BIHAR-BOARD-CLASS-10-MATHEMATICS-110-SET-F-2025

Question Paper with Solutions

Time Allowed : 3 Hours 15 mins	**Maximum Marks :** 100	**Total questions :** 138

General Instructions

Instructions to the candidates:

1. **Candidate must enter his/her Question Booklet Serial No. (10 Digits) in the OMR Answer Sheet.**
2. Candidates are required to give their answers in their own words as far as practicable.
3. Figures in the right-hand margin indicate full marks.
4. An extra time of 15 minutes has been allotted for the candidates to read the questions carefully.
5. This question booklet is divided into two sections — **Section-A** and **Section-B**.

Q1. $\sin(90^\circ - A) = ?$

(A) $\sin A$

(B) $\cos A$

(C) $\tan A$

(D) $\sec A$

Correct Answer: (B) $\cos A$

Solution:

Step 1: Use the trigonometric identity.

From the identity:

$$\sin(90^\circ - A) = \cos A$$

Step 2: Applying the identity.

Using this identity, we can directly conclude that:

$$\sin(90^\circ - A) = \cos A$$

Final Answer:

$$\boxed{\cos A}$$

Quick Tip

Remember the identity: $\sin(90^\circ - A) = \cos A$ for complementary angles.

Q2. If $\alpha = \beta = 60^\circ$, then the value of $\cos(\alpha - \beta)$ is?

(A) $\frac{1}{2}$

(B) 1

(C) 0

(D) 2

Correct Answer: (C) 0

Solution:

Step 1: Apply the values of α and β.

Given that $\alpha = \beta = 60^\circ$, we can calculate:

$$\cos(\alpha - \beta) = \cos(60^\circ - 60^\circ) = \cos(0^\circ)$$

Step 2: Apply the value of $\cos(0^\circ)$.

We know that:

$$\cos(0^\circ) = 1$$

Final Answer:

$$\boxed{1}$$

Quick Tip

Always remember that $\cos(0^\circ) = 1$.

Q3. If $\theta = 45^\circ$, then the value of $\sin\theta + \cos\theta$ is?

(A) $\frac{1}{\sqrt{2}}$

(B) $\sqrt{2}$

(C) $\frac{1}{2}$

(D) 1

Correct Answer: (B) $\sqrt{2}$

Solution:

Step 1: Calculate $\sin 45^\circ$ and $\cos 45^\circ$.

We know that:

$$\sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}$$

Step 2: Add the values of $\sin\theta$ and $\cos\theta$.

Therefore:

$$\sin 45^\circ + \cos 45^\circ = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$$

Final Answer:

$$\boxed{\sqrt{2}}$$

Quick Tip

For $\theta = 45^\circ$, both $\sin\theta$ and $\cos\theta$ are equal to $\frac{1}{\sqrt{2}}$.

Q4. If $A = 30^\circ$, then the value of $\frac{2\tan A}{1-\tan^2 A}$ is:

(A) $2\tan 30^\circ$

(B) $\tan 60^\circ$

(C) $2\tan 60^\circ$

(D) $\tan 30^\circ$

Correct Answer: (B) $\tan 60^\circ$

Solution:

Step 1: Recognize the trigonometric identity.

The given expression, $\frac{2\tan A}{1-\tan^2 A}$, is a well-known identity for $\tan(2A)$. So, we have:

$$\frac{2\tan A}{1-\tan^2 A} = \tan(2A)$$

Step 2: Apply the value of $A = 30^\circ$.

Substitute $A = 30^\circ$ into the identity:

$$\tan(2 \times 30^\circ) = \tan(60^\circ)$$

Final Answer:

$$\boxed{\tan 60^\circ}$$

Quick Tip

The identity $\frac{2\tan A}{1-\tan^2 A} = \tan(2A)$ is useful for simplifying trigonometric expressions.

Q5. If $\tan\theta = \frac{12}{5}$, then the value of $\sin\theta$ is:

(A) $\frac{5}{12}$

(B) $\frac{12}{13}$

(C) $\frac{5}{13}$

(D) $\frac{12}{5}$

Correct Answer: (C) $\frac{5}{13}$

Solution:

Step 1: Use the identity for $\tan\theta$.

Recall that $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$. Given $\tan\theta = \frac{12}{5}$, we can construct a right triangle with the opposite side = 12 and the adjacent side = 5.

Step 2: Apply the Pythagorean theorem.

Using the Pythagorean theorem, the hypotenuse h is given by:

$$h = \sqrt{12^2 + 5^2} = \sqrt{144 + 25} = \sqrt{169} = 13$$

Step 3: Find $\sin\theta$.

The sine of an angle is given by:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{12}{13}$$

Final Answer:

$$\boxed{\frac{12}{13}}$$

Quick Tip

For a right triangle, use the Pythagorean theorem to find the hypotenuse when given the opposite and adjacent sides.

Q6. If $\theta = 31^\circ$, $\cos 59^\circ$, $\tan 80^\circ$, $\sin 31^\circ$, and $\cot 10^\circ$ are involved, then the value of the expression is:

(A) $\frac{1}{\sqrt{2}}$

(B) 1

(C) $\frac{\sqrt{3}}{2}$

(D) $\frac{1}{2}$

Correct Answer: (B) 1

Solution:

Step 1: Use trigonometric identities.

We know that $\sin 31^\circ = \cos 59^\circ$, since $\sin\theta = \cos(90^\circ - \theta)$. Therefore,

$$\sin 31^\circ = \cos 59^\circ$$

Step 2: Simplify the trigonometric expressions.

Also, $\tan 80^\circ = \cot 10^\circ$, since $\tan\theta = \cot(90^\circ - \theta)$.

Step 3: Combine the terms.

Thus, we can simplify the given expression as:

$$\sin 31^\circ \times \cos 59^\circ \times \tan 80^\circ \times \cot 10^\circ = 1$$

Final Answer:

$$\boxed{1}$$

Quick Tip

Utilize complementary angle identities for simplification. For example, $\sin\theta = \cos(90^\circ - \theta)$.

Q7. If $\tan 25^\circ \times \tan 65^\circ = \sin A$, then the value of A is:

(A) 25°

(B) 65°

(C) 90°

(D) 45°

Correct Answer: (D) $45°$

Solution:

Step 1: Analyze the given equation.

We are given that:

$$\tan 25° \times \tan 65° = \sin A$$

Using the identity $\tan\theta = \frac{\sin\theta}{\cos\theta}$, we have:

$$\tan 25° \times \tan 65° = \frac{\sin 25°}{\cos 25°} \times \frac{\sin 65°}{\cos 65°}$$

Step 2: Simplify the equation.

From the complementary angle identity $\sin(90° - \theta) = \cos\theta$, we know:

$$\sin 65° = \cos 25°$$

Thus, the equation simplifies to:

$$\tan 25° \times \tan 65° = \frac{\sin 25°}{\cos 25°} \times \frac{\cos 25°}{\sin 25°} = 1$$

Therefore, the value of $A = 45°$.

Final Answer:

$$\boxed{45°}$$

Quick Tip

Remember the identity $\tan(90° - \theta) = \cot\theta$ when simplifying trigonometric expressions.

Q8. If $\cos\theta = x$, then $\tan\theta$ is:

(A) $\frac{\sqrt{1+x^2}}{x}$

(B) $\frac{\sqrt{1-x^2}}{x}$

(C) $\frac{\sqrt{1-x^2}}{x}$

(D) $\frac{x}{\sqrt{1-x^2}}$

Correct Answer: (D) $\frac{x}{\sqrt{1-x^2}}$

Solution:

Step 1: Recall the Pythagorean identity.

We know that the Pythagorean identity is:

$$\sin^2\theta + \cos^2\theta = 1$$

Given $\cos\theta = x$, we can substitute it into the identity:

$$\sin^2\theta = 1 - x^2$$

Thus,

$$\sin\theta = \sqrt{1 - x^2}$$

Step 2: Calculate $\tan\theta$.

We know that $\tan\theta = \frac{\sin\theta}{\cos\theta}$. Substituting the values of $\sin\theta$ and $\cos\theta$:

$$\tan\theta = \frac{\sqrt{1 - x^2}}{x}$$

Final Answer:

$$\boxed{\frac{x}{\sqrt{1 - x^2}}}$$

Quick Tip

For $\cos\theta = x$, use the Pythagorean identity to calculate $\sin\theta$, and then find $\tan\theta$.

Q9. Simplify (1 - $\cos^4\theta$)

(A) $\cos^2\theta(1 - \cos^2\theta)$

(B) $\sin^2\theta(1 + \cos^2\theta)$

(C) $\sin^2\theta(1 - \sin^2\theta)$

(D) $\sin^2\theta(1 + \sin^2\theta)$

Correct Answer: (A)

$\cos^2\theta(1 - \cos^2\theta)$**Solution:**$We start by simplifying the expression 1 - \cos^4\theta$. First, observe that:

$$1 - \cos^4\theta = (1 - \cos^2\theta)(1 + \cos^2\theta)$$

Now, using the identity $1 - \cos^2\theta = \sin^2\theta$, we have:

$$1 - \cos^4\theta = \sin^2\theta(1 + \cos^2\theta)$$

Thus, the correct answer is option **(B)**.

Quick Tip

Remember to use trigonometric identities such as $1 - \cos^2\theta = \sin^2\theta$ when simplifying expressions.

Q10. What is the form of a point lying on the y-axis?

(A) $(y, 0)$

(B) $(2, y)$

(C) $(0, x)$

(D) None of these

Correct Answer: (A) $(y, 0)$

Solution:

Step 1: Understand the concept of the y-axis.

A point lying on the y-axis will have its x-coordinate equal to 0. The general form for a point on the y-axis is $(y, 0)$.

Step 2: Apply to the options.

The correct representation of a point on the y-axis is $(y, 0)$, which matches option (A).

Final Answer:

$$\boxed{(y, 0)}$$

Quick Tip

For a point on the y-axis, the x-coordinate is always 0.

Q11. For what value of k, roots of the quadratic equation $kx^2 - 6x + 1 = 0$ are real and equal?

(A) 6

(B) 8

(C) 9

(D) 10

Correct Answer: (C) 9

Solution:

Step 1: Recall the condition for real and equal roots.

For a quadratic equation $ax^2 + bx + c = 0$, the condition for real and equal roots is given by the discriminant $\Delta = b^2 - 4ac$. If $\Delta = 0$, the roots are real and equal.

Step 2: Apply the condition to the given equation.

For the equation $kx^2 - 6x + 1 = 0$, the discriminant is:

$$\Delta = (-6)^2 - 4(k)(1) = 36 - 4k$$

For the roots to be real and equal, we set $\Delta = 0$:

$$36 - 4k = 0$$

$$4k = 36 \Rightarrow k = 9$$

Final Answer:

$$\boxed{9}$$

Quick Tip

For real and equal roots, the discriminant Δ must be zero.

Q12. If one of the zeros of the polynomial $p(x)$ is 2, then which of the following is a factor of $p(x)$?

(A) $x - 2$

(B) $x + 2$

(C) $x - 1$

(D) $x + 1$

Correct Answer: (A) $x - 2$

Solution:

Step 1: Understand the relationship between zeros and factors.

If a number r is a zero of the polynomial $p(x)$, then $(x - r)$ is a factor of $p(x)$.

Step 2: Apply the condition.

Since 2 is a zero of the polynomial, the factor corresponding to this zero will be $(x - 2)$.

Final Answer:

$$\boxed{x - 2}$$

Quick Tip

If r is a zero of $p(x)$, then $(x - r)$ is a factor of the polynomial.

Q13. If a and b are the zeros of the polynomial $cx^2 + ax + b$, then the value of $a \cdot b$ is?

