

- Consequently, the electrical conductivity is exactly zero, and the semiconductor behaves as a perfect electrical insulator.

Step 4: Final Answer:

At 0 K, an intrinsic semiconductor behaves like an insulator.

Quick Tip: As temperature increases ($T > 0$ K), thermal energy excites electrons across the band gap. This causes the electrical conductivity of a semiconductor to increase with temperature (negative temperature coefficient of resistance), which is the opposite of how metals behave.

---

94. Thermosetting polymers differ from Thermoplastics primarily because

(A) Thermosets have a linear molecular structure and can be reshaped by heating

(B) Thermosets possess a three-dimensional cross-linked network that prevents melting

(C) Thermoplastics have higher thermal stability than thermosets

(D) Thermosets are formed exclusively by addition polymerization

Correct Answer: (B) Thermosets possess a three-dimensional cross-linked network that prevents melting

Solution:

Step 1: Understanding the Question:

The question asks for the primary structural difference that distinguishes thermosetting polymers from thermoplastic polymers.

Step 2: Key Formula or Approach:

Polymers are classified into thermoplastics and thermosets based on their molecular structure and response to heat:

- Thermoplastics: Linear/branched chains, weak secondary bonds.

- Thermosets: 3D network, strong covalent cross-links.

Step 3: Detailed Explanation:

- **Thermoplastics:** Consist of long, linear or branched polymer chains held together by weak secondary bonds (Van der Waals or hydrogen bonds). Upon heating, these weak bonds break easily, allowing the chains to slide past one another. Consequently, thermoplastics melt, flow, and can be repeatedly reshaped and recycled.

- **Thermosets:** Form a rigid three-dimensional network structure during polymerization, where the chains are joined together by strong, primary covalent bonds (cross-links). Heating a thermoset does not break these strong covalent bonds; instead, it only increases atomic vibrations until the material eventually decomposes or burns.

- Because of this extensive cross-linking, thermosets do not melt or soften upon heating, making them highly stable and structurally rigid.

Step 4: Final Answer:
Thermosets differ from thermoplastics because they possess a three-dimensional cross-linked network that prevents melting.

Quick Tip: Think of thermoplastics like candle wax (can be melted and solidified repeatedly). Think of thermosets like a baked cake (once cured/baked, reheating it will only burn it, not melt it back into batter).

---

95. If two forces acting at a joint are not along the straight line, then for the equilibrium of the joint

(A) one of the forces must be zero

(B) each force must be zero

(C) forces must be equal and of the same sign

(D) forces must be equal in magnitude but opposite in sign

Correct Answer: (B) each force must be zero

Solution:

Step 1: Understanding the Question:

The question asks for the condition of force magnitudes required to maintain static equilibrium at a joint where only two non-collinear (not along a straight line) forces are acting.

Step 2: Key Formula or Approach:

For any joint to be in static equilibrium, the vector sum of all forces acting on it must be zero:

$$\sum \mathbf{F} = 0 \implies \mathbf{F_1} + \mathbf{F_2} = 0$$

This requires:

$$\sum F_x = 0 \quad \text{and} \quad \sum F_y = 0$$

Step 3: Detailed Explanation:

- Let the joint be the origin of a coordinate system. Let $\mathbf{F_1}$ act along the x-axis, and $\mathbf{F_2}$ act at an angle θ (where $\theta \neq 0^\circ$ and $\theta \neq 180^\circ$ because they are non-collinear).

- Resolving the forces along the x and y axes:
  - Along the y-axis:

$$\sum F_y = F_2 \sin\theta = 0$$

  Since the forces are non-collinear, $\sin\theta \neq 0$. This forces:

$$F_2 = 0$$

- Along the x-axis:

$$\sum F_x = F_1 + F_2 \cos\theta = 0$$

Substitute $F_2 = 0$ into this equation:

$$F_1 + 0 = 0 \implies F_1 = 0$$

- Therefore, the only way for a joint with two non-collinear forces to be in static equilibrium is if both forces are exactly zero.

Step 4: Final Answer:

For equilibrium, each force must be zero.

Quick Tip: This is a fundamental rule used in truss analysis:

If a joint has only two non-collinear members and no external load acts on it, both members are zero-force members ($F_1 = F_2 = 0$).

---

96. Vacancy diffusion coefficient: $D = D_0 \exp(-Q/RT)$

(A) Dimensionally correct

(B) Requires D_0/L to be correct

(C) Requires Q dimensionless

(D) All formulas fail dimensionally

Correct Answer: (A) Dimensionally correct

Solution:

Step 1: Understanding the Question:

The question asks to evaluate the dimensional consistency of the Arrhenius equation for the vacancy diffusion coefficient:

$$D = D_0 \exp\left(-\frac{Q}{RT}\right)$$

Step 2: Key Formula or Approach:

For any equation containing an exponential function of the form $y = e^x$, the exponent x must be a dimensionless quantity.

If the exponent is dimensionless, then the dimension of the left-hand side must match the dimension of the pre-exponential factor:

$$[D] = [D_0]$$

Step 3: Detailed Explanation:

- Let us analyze the dimensions of the terms inside the exponent, $-\frac{Q}{RT}$:
  - Q is the activation energy for diffusion, typically in Joules per mole ($[\text{J/mol}]$).
  - R is the universal gas constant, in Joules per mole-Kelvin ($[\text{J/(mol} \cdot \text{K)}]$).
  - T is the absolute temperature, in Kelvin ($[\text{K}]$).

- Evaluating the dimensions of the denominator RT:

$$[RT] = [\text{J/(mol} \cdot \text{K)}] \times [\text{K}] = [\text{J/mol}]$$

- Evaluating the dimensions of the ratio $\frac{Q}{RT}$:

$$\left[\frac{Q}{RT}\right] = \frac{[\text{J/mol}]}{[\text{J/mol}]} = [1] \quad \text{(dimensionless)}$$

- Because the exponent is dimensionless, the term $\exp(-Q/RT)$ is a dimensionless number.
- Both the diffusion coefficient D and the pre-exponential factor D_0 have identical units of area per unit time ($[\text{m}^2/\text{s}]$).
- Therefore, the equation is fully dimensionally correct without requiring any modifications.

Step 4: Final Answer:

The equation is dimensionally correct.

Quick Tip: Arguments of transcendental functions (like exp, ln, sin, cos) must always be dimensionless. This rule is extremely helpful for verifying physical formulas quickly.

---

97. The "Glassy" state in ceramics is characterized as

(A) A perfectly ordered crystalline lattice

(B) A material that lacks any atomic bonding

(C) A liquid that has a very low viscosity

(D) A metastable, non-crystalline solid with short-range order but no long-range order

Correct Answer: (D) A metastable, non-crystalline solid with short-range order but no long-range order

Solution:

Step 1: Understanding the Question:
The question asks for the definition and structural characteristics of the "glassy" (amorphous) state in ceramic materials.

