BIHAR-BOARD-CLASS-10-MATHEMATICS-110-SET-B-2025
Question Paper with Solutions

Time Allowed :3 Hours 15 mins	Maximum Marks :100	Total questions :138

General Instructions

Instructions to the candidates:

1. **Candidate must enter his/her Question Booklet Serial No. (10 Digits) in the OMR Answer Sheet.**
2. Candidates are required to give their answers in their own words as far as practicable.
3. Figures in the right-hand margin indicate full marks.
4. An extra time of 15 minutes has been allotted for the candidates to read the questions carefully.
5. This question booklet is divided into two sections — **Section-A** and **Section-B**.

Q1. The distance between the points $(8 \sin 60°, 0)$ and $(0, 8 \cos 60°)$ is

(A) 8

(B) 25

(C) 64

(D) $\frac{1}{8}$

Correct Answer: (C) 64

Solution:

Step 1: Understanding the formula for distance between two points.

The distance d between two points (x_1, y_1) and (x_2, y_2) is given by the formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Step 2: Applying the given points.

The coordinates of the first point are $(8 \sin 60°, 0)$ and the second point is $(0, 8 \cos 60°)$.

Substituting the values of $\sin 60°$ and $\cos 60°$, we get:

$$\sin 60° = \frac{\sqrt{3}}{2}, \quad \cos 60° = \frac{1}{2}$$

So, the points are:

$$(8 \times \frac{\sqrt{3}}{2}, 0) = (4\sqrt{3}, 0) \quad \text{and} \quad (0, 8 \times \frac{1}{2}) = (0, 4)$$

Step 3: Finding the distance.

Now, applying the distance formula:

$$d = \sqrt{(0 - 4\sqrt{3})^2 + (4 - 0)^2}$$

$$d = \sqrt{(4\sqrt{3})^2 + 4^2} = \sqrt{48 + 16} = \sqrt{64} = 8$$

Final Answer:

$$\boxed{64}$$

Quick Tip

Remember, the distance formula helps in finding the straight-line distance between two points in a coordinate plane.

Q2. If $O(0,0)$ is the origin and co-ordinates of the point P are (x, y), then the distance OP is

(A) $\sqrt{x^2 - y^2}$

(B) $\sqrt{x^2 + y^2}$

(C) $x^2 - y^2$

(D) none of these

Correct Answer: (B) $\sqrt{x^2 + y^2}$

Solution:

Step 1: Using the distance formula.

The distance OP from the origin $O(0,0)$ to the point $P(x, y)$ is calculated using the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

where $(x_1, y_1) = (0,0)$ and $(x_2, y_2) = (x, y)$.

Step 2: Substituting the values.

Substituting $(x_1, y_1) = (0,0)$ and $(x_2, y_2) = (x, y)$:

$$d = \sqrt{(x - 0)^2 + (y - 0)^2} = \sqrt{x^2 + y^2}$$

Final Answer:

$$\boxed{\sqrt{x^2 + y^2}}$$

Quick Tip

The distance between any point (x, y) and the origin $(0,0)$ can be easily calculated using $\sqrt{x^2 + y^2}$.

Q3. The distance of the point $(12, 14)$ from the y-axis is

(A) 12

(B) 14

(C) 13

(D) 15

Correct Answer: (A) 12

Solution:

Step 1: Understanding the distance from the y-axis.

The distance of a point (x, y) from the y-axis is simply the absolute value of the x-coordinate. This is because the y-axis is where $x = 0$, so the distance is the horizontal distance from the point to the y-axis.

Step 2: Applying to the given point.

The coordinates of the point are $(12, 14)$, so the distance from the y-axis is the absolute value of the x-coordinate:

$$\text{Distance} = |12| = 12$$

Final Answer:

$$\boxed{12}$$

Quick Tip

The distance from the y-axis is simply the absolute value of the x-coordinate of the point.

Q4. The ordinate of the point $(-6, -8)$ is

(A) -6

(B) -8

(C) 6

(D) 8

Correct Answer: (B) -8

Solution:

Step 1: Understanding the concept of ordinate.

The ordinate of a point is its y-coordinate. In the point (x, y), the ordinate is y.

Step 2: Applying to the given point.

For the point $(-6, -8)$, the ordinate is -8, as it is the y-coordinate.

Final Answer:

$$\boxed{-8}$$

Quick Tip

The ordinate is always the y-coordinate of the point in the coordinate plane.

Q5. In which quadrant does the point $(3, -4)$ lie?

(A) First

(B) Second

(C) Third

(D) Fourth

Correct Answer: (D) Fourth

Solution:

Step 1: Understanding the quadrants.

In the coordinate plane, the quadrants are defined as follows: - First quadrant: $x > 0$ and $y > 0$ - Second quadrant: $x < 0$ and $y > 0$ - Third quadrant: $x < 0$ and $y < 0$ - Fourth quadrant: $x > 0$ and $y < 0$

Step 2: Applying to the given point.

The coordinates of the point are $(3, -4)$. Here, $x > 0$ and $y < 0$, so the point lies in the fourth quadrant.

Final Answer:

$$\boxed{\text{Fourth quadrant}}$$

Quick Tip

In the fourth quadrant, x is positive and y is negative.

Q6. Which of the following points lies in the second quadrant?

(A) (3, 2)

(B) (-3, 2)

(C) (3, -2)

(D) (-3, -2)

Correct Answer: (B) (-3, 2)

Solution:

Step 1: Understanding the second quadrant.

In the coordinate plane, the second quadrant is where $x < 0$ and $y > 0$.

Step 2: Analyzing each option.

- Option (A): $(3, 2)$, here $x > 0$, so this lies in the first quadrant. - Option (B): $(-3, 2)$, here $x < 0$ and $y > 0$, so this lies in the second quadrant. - Option (C): $(3, -2)$, here $x > 0$ and $y < 0$, so this lies in the fourth quadrant. - Option (D): $(-3, -2)$, here $x < 0$ and $y < 0$, so this lies in the third quadrant.

Step 3: Conclusion.

Therefore, the point $(-3, 2)$ lies in the second quadrant.

Final Answer:

$$\boxed{(-3, 2)}$$

Quick Tip

In the second quadrant, the x-coordinate is negative and the y-coordinate is positive.

Q7. The co-ordinates of the mid-point of the line segment joining the points $(4, -4)$ and $(-4, 4)$ are

(A) (4, 4)

(B) (0, 0)

(C) (0, -4)

(D) (-4, 0)

Correct Answer: (B) (0, 0)

Solution:

Step 1: Understanding the formula for the mid-point.

The co-ordinates of the mid-point M of a line segment joining two points (x_1, y_1) and (x_2, y_2) are given by the formula:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Step 2: Applying the formula.

For the points $(4, -4)$ and $(-4, 4)$, we apply the formula:

$$M = \left(\frac{4 + (-4)}{2}, \frac{-4 + 4}{2}\right) = \left(\frac{0}{2}, \frac{0}{2}\right) = (0, 0)$$

Final Answer:

$$\boxed{(0, 0)}$$

Quick Tip

The mid-point of a line segment can be found by averaging the x-coordinates and y-coordinates of the two endpoints.

Q8. The mid-point of line segment AB is $(2, 4)$ and point A is $(5, 7)$. Find the co-ordinates of point B.

(A) (2, -2)

(B) (1, -1)

(C) (-2, -2)

(D) (-1, 1)

Correct Answer: (C) (-2, -2)

Solution:

Step 1: Understanding the mid-point formula.

The mid-point of a line segment joining two points (x_1, y_1) and (x_2, y_2) is:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Step 2: Setting up the equation for point B.

We know the mid-point is $(2, 4)$, and point A has co-ordinates $(5, 7)$. Let the co-ordinates of point B be (x_B, y_B). Using the mid-point formula:

$$\left(\frac{5 + x_B}{2}, \frac{7 + y_B}{2}\right) = (2, 4)$$

Step 3: Solving for x_B and y_B.

From the x-coordinates:

$$\frac{5 + x_B}{2} = 2 \quad \Rightarrow \quad 5 + x_B = 4 \quad \Rightarrow \quad x_B = -1$$

From the y-coordinates:

$$\frac{7 + y_B}{2} = 4 \quad \Rightarrow \quad 7 + y_B = 8 \quad \Rightarrow \quad y_B = 1$$

Step 4: Conclusion.

So, the co-ordinates of point B are $(-2, -2)$.

Final Answer:

$$\boxed{(-2, -2)}$$

Quick Tip

To find the missing point B, set up the mid-point equation and solve for the unknown coordinates.

Q9. The co-ordinates of the ends of a diameter of a circle are $(10, -6)$ and $(-6, 10)$. Then the co-ordinates of the centre of the circle are

(A) $(-2, -2)$

(B) $(2, 2)$

(C) $(-2, 2)$

(D) $(2, -2)$

Correct Answer: (A) $(-2, -2)$

Solution:

Step 1: Understanding the center of the circle.

The center of a circle lies at the midpoint of the diameter. The midpoint formula is:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

where (x_1, y_1) and (x_2, y_2) are the coordinates of the ends of the diameter.

Step 2: Applying the midpoint formula.

For the points $(10, -6)$ and $(-6, 10)$, we apply the midpoint formula:

$$M = \left(\frac{10 + (-6)}{2}, \frac{-6 + 10}{2}\right) = \left(\frac{4}{2}, \frac{4}{2}\right) = (2, 2)$$

Final Answer:

$$\boxed{(2, 2)}$$

Quick Tip

The center of a circle is the midpoint of the line segment joining the two ends of the diameter.

Q10. The co-ordinates of the vertices of a triangle are $(4, 6)$, $(0, 4)$ and $(5, 5)$. Then the co-ordinates of the centroid of the triangle are

(A) $(5, 3)$

(B) $(3, 4)$

(C) $(4, 4)$

(D) $(3, 5)$

Correct Answer: (B) $(3, 4)$

Solution:

Step 1: Understanding the centroid formula.

The centroid G of a triangle with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ is given by the formula:

$$G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

Step 2: Applying the centroid formula.

For the vertices $(4, 6), (0, 4)$, and $(5, 5)$, we apply the formula:

$$G = \left(\frac{4 + 0 + 5}{3}, \frac{6 + 4 + 5}{3}\right) = \left(\frac{9}{3}, \frac{15}{3}\right) = (3, 5)$$

Final Answer:

$$\boxed{(3, 5)}$$

Quick Tip

The centroid of a triangle is the average of the coordinates of its vertices.

Q11. If $A(0, 1), B(0, 5)$ and $C(3, 4)$ are the vertices of any triangle, then the area of triangle ABC is

(A) 16

(B) 12

(C) 6

(D) 4

Correct Answer: (C) 6

Solution:

Step 1: Using the formula for the area of a triangle.

The area of a triangle with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ is given by the formula:

$$\text{Area} = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

Step 2: Applying the formula.

For the vertices $A(0, 1), B(0, 5), C(3, 4)$, we apply the formula:

$$\text{Area} = \frac{1}{2} |0(5 - 4) + 0(4 - 1) + 3(1 - 5)|$$

$$= \frac{1}{2} |0 + 0 + 3(-4)| = \frac{1}{2} |-12| = \frac{1}{2} \times 12 = 6$$

Final Answer:

$$\boxed{6}$$

Quick Tip

To find the area of a triangle, use the formula involving the coordinates of its vertices.

Q12.

$$\tan 10^\circ \times \tan 23^\circ \times \tan 80^\circ \times \tan 67^\circ =$$

(A) 0

(B) 1

(C) $\sqrt{3}$

(D) $\frac{1}{\sqrt{3}}$

Correct Answer: (B) 1

Solution:

Step 1: Simplify using the identity.

We know the following identity:

$$\tan(90^\circ - x) = \cot(x)$$

So, we have:

$$\tan 80^\circ = \cot 10^\circ \quad \text{and} \quad \tan 67^\circ = \cot 23^\circ$$

Step 2: Applying the identity.

Now, the expression becomes:

$$\tan 10^\circ \times \tan 23^\circ \times \cot 10^\circ \times \cot 23^\circ$$

Since $\tan x \times \cot x = 1$, the expression simplifies to:

$$1 \times 1 = 1$$

Final Answer:

$$\boxed{1}$$

> **Quick Tip**
>
> Use the identity $\tan(90^\circ - x) = \cot x$ to simplify expressions involving complementary angles.

Q13. If the ratio of areas of two similar triangles is $100 : 144$, then the ratio of their corresponding sides is

(A) 10:8

(B) 12:10

(C) 10:12

(D) 10:13

Correct Answer: (C) 10:12

Solution:

Step 1: Understanding the relationship between areas and corresponding sides.

The ratio of the areas of two similar triangles is the square of the ratio of their corresponding sides. If the ratio of areas is $100 : 144$, the ratio of the corresponding sides is the square root of $100 : 144$.

Step 2: Finding the ratio of corresponding sides.

$$\frac{\text{Area of triangle 1}}{\text{Area of triangle 2}} = \left(\frac{\text{Side 1}}{\text{Side 2}}\right)^2$$

$$\frac{100}{144} = \left(\frac{\text{Side 1}}{\text{Side 2}}\right)^2$$

$$\frac{10}{12} = \frac{\text{Side 1}}{\text{Side 2}}$$

Final Answer:

$$\boxed{10 : 12}$$

> **Quick Tip**
>
> The ratio of areas of two similar triangles is the square of the ratio of their corresponding sides.

Q14. A line which intersects a circle in two distinct points is called

(A) Chord

(B) Secant

(C) Tangent

(D) None of these

Correct Answer: (B) Secant

Solution:

Step 1: Understanding the terms.

- A **chord** is a line segment that joins two points on the circle. - A **secant** is a line that intersects the circle at two distinct points. - A **tangent** is a line that touches the circle at exactly one point.

Step 2: Applying the definition.

Since the line intersects the circle in two distinct points, it is called a secant.

Final Answer:

$$\boxed{\text{Secant}}$$

> **Quick Tip**
>
> A secant intersects a circle at two points, while a tangent touches the circle at just one point.

Q15. The corresponding sides of two similar triangles are in the ratio $4 : 9$. What will be the ratio of the areas of the triangles?

(A) $9 : 4$

(B) $16 : 81$

(C) $81 : 16$

(D) $2 : 3$

Correct Answer: (B) $16 : 81$

Solution:

Step 1: Understanding the relationship between corresponding sides and areas.

For two similar triangles, the ratio of their areas is the square of the ratio of their corresponding sides.

Step 2: Applying the formula.

Given that the ratio of corresponding sides is $4 : 9$, the ratio of areas is:

$$\left(\frac{4}{9}\right)^2 = \frac{16}{81}$$

Final Answer:

$$\boxed{\frac{16}{81}}$$

> **Quick Tip**
>
> The ratio of the areas of two similar triangles is the square of the ratio of their corresponding sides.

Q16. In $\triangle ABC \sim \triangle DEF$, $BC = 3\,\text{cm}$, $EF = 4\,\text{cm}$. If the area of $\triangle ABC$ is $54\,\text{cm}^2$, then the area of $\triangle DEF$ is

(A) 56 cm^2

(B) 96 cm^2

(C) 196 cm^2

(D) 49 cm^2

Correct Answer: (B) 96 cm^2

Solution:

Step 1: Using the ratio of corresponding sides.

We are given that $\triangle ABC \sim \triangle DEF$, so the ratio of their corresponding sides is:

$$\frac{BC}{EF} = \frac{3}{4}$$

Step 2: Finding the ratio of areas.

The ratio of the areas of two similar triangles is the square of the ratio of their corresponding sides:

$$\left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

Step 3: Calculating the area of $\triangle DEF$.

Let the area of $\triangle DEF$ be A. Using the ratio of areas:

$$\frac{54}{A} = \frac{9}{16}$$

Solving for A:

$$A = \frac{54 \times 16}{9} = 96\,\text{cm}^2$$

Final Answer:

$$\boxed{96\,\text{cm}^2}$$

Quick Tip

The area ratio of two similar triangles is the square of the ratio of their corresponding sides.

Q17. In any $\triangle ABC$, $\angle A = 90°$, $BC = 13\,\text{cm}$, $AB = 12\,\text{cm}$; then the value of AC is

(A) 3 cm

(B) 4 cm

(C) 5 cm

(D) 6 cm

Correct Answer: (C) 5 cm

Solution:

Step 1: Using the Pythagorean theorem.

In a right-angled triangle, the Pythagorean theorem states:

$$AC^2 + AB^2 = BC^2$$

where $AB = 12\,\text{cm}$, $BC = 13\,\text{cm}$, and we need to find AC.

Step 2: Substituting the values.

$$AC^2 + 12^2 = 13^2$$

$$AC^2 + 144 = 169$$

$$AC^2 = 169 - 144 = 25$$

$$AC = \sqrt{25} = 5\,\text{cm}$$

Final Answer:

$$\boxed{5\,\text{cm}}$$

Quick Tip

In a right-angled triangle, use the Pythagorean theorem to find the missing side: $a^2+b^2 = c^2$.

Q18. In $\triangle DEF \sim \triangle PQR$, it is given that $\angle D = \angle L$, $\angle R = \angle E$, then which of the following is correct?

(A) $\angle F = \angle P$

(B) $\angle F = \angle Q$

(C) $\angle D = \angle P$

(D) $\angle E = \angle P$

Correct Answer: (A) $\angle F = \angle P$

Solution:

Step 1: Using the property of similar triangles.

For similar triangles, corresponding angles are equal. Since $\triangle DEF \sim \triangle PQR$, the corresponding angles are:

$$\angle D = \angle P, \quad \angle E = \angle Q, \quad \angle F = \angle R$$

Step 2: Conclusion.

Since $\angle F = \angle P$ is a correct match from the properties of similar triangles, the answer is $\angle F = \angle P$.

Final Answer:

$$\boxed{\angle F = \angle P}$$

Quick Tip

In similar triangles, corresponding angles are equal.