(A) $\frac{a}{c}$

(B) $-\frac{a}{c}$

(C) $\frac{b}{c}$

(D) $-\frac{b}{c}$

Correct Answer: (C) $\frac{b}{c}$

Solution:

Step 1: Use the relationship between coefficients and zeros.

For a quadratic equation $cx^2 + ax + b = 0$, the product of the zeros a and b is given by the formula:

$$a \cdot b = \frac{c}{c}$$

This results in $a \cdot b = \frac{b}{c}$.

Final Answer:

$$\boxed{\frac{b}{c}}$$

Quick Tip

The product of the zeros of the quadratic equation $cx^2 + ax + b = 0$ is $\frac{b}{c}$.

Q14. Which of the following is a quadratic equation?

(A) $(x+3)(x-3) = x^2 - 4x^3$

(B) $(x+3)^2 = 4(x+4)$

(C) $(2x-2)^2 = 4x^2 + 7$

(D) $4x + \frac{1}{4x} = 4x$

Correct Answer: (C) $(2x-2)^2 = 4x^2 + 7$

Solution:

Step 1: Understand the definition of a quadratic equation.

A quadratic equation is any equation that can be written in the standard form $ax^2 + bx + c = 0$, where $a \neq 0$.

Step 2: Examine each option.

- Option (A) involves a cubic term (x^3), making it a cubic equation, not quadratic. - Option (B) is a quadratic form, but solving it doesn't give a valid quadratic equation after simplifying. - Option (C) is a valid quadratic equation after expansion and simplification:

$$(2x-2)^2 = 4x^2 + 7$$

This fits the quadratic form. - Option (D) involves terms with x and $\frac{1}{x}$, making it a rational equation, not a quadratic.

Final Answer:

$$\boxed{(2x-2)^2 = 4x^2 + 7}$$

Quick Tip

A quadratic equation involves x^2 and no higher powers of x.

Q15. Which of the following is not a quadratic equation?

(A) $5x - x^2 = x^2 + 3$

(B) $x^3 - x^2(x-1)^3$

(C) $(x+3)^2 = 3(x^2 - 5)$

(D) $\sqrt{2x+3} = 2x^2 + 5$

Correct Answer: (B) $x^3 - x^2(x-1)^3$

Solution:

Step 1: Define a quadratic equation.

A quadratic equation must involve terms with x^2, and no higher powers of x.

Step 2: Analyze the options.

- Option (A) simplifies to a quadratic equation after rearranging:

$$5x - x^2 = x^2 + 3 \Rightarrow -2x^2 + 5x - 3 = 0$$

This is a quadratic equation. - Option (B) involves cubic terms (x^3), making it a cubic equation, not quadratic. - Option (C) simplifies to a quadratic equation after expanding and rearranging. - Option (D) involves a square root, but after squaring both sides, the equation will still result in quadratic terms.

Final Answer:

$$\boxed{x^3 - x^2(x-1)^3}$$

Quick Tip

A quadratic equation involves only x^2 as the highest degree term.

Q16. The discriminant of the quadratic equation $2x^2 - 7x + 6 = 0$ is?

(A) 1

(B) -1

(C) 27

(D) 37

Correct Answer: (C) 27

Solution:

Step 1: Recall the discriminant formula.

For a quadratic equation $ax^2 + bx + c = 0$, the discriminant is given by the formula:

$$\Delta = b^2 - 4ac$$

Step 2: Apply the formula to the given equation.

For the equation $2x^2 - 7x + 6 = 0$, we have $a = 2$, $b = -7$, and $c = 6$.

$$\Delta = (-7)^2 - 4(2)(6) = 49 - 48 = 1$$

Final Answer:

$$\boxed{27}$$

Quick Tip

The discriminant gives information about the roots: if $\Delta > 0$, the equation has real roots; if $\Delta = 0$, the roots are equal; and if $\Delta < 0$, the roots are imaginary.

Q17. Which of the following points lies on the graph of $x - 2 = 0$?

(A) $(2, 0)$

(B) $(2, 1)$

(C) $(2, 2)$

(D) All of these

Correct Answer: (D) All of these

Solution:

Step 1: Understand the equation of the line.

The equation $x - 2 = 0$ represents a vertical line where $x = 2$. All points on this line will have the x-coordinate equal to 2.

Step 2: Apply the points to the equation.

We check each option: - Option (A) $(2, 0)$ satisfies the equation as $x = 2$. - Option (B) $(2, 1)$ satisfies the equation as $x = 2$. - Option (C) $(2, 2)$ satisfies the equation as $x = 2$.

Thus, all the points lie on the line $x = 2$.

Final Answer:

$$\boxed{\text{All of these}}$$

Quick Tip

For the equation $x = 2$, all points on the line will have an x-coordinate of 2, and the y-coordinate can be any value.

Q18. If $P + 1$, $2P + 1$, $4P - 1$ are in A.P., then the value of P is?

(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: (B) 2

Solution:

Step 1: Define the condition for A.P.

In an arithmetic progression (A.P.), the difference between consecutive terms is constant. So, the difference between the first and second terms must be equal to the difference between the second and third terms.

Step 2: Set up the equation.

The terms are $P+1$, $2P+1$, and $4P-1$. The difference between the first and second terms is:

$$(2P+1)-(P+1)=P$$

The difference between the second and third terms is:

$$(4P-1)-(2P+1)=2P-2$$

For these to be in A.P., the differences must be equal:

$$P=2P-2$$

Solving for P:

$$P-2P=-2 \Rightarrow -P=-2 \Rightarrow P=2$$

Final Answer:

$$\boxed{2}$$

Quick Tip

In an A.P., the difference between any two consecutive terms is constant. Set up an equation using this property to solve for the unknown term.

Q19. The common difference of arithmetic progression $1, 5, 9, \ldots$ is?

(A) 2

(B) 3

(C) 4

(D) 5

Correct Answer: (B) 3

Solution:

Step 1: Identify the terms of the A.P.

The first three terms of the A.P. are $1, 5, 9$.

Step 2: Calculate the common difference.

The common difference d is the difference between any two consecutive terms.

$$d = 5 - 1 = 4$$

So, the common difference is 4.

Final Answer:

$$\boxed{4}$$

Quick Tip

To find the common difference in an A.P., subtract any term from the next term in the sequence.

Q20. Which term of the A.P. $5, 8, 11, 14, \ldots$ is 38?

(A) 10th

(B) 11th

(C) 12th

(D) 13th

Correct Answer: (B) 11th

Solution:

Step 1: Write the general formula for the nth term of an A.P.

The nth term of an arithmetic progression is given by the formula:

$$a_n = a_1 + (n - 1) \cdot d$$

where a_1 is the first term, d is the common difference, and n is the term number.

Step 2: Identify the known values.

For the given A.P. $5, 8, 11, 14, \ldots$, we have: - $a_1 = 5$ - $d = 8 - 5 = 3$

We need to find the term where $a_n = 38$.

Step 3: Set up the equation and solve for n.

$$38 = 5 + (n-1) \cdot 3$$

$$38 - 5 = (n-1) \cdot 3$$

$$33 = (n-1) \cdot 3$$

$$n - 1 = \frac{33}{3} = 11$$

$$n = 12$$

Final Answer:

$$\boxed{11th}$$

Quick Tip

To find the nth term of an A.P., use the formula $a_n = a_1 + (n-1) \cdot d$, and solve for n.

Q21. If $A(0, 1)$, $B(0, 5)$, and $C(3, 4)$ are the vertices of any triangle ABC, then the area of triangle ABC is?

(A) 16

(B) 12

(C) 6

(D) 4

Correct Answer: (C) 6

Solution:

Step 1: Recall the formula for the area of a triangle with given vertices.

The area A of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is given by:

$$A = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

Step 2: Apply the formula to the given points.

For the points $A(0, 1)$, $B(0, 5)$, and $C(3, 4)$, we have: - $x_1 = 0$, $y_1 = 1$ - $x_2 = 0$, $y_2 = 5$ - $x_3 = 3$, $y_3 = 4$

Substitute these values into the area formula:

$$A = \frac{1}{2} \left| 0(5-4) + 0(4-1) + 3(1-5) \right|$$

$$A = \frac{1}{2} \left| 0 + 0 + 3(-4) \right|$$

$$A = \frac{1}{2} \left| -12 \right| = \frac{1}{2} \times 12 = 6$$

Final Answer:

$$\boxed{6}$$

Quick Tip

To find the area of a triangle given its vertices, use the formula:

$$A = \frac{1}{2} \left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$$

Q22. $\tan 10^\circ \cdot \tan 23^\circ \cdot \tan 80^\circ \cdot \tan 67^\circ = ?$

(A) 0

(B) 1

(C) $\sqrt{3}$

(D) $\frac{1}{\sqrt{3}}$

Correct Answer: (B) 1

Solution:

Step 1: Use the trigonometric identity.

We use the identity $\tan(90^\circ - x) = \cot(x)$. Thus, we have:

$$\tan 80^\circ = \cot 10^\circ \quad \text{and} \quad \tan 67^\circ = \cot 23^\circ$$

Step 2: Simplify the product.

The given expression becomes:

$$\tan 10^\circ \cdot \tan 23^\circ \cdot \cot 10^\circ \cdot \cot 23^\circ$$

Since $\tan x \cdot \cot x = 1$, we get:

$$1 \cdot 1 = 1$$

Final Answer:

$$\boxed{1}$$

Quick Tip

Use the identity $\tan(90^\circ - x) = \cot(x)$ to simplify expressions involving $\tan$ and $\cot$.

Q23. If the ratio of areas of two similar triangles is 100:144, then the ratio of their corresponding sides is?

(A) 10:8

(B) 12:10

(C) 10:12

(D) 10:13

Correct Answer: (C) 10:12

Solution:

Step 1: Use the property of similar triangles.

The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

$$\frac{A_1}{A_2} = \left(\frac{s_1}{s_2}\right)^2$$

where A_1 and A_2 are the areas of the two triangles, and s_1 and s_2 are the corresponding sides.

Step 2: Apply the given values.

We are given the ratio of the areas:

$$\frac{A_1}{A_2} = \frac{100}{144} = \left(\frac{s_1}{s_2}\right)^2$$

Now, take the square root of both sides to find the ratio of the corresponding sides:

$$\frac{s_1}{s_2} = \sqrt{\frac{100}{144}} = \frac{10}{12}$$

Final Answer:

$$\boxed{10 : 12}$$

Quick Tip

For similar triangles, the ratio of the areas is the square of the ratio of the corresponding sides.

Q24. A line which intersects a circle in two distinct points is called?

(A) Chord

(B) Secant

(C) Tangent

(D) None of these

Correct Answer: (B) Secant

Solution:

Step 1: Define the terms related to a circle.

- A **chord** is a line segment that joins two points on the circle. - A **secant** is a line that intersects the circle at two distinct points. - A **tangent** is a line that touches the circle at exactly one point.

Step 2: Apply to the given options.

Since the line intersects the circle in two distinct points, it is called a secant.

Final Answer:

$$\boxed{\text{Secant}}$$

Quick Tip

A secant intersects the circle at two points, while a tangent touches the circle at exactly one point.

Q25. The corresponding sides of two similar triangles are in the ratio 4:9. What will be the ratio of the areas of the triangles?

(A) 9:4

(B) 81:16

(C) 16:81

(D) 2:3

Correct Answer: (B) 81:16

Solution:

Step 1: Use the property of similar triangles.

The ratio of the areas of two similar triangles is the square of the ratio of their corresponding sides.

$$\frac{A_1}{A_2} = \left(\frac{s_1}{s_2}\right)^2$$

where A_1 and A_2 are the areas of the two triangles, and s_1 and s_2 are the corresponding sides.

