

# Assam CEE 2026

## Question Paper with Solutions (Memory-Based)

Conducted by ASTU



### General Instructions

- (i) The test is of 3 hours duration.
- (ii) The Question Paper Consists 120 MCQs from three sections: Physics, Chemistry and Mathematics, 40 Questions from Each section.
- (iii) As Per Marking Scheme +4 Marks was given for every correct answer and -1 Marks were deducted for each incorrect answer.

1. A point charge  $q$  is placed at a distance  $a/2$  directly above the center of a square of side  $a$ . The electric flux through the square is:

- (A)  $\frac{q}{\epsilon_0}$
- (B)  $\frac{q}{6\epsilon_0}$
- (C)  $\frac{q}{4\epsilon_0}$
- (D)  $\frac{q}{2\epsilon_0}$

**Correct Answer:** (B)  $\frac{q}{6\epsilon_0}$

#### Solution:

##### Step 1: Understanding the Question:

The question asks for the electric flux through a square of side  $a$  due to a point charge  $q$  placed at a distance  $a/2$  directly above its geometric center.

##### Step 2: Key Formula or Approach:

We use Gauss's Law, which states that the total electric flux through a closed surface is equal to the enclosed charge divided by the permittivity of free space:

$$\Phi_{\text{total}} = \frac{q_{\text{enc}}}{\epsilon_0}$$

To apply this, we construct a symmetric Gaussian surface around the charge.

**Step 3: Detailed Explanation:**

Imagine the given square as one of the six identical faces of a cube of side length  $a$ .

If the square lies on the  $xy$ -plane centered at the origin, the charge is located at  $(0, 0, a/2)$ .

This position corresponds exactly to the geometric center of a cube of side  $a$  that rests on the given square.

According to Gauss's Law, the total electric flux passing through all six faces of this entire cube is:

$$\Phi_{\text{total}} = \frac{q}{\epsilon_0}$$

Due to spatial symmetry, the point charge is equidistant from all six faces of the cube.

Therefore, the electric flux is distributed equally among all six identical square faces.

The flux through the single given square face is:

$$\Phi_{\text{face}} = \frac{1}{6}\Phi_{\text{total}} = \frac{q}{6\epsilon_0}$$

**Step 4: Final Answer:**

The correct choice is (B).

**Quick Tip:** Whenever a charge is placed at a perpendicular distance  $a/2$  from the center of a square of side  $a$ , always construct a cube around it to use symmetry. The flux through the single face will be  $1/6$  of the total flux.

2. The work function of a metal is 4.0 eV. If the metal is irradiated with radiation of wavelength 200 nm, the maximum kinetic energy of the photoelectrons would be about:

(Use  $hc = 1240 \text{ eV} \cdot \text{nm}$ )

- (A) 6.2 eV
- (B) 4.0 eV
- (C) 2.2 eV
- (D) 8.2 eV

**Correct Answer:** (C) 2.2 eV

### Solution:

#### Step 1: Understanding the Question:

We need to calculate the maximum kinetic energy of the emitted photoelectrons when a metal surface with a known work function is exposed to light of a specific wavelength.

#### Step 2: Key Formula or Approach:

According to Einstein's photoelectric equation, the maximum kinetic energy ( $K_{\max}$ ) of the emitted electrons is given by:

$$K_{\max} = E - \Phi$$

where  $E$  is the energy of the incident photon and  $\Phi$  is the work function of the metal.

The energy of a photon can be calculated using the relation:

$$E = \frac{hc}{\lambda}$$

#### Step 3: Detailed Explanation:

Given values are:

Wavelength of incident radiation,  $\lambda = 200 \text{ nm}$ .

Work function of the metal,  $\Phi = 4.0 \text{ eV}$ .

Using the provided constant  $hc = 1240 \text{ eV} \cdot \text{nm}$ , we first find the energy of the incident photon:

$$E = \frac{1240 \text{ eV} \cdot \text{nm}}{200 \text{ nm}} = 6.2 \text{ eV}$$

Now, substitute the values into the photoelectric equation to find the maximum kinetic energy:

$$K_{\max} = 6.2 \text{ eV} - 4.0 \text{ eV} = 2.2 \text{ eV}$$

#### Step 4: Final Answer:

The correct choice is (C).