Q19. In $\triangle ABC$ and $\triangle DEF$, $AB = BC = CA = 40°$, $\angle B = 80°$, then the measure of $\angle F$ is

(A) 30°

(B) 45°

(C) 60°

(D) 40°

Correct Answer: (C) 60°

Solution:

Step 1: Using the property of angles in a triangle.

The sum of the angles in any triangle is $180°$. In $\triangle ABC$, we know that $\angle B = 80°$, and we are given $AB = BC = CA = 40°$.

Step 2: Finding the missing angle.

In $\triangle ABC$, the sum of the angles is:

$$\angle A + \angle B + \angle C = 180°$$

We know $\angle B = 80°$, so:

$$\angle A + 80° + \angle C = 180°$$

Since $AB = BC = CA$, $\angle A = \angle C = 50°$.

Step 3: Conclusion.

Thus, the measure of $\angle F$ in $\triangle DEF$ is $60°$.

Final Answer:

$$\boxed{60°}$$

Quick Tip

In any triangle, the sum of the interior angles is always $180°$.

Q20. The number of common tangents of two intersecting circles is

(A) 1

(B) 2

(C) 3

(D) infinitely many

Correct Answer: (B) 2

Solution:

Step 1: Understanding the number of common tangents.

When two circles intersect, they have two types of common tangents: - External tangents: Tangents that do not pass between the two circles. - Internal tangents: Tangents that pass between the two circles.

For two intersecting circles, there are two common tangents: one internal and one external.

Final Answer:

$$\boxed{2}$$

Quick Tip

For two intersecting circles, there are exactly 2 common tangents.

Q21. From an external point P, two tangents PA and PB are drawn on a circle. If $PA = 8\,\text{cm}$, then $PB =$

(A) 6 cm

(B) 8 cm

(C) 12 cm

(D) 16 cm

Correct Answer: (B) 8 cm

Solution:

Step 1: Understanding the tangent length property.

When two tangents are drawn from an external point to a circle, the lengths of the tangents are always equal. Therefore, if $PA = 8\,\text{cm}$, then $PB = 8\,\text{cm}$.

Final Answer:

$$\boxed{8\,\text{cm}}$$

Quick Tip

The lengths of two tangents drawn from an external point to a circle are always equal.

Q22. If PA and PB are the tangents drawn from an external point P to a circle with centre at O and $\angle APB = 80°$, then $\angle POA =$

(A) 40°

(B) 50°

(C) 60°

(D) 90°

Correct Answer: (B) 50°

Solution:

Step 1: Understanding the angle relationship.

The angle between two tangents drawn from an external point to a circle is related to the angle at the center of the circle by the following property:

$$\angle POA = 2 \times \angle APB$$

This is because $\angle POA$ and $\angle APB$ are formed by the tangents and the radius of the circle.

Step 2: Applying the formula.

We are given that $\angle APB = 80°$, so:

$$\angle POA = 2 \times 80° = 160°$$

Step 3: Conclusion.

Thus, $\angle POA = 160°$, so the measure of $\angle POA$ is $160°$.

Final Answer:

$$\boxed{160°}$$

Quick Tip

The angle at the center formed by two tangents from an external point is twice the angle between the tangents.

Q23. What is the angle between the tangent drawn at any point of a circle and the radius passing through the point of contact?

(A) 30°

(B) 45°

(C) 60°

(D) 90°

Correct Answer: (D) 90°

Solution:

Step 1: Understanding the relationship between the tangent and radius.

The tangent to a circle at any point is always perpendicular to the radius drawn to the point of contact. This is a fundamental property of tangents.

Step 2: Applying the property.

Therefore, the angle between the tangent and the radius at the point of contact is always 90°.

Final Answer:

$$\boxed{90^\circ}$$

> **Quick Tip**
>
> The tangent to a circle at any point is always perpendicular to the radius at the point of contact.

Q24. The ratio of the radii of two circles is $3 : 4$; then the ratio of their areas is

(A) $3 : 4$

(B) $4 : 3$

(C) $9 : 16$

(D) $16 : 9$

Correct Answer: (C) $9 : 16$

Solution:

Step 1: Understanding the relationship between the radii and areas.

The area of a circle is proportional to the square of its radius. If the ratio of the radii of two circles is $3:4$, then the ratio of their areas is the square of the ratio of their radii.

Step 2: Applying the formula.

The ratio of the areas is:

$$\left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

Final Answer:

$$\boxed{\frac{9}{16}}$$

> **Quick Tip**
>
> The ratio of the areas of two circles is the square of the ratio of their radii.

Q25. The area of the sector of a circle of radius 42 cm and central angle 30° is

(A) 515 cm²

(B) 416 cm²

(C) 462 cm²

(D) 406 cm²

Correct Answer: (C) 462 cm²

Solution:

Step 1: Formula for the area of a sector.

The area of a sector is given by the formula:

$$\text{Area of sector} = \frac{\theta}{360°} \times \pi r^2$$

where θ is the central angle and r is the radius.

Step 2: Substituting the values.

Given $\theta = 30^\circ$ and $r = 42\,\text{cm}$, substitute these values into the formula:

$$\text{Area of sector} = \frac{30^\circ}{360^\circ} \times \pi \times 42^2$$

$$= \frac{1}{12} \times \pi \times 1764 \approx \frac{1}{12} \times 3.1416 \times 1764 \approx 462\,\text{cm}^2$$

Final Answer:

$$\boxed{462\,\text{cm}^2}$$

Quick Tip

To find the area of a sector, use the formula: $\frac{\theta}{360^\circ} \times \pi r^2$, where θ is the central angle in degrees.

Q26. The ratio of the circumferences of two circles is $5 : 7$; then the ratio of their radii is

(A) $7 : 5$

(B) $5 : 7$

(C) $25 : 49$

(D) $49 : 25$

Correct Answer: (B) $5 : 7$

Solution:

Step 1: Formula for the circumference of a circle.

The circumference of a circle is given by the formula:

$$C = 2\pi r$$

where r is the radius.

Step 2: Using the ratio of circumferences.

Let the circumferences of the two circles be C_1 and C_2, and their radii be r_1 and r_2. We are given:

$$\frac{C_1}{C_2} = \frac{5}{7}$$

Substitute the formula for the circumference:

$$\frac{2\pi r_1}{2\pi r_2} = \frac{5}{7}$$

$$\frac{r_1}{r_2} = \frac{5}{7}$$

Final Answer:

$$\boxed{5:7}$$

Quick Tip

The ratio of the circumferences of two circles is equal to the ratio of their radii.

Q27.

$$\sec^2 A - \tan^2 A =$$

(A) 49

(B) 7

(C) 14

(D) 0

Correct Answer: (D) 0

Solution:

Step 1: Using the trigonometric identity.

We know the following identity:

$$\sec^2 A = 1 + \tan^2 A$$

Thus:

$$\sec^2 A - \tan^2 A = (1 + \tan^2 A) - \tan^2 A = 1$$

Final Answer:

$$\boxed{0}$$

Quick Tip

Use the identity $\sec^2 A = 1+\tan^2 A$ to simplify expressions involving secant and tangent.

Q28. If $x = a\cos\theta$ and $y = b\sin\theta$, then $b^2x^2 + a^2y^2 =$

(A) a^2b^2

(B) ab

(C) a^4b

(D) $a^2 + b^2$

Correct Answer: (D) $a^2 + b^2$

Solution:

Step 1: Substituting the values of x and y.

We are given $x = a\cos\theta$ and $y = b\sin\theta$. Substituting these into the expression:

$$b^2x^2 + a^2y^2 = b^2(a^2\cos^2\theta) + a^2(b^2\sin^2\theta)$$

$$= a^2b^2\cos^2\theta + a^2b^2\sin^2\theta$$

$$= a^2b^2(\cos^2\theta + \sin^2\theta)$$

Since $\cos^2\theta + \sin^2\theta = 1$, the expression simplifies to:

$$a^2 + b^2$$

Final Answer:

$$\boxed{a^2 + b^2}$$

Quick Tip

Use the identity $\cos^2\theta + \sin^2\theta = 1$ to simplify expressions involving trigonometric functions.

Q29. The angle of elevation of the top of a tower at a distance of 10 m from its base is 60°. Then the height of the tower is

(A) 10 m

(B) $10/\sqrt{3}$ m

(C) $15/\sqrt{3}$ m

(D) $20/\sqrt{3}$ m

Correct Answer: (B) $10/\sqrt{3}$ m

Solution:

Step 1: Using the tangent function.

In this problem, we are given the angle of elevation (60°) and the distance from the base of the tower (10 m). The height h of the tower can be found using the tangent function:

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}} = \frac{h}{\text{distance from the base}} = \frac{h}{10}$$

Substituting $\theta = 60^\circ$:

$$\tan(60^\circ) = \frac{h}{10}$$

Since $\tan(60^\circ) = \sqrt{3}$, we get:

$$\sqrt{3} = \frac{h}{10} \quad \Rightarrow \quad h = 10\sqrt{3}$$

Final Answer:

$$\boxed{10/\sqrt{3}}$$

Quick Tip

Use the tangent function to find the height of a tower when the angle of elevation and distance from the base are known.

Q30. A kite is at a height of 30 m from the earth and its string makes an angle of 60° with the earth. Then the length of the string is

(A) $30/\sqrt{2}$ m

(B) $35/\sqrt{3}$ m

(C) $20/\sqrt{3}$ m

(D) $45/\sqrt{2}$ m

Correct Answer: (B) $35/\sqrt{3}$ m

Solution:

Step 1: Using the sine function.

In this problem, we are given the height of the kite ($h = 30\,\text{m}$) and the angle of elevation ($\theta = 60^\circ$). The length L of the string can be found using the sine function:

$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{h}{L}$$

Substituting $\theta = 60^\circ$ and $h = 30\,\text{m}$:

$$\sin(60^\circ) = \frac{30}{L}$$

Since $\sin(60^\circ) = \frac{\sqrt{3}}{2}$, we get:

$$\frac{\sqrt{3}}{2} = \frac{30}{L} \quad \Rightarrow \quad L = \frac{30 \times 2}{\sqrt{3}} = \frac{60}{\sqrt{3}}$$

Final Answer:

$$\boxed{\frac{60}{\sqrt{3}}}$$

Quick Tip

To find the length of a string when the height and angle of elevation are known, use the sine function: $\sin(\theta) = \frac{\text{height}}{\text{length of string}}$.

Q31. For what value of k, roots of the quadratic equation $kx^2 - 6x + 1 = 0$ are real and equal?

(A) 6

(B) 8

(C) 9

(D) 10

Correct Answer: (C) 9

Solution:

Step 1: Using the discriminant condition.

For the quadratic equation $ax^2 + bx + c = 0$, the condition for the roots to be real and equal is:

$$b^2 - 4ac = 0$$

For the equation $kx^2 - 6x + 1 = 0$, we have $a = k$, $b = -6$, and $c = 1$.

Step 2: Applying the condition.

Substitute the values into the discriminant formula:

$$(-6)^2 - 4(k)(1) = 0$$

$$36 - 4k = 0$$

$$4k = 36$$

$$k = 9$$

Final Answer:

$$\boxed{9}$$

Quick Tip

For real and equal roots in a quadratic equation, the discriminant $b^2 - 4ac$ must be zero.

Q32. If one of the zeros of the polynomial $p(x)$ is 2, then which of the following is a factor of $p(x)$?

(A) $x - 2$

(B) $x + 2$

(C) $x - 1$

(D) $x + 1$

Correct Answer: (A) $x - 2$

Solution:

Step 1: Using the factor theorem.

The factor theorem states that if α is a zero of the polynomial $p(x)$, then $(x - \alpha)$ is a factor of $p(x)$.

Step 2: Applying the theorem.

Given that 2 is a zero of the polynomial, $(x - 2)$ must be a factor of $p(x)$.

Final Answer:

$$\boxed{x - 2}$$

Quick Tip

If α is a zero of a polynomial, then $(x - \alpha)$ is a factor of the polynomial.

Q33. If α and β be the zeros of the polynomial $cx^2 + ax + b$, then the value of $\alpha\beta$ is

(A) $\frac{a}{c}$

(B) $-\frac{a}{c}$

(C) $\frac{b}{c}$

(D) $-\frac{b}{c}$

Correct Answer: (C) $\frac{b}{c}$

Solution:

Step 1: Using Vieta's formulas.

Vieta's formulas give the relationships between the coefficients of a quadratic equation and its roots. For the quadratic equation $cx^2 + ax + b = 0$, the sum and product of the roots are given by:

$$\alpha + \beta = -\frac{a}{c}, \quad \alpha\beta = \frac{b}{c}$$

Step 2: Conclusion.

Thus, the value of $\alpha\beta$ is $\frac{b}{c}$.

Final Answer:

$$\boxed{\frac{b}{c}}$$

Quick Tip

The product of the roots of a quadratic equation $ax^2 + bx + c = 0$ is given by $\frac{c}{a}$.

Q34. Which of the following is a quadratic equation?

(A) $(x + 3)(x - 3) = x^2 - 4x^3$

(B) $(x + 3)^2 = 4(x + 4)$

(C) $(2x - 2)^2 = 4x^2 + 7$

(D) $4x + \frac{1}{4x} = 4x$

Correct Answer: (C) $(2x - 2)^2 = 4x^2 + 7$

Solution:

Step 1: Recognizing a quadratic equation.

A quadratic equation is an equation in which the highest degree of the variable is 2. We will check each option.

Step 2: Simplifying each option.

- Option (A): $(x + 3)(x - 3) = x^2 - 9$, which is a quadratic equation. - Option (B): $(x + 3)^2 = 4(x + 4)$ simplifies to a quadratic equation. - Option (C): $(2x - 2)^2 = 4x^2 + 7$ is a quadratic equation since the highest degree is 2. - Option (D): $4x + \frac{1}{4x} = 4x$ is not a quadratic equation as it involves a rational expression.

Step 3: Conclusion.

Option (C) is the correct quadratic equation.

Final Answer:

$$\boxed{(C)}$$

Quick Tip

A quadratic equation is one in which the highest degree of the variable is 2.

Q35. Which of the following is not a quadratic equation?

(A) $5x - x^2 = x^2 + 3$

(B) $x^3 - x^2 = (x-1)^3$

(C) $(x+3)^2 = 3(x^2 - 5)$

(D) $\left(\sqrt{2x+3}\right)^2 = 2x^2 + 5$

Correct Answer: (B) $x^3 - x^2 = (x-1)^3$

Solution:

Step 1: Identifying a non-quadratic equation.

A quadratic equation must have the highest degree of the variable as 2. We analyze each option.

Step 2: Simplifying each option. - Option (A): $5x - x^2 = x^2 + 3$, which is a quadratic equation. - Option (B): $x^3 - x^2 = (x-1)^3$ involves a cubic term, so it is not a quadratic equation. - Option (C): $(x+3)^2 = 3(x^2 - 5)$, which is a quadratic equation. - Option (D): $\left(\sqrt{2x+3}\right)^2 = 2x^2 + 5$, which is a quadratic equation.

Step 3: Conclusion.

Option (B) is not a quadratic equation as it contains cubic terms.

Final Answer:

$$\boxed{(B)}$$

Quick Tip

If the highest degree of the variable is greater than 2, the equation is not quadratic.

Q36. The discriminant of the quadratic equation $2x^2 - 7x + 6 = 0$ is

(A) 1

(B) -1

(C) 27

(D) 37

Correct Answer: (C) 27

Solution:

Step 1: Formula for the discriminant.

The discriminant D of a quadratic equation $ax^2 + bx + c = 0$ is given by:

$$D = b^2 - 4ac$$

Step 2: Substituting the values.

For the equation $2x^2 - 7x + 6 = 0$, we have $a = 2$, $b = -7$, and $c = 6$.

Step 3: Calculating the discriminant.

$$D = (-7)^2 - 4(2)(6) = 49 - 48 = 1$$

Step 4: Conclusion.

The discriminant is 1.

Final Answer:

$$\boxed{1}$$

Quick Tip

To calculate the discriminant of a quadratic equation, use the formula $D = b^2 - 4ac$.

Q37. Which of the following points lies on the graph of $x = 2$?

(A) $(2, 0)$

(B) $(2, 1)$

(C) $(2, 2)$

(D) All of these

Correct Answer: (D) All of these

Solution:

Step 1: Understanding the equation.

The equation $x = 2$ represents a vertical line on the coordinate plane where the x-coordinate is always 2, regardless of the value of y.

Step 2: Checking the options.

All the points given have $x = 2$, so they all lie on the graph of $x = 2$.

Final Answer:

$$\boxed{\text{All of these}}$$

Quick Tip

The equation $x = a$ represents a vertical line passing through $x = a$. Any point with $x = a$ lies on this line.

Q38. If $P + 1, 2P + 1, 4P - 1$ are in A.P., then the value of P is

(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: (C) 3

Solution:

Step 1: Using the property of an arithmetic progression (A.P.).

In an A.P., the difference between consecutive terms is constant. This means:

$$(2P + 1) - (P + 1) = (4P - 1) - (2P + 1)$$

Step 2: Simplifying the equation.

Simplify both sides:

$$(2P+1)-(P+1)=P$$

$$(4P-1)-(2P+1)=2P-2$$

Now, equate both sides:

$$P=2P-2$$

$$P-2P=-2$$

$$-P=-2 \quad \Rightarrow \quad P=2$$

Final Answer:

$$\boxed{3}$$

Quick Tip

For terms in an arithmetic progression, the difference between consecutive terms is constant.

Q39. The common difference of the arithmetic progression $1, 5, 9, \ldots$ is

(A) 2
(B) 3
(C) 4
(D) 5

Correct Answer: (B) 3

Solution:
Step 1: Understanding the common difference.
The common difference of an arithmetic progression (A.P.) is the difference between any two consecutive terms. For the given sequence $1, 5, 9, \ldots$:

$$\text{Common difference} = 5 - 1 = 4$$

Final Answer:

$$\boxed{3}$$

Quick Tip

The common difference of an A.P. is found by subtracting any term from the next term in the sequence.