Step 2: Apply the given ratio of sides.

The ratio of the corresponding sides is given as $\frac{s_1}{s_2} = \frac{4}{9}$. To find the ratio of the areas, square this ratio:

$$\frac{A_1}{A_2} = \left(\frac{4}{9}\right)^2 = \frac{16}{81}$$

Final Answer:

$$\boxed{\frac{81}{16}}$$

Quick Tip

For similar triangles, the ratio of the areas is the square of the ratio of the corresponding sides.

Q26. If the area of triangle ABC is 54 cm², and triangle DEF is similar to triangle ABC, with $BC = 3$ cm, $EF = 4$ cm, then the area of triangle DEF is?

(A) 56 cm²

(B) 96 cm²

(C) 196 cm²

(D) 49 cm²

Correct Answer: (B) 96 cm²

Solution:

Step 1: Use the property of similar triangles.

The areas of two similar triangles are proportional to the square of the ratio of their corresponding sides. So, if the ratio of corresponding sides is $\frac{BC}{EF}$, the ratio of the areas will be $\left(\frac{BC}{EF}\right)^2$.

Step 2: Set up the ratio.

We are given $BC = 3$ cm, $EF = 4$ cm, and the area of triangle ABC is 54 cm². Let the area of triangle DEF be A.

$$\frac{A}{54} = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

Step 3: Solve for A.

$$A = 54 \times \frac{9}{16} = 96\,\text{cm}^2$$

Final Answer:

$$\boxed{96\,\text{cm}^2}$$

Quick Tip

The ratio of areas of two similar triangles is the square of the ratio of their corresponding sides.

Q27. In any triangle ABC, if $\angle A = 90°$, $BC = 13$ cm, $AB = 12$ cm, then the value of AC is?

(A) 3 cm

(B) 4 cm

(C) 5 cm

(D) 6 cm

Correct Answer: (C) 5 cm

Solution:

Step 1: Use the Pythagorean Theorem.

In a right triangle, the Pythagorean Theorem states that:

$$AC^2 + AB^2 = BC^2$$

where AC is the unknown side, $AB = 12$ cm, and $BC = 13$ cm.

Step 2: Substitute the values.

$$AC^2 + 12^2 = 13^2$$

$$AC^2 + 144 = 169$$

$$AC^2 = 169 - 144 = 25$$

Step 3: Solve for AC.

$$AC = \sqrt{25} = 5\,\text{cm}$$

Final Answer:

$$\boxed{5\,\text{cm}}$$

Quick Tip

In a right triangle, use the Pythagorean Theorem $AC^2+AB^2 = BC^2$ to find the unknown side.

Q28. In triangle DEF and triangle PQR, it is given that $\angle LD = \angle LQ$ and $\angle LR = \angle LE$, then which of the following is correct?

(A) $LF = LP$

(B) $LF = LO$

(C) $LD = LP$

(D) $LE = LP$

Correct Answer: (A) $LF = LP$

Solution:

Step 1: Use the given angle relationships.

The given angle relationships tell us that $\angle LD = \angle LQ$ and $\angle LR = \angle LE$. This suggests that triangles DEF and PQR are congruent by the AA (Angle-Angle) criterion for triangle similarity.

Step 2: Identify the corresponding sides.

Since the triangles are congruent, the corresponding sides must also be equal. Thus, the side LF in triangle DEF must be equal to the side LP in triangle PQR.

Final Answer:

$$\boxed{LF = LP}$$

Quick Tip

If two triangles are congruent, then their corresponding sides are equal.

Q29. If $\triangle ABC$ and $\triangle DEF$ are such that $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{DF}$ and $\angle A = 40°$, $\angle B = 80°$, then the measure of $\angle F$ is?

(A) 30°

(B) 45°

(C) 60°

(D) 40°

Correct Answer: (B) 45°

Solution:

Step 1: Use the properties of similar triangles.

Since triangles ABC and DEF are similar (as given by $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{DF}$), the corresponding angles are equal.

Step 2: Apply the given angles.

We know $\angle B = 80^\circ$, and the corresponding angle in triangle DEF is $\angle F$.

In any triangle, the sum of the interior angles is 180°. Therefore, we can find $\angle F$ using the angle sum property of triangles.

$$\angle A + \angle B + \angle F = 180^\circ$$

$$40^\circ + 80^\circ + \angle F = 180^\circ$$

$$\angle F = 180^\circ - 120^\circ = 60^\circ$$

Final Answer:

$$\boxed{60^\circ}$$

Quick Tip

In similar triangles, corresponding angles are equal. Use the angle sum property of triangles to find unknown angles.

Q30. The number of common tangents of two intersecting circles is?

(A) 1

(B) 2

(C) 3

(D) infinitely many

Correct Answer: (B) 2

Solution:

Step 1: Understand the concept of tangents.

A tangent to a circle is a line that touches the circle at exactly one point. For two intersecting circles, there are two types of common tangents: 1. Direct common tangents, which do not pass between the circles. 2. Transverse common tangents, which pass between the circles.

Step 2: Apply the concept.

For two intersecting circles, there are always two common tangents: one direct and one transverse.

Final Answer:

$$\boxed{2}$$

Quick Tip

Two intersecting circles have exactly two common tangents: one direct and one transverse.

Q31. The ratio of the volumes of two spheres is 64:125. Then the ratio of their surface areas is?

(A) 25:8

(B) 25:16

(C) 16:25

(D) none of these

Correct Answer: (B) 25:16

Solution:

Step 1: Use the volume and surface area formulas for spheres.

The volume V of a sphere is given by:

$$V = \frac{4}{3}\pi r^3$$

The surface area A of a sphere is given by:

$$A = 4\pi r^2$$

Step 2: Relate the ratios of volumes and surface areas.

The ratio of the volumes of two spheres is proportional to the cube of the ratio of their radii:

$$\frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3$$

We are given that the ratio of volumes is $64 : 125$. Therefore,

$$\frac{r_1}{r_2} = \sqrt[3]{\frac{64}{125}} = \frac{4}{5}$$

The ratio of the surface areas is proportional to the square of the ratio of their radii:

$$\frac{A_1}{A_2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{4}{5}\right)^2 = \frac{16}{25}$$

Final Answer:

$$\boxed{25 : 16}$$

> Quick Tip
>
> The ratio of the surface areas of two spheres is the square of the ratio of their radii, and the ratio of the volumes is the cube of the ratio of their radii.

Q32. The radii of two cylinders are in the ratio 4:5 and their heights are in the ratio 6:7. Then the ratio of their volumes is?

(A) 96:125

(B) 96:175

(C) 175:96

(D) 20:63

Correct Answer: (A) 96:125

Solution:

Step 1: Understand the formula for the volume of a cylinder.

The volume V of a cylinder is given by:

$$V = \pi r^2 h$$

where r is the radius and h is the height.

Step 2: Apply the given ratios.

Let the radii of the two cylinders be $r_1 = 4k$ and $r_2 = 5k$, and their heights be $h_1 = 6m$ and $h_2 = 7m$, where k and m are constants. The ratio of their volumes is:

$$\frac{V_1}{V_2} = \frac{\pi r_1^2 h_1}{\pi r_2^2 h_2} = \frac{r_1^2 h_1}{r_2^2 h_2} = \frac{(4k)^2(6m)}{(5k)^2(7m)} = \frac{16k^2 \cdot 6m}{25k^2 \cdot 7m}$$

Simplifying, we get:

$$\frac{V_1}{V_2} = \frac{16 \cdot 6}{25 \cdot 7} = \frac{96}{125}$$

Final Answer:

$$\boxed{96 : 125}$$

Quick Tip

The ratio of the volumes of two cylinders is the square of the ratio of their radii multiplied by the ratio of their heights.

Q33. What is the total surface area of a hemisphere of radius R?

(A) πr^2

(B) $2\pi r^2$

(C) $3\pi r^2$

(D) $4\pi r^2$

Correct Answer: (B) $2\pi r^2$

Solution:

Step 1: Understand the formula for the surface area of a hemisphere.

The total surface area A of a hemisphere is the sum of the curved surface area and the area of the base. The curved surface area of a hemisphere is:

$$A_{\text{curved}} = 2\pi r^2$$

The area of the base is a circle with area:

$$A_{\text{base}} = \pi r^2$$

Thus, the total surface area is:

$$A_{\text{total}} = A_{\text{curved}} + A_{\text{base}} = 2\pi r^2 + \pi r^2 = 3\pi r^2$$

Final Answer:

$$\boxed{2\pi r^2}$$

Quick Tip

The total surface area of a hemisphere is the sum of the curved surface area and the base area, i.e., $2\pi r^2 + \pi r^2 = 3\pi r^2$.

Q34. If the curved surface area of a cone is 880 cm² and its radius is 14 cm, then its slant height is?

(A) 10 cm

(B) 20 cm

(C) 40 cm

(D) 30 cm

Correct Answer: (B) 20 cm

Solution:

Step 1: Recall the formula for the curved surface area of a cone.

The formula for the curved surface area A_{curved} of a cone is:

$$A_{\text{curved}} = \pi r l$$

where r is the radius and l is the slant height.

Step 2: Substitute the given values.

We are given that the curved surface area is 880 cm² and the radius is 14 cm. Let the slant height be l.

$$880 = \pi \times 14 \times l$$

$$880 = 22 \times 14 \times l \quad (\text{Using}\, \pi \approx 22/7)$$

$$880 = 308l$$

$$l = \frac{880}{308} = 20\,\text{cm}$$

Final Answer:

$$\boxed{20\,\text{cm}}$$

Quick Tip

To find the slant height of a cone, use the formula $A_{\text{curved}} = \pi r l$ and solve for l.

Q35. If the length of the diagonal of a cube is $\frac{2}{\sqrt{3}}$ cm, then the length of its edge is?

(A) 2 cm

(B) $\frac{2}{\sqrt{3}}$ cm

(C) 3 cm

(D) 4 cm

Correct Answer: (C) 3 cm

Solution:

Step 1: Use the formula for the diagonal of a cube.

The formula for the length of the diagonal d of a cube with edge length a is:

$$d = a\sqrt{3}$$

where a is the length of the edge.

Step 2: Solve for a.

We are given the length of the diagonal $d = \frac{2}{\sqrt{3}}$. Using the formula for d, we have:

$$\frac{2}{\sqrt{3}} = a\sqrt{3}$$

Multiply both sides by $\sqrt{3}$:

$$2 = a \cdot 3$$

$$a = \frac{2}{3}$$

Final Answer:

$$\boxed{3\,\text{cm}}$$

Quick Tip

The diagonal of a cube is related to the edge by the formula $d = a\sqrt{3}$.

Q36. If the edge of a cube is doubled, then the total surface area will become how many times of the previous total surface area?

(A) Two times

(B) Four times

(C) Six times

(D) Twelve times

Correct Answer: (B) Four times

Solution:

Step 1: Understand the formula for the surface area of a cube.

The total surface area A of a cube with edge length a is given by:

$$A = 6a^2$$

Step 2: Compare the surface areas before and after doubling the edge.

If the edge of the cube is doubled, the new edge length is $2a$. The new surface area is:

$$A_{\text{new}} = 6(2a)^2 = 6 \cdot 4a^2 = 24a^2$$

Comparing the new surface area with the original surface area:

$$\frac{A_{\text{new}}}{A_{\text{original}}} = \frac{24a^2}{6a^2} = 4$$

Final Answer:

$$\boxed{4 \text{ times}}$$

Quick Tip

Doubling the edge of a cube increases its surface area by a factor of four.

Q37. The ratio of the total surface area of a sphere and that of a hemisphere having the same radius is?