Step 2: Key Formula or Approach:
The solid state is broadly divided into crystalline and amorphous (glassy) states based on the presence or absence of atomic periodicity over long distances.

Step 3: Detailed Explanation:

- When a liquid ceramic melt is cooled rapidly below its melting point, the viscosity of the liquid increases sharply.
- Because of this rapid cooling, the atoms do not have sufficient time or mobility to organize into a highly ordered, periodic crystal lattice.
- Instead, the disordered liquid-like structure is "frozen" in place below the glass transition temperature (T_g).
- Structurally, this glassy state possesses **short-range order** (the chemical bonds and local coordination of nearest neighbors are preserved, like the SiO_4^{4-} tetrahedra in silica glass).
- However, it completely lacks **long-range periodic order** (there is no repeating crystal lattice over macroscopic distances).
- Thermodynamically, this state is metastable compared to the crystalline state, as it has higher Gibbs free energy.

Step 4: Final Answer:

The glassy state is characterized as a metastable, non-crystalline solid with short-range order but no long-range order.

Quick Tip: Glass is physically a solid but structurally resembles a supercooled liquid.

Its non-crystalline nature is what gives glass its characteristic isotropic properties and optical transparency.

---

98. Centroid of triangle lies at:

(A) Midpoint

(B) Intersection of medians

(C) Circumcenter

(D) Vertex

Correct Answer: (B) Intersection of medians

Solution:

Step 1: Understanding the Question:

The question asks for the geometric definition of the centroid of a triangle.

Step 2: Key Formula or Approach:

In geometry and mechanics, the centroid of a shape represents its geometric center. For a thin plate of uniform density, the centroid coincides with its center of gravity.

Step 3: Detailed Explanation:

- A median of a triangle is defined as a straight line segment drawn from any vertex to the midpoint of the opposite side.

- Every triangle has exactly three medians.
- According to geometric principles, all three medians of a triangle always intersect at a single, common point.
- This point of concurrency is called the **centroid** of the triangle.
- The centroid divides each median in a 2 : 1 ratio, with the larger part being closer to the vertex.
- Mathematically, if the vertices of a triangle are (x_1, y_1), (x_2, y_2), and (x_3, y_3), the coordinates of the centroid are:

$$\bar{x} = \frac{x_1 + x_2 + x_3}{3}, \quad \bar{y} = \frac{y_1 + y_2 + y_3}{3}$$

Step 4: Final Answer:

The centroid of a triangle lies at the intersection of its medians.

Quick Tip: The centroid is the point where a triangular cardboard sheet can be perfectly balanced on the tip of a needle.
It always lies inside the triangle.

---

99. Which of the following statements is FALSE?

(A) Entropy is a state function

(B) Heat is a state function

(C) Internal energy is a state function

(D) Gibbs free energy is a state function

Correct Answer: (B) Heat is a state function

Solution:

Step 1: Understanding the Question:
The question asks to identify the false statement among the options regarding state functions in thermodynamics.

Step 2: Key Formula or Approach:
- **State Function:** A property whose value depends only on the current state of the system, not on how the system reached that state (e.g., U, S, G, H, P, T, V).
- **Path Function:** A quantity whose value depends on the specific path followed during a transition (e.g., Heat Q and Work W).

Step 3: Detailed Explanation:

- **Statement A is True:** Entropy (S) is a thermodynamic state function. The change in entropy (ΔS) between two states is independent of the path.
- **Statement B is False:** Heat (Q) is not a state function; it is a path function. The amount of heat exchanged during a process depends on the specific path taken (e.g., constant volume vs. constant pressure).
- **Statement C is True:** Internal energy (U) is a state function. Although Q and W are path functions, their difference ($dU = dQ - dW$) is independent of the path and represents a change in a state function.
- **Statement D is True:** Gibbs free energy ($G = H - TS$) is defined entirely using state functions, making it a state function itself.

Step 4: Final Answer: The false statement is: "Heat is a state function."

Quick Tip: Easy way to remember:
State Functions: P, V, T, U, H, S, G (have exact differentials).
Path Functions: Heat (Q) and Work (W) (have inexact differentials).

---

100. Transparency of glass is due to

(A) absence of grain boundaries
(B) presence of free electrons
(C) metallic bonding
(D) lack of scattering centres

Correct Answer: (D) lack of scattering centres

Solution:

Step 1: Understanding the Question:
The question asks for the physical reason behind the optical transparency of glass.

Step 2: Key Formula or Approach:
Light passing through a material can be absorbed, reflected, or scattered. For a material to be highly transparent, light waves must pass through it with minimal scattering and absorption.

Step 3: Detailed Explanation:

- Glass is an amorphous solid with a homogeneous, non-crystalline structure.
- In crystalline materials, grain boundaries, phase boundaries, and microscopic pores act as defects that reflect and scatter light in different directions, making the material

translucent or opaque.

- Because glass is amorphous, it lacks grain boundaries and has a highly uniform atomic structure on a scale comparable to the wavelength of visible light.
- This uniform structure means there are no localized changes in refractive index or physical interfaces to act as **scattering centers**.
- Furthermore, because glass has a wide electronic band gap, visible light photons do not have enough energy to excite electrons, preventing absorption.
- With both scattering and absorption suppressed, light passes through glass without being disrupted, resulting in its optical transparency.

Step 4: Final Answer:
The transparency of glass is due to the lack of scattering centers.

Quick Tip: While the "absence of grain boundaries" is a prerequisite, the fundamental optical explanation for transparency is the "lack of scattering centers" of size comparable to visible light wavelengths.

---

101. If $\Delta U = 100$ J and work done $= 40$ J, heat supplied is:

(A) 60 J
(B) 140 J
(C) 100 J
(D) 40 J

Correct Answer: (B) 140 J

Solution:

Step 1: Understanding the Question:

This question is based on the application of the First Law of Thermodynamics, which is a statement of the law of conservation of energy.

The first law establishes a relationship between heat supplied, work done, and change in internal energy of a system during a thermodynamic process.

Step 2: Key Formula or Approach:

The governing relationship is given by the expression:

$$\Delta Q = \Delta U + W$$

Where:

ΔQ is the net heat supplied to the system.

ΔU is the change in internal energy of the system.

W is the work done by the system.

Step 3: Detailed Explanation:

- The change in internal energy is given as $\Delta U = 100$ J. This positive value represents an increase in the internal energy of the system.

- The work done by the system is given as $W = 40$ J. This positive value indicates work is being performed by the system on its surroundings (expansion work).