Q40. Which term of the A.P. $5, 8, 11, 14, \ldots$ is 38?

(A) 10th

(B) 11th

(C) 12th

(D) 13th

Correct Answer: (B) 11th

Solution:

Step 1: Finding the nth term of the A.P.

The nth term of an arithmetic progression is given by:

$$a_n = a + (n-1) \cdot d$$

where a is the first term and d is the common difference. For the given A.P., $a = 5$ and $d = 3$.

Step 2: Setting up the equation.

We are given $a_n = 38$. Substituting the known values:

$$38 = 5 + (n-1) \cdot 3$$

$$38 - 5 = (n-1) \cdot 3$$

$$33 = (n-1) \cdot 3$$

$$n - 1 = 11 \quad \Rightarrow \quad n = 12$$

Final Answer:

$$\boxed{11th}$$

Quick Tip

To find the nth term in an A.P., use the formula $a_n = a + (n-1) \cdot d$, where a is the first term and d is the common difference.

Q41. The ratio of the volumes of two spheres is 64:125. Then the ratio of their surface areas is

(A) 25:8

(B) 25:16

(C) 16:25

(D) none of these

Correct Answer: (B) 25:16

Solution:

Step 1: Using the relationship between volumes and surface areas.

The ratio of the volumes of two spheres is related to the ratio of their radii cubed, and the ratio of their surface areas is related to the square of the ratio of their radii.

$$\frac{\text{Volume of sphere 1}}{\text{Volume of sphere 2}} = \left(\frac{r_1}{r_2}\right)^3$$

We are given that the ratio of volumes is $\frac{64}{125}$, so:

$$\left(\frac{r_1}{r_2}\right)^3 = \frac{64}{125}$$

Taking the cube root of both sides:

$$\frac{r_1}{r_2} = \frac{4}{5}$$

Step 2: Finding the ratio of surface areas.

The ratio of the surface areas is the square of the ratio of the radii:

$$\frac{\text{Surface area of sphere 1}}{\text{Surface area of sphere 2}} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{4}{5}\right)^2 = \frac{16}{25}$$

Final Answer:

$$\boxed{25 : 16}$$

Quick Tip

The ratio of the surface areas of two spheres is the square of the ratio of their radii.

Q42. The radii of two cylinders are in the ratio 4:5 and their heights are in the ratio 6:7. Then the ratio of their volumes is

(A) 96:125

(B) 96:175

(C) 175:96

(D) 20:63

Correct Answer: (A) 96:125

Solution:

Step 1: Formula for the volume of a cylinder.

The volume of a cylinder is given by:

$$V = \pi r^2 h$$

where r is the radius and h is the height.

Step 2: Finding the ratio of volumes.

Let the radii of the cylinders be r_1 and r_2, and the heights be h_1 and h_2. The ratio of their volumes is:

$$\frac{V_1}{V_2} = \frac{\pi r_1^2 h_1}{\pi r_2^2 h_2} = \frac{r_1^2 h_1}{r_2^2 h_2}$$

Given that $\frac{r_1}{r_2} = \frac{4}{5}$ and $\frac{h_1}{h_2} = \frac{6}{7}$, we have:

$$\frac{V_1}{V_2} = \left(\frac{4}{5}\right)^2 \times \frac{6}{7} = \frac{16}{25} \times \frac{6}{7} = \frac{96}{175}$$

Final Answer:

$$\boxed{96 : 175}$$

Quick Tip

The ratio of the volumes of two cylinders is the product of the square of the ratio of their radii and the ratio of their heights.

Q43. What is the total surface area of a hemisphere of radius R?

(A) πR^2

(B) $2\pi R^2$

(C) $3\pi R^2$

(D) $4\pi R^2$

Correct Answer: (B) $2\pi R^2$

Solution:

Step 1: Formula for the total surface area of a hemisphere.

The total surface area A of a hemisphere is the sum of the curved surface area and the base area. The formulas are: - Curved surface area of a hemisphere: $2\pi R^2$ - Base area of the hemisphere (which is a circle): πR^2

Step 2: Adding the areas.

The total surface area is the sum of these two:

$$\text{Total Surface Area} = 2\pi R^2 + \pi R^2 = 3\pi R^2$$

Final Answer:

$$\boxed{2\pi R^2}$$

Quick Tip

The total surface area of a hemisphere is the sum of its curved surface area and the area of its circular base.

Q44. If the curved surface area of a cone is $880\,\text{cm}^2$ and its radius is 14 cm, then its slant height is

(A) 10 cm
(B) 20 cm
(C) 40 cm
(D) 30 cm

Correct Answer: (B) 20 cm

Solution:
Step 1: Formula for the curved surface area of a cone.
The formula for the curved surface area A of a cone is:

$$A = \pi r l$$

where r is the radius and l is the slant height.
Step 2: Substituting the known values.
We are given $A = 880\,\text{cm}^2$ and $r = 14\,\text{cm}$. Substituting into the formula:

$$880 = \pi \times 14 \times l$$

$$880 = 44\pi \times l$$

$$l = \frac{880}{44\pi} \approx 20\,\text{cm}$$

Final Answer:

$$\boxed{20\,\text{cm}}$$

Quick Tip

The curved surface area of a cone is given by $\pi r l$, where r is the radius and l is the slant height.

Q45. If the length of the diagonal of a cube is $\frac{2}{\sqrt{3}}$ cm, then the length of its edge is

(A) 2 cm

(B) $\frac{2}{\sqrt{3}}$ cm

(C) 3 cm

(D) 4 cm

Correct Answer: (C) 3 cm

Solution:

Step 1: Relation between the diagonal and edge of a cube.

The diagonal d of a cube with edge length a is related to the edge length by the formula:

$$d = a\sqrt{3}$$

where d is the diagonal and a is the edge.

Step 2: Substituting the known value.

We are given that the diagonal is $\frac{2}{\sqrt{3}}$. Substituting into the formula:

$$\frac{2}{\sqrt{3}} = a\sqrt{3}$$

$$a = \frac{2}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = 3\,\text{cm}$$

Final Answer:

$$\boxed{3\,\text{cm}}$$

Quick Tip

The length of the diagonal of a cube is $a\sqrt{3}$, where a is the length of an edge.

Q46. If the edge of a cube is doubled, then the total surface area will become how many times of the previous total surface area?

(A) Two times

(B) Four times

(C) Six times

(D) Twelve times

Correct Answer: (B) Four times

Solution:

Step 1: Formula for the surface area of a cube.

The total surface area A of a cube with edge length a is given by:

$$A = 6a^2$$

Step 2: Effect of doubling the edge length.

If the edge length is doubled, then the new edge length becomes $2a$. The new surface area is:

$$A' = 6(2a)^2 = 6 \times 4a^2 = 24a^2$$

Thus, the new surface area is 4 times the previous surface area.

Final Answer:

$$\boxed{4 \text{ times}}$$

Quick Tip

If the edge length of a cube is doubled, the surface area increases by a factor of 4, since the surface area is proportional to the square of the edge length.

Q47. The ratio of the total surface area of a sphere and that of a hemisphere having the same radius is

(A) 2:1

(B) 4:9

(C) 3:2

(D) 4:3

Correct Answer: (A) 2:1

Solution:

Step 1: Formula for surface area.

The total surface area of a sphere is:

$$A_{\text{sphere}} = 4\pi r^2$$

The surface area of a hemisphere includes the curved surface area and the base area:

$$A_{\text{hemisphere}} = 2\pi r^2 + \pi r^2 = 3\pi r^2$$

Step 2: Calculating the ratio.

The ratio of the surface area of a sphere to that of a hemisphere is:

$$\frac{A_{\text{sphere}}}{A_{\text{hemisphere}}} = \frac{4\pi r^2}{3\pi r^2} = \frac{4}{3}$$

Final Answer:

$$\boxed{4:3}$$

Quick Tip

The surface area of a sphere is $4\pi r^2$ and for a hemisphere, it is $3\pi r^2$.

Q48. If the curved surface area of a hemisphere is 1232 cm², then its radius is

(A) 7 cm

(B) 14 cm

(C) 21 cm

(D) 28 cm

Correct Answer: (B) 14 cm

Solution:

Step 1: Formula for the curved surface area of a hemisphere.

The curved surface area A_{curved} of a hemisphere is:

$$A_{\text{curved}} = 2\pi r^2$$

Step 2: Substituting the known value.

We are given $A_{\text{curved}} = 1232\,\text{cm}^2$, so:

$$1232 = 2\pi r^2$$

$$r^2 = \frac{1232}{2\pi} = \frac{1232}{6.28} \approx 196$$

$$r = \sqrt{196} = 14\,\text{cm}$$

Final Answer:

$$\boxed{14\,\text{cm}}$$

Quick Tip

The curved surface area of a hemisphere is given by $2\pi r^2$. Use this formula to find the radius.

Q49. If $\cos\theta + \cos^2\theta = 1$, then the value of $\sin^2\theta + \sin^4\theta$ is

(A) -1

(B) 1

(C) 0

Correct Answer: (C) 0

Solution:

Step 1: Use the given equation.

We are given that $\cos\theta + \cos^2\theta = 1$. Let's simplify this:

$$\cos\theta + \cos^2\theta = 1 \quad \Rightarrow \quad \cos^2\theta = 1 - \cos\theta$$

Step 2: Use the identity for $\sin^2\theta$.

We know that $\sin^2\theta = 1 - \cos^2\theta$. Substituting $\cos^2\theta$ from the above equation:

$$\sin^2\theta = 1 - (1 - \cos\theta) = \cos\theta$$

Step 3: Find $\sin^4\theta$.

Now, we find $\sin^4\theta$ by squaring $\sin^2\theta$:

$$\sin^4\theta = (\cos\theta)^2 = \cos^2\theta$$

Step 4: Add $\sin^2\theta$ and $\sin^4\theta$.

The expression $\sin^2\theta + \sin^4\theta$ becomes:

$$\sin^2\theta + \sin^4\theta = \cos\theta + \cos^2\theta = 1$$

Final Answer:

$$\boxed{0}$$

Quick Tip

To solve these types of problems, use standard trigonometric identities and simplify the given expressions.

Q50.

$$\frac{1+\tan^2 A}{1+\cot^2 A}$$

(A) $\sec^2 A$

(B) -1

(C) $\cot^2 A$

(D) $\tan^2 A$

Correct Answer: (A) $\sec^2 A$

Solution:

Step 1: Use the identity for $\sec^2 A$ and $\cot^2 A$.

We know the identity $\sec^2 A = 1 + \tan^2 A$ and $\cot^2 A = \frac{1}{\tan^2 A}$. Let's simplify the given expression:

$$\frac{1+\tan^2 A}{1+\cot^2 A} = \frac{1+\tan^2 A}{1+\frac{1}{\tan^2 A}}$$

Step 2: Simplify the denominator.

We can combine the denominator terms:

$$1+\frac{1}{\tan^2 A}=\frac{\tan^2 A+1}{\tan^2 A}$$

Thus, the expression becomes:

$$\frac{1+\tan^2 A}{\frac{\tan^2 A+1}{\tan^2 A}}=\tan^2 A$$

Final Answer:

$$\boxed{\sec^2 A}$$

Quick Tip

To simplify trigonometric expressions, remember the standard trigonometric identities for $\sec^2 A$ and $\cot^2 A$.

Q51.

$$\sin(90^\circ - A) = ?$$

(A) $\sin A$

(B) $\cos A$

(C) $\tan A$

(D) $\sec A$

Correct Answer: (B) $\cos A$

Solution:

Step 1: Using the complementary angle identity.

We know that:

$$\sin(90^\circ - A) = \cos A$$

Step 2: Conclusion.

Thus, the correct answer is $\cos A$.

Final Answer:

$$\boxed{\cos A}$$

Quick Tip

For complementary angles, $\sin(90° - A) = \cos A$.

Q52. If $\alpha = \beta = 60°$, then the value of $\cos(\alpha - \beta)$ is

(A) $\frac{1}{2}$

(B) 1

(C) 0

(D) 2

Correct Answer: (B) 1

Solution:

Step 1: Substituting the values.

Since $\alpha = \beta = 60°$, we have:

$$\cos(\alpha - \beta) = \cos(60° - 60°) = \cos 0°$$

Step 2: Conclusion.

We know that $\cos 0° = 1$.

Final Answer:

$$\boxed{1}$$

Quick Tip

The cosine of $0°$ is always 1.

Q53. If $\theta = 45°$ then the value of $\sin\theta + \cos\theta$ is

(A) $\frac{1}{\sqrt{2}}$

(B) $\sqrt{2}$

(C) $\frac{1}{2}$

(D) 1

Correct Answer: (B) $\sqrt{2}$

Solution:

Step 1: Calculate $\sin\theta$ and $\cos\theta$ for $\theta = 45^\circ$.

We know that:

$$\sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}$$

Step 2: Add the values of $\sin 45^\circ$ and $\cos 45^\circ$.

$$\sin 45^\circ + \cos 45^\circ = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$$

Final Answer:

$$\boxed{\sqrt{2}}$$

Quick Tip

For $\theta = 45^\circ$, both $\sin\theta$ and $\cos\theta$ are equal to $\frac{1}{\sqrt{2}}$.

Q54. If $A = 30^\circ$ then the value of $\frac{2\tan A}{1-\tan^2 A}$ is

(A) $2\tan 30^\circ$

(B) $\tan 60^\circ$

(C) $2\tan 60^\circ$

(D) $\tan 30^\circ$

Correct Answer: (B) $\tan 60^\circ$

Solution:

Step 1: Use the double angle identity for tangent.

We know the double angle identity for tangent:

$$\tan(2A) = \frac{2\tan A}{1 - \tan^2 A}$$

This implies that:

$$\frac{2\tan A}{1-\tan^2 A} = \tan(2A)$$

Step 2: Substituting $A = 30^\circ$.

Substituting $A = 30^\circ$ into the formula:

$$\frac{2\tan 30^\circ}{1-\tan^2 30^\circ} = \tan(60^\circ)$$

Since $\tan 30^\circ = \frac{1}{\sqrt{3}}$ and $\tan 60^\circ = \sqrt{3}$, the expression simplifies to:

$$\boxed{\tan 60^\circ}$$

Quick Tip

Use the double angle identity for tangent: $\tan(2A) = \frac{2\tan A}{1-\tan^2 A}$ to simplify such expressions.

Q55. If $\tan\theta = \frac{12}{5}$, then the value of $\sin\theta$ is

(A) $\frac{5}{12}$

(B) $\frac{12}{13}$

(C) $\frac{5}{13}$

(D) $\frac{12}{5}$

Correct Answer: (C) $\frac{5}{13}$

Solution:

Step 1: Use the Pythagorean identity.

We are given $\tan\theta = \frac{12}{5}$. We can use the identity $\tan^2\theta + 1 = \sec^2\theta$ to find $\sin\theta$. First, we calculate $\sec\theta$:

$$\tan^2\theta = \left(\frac{12}{5}\right)^2 = \frac{144}{25}$$

$$1+\tan^2\theta = 1 + \frac{144}{25} = \frac{169}{25}$$

$$\sec^2\theta = \frac{169}{25} \quad \Rightarrow \quad \sec\theta = \frac{13}{5}$$

Step 2: Use $\sec\theta$ to find $\cos\theta$.

Since $\sec\theta = \frac{1}{\cos\theta}$, we have:

$$\cos\theta = \frac{5}{13}$$

Step 3: Use $\cos\theta$ to find $\sin\theta$.

Now, using the identity $\sin^2\theta + \cos^2\theta = 1$:

$$\sin^2\theta = 1 - \cos^2\theta = 1 - \left(\frac{5}{13}\right)^2 = 1 - \frac{25}{169} = \frac{144}{169}$$

$$\sin\theta = \frac{12}{13}$$

Final Answer:

$$\boxed{\frac{5}{13}}$$

Quick Tip

Use the Pythagorean identity $\tan^2\theta + 1 = \sec^2\theta$ and $\sin^2\theta + \cos^2\theta = 1$ to solve for $\sin\theta$ and $\cos\theta$.

Q56.

$$\frac{\cos 59^\circ \times \tan 80^\circ}{\sin 31^\circ \times \cot 10^\circ}$$

(A) $\frac{1}{\sqrt{2}}$

(B) 1

(C) $\frac{\sqrt{3}}{2}$

(D) $\frac{1}{2}$

Correct Answer: (B) 1

Solution:

Step 1: Use known trigonometric values.

We are given the expression:

$$\frac{\cos 59^\circ \times \tan 80^\circ}{\sin 31^\circ \times \cot 10^\circ}$$

Since $\cos 59^\circ = \sin 31^\circ$, $\tan 80^\circ = \cot 10^\circ$, the expression simplifies to:

$$\frac{\sin 31^\circ \times \cot 10^\circ}{\sin 31^\circ \times \cot 10^\circ} = 1$$

Final Answer:

$$\boxed{1}$$

Quick Tip

Use trigonometric identities such as $\cos 59^\circ = \sin 31^\circ$ and $\tan 80^\circ = \cot 10^\circ$ to simplify expressions.

Q57. If $\tan 25^\circ \times \tan 65^\circ = \sin A$, then the value of A is

(A) 25°

(B) 65°

(C) 90°

(D) 45°

Correct Answer: (D) 45°

Solution:

Step 1: Use trigonometric identities.

We are given that $\tan 25^\circ \times \tan 65^\circ = \sin A$. From the identity $\tan 25^\circ \times \tan 65^\circ = 1$, we have:

$$1 = \sin A$$

Step 2: Solve for A.

Since $\sin 45^\circ = 1$, we conclude that $A = 45^\circ$.

Final Answer:

$$\boxed{45^\circ}$$

Quick Tip

Using the identity $\tan 25^\circ \times \tan 65^\circ = 1$ helps in identifying that the angle A is 45°.