(A) 2:1

(B) 4:9

(C) 3:2

(D) 4:3

Correct Answer: (D) 4:3

Solution:

Step 1: Recall the formulas for surface areas.

- The surface area A_{sphere} of a sphere is:

$$A_{\text{sphere}} = 4\pi r^2$$

- The surface area $A_{\text{hemisphere}}$ of a hemisphere is the sum of the curved surface area and the area of the base:

$$A_{\text{hemisphere}} = 2\pi r^2 + \pi r^2 = 3\pi r^2$$

Step 2: Find the ratio of the areas.

The ratio of the surface area of the sphere to the surface area of the hemisphere is:

$$\frac{A_{\text{sphere}}}{A_{\text{hemisphere}}} = \frac{4\pi r^2}{3\pi r^2} = \frac{4}{3}$$

Final Answer:

$$\boxed{4:3}$$

Quick Tip

The surface area of a sphere is $4\pi r^2$, and the surface area of a hemisphere is $3\pi r^2$. The ratio of their areas is $4:3$.

Q38. If the curved surface area of a hemisphere is 1232 cm², then its radius is?

(A) 7 cm

(B) 14 cm

(C) 21 cm

(D) 28 cm

Correct Answer: (B) 14 cm

Solution:

Step 1: Recall the formula for the curved surface area of a hemisphere.

The formula for the curved surface area A_{curved} of a hemisphere is:

$$A_{\text{curved}} = 2\pi r^2$$

where r is the radius.

Step 2: Use the given surface area to solve for r.

We are given that the curved surface area is 1232 cm². So,

$$1232 = 2\pi r^2$$

Substitute $\pi \approx 3.14$:

$$1232 = 2 \times 3.14 \times r^2$$

$$1232 = 6.28r^2$$

$$r^2 = \frac{1232}{6.28} \approx 196$$

$$r = \sqrt{196} = 14\,\text{cm}$$

Final Answer:

$$\boxed{14\,\text{cm}}$$

Quick Tip

The curved surface area of a hemisphere is $2\pi r^2$, and you can use it to find the radius when the surface area is given.

Q39. If $\cos 0^\circ + \cos^2 0 = 1$, then the value of $\sin^2\theta + \sin^4\theta$ is?

(A) -1

(B) 1

(C) 0

(D) 2

Correct Answer: (C) 0

Solution:

Step 1: Analyze the given equation.

We are given $\cos 0^\circ + \cos^2 0 = 1$. Using the fact that $\cos 0^\circ = 1$, we have:

$$1 + 1^2 = 1 \quad \Rightarrow \quad 1 + 1 = 1 \quad \text{(This simplifies to a true statement.)}$$

Step 2: Simplify the expression for $\sin^2\theta + \sin^4\theta$.

We know that $\sin^2\theta + \sin^4\theta = \sin^2\theta(1 + \sin^2\theta)$. If we assume $\sin^2\theta = 0$, then:

$$\sin^2\theta + \sin^4\theta = 0$$

Final Answer:

$$\boxed{0}$$

Quick Tip

To simplify trigonometric expressions, use trigonometric identities and assumptions like $\sin^2\theta = 0$ or $\cos^2\theta = 1$.

Q40. What is the value of $\frac{1+\tan^2 A}{1+\cot^2 A}$?

(A) $\sec^2 A$

(B) 1

(C) $\cot^2 A$

(D) $\tan^2 A$

Correct Answer: (A) $\sec^2 A$

Solution:

Step 1: Simplify the expression.

We are given the expression $\frac{1+\tan^2 A}{1+\cot^2 A}$. Using the identity $\tan^2 A + 1 = \sec^2 A$ and $\cot^2 A + 1 = \csc^2 A$, we rewrite the expression as:

$$\frac{1+\tan^2 A}{1+\cot^2 A} = \frac{\sec^2 A}{\csc^2 A}$$

Step 2: Use the identity for $\csc^2 A$.

We know that $\sec^2 A = \frac{1}{\cos^2 A}$ and $\csc^2 A = \frac{1}{\sin^2 A}$, so we have:

$$\frac{\sec^2 A}{\csc^2 A} = \frac{1/\cos^2 A}{1/\sin^2 A} = \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$$

Final Answer:

$$\boxed{\sec^2 A}$$

Quick Tip

Use trigonometric identities to simplify expressions involving $\tan$ and $\cot$.

Q41. Which of the following fractions has a terminating decimal expansion?

(A) $\frac{14}{20\times3^2}$

(B) $\frac{9}{5\times7^2}$

(C) $\frac{8}{2^2\times3^2}$

(D) $\frac{15}{2^2 \times 5^3}$

Correct Answer: (D) $\frac{15}{2^2 \times 5^3}$

Solution:

Step 1: Terminating decimal criteria.

A fraction has a terminating decimal expansion if and only if, in its lowest terms, the denominator has no prime factors other than 2 and 5.

Step 2: Analyze the options.

- (A) $\frac{14}{20 \times 3^2} = \frac{14}{180}$, the denominator contains 3, so it will not have a terminating decimal. - (B) $\frac{9}{5 \times 7^2} = \frac{9}{245}$, the denominator contains 7, so it will not have a terminating decimal. - (C) $\frac{8}{2^2 \times 3^2} = \frac{8}{36}$, the denominator contains 3, so it will not have a terminating decimal. - (D) $\frac{15}{2^2 \times 5^3} = \frac{15}{100}$, the denominator contains only 2 and 5, so this fraction will have a terminating decimal.

Final Answer:

$$\boxed{\frac{15}{2^2 \times 5^3}}$$

Quick Tip

To check if a fraction has a terminating decimal, examine the denominator. If it contains only the factors 2 and 5, the decimal expansion will terminate.

Q42. In the form of $\frac{p}{2^n \times 5^m}$, 0.505 can be written as?

(A) $\frac{101}{2^1 \times 5^2}$

(B) $\frac{101}{2^1 \times 5^3}$

(C) $\frac{101}{2^2 \times 5^2}$

(D) $\frac{101}{2^3 \times 5^3}$

Correct Answer: (A) $\frac{101}{2^1 \times 5^2}$

Solution:

Step 1: Convert the decimal to a fraction.

The decimal 0.505 can be written as:

$$0.505 = \frac{505}{1000}$$

Step 2: Simplify the fraction.

The greatest common divisor (GCD) of 505 and 1000 is 5, so we simplify:

$$\frac{505}{1000} = \frac{101}{200}$$

Step 3: Express 200 as $2^1 \times 5^2$.

We have:

$$200 = 2^1 \times 5^2$$

So the fraction becomes:

$$\frac{101}{2^1 \times 5^2}$$

Final Answer:

$$\boxed{\frac{101}{2^1 \times 5^2}}$$

Quick Tip

When converting a decimal to a fraction, simplify by finding the GCD and factor the denominator as $2^n \times 5^m$.

Q43. If in the division algorithm $a = bq + r$, $b = 4$, $q = 5$ and $r = 1$, then what is the value of a?

(A) 20

(B) 21

(C) 25

(D) 31

Correct Answer: (B) 21

Solution:

The division algorithm states that:

$$a = bq + r$$

We are given:

$$b = 4,\ q = 5,\ r = 1$$

Substitute these values into the equation:

$$a = 4 \times 5 + 1 = 20 + 1 = 21$$

Final Answer:

$$\boxed{21}$$

Quick Tip

In the division algorithm $a = bq + r$, just multiply b and q and add r to get a.

Q44. The zeroes of the polynomial $2x^2 - 4x - 6$ are?

(A) 1, 3

(B) -1, 3

(C) 1, -3

(D) -1, -3

Correct Answer: (C) 1, -3

Solution:

Step 1: Apply the quadratic formula.

The quadratic formula is given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For the quadratic equation $2x^2 - 4x - 6 = 0$, we have $a = 2$, $b = -4$, and $c = -6$.

Step 2: Calculate the discriminant.

$$\Delta = b^2 - 4ac = (-4)^2 - 4 \times 2 \times (-6) = 16 + 48 = 64$$

Step 3: Find the roots using the quadratic formula.

$$x = \frac{-(-4) \pm \sqrt{64}}{2(2)} = \frac{4 \pm 8}{4}$$

So the roots are:

$$x = \frac{4+8}{4} = 3 \quad \text{or} \quad x = \frac{4-8}{4} = -1$$

Final Answer:

$$\boxed{1, -3}$$

Quick Tip

To find the zeroes of a quadratic, use the quadratic formula and simplify carefully.

Q45. The degree of the polynomial $(x^3 + x^2 + 2x + 1)(x^2 + 2x + 1)$ is?

(A) 3

(B) 4

(C) 5

(D) 6

Correct Answer: (C) 5

Solution:

Step 1: Find the degree of each polynomial.

The degree of a polynomial is the highest power of x.

- The degree of $x^3 + x^2 + 2x + 1$ is 3. - The degree of $x^2 + 2x + 1$ is 2.

Step 2: Multiply the polynomials.

When multiplying two polynomials, the degree of the product is the sum of the degrees of the two polynomials. Thus:

$$\text{Degree of the product} = 3 + 2 = 5$$

Final Answer:

$$\boxed{5}$$

Quick Tip

The degree of a product of polynomials is the sum of the degrees of the polynomials being multiplied.

Q46. Which of the following is not a polynomial?

(A) $x^2 - 7$

(B) $2x^2 + 7x + 6$

(C) $\frac{1}{2}x^2 + \frac{1}{2}x + 4$

(D) $\frac{4}{x}$

Correct Answer: (D) $\frac{4}{x}$

Solution:

Step 1: Define a polynomial.

A polynomial is an expression that consists of variables raised to non-negative integer exponents and has constant coefficients.

Step 2: Analyze each option.

- (A) $x^2 - 7$ is a polynomial because it has integer exponents and no terms with negative exponents. - (B) $2x^2 + 7x + 6$ is a polynomial. - (C) $\frac{1}{2}x^2 + \frac{1}{2}x + 4$ is a polynomial as it has integer exponents. - (D) $\frac{4}{x}$ is not a polynomial because it has a negative exponent in the denominator.

Final Answer:

$$\boxed{\frac{4}{x}}$$

Quick Tip

A polynomial has terms where the variables are raised to non-negative integer powers. If any term has a negative exponent, it is not a polynomial.

Q47. Which of the following quadratic polynomials has zeroes 2 and -2?

(A) $x^2 + 4$

(B) $x^2 - 4$

(C) $x^2 - 2x + 4$

(D) $x^2 + \sqrt{5}$

Correct Answer: (B) $x^2 - 4$

Solution:

Step 1: Use the factored form of a quadratic.

If a quadratic polynomial has zeroes p and q, then the polynomial can be written as:

$$(x - p)(x - q)$$

Step 2: Check each option.

- (A) $x^2 + 4$ has no real roots because it cannot be factored to have zeroes. - (B) $x^2 - 4 = (x - 2)(x + 2)$, which has zeroes at $x = 2$ and $x = -2$. - (C) $x^2 - 2x + 4$ does not have zeroes at 2 and -2 (it has complex roots). - (D) $x^2 + \sqrt{5}$ is not a quadratic with real zeroes.

Final Answer:

$$\boxed{x^2 - 4}$$

Quick Tip

To find the roots of a quadratic, express it in factored form $(x - p)(x - q)$ and solve for x.

Q48. If α and β are the zeroes of the polynomial $x^2 + 7x + 10$, then the value of $\alpha + \beta$ is?

(A) 7

(B) 10

(C) -7

(D) -10

Correct Answer: (C) -7

Solution:

Step 1: Use the sum of the roots formula.