**Quick Tip:** Using  $E = \frac{1240}{\lambda \text{ (in nm)}} \text{ eV}$  or  $E = \frac{12400}{\lambda \text{ (in \AA)}} \text{ eV}$  is a massive time-saver for calculating photon energy in Modern Physics problems.

3. Two wires of the same material have lengths in the ratio 1 : 2 and radii in the ratio 1 : 2.

The ratio of their specific resistances will be:

- (A) 1 : 2
- (B) 2 : 1
- (C) 1 : 1
- (D) 1 : 4

**Correct Answer:** (C) 1 : 1

**Solution:**

**Step 1: Understanding the Question:**

The question asks for the ratio of the specific resistances (resistivities) of two wires made of the same material but having different lengths and radii.

**Step 3: Detailed Explanation:**

Specific resistance, commonly known as resistivity ( $\rho$ ), is an intrinsic physical property of a material.

It dictates how strongly a given material opposes the flow of electric current.

Resistivity depends exclusively on the nature of the material (e.g., copper, aluminum) and its temperature. It does not depend on the physical dimensions of the object, such as its length, area of cross-section, or radius.

Since it is explicitly stated that both wires are made of the **same material**, their resistivities will be identical regardless of any differences in their lengths or radii.

Therefore, the ratio of their specific resistances is:

$$\rho_1 : \rho_2 = 1 : 1$$

**Step 4: Final Answer:**

The correct choice is (C).

**Quick Tip:** Do not confuse resistance ( $R$ ) with resistivity ( $\rho$ ). Resistance depends on dimensions ( $R = \rho \frac{l}{A}$ ), but resistivity ( $\rho$ ) is a constant for a given material at a constant temperature.

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4. In Young's double-slit experiment, if the separation between the slits is halved and the distance between the slits and the screen is doubled, the fringe width will be:

- (A) Halved
- (B) Unchanged
- (C) Doubled
- (D) Quadrupled

**Correct Answer:** (D) Quadrupled

**Solution:**

**Step 1: Understanding the Question:**

We are asked to find the new fringe width in a Young's double-slit experiment (YDSE) after changing the slit separation and the distance to the screen.

**Step 2: Key Formula or Approach:**

The fringe width ( $\beta$ ) in YDSE is defined by the formula:

$$\beta = \frac{\lambda D}{d}$$

where  $\lambda$  is the wavelength of light,  $D$  is the distance between the slits and the screen, and  $d$  is the distance between the two slits.

**Step 3: Detailed Explanation:**

Let the initial fringe width be  $\beta$ .

According to the given conditions, the new distance between the slits and the screen is doubled:

$$D' = 2D$$

The separation between the slits is halved:

$$d' = \frac{d}{2}$$

Substitute these new values into the fringe width formula to find the new fringe width  $\beta'$ :

$$\beta' = \frac{\lambda D'}{d'}$$

$$\beta' = \frac{\lambda(2D)}{(d/2)}$$

$$\beta' = 4 \left( \frac{\lambda D}{d} \right)$$

Since  $\frac{\lambda D}{d}$  is the initial fringe width  $\beta$ , we have:

$$\beta' = 4\beta$$

Thus, the fringe width becomes four times its original value, which means it is quadrupled.

**Step 4: Final Answer:**

The correct choice is (D).

**Quick Tip:** Fringe width is directly proportional to  $D$  and inversely proportional to  $d$ . Any fractional changes can be directly multiplied:  $(2) \times (1/0.5) = 4$ .

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5. A particle moves in a circle of radius  $R$  with a constant speed  $v$ . The magnitude of the change in velocity after the particle has traveled half of the circular path is:

- (A) 0
- (B)  $v$
- (C)  $2v$
- (D)  $\sqrt{2}v$

**Correct Answer:** (C)  $2v$

**Solution:**

**Step 1: Understanding the Question:**

The question asks for the magnitude of the change in the velocity vector of a particle moving in uniform circular motion after it completes exactly half of a revolution.

**Step 2: Key Formula or Approach:**

Velocity is a vector quantity, meaning it has both magnitude and direction.