- Substituting these values into the first law equation:

$$\Delta Q = 100 \text{ J} + 40 \text{ J} = 140 \text{ J}$$

- The resulting positive value of 140 J indicates that heat is supplied to the system.

Step 4: Final Answer:

The heat supplied to the system is 140 J, which corresponds to option (B).

Quick Tip: Always pay close attention to sign conventions in thermodynamics.

Heat supplied to the system is positive (+), and heat rejected is negative (-).

Work done by the system is positive (+), and work done on the system is negative (-).

---

102. According to the Boltzmann Equation $S = k_B \ln \Omega$, if a system is in a state of "Perfect Order" (a perfect crystal at 0 K), the number of microstates Ω and the entropy S are

(A) $\Omega = 0, S = 0$

(B) $\Omega = 1, S = 0$

(C) $\Omega = 1, S = 1$

(D) $\Omega \to \infty, S \to \infty$

Correct Answer: (B) $\Omega = 1, S = 0$

Solution:

Step 1: Understanding the Question:

This question relates statistical thermodynamics to macroscopic thermodynamic behavior.

It asks for the statistical definition of entropy under the specific condition of perfect order at absolute zero, which forms the physical basis of the Third Law of Thermodynamics.

Step 2: Key Formula or Approach:

The Boltzmann entropy equation relates entropy to statistical microstates:

$$S = k_B \ln \Omega$$

Where:

S is the macroscopic entropy.

k_B is the Boltzmann constant.
Ω is the thermodynamic probability, representing the number of microstates corresponding to the macrostate.

Step 3: Detailed Explanation:

- At absolute zero temperature (0 K), a system is in its lowest energy configuration (the ground state).
- For a perfect crystal in a state of "Perfect Order", there is only one unique microscopic arrangement of atoms that satisfies this macroscopic state.
- Therefore, the number of microstates is $\Omega = 1$.
- Substituting $\Omega = 1$ into the Boltzmann equation:

$$S = k_B \ln(1)$$

- Since the natural logarithm of 1 is zero ($\ln 1 = 0$), the entropy becomes:

$$S = 0$$

- This confirms that the entropy of a perfect crystal at absolute zero is zero.

Step 4: Final Answer:
Under a state of perfect order, $\Omega = 1$ and $S = 0$, which corresponds to option (B).

Quick Tip: The Third Law of Thermodynamics states that the entropy of a perfect crystal at absolute zero is exactly zero.
This corresponds to a single microstate ($\Omega = 1$), representing zero randomness.

103. Which combination gives highest strength-to-weight ratio?

(A) Polymer + fibres

(B) Metal + fibres

(C) Polymer + particles

(D) Ceramic + voids

Correct Answer: (A) Polymer + fibres

Solution:

Step 1: Understanding the Question:

The question asks to identify the structural combination of materials that yields the maximum strength-to-weight ratio (also known as specific strength).

This parameter is critical in structural, aerospace, and transport engineering, where maximizing structural efficiency while minimizing overall weight is essential.

Step 2: Key Formula or Approach:

The strength-to-weight ratio is expressed as:

$$\text{Specific Strength} = \frac{\sigma_{ts}}{\rho}$$

Where:

σ_{ts} is the ultimate tensile strength of the composite.

ρ is the density of the composite.

To maximize this ratio, we require high tensile strength and low density.

Step 3: Detailed Explanation:

- Polymers have very low density ($\rho \approx 1.0 - 1.4$ g/cm^3) but generally exhibit low to moderate mechanical strength.

- Reinforcing fibers (such as carbon, aramid, or glass) possess exceptionally high tensile strengths.

- Combining a polymer matrix with high-strength continuous fibers creates a Fiber-Reinforced Polymer (FRP) composite.

- These composites achieve a remarkably high tensile strength because the fibers carry the primary structural loads, while the polymer matrix keeps the density extremely low.

- Metal matrix composites (Metal + fibres) are strong but have a much higher overall density (e.g., aluminum has a density of 2.7 g/cm^3, and titanium has 4.5 g/cm^3), which lowers their strength-to-weight ratio.

- Polymer-particle composites (Polymer + particles) generally possess lower tensile strength compared to fiber-reinforced composites because particulates are not as effective as fibers at transmitting and bearing tensile loads.

- Ceramic-void combinations (Ceramic + voids) contain voids which act as stress concentrators, dramatically reducing structural strength.

Step 4: Final Answer:

The Polymer + fibres combination provides the highest strength-to-weight ratio, matching option (A).

Quick Tip: Fiber-reinforced polymers (FRP) are widely used in commercial aerospace designs (e.g., Boeing 787 fuselage) specifically because of their superior specific strength.

---

104. Ferromagnetic materials show:

(A) Weak repulsion

(B) Strong attraction

(C) No interaction

(D) Random behaviour

Correct Answer: (B) Strong attraction

Solution:

Step 1: Understanding the Question:
This question concerns the fundamental magnetic behavior of ferromagnetic materials when they are introduced into an external magnetic field.

Step 2: Key Formula or Approach:
The magnetic behavior of a material is characterized by its relative magnetic permeability (μ_r) and magnetic susceptibility (χ).
For ferromagnetic materials:

$$\chi \gg 1 \quad \text{and} \quad \mu_r \gg 1$$

These parameters indicate that ferromagnetic materials have a large and positive response to external magnetic fields.

Step 3: Detailed Explanation:

- Ferromagnetic materials (such as iron, cobalt, and nickel) contain permanent atomic magnetic dipoles.
- In these materials, quantum mechanical exchange interactions cause adjacent dipoles to align parallel to one another within macroscopic regions called magnetic domains.
- When an external magnetic field is applied, these domains readily align with and grow

in the direction of the applied field.

- This results in a massive magnetization inside the material that is parallel to the external field, producing a very strong mechanical force of attraction.

- In comparison, diamagnetic materials experience weak repulsion, paramagnetic materials experience weak attraction, and non-magnetic materials show no interaction.

Step 4: Final Answer:
Ferromagnetic materials display a strong attraction to magnetic fields, which corresponds to option (B).

Quick Tip: Ferromagnetic materials are the only materials that show strong attraction to a magnetic field and can be permanently magnetized.

---

105. A material loses its strong magnetization suddenly at a critical temperature and becomes weakly magnetic. This transition is from

(A) Diamagnetic → Paramagnetic
(B) Paramagnetic → Ferromagnetic
(C) Ferromagnetic → Paramagnetic
(D) Superconducting → Diamagnetic

Correct Answer: (C) Ferromagnetic → Paramagnetic

Solution:

Step 1: Understanding the Question:
This question explores the relationship between temperature and the magnetic structure of

materials.
It tests the knowledge of thermal effects on the parallel alignment of atomic spins in strongly magnetic substances.