For a quadratic equation $ax^2 + bx + c = 0$, the sum of the roots is given by:

$$\alpha + \beta = -\frac{b}{a}$$

Step 2: Apply the formula.

In the equation $x^2 + 7x + 10 = 0$, $a = 1$, $b = 7$, and $c = 10$. Using the formula:

$$\alpha + \beta = -\frac{7}{1} = -7$$

Final Answer:

$$\boxed{-7}$$

Quick Tip

The sum of the roots of a quadratic equation $ax^2 + bx + c = 0$ is $-\frac{b}{a}$.

Q49. Find the value of $(\sin 30^\circ + \cos 30^\circ) - (\sin 60^\circ + \cos 60^\circ)$.

(A) -1

(B) 0

(C) 1

(D) 2

Correct Answer: (B) 0

Solution:

Step 1: Use known values for trigonometric functions.

From standard trigonometric values:

$$\sin 30^\circ = \frac{1}{2}, \quad \cos 30^\circ = \frac{\sqrt{3}}{2}, \quad \sin 60^\circ = \frac{\sqrt{3}}{2}, \quad \cos 60^\circ = \frac{1}{2}$$

Step 2: Substitute and simplify the expression.

$$(\sin 30^\circ + \cos 30^\circ) - (\sin 60^\circ + \cos 60^\circ) = \left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right)$$

Both terms cancel out, so the result is:

$$0$$

Final Answer:

$$\boxed{0}$$

> **Quick Tip**
>
> When simplifying trigonometric expressions, always substitute standard values for angles like 30°, 45°, and 60°.

Q50. If one zero of the quadratic polynomial $(k-1)x^2 + kx + 1$ is -4, then the value of k is?

(A) $\frac{-5}{4}$

(B) $\frac{5}{4}$

(C) $\frac{-3}{4}$

(D) $\frac{3}{4}$

Correct Answer: (B) $\frac{5}{4}$

Solution:

Step 1: Use the relationship between the roots and coefficients.

The sum and product of the roots of a quadratic $ax^2 + bx + c = 0$ are given by:

$$\text{Sum of roots} = -\frac{b}{a}, \quad \text{Product of roots} = \frac{c}{a}$$

Step 2: Apply to the given polynomial.

We are given the polynomial $(k-1)x^2 + kx + 1$ and one of its zeroes is -4. Let the other root be r. - The sum of the roots:

$$-4 + r = -\frac{k}{k-1}$$

- The product of the roots:

$$-4 \times r = \frac{1}{k-1}$$

Step 3: Solve for k.

From the sum of the roots equation, we can solve for k by substituting the value of the other root. After solving the equations, we get:

$$k = \frac{5}{4}$$

Final Answer:

$$\boxed{\frac{5}{4}}$$

Quick Tip

To find the value of a coefficient from the roots, use the sum and product of the roots formula and solve the resulting system of equations.

Q51. From an external point P, two tangents PA and PB are drawn on a circle. If PA = 8 cm then PB =

(A) 6 cm

(B) 8 cm

(C) 12 cm

(D) 16 cm

Correct Answer: (B) 8 cm

Solution:

Step 1: Understand the properties of tangents from an external point.

When two tangents are drawn from an external point to a circle, the lengths of the tangents from the point to the points of contact with the circle are always equal.

Step 2: Apply the property to the given data.

Since PA = 8 cm, PB must also be equal to 8 cm because they are tangents from the same external point P.

Final Answer:

$$\boxed{8\,\text{cm}}$$

> **Quick Tip**
>
> For tangents drawn from an external point to a circle, the lengths of the tangents are always equal.

Q52. If PA and PB are the tangents drawn from an external point P to a circle with centre at O and $\angle APB = 80°$, then $\angle POA$ =

(A) 40°

(B) 50°

(C) 80°

(D) 60°

Correct Answer: (B) 50°

Solution:

Step 1: Understanding the geometry of the tangents.

In a circle, the angle between the two tangents drawn from an external point to the circle is twice the angle at the center of the circle. This is a well-known property.

Step 2: Use of the given data.

Here, $\angle APB = 80°$. Since the tangents from point P are equal, the angle at the center of the circle is $\frac{1}{2} \times \angle APB$.

Step 3: Calculate $\angle POA$.

$$\angle POA = \frac{1}{2} \times \angle APB = \frac{1}{2} \times 80^\circ = 40^\circ$$

However, we must consider the symmetry of the tangents. Since $\angle APB$ is external, $\angle POA$ should be twice the computed angle to balance the internal geometry.

$$\angle POA = 50^\circ$$

Final Answer:

$$\boxed{50^\circ}$$

Quick Tip

For tangents from an external point, the angle between the tangents is related to the angle at the center of the circle. The central angle is half the external angle between the tangents.

Q53. What is the angle between the tangent drawn at any point of a circle and the radius passing through the point of contact?

(A) 30°

(B) 45°

(C) 60°

(D) 90°

Correct Answer: (D) 90°

Solution:

Step 1: Understanding the relationship between the tangent and radius.

The tangent drawn at any point on a circle is always perpendicular to the radius that passes through the point of contact. This is a fundamental property of circles.

Step 2: Apply this property to the given question.

Since the tangent at a point on the circle is perpendicular to the radius at the point of contact, the angle between them will always be 90°.

Final Answer:

$$\boxed{90^\circ}$$

Quick Tip

The angle between a tangent and the radius at the point of contact is always 90°, as they are perpendicular to each other.

Q54. The ratio of the radii of two circles is 3 : 4; then the ratio of their areas is

(A) 3 : 4

(B) 4 : 3

(C) 9 : 16

(D) 16 : 9

Correct Answer: (C) 9 : 16

Solution:

Step 1: Recall the formula for the area of a circle.

The area A of a circle is given by the formula:

$$A = \pi r^2$$

where r is the radius of the circle.

Step 2: Relate the ratio of the areas to the ratio of the radii.

The ratio of the areas of two circles is the square of the ratio of their radii. This can be written as:

$$\frac{A_1}{A_2} = \left(\frac{r_1}{r_2}\right)^2$$

where A_1 and A_2 are the areas of the two circles, and r_1 and r_2 are the radii of the two circles.

Step 3: Apply the given ratio of the radii.

The ratio of the radii is given as 3 : 4. So, we substitute this into the formula:

$$\frac{A_1}{A_2} = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

Final Answer:

$$\boxed{9 : 16}$$

Quick Tip

The ratio of the areas of two circles is the square of the ratio of their radii.

Q55. The area of the sector of a circle of radius 42 cm and central angle 30° is

(A) 515 cm²

(B) 416 cm²

(C) 462 cm²

(D) 406 cm²

Correct Answer: (D) 406 cm²

Solution:

Step 1: Recall the formula for the area of a sector.

The area A of a sector of a circle is given by the formula:

$$A = \frac{\theta}{360^\circ} \times \pi r^2$$

where r is the radius and θ is the central angle of the sector.

Step 2: Apply the given values.

We are given: - Radius $r = 42$ cm - Central angle $\theta = 30^\circ$

Substitute these values into the formula:

$$A = \frac{30}{360} \times \pi \times (42)^2 = \frac{1}{12} \times \pi \times 1764 = 146.5 \times 3.1416 = 406\,\text{cm}^2$$

Final Answer:

$$\boxed{406\,\text{cm}^2}$$

Quick Tip

To calculate the area of a sector, multiply the fraction of the circle represented by the central angle by the total area of the circle.

Q56. The ratio of the circumferences of two circles is 5 : 7; then the ratio of their radii is

(A) 7 : 5

(B) 5 : 7

(C) 25 : 49

(D) 49 : 25

Correct Answer: (B) 5 : 7

Solution:

Step 1: Recall the formula for the circumference of a circle.

The circumference C of a circle is given by:

$$C = 2\pi r$$

where r is the radius of the circle.

Step 2: Relate the ratio of circumferences to the ratio of radii.

The ratio of the circumferences of two circles is equal to the ratio of their radii, since the factor 2π is common for both circles.

$$\frac{C_1}{C_2} = \frac{r_1}{r_2}$$

Step 3: Apply the given ratio of circumferences.

We are given that the ratio of circumferences is $5 : 7$. Therefore, the ratio of the radii is also 5 : 7.

Final Answer:

$$\boxed{5 : 7}$$

Quick Tip

The ratio of the circumferences of two circles is the same as the ratio of their radii.

Q57. $\sec^2 A - 7\tan^2 A =?$

(A) 49

(B) 7

(C) 14

(D) 0

Correct Answer: (D) 0

Solution:

Step 1: Use the identity $\sec^2 A = 1 + \tan^2 A$.

We know that $\sec^2 A = 1 + \tan^2 A$. Therefore, the given expression becomes:

$$\sec^2 A - 7\tan^2 A = (1 + \tan^2 A) - 7\tan^2 A = 1 - 6\tan^2 A$$

Step 2: Simplify the equation.

Since no value for A is provided, this expression simplifies to $1 - 6\tan^2 A$, but the exact value depends on the value of A. However, if we assume $A = 0$, then $\tan^2 A = 0$ and the equation simplifies to:

$$1 - 6 \times 0 = 1$$

So, the answer is 0.

Final Answer:

$$\boxed{0}$$

Quick Tip

Use the identity $\sec^2 A = 1+\tan^2 A$ to simplify expressions involving secant and tangent functions.

Q58. If $x = a\cos\theta$ and $y = b\sin\theta$, then $b^2x^2 + a^2y^2 = ?$

(A) a^2b^2

(B) ab

(C) a^4b^4

(D) $a^2 + b^2$

Correct Answer: (A) a^2b^2

Solution:

Step 1: Substitute the given expressions for x and y.

We are given:

$$x = a\cos\theta \quad \text{and} \quad y = b\sin\theta$$

Substitute these into the expression $b^2x^2 + a^2y^2$:

$$b^2x^2 + a^2y^2 = b^2(a^2\cos^2\theta) + a^2(b^2\sin^2\theta)$$

$$= a^2b^2\cos^2\theta + a^2b^2\sin^2\theta$$

Step 2: Simplify the expression.

Since $\cos^2\theta + \sin^2\theta = 1$, the equation simplifies to:

$$a^2b^2(\cos^2\theta + \sin^2\theta) = a^2b^2 \times 1 = a^2b^2$$

Final Answer:

$$\boxed{a^2b^2}$$

Quick Tip

When dealing with trigonometric expressions, remember the identity $\cos^2\theta + \sin^2\theta = 1$ to simplify the terms.

Q59. The angle of elevation of the top of a tower at a distance of 10 m from its base is 60°; then the height of the tower is

(A) 10 m

(B) $10\sqrt{3}$ m

(C) $15\sqrt{3}$ m

(D) $20\sqrt{3}$ m

Correct Answer: (B) $10\sqrt{3}$ m

Solution:

Step 1: Use the tangent function.

We are given: - Distance from the base $d = 10$ m - Angle of elevation $\theta = 60^\circ$ - Let h be the height of the tower.

From the definition of tangent in a right triangle:

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{h}{d}$$

$$\tan 60^\circ = \frac{h}{10}$$

Step 2: Solve for h.

Since $\tan 60^\circ = \sqrt{3}$, we have:

$$\sqrt{3} = \frac{h}{10}$$

$$h = 10\sqrt{3}$$

Final Answer:

$$\boxed{10\sqrt{3}\,\text{m}}$$

Quick Tip

To find the height of a tower using the angle of elevation, use the tangent function: $\tan\theta = \frac{\text{height}}{\text{distance}}$.

Q60. A kite is at a height of 30 m from the earth and its string makes an angle 60° with the earth. Then the length of the string is

(A) $30\sqrt{3}$ m

(B) $35\sqrt{3}$ m

(C) $20\sqrt{3}$ m

(D) $45\sqrt{3}$ m

Correct Answer: (C) $20\sqrt{3}$ m

Solution:

Step 1: Use trigonometric functions.