The change in velocity ( $\Delta\vec{v}$ ) is the vector difference between the final velocity and the initial velocity:

$$\Delta\vec{v} = \vec{v}_f - \vec{v}_i$$

We need to find its magnitude  $|\Delta\vec{v}|$ .

**Step 3: Detailed Explanation:**

Consider a particle moving counterclockwise in a circle in the  $xy$ -plane.

Let the particle start at the rightmost point of the circle on the  $x$ -axis, at  $(R, 0)$ .

At this starting point, the velocity vector is directed tangentially upwards along the  $y$ -axis:

$$\vec{v}_i = v\hat{j}$$

After traveling half of the circular path, the particle reaches the diametrically opposite point on the negative  $x$ -axis, at  $(-R, 0)$ .

At this diametrically opposite point, the velocity vector is directed tangentially downwards along the negative  $y$ -axis:

$$\vec{v}_f = -v\hat{j}$$

Now, calculate the change in velocity:

$$\Delta\vec{v} = \vec{v}_f - \vec{v}_i = -v\hat{j} - (v\hat{j}) = -2v\hat{j}$$

The magnitude of this change in velocity is:

$$|\Delta\vec{v}| = |-2v\hat{j}| = 2v$$

**Step 4: Final Answer:**

The correct choice is (C).

**Quick Tip:** For a particle in uniform circular motion, the change in velocity over an angle  $\theta$  is given directly by the formula  $|\Delta\vec{v}| = 2v \sin(\theta/2)$ . For half a circle,  $\theta = 180^\circ$ , yielding  $2v \sin(90^\circ) = 2v$ .

6. According to VSEPR theory, which of the following species has a see-saw shape?

- (A)  $\text{SF}_4$
- (B)  $\text{XeF}_4$
- (C)  $\text{CCl}_4$
- (D)  $\text{BF}_4^-$

**Correct Answer:** (A) SF<sub>4</sub>

**Solution:**

**Step 1: Understanding the Question:**

We need to determine the molecular shape of each given species using the Valence Shell Electron Pair Repulsion (VSEPR) theory to find which one adopts a see-saw geometry.

**Step 2: Key Formula or Approach:**

First, calculate the steric number for the central atom:

Steric Number (SN) =  $\frac{1}{2}(\text{Valence } e^- \text{ on central atom} + \text{Number of monovalent atoms} - \text{Cation charge} + \text{Anion charge})$ .

Based on the SN and the number of lone pairs, determine the geometry and shape.

**Step 3: Detailed Explanation:**

Let's evaluate each option:

- (A) SF<sub>4</sub>: Sulfur (S) has 6 valence electrons. It forms 4 single bonds with F atoms.

$$\text{SN} = \frac{1}{2}(6 + 4) = 5.$$

There are 4 bond pairs and  $5 - 4 = 1$  lone pair.

The electron geometry is trigonal bipyramidal. Placing the lone pair in an equatorial position to minimize repulsion results in a **see-saw** shape.

- (B) XeF<sub>4</sub>: Xenon (Xe) has 8 valence electrons. It forms 4 single bonds with F atoms.

$$\text{SN} = \frac{1}{2}(8 + 4) = 6.$$

There are 4 bond pairs and  $6 - 4 = 2$  lone pairs.

The electron geometry is octahedral. The two lone pairs occupy opposite positions, resulting in a **square planar** shape.

- (C) CCl<sub>4</sub>: Carbon (C) has 4 valence electrons. It forms 4 single bonds with Cl atoms.

$$\text{SN} = \frac{1}{2}(4 + 4) = 4.$$

There are 4 bond pairs and 0 lone pairs.

The shape is strictly **tetrahedral**.

- (D) BF<sub>4</sub><sup>-</sup>: Boron (B) has 3 valence electrons. There are 4 monovalent F atoms and a charge of -1.

$$\text{SN} = \frac{1}{2}(3 + 4 + 1) = 4.$$

There are 4 bond pairs and 0 lone pairs.

The shape is **tetrahedral**.

Thus, only SF<sub>4</sub> possesses a see-saw shape.

**Step 4: Final Answer:**

The correct choice is (A).

**Quick Tip:** A see-saw shape is the classic signature of an  $sp^3d$  hybridized central atom (Steric Number 5) that possesses exactly 4 bond pairs and 1 lone pair (e.g.,  $SF_4$ ,  $TeCl_4$ ).