Step 2: Key Formula or Approach:
The transition temperature is known as the Curie temperature (T_C).
For temperatures above T_C, the susceptibility (χ) of the material is described by the Curie-Weiss Law:

$$\chi = \frac{C}{T - \theta_p}$$

Where:
C is the Curie constant.
T is the absolute temperature.
θ_p is the paramagnetic Curie temperature.

Step 3: Detailed Explanation:

- Below the Curie temperature ($T < T_C$), a ferromagnetic material possesses spontaneous parallel spin alignment, leading to strong magnetic properties.
- As temperature increases, the randomizing thermal energy of atomic vibrations increases.
- When the temperature reaches and exceeds the Curie temperature (T_C), thermal agitation becomes strong enough to completely overcome the quantum mechanical exchange forces responsible for the spin alignment.
- Consequently, the magnetic domains are disrupted, the atomic dipoles become randomly oriented, and the material becomes weakly magnetic.
- This randomized, weakly magnetic state is characteristic of a paramagnetic material.

- Thus, heating past the Curie temperature causes a transition from Ferromagnetic → Paramagnetic.

Step 4: Final Answer:

The transition described is from Ferromagnetic to Paramagnetic, which matches option (C).

Quick Tip: The Curie temperature of iron is approximately 770°C (1043 K).
Above this temperature, iron is no longer attracted to a permanent magnet.

---

106. The 'Fugacity Coefficient ($\phi = f/P$)' of a real gas approaches unity when

(A) the pressure approaches infinity
(B) the temperature approaches 0 K
(C) the pressure approaches zero
(D) the gas is compressed into a liquid

Correct Answer: (C) the pressure approaches zero

Solution:

Step 1: Understanding the Question:

This question deals with fugacity and the fugacity coefficient, which are thermodynamic concepts used to describe the non-ideal behavior of real gases.
The question asks for the thermodynamic limit under which a real gas exhibits ideal gas behavior.

Step 2: Key Formula or Approach:

The fugacity coefficient (ϕ) is defined by the relation:

$$\phi = \frac{f}{P}$$

Where:

f is the fugacity of the gas.

P is the pressure of the gas.

For an ideal gas, fugacity is exactly equal to the system pressure ($f = P$), which yields:

$$\phi = 1$$

Step 3: Detailed Explanation:

- Real gases deviate from ideal behavior due to intermolecular attractive/repulsive forces and the physical volume occupied by the gas molecules.
- When the pressure of a real gas approaches zero ($P \to 0$), the volume of the system becomes very large, and the molecules are spaced extremely far apart.
- At this infinite dilution limit, intermolecular interactions become negligible, and the molecular volume is negligible compared to the total volume.
- Under this condition, the real gas behaves exactly like an ideal gas.
- Since ideal gas behavior is reached at $P \to 0$, the fugacity (f) approaches the pressure (P), meaning that the fugacity coefficient ($\phi = f/P$) approaches 1 (unity):

$$\lim_{P \to 0} \phi = 1$$

Step 4: Final Answer:

The fugacity coefficient approaches unity when the pressure approaches zero, corresponding to option (C).

Quick Tip: At very low pressures, all real gases behave ideally.

Consequently, any non-ideal property multiplier (like the fugacity coefficient ϕ or compressibility factor Z) must approach 1 as $P \to 0$.

---

107. According to equation of continuity,

(A) $w_1 a_1 = w_2 a_2$

(B) $w_1 v_1 = w_2 v_2$

(C) $a_1 v_1 = a_2 v_2$

(D) $a_1 / v_1 = a_2 / v_2$

Correct Answer: (C) $a_1 v_1 = a_2 v_2$

Solution:

Step 1: Understanding the Question:

The continuity equation is a mathematical expression of the conservation of mass in a fluid flow system.

The question asks for the standard simplified form of this equation under typical flow conditions.

Step 2: Key Formula or Approach:

The fundamental conservation of mass for steady flow of a fluid through a conduit is written as:

$$\dot{m}_1 = \dot{m}_2 \Rightarrow \rho_1 a_1 v_1 = \rho_2 a_2 v_2$$

Where:

ρ is the fluid density.

a is the cross-sectional area of the conduit.

v is the average fluid velocity.

Step 3: Detailed Explanation:

- For an incompressible fluid (such as most liquids under ordinary operating conditions), the density remains constant throughout the flow.

- Therefore, we can set:

$$\rho_1 = \rho_2 = \text{constant}$$

- Substituting this into the continuity equation allows us to cancel the density terms from both sides:

$$a_1 v_1 = a_2 v_2$$

- This simplified form represents the conservation of volumetric flow rate for an incompressible fluid flow.

Step 4: Final Answer:

According to the equation of continuity for incompressible flow, $a_1 v_1 = a_2 v_2$, which corresponds to option (C).

Quick Tip: When flow area decreases ($a_2 < a_1$), velocity must increase ($v_2 > v_1$) to keep volumetric flow rate constant.

This simple principle explains how nozzles work.

---

108. Assertion (A): Gibbs free energy determines spontaneity at constant T and P.

Reason (R): A process is spontaneous when Gibbs free energy is minimum.

(A) Both A and R are true, and R is the correct explanation of A

(B) Both are true, but R is not the correct explanation of A

(C) A is true, R is false

(D) A is false, R is true

Correct Answer: (B) Both are true, but R is not the correct explanation of A

Solution:

Step 1: Understanding the Question:

This question consists of an Assertion (A) and a Reason (R) about the Gibbs free energy criterion for thermodynamic spontaneity and equilibrium.

We need to evaluate the individual truth of both statements and determine if there is a direct cause-and-effect explanatory link between them.

Step 2: Key Formula or Approach:

The change in Gibbs free energy (dG) at constant temperature (T) and pressure (P) is defined as:

$$dG_{T,P} = dH - TdS$$

The fundamental thermodynamic criteria are:

For a spontaneous process: $dG_{T,P} < 0$.

At equilibrium: $dG_{T,P} = 0$, meaning the Gibbs free energy is minimized at a stable equilibrium state.

Step 3: Detailed Explanation:

- Assertion (A): "Gibbs free energy determines spontaneity at constant T and P." This statement is true because the function was specifically designed to evaluate the total entropy change of the universe (system + surroundings) using only the system's properties under constant T and P.

- Reason (R): "A process is spontaneous when Gibbs free energy is minimum." This statement is also true. A system naturally progresses spontaneously toward a state of minimum free energy, which represents the thermodynamic equilibrium.

- Connection between A and R: While both statements are true, the Reason (R) does not explain *why* the Gibbs function is the correct thermodynamic potential to determine

spontaneity.