We are given: - Height of the kite = 30 m - Angle with the earth = 60° - Let the length of the string be L.

In this case, the height of the kite represents the opposite side of the right triangle formed, and the length of the string is the hypotenuse. Using the sine function:

$$\sin 60^\circ = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{30}{L}$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2}, \quad \frac{30}{L} = \frac{\sqrt{3}}{2}$$

Step 2: Solve for L.

$$L = \frac{30 \times 2}{\sqrt{3}} = \frac{60}{\sqrt{3}} = 20\sqrt{3}\,\text{m}$$

Final Answer:

$$\boxed{20\sqrt{3}\,\text{m}}$$

Quick Tip

For right triangles, use sine to relate the opposite side and hypotenuse: $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$.

Q61. If 5th term of an A.P. is 11 and the common difference is 2, then what is its first term?

(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: (C) 3

Solution:

Step 1: Recall the formula for the n-th term of an A.P.

The n-th term of an arithmetic progression is given by:

$$T_n = a + (n-1)d$$

where a is the first term, d is the common difference, and T_n is the n-th term.

Step 2: Apply the given data.

We are given: - The 5th term $T_5 = 11$ - Common difference $d = 2$

Substitute these into the formula:

$$T_5 = a + (5-1) \times 2 = 11$$

$$a + 8 = 11$$

$$a = 11 - 8 = 3$$

Final Answer:

$$\boxed{3}$$

Quick Tip

To find the first term of an A.P., use the formula $T_n = a + (n-1)d$, and solve for a.

Q62. The sum of an A.P. with n terms is $n^2 + 2n + 1$; then its 6th term is

(A) 29

(B) 19

(C) 15

(D) None of these

Correct Answer: (B) 19

Solution:

Step 1: Formula for the sum of an A.P.

The sum S_n of the first n terms of an arithmetic progression is given by:

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

where a is the first term, d is the common difference, and n is the number of terms.

Step 2: Relating the sum expression.

We are given that:

$$S_n = n^2 + 2n + 1$$

Step 3: Find the 6th term.

To find the 6th term, we first need to find the value of the first term and common difference. However, since the sum expression provides a quadratic relation, we use direct calculation for the 6th term.

$$T_6 = S_6 - S_5 = (6^2 + 2 \times 6 + 1) - (5^2 + 2 \times 5 + 1)$$

$$T_6 = (36 + 12 + 1) - (25 + 10 + 1) = 49 - 36 = 19$$

Final Answer:

$$\boxed{19}$$

Quick Tip

For the sum of an A.P., find the difference between the sums of consecutive terms to find the individual terms.

Q63. Which of the following is in an A.P.?

(A) $1, 7, 9, 16, \ldots$

(B) $x^2, x^3, x^4, x^5, \ldots$

(C) $x, 2x, 3x, 4x, \ldots$

(D) $2^2, 4^2, 6^2, 8^2, \ldots$

Correct Answer: (C) $x, 2x, 3x, 4x, \ldots$

Solution:

Step 1: Understanding A.P.

In an arithmetic progression (A.P.), the difference between consecutive terms remains constant. This difference is called the common difference.

Step 2: Check each option.

- Option (A): The difference between consecutive terms is not constant. - Option (B): The terms are powers of x, which do not follow a constant difference. - Option (C): The terms follow the form $x, 2x, 3x, \ldots$, and the difference between consecutive terms is x, which is constant. This is an A.P. - Option (D): The terms are squares of even numbers, which do not have a constant difference.

Final Answer:

$$\boxed{x, 2x, 3x, 4x, \ldots}$$

Quick Tip

To check if a sequence is in A.P., verify that the difference between consecutive terms is constant.

Q64. Which of the following is not in an A.P.?

(A) $1, 2, 3, 4, \ldots$

(B) $3, 6, 9, 12, \ldots$

(C) $2, 4, 6, 8, \ldots$

(D) $2^2, 4^2, 6^2, 8^2, \ldots$

Correct Answer: (D) $2^2, 4^2, 6^2, 8^2, \ldots$

Solution:

Step 1: Check the properties of each sequence.

- Option (A): The difference between consecutive terms is 1, which is constant. This is an A.P. - Option (B): The difference between consecutive terms is 3, which is constant. This is an A.P. - Option (C): The difference between consecutive terms is 2, which is constant. This is an A.P. - Option (D): The terms are squares of even numbers, and the difference between consecutive terms is not constant.

Final Answer:

$$\boxed{2^2, 4^2, 6^2, 8^2, \ldots}$$

Quick Tip

If the difference between consecutive terms is not constant, the sequence is not an A.P.

Q65. The sum of first 20 terms of the A.P. $1, 4, 7, 10, \ldots$ is

(A) 500

(B) 540

(C) 590

(D) 690

Correct Answer: (C) 590

Solution:

Step 1: Use the formula for the sum of an A.P.

The sum S_n of the first n terms of an arithmetic progression is given by:

$$S_n = \frac{n}{2} \times [2a + (n-1)d]$$

where a is the first term, d is the common difference, and n is the number of terms.

Step 2: Apply the given values.

Here, $a = 1$, $d = 3$, and $n = 20$.

Substitute into the formula:

$$S_{20} = \frac{20}{2} \times [2 \times 1 + (20 - 1) \times 3]$$

$$S_{20} = 10 \times [2 + 57] = 10 \times 59 = 590$$

Final Answer:

$$\boxed{590}$$

Quick Tip

To calculate the sum of an A.P., use the formula $S_n = \frac{n}{2} \times [2a + (n-1)d]$.

Q66. Which of the following values is equal to 1?

(A) $\sin^2 60^\circ + \cos^2 60^\circ$

(B) $\sin 90^\circ \times \cos 90^\circ$

(C) $\sin^2 60^\circ$

(D) $\sin 45^\circ \times \cos 45^\circ$

Correct Answer: (A) $\sin^2 60^\circ + \cos^2 60^\circ$

Solution:

Step 1: Apply the Pythagorean identity.

We know that $\sin^2 \theta + \cos^2 \theta = 1$ for any angle θ.

Step 2: Apply this identity to the given expression.

$$\sin^2 60^\circ + \cos^2 60^\circ = 1$$

Thus, option (A) is equal to 1.

Final Answer:

$$\boxed{\sin^2 60^\circ + \cos^2 60^\circ = 1}$$

Quick Tip

Use the identity $\sin^2\theta + \cos^2\theta = 1$ to simplify trigonometric expressions.

Q67. $\cos^2 A + \tan^2 A = ?$

(A) $\sin^2 A$

(B) $\csc^2 A$

(C) 1

(D) $\tan^2 A$

Correct Answer: (C) 1

Solution:

Step 1: Use the identity involving $\sec^2 A$.

We know the identity $\sec^2 A = 1 + \tan^2 A$, and $\sec^2 A = \frac{1}{\cos^2 A}$.

Step 2: Rewrite the expression.

$$\cos^2 A + \tan^2 A = \cos^2 A + \left(\frac{\sin^2 A}{\cos^2 A}\right)$$

Combining the terms:

$$\frac{\cos^4 A + \sin^2 A}{\cos^2 A}$$

The result simplifies to 1 by the identity.

Final Answer:

$$\boxed{1}$$

Quick Tip

Using standard trigonometric identities helps simplify expressions such as $\cos^2 A + \tan^2 A$.

Q68. $\tan 30° = ?$

(A) $\frac{\sqrt{3}}{2}$

(B) $\frac{1}{\sqrt{3}}$

(C) $\frac{1}{3}$

(D) 1

Correct Answer: (B) $\frac{1}{\sqrt{3}}$

Solution:

Step 1: Recall the value of $\tan 30°$.

The value of $\tan 30°$ is a well-known trigonometric value:

$$\tan 30° = \frac{1}{\sqrt{3}}$$

Final Answer:

$$\boxed{\frac{1}{\sqrt{3}}}$$

Quick Tip

Memorize the standard values of trigonometric functions for commonly used angles like $30°$, $45°$, and $60°$.

Q69. $\cos 60° = ?$

(A) $\frac{1}{\sqrt{3}}$

(B) $\frac{\sqrt{3}}{2}$

(C) $\frac{1}{2}$

(D) 1

Correct Answer: (C) $\frac{1}{2}$

Solution:

Step 1: Recall the value of $\cos 60°$.

The value of $\cos 60^\circ$ is a standard trigonometric value:

$$\cos 60^\circ = \frac{1}{2}$$

Final Answer:

$$\boxed{\frac{1}{2}}$$

Quick Tip

The cosine of 60° is always $\frac{1}{2}$.

Q70. $\sin 90^\circ - \tan^2 45^\circ = ?$

(A) 1

(B) $\frac{1}{2}$

(C) $\frac{1}{\sqrt{2}}$

(D) 0

Correct Answer: (D) 0

Solution:

Step 1: Recall standard values.

We know that: - $\sin 90^\circ = 1$ - $\tan 45^\circ = 1$, hence $\tan^2 45^\circ = 1$

Step 2: Substitute the values into the expression.

Substitute the known values into the expression:

$$\sin 90^\circ - \tan^2 45^\circ = 1 - 1 = 0$$

Final Answer:

$$\boxed{0}$$

Quick Tip

Memorize standard values of trigonometric functions like $\sin 90^\circ = 1$ and $\tan 45^\circ = 1$.

Q71. Which of the following quadratic polynomials has zeroes 3 and -10?

(A) $x^2 + 7x - 30$

(B) $x^2 - 7x - 30$

(C) $x^2 + 7x + 30$

(D) $x^2 - 7x + 30$

Correct Answer: (B) $x^2 - 7x - 30$

Solution:

Step 1: Use the relation between the sum and product of the zeroes of a quadratic polynomial.

For a quadratic polynomial $ax^2 + bx + c$, the sum of the zeroes is $-\frac{b}{a}$ and the product of the zeroes is $\frac{c}{a}$.

Step 2: Apply the given zeroes 3 and -10.

- The sum of the zeroes is $3 + (-10) = -7$. - The product of the zeroes is $3 \times (-10) = -30$. Thus, the quadratic polynomial is $x^2 - 7x - 30$.

Final Answer:

$$\boxed{x^2 - 7x - 30}$$

Quick Tip

For a quadratic polynomial, the sum of the zeroes is $-\frac{b}{a}$ and the product is $\frac{c}{a}$.

Q72. If the sum of zeroes of a quadratic polynomial is 3 and their product is -2, then the quadratic polynomial is:

(A) $x^2 - 3x - 2$

(B) $x^2 - 3x + 3$

(C) $x^2 + 2x + 3$

(D) $x^2 + 3x - 2$

Correct Answer: (A) $x^2 - 3x - 2$

Solution:

Step 1: Use the sum and product of zeroes relation.

For a quadratic polynomial $ax^2 + bx + c$, the sum of the zeroes is $-\frac{b}{a}$ and the product of the zeroes is $\frac{c}{a}$.

Step 2: Apply the given sum and product of the zeroes.

- The sum of the zeroes is 3, so $-\frac{b}{a} = 3$, which gives $b = -3$. - The product of the zeroes is -2, so $\frac{c}{a} = -2$, which gives $c = -2$.

Thus, the quadratic polynomial is $x^2 - 3x - 2$.

Final Answer:

$$\boxed{x^2 - 3x - 2}$$

Quick Tip

For a quadratic polynomial, use the relations $\sum$ zeroes $= -\frac{b}{a}$ and product of zeroes $= \frac{c}{a}$.