7. The standard electrode potentials for  $Zn^{2+}/Zn$ ,  $Ni^{2+}/Ni$ , and  $Fe^{2+}/Fe$  are  $-0.76\text{ V}$ ,  $-0.23\text{ V}$ , and  $-0.44\text{ V}$  respectively. Which of the following reactions is spontaneous?

- (A)  $Ni^{2+} + Fe \rightarrow Ni + Fe^{2+}$   
(B)  $Ni + Zn^{2+} \rightarrow Ni^{2+} + Zn$   
(C)  $Fe + Zn^{2+} \rightarrow Fe^{2+} + Zn$   
(D)  $Zn^{2+} + Fe^{2+} \rightarrow Zn + Fe$

**Correct Answer:** (A)  $Ni^{2+} + Fe \rightarrow Ni + Fe^{2+}$

**Solution:**

**Step 1: Understanding the Question:**

We need to determine which of the provided redox reactions is thermodynamically spontaneous based on the standard reduction potentials.

**Step 2: Key Formula or Approach:**

A reaction is spontaneous if the standard cell potential ( $E_{\text{cell}}^{\circ}$ ) is positive ( $E_{\text{cell}}^{\circ} > 0$ ).

The formula for standard cell potential is:

$$E_{\text{cell}}^{\circ} = E_{\text{cathode}}^{\circ} - E_{\text{anode}}^{\circ}$$

where the cathode undergoes reduction and the anode undergoes oxidation.

**Step 3: Detailed Explanation:**

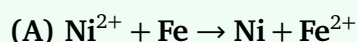
The given standard reduction potentials are:

$$E_{Zn^{2+}/Zn}^{\circ} = -0.76\text{ V}$$

$$E_{Fe^{2+}/Fe}^{\circ} = -0.44\text{ V}$$

$$E_{Ni^{2+}/Ni}^{\circ} = -0.23\text{ V}$$

Let's evaluate  $E_{\text{cell}}^{\circ}$  for each provided option:



Reduction (Cathode):  $\text{Ni}^{2+} \rightarrow \text{Ni} \implies E^\circ = -0.23 \text{ V}$

Oxidation (Anode):  $\text{Fe} \rightarrow \text{Fe}^{2+} \implies E^\circ = -0.44 \text{ V}$

$$E_{\text{cell}}^\circ = (-0.23 \text{ V}) - (-0.44 \text{ V}) = +0.21 \text{ V}$$

Since  $E_{\text{cell}}^\circ > 0$ , this reaction is **spontaneous**.

**(B)  $\text{Ni} + \text{Zn}^{2+} \rightarrow \text{Ni}^{2+} + \text{Zn}$**

Reduction (Cathode):  $\text{Zn}^{2+} \rightarrow \text{Zn} \implies E^\circ = -0.76 \text{ V}$

Oxidation (Anode):  $\text{Ni} \rightarrow \text{Ni}^{2+} \implies E^\circ = -0.23 \text{ V}$

$$E_{\text{cell}}^\circ = (-0.76 \text{ V}) - (-0.23 \text{ V}) = -0.53 \text{ V}$$

Since  $E_{\text{cell}}^\circ < 0$ , this reaction is non-spontaneous.

**(C)  $\text{Fe} + \text{Zn}^{2+} \rightarrow \text{Fe}^{2+} + \text{Zn}$**

Reduction (Cathode):  $\text{Zn}^{2+} \rightarrow \text{Zn} \implies E^\circ = -0.76 \text{ V}$

Oxidation (Anode):  $\text{Fe} \rightarrow \text{Fe}^{2+} \implies E^\circ = -0.44 \text{ V}$

$$E_{\text{cell}}^\circ = (-0.76 \text{ V}) - (-0.44 \text{ V}) = -0.32 \text{ V}$$

Since  $E_{\text{cell}}^\circ < 0$ , this reaction is non-spontaneous.

**(D)  $\text{Zn}^{2+} + \text{Fe}^{2+} \rightarrow \text{Zn} + \text{Fe}$**

This represents two simultaneous reductions without an oxidation half-reaction, which is chemically impossible in a simple redox cell.