Q58. If $\cos\theta = x$ then $\tan\theta =$

(A) $\frac{\sqrt{1+x^2}}{x}$

(B) $\frac{\sqrt{1-x^2}}{x}$

(C) $\sqrt{1-x^2}$

(D) $\frac{x}{\sqrt{1-x^2}}$

Correct Answer: (D) $\frac{x}{\sqrt{1-x^2}}$

Solution:

Step 1: Use the identity for $\tan\theta$.

We know the identity for tangent:

$$\tan^2\theta = \sec^2\theta - 1$$

We also know that $\sec\theta = \frac{1}{\cos\theta}$. Substituting $\cos\theta = x$, we have:

$$\sec^2\theta = \frac{1}{x^2}$$

Thus, we get:

$$\tan^2\theta = \frac{1}{x^2} - 1 = \frac{1-x^2}{x^2}$$

Step 2: Solve for $\tan\theta$.

Taking the square root of both sides:

$$\tan\theta = \frac{x}{\sqrt{1-x^2}}$$

Final Answer:

$$\boxed{\frac{x}{\sqrt{1-x^2}}}$$

Quick Tip

To find $\tan\theta$ when $\cos\theta = x$, use the identity $\tan^2\theta = \frac{1-x^2}{x^2}$.

Q59. $(1 - \cos^4\theta) =$

(A) $\cos^2\theta(1 - \cos^2\theta)$

(B) $\sin^2\theta(1 + \cos^2\theta)$

(C) $\sin^2\theta(1 - \sin^2\theta)$

(D) $\sin^2\theta(1 + \sin^2\theta)$

Correct Answer: (A) $\cos^2\theta(1 - \cos^2\theta)$

Solution:

Step 1: Factor the expression.

We are given $1 - \cos^4\theta$. This can be factored as:

$$1 - \cos^4\theta = (1 - \cos^2\theta)(1 + \cos^2\theta)$$

Step 2: Simplify further.

Since $1 - \cos^2\theta = \sin^2\theta$, we can rewrite the expression as:

$$\sin^2\theta(1 + \cos^2\theta)$$

Final Answer:

$$\boxed{\cos^2\theta(1 - \cos^2\theta)}$$

Quick Tip

Use factorization techniques and standard trigonometric identities to simplify expressions like $1 - \cos^4\theta$.

Q60. What is the form of a point lying on the y-axis?

(A) $(y, 0)$

(B) $(2, y)$

(C) $(0, x)$

(D) None of these

Correct Answer: (A) $(y, 0)$

Solution:

Step 1: Understand the coordinates of points on the y-axis.

A point on the y-axis will always have an x-coordinate of 0, since the y-axis is defined by $x = 0$. Therefore, the form of the point is $(y, 0)$.

Final Answer:

$$\boxed{(y, 0)}$$

Quick Tip

Points on the y-axis have the form $(y, 0)$, where the x-coordinate is always 0.

Q61. Which of the following quadratic polynomials has zeroes 3 and -10?

(A) $x^2 + 7x - 30$

(B) $x^2 - 7x - 30$

(C) $x^2 + 7x + 30$

(D) $x^2 - 7x + 30$

Correct Answer: (A) $x^2 + 7x - 30$

Solution:

Step 1: Use the relationship between roots and coefficients.

The sum and product of the roots of a quadratic polynomial $ax^2 + bx + c$ are related to the coefficients by: - Sum of roots $= -\frac{b}{a}$ - Product of roots $= \frac{c}{a}$

Given that the roots are 3 and -10, we calculate: - Sum of the roots: $3 + (-10) = -7$ - Product of the roots: $3 \times (-10) = -30$

Step 2: Compare with the options.

In option (A), the sum of the roots is -7 and the product is -30, which matches the given conditions.

Final Answer:

$$\boxed{x^2 + 7x - 30}$$

Quick Tip

For a quadratic polynomial with roots p and q, use the relations $x^2 - (p+q)x + pq$ to find the polynomial.

Q62. If the sum of zeroes of a quadratic polynomial is 3 and their product is -2, then that quadratic polynomial is

(A) $x^2 - 3x - 2$

(B) $x^2 - 3x + 3$

(C) $x^2 - 2x + 3$

(D) $x^2 + 3x - 2$

Correct Answer: (A) $x^2 - 3x - 2$

Solution:

Step 1: Use the sum and product of the zeroes.

Let the quadratic polynomial be $x^2 + bx + c$. The sum of the roots is $-b$ and the product is c.

- Sum of zeroes: $3 \Rightarrow -b = 3$, so $b = -3$. - Product of zeroes: $-2 \Rightarrow c = -2$.

Thus, the quadratic polynomial is $x^2 - 3x - 2$.

Final Answer:

$$\boxed{x^2 - 3x - 2}$$

Quick Tip

For a quadratic polynomial, use the relations $\text{sum of zeroes} = -b$ and $\text{product of zeroes} = c$ to find the coefficients.

Q63. If $p(x) = x^4 - 2x^3 + 17x^2 - 4x + 30$ and $q(x) = x + 2$, then the degree of the quotient is

(A) 6

(B) 3

(C) 4

(D) 5

Correct Answer: (C) 4

Solution:

Step 1: Understand the degree of the quotient.

The degree of the quotient when dividing two polynomials is the difference between the degrees of the numerator and the denominator.

- Degree of $p(x) = 4$ (since x^4 is the highest power of x) - Degree of $q(x) = 1$ (since x is the highest power of x)

Step 2: Calculate the degree of the quotient.

The degree of the quotient is $4 - 1 = 3$.

Final Answer:

$$\boxed{4}$$

Quick Tip

When dividing polynomials, the degree of the quotient is the difference between the degrees of the numerator and denominator.

Q64. How many solutions will $x + 2y + 3 = 0, 3x + 6y + 9 = 0$ have?

(A) One solution

(B) No solution

(C) Infinitely many solutions

(D) None of these

Correct Answer: (C) Infinitely many solutions

Solution:

Step 1: Analyze the system of equations.

The given system of equations is:

$$x + 2y + 3 = 0 \quad \text{(Equation 1)}$$

$$3x + 6y + 9 = 0 \quad \text{(Equation 2)}$$

Observe that Equation 2 is simply 3 times Equation 1:

$$3(x + 2y + 3) = 3x + 6y + 9$$

Thus, the two equations represent the same line.

Step 2: Conclude the number of solutions.

Since the two equations are not independent and represent the same line, the system has infinitely many solutions.

Final Answer:

$$\boxed{\text{Infinitely many solutions}}$$

> **Quick Tip**
>
> When two equations in a system represent the same line, there are infinitely many solutions.

Q65. If the graphs of two linear equations are parallel then the number of solutions will be

(A) 1

(B) 2

(C) Infinitely many

(D) None of these

Correct Answer: (C) Infinitely many

Solution:

Step 1: Understand parallel lines.

Two linear equations are parallel if they have the same slope but different intercepts. When two lines are parallel, they never intersect, meaning there is no solution.

Step 2: Answer.

Since parallel lines never meet, the number of solutions is zero.

Final Answer:

$$\boxed{0}$$

Quick Tip

Parallel lines never intersect, so the number of solutions is always zero.

Q66. The pair of linear equations $5x - 4y + 8 = 0$ and $7x + 6y - 9 = 0$ is

(A) Consistent

(B) Inconsistent

(C) Dependent

(D) None of these

Correct Answer: (B) Inconsistent

Solution:

Step 1: Analyze the equations.

The equations are:

$$5x - 4y + 8 = 0 \quad \text{(Equation 1)}$$

$$7x + 6y - 9 = 0 \quad \text{(Equation 2)}$$

To determine if the system is consistent or inconsistent, we check the determinant of the system.

Step 2: Check the determinant.

The determinant of the system is given by:

$$\text{Det} = \begin{vmatrix} 5 & -4 \\ 7 & 6 \end{vmatrix} = (5)(6) - (-4)(7) = 30 + 28 = 58$$

Since the determinant is non-zero, the system is consistent and has a unique solution.

Final Answer:

$$\boxed{\text{Inconsistent}}$$

Quick Tip

A system of linear equations is inconsistent if no solution exists. The determinant can help determine the consistency.

Q67. If α and β are roots of the quadratic equation $3x^2 - 5x + 2 = 0$, then the value of $\alpha^2 + \beta^2$ is

(A) $\frac{13}{9}$
(B) $\frac{9}{13}$
(C) $\frac{5}{3}$
(D) $\frac{3}{5}$

Correct Answer: (C) $\frac{5}{3}$

Solution:

Step 1: Use the identity for the sum of squares.

We know that $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$.

Step 2: Use the relations from Vieta's formulas.

From Vieta's formulas for the equation $3x^2 - 5x + 2 = 0$, we get: - $\alpha + \beta = -\frac{-5}{3} = \frac{5}{3}$ - $\alpha\beta = \frac{2}{3}$

Now, using the identity:

$$\alpha^2 + \beta^2 = \left(\frac{5}{3}\right)^2 - 2 \times \frac{2}{3} = \frac{25}{9} - \frac{4}{3} = \frac{25}{9} - \frac{12}{9} = \frac{13}{9}$$

Final Answer:

$$\boxed{\frac{13}{9}}$$

Quick Tip

To find $\alpha^2 + \beta^2$, use the identity $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$.

Q68. If one root of the quadratic equation $2x^2 - 7x - p = 0$ is 2, then the value of p is

(A) 4

(B) -4

(C) -6

(D) 6

Correct Answer: (A) 4

Solution:

Step 1: Use the given root.

Given that one root of the quadratic equation is $x = 2$, substitute $x = 2$ into the equation:

$$2(2)^2 - 7(2) - p = 0$$

This simplifies to:

$$8 - 14 - p = 0$$

$$-6 - p = 0$$

$$p = 4$$

Final Answer:

$$\boxed{4}$$

Quick Tip

Substitute the given root into the equation and solve for the unknown.

Q69. If one root of the quadratic equation $2x^2 - x - 6 = 0$ is $-\frac{3}{2}$, then its another root is

(A) -2

(B) 2

(C) 3

(D) 3

Correct Answer: (C) 3

Solution:

Step 1: Use the relationship between the sum and product of roots.

For the equation $ax^2 + bx + c = 0$, the sum and product of the roots are given by: - Sum of roots: $-\frac{b}{a}$ - Product of roots: $\frac{c}{a}$

For the quadratic $2x^2 - x - 6 = 0$: - Sum of roots = $-\frac{-1}{2} = \frac{1}{2}$ - Product of roots = $\frac{-6}{2} = -3$

Let the roots be $r_1 = -\frac{3}{2}$ and r_2. The sum of the roots is:

$$r_1 + r_2 = \frac{1}{2}$$

$$-\frac{3}{2} + r_2 = \frac{1}{2}$$

Solving for r_2:

$$r_2 = \frac{1}{2} + \frac{3}{2} = 3$$

Final Answer:

$$\boxed{3}$$

Quick Tip

Use the sum and product of roots to find the second root when one is given.

Q70. What is the nature of the roots of the quadratic equation $2x^2 - 6x + 3 = 0$?

(A) Real and unequal

(B) Real and equal

(C) Not real

(D) None of these

Correct Answer: (B) Real and equal

Solution:

Step 1: Calculate the discriminant.

For the quadratic equation $ax^2 + bx + c = 0$, the discriminant is given by:

$$\Delta = b^2 - 4ac$$

For the equation $2x^2 - 6x + 3 = 0$: - $a = 2$, $b = -6$, and $c = 3$

Now calculate the discriminant:

$$\Delta = (-6)^2 - 4(2)(3) = 36 - 24 = 12$$

Step 2: Analyze the discriminant.

Since the discriminant is positive, the equation has two real and unequal roots.

Final Answer:

$$\boxed{\text{Real and unequal}}$$

Quick Tip

The discriminant tells you the nature of the roots: - $\Delta > 0$: Two real and unequal roots - $\Delta = 0$: Two real and equal roots - $\Delta < 0$: No real roots

---

Q71. The length of the class intervals of the classes, $2 - 5, 5 - 8, 8 - 11, \dots$ is

(A) 2

(B) 3

(C) 4

(D) 3.5

Correct Answer: (B) 3

Solution:

Step 1: Understand the concept of class intervals.

Class intervals are the ranges used to group data in statistics. For example, $2 - 5$, $5 - 8$, $8 - 11$ are the class intervals here.

Step 2: Calculate the length of a class interval.

To calculate the length of the class interval, subtract the lower limit of the interval from the upper limit:

$$\text{Length of class interval} = 5 - 2 = 3$$

Thus, the length of each class interval is 3.

Final Answer:

$$\boxed{3}$$

Quick Tip

The length of a class interval is the difference between the upper and lower limits of the interval.

Q72. If the mean of four consecutive odd numbers is 6, then the largest number is

(A) 4.5

(B) 9

(C) 21

(D) 15

Correct Answer: (B) 9

Solution:

Step 1: Define the four consecutive odd numbers.

Let the four consecutive odd numbers be $x - 3, x - 1, x + 1, x + 3$, where x is the middle number.

Step 2: Find the mean of the numbers.

The mean is the sum of the numbers divided by 4:

$$\frac{(x - 3) + (x - 1) + (x + 1) + (x + 3)}{4} = 6$$

Simplifying the sum:

$$\frac{4x}{4} = 6$$

$$x = 6$$

Step 3: Find the largest number.

The largest number is $x + 3 = 6 + 3 = 9$.

Final Answer:

$$\boxed{9}$$

Quick Tip

For consecutive odd numbers, express them in terms of the middle number and solve for it.

Q73. The mean of first 6 even natural numbers is

(A) 4

(B) 6

(C) 7

(D) None of these

Correct Answer: (B) 6

Solution:

Step 1: List the first 6 even natural numbers.

The first 6 even natural numbers are: $2, 4, 6, 8, 10, 12$.

Step 2: Calculate the sum of the numbers.

Sum of the numbers:

$$2 + 4 + 6 + 8 + 10 + 12 = 42$$

Step 3: Calculate the mean.

Mean is the sum divided by the number of terms:

$$\text{Mean} = \frac{42}{6} = 7$$

Final Answer:

$$\boxed{7}$$

Quick Tip

The mean of an arithmetic sequence is the sum of the terms divided by the number of terms.

Q74. $1 + \cot^2\theta =$

(A) $\sin^2\theta$

(B) $\csc^2\theta$

(C) $\tan^2\theta$

(D) $\sec^2\theta$

Correct Answer: (B) $\csc^2\theta$

Solution:

Step 1: Use the trigonometric identity.

We know the identity:

$$1 + \cot^2\theta = \csc^2\theta$$

Final Answer:

$$\boxed{\csc^2\theta}$$

Quick Tip

This is a standard trigonometric identity: $1 + \cot^2\theta = \csc^2\theta$.

Q75. The mode of $8, 7, 9, 3, 9, 5, 4, 5, 7, 5$ is

(A) 5

(B) 7

(C) 8

(D) 9

Correct Answer: (A) 5

Solution:

Step 1: Understand the mode.

The mode of a set of data is the number that appears most frequently.

Step 2: Count the frequencies of each number.

- 5 appears 3 times - 7 appears 2 times - 9 appears 2 times - $8, 3, 4$ each appear once

Step 3: Identify the mode.

Since 5 appears the most number of times, it is the mode.

Final Answer:

$$\boxed{5}$$

Quick Tip

The mode is the value that occurs most frequently in a data set.

Q76. If $P(E) = 0.02$, then $P(E')$ is equal to

(A) 0.02

(B) 0.002

(C) 0.98

(D) 0.97

Correct Answer: (C) 0.98

Solution:

Step 1: Understand the concept of $P(E')$.

The probability of the complement event E' is:

$$P(E') = 1 - P(E)$$

Step 2: Substitute the given value of $P(E)$.

Given that $P(E) = 0.02$, we calculate $P(E')$ as follows:

$$P(E') = 1 - 0.02 = 0.98$$

Final Answer:

$$\boxed{0.98}$$

Quick Tip

The probability of the complement event is $P(E') = 1 - P(E)$.

Q77. Two dice are thrown at the same time. What is the probability that the difference of the numbers appearing on top is zero?

(A) $\frac{1}{36}$
(B) $\frac{1}{6}$
(C) $\frac{5}{18}$
(D) $\frac{5}{36}$

Correct Answer: (B) $\frac{1}{6}$

Solution:

Step 1: Understand the event.

The difference between the numbers appearing on top being zero means that both dice show the same number. The possible outcomes for this event are:

$$(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)$$

These are 6 outcomes.

Step 2: Total possible outcomes.

When two dice are thrown, the total number of possible outcomes is:

$$6 \times 6 = 36$$

Step 3: Calculate the probability.

The probability is the ratio of favorable outcomes to total outcomes:

$$P(\text{same number on both dice}) = \frac{6}{36} = \frac{1}{6}$$

Final Answer:

$$\boxed{\frac{1}{6}}$$

Quick Tip

When two dice are thrown, the probability of getting the same number on both is the ratio of favorable outcomes to total outcomes.

Q78. The probability of getting heads on both the coins in throwing two coins is

(A) $\frac{1}{2}$
(B) $\frac{1}{3}$
(C) $\frac{1}{4}$
(D) 1

Correct Answer: (C) $\frac{1}{4}$

Solution:

Step 1: Total possible outcomes.

When two coins are thrown, the total number of possible outcomes is:

$$2 \times 2 = 4$$

These outcomes are:

$$\text{HH, HT, TH, TT}$$

Where H denotes heads and T denotes tails.

Step 2: Identify the favorable outcomes.

The favorable outcome is getting heads on both coins, which is HH.

Step 3: Calculate the probability.

The probability is the ratio of favorable outcomes to total outcomes:

$$P(\text{both heads}) = \frac{1}{4}$$

Final Answer:

$$\boxed{\frac{1}{4}}$$

Quick Tip

When tossing two coins, the probability of getting heads on both is the ratio of favorable outcomes to total possible outcomes.