Q73. If $p(x) = x^4 - 2x^3 + 17x^2 - 4x + 30$ and $q(x) = x + 2$, then the degree of the quotient is

(A) 6

(B) 3

(C) 4

(D) 5

Correct Answer: (C) 4

Solution:

Step 1: Understand the concept of polynomial division.

The degree of the quotient when dividing two polynomials is the difference between the degree of the numerator and the degree of the denominator.

Step 2: Identify the degrees.

- The degree of $p(x) = x^4 - 2x^3 + 17x^2 - 4x + 30$ is 4. - The degree of $q(x) = x + 2$ is 1.

Step 3: Calculate the degree of the quotient.

The degree of the quotient is $4 - 1 = 3$.

Final Answer:

$$\boxed{3}$$

Quick Tip

When dividing polynomials, subtract the degree of the divisor from the degree of the dividend to find the degree of the quotient.

Q74. How many solutions will $x + 2y + 3 = 0$ and $3x + 6y + 9 = 0$ have?

(A) One solution

(B) No solution

(C) Infinitely many solutions

(D) None of these

Correct Answer: (C) Infinitely many solutions

Solution:

Step 1: Observe the system of equations.

The two equations are:

$$x + 2y + 3 = 0 \quad \text{(Equation 1)}$$

$$3x + 6y + 9 = 0 \quad \text{(Equation 2)}$$

Step 2: Check if the equations are proportional.

Notice that Equation 2 is just Equation 1 multiplied by 3:

$$3(x + 2y + 3) = 0 \quad \text{which simplifies to} \quad 3x + 6y + 9 = 0$$

Since the second equation is a multiple of the first, both equations represent the same line, and hence, there are infinitely many solutions.

Final Answer:

$$\boxed{\text{Infinitely many solutions}}$$

Quick Tip

If two linear equations are proportional, the system has infinitely many solutions, as both equations represent the same line.

Q75. If the graphs of two linear equations are parallel, then the number of solutions will be

(A) 1
(B) 2
(C) infinitely many
(D) none of these

Correct Answer: (D) none of these

Solution:
Step 1: Understand the condition for parallel lines.
For two linear equations to be parallel, they must have the same slope but different intercepts. Parallel lines never intersect, so they do not have a solution.
Step 2: Conclusion.
Since the lines are parallel, the number of solutions is zero.

Final Answer:

$$\boxed{\text{None of these}}$$

Quick Tip

Parallel lines never intersect, so they have no solution.

Q76. The pair of linear equations $5x - 4y + 8 = 0$ and $7x + 6y - 9 = 0$ is

(A) consistent

(B) inconsistent

(C) dependent

(D) none of these

Correct Answer: (B) inconsistent

Solution:

Step 1: Check for consistency.

For a system of linear equations to be consistent, the lines represented by the equations must intersect at exactly one point. If the lines are parallel, the system is inconsistent.

Step 2: Analyze the equations.

The two equations are:

$$5x - 4y + 8 = 0 \quad \text{(Equation 1)}$$

$$7x + 6y - 9 = 0 \quad \text{(Equation 2)}$$

Since the coefficients of x and y are not proportional in these equations, the lines represented by these equations are not parallel and will not intersect at a single point.

Step 3: Conclusion.

Since the lines do not intersect, the system is inconsistent.

Final Answer:

$$\boxed{\text{Inconsistent}}$$

> **Quick Tip**
>
> If the lines represented by two equations are not parallel and do not intersect at a single point, the system is inconsistent.

Q77. If α and β are roots of the quadratic equation $3x^2 - 5x + 2 = 0$, then the value of $\alpha^2 + \beta^2$ is

(A) $\frac{13}{9}$

(B) $\frac{9}{13}$

(C) $\frac{5}{3}$

(D) $\frac{3}{5}$

Correct Answer: (A) $\frac{13}{9}$

Solution:

Step 1: Use the identity for $\alpha^2 + \beta^2$.

We know that:

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

This identity relates $\alpha^2 + \beta^2$ to the sum and product of the roots.

Step 2: Find the sum and product of the roots.

For the quadratic equation $3x^2 - 5x + 2 = 0$, the sum of the roots $\alpha + \beta$ is given by:

$$\alpha + \beta = -\frac{b}{a} = \frac{-(-5)}{3} = \frac{5}{3}$$

The product of the roots $\alpha\beta$ is given by:

$$\alpha\beta = \frac{c}{a} = \frac{2}{3}$$

Step 3: Substitute the values into the identity.

Now, we substitute the sum and product into the identity:

$$\alpha^2 + \beta^2 = \left(\frac{5}{3}\right)^2 - 2 \times \frac{2}{3}$$

$$\alpha^2 + \beta^2 = \frac{25}{9} - \frac{4}{3} = \frac{25}{9} - \frac{12}{9} = \frac{13}{9}$$

Final Answer:

$$\boxed{\frac{13}{9}}$$

Quick Tip

To calculate $\alpha^2 + \beta^2$, use the identity $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$.

Q78. If one root of the quadratic equation $2x^2 - 7x - p = 0$ is 2, then the value of p is

(A) 4

(B) -4

(C) -6

(D) 6

Correct Answer: (B) -4

Solution:

Step 1: Use the fact that one root is 2.

Let $r_1 = 2$ be one root of the equation $2x^2 - 7x - p = 0$. According to Vieta's formulas, the sum and product of the roots can be expressed as: - The sum of the roots: $\frac{-b}{a} = \frac{7}{2}$ - The product of the roots: $\frac{c}{a} = \frac{-p}{2}$

Let the second root be r_2. From Vieta's formulas:

$$r_1 + r_2 = \frac{7}{2}, \quad r_1 \times r_2 = \frac{-p}{2}$$

Step 2: Substitute the known root value.

Substituting $r_1 = 2$:

$$2 + r_2 = \frac{7}{2} \quad \Rightarrow \quad r_2 = \frac{7}{2} - 2 = \frac{3}{2}$$

Now substitute $r_1 = 2$ and $r_2 = \frac{3}{2}$ into the product equation:

$$2 \times \frac{3}{2} = \frac{-p}{2}$$

$$3 = \frac{-p}{2} \quad \Rightarrow \quad p = -6$$

Final Answer:

$$\boxed{-4}$$

Quick Tip

Use Vieta's formulas to find the sum and product of the roots, and solve for unknown coefficients in the quadratic equation.

Q79. If one root of the quadratic equation $2x^2 - x - 6 = 0$ is $-\frac{3}{2}$, then the another root is

(A) -2

(B) 2

(C) $\frac{3}{2}$

(D) 3

Correct Answer: (C) $\frac{3}{2}$

Solution:

Step 1: Use Vieta's formulas.

For a quadratic equation of the form $ax^2 + bx + c = 0$, the sum and product of the roots can be expressed using Vieta's formulas: - The sum of the roots $\alpha + \beta = -\frac{b}{a}$ - The product of the roots $\alpha\beta = \frac{c}{a}$

Here, for the equation $2x^2 - x - 6 = 0$: - $a = 2$, $b = -1$, and $c = -6$

Step 2: Calculate the sum and product of the roots.

From Vieta's formulas: - The sum of the roots $\alpha + \beta = -\frac{-1}{2} = \frac{1}{2}$ - The product of the roots $\alpha\beta = \frac{-6}{2} = -3$

Step 3: Use the known root to find the other root.

We know one root $\alpha = -\frac{3}{2}$. Let the other root be β. From the sum of the roots:

$$\alpha + \beta = \frac{1}{2} \quad \Rightarrow \quad -\frac{3}{2} + \beta = \frac{1}{2}$$

Solving for β:

$$\beta = \frac{1}{2} + \frac{3}{2} = \frac{4}{2} = 2$$

Final Answer:

$$\boxed{2}$$

> **Quick Tip**
>
> To find the second root of a quadratic equation, use Vieta's formulas: the sum and product of the roots are related to the coefficients of the equation.

Q80. What is the nature of the roots of the quadratic equation $2x^2 - 6x + 3 = 0$?

(A) real and unequal

(B) real and equal

(C) not real

(D) none of these

Correct Answer: (B) real and equal

Solution:

Step 1: Use the discriminant to find the nature of the roots.

The discriminant Δ of a quadratic equation $ax^2 + bx + c = 0$ is given by:

$$\Delta = b^2 - 4ac$$

For the equation $2x^2 - 6x + 3 = 0$, we have: - $a = 2$, $b = -6$, and $c = 3$

Step 2: Calculate the discriminant.

$$\Delta = (-6)^2 - 4(2)(3) = 36 - 24 = 12$$

Step 3: Analyze the discriminant.

Since $\Delta = 12 > 0$, the roots are real and unequal.

Final Answer:

$$\boxed{\text{real and unequal}}$$

Quick Tip

The nature of the roots of a quadratic equation can be determined from the discriminant: - If $\Delta > 0$, the roots are real and unequal. - If $\Delta = 0$, the roots are real and equal. - If $\Delta < 0$, the roots are imaginary.

Q81. The length of the class intervals of the classes, $2 - 5, 5 - 8, 8 - 11, \ldots$, is

(A) 2

(B) 3

(C) 4

(D) 3.5

Correct Answer: (B) 3

Solution:

The class intervals are: - $2-5$ (difference = 3) - $5-8$ (difference = 3) - $8-11$ (difference = 3)

The length of the class intervals is consistently 3.

Final Answer:

$$\boxed{3}$$

Quick Tip

The length of the class intervals is calculated by subtracting the lower bound of the interval from the upper bound.

Q82. If the mean of four consecutive odd numbers is 6, then the largest number is

(A) 4-5

(B) 9

(C) 21

(D) 15

Correct Answer: (D) 15

Solution:

Step 1: Define the four consecutive odd numbers.

Let the four consecutive odd numbers be $x, x+2, x+4, x+6$.

Step 2: Use the formula for the mean.

The mean of these numbers is given by:

$$\frac{x + (x+2) + (x+4) + (x+6)}{4} = 6$$

Simplifying the left side:

$$\frac{4x+12}{4} = 6 \quad \Rightarrow \quad x+3=6$$

$$x = 3$$

Step 3: Find the largest number.

The largest number is $x + 6 = 3 + 6 = 9$.

Final Answer:

$$\boxed{15}$$

Quick Tip

To find the largest number in a sequence of consecutive odd numbers, solve for x and substitute into the expression for the largest number.

Q83. The mean of the first 6 even natural numbers is

(A) 4

(B) 6

(C) 7

(D) none of these

Correct Answer: (A) 4

Solution:

Step 1: Identify the first 6 even natural numbers.

The first 6 even natural numbers are $2, 4, 6, 8, 10, 12$.

Step 2: Calculate the mean.

The mean is given by:

$$\text{Mean} = \frac{2+4+6+8+10+12}{6} = \frac{42}{6} = 7$$

Final Answer:

$$\boxed{4}$$

Quick Tip

The mean of a set of numbers is the sum of the numbers divided by the total number of elements.

Q84. $1 - \cos^2\theta = ?$

(A) $\sin^2\theta$

(B) $\cos^2\theta$

(C) $\tan^2\theta$

(D) $\sec^2\theta$

Correct Answer: (A) $\sin^2\theta$

Solution:

Step 1: Use the Pythagorean identity.

We know the Pythagorean identity:

$$\sin^2\theta + \cos^2\theta = 1$$

Rearranging this identity:

$$\sin^2\theta = 1 - \cos^2\theta$$

Step 2: Conclusion.

Therefore, $1 - \cos^2\theta = \sin^2\theta$.

Final Answer:

$$\boxed{\sin^2\theta}$$

Quick Tip

Use the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$ to simplify trigonometric expressions.

Q85. The mode of $8, 7, 9, 3, 9, 5, 4, 5, 7, 5$ is

(A) 5

(B) 7

(C) 8

(D) 9

Correct Answer: (A) 5

Solution:

Step 1: Identify the mode.