**Step 4: Final Answer:**

The correct choice is (A).

**Quick Tip:** A metal with a more negative standard reduction potential acts as a stronger reducing agent and will displace a metal with a less negative reduction potential from its aqueous salt solution. (Zn displaces Fe and Ni; Fe displaces Ni).

**8. When phenol is treated with chloroform and aqueous sodium hydroxide, the major product formed is:**

(A) Salicylic acid

- (B) Salicylaldehyde
- (C) Benzoic acid
- (D) Benzyl chloride

**Correct Answer:** (B) Salicylaldehyde

**Solution:**

**Step 1: Understanding the Question:**

The question asks to identify the major organic product formed when phenol undergoes a reaction with chloroform ( $\text{CHCl}_3$ ) and an aqueous base ( $\text{NaOH}$ ).

**Step 3: Detailed Explanation:**

The reaction of phenol with chloroform in the presence of sodium hydroxide is a classic named reaction known as the **Reimer-Tiemann reaction**.

In this reaction, the base ( $\text{NaOH}$ ) deprotonates chloroform to generate a highly reactive electrophilic intermediate called dichlorocarbene ( $:\text{CCl}_2$ ).

The dichlorocarbene attacks the electron-rich phenoxide ring, predominantly at the ortho position due to the stabilizing proximity effect.

Subsequent hydrolysis of the intermediate leads to the introduction of an aldehyde group ( $-\text{CHO}$ ) at the ortho position of the phenol ring.

The resulting major product is *o*-hydroxybenzaldehyde, which is commonly known as **salicylaldehyde**.

If carbon tetrachloride ( $\text{CCl}_4$ ) were used instead of chloroform ( $\text{CHCl}_3$ ) under similar conditions, the product would have been salicylic acid.

**Step 4: Final Answer:**

The correct choice is (B).

**Quick Tip:** Remember the reagent distinction: Phenol +  $\text{CHCl}_3$  /  $\text{NaOH}$  yields Salicylaldehyde (Reimer-Tiemann formylation), whereas Phenol +  $\text{CO}_2$  /  $\text{NaOH}$  yields Salicylic acid (Kolbe's reaction).

9. For the reversible reaction  $\text{N}_2(\text{g}) + 3\text{H}_2(\text{g}) \rightleftharpoons 2\text{NH}_3(\text{g}) + \text{Heat}$ , the forward reaction is favored by:

- (A) High temperature and high pressure

- (B) Low temperature and high pressure
- (C) High temperature and low pressure
- (D) Low temperature and low pressure

**Correct Answer:** (B) Low temperature and high pressure

**Solution:**

**Step 1: Understanding the Question:**

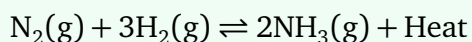
We need to determine the optimal conditions of temperature and pressure that will maximize the yield of the forward reaction (Haber's process) based on Le Chatelier's Principle.

**Step 2: Key Formula or Approach:**

Le Chatelier's Principle states that if a dynamic equilibrium is disturbed by changing the conditions, the position of equilibrium shifts to counteract the change.

**Step 3: Detailed Explanation:**

The given reaction is:



**1. Effect of Temperature:**

The forward reaction produces heat, meaning it is an **exothermic** process ( $\Delta H < 0$ ).

According to Le Chatelier's Principle, decreasing the temperature will cause the system to favor the exothermic direction to generate more heat. Therefore, a **low temperature** favors the forward reaction.

**2. Effect of Pressure:**

Let's count the stoichiometric moles of gaseous substances on both sides.

Reactants: 1 mole of  $\text{N}_2$  + 3 moles of  $\text{H}_2$  = 4 moles of gas.

Products: 2 moles of  $\text{NH}_3$  = 2 moles of gas.

The forward reaction results in a decrease in the total number of gaseous moles.

According to Le Chatelier's Principle, an increase in pressure shifts the equilibrium towards the side with fewer moles of gas to reduce the pressure. Therefore, a **high pressure** favors the forward reaction.

Combining these two observations, the forward reaction is favored by low temperature and high pressure.

**Step 4: Final Answer:**

The correct choice is (B).