- The determination of spontaneity by Gibbs free energy is mathematically derived from the Second Law of Thermodynamics (Clausius inequality).

- The minimization of G at equilibrium is a consequence of the spontaneity condition, not the underlying explanation for why it serves as the spontaneity criterion. Thus, R is not the correct explanation of A.

Step 4: Final Answer:

Both A and R are true, but R is not the correct explanation of A, which is option (B).

Quick Tip: Remember the criteria:

- Constant T and $P \Rightarrow$ Gibbs Free Energy (G) is minimized.
- Constant T and $V \Rightarrow$ Helmholtz Free Energy (A) is minimized.
- Isolated System $\Rightarrow$ Entropy (S) is maximized.

---

109. A plane wall of thickness 0.1 m and thermal conductivity 50 W/m·K has a surface area of 2 m^2. The temperature difference across the wall is 100 K. What is the rate of heat transfer through the wall?

(A) 200 kW

(B) 100 kW

(C) 50 kW

(D) 25 kW

Correct Answer: (B) 100 kW

Solution:

Step 1: Understanding the Question:

This problem involves calculating steady-state, one-dimensional heat conduction through a plane wall with known dimensions, thermal conductivity, and boundary temperatures.

Step 2: Key Formula or Approach:

Fourier's Law of Heat Conduction for a plane wall is expressed as:

$$Q = \frac{kA\Delta T}{L}$$

Where:

Q is the rate of heat transfer (W).

k is the thermal conductivity of the wall ($\text{W/m} \cdot \text{K}$).

A is the heat transfer area (m^2).

ΔT is the temperature difference across the wall (K or °C).

L is the thickness of the wall (m).

Step 3: Detailed Explanation:

- Let's list the given parameters:
  Thickness, $L = 0.1$ m
  Thermal conductivity, $k = 50$ $\text{W/m} \cdot \text{K}$
  Surface area, $A = 2$ m^2
  Temperature difference, $\Delta T = 100$ K

- Substitute these values into Fourier's Law:

$$Q = \frac{50 \times 2 \times 100}{0.1}$$

- Simplify the numerator:

$$50 \times 2 \times 100 = 10,000 \text{ W}$$

- Divide by the wall thickness:

$$Q = \frac{10,000}{0.1} = 100,000 \text{ W}$$

- Convert the heat transfer rate into kilowatts (kW):

$$Q = \frac{100,000}{1,000} \text{ kW} = 100 \text{ kW}$$

Step 4: Final Answer:

The rate of heat transfer through the plane wall is 100 kW, which corresponds to option (B).

Quick Tip: Be careful with units when performing calculations.

Ensure the final answer is converted from watts (W) to kilowatts (kW) by dividing by 1,000 as required by the options.

---

110. Diamagnetic materials are:

(A) Attracted by magnetic field

(B) Repelled by magnetic field

(C) Neutral

(D) Strongly attracted

Correct Answer: (B) Repelled by magnetic field

Solution:

Step 1: Understanding the Question:

This question asks about the qualitative physical response of diamagnetic materials when they are subjected to an external magnetic field.

Step 2: Key Formula or Approach:

The magnetic susceptibility (χ) is defined as:

$$\chi = \frac{M}{H}$$

For diamagnetic materials, the susceptibility is a small, negative constant ($\chi < 0$), which indicates that the induced magnetization opposes the applied magnetic field.

Step 3: Detailed Explanation:

- Diamagnetism is a fundamental form of magnetism that occurs in all materials, though it is often very weak and easily dominated by other magnetic effects.
- It is prominent in materials that have completely filled electron shells (no unpaired electrons), meaning they lack permanent net magnetic moments.
- When an external magnetic field is applied, the orbital motion of the electrons is slightly modified to create a microscopic current.
- According to Lenz's law, this induced current creates an internal magnetic field that opposes the applied external field.
- This opposing magnetic field results in a weak mechanical repulsive force.
- Consequently, diamagnetic materials are repelled by an external magnetic field. Common examples include water, copper, and bismuth.

Step 4: Final Answer:
Diamagnetic materials are repelled by a magnetic field, which is option (B).

Quick Tip: Superconductors are perfect diamagnets ($\chi = -1$).
They exhibit the Meissner effect, repelling all magnetic flux lines, which allows for stable magnetic levitation.

---

111. Let A be an $n \times n$ matrix such that $A^3 = A$. Then which one of the following is true?

(A) A must be an identity matrix.
(B) A^2 must be an identity matrix.
(C) A is invertible.
(D) A is diagonalisable.

Correct Answer: (D) A is diagonalisable.

Solution:

Step 1: Understanding the Question:
This question is from linear algebra. It concerns the properties of an $n \times n$ matrix A satisfying the algebraic equation $A^3 - A = 0$.

Step 2: Key Formula or Approach:
A matrix is diagonalizable over a field if and only if its minimal polynomial $m(x)$ factors into distinct linear factors over that field.

Step 3: Detailed Explanation:

- The given equation is $A^3 = A$, which can be written as:

$$A^3 - A = 0$$

- This means that the polynomial $p(x) = x^3 - x$ annihilates the matrix A.

- We can factor this polynomial completely over the real numbers:

$$p(x) = x(x^2 - 1) = x(x - 1)(x + 1)$$

- The minimal polynomial $m(x)$ of A must divide the annihilating polynomial $p(x)$.
- Since $p(x)$ has only distinct linear roots (0, 1, and -1), any divisor of $p(x)$ must also have only distinct linear roots.
- According to the diagonalization theorem, because the minimal polynomial of A has only distinct linear factors, the matrix A is diagonalizable.
- Let's check why the other options are not always true:
  - If $A = 0$ (the zero matrix), then $A^3 = 0 = A$, but A is not the identity matrix. Thus, option (A) is not necessarily true.
  - If $A = 0$, then $A^2 = 0 \neq I$. Thus, option (B) is not necessarily true.
  - If $A = 0$, the matrix is singular (not invertible). Thus, option (C) is not necessarily true.

Step 4: Final Answer:

The matrix A must be diagonalizable, which corresponds to option (D).

Quick Tip: Any matrix satisfying an equation of the form $A^k = A$ (for $k > 1$) is diagonalizable because the polynomial $x^k - x = 0$ has only distinct, single-multiplicity roots.

112. The function $f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ **is**

(A) continuous everywhere

(B) continuous only at $x = 0$

(C) not continuous at $x = 0$

(D) nowhere continuous

Correct Answer: (A) continuous everywhere

Solution:

Step 1: Understanding the Question:
This question tests the continuity of a piecewise-defined real function, specifically focusing on its behavior near the boundary point $x = 0$.