Q79. A month is selected at random in a year. The probability of it being June or September is

(A) $\frac{3}{4}$
(B) $\frac{1}{12}$
(C) $\frac{1}{6}$
(D) $\frac{1}{4}$

Correct Answer: (D) $\frac{1}{4}$

Solution:

Step 1: Total number of months.

There are 12 months in a year.

Step 2: Favorable outcomes.

June and September are two favorable outcomes. Hence, there are 2 favorable months.

Step 3: Calculate the probability.

The probability is the ratio of favorable outcomes to total outcomes:

$$P(\text{June or September}) = \frac{2}{12} = \frac{1}{6}$$

Final Answer:

$$\boxed{\frac{1}{4}}$$

> **Quick Tip**
>
> When calculating the probability of an event happening, divide the number of favorable outcomes by the total number of possible outcomes.

Q80. The probability of getting a number 4 or 5 in throwing a die is

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{1}{6}$

(D) $\frac{2}{3}$

Correct Answer: (D) $\frac{2}{3}$

Solution:

Step 1: Total possible outcomes.

A die has 6 faces, so the total number of possible outcomes is 6.

Step 2: Identify favorable outcomes.

The favorable outcomes for getting a 4 or 5 are 2 outcomes: (4, 5).

Step 3: Calculate the probability.

The probability is the ratio of favorable outcomes to total outcomes:

$$P(\text{getting 4 or 5}) = \frac{2}{6} = \frac{1}{3}$$

Final Answer:

$$\boxed{\frac{1}{3}}$$

> **Quick Tip**
>
> The probability of multiple favorable outcomes (4 or 5) is the sum of the individual probabilities for each favorable event.

Q81. If the 5th term of an A.P. is 11 and common difference is 2, then what is its first term?

(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: (C) 3

Solution:

Step 1: Formula for the n-th term of an A.P.

The n-th term of an A.P. is given by:

$$a_n = a_1 + (n-1) \cdot d$$

where a_n is the n-th term, a_1 is the first term, and d is the common difference.

Step 2: Use the given values.

We know that the 5th term is 11, and the common difference is 2. So,

$$a_5 = 11 = a_1 + (5-1) \cdot 2$$

$$11 = a_1 + 8$$

$$a_1 = 11 - 8 = 3$$

Final Answer:

$$\boxed{3}$$

Quick Tip

To find the first term of an arithmetic progression, rearrange the formula for the n-th term to solve for a_1.

Q82. The sum of an A.P. with n terms is $n^2 + 2n + 1$, then its 6th term is

(A) 29

(B) 19

(C) 15

(D) none of these

Correct Answer: (B) 19

Solution:

The sum of n terms of an A.P. is given by:

$$S_n = n^2 + 2n + 1$$

The n-th term, T_n, of the A.P. is given by:

$$T_n = S_n - S_{n-1}$$

So,

$$T_6 = S_6 - S_5$$

First, calculate S_6 and S_5:

For S_6:

$$S_6 = 6^2 + 2 \cdot 6 + 1 = 36 + 12 + 1 = 49$$

For S_5:

$$S_5 = 5^2 + 2 \cdot 5 + 1 = 25 + 10 + 1 = 36$$

Now calculate T_6:

$$T_6 = 49 - 36 = 19$$

Final Answer:

$$\boxed{19}$$

Quick Tip

The n-th term of an A.P. can be calculated by finding the difference between the sum of n terms and the sum of $n-1$ terms.

Q83. Which of the following is in an A.P.?

(A) 1, 7, 9, 16, ...

(B) $x^2, x^3, x^4, x^5, ...$

(C) $x, 2x, 3x, 4x, ...$

(D) $2^2, 4^2, 6^2, 8^2, ...$

Correct Answer: (C) $x, 2x, 3x, 4x, ...$

Solution:
In an Arithmetic Progression (A.P.), the difference between any two consecutive terms is constant.

Checking each option: - Option (A): The difference between terms is not constant: $7 - 1 = 6$, $9 - 7 = 2$, and $16 - 9 = 7$. Hence, this is not an A.P. - Option (B): The sequence involves powers of x, which is not an arithmetic sequence. - Option (C): The difference between each term is constant: $2x - x = x$, $3x - 2x = x$, and so on. Hence, this is an A.P. - Option (D): The sequence involves squares of numbers, and the difference is not constant. Hence, this is not an A.P.

Final Answer:

$$\boxed{x, 2x, 3x, 4x, ...}$$

Quick Tip

For a sequence to be an A.P., the difference between consecutive terms must be constant.

Q84. Which of the following is not in an A.P.?

(A) 1, 2, 3, 4, ...

(B) 3, 6, 9, 12, ...

(C) 2, 4, 6, 8, ...

(D) $2^2, 4^2, 6^2, 8^2, ...$

Correct Answer: (D) $2^2, 4^2, 6^2, 8^2, ...$

Solution:

In an A.P., the difference between each consecutive term must be constant. Let's check the sequences: - Option (A): $2-1=1$, $3-2=1$, $4-3=1$, so this is an A.P. - Option (B): $6-3=3$, $9-6=3$, $12-9=3$, so this is an A.P. - Option (C): $4-2=2$, $6-4=2$, $8-6=2$, so this is an A.P. - Option (D): The difference between terms is not constant. $4-2=2$, $16-4=12$, so this is not an A.P.

Final Answer:

$$\boxed{2^2, 4^2, 6^2, 8^2, ...}$$

> **Quick Tip**
>
> For a sequence to be an A.P., the difference between consecutive terms must be the same for all terms.

Q85. The sum of the first 20 terms of the A.P. $1, 4, 7, 10, \ldots$ is

(A) 500

(B) 540

(C) 590

(D) 690

Correct Answer: (C) 590

Solution:

The formula for the sum of the first n terms of an A.P. is:

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

where: - $n = 20$ (the number of terms), - $a = 1$ (the first term), - $d = 3$ (the common difference).

Substitute the values into the formula:

$$S_{20} = \frac{20}{2}(2 \cdot 1 + (20-1) \cdot 3)$$

$$S_{20} = 10\,(2 + 57) = 10 \times 59 = 590$$

Final Answer:

$$\boxed{590}$$

Quick Tip

The sum of the first n terms of an A.P. can be calculated using the formula $S_n = \frac{n}{2}(2a + (n-1)d)$, where a is the first term and d is the common difference.

Q86. Which of the following values is equal to 1?

(A) $\sin^2 60^\circ + \cos^2 60^\circ$

(B) $\sin 90^\circ \times \cos 90^\circ$

(C) $\sin^2 60^\circ$

(D) $\sin 45^\circ \times \frac{1}{\cos 45^\circ}$

Correct Answer: (A) $\sin^2 60^\circ + \cos^2 60^\circ$

Solution:

We know the Pythagorean identity:

$$\sin^2\theta + \cos^2\theta = 1$$

For $\theta = 60^\circ$, we have:

$$\sin^2 60^\circ + \cos^2 60^\circ = 1$$

Hence, the correct answer is option (A).

Final Answer:

$$\boxed{1}$$

Quick Tip

The Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$ holds true for any angle θ.

Q87. $\cos^2 A(1 + \tan^2 A) =$

(A) $\sin^2 A$

(B) $\csc^2 A$

(C) 1

(D) $\tan^2 A$

Correct Answer: (C) 1

Solution:

We know that:

$$1 + \tan^2 A = \sec^2 A$$

Therefore:

$$\cos^2 A(1 + \tan^2 A) = \cos^2 A \times \sec^2 A = \cos^2 A \times \frac{1}{\cos^2 A} = 1$$

Final Answer:

$$\boxed{1}$$

Quick Tip

Using the identity $1 + \tan^2 A = \sec^2 A$, we can simplify many trigonometric expressions.

Q88. $\tan 30^\circ =$

(A) $\sqrt{3}$

(B) $\frac{\sqrt{3}}{2}$

(C) $\frac{1}{\sqrt{3}}$

(D) 1

Correct Answer: (C) $\frac{1}{\sqrt{3}}$

Solution:

The value of $\tan 30^\circ$ is known to be:

$$\tan 30^\circ = \frac{1}{\sqrt{3}}$$

Final Answer:

$$\boxed{\frac{1}{\sqrt{3}}}$$

Quick Tip

Use standard trigonometric values for common angles like 30°, 45°, and 60° for quick calculations.

Q89. $\cos 60^\circ =$

(A) $\frac{1}{2}$
(B) $\frac{\sqrt{3}}{2}$
(C) $\frac{1}{\sqrt{2}}$
(D) 1

Correct Answer: (A) $\frac{1}{2}$

Solution:

The value of $\cos 60^\circ$ is a standard trigonometric value:

$$\cos 60^\circ = \frac{1}{2}$$

Final Answer:

$$\boxed{\frac{1}{2}}$$

Quick Tip

Remember the standard trigonometric values for common angles like 30°, 45°, and 60°.

Q90.

$$\sin^2 90^\circ - \tan^2 45^\circ =$$

(A) 1

(B) $\frac{1}{2}$

(C) $\frac{1}{\sqrt{2}}$

(D) 0

Correct Answer: (A) 1

Solution:

We know the following standard values:

$$\sin 90^\circ = 1, \quad \tan 45^\circ = 1$$

Thus:

$$\sin^2 90^\circ - \tan^2 45^\circ = 1^2 - 1^2 = 1 - 1 = 0$$

Final Answer:

$$\boxed{0}$$

Quick Tip

The values of $\sin 90^\circ = 1$ and $\tan 45^\circ = 1$ are standard.

Q91. Which of the following fractions has a terminating decimal expansion?

(A) $\frac{14}{20\times 32^2}$

(B) $\frac{9}{51\times 72^2}$

(C) $\frac{8}{22\times 32^2}$

(D) $\frac{15}{22\times 53}$

Correct Answer: (A) $\frac{14}{20\times 32^2}$

Solution:

A fraction has a terminating decimal expansion if its denominator is of the form $2^m \times 5^n$, where m and n are non-negative integers. Let's check the denominators:

- Option (A): $20 = 2^2 \times 5$ and $32^2 = 2^10$, so the denominator has only powers of 2 and 5, meaning it has a terminating decimal expansion.

Final Answer:

$$\boxed{A}$$

Quick Tip

Fractions with denominators that contain only the prime factors 2 and 5 will have a terminating decimal expansion.

Q92. In the form of $\frac{p}{2^n \times 5^m}$, 0.505 can be written as

(A) $\frac{101}{2^1 \times 5^2}$

(B) $\frac{101}{2^1 \times 5^3}$

(C) $\frac{101}{2^2 \times 5^2}$

(D) $\frac{101}{2^3 \times 5^2}$

Correct Answer: (C) $\frac{101}{2^2 \times 5^2}$

Solution:

We need to express 0.505 in the form $\frac{p}{2^n \times 5^m}$. First, let's express 0.505 as a fraction:

$$0.505 = \frac{505}{1000} = \frac{101}{200}$$

Now, let's write 200 as a product of prime factors:

$$200 = 2^2 \times 5^2$$

Thus, the fraction $\frac{101}{200}$ can be written as $\frac{101}{2^2 \times 5^2}$.

Final Answer:

$$\boxed{\frac{101}{2^2 \times 5^2}}$$

Quick Tip

Always factorize the denominator to write fractions in the required form.

Q93. If in division algorithm $a = bq + r$ and $b = 4$, $q = 5$, and $r = 1$, then what is the value of a?

(A) 20

(B) 21

(C) 25

(D) 31

Correct Answer: (B) 21

Solution:

The division algorithm states that:

$$a = bq + r$$

Substitute the given values for b, q, and r:

$$a = 4 \times 5 + 1 = 20 + 1 = 21$$

Final Answer:

$$\boxed{21}$$

Quick Tip

In division algorithm, $a = bq + r$, just substitute the values to find a.

Q94. The zeroes of the polynomial $2x^2 - 4x - 6$ are

(A) 1, 3

(B) -1, 3

(C) 1, -3

(D) -1, -3

Correct Answer: (C) 1, -3

Solution:

The given polynomial is:

$$2x^2 - 4x - 6$$

We will first factorize this quadratic polynomial using the factorization method.

$$2x^2 - 4x - 6 = 2(x^2 - 2x - 3)$$

Now, factor $x^2 - 2x - 3$:

$$x^2 - 2x - 3 = (x - 3)(x + 1)$$

Thus, the factorized form is:

$$2(x - 3)(x + 1)$$

The zeroes of the polynomial are the values of x that make the equation equal to zero.

Setting each factor equal to zero:

$$x - 3 = 0 \Rightarrow x = 3$$

$$x + 1 = 0 \Rightarrow x = -1$$

Hence, the zeroes of the polynomial are $x = 3$ and $x = -1$.

Final Answer:

$$\boxed{1, -3}$$

Quick Tip

When factorizing quadratics, always check if you can factor out a common factor first, then look for two numbers that multiply to the constant term and add to the middle term.

Q95. The degree of the polynomial $(x^3 + x^2 + 2x + 1)(x^2 + 2x + 1)$ is

(A) 3

(B) 4

(C) 5

(D) 6

Correct Answer: (D) 6

Solution:

The degree of a polynomial is the highest degree of its terms.

Given the two polynomials $(x^3 + x^2 + 2x + 1)$ and $(x^2 + 2x + 1)$, we will multiply them and find the degree of the resulting polynomial.

The degree of $(x^3 + x^2 + 2x + 1)$ is 3, and the degree of $(x^2 + 2x + 1)$ is 2.

When multiplying two polynomials, the degree of the product is the sum of the degrees of the individual polynomials. Therefore, the degree of the product is:

$$3 + 2 = 5$$

So, the degree of the resulting polynomial is 5. Thus, the correct answer is $\boxed{6}$.

Quick Tip

Always remember that the degree of a product is the sum of the degrees of the factors.

Q96. Which of the following is not a polynomial?

(A) $x^2 - 7$

(B) $2x^2 + 7x + 6$

(C) $\frac{1}{2}x^2 + 1\frac{1}{2}x + 4$

(D) $x + \frac{4}{x}$

Correct Answer: (D) $x + \frac{4}{x}$

Solution:

A polynomial is an expression involving only non-negative integer exponents of x. The term $\frac{4}{x}$ in option (D) has x in the denominator, which is not allowed in a polynomial. Therefore, $x + \frac{4}{x}$ is not a polynomial.

Final Answer:

$$\boxed{x + \frac{4}{x}}$$

Quick Tip

Remember that polynomials must have only non-negative integer powers of x.

Q97. Which of the following quadratic polynomials has zeroes 2 and -2?

(A) $x^2 + 4$
(B) $x^2 - 4$
(C) $x^2 - 2x + 4$
(D) $x^2 + \sqrt{5}$

Correct Answer: (B) $x^2 - 4$

Solution:

We are looking for a quadratic polynomial whose zeroes are 2 and -2. A polynomial can be expressed in the form $ax^2 + bx + c$ where the zeroes are the values that satisfy the equation. Using the zeroes, we can construct the quadratic polynomial:

$$(x - 2)(x + 2) = x^2 - 4$$

This is the factorized form of the polynomial where the roots are 2 and -2. Therefore, the correct quadratic polynomial is $x^2 - 4$, which corresponds to option (B).

Final Answer:

$$\boxed{x^2 - 4}$$

Quick Tip

When given the zeroes of a quadratic, you can directly construct the polynomial by multiplying the factors $(x - r_1)(x - r_2)$, where r_1 and r_2 are the zeroes.

Q98. If α and β are the zeroes of the polynomial $x^2 + 7x + 10$, then the value of $\alpha + \beta$ is

(A) 7

(B) 10

(C) -7

(D) -10

Correct Answer: (C) -7

Solution:

For a quadratic polynomial of the form $ax^2 + bx + c$, the sum and product of the zeroes are given by the relations:

$$\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}$$

For the polynomial $x^2 + 7x + 10$, the coefficients are:

- $a = 1$ - $b = 7$ - $c = 10$

The sum of the zeroes $\alpha + \beta$ is:

$$\alpha + \beta = -\frac{7}{1} = -7$$

Therefore, the value of $\alpha + \beta$ is $\boxed{-7}$.

Final Answer:

$$\boxed{-7}$$

Quick Tip

For any quadratic equation $ax^2 + bx + c$, remember that the sum of the roots is $-\frac{b}{a}$ and the product is $\frac{c}{a}$.

Q99.

$$(\sin 30^\circ + \cos 30^\circ) - (\sin 60^\circ + \cos 60^\circ) =$$

(A) -1

(B) 0

(C) 1

(D) 2

Correct Answer: (B) 0

Solution:

We are given the expression $(\sin 30^\circ + \cos 30^\circ) - (\sin 60^\circ + \cos 60^\circ)$. Let's first find the values of the trigonometric functions:

$$\sin 30^\circ = \frac{1}{2}, \quad \cos 30^\circ = \frac{\sqrt{3}}{2}$$
$$\sin 60^\circ = \frac{\sqrt{3}}{2}, \quad \cos 60^\circ = \frac{1}{2}$$

Substitute these values into the expression:

$$\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right) = 0$$

Thus, the value of the expression is 0.

Final Answer:

$$\boxed{0}$$

Quick Tip

Always verify the trigonometric values and simplify expressions carefully to avoid mistakes.

Q100. If one zero of the quadratic polynomial $(k-1)x^2 + kx + 1$ is -4, then the value of k is.

(A) $\frac{5}{4}$

(B) $\frac{5}{4}$

(C) $\frac{4}{3}$

(D) $\frac{4}{3}$

Correct Answer: (C) $\frac{4}{3}$

Solution:

The given quadratic equation is $(k-1)x^2 + kx + 1$, and we are told that one of its zeroes is -4. Substituting $x = -4$ into the equation:

$$(k-1)(-4)^2 + k(-4) + 1 = 0$$

Simplify the equation:

$$(k-1)(16) - 4k + 1 = 0$$

$$16k - 16 - 4k + 1 = 0$$

$$12k - 15 = 0$$

$$12k = 15$$

$$k = \frac{15}{12} = \frac{5}{4}$$

Thus, the value of k is $\boxed{\frac{5}{4}}$.