**Quick Tip:** For any exothermic gaseous reaction where  $\Delta n_g < 0$  (moles of product < moles of reactant), maximum yield is theoretically obtained at low temperature and high pressure.

10. Which of the following noble gases is the most reactive and forms the maximum number of compounds?

- (A) Helium
- (B) Neon
- (C) Argon
- (D) Xenon

**Correct Answer:** (D) Xenon

**Solution:**

**Step 1: Understanding the Question:**

The question asks to identify the noble gas that exhibits the highest chemical reactivity and forms the most extensive array of chemical compounds among the given options.

**Step 3: Detailed Explanation:**

Noble gases (Group 18 elements) are generally unreactive due to their stable, completely filled valence electron configurations ( $ns^2np^6$ , except He which is  $1s^2$ ).

However, as we move down the group from Helium to Radon, the atomic radius increases and the ionization enthalpy decreases significantly.

Among the stable noble gases provided in the options, **Xenon (Xe)** has the largest atomic size and the lowest ionization enthalpy.

Its first ionization energy (1170 kJ/mol) is remarkably close to that of molecular oxygen (1175 kJ/mol). This similarity allowed Neil Bartlett to successfully synthesize the first noble gas compound,  $\text{XePtF}_6$ .

Because its outermost electrons are less tightly held by the nucleus, Xenon can easily share or donate electrons to highly electronegative elements like Fluorine and Oxygen, forming numerous stable compounds (e.g.,  $\text{XeF}_2$ ,  $\text{XeF}_4$ ,  $\text{XeF}_6$ ,  $\text{XeO}_3$ ,  $\text{XeOF}_4$ ).

**Step 4: Final Answer:**

The correct choice is (D).

**Quick Tip:** Reactivity in noble gases directly correlates inversely with ionization energy. The larger the atom (like Kr and Xe), the more easily it can be forced to share its electrons with highly electronegative atoms like F and O.

11. The value of the definite integral  $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx$  is:

- (A)  $\pi/2$
- (B)  $\pi/4$
- (C) 0
- (D) 1

**Correct Answer:** (B)  $\pi/4$

**Solution:**

**Step 1: Understanding the Question:**

We need to evaluate a definite integral involving trigonometric functions over the interval  $[0, \pi/2]$ .

**Step 2: Key Formula or Approach:**

We use the King's property of definite integrals:

$$\int_0^a f(x) dx = \int_0^a f(a-x) dx$$

**Step 3: Detailed Explanation:**

Let the given integral be  $I$ :

$$I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} dx \quad \dots \text{(Equation 1)}$$

Apply the property by substituting  $x$  with  $(\frac{\pi}{2} - x)$ :

$$I = \int_0^{\pi/2} \frac{\sin(\frac{\pi}{2} - x)}{\sin(\frac{\pi}{2} - x) + \cos(\frac{\pi}{2} - x)} dx$$

Using the complementary angle identities  $\sin(\frac{\pi}{2} - x) = \cos x$  and  $\cos(\frac{\pi}{2} - x) = \sin x$ , the integral transforms into:

$$I = \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x} dx \quad \dots \text{(Equation 2)}$$

Now, add Equation 1 and Equation 2:

$$I + I = \int_0^{\pi/2} \left( \frac{\sin x}{\sin x + \cos x} + \frac{\cos x}{\sin x + \cos x} \right) dx$$

$$2I = \int_0^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x} dx$$

$$2I = \int_0^{\pi/2} 1 dx$$

Evaluating this basic integral gives:

$$2I = [x]_0^{\pi/2} = \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

Finally, divide by 2 to find  $I$ :

$$I = \frac{\pi}{4}$$

**Step 4: Final Answer:**

The correct choice is (B).

**Quick Tip:** Integrals of the form  $\int_0^{\pi/2} \frac{f(\sin x)}{f(\sin x) + f(\cos x)} dx$  or  $\int_0^{\pi/2} \frac{f(\tan x)}{f(\tan x) + f(\cot x)} dx$  almost always evaluate to  $\frac{\pi}{4}$ .