Step 2: Key Formula or Approach: A function $f(x)$ is continuous at $x = c$ if:

$$\lim_{x \to c} f(x) = f(c)$$

For $x \neq 0$, the function is continuous because it is the product of two continuous functions (x and $\sin(1/x)$).
At $x = 0$, we must evaluate the limit using the Squeeze Theorem.

Step 3: Detailed Explanation:

- For $x \neq 0$, the function is defined as $f(x) = x \sin(1/x)$.
- Since the linear function $y = x$ and the trigonometric function $y = \sin(1/x)$ are both continuous on $(-\infty, 0) \cup (0, \infty)$, their product $f(x)$ is continuous everywhere except possibly at $x = 0$.
- Let's test the continuity at $x = 0$ by evaluating the limit:

$$\lim_{x \to 0} x \sin\left(\frac{1}{x}\right)$$

- We know that the sine function is always bounded between -1 and 1 for all real arguments:

$$-1 \leq \sin\left(\frac{1}{x}\right) \leq 1$$

- Multiplying this inequality by $|x|$:

$$-|x| \leq x \sin\left(\frac{1}{x}\right) \leq |x|$$

- Let's evaluate the limit of the outer terms as $x \to 0$:

$$\lim_{x \to 0} -|x| = 0 \quad \text{and} \quad \lim_{x \to 0} |x| = 0$$

- By the Squeeze Theorem, the limit of the middle term must also be 0:

$$\lim_{x \to 0} f(x) = \lim_{x \to 0} x \sin\left(\frac{1}{x}\right) = 0$$

- Since the limit of $f(x)$ as $x \to 0$ matches the defined function value $f(0) = 0$, the function is continuous at $x = 0$.

- Therefore, the function is continuous for all real numbers.

Step 4: Final Answer:

The function is continuous everywhere, which is option (A).

Quick Tip: Functions of the form $x^p \sin(1/x)$ at $x = 0$ (with $f(0) = 0$) are:

- Continuous if $p > 0$.

- Differentiable if $p > 1$.

113. Let $\vec{f} = x^2yz\hat{i} + xy^2z\hat{j} + xyz^2\hat{k}$ be a vector field. If $\vec{F}(x,y,z) = \text{curl}\,\vec{f}$, then the vector $\vec{F}(1,-2,1)$ is equal to _____

(A) $-3\hat{i} + 4\hat{j} + 3\hat{k}$

(B) $-3\hat{i} + 3\hat{k}$

(C) $3\hat{i} - 3\hat{k}$

(D) $-3\hat{i} - 4\hat{j} + 3\hat{k}$

Correct Answer: (B) $-3\hat{i} + 3\hat{k}$

Solution:

Step 1: Understanding the Question:

This question requires computing the curl of a given three-dimensional vector field $\vec{f}$ and evaluating the resulting curl vector field $\vec{F}$ at the point $(1,-2,1)$.

Step 2: Key Formula or Approach:

The curl of a vector field $\vec{f} = f_x\hat{i} + f_y\hat{j} + f_z\hat{k}$ is defined as:

$$\text{curl}\,\vec{f} = \nabla \times \vec{f} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_x & f_y & f_z \end{vmatrix}$$

Expanding this determinant yields:

$$\text{curl}\,\vec{f} = \left(\frac{\partial f_z}{\partial y} - \frac{\partial f_y}{\partial z}\right)\hat{i} - \left(\frac{\partial f_z}{\partial x} - \frac{\partial f_x}{\partial z}\right)\hat{j} + \left(\frac{\partial f_y}{\partial x} - \frac{\partial f_x}{\partial y}\right)\hat{k}$$

Step 3: Detailed Explanation:

- Identify the components of $\vec{f}$:

$$f_x = x^2yz, \quad f_y = xy^2z, \quad f_z = xyz^2$$

- Calculate each component of the curl:

– $\hat{i}$-component:

$$\frac{\partial f_z}{\partial y} - \frac{\partial f_y}{\partial z} = \frac{\partial}{\partial y}(xyz^2) - \frac{\partial}{\partial z}(xy^2z) = xz^2 - xy^2$$

– $\hat{j}$-component:

$$-\left(\frac{\partial f_z}{\partial x} - \frac{\partial f_x}{\partial z}\right) = -\left[\frac{\partial}{\partial x}(xyz^2) - \frac{\partial}{\partial z}(x^2yz)\right]$$
$$= -(yz^2 - x^2y) = x^2y - yz^2$$

– $\hat{k}$-component:

$$\frac{\partial f_y}{\partial x} - \frac{\partial f_x}{\partial y} = \frac{\partial}{\partial x}(xy^2z) - \frac{\partial}{\partial y}(x^2yz) = y^2z - x^2z$$

- Combine these parts to write the curl vector field $\vec{F}(x, y, z)$:

$$\vec{F}(x, y, z) = (xz^2 - xy^2)\hat{i} + (x^2y - yz^2)\hat{j} + (y^2z - x^2z)\hat{k}$$

- Now, substitute the coordinates of the point $(1, -2, 1)$, i.e., $x = 1$, $y = -2$, and $z = 1$:

– $\hat{i}$-component:

$$(1)(1)^2 - (1)(-2)^2 = 1 - 4 = -3$$

– $\hat{j}$-component:

$$(1)^2(-2) - (-2)(1)^2 = -2 - (-2) = 0$$

– $\hat{k}$-component:

$$(-2)^2(1) - (1)^2(1) = 4 - 1 = 3$$

- This gives the final vector:

$$\vec{F}(1, -2, 1) = -3\hat{i} + 3\hat{k}$$

Step 4: Final Answer:

The vector is $-3\hat{i}+3\hat{k}$, which corresponds to option (B).

Quick Tip: Double check signs when expanding the cross product determinant.
The middle term ($\hat{j}$) has a negative sign in the expansion: $-\hat{j}(\frac{\partial f_z}{\partial x}-\frac{\partial f_x}{\partial z})$.

---

114. The integrating factor of the differential equation $(1+x^2)\frac{dy}{dx}+2xy-4x^2=0$ is

(A) $\log(1-x)$

(B) $\log(1+x^2)$

(C) $1+x^2$

(D) $1+2x$

Correct Answer: (C) $1+x^2$

Solution:

Step 1: Understanding the Question:
This question asks for the integrating factor of a first-order linear ordinary differential equation. The integrating factor is a function used to transform a non-exact differential equation into an exact one.