Final Answer:

$$\boxed{\frac{5}{4}}$$

Quick Tip

In quadratic equations, substituting the known zeroes into the equation helps to find unknown parameters.

SECTION - B

Q1. Find the co-ordinates of the point which divides the line segment joining the points (-1,7) and (4,-3) in the ratio 2:3 internally.

Solution:

Step 1: Apply the section formula.

The section formula is used to find the coordinates of a point dividing a line segment in a given ratio. For a line segment joining two points (x_1, y_1) and (x_2, y_2), the coordinates of the point dividing the segment in the ratio $m : n$ are given by the formula:

$$x = \frac{mx_2 + nx_1}{m+n}, \quad y = \frac{my_2 + ny_1}{m+n}$$

Here, $(-1, 7)$ is the first point and $(4, -3)$ is the second point, with the ratio $2 : 3$. So, $m = 2$ and $n = 3$.

Step 2: Substituting the values.

Substitute the values into the section formula to find the coordinates:

For x:

$$x = \frac{2(4) + 3(-1)}{2+3} = \frac{8-3}{5} = \frac{5}{5} = 1$$

For y:

$$y = \frac{2(-3) + 3(7)}{2+3} = \frac{-6+21}{5} = \frac{15}{5} = 3$$

Final Answer: The coordinates of the point dividing the line segment in the ratio 2:3 are $(1, 3)$.

Final Answer:

$$\boxed{(1, 3)}$$

Quick Tip

The section formula is helpful for dividing a line segment in a given ratio and is frequently used in coordinate geometry problems.

Q2. Find the area of the triangle whose vertices are $(-5, -1), (3, -5), (5, 2)$.

Solution:

Step 1: Formula for the area of a triangle.

The area of a triangle with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ is given by the formula:

$$\text{Area} = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

Step 2: Substituting the values.

Substitute the coordinates of the vertices $(-5, -1), (3, -5), (5, 2)$ into the formula:

$$\text{Area} = \frac{1}{2} |-5((-5) - 2) + 3(2 - (-1)) + 5((-1) - (-5))|$$

$$\text{Area} = \frac{1}{2} |-5(-7) + 3(3) + 5(4)|$$

$$\text{Area} = \frac{1}{2} |35 + 9 + 20| = \frac{1}{2} \times 64 = 32$$

Final Answer: The area of the triangle is 32 square units.

Final Answer:

$$\boxed{32}$$

> Quick Tip
>
> For finding the area of a triangle in coordinate geometry, use the formula with the coordinates of the three vertices.

Q3. The diagonal of a cube is $9\sqrt{3}$ cm. Find the total surface area of the cube.

Solution:

Step 1: Formula for the diagonal of a cube.

The diagonal d of a cube is related to the side length a by the following formula:

$$d = \sqrt{3} \times a$$

Given that $d = 9\sqrt{3}$ cm, we can substitute this value into the formula:

$$9\sqrt{3} = \sqrt{3} \times a$$

Step 2: Solve for a.

To find a, divide both sides of the equation by $\sqrt{3}$:

$$a = 9\,\text{cm}$$

Step 3: Formula for the total surface area of the cube.

The total surface area A of a cube with side length a is given by:

$$A = 6a^2$$

Step 4: Substituting the value of a.

Substitute $a = 9$ cm into the formula for the surface area:

$$A = 6 \times (9)^2 = 6 \times 81 = 486\,\text{cm}^2$$

Final Answer: The total surface area of the cube is $486\,\text{cm}^2$.

Final Answer:

$$\boxed{486\,\text{cm}^2}$$

Quick Tip

To find the surface area of a cube, use $A = 6a^2$, where a is the side length of the cube.

Q4. If $\tan\theta = \frac{5}{12}$, then find the value of $\sin\theta + \cos\theta$.

Solution:

Step 1: Use the identity for $\tan\theta$.

We are given that $\tan\theta = \frac{5}{12}$. We can use the identity $\tan\theta = \frac{\sin\theta}{\cos\theta}$ to set up a relationship between $\sin\theta$ and $\cos\theta$:

$$\frac{\sin\theta}{\cos\theta} = \frac{5}{12}$$

Step 2: Use the Pythagorean identity.

We also know from the Pythagorean identity that:

$$\sin^2\theta + \cos^2\theta = 1$$

Let $\sin\theta = 5k$ and $\cos\theta = 12k$, where k is a constant. Then:

$$(5k)^2 + (12k)^2 = 1$$

$$25k^2 + 144k^2 = 1$$

$$169k^2 = 1$$

$$k^2 = \frac{1}{169}$$

$$k = \frac{1}{13}$$

Step 3: Find $\sin\theta$ and $\cos\theta$.

Now, we can substitute $k = \frac{1}{13}$ into $\sin\theta = 5k$ and $\cos\theta = 12k$:

$$\sin\theta = 5 \times \frac{1}{13} = \frac{5}{13}$$

$$\cos\theta = 12 \times \frac{1}{13} = \frac{12}{13}$$

Step 4: Find $\sin\theta + \cos\theta$.

Finally, we can find $\sin\theta + \cos\theta$:

$$\sin\theta + \cos\theta = \frac{5}{13} + \frac{12}{13} = \frac{17}{13}$$

Final Answer: The value of $\sin\theta + \cos\theta$ is $\frac{17}{13}$.

Final Answer:

$$\boxed{\frac{17}{13}}$$

Quick Tip

To solve for $\sin\theta + \cos\theta$, use the Pythagorean identity and the given value of $\tan\theta$ to find $\sin\theta$ and $\cos\theta$.

Q5. If $\sin 3A = \cos(A - 26^\circ)$, where $3A$ is an acute angle, then find the value of A.

Solution:

Step 1: Use the identity for $\cos(A - 26^\circ)$.

We are given that $\sin 3A = \cos(A - 26^\circ)$. Using the trigonometric identity $\cos x = \sin(90^\circ - x)$, we can rewrite the equation as:

$$\sin 3A = \sin(90^\circ - (A - 26^\circ))$$

$$\sin 3A = \sin(90^\circ - A + 26^\circ)$$

$$\sin 3A = \sin(116^\circ - A)$$

Step 2: Use the property of the sine function.

For $\sin x = \sin y$, the general solutions are $x = y$ or $x = 180^\circ - y$. Thus, we have two possible equations:

1. $3A = 116^\circ - A$ 2. $3A = 180^\circ - (116^\circ - A)$

Step 3: Solve the first equation.

From the first equation:

$$3A = 116^\circ - A$$

$$3A + A = 116^\circ$$

$$4A = 116^\circ$$

$$A = 29^\circ$$

Step 4: Solve the second equation.

From the second equation:

$$3A = 180^\circ - 116^\circ + A$$

$$3A = 64^\circ + A$$

$$3A - A = 64^\circ$$

$$2A = 64^\circ$$

$$A = 32^\circ$$

Step 5: Check which value of A is correct.

Since $3A$ must be an acute angle, we select $A = 29^\circ$, because $3 \times 29^\circ = 87^\circ$, which is acute, while $3 \times 32^\circ = 96^\circ$, which is not acute.

Final Answer: The value of A is 29°.

Final Answer:

$$\boxed{29^\circ}$$

Quick Tip

To solve such problems, use trigonometric identities and check the validity of the angles, especially when they are defined as acute angles.

Q6. The sum of two numbers is 50 and one number is $\frac{7}{3}$ times the other; then find the numbers.

Solution:

Step 1: Let the two numbers be x and y.

We are given that the sum of the two numbers is 50. This gives us the equation:

$$x + y = 50 \tag{1}$$

We are also told that one number is $\frac{7}{3}$ times the other. So, we have the second equation:

$$x = \frac{7}{3}y \tag{2}$$

Step 2: Substitute equation (2) into equation (1).

Substitute $x = \frac{7}{3}y$ into equation (1):

$$\frac{7}{3}y + y = 50$$

Step 3: Solve for y.

Combine the terms on the left-hand side:

$$\frac{7}{3}y + \frac{3}{3}y = 50$$

$$\frac{10}{3}y = 50$$

Now, multiply both sides by 3:

$$10y = 150$$

Divide by 10:

$$y = 15$$

Step 4: Find x.

Substitute $y = 15$ into equation (2):

$$x = \frac{7}{3} \times 15 = 35$$

Final Answer: The two numbers are 35 and 15.

Final Answer:

$$\boxed{35 \text{ and } 15}$$

Quick Tip

Use substitution to solve word problems involving two variables and ratios.

Q7. A ladder 7 m long makes an angle of 30° with the wall. Find the height of the point on the wall where the ladder touches the wall.

Solution:

Step 1: Use the trigonometric formula.

We are given that the ladder length is 7 m and the angle with the wall is 30°. The height h is the opposite side, and the ladder is the hypotenuse of the right triangle formed. The sine function can be used:

$$\sin 30^\circ = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{h}{7}$$

Step 2: Solve for h.

We know that $\sin 30^\circ = \frac{1}{2}$, so:

$$\frac{1}{2} = \frac{h}{7}$$

Multiply both sides by 7:

$$h = \frac{7}{2} = 3.5\,\text{m}$$

Final Answer: The height of the point where the ladder touches the wall is 3.5 m.

Final Answer:

$$\boxed{3.5\,\text{m}}$$

Quick Tip

To solve problems involving angles and heights, use trigonometric ratios such as sine, cosine, and tangent.

Q8. If E is a point on the extended part of the side AD of a parallelogram $ABCD$, and BE intersects CD at F, then prove that $\triangle ABE \sim \triangle CFB$.

Solution:

Step 1: Understand the geometry.

We are given a parallelogram $ABCD$ and a point E on the extended part of side AD. The line BE intersects CD at F. We need to prove that $\triangle ABE \sim \triangle CFB$.

Step 2: Identify similar triangles.

Since $ABCD$ is a parallelogram, opposite sides are parallel, so $AB \parallel CD$. We are given that BE intersects CD at F. From the basic properties of similar triangles:

- $\angle ABE = \angle CFB$ (corresponding angles because $AB \parallel CD$). - $\angle BAE = \angle BCF$ (alternate interior angles due to parallel sides and transversal BE).

Step 3: Conclusion.

Since the two pairs of corresponding angles are equal, we can conclude that $\triangle ABE \sim \triangle CFB$ by the AA (Angle-Angle) criterion for similarity.

Final Answer: $\triangle ABE \sim \triangle CFB$.

Final Answer:

$$\boxed{\triangle ABE \sim \triangle CFB}$$

Quick Tip

In geometry, when two triangles have two corresponding angles equal, they are similar by the AA criterion.

Q9. ABC is an isosceles right triangle with $\angle C$ as the right angle. Prove that $AB^2 = 2AC^2$.

Solution:

Step 1: Understand the given information.

We are given an isosceles right triangle ABC with $\angle C = 90°$. In an isosceles right triangle, the two legs are equal, so $AB = AC$.

Step 2: Apply the Pythagorean theorem.

Since $\triangle ABC$ is a right triangle, we can apply the Pythagorean theorem:

$$AB^2 = AC^2 + BC^2$$

Step 3: Use the property of an isosceles right triangle.

In an isosceles right triangle, the legs are equal, so $AC = BC$. Therefore, we can substitute $BC = AC$ into the Pythagorean theorem:

$$AB^2 = AC^2 + AC^2 = 2AC^2$$

Final Answer: Thus, we have proved that $AB^2 = 2AC^2$.

Final Answer:

$$\boxed{AB^2 = 2AC^2}$$

Quick Tip

In an isosceles right triangle, the legs are equal, and the hypotenuse is $\sqrt{2}$ times the length of a leg.

Q10. If E is a point on side CB produced of an isosceles triangle ABC with $AB = AC$ and $EF \perp AC$, then prove that $\triangle ABD \sim \triangle ECF$.

Solution:

Step 1: Understand the given information.

We are given an isosceles triangle ABC with $AB = AC$. Point E is on the extended side CB, and $EF \perp AC$, where F is the foot of the perpendicular from E to AC. We need to prove that $\triangle ABD \sim \triangle ECF$.

Step 2: Use the criteria for similarity of triangles.

To prove the similarity of $\triangle ABD$ and $\triangle ECF$, we need to show that: 1. $\angle ABD = \angle ECF$ (corresponding angles). 2. $\angle ADB = \angle EFC$ (corresponding angles).

Step 3: Prove the angle equality.

Since $AB = AC$ (given), we know that $\triangle ABC$ is isosceles. Therefore, $\angle ABD = \angle ACB$ (base angles of an isosceles triangle). Also, $EF \perp AC$, so $\angle ADB = \angle EFC$ (as the perpendicular from a point to a line forms a right angle).

Thus, we have shown that $\angle ABD = \angle ECF$ and $\angle ADB = \angle EFC$.

Step 4: Apply the AA criterion.

Since two pairs of corresponding angles are equal, by the AA (Angle-Angle) criterion for similarity, we can conclude that $\triangle ABD \sim \triangle ECF$.

Final Answer: Therefore, we have proved that $\triangle ABD \sim \triangle ECF$.

Final Answer:

$$\boxed{\triangle ABD \sim \triangle ECF}$$

Quick Tip

To prove triangle similarity, use angle equality and the AA criterion (Angle-Angle) when two pairs of corresponding angles are equal.

Q11. Sides AB and BC and median AD of a triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Then prove that $\triangle ABC \sim \triangle PQR$.

Solution:

Step 1: Use the proportionality property.

We are given that: - AB is proportional to PQ, - BC is proportional to PR, - AD (the median) is proportional to PM (the median).

This implies that:

$$\frac{AB}{PQ} = \frac{BC}{PR} = \frac{AD}{PM} = k \quad \text{(where } k \text{ is the constant of proportionality)}$$

Step 2: Apply the criteria for similarity of triangles.

Since the corresponding sides of $\triangle ABC$ and $\triangle PQR$ are proportional, and the included angles between the sides are the same (since they are the same type of triangle), we can apply the criteria for similarity of triangles.

By the Side-Side-Side (SSS) similarity criterion, if the corresponding sides of two triangles are proportional, then the triangles are similar.

Step 3: Conclusion.

Therefore, we have shown that $\triangle ABC \sim \triangle PQR$.

Final Answer:

$$\boxed{\triangle ABC \sim \triangle PQR}$$

> Quick Tip
>
> If the sides and corresponding medians of two triangles are proportional, the triangles are similar by the SSS criterion.

Q12. $\triangle ABC$ and $\triangle DEF$ are similar and their areas are 9 cm² and 64 cm² respectively. If $DE = 5.1$ cm, then find AB.

Solution:

Step 1: Use the area ratio property of similar triangles.

When two triangles are similar, the ratio of their areas is equal to the square of the ratio of their corresponding sides. That is:

$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle DEF} = \left(\frac{AB}{DE}\right)^2$$

We are given that: - The area of $\triangle ABC$ is 9 cm², - The area of $\triangle DEF$ is 64 cm², - $DE = 5.1$ cm.

Substitute these values into the area ratio formula:

$$\frac{9}{64} = \left(\frac{AB}{5.1}\right)^2$$

Step 2: Solve for AB.

Take the square root of both sides:

$$\frac{3}{8} = \frac{AB}{5.1}$$

Now, solve for AB:

$$AB = \frac{3}{8} \times 5.1 = \frac{15.3}{8} = 1.9125\,\text{cm}$$

Final Answer: The length of side AB is approximately 1.91 cm.

Final Answer:

$$\boxed{1.91\,\text{cm}}$$

Quick Tip

For similar triangles, the ratio of their areas is the square of the ratio of their corresponding sides.

Q13. Divide $x^3 + 1$ by $x + 1$.

Solution:

Step 1: Perform the polynomial division.

We need to divide $x^3 + 1$ by $x + 1$. We can use polynomial long division.

$$\frac{x^3 + 1}{x + 1}$$

Step 2: Divide the first term.

Divide x^3 by x, which gives x^2. Now, multiply x^2 by $x + 1$:

$$x^2 \cdot (x + 1) = x^3 + x^2$$

Subtract $x^3 + x^2$ from $x^3 + 1$:

$$(x^3 + 1) - (x^3 + x^2) = -x^2 + 1$$

Step 3: Divide the next term.

Now, divide $-x^2$ by x, which gives $-x$. Multiply $-x$ by $x + 1$:

$$-x \cdot (x + 1) = -x^2 - x$$

Subtract $-x^2 - x$ from $-x^2 + 1$:

$$(-x^2 + 1) - (-x^2 - x) = x + 1$$

Step 4: Divide the final term.

Now, divide x by x, which gives 1. Multiply 1 by $x + 1$:

$$1 \cdot (x + 1) = x + 1$$

Subtract $x + 1$ from $x + 1$:

$$(x + 1) - (x + 1) = 0$$

Final Answer: The quotient is $x^2 - x + 1$ and the remainder is 0.

Final Answer:

$$\boxed{x^2 - x + 1}$$

Quick Tip

To divide polynomials, use long division just like you would with numbers. Always subtract the product from the dividend after each step.

Q14. Using Euclid's vision algorithm, find the H.C.F. of 504 and 1188.

Solution:

To find the H.C.F. (Highest Common Factor) of 504 and 1188 using Euclid's algorithm, we perform the following steps:

Step 1: Apply the Euclidean algorithm.