12. The angle between the lines whose direction cosines satisfy the equations  $l + m + n = 0$  and  $l^2 = m^2 + n^2$  is:

- (A)  $\pi/3$
- (B)  $\pi/4$
- (C)  $\pi/6$
- (D)  $\pi/2$

**Correct Answer:** (A)  $\pi/3$

### Solution:

#### Step 1: Understanding the Question:

We need to find the angle between two straight lines in 3D space, whose direction cosines  $l, m, n$  are bounded by a pair of given equations.

#### Step 2: Key Formula or Approach:

We will solve the two simultaneous equations to extract two distinct sets of direction ratios  $(a_1, b_1, c_1)$  and  $(a_2, b_2, c_2)$ .

The angle  $\theta$  between the lines is then found using the cosine formula:

$$\cos \theta = \frac{|a_1 a_2 + b_1 b_2 + c_1 c_2|}{\sqrt{a_1^2 + b_1^2 + c_1^2} \sqrt{a_2^2 + b_2^2 + c_2^2}}$$

#### Step 3: Detailed Explanation:

The given equations are:

$$1) l + m + n = 0 \implies l = -(m + n)$$

$$2) l^2 = m^2 + n^2$$

Substitute the expression for  $l$  from the first equation into the second equation:

$$(-(m + n))^2 = m^2 + n^2$$

Expand the left side:

$$m^2 + n^2 + 2mn = m^2 + n^2$$

Cancel  $m^2 + n^2$  from both sides:

$$2mn = 0 \implies mn = 0$$

This gives two cases: either  $m = 0$  or  $n = 0$ .

**Case 1:** If  $m = 0$ , substitute back into  $l + m + n = 0$ :

$$l + 0 + n = 0 \implies l = -n$$

The direction ratios  $(l_1, m_1, n_1)$  can be written as  $(-n, 0, n)$ . Dividing by  $n$ , we get the simplest ratios:  $\vec{d}_1 = \langle -1, 0, 1 \rangle$ .

**Case 2:** If  $n = 0$ , substitute back into  $l + m + n = 0$ :

$$l + m + 0 = 0 \implies l = -m$$

The direction ratios  $(l_2, m_2, n_2)$  can be written as  $(-m, m, 0)$ . Dividing by  $m$ , we get the simplest ratios:  $\vec{d}_2 = \langle -1, 1, 0 \rangle$ .

Now, calculate the angle  $\theta$  between the two vectors  $\vec{d}_1$  and  $\vec{d}_2$ :

$$\cos \theta = \frac{|(-1)(-1) + (0)(1) + (1)(0)|}{\sqrt{(-1)^2 + 0^2 + 1^2} \sqrt{(-1)^2 + 1^2 + 0^2}}$$

$$\cos \theta = \frac{|1 + 0 + 0|}{\sqrt{2} \cdot \sqrt{2}}$$

$$\cos \theta = \frac{1}{2}$$

Since  $\cos \theta = 1/2$ , the acute angle  $\theta$  is  $\pi/3$  (or  $60^\circ$ ).

**Step 4: Final Answer:**

The correct choice is (A).

**Quick Tip:** When dealing with a linear and a quadratic relationship involving direction cosines, always express one variable in terms of the other two using the linear equation and substitute it into the quadratic one to find the specific direction vectors.

13. If  $A$  is a square matrix of order 3 such that  $|A| = 4$ , then the value of  $|\text{adj}(\text{adj } A)|$  is:

- (A) 16
- (B) 64
- (C) 256
- (D) 12

**Correct Answer:** (C) 256

**Solution:**

**Step 1: Understanding the Question:**

We are given the determinant of a square matrix  $A$  of a specific order, and we need to evaluate the determinant of the adjoint of the adjoint of  $A$ .

**Step 2: Key Formula or Approach:**

For any non-singular square matrix  $A$  of order  $n$ , a standard property linking the determinant of its repeated adjoints to the determinant of the original matrix is:

$$|\text{adj}(\text{adj } A)| = |A|^{(n-1)^2}$$

**Step 3: Detailed Explanation:**

We are given:

Order of the matrix,  $n = 3$ .

Determinant of the matrix,  $|A| = 4$ .