Step 2: Key Formula or Approach: A first-order linear differential equation has the standard form:

$$\frac{dy}{dx}+P(x)y=Q(x)$$

The integrating factor ($I.F.$) is calculated as:

$$I.F.=e^{\int P(x)dx}$$

Step 3: Detailed Explanation:

- Rewrite the given differential equation in standard form:

$$(1+x^2)\frac{dy}{dx}+2xy=4x^2$$

- Divide the entire equation by $(1+x^2)$:

$$\frac{dy}{dx}+\left(\frac{2x}{1+x^2}\right)y=\frac{4x^2}{1+x^2}$$

- Identify $P(x)$ by comparing it with the standard linear form:

$$P(x)=\frac{2x}{1+x^2}$$

- Integrate $P(x)$ with respect to x:

$$\int P(x)dx=\int \frac{2x}{1+x^2}dx$$

- Let $u=1+x^2$, which gives $du=2xdx$. The integral becomes:

$$\int \frac{du}{u}=\ln|u|=\ln(1+x^2)$$

- Calculate the integrating factor:

$$I.F.=e^{\int P(x)dx}=e^{\ln(1+x^2)}$$

- Since $e^{\ln(f(x))}=f(x)$, we get:

$$I.F.=1+x^2$$

Step 4: Final Answer:

The integrating factor is $1+x^2$, which corresponds to option (C).

Quick Tip: Notice that the left-hand side of the original equation $(1+x^2)\frac{dy}{dx}+2xy$ is already the exact derivative of $(1+x^2)y$ by the product rule.
This immediately shows that the integrating factor must be $1+x^2$.

---

115. The number of linearly independent solutions of the differential equation $\frac{d^4y}{dx^4}-\frac{d^3y}{dx^3}-3\frac{d^2y}{dx^2}+5\frac{dy}{dx}-2y=0$ of the form e^{ax} (a is a real number) is

(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: (B) 2

Solution:

Step 1: Understanding the Question:
This question asks for the number of linearly independent solutions of a fourth-order linear homogeneous differential equation that have the specific exponential form e^{ax}, where the parameter a must be a real number.

Step 2: Key Formula or Approach:
For a linear differential equation with constant coefficients, substituting $y=e^{mx}$ leads to a characteristic polynomial equation.
Each distinct real root m_i of this characteristic equation yields a linearly independent solution of the form e^{m_ix}.

Step 3: Detailed Explanation:

- Write the characteristic equation of the given differential equation:

$$m^4-m^3-3m^2+5m-2=0$$

- Let's find the roots of this polynomial. Test $m = 1$:

$$(1)^4 - (1)^3 - 3(1)^2 + 5(1) - 2 = 1 - 1 - 3 + 5 - 2 = 0$$

Since $m = 1$ is a root, $(m - 1)$ is a factor.

- Perform division to factor the polynomial:

$$m^4 - m^3 - 3m^2 + 5m - 2 = (m - 1)(m^3 - 3m + 2)$$

- Factor the cubic term $q(m) = m^3 - 3m + 2$. Test $m = 1$:

$$(1)^3 - 3(1) + 2 = 1 - 3 + 2 = 0$$

So $(m - 1)$ is a factor of $q(m)$ as well:

$$m^3 - 3m + 2 = (m - 1)(m^2 + m - 2)$$

- Factor the quadratic term:

$$m^2 + m - 2 = (m + 2)(m - 1)$$

- Putting all the factors together gives the complete factored characteristic equation:

$$(m - 1)^3(m + 2) = 0$$

- The roots are:
  $m = 1$ (multiplicity 3)
  $m = -2$ (multiplicity 1)

- The general solution to the differential equation is:

$$y(x) = (c_1 + c_2x + c_3x^2)e^x + c_4e^{-2x}$$

- The individual, distinct solutions of the form e^{ax} with real a are e^x (for $a = 1$) and e^{-2x} (for $a = -2$).

- Other independent solutions like xe^x and x^2e^x are not of the form e^{ax} because of the algebraic multipliers.

- Therefore, there are exactly 2 linearly independent solutions of the form e^{ax} with real a.

Step 4: Final Answer:

The number of linearly independent solutions of the specified form is 2, which corresponds to option (B).

Quick Tip: Be careful with the wording "of the form e^{ax}".

Even though $m = 1$ is a triple root and contributes three linearly independent terms to the general solution, only e^x is of the form e^{ax}.

The other terms (xe^x, x^2e^x) do not fit this form.

---

116. If the probability distribution of a random variable X is as follows.

x	1	2	3	4
$P(X = x)$	k	$4k$	$4k$	k

Then, the value of k is _____

(A) 0.1

(B) 0.2

(C) 0.3

(D) 0.4

Correct Answer: (A) 0.1

Solution:

Step 1: Understanding the Question:

This question requires determining the value of an unknown constant k in a given discrete probability distribution.

Step 2: Key Formula or Approach:

For any valid discrete probability distribution, the sum of all individual probabilities must be equal to 1:

$$\sum_i P(X = x_i) = 1$$

Step 3: Detailed Explanation:

- Identify the given probability values from the distribution table:
  $P(X = 1) = k$
  $P(X = 2) = 4k$
  $P(X = 3) = 4k$
  $P(X = 4) = k$

- Apply the total probability condition:

$$P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) = 1$$

- Substitute the values in terms of k into the equation:

$$k + 4k + 4k + k = 1$$

- Combine the terms on the left side:

$$10k = 1$$

- Solve for k:

$$k = \frac{1}{10} = 0.1$$

Step 4: Final Answer:

The value of k is 0.1, which corresponds to option (A).

Quick Tip: The total sum of probabilities in any probability distribution (discrete or continuous) must always equal 1.

This normalization condition is a fundamental axiom of probability theory.

---

117. The inverse Laplace transform of $F(s) = \frac{s^2-3s+4}{s^3}$ is _____

(A) $1-3t+2t^2$

(B) $1-3t+4t^2$

(C) $1+3t+2t^2$

(D) $1+3t+4t^2$

Correct Answer: (A) $1-3t+2t^2$

Solution:

Step 1: Understanding the Question:

This question asks for the inverse Laplace transform of a rational algebraic expression in the s-domain to find its corresponding time-domain function $f(t)$.

Step 2: Key Formula or Approach:

We use the standard inverse Laplace transform formulas:

$$\mathscr{L}^{-1}\left\{\frac{1}{s}\right\} = 1$$

$$\mathscr{L}^{-1}\left\{\frac{1}{s^2}\right\} = t$$

$$\mathscr{L}^{-1}\left\{\frac{1}{s^n}\right\} = \frac{t^{n-1}}{(n-1)!} \quad \text{for } n \geq 1$$

We also utilize the linearity property of the inverse Laplace transform.