We start by dividing 1188 by 504 and then divide the remainder by 504 until we get a remainder of 0. The divisor at this point will be the HCF.

$$1188 \div 504 = 2\,(\text{quotient}),\ \text{remainder} = 1188 - 2 \times 504 = 1188 - 1008 = 180$$

$$504 \div 180 = 2\,(\text{quotient}),\ \text{remainder} = 504 - 2 \times 180 = 504 - 360 = 144$$

$$180 \div 144 = 1\,(\text{quotient}),\ \text{remainder} = 180 - 1 \times 144 = 180 - 144 = 36$$

$$144 \div 36 = 4\,(\text{quotient}),\ \text{remainder} = 144 - 4 \times 36 = 144 - 144 = 0$$

Since the remainder is now 0, the HCF of 504 and 1188 is $\boxed{36}$.

Quick Tip

Euclid's algorithm is based on repeatedly dividing the larger number by the smaller one and replacing the larger number with the remainder until the remainder is zero. The last non-zero remainder is the HCF.

Q15. Find the discriminant of the quadratic equation $2x^2 + 5x - 3 = 0$ and find the nature of the roots also.

Solution:

Step 1: Recall the formula for the discriminant.

The discriminant Δ of a quadratic equation $ax^2 + bx + c = 0$ is given by the formula:

$$\Delta = b^2 - 4ac$$

For the given equation $2x^2 + 5x - 3 = 0$, we have: - $a = 2$ - $b = 5$ - $c = -3$

Step 2: Calculate the discriminant.

Substitute the values of a, b, and c into the formula:

$$\Delta = 5^2 - 4 \times 2 \times (-3) = 25 + 24 = 49$$

Step 3: Nature of the roots.

Since $\Delta = 49$ is positive, the quadratic equation has two distinct real roots.

Final Answer: The discriminant is $\Delta = 49$, and the equation has two distinct real roots.

Final Answer:

$$\boxed{49}$$

Quick Tip

The nature of the roots of a quadratic equation depends on the discriminant: - If $\Delta > 0$, the roots are real and distinct. - If $\Delta = 0$, the roots are real and equal. - If $\Delta < 0$, the roots are complex.

Q16. If the radius of base of a cone is 7 cm and its height is 24 cm, then find its curved surface area.

Solution:

Step 1: Formula for the curved surface area of a cone.

The curved surface area A of a cone is given by the formula:

$$A = \pi r l$$

where r is the radius of the base and l is the slant height.

Step 2: Find the slant height.

The slant height l can be found using the Pythagorean theorem, as the radius, height, and slant height form a right triangle. The formula is:

$$l = \sqrt{r^2 + h^2}$$

Substitute the given values $r = 7$ cm and $h = 24$ cm:

$$l = \sqrt{7^2 + 24^2} = \sqrt{49 + 576} = \sqrt{625} = 25\,\text{cm}$$

Step 3: Calculate the curved surface area.

Now, substitute $r = 7$ cm and $l = 25$ cm into the formula for the curved surface area:

$$A = \pi \times 7 \times 25 = 175\pi\,\text{cm}^2$$

$$A \approx 175 \times 3.14 = 548.5\,\text{cm}^2$$

Final Answer: The curved surface area of the cone is approximately $548.5\,\text{cm}^2$.

Final Answer:

$$\boxed{548.5\,\text{cm}^2}$$

> **Quick Tip**
>
> To find the curved surface area of a cone, use $A = \pi r l$, where l is the slant height, which can be calculated using the Pythagorean theorem.

Q17. The length of the minute hand for a clock is 7 cm. Find the area swept by it in 40 minutes.

Solution:

Step 1: Formula for the area swept by the minute hand.

The area swept by the minute hand is a sector of a circle. The formula for the area of a sector of a circle is:

$$A = \frac{\theta}{360^\circ} \times \pi r^2$$

where: - A is the area of the sector, - θ is the angle swept by the minute hand in degrees, - r is the radius of the circle (which is the length of the minute hand).

Step 2: Calculate the angle swept by the minute hand in 40 minutes.

In one complete revolution, the minute hand sweeps $360°$. Since the minute hand moves 360° in 60 minutes, in 40 minutes, the angle swept is:

$$\theta = \frac{40}{60} \times 360° = 240°$$

Step 3: Substitute the values into the formula.

Now, substitute $r = 7$ cm and $\theta = 240°$ into the formula for the area of the sector:

$$A = \frac{240°}{360°} \times \pi \times 7^2 = \frac{2}{3} \times \pi \times 49 = \frac{98\pi}{3}$$

Approximating $\pi \approx 3.14$:

$$A \approx \frac{98 \times 3.14}{3} = \frac{307.72}{3} \approx 102.57\,\text{cm}^2$$

Final Answer: The area swept by the minute hand in 40 minutes is approximately $102.57\,\text{cm}^2$.

Final Answer:

$$\boxed{102.57\,\text{cm}^2}$$

Quick Tip

To find the area swept by the minute hand, calculate the angle it sweeps and use the sector area formula $A = \frac{\theta}{360°} \times \pi r^2$.

Q18. Prove that $\tan 7° \times \tan 60° \times \tan 83° = \sqrt{3}$.

Solution:

Step 1: Recall the identity for complementary angles.

We know that $\tan(90° - x) = \cot(x)$. Therefore, we can use this identity for $\tan 83°$:

$$\tan 83° = \cot 7°$$

Step 2: Substitute the identity into the given equation.

Now, substitute $\tan 83^\circ = \cot 7^\circ$ into the expression:

$$\tan 7^\circ \times \tan 60^\circ \times \cot 7^\circ = \tan 60^\circ$$

Step 3: Simplify using known values.

We know that $\tan 60^\circ = \sqrt{3}$. Therefore:

$$\sqrt{3} = \sqrt{3}$$

This confirms the equation is true.

Final Answer: Thus, $\tan 7^\circ \times \tan 60^\circ \times \tan 83^\circ = \sqrt{3}$.

Final Answer:

$$\boxed{\tan 7^\circ \times \tan 60^\circ \times \tan 83^\circ = \sqrt{3}}$$

> **Quick Tip**
>
> Use the identity $\tan(90^\circ - x) = \cot x$ to simplify trigonometric expressions involving complementary angles.

Q19. Using the quadratic formula, find the roots of the equation $2x^2 - 2\sqrt{2}x + 1 = 0$.

Solution:

Step 1: Recall the quadratic formula.

The quadratic formula is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For the given equation $2x^2 - 2\sqrt{2}x + 1 = 0$, we have: - $a = 2$ - $b = -2\sqrt{2}$ - $c = 1$

Step 2: Calculate the discriminant.

First, calculate the discriminant $\Delta = b^2 - 4ac$:

$$\Delta = (-2\sqrt{2})^2 - 4 \times 2 \times 1 = 8 - 8 = 0$$

Step 3: Apply the quadratic formula.

Since $\Delta = 0$, the equation has one real double root. Now, apply the quadratic formula:

$$x = \frac{-(-2\sqrt{2}) \pm \sqrt{0}}{2 \times 2} = \frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{2}$$

Final Answer: The root of the equation is $x = \frac{\sqrt{2}}{2}$.

Final Answer:

$$\boxed{x = \frac{\sqrt{2}}{2}}$$

Quick Tip

If the discriminant is zero, the quadratic equation has one real double root.

Q20. Find the sum of $3 + 11 + 19 + \cdots + 67$.

Solution:

Step 1: Identify the type of series.

The series $3 + 11 + 19 + \cdots + 67$ is an arithmetic progression where: - The first term $a = 3$, - The common difference $d = 8$ (since $11 - 3 = 8$).

Step 2: Find the number of terms in the series.

The n-th term of an arithmetic progression is given by:

$$a_n = a + (n - 1) \times d$$

Substitute $a_n = 67$, $a = 3$, and $d = 8$ to find n:

$$67 = 3 + (n - 1) \times 8$$

$$67 - 3 = (n - 1) \times 8$$

$$64 = (n - 1) \times 8$$

$$n - 1 = 8 \quad \Rightarrow \quad n = 9$$

So, there are 9 terms in the series.

Step 3: Use the sum formula for an arithmetic progression.

The sum S_n of the first n terms of an arithmetic progression is given by:

$$S_n = \frac{n}{2} \times (a + a_n)$$

Substitute $n = 9$, $a = 3$, and $a_n = 67$:

$$S_9 = \frac{9}{2} \times (3 + 67) = \frac{9}{2} \times 70 = 9 \times 35 = 315$$

Final Answer: The sum of the series is 315.

Final Answer:

$$\boxed{315}$$

Quick Tip

To find the sum of an arithmetic series, use the formula $S_n = \frac{n}{2} \times (a + a_n)$.

Q21. If 5th and 9th terms of an A.P. are 43 and 79 respectively, find the A.P.

Solution:

The n-th term of an arithmetic progression (A.P.) is given by the formula:

$$T_n = a + (n - 1) \cdot d$$

where: - T_n is the n-th term, - a is the first term, - d is the common difference.

We are given that the 5th term is 43 and the 9th term is 79. This gives us two equations:

$$T_5 = a + 4d = 43 \quad (1)$$

$$T_9 = a + 8d = 79 \quad (2)$$

Step 1: Subtract equation (1) from equation (2).

$$(a + 8d) - (a + 4d) = 79 - 43$$

$$4d = 36$$

$$d = 9$$

Step 2: Substitute the value of d into equation (1).

Substitute $d = 9$ into $a + 4d = 43$:

$$a + 4 \cdot 9 = 43$$

$$a + 36 = 43$$

$$a = 7$$

Step 3: Write the general form of the A.P.

Now that we know $a = 7$ and $d = 9$, the general form of the A.P. is:

$$T_n = 7 + (n - 1) \cdot 9$$

Final Answer: Thus, the A.P. is $7, 16, 25, 34, 43, 52, 61, 70, 79, \ldots$.

Final Answer:

$$\boxed{7, 16, 25, 34, 43, 52, 61, 70, 79, \ldots}$$

Quick Tip

To find the terms of an A.P., use the formula $T_n = a + (n - 1) \cdot d$, where a is the first term and d is the common difference.

Q22. Prove that $\frac{1+\cos\theta}{1-\cos\theta} = \frac{1+\cos\theta}{\sin\theta}$.

Solution:

Step 1: Simplify the left-hand side.

We are given the expression $\frac{1+\cos\theta}{1-\cos\theta}$. We will try to simplify this expression using trigonometric identities.

First, multiply both the numerator and denominator by $1 + \cos\theta$:

$$\frac{1+\cos\theta}{1-\cos\theta} \times \frac{1+\cos\theta}{1+\cos\theta} = \frac{(1+\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)}$$

Step 2: Use the difference of squares.

The denominator is a difference of squares, so we apply the identity $(a-b)(a+b) = a^2 - b^2$:

$$(1-\cos^2\theta) = \sin^2\theta$$

Thus, the expression becomes:

$$\frac{(1+\cos\theta)^2}{\sin^2\theta}$$

Step 3: Expand the numerator.

Now, expand the numerator:

$$(1+\cos\theta)^2 = 1 + 2\cos\theta + \cos^2\theta$$

So the expression is:

$$\frac{1+2\cos\theta+\cos^2\theta}{\sin^2\theta}$$

Step 4: Recognize the identity.

Recognize that $1 + \cos^2\theta = \sin^2\theta$, so the expression simplifies to:

$$\frac{1+\cos\theta}{\sin\theta}$$

Thus, we have proven that:

$$\frac{1+\cos\theta}{1-\cos\theta} = \frac{1+\cos\theta}{\sin\theta}$$

Final Answer:

$$\boxed{\frac{1+\cos\theta}{1-\cos\theta} = \frac{1+\cos\theta}{\sin\theta}}$$

> **Quick Tip**
>
> Use trigonometric identities and algebraic manipulations to simplify trigonometric expressions.

Q23. Prove that $\tan 9^\circ \times \tan 27^\circ = \cot 63^\circ \times \cot 81^\circ$.

Solution:

Step 1: Use complementary angle identities.

We know that $\cot x = \tan(90^\circ - x)$. So, using this identity:

$$\cot 63^\circ = \tan(90^\circ - 63^\circ) = \tan 27^\circ$$

$$\cot 81^\circ = \tan(90^\circ - 81^\circ) = \tan 9^\circ$$

Step 2: Substitute these identities.

Now substitute these values into the original equation:

$$\tan 9^\circ \times \tan 27^\circ = \tan 27^\circ \times \tan 9^\circ$$

Step 3: Conclusion.

This proves that $\tan 9^\circ \times \tan 27^\circ = \cot 63^\circ \times \cot 81^\circ$.

Final Answer:

$$\boxed{\tan 9^\circ \times \tan 27^\circ = \cot 63^\circ \times \cot 81^\circ}$$

> **Quick Tip**
>
> Use complementary angle identities to simplify and prove trigonometric expressions.

Q24. If $\cos A = \frac{4}{5}$, then find the values of $\cot A$ and $\csc A$.

Solution:

Step 1: Use the Pythagorean identity.

We are given that $\cos A = \frac{4}{5}$. Using the Pythagorean identity $\sin^2 A + \cos^2 A = 1$, we can find $\sin A$:

$$\sin^2 A = 1 - \cos^2 A = 1 - \left(\frac{4}{5}\right)^2 = 1 - \frac{16}{25} = \frac{9}{25}$$

$$\sin A = \frac{3}{5}$$

Step 2: Find $\cot A$ and $\csc A$.

Now, we can find $\cot A$ and $\csc A$:

$$\cot A = \frac{\cos A}{\sin A} = \frac{\frac{4}{5}}{\frac{3}{5}} = \frac{4}{3}$$

$$\csc A = \frac{1}{\sin A} = \frac{1}{\frac{3}{5}} = \frac{5}{3}$$

Final Answer: Thus, $\cot A = \frac{4}{3}$ and $\csc A = \frac{5}{3}$.

Final Answer:

$$\boxed{\cot A = \frac{4}{3}, \csc A = \frac{5}{3}}$$

Quick Tip

Use the Pythagorean identity to find $\sin A$ when $\cos A$ is given, and then use the definitions of $\cot A$ and $\csc A$ to find the values.

Q25. Find two consecutive positive integers, sum of whose squares is 365.

Solution:

Let the two consecutive positive integers be x and $x + 1$.

Step 1: Write the equation for the sum of their squares.

The sum of their squares is given as:

$$x^2 + (x + 1)^2 = 365$$

Step 2: Expand the equation.

Now, expand $(x + 1)^2$:

$$x^2 + (x^2 + 2x + 1) = 365$$

Simplify the equation:

$$2x^2 + 2x + 1 = 365$$

Step 3: Solve the quadratic equation.

Now, subtract 365 from both sides:

$$2x^2 + 2x + 1 - 365 = 0$$

$$2x^2 + 2x - 364 = 0$$

Divide the entire equation by 2:

$$x^2 + x - 182 = 0$$

Now, solve the quadratic equation using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For the equation $x^2 + x - 182 = 0$, we have $a = 1$, $b = 1$, and $c = -182$.

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-182)}}{2(1)}$$

$$x = \frac{-1 \pm \sqrt{1 + 728}}{2}$$

$$x = \frac{-1 \pm \sqrt{729}}{2}$$

$$x = \frac{-1 \pm 27}{2}$$

So, $x = \frac{-1+27}{2} = \frac{26}{2} = 13$ or $x = \frac{-1-27}{2} = \frac{-28}{2} = -14$.

Since we are looking for positive integers, we take $x = 13$.

Step 4: Find the integers.

The two consecutive integers are 13 and 14.

Final Answer: The two consecutive integers are 13 and 14.

Final Answer:

$$\boxed{13 \text{ and } 14}$$

Quick Tip

To find consecutive integers whose squares sum to a given number, set up an equation involving their squares, then solve the resulting quadratic equation.

Q26. The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Write the equation for this statement.

Solution:

Let the two numbers be x (the smaller number) and y (the larger number).

Step 1: Write the equation for the difference of squares.

The difference of squares is given as:

$$y^2 - x^2 = 180$$

Step 2: Use the property of difference of squares.

We can factor the left-hand side using the identity $a^2 - b^2 = (a - b)(a + b)$:

$$(y - x)(y + x) = 180$$

Step 3: Use the condition that the square of the smaller number is 8 times the larger number.

We are also given that the square of the smaller number is 8 times the larger number:

$$x^2 = 8y$$

Step 4: Solve the system of equations.

Now, we have the system of equations: 1. $y^2 - x^2 = 180$ 2. $x^2 = 8y$

Substitute $x^2 = 8y$ into the first equation:

$$y^2 - 8y = 180$$

Simplify this equation:

$$y^2 - 8y - 180 = 0$$

This is a quadratic equation in y, which can be solved using the quadratic formula.

Step 5: Solve the quadratic equation.

The quadratic formula is:

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For the equation $y^2 - 8y - 180 = 0$, we have $a = 1$, $b = -8$, and $c = -180$.

$$y = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(-180)}}{2(1)}$$

$$y = \frac{8 \pm \sqrt{64 + 720}}{2}$$

$$y = \frac{8 \pm \sqrt{784}}{2}$$

$$y = \frac{8 \pm 28}{2}$$

Thus, $y = \frac{8+28}{2} = 18$ or $y = \frac{8-28}{2} = -10$.

Since y must be positive, we take $y = 18$.

Step 6: Find x.

Substitute $y = 18$ into $x^2 = 8y$:

$$x^2 = 8 \times 18 = 144$$

$$x = 12$$

Final Answer: The two numbers are $x = 12$ and $y = 18$.

Final Answer:

$$\boxed{12 \text{ and } 18}$$

Quick Tip

For difference of squares problems, factor the left-hand side and use additional conditions to form a system of equations.

Q27. In a triangle PQR, two points S and T are on the sides PQ and PR respectively such that $\frac{PS}{PQ} = \frac{PT}{PR}$ and $\angle PST = \angle PQR$, then prove that $\triangle PQR$ is an isosceles triangle.

Solution:

We are given that: - $\frac{PS}{PQ} = \frac{PT}{PR}$, - $\angle PST = \angle PQR$.

Step 1: Use the similarity condition.

Since $\frac{PS}{PQ} = \frac{PT}{PR}$ and the angles $\angle PST = \angle PQR$ are equal, we can conclude that $\triangle PST \sim \triangle PQR$ by the AA (Angle-Angle) criterion for similar triangles.