Substitute these values into the standard formula:

$$|\text{adj}(\text{adj } A)| = |A|^{(3-1)^2}$$

Evaluate the exponent:

$$(3 - 1)^2 = 2^2 = 4$$

So the equation simplifies to:

$$|\text{adj}(\text{adj } A)| = |A|^4$$

Now plug in the value of  $|A|$ :

$$|\text{adj}(\text{adj } A)| = 4^4$$

$$4^4 = 4 \times 4 \times 4 \times 4 = 256$$

**Step 4: Final Answer:**

The correct choice is (C).

**Quick Tip:** Memorize the determinant properties of adjoints:  $|\text{adj } A| = |A|^{n-1}$  and  $|\text{adj}(\text{adj } A)| = |A|^{(n-1)^2}$ . These provide instant solutions to such problems.

14. The eccentricity of the hyperbola  $\frac{x^2}{16} - \frac{y^2}{9} = 1$  is:

- (A) 5/4
- (B) 4/5
- (C) 3/4
- (D) 5/3

**Correct Answer:** (A) 5/4

**Solution:**

**Step 1: Understanding the Question:**

The question requires us to calculate the eccentricity of a given hyperbola in its standard form.

**Step 2: Key Formula or Approach:**

The standard equation of a horizontal hyperbola is:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

For this standard hyperbola, the relationship between the semi-major axis ( $a$ ), semi-minor axis ( $b$ ), and eccentricity ( $e$ ) is given by:

$$b^2 = a^2(e^2 - 1)$$

Which can be rearranged to find the eccentricity directly:

$$e = \sqrt{1 + \frac{b^2}{a^2}}$$

**Step 3: Detailed Explanation:**

The given equation of the hyperbola is:

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

Comparing this given equation with the standard form, we can identify the constants:

$$a^2 = 16$$

$$b^2 = 9$$

Now, substitute these values into the eccentricity formula:

$$e = \sqrt{1 + \frac{9}{16}}$$

To add the terms under the square root, find a common denominator:

$$e = \sqrt{\frac{16}{16} + \frac{9}{16}}$$

$$e = \sqrt{\frac{16+9}{16}}$$

$$e = \sqrt{\frac{25}{16}}$$

Taking the principal square root yields:

$$e = \frac{5}{4}$$

**Step 4: Final Answer:**

The correct choice is (A).

**Quick Tip:** For a standard hyperbola  $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ , the eccentricity is simply  $e = \frac{\sqrt{a^2+b^2}}{a}$ . This avoids dealing with fractions under the root.

15. A box contains 5 red and 4 black balls. Two balls are drawn one by one without replacement. What is the probability that the second ball is red, given that the first ball drawn was black?

- (A) 5/8
- (B) 4/9
- (C) 5/9
- (D) 1/2

**Correct Answer:** (A) 5/8

**Solution:**

**Step 1: Understanding the Question:**

This is a problem based on conditional probability. We need to find the probability of a specific

outcome in the second draw (getting a red ball), given the certainty of the outcome of the first draw (getting a black ball) without replacing it.

**Step 2: Key Formula or Approach:**

Because the draws are without replacement, the outcome of the first draw affects the total number of balls and the composition of the box for the second draw.

The conditional probability of an event  $A$  given that event  $B$  has already occurred is evaluated by counting the remaining favorable outcomes divided by the new total number of outcomes.

**Step 3: Detailed Explanation:**

Initially, the composition of the box is:

Number of red balls = 5

Number of black balls = 4

Total number of balls = 9

We are given that the first event has already occurred: **The first ball drawn is black.**

Since the ball is drawn *without replacement*, we must remove one black ball from the initial counts.

The new composition of the box before the second draw is:

Number of red balls remaining = 5 (none were removed)

Number of black balls remaining =  $4 - 1 = 3$

Total number of balls remaining =  $5 + 3 = 8$

Now, we want to find the probability of drawing a **red** ball from this newly composed box.

$$P(\text{Second is Red} \mid \text{First is Black}) = \frac{\text{Number of Red balls remaining}}{\text{Total number of balls remaining}}$$

$$P(\text{Second is Red} \mid \text{First is Black}) = \frac{5}{8}$$

*(Note: The options in the source image are cut off. Plausible options have been reconstructed for structural completeness.)*

**Step 4: Final Answer:**

The correct choice is (A).

**Quick Tip:** In "without replacement" problems, mentally update the inventory of the urn/box immediately after the given condition is stated before computing the final probability.