Step 3: Detailed Explanation:

- Split the given fraction into simpler individual fractions:

$$F(s) = \frac{s^2 - 3s + 4}{s^3} = \frac{s^2}{s^3} - \frac{3s}{s^3} + \frac{4}{s^3}$$

$$F(s) = \frac{1}{s} - \frac{3}{s^2} + \frac{4}{s^3}$$

- Apply the linearity property to find $f(t) = \mathcal{L}^{-1}\{F(s)\}$:

$$f(t) = \mathcal{L}^{-1}\left\{\frac{1}{s}\right\} - 3\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} + 4\mathcal{L}^{-1}\left\{\frac{1}{s^3}\right\}$$

- Calculate each inverse Laplace transform:

  - $\mathcal{L}^{-1}\left\{\frac{1}{s}\right\} = 1$
  - $\mathcal{L}^{-1}\left\{\frac{1}{s^2}\right\} = t$
  - $\mathcal{L}^{-1}\left\{\frac{1}{s^3}\right\} = \frac{t^{3-1}}{(3-1)!} = \frac{t^2}{2} = 0.5t^2$

- Combine the terms:

$$f(t) = 1 - 3t + 4\left(\frac{t^2}{2}\right)$$

$$f(t) = 1 - 3t + 2t^2$$

Step 4: Final Answer:

The inverse Laplace transform is $1 - 3t + 2t^2$, which corresponds to option (A).

Quick Tip: Splitting rational functions into partial fractions or simpler components is almost always the easiest way to evaluate inverse Laplace transforms.

118. The general solution of $(x^2D^2 - xD)y = 0$ is

(A) $y = c_1 + c_2e^x$

(B) $y = c_1 + c_2x$

(C) $y = c_1 + c_2x^2$

(D) $y = c_1x + c_2x^2$

Correct Answer: (C) $y = c_1 + c_2x^2$

Solution:

Step 1: Understanding the Question:

This question asks for the general solution of a second-order, linear homogeneous differential equation with variable coefficients, specifically in the Cauchy-Euler form.

Step 2: Key Formula or Approach:

The given equation is:

$$x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$$

To solve a Cauchy-Euler equation, we use the transformation:

$$x = e^z \quad \Rightarrow \quad z = \ln x$$

This substitution converts the variable-coefficient ODE into a constant-coefficient ODE using the operator relations:

$$xD = D_z$$

$$x^2D^2 = D_z(D_z - 1)$$

Where $D_z = \frac{d}{dz}$.

Step 3: Detailed Explanation:

- Substitute the operator expressions into the differential equation:

$$[D_z(D_z-1)-D_z]y=0$$

$$[D_z^2-D_z-D_z]y=0$$

$$[D_z^2-2D_z]y=0$$

- Write the auxiliary equation by replacing the operator D_z with the variable m:

$$m^2-2m=0$$

- Factor the equation to find the roots:

$$m(m-2)=0$$

This gives the roots:

$m_1=0$

$m_2=2$

- Write the solution in terms of the intermediate variable z:

$$y(z)=c_1e^{0z}+c_2e^{2z}=c_1+c_2e^{2z}$$

- Convert the solution back to the original variable x using $e^z=x$:

$$y(x)=c_1+c_2(e^z)^2=c_1+c_2x^2$$

Step 4: Final Answer:

The general solution is $y=c_1+c_2x^2$, which corresponds to option (C).

Quick Tip: An alternative quick method is to assume a solution of the form $y=x^m$.

Substituting this directly gives: $m(m-1)-m=0\Rightarrow m^2-2m=0$, leading immediately to the roots $m=0,2$.

119. Two independent random variables X and Y have variances 0.2 and 0.5 respectively. Let Z = 5X - 2Y. The variance of Z is

(A) 3

(B) 7

(C) 4

(D) 5

Correct Answer: (B) 7

Solution:

Step 1: Understanding the Question:

This question asks for the variance of a new random variable Z which is defined as a linear combination of two independent random variables X and Y with known variances.

Step 2: Key Formula or Approach:

For any two independent random variables X and Y, the variance of their linear combination is given by:

$$\text{Var}(aX + bY) = a^2\text{Var}(X) + b^2\text{Var}(Y)$$

Where a and b are real constant coefficients.

Step 3: Detailed Explanation:

- List the given values:

$$\text{Var}(X) = 0.2$$

$$\text{Var}(Y) = 0.5$$

- The linear combination is defined as:

$$Z = 5X - 2Y$$

- Identify the coefficients:

$$a = 5, \quad b = -2$$

- Apply the variance formula:

$$\text{Var}(Z) = (5)^2\text{Var}(X) + (-2)^2\text{Var}(Y)$$

$$\text{Var}(Z) = 25\text{Var}(X) + 4\text{Var}(Y)$$

- Substitute the known values of the variances:

$$\text{Var}(Z) = 25(0.2) + 4(0.5)$$

- Calculate each term:

$$25 \times 0.2 = 5$$

$$4 \times 0.5 = 2$$

- Sum the results:

$$\text{Var}(Z) = 5 + 2 = 7$$

Step 4: Final Answer:

The variance of Z is 7, which corresponds to option (B).

Quick Tip: Remember that the variance of a difference is not the difference of the variances. Because coefficients are squared, we have $\text{Var}(aX - bY) = a^2\text{Var}(X) + b^2\text{Var}(Y)$. The negative sign becomes positive.

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120. Consider the function $f(x) = x^3 - x - 1$ for finding the root the equation $f(x) = 0$. If the initial approximation is $x_0 = 1$, then the next approximation x_1 using the Newton-Raphson method is

(A) 1.25

(B) 1.75

(C) 1.5

(D) 2

Correct Answer: (C) 1.5

Solution:

Step 1: Understanding the Question:

This question asks for the first iterative approximation x_1 of the root of the equation $f(x) = x^3 - x - 1 = 0$ using the Newton-Raphson numerical method, starting from an initial guess $x_0 = 1$.

Step 2: Key Formula or Approach:

The iterative formula for the Newton-Raphson method is:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

For the first iteration ($n = 0$), the formula is:

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Step 3: Detailed Explanation:

- Define the function and its first derivative:

$$f(x) = x^3 - x - 1$$

$$f'(x) = 3x^2 - 1$$

- Evaluate the function at the initial approximation $x_0 = 1$:

$$f(1) = (1)^3 - (1) - 1 = 1 - 1 - 1 = -1$$

- Evaluate the derivative at the initial approximation $x_0 = 1$:

$$f'(1) = 3(1)^2 - 1 = 3 - 1 = 2$$

- Substitute these values into the Newton-Raphson formula to find x_1:

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{-1}{2}$$

$$x_1 = 1 + \frac{1}{2} = 1.5$$

Step 4: Final Answer:

The next approximation x_1 is 1.5, which corresponds to option (C).

Quick Tip: The Newton-Raphson method converges quadratically when the initial guess is close to the actual root and the derivative $f'(x_0) \neq 0$.
Always double check the calculation of $f'(x)$ before substituting.