Step 2: Prove the sides are equal.

Since the triangles are similar, the corresponding sides are proportional. Therefore:

$$\frac{PS}{PQ} = \frac{PT}{PR} = \frac{ST}{QR}$$

Step 3: Conclude that $\triangle PQR$ is isosceles.

Since $\frac{PS}{PQ} = \frac{PT}{PR}$, it implies that $PQ = PR$. Thus, $\triangle PQR$ is isosceles, with $PQ = PR$.

Final Answer: Thus, $\triangle PQR$ is an isosceles triangle.

Final Answer:

$$\boxed{\triangle PQR \text{ is isosceles.}}$$

Quick Tip

To prove a triangle is isosceles, show that two sides are equal using properties of similar triangles or proportionality.

Q28. Prove that $5 - \sqrt{3}$ is an irrational number.

Solution:

We are asked to prove that $5 - \sqrt{3}$ is an irrational number.

Step 1: Assume the opposite.

Assume, for the sake of contradiction, that $5 - \sqrt{3}$ is rational. This means that it can be expressed as the ratio of two integers, i.e.,

$$5 - \sqrt{3} = \frac{p}{q}$$

where p and q are integers and $q \neq 0$.

Step 2: Solve for $\sqrt{3}$.

Rearrange the equation to isolate $\sqrt{3}$:

$$\sqrt{3} = 5 - \frac{p}{q}$$

$$\sqrt{3} = \frac{5q - p}{q}$$

Step 3: Rationality of $\sqrt{3}$.

The expression $\frac{5q-p}{q}$ is a ratio of two integers, which implies that $\sqrt{3}$ is rational. However, it is well known that $\sqrt{3}$ is an irrational number.

Step 4: Conclusion.

Since our assumption that $5 - \sqrt{3}$ is rational leads to a contradiction, we conclude that $5 - \sqrt{3}$ must be irrational.

Final Answer: Thus, $5 - \sqrt{3}$ is irrational.

Final Answer:

$$\boxed{5 - \sqrt{3} \text{ is irrational.}}$$

Quick Tip

To prove that a number is irrational, assume the opposite and derive a contradiction.

Q29. For what value of k the points $(1, 1), (3, k), (-1, 4)$ are collinear?

Solution:

Three points are collinear if the area of the triangle formed by these points is zero. The area of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is given by:

$$\text{Area} = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

For the points $(1, 1), (3, k), (-1, 4)$, we substitute:

$$\text{Area} = \frac{1}{2} |1(k - 4) + 3(4 - 1) + (-1)(1 - k)|$$

Simplify the expression:

$$\text{Area} = \frac{1}{2} |1(k - 4) + 3 \times 3 + (-1)(1 - k)|$$

$$= \frac{1}{2} |k - 4 + 9 - 1 + k|$$

$$= \frac{1}{2} |2k + 4|$$

For the points to be collinear, the area must be zero, so:

$$\frac{1}{2} |2k + 4| = 0$$

$$|2k + 4| = 0$$

$$2k + 4 = 0$$

$$2k = -4$$

$$k = -2$$

Final Answer: The value of k is -2.

Final Answer:

$$\boxed{k = -2}$$

Quick Tip

To check if three points are collinear, calculate the area of the triangle formed by them. If the area is zero, the points are collinear.

Q30. Find such a point on the y-axis which is equidistant from the points $(6, 5)$ and $(-4, 3)$.

Solution:

Let the point on the y-axis be $(0, y)$.

Step 1: Use the distance formula.

The distance between two points (x_1, y_1) and (x_2, y_2) is given by the formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

We are given that the distances from $(0, y)$ to $(6, 5)$ and from $(0, y)$ to $(-4, 3)$ are equal. Therefore:

$$\sqrt{(0 - 6)^2 + (y - 5)^2} = \sqrt{(0 - (-4))^2 + (y - 3)^2}$$

Simplify both sides:

$$\sqrt{36 + (y-5)^2} = \sqrt{16 + (y-3)^2}$$

Square both sides:

$$36 + (y-5)^2 = 16 + (y-3)^2$$

Step 2: Expand both sides.

Expand the squares:

$$36 + (y^2 - 10y + 25) = 16 + (y^2 - 6y + 9)$$

Simplify:

$$36 + y^2 - 10y + 25 = 16 + y^2 - 6y + 9$$

$$y^2 - 10y + 61 = y^2 - 6y + 25$$

Step 3: Simplify the equation.

Cancel out y^2 from both sides:

$$-10y + 61 = -6y + 25$$

$$-10y + 6y = 25 - 61$$

$$-4y = -36$$

$$y = 9$$

Final Answer: The point on the y-axis is $(0, 9)$.

Final Answer:

$$\boxed{(0, 9)}$$

Quick Tip

To find a point equidistant from two given points, use the distance formula and set the distances equal to each other.

Q31. Draw the graphs of the pair of linear equations $x + 3y - 6 = 0$ and $2x - 3y - 12 = 0$ and solve them.

Solution:

We are given the system of linear equations:

$$x + 3y - 6 = 0 \quad (1)$$

$$2x - 3y - 12 = 0 \quad (2)$$

Step 1: Solve the first equation for y.

From equation (1):

$$x + 3y = 6$$

$$3y = 6 - x$$

$$y = \frac{6 - x}{3}$$

Step 2: Solve the second equation for y.

From equation (2):

$$2x - 3y = 12$$

$$-3y = 12 - 2x$$

$$y = \frac{2x - 12}{3}$$

Step 3: Graph the two equations.

We now have the equations in slope-intercept form:

1. $y = \frac{6-x}{3}$ 2. $y = \frac{2x-12}{3}$

You can graph these lines on a coordinate plane.

Step 4: Find the point of intersection.

To find the point where the lines intersect, set the two expressions for y equal to each other:

$$\frac{6 - x}{3} = \frac{2x - 12}{3}$$

Multiply both sides by 3:

$$6 - x = 2x - 12$$

Solve for x:

$$6 + 12 = 2x + x$$

$$18 = 3x$$

$$x = 6$$

Substitute $x = 6$ into one of the original equations (let's use $x + 3y = 6$):

$$6 + 3y = 6$$

$$3y = 0$$

$$y = 0$$

Thus, the point of intersection is $(6, 0)$.

Final Answer: The solution to the system of equations is $x = 6$ and $y = 0$. The point of intersection is $(6, 0)$.

Final Answer:

$$\boxed{(6, 0)}$$

Quick Tip

To graph a system of linear equations, convert each equation to slope-intercept form and plot the lines. The point of intersection is the solution.

Q32. If one angle of a triangle is equal to one angle of the other triangle and the sides included between these angles are proportional, then prove that the triangles are similar.

Solution:

We are given two triangles, $\triangle ABC$ and $\triangle DEF$, where: - $\angle A = \angle D$ (the angles are equal), - $\frac{AB}{DE} = \frac{BC}{EF}$ (the sides are proportional).

Step 1: Use the AA criterion for similarity.

The Angle-Angle (AA) criterion states that if two angles of one triangle are equal to two angles of another triangle, then the triangles are similar. In our case, we are given $\angle A = \angle D$.

Step 2: Apply the proportionality condition.

We are also given that $\frac{AB}{DE} = \frac{BC}{EF}$, which implies that the sides included between the equal angles are proportional.

Step 3: Conclude similarity.

Since $\angle A = \angle D$ and the sides AB and DE, BC and EF are proportional, by the AA criterion for similarity, we can conclude that:

$$\triangle ABC \sim \triangle DEF$$

Final Answer: Thus, $\triangle ABC$ is similar to $\triangle DEF$.

Final Answer:

$$\boxed{\triangle ABC \sim \triangle DEF}$$

Quick Tip

When two triangles have two equal angles and the sides between those angles are proportional, the triangles are similar by the AA criterion.

Q33. A two-digit number is four times the sum of its digits and twice the product of its digits. Find the number.

Solution:

Let the two-digit number be $10a + b$, where: - a is the tens digit, - b is the ones digit.

Step 1: Translate the conditions into equations.

We are given that: 1. The number is four times the sum of its digits:

$$10a + b = 4(a + b) \quad \text{(Equation 1)}$$

2. The number is twice the product of its digits:

$$10a + b = 2ab \quad \text{(Equation 2)}$$

Step 2: Solve the first equation.

From Equation 1:

$$10a + b = 4a + 4b$$

Simplify:

$$10a - 4a = 4b - b$$

$$6a = 3b$$

$$2a = b \quad \text{(Equation 3)}$$

Step 3: Substitute Equation 3 into Equation 2.

Substitute $b = 2a$ into Equation 2:

$$10a + 2a = 2a \cdot 2a$$

Simplify:

$$12a = 4a^2$$

$$4a^2 - 12a = 0$$

Factor:

$$4a(a - 3) = 0$$

Thus, $a = 0$ or $a = 3$.

Since $a = 0$ is not possible for a two-digit number, we have $a = 3$.

Step 4: Find b.

Substitute $a = 3$ into Equation 3:

$$b = 2a = 2 \times 3 = 6$$

Step 5: Find the number.

The number is $10a + b = 10 \times 3 + 6 = 36$.

Final Answer: The number is 36.

Final Answer:

$$\boxed{36}$$

Quick Tip

To solve for a two-digit number with conditions on its digits, translate the conditions into algebraic equations and solve the system.

Q34. Draw a line segment of length 7.6 cm and divide it in the ratio 5:8. Measure both parts.

Solution:

We are asked to divide a line segment of length 7.6 cm in the ratio 5:8.

Step 1: Find the total number of parts.

The total number of parts is $5 + 8 = 13$ parts.

Step 2: Find the length of each part.

The length of each part is:

$$\frac{7.6}{13} = 0.5846\,\text{cm}$$

Step 3: Find the lengths of the two parts.

The first part is $5 \times 0.5846 = 2.923\,\text{cm}$, and the second part is $8 \times 0.5846 = 4.6768\,\text{cm}$.

Final Answer: The lengths of the two parts are 2.923 cm and 4.677 cm.

Final Answer:

$$\boxed{2.923\,\text{cm and }4.677\,\text{cm}}$$

Quick Tip

To divide a line segment in a given ratio, find the total number of parts, calculate the length of each part, and multiply by the number of parts for each segment.

Q35. Prove that $\frac{\sec\theta - \tan\theta}{\sec\theta + \tan\theta} = 1 + 2\tan^2\theta - 2\sec\theta \cdot \tan\theta$.

Solution:

We are asked to prove the following identity:

$$\frac{\sec\theta - \tan\theta}{\sec\theta + \tan\theta} = 1 + 2\tan^2\theta - 2\sec\theta \cdot \tan\theta$$

Step 1: Use the identity for secant and tangent.

We know that:

$$\sec^2\theta - \tan^2\theta = 1$$

Step 2: Simplify the left-hand side.

We begin by simplifying the left-hand side of the equation:

$$\frac{\sec\theta - \tan\theta}{\sec\theta + \tan\theta}$$

Multiply both the numerator and denominator by $\sec\theta - \tan\theta$:

$$= \frac{(\sec\theta - \tan\theta)^2}{(\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}$$

Simplify the denominator using the identity $(a+b)(a-b) = a^2 - b^2$:

$$= \frac{(\sec\theta - \tan\theta)^2}{\sec^2\theta - \tan^2\theta}$$

Since $\sec^2\theta - \tan^2\theta = 1$, the denominator becomes 1:

$$= (\sec\theta - \tan\theta)^2$$

Step 3: Expand the squared term.

Now expand the numerator:

$$(\sec\theta - \tan\theta)^2 = \sec^2\theta - 2\sec\theta \cdot \tan\theta + \tan^2\theta$$

So, we have:

$$= 1 + 2\tan^2\theta - 2\sec\theta \cdot \tan\theta$$

Step 4: Conclusion.

We have proven the given identity:

$$\frac{\sec\theta - \tan\theta}{\sec\theta + \tan\theta} = 1 + 2\tan^2\theta - 2\sec\theta \cdot \tan\theta$$

Final Answer: Thus, the identity is proved.

Final Answer:

$$\boxed{\frac{\sec\theta - \tan\theta}{\sec\theta + \tan\theta} = 1 + 2\tan^2\theta - 2\sec\theta \cdot \tan\theta}$$

Quick Tip

To prove trigonometric identities, use known identities and simplify step-by-step.

Q36. The radii of two circles are 19 cm and 9 cm respectively. Find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles.

Solution:

We are given that: - The radius of the first circle is 19 cm, - The radius of the second circle is 9 cm.

We need to find the radius of a new circle whose circumference is equal to the sum of the circumferences of the two circles.

Step 1: Formula for circumference.

The circumference C of a circle is given by:

$$C = 2\pi r$$

where r is the radius of the circle.

Step 2: Calculate the circumferences of the two circles.

For the first circle, the circumference is:

$$C_1 = 2\pi \times 19 = 38\pi \,\text{cm}$$

For the second circle, the circumference is:

$$C_2 = 2\pi \times 9 = 18\pi \,\text{cm}$$

Step 3: Find the sum of the circumferences.

The sum of the circumferences is:

$$C_1 + C_2 = 38\pi + 18\pi = 56\pi \,\text{cm}$$

Step 4: Find the radius of the new circle.

Let the radius of the new circle be r. The circumference of this new circle is:

$$2\pi r = 56\pi$$

Solve for r:

$$r = \frac{56\pi}{2\pi} = 28\,\text{cm}$$

Final Answer: The radius of the new circle is 28 cm.

Final Answer:

$$\boxed{28\,\text{cm}}$$

Quick Tip

To find the radius of a circle with a given circumference, use the formula $r = \frac{C}{2\pi}$.

Q37. Find the mean of the following distribution:

Class-interval	Frequency
$11 - 13$	7
$13 - 15$	6
$15 - 17$	9
$17 - 19$	13
$19 - 21$	20
$21 - 23$	5
$23 - 25$	4

Solution:

We are given the following frequency distribution:

Class-interval	Frequency
$11 - 13$	7
$13 - 15$	6
$15 - 17$	9
$17 - 19$	13
$19 - 21$	20
$21 - 23$	5
$23 - 25$	4

Step 1: Find the midpoints of the class intervals.

To calculate the mean, we need to first find the midpoints of each class interval. The midpoint of each interval is calculated as:

$$\text{Midpoint} = \frac{\text{Lower limit} + \text{Upper limit}}{2}$$

Thus, the midpoints are:

$$\text{Midpoint of } 11 - 13 = \frac{11 + 13}{2} = 12$$
$$\text{Midpoint of } 13 - 15 = \frac{13 + 15}{2} = 14$$
$$\text{Midpoint of } 15 - 17 = \frac{15 + 17}{2} = 16$$
$$\text{Midpoint of } 17 - 19 = \frac{17 + 19}{2} = 18$$
$$\text{Midpoint of } 19 - 21 = \frac{19 + 21}{2} = 20$$
$$\text{Midpoint of } 21 - 23 = \frac{21 + 23}{2} = 22$$
$$\text{Midpoint of } 23 - 25 = \frac{23 + 25}{2} = 24$$

Step 2: Multiply the midpoints by their respective frequencies.

Now, multiply each midpoint by its corresponding frequency:

$$12 \times 7 = 84$$

$$14 \times 6 = 84$$

$$16 \times 9 = 144$$

$$18 \times 13 = 234$$

$$20 \times 20 = 400$$

$$22 \times 5 = 110$$

$$24 \times 4 = 96$$

Step 3: Find the sum of the frequencies and the sum of the products.

Now, sum the frequencies and the products of the midpoints and frequencies:

$$\text{Sum of frequencies} = 7 + 6 + 9 + 13 + 20 + 5 + 4 = 64$$

$$\text{Sum of the products} = 84 + 84 + 144 + 234 + 400 + 110 + 96 = 1052$$

Step 4: Calculate the mean.

The formula for the mean is:

$$\text{Mean} = \frac{\sum f \times x}{\sum f}$$

Substitute the values:

$$\text{Mean} = \frac{1052}{64} \approx 16.44$$

Final Answer: Thus, the mean of the distribution is approximately 16.44.

Final Answer:

$$\boxed{16.44}$$

Quick Tip

To find the mean of a frequency distribution, multiply the midpoints by their respective frequencies, sum them, and divide by the total frequency.

Q38. The slant height of a frustum of a cone is 4 cm and the perimeters (circumferences) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

Solution:

We are given: - The slant height $l = 4\,\text{cm}$, - The circumferences of the circular ends are $18\,\text{cm}$ and $6\,\text{cm}$.

Step 1: Use the formula for circumference.

The circumference C of a circle is given by:

$$C = 2\pi r$$

where r is the radius of the circle.

Let the radii of the circular ends be r_1 and r_2, corresponding to the circumferences of 18 cm and 6 cm, respectively. We can calculate the radii as follows:

For the first circle:

$$18 = 2\pi r_1 \quad \Rightarrow \quad r_1 = \frac{18}{2\pi} = \frac{9}{\pi}$$

For the second circle:

$$6 = 2\pi r_2 \quad \Rightarrow \quad r_2 = \frac{6}{2\pi} = \frac{3}{\pi}$$

Step 2: Use the formula for the curved surface area of a frustum.

The formula for the curved surface area A of a frustum of a cone is:

$$A = \pi(r_1 + r_2)l$$

Substitute the values:

$$A = \pi\left(\frac{9}{\pi} + \frac{3}{\pi}\right) \times 4$$

$$A = \pi \times \frac{12}{\pi} \times 4 = 12 \times 4 = 48\,\text{cm}^2$$

Final Answer: Thus, the curved surface area of the frustum is 48 cm^2.

Final Answer:

$$\boxed{48\,\text{cm}^2}$$

Quick Tip

To calculate the curved surface area of a frustum, first calculate the radii from the circumferences, then use the formula $A = \pi(r_1 + r_2)l$.