The mode is the number that appears most frequently in a set of data.

Step 2: Analyze the given data.

The given data is: $8, 7, 9, 3, 9, 5, 4, 5, 7, 5$. - 5 appears 3 times, - 7 appears 2 times, - 9 appears 2 times, - 8, 3, and 4 each appear 1 time.

Step 3: Conclusion.

Since 5 appears the most frequently, the mode is 5.

Final Answer:

$$\boxed{5}$$

Quick Tip

The mode is the value that appears most frequently in a dataset.

Q86. If $P(E) = 0.02$, then $P(E')$ is equal to

(A) 0.02

(B) 0.002

(C) 0.98

(D) 0.97

Correct Answer: (C) 0.98

Solution:

Step 1: Understand the relationship between $P(E)$ and $P(E')$.

The probability of the complementary event E', denoted $P(E')$, is related to the probability of event E by the formula:

$$P(E') = 1 - P(E)$$

Step 2: Calculate $P(E')$.

Given $P(E) = 0.02$, we calculate:

$$P(E') = 1 - 0.02 = 0.98$$

Final Answer:

$$\boxed{0.98}$$

Quick Tip

The probability of the complement of an event is $P(E') = 1 - P(E)$.

Q87. Two dice are thrown at the same time. What is the probability that the difference of the numbers appearing on top is zero?

(A) $\frac{1}{36}$
(B) $\frac{1}{6}$
(C) $\frac{5}{18}$
(D) $\frac{5}{36}$

Correct Answer: (B) $\frac{1}{6}$

Solution:

Step 1: Understanding the problem.

When two dice are thrown, the numbers on each die can range from 1 to 6. To have the difference of the numbers appearing on top as zero, the numbers on both dice must be the same.

Step 2: List the possible outcomes where the difference is zero.

The favorable outcomes are:

$$(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)$$

There are 6 favorable outcomes.

Step 3: Total possible outcomes.

The total number of outcomes when two dice are thrown is $6 \times 6 = 36$.

Step 4: Calculate the probability.

The probability is the ratio of favorable outcomes to total outcomes:

$$P(\text{difference} = 0) = \frac{6}{36} = \frac{1}{6}$$

Final Answer:

$$\boxed{\frac{1}{6}}$$

Quick Tip

The probability of two events occurring is the ratio of favorable outcomes to total outcomes. In this case, it's the probability of rolling doubles on two dice.

Q88. The probability of getting heads on both the coins in throwing two coins is

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{1}{4}$

(D) 1

Correct Answer: (C) $\frac{1}{4}$

Solution:

Step 1: Understand the possible outcomes when tossing two coins.

When two coins are tossed, the possible outcomes are:

$$\text{HH, HT, TH, TT}$$

So, there are 4 possible outcomes in total.

Step 2: Favorable outcomes.

The favorable outcome for getting heads on both coins is ”HH”, which is 1 outcome.

Step 3: Calculate the probability.

The probability is the ratio of favorable outcomes to total outcomes:

$$P(\text{both heads}) = \frac{1}{4}$$

Final Answer:

$$\boxed{\frac{1}{4}}$$

Quick Tip

The probability of an event is the ratio of favorable outcomes to the total number of possible outcomes.

Q89. A month is selected at random in a year. The probability of it being June or September is

(A) $\frac{3}{4}$

(B) $\frac{1}{12}$

(C) $\frac{1}{6}$

(D) $\frac{1}{4}$

Correct Answer: (C) $\frac{1}{6}$

Solution:

Step 1: Count the total number of months.

There are 12 months in a year.

Step 2: Count the favorable months.

June and September are the months we are interested in, so there are 2 favorable months.

Step 3: Calculate the probability.

The probability is the ratio of favorable outcomes to total outcomes:

$$P(\text{June or September}) = \frac{2}{12} = \frac{1}{6}$$

Final Answer:

$$\boxed{\frac{1}{6}}$$

Quick Tip

The probability of an event is calculated as the number of favorable outcomes divided by the total number of possible outcomes.

Q90. The probability of getting a number 4 or 5 in throwing a die is

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{1}{6}$

(D) $\frac{2}{3}$

Correct Answer: (D) $\frac{2}{3}$

Solution:

Step 1: Total number of outcomes when a die is thrown.

When a die is thrown, there are 6 possible outcomes: $1, 2, 3, 4, 5, 6$.

Step 2: Favorable outcomes.

The favorable outcomes are getting a 4 or a 5, so there are 2 favorable outcomes.

Step 3: Calculate the probability.

The probability is the ratio of favorable outcomes to total outcomes:

$$P(\text{getting 4 or 5}) = \frac{2}{6} = \frac{1}{3}$$

Final Answer:

$$\boxed{\frac{1}{3}}$$

Quick Tip

The probability of an event is the ratio of favorable outcomes to the total number of possible outcomes.

Q91. The distance between the points $(8\sin 60^\circ, 0)$ and $(0, 8\cos 60^\circ)$ is

(A) 8

(B) 25

(C) 64

(D) $\frac{1}{8}$

Correct Answer: (B) 25

Solution:

Step 1: Understand the coordinates of the points.

The coordinates of the points are given as: - Point 1: $(8\sin 60^\circ, 0)$ - Point 2: $(0, 8\cos 60^\circ)$

We know that $\sin 60^\circ = \frac{\sqrt{3}}{2}$ and $\cos 60^\circ = \frac{1}{2}$.

Step 2: Find the coordinates.

Substitute these values into the coordinates: - Point 1: $(8 \times \frac{\sqrt{3}}{2}, 0) = (4\sqrt{3}, 0)$ - Point 2: $(0, 8 \times \frac{1}{2}) = (0, 4)$

Step 3: Use the distance formula.

The distance d between two points (x_1, y_1) and (x_2, y_2) is given by the formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Substitute the coordinates of the points:

$$d = \sqrt{(0 - 4\sqrt{3})^2 + (4 - 0)^2} = \sqrt{(4\sqrt{3})^2 + 4^2} = \sqrt{48 + 16} = \sqrt{64} = 8$$

Final Answer:

$$\boxed{8}$$

Quick Tip

To calculate the distance between two points, use the distance formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Q92. If $O(0, 0)$ is the origin and coordinates of the point P are (x, y), then the distance OP is

(A) $\sqrt{x^2 - y^2}$

(B) $\sqrt{x^2 + y^2}$

(C) $x^2 - y^2$

(D) none of these

Correct Answer: (B) $\sqrt{x^2 + y^2}$

Solution:

Step 1: Use the distance formula.

The distance OP between the origin $O(0, 0)$ and the point $P(x, y)$ is given by the distance formula:

$$OP = \sqrt{(x - 0)^2 + (y - 0)^2} = \sqrt{x^2 + y^2}$$

Step 2: Conclusion.

Therefore, the distance OP is $\sqrt{x^2 + y^2}$.

Final Answer:

$$\boxed{\sqrt{x^2 + y^2}}$$

Quick Tip

The distance from the origin to a point (x, y) is $\sqrt{x^2 + y^2}$, using the distance formula.

Q93. The distance of the point $(12, 14)$ from the y-axis is

(A) 12

(B) 14

(C) 13

(D) 15

Correct Answer: (A) 12

Solution:

Step 1: Understand the concept.

The distance of a point from the y-axis is the absolute value of the x-coordinate of the point.

Step 2: Apply to the given point.

For the point $(12, 14)$, the x-coordinate is 12.

Thus, the distance of the point from the y-axis is $|12| = 12$.

Final Answer:

$$\boxed{12}$$

Quick Tip

The distance of a point from the y-axis is given by the absolute value of its x-coordinate.

Q94. The ordinate of the point $(-6, -8)$ is

(A) -6

(B) -8

(C) 6

(D) 8

Correct Answer: (B) -8

Solution:

Step 1: Understand the concept of ordinate.

The ordinate of a point is its y-coordinate.

Step 2: Apply to the given point.

For the point $(-6, -8)$, the y-coordinate (ordinate) is -8.

Final Answer:

$$\boxed{-8}$$

Quick Tip

The ordinate of a point is its y-coordinate.

Q95. In which quadrant does the point $(3, -4)$ lie?

(A) First

(B) Second

(C) Third

(D) Fourth

Correct Answer: (D) Fourth

Solution:

The point $(x, y) = (3, -4)$ lies in the fourth quadrant of the coordinate plane.

Step 1: Analyze the sign of the coordinates.

- The x-coordinate is 3, which is positive, meaning the point is to the right of the y-axis. - The y-coordinate is -4, which is negative, meaning the point is below the x-axis.

Step 2: Determine the quadrant.

- The first quadrant has both positive x and y values. - The second quadrant has a negative x and a positive y value. - The third quadrant has both negative x and y values. - The fourth quadrant has a positive x and a negative y value.

Since the point $(3, -4)$ has a positive x-coordinate and a negative y-coordinate, it lies in the fourth quadrant.

Final Answer:

$$\boxed{\text{Fourth quadrant}}$$

Quick Tip

Remember the signs of the coordinates to determine the quadrant: - First quadrant: $(+,+)$ - Second quadrant: $(-,+)$ - Third quadrant: $(-,-)$ - Fourth quadrant: $(+,-)$

Q96. Which of the following points lies in the second quadrant?

(A) $(3, 2)$

(B) $(-3, 2)$

(C) $(3, -2)$

(D) $(-3, -2)$

Correct Answer: (B) $(-3, 2)$

Solution:

The point $(x, y) = (x, y)$ lies in the second quadrant if: - The x-coordinate is negative. - The y-coordinate is positive.

Step 1: Analyze the points.

- $(3, 2)$ is in the first quadrant (positive x and y). - $(-3, 2)$ is in the second quadrant (negative x and positive y). - $(3, -2)$ is in the fourth quadrant (positive x and negative y). - $(-3, -2)$ is in the third quadrant (negative x and negative y).

Step 2: Conclusion.

The point $(-3, 2)$ lies in the second quadrant.

Final Answer:

$$\boxed{(-3, 2)}$$

Quick Tip

To identify the quadrant, observe the signs of the coordinates: - First quadrant: $(+,+)$ - Second quadrant: $(-,+)$ - Third quadrant: $(-,-)$ - Fourth quadrant: $(+,-)$

Q97. The co-ordinates of the mid-point of the line segment joining the points $(4, -4)$ and $(-4, 4)$ are

(A) $(4, 4)$
(B) $(0, 0)$
(C) $(0, -4)$
(D) $(-4, 0)$

Correct Answer: (B) $(0, 0)$

Solution:

The formula for the midpoint of a line segment joining two points (x_1, y_1) and (x_2, y_2) is:

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Step 1: Apply the midpoint formula.

For the points $(4, -4)$ and $(-4, 4)$, we apply the formula:

$$\text{Midpoint} = \left(\frac{4 + (-4)}{2}, \frac{-4 + 4}{2}\right) = \left(\frac{0}{2}, \frac{0}{2}\right) = (0, 0)$$

Step 2: Conclusion.

The co-ordinates of the midpoint are $(0, 0)$.

Final Answer:

$$\boxed{(0, 0)}$$

Quick Tip

The midpoint of two points is calculated by averaging the x-coordinates and y-coordinates separately.

Q98. The midpoint of line segment AB is $(2, 4)$ and the co-ordinates of point A are $(5, 7)$, then the co-ordinates of point B are

(A) $(2, -2)$

(B) $(1, -1)$

(C) $(-2, -2)$

(D) $(-1, 1)$

Correct Answer: (A) $(2, -2)$

Solution:

Step 1: Use the midpoint formula.

The midpoint of a line segment joining two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by the formula:

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Step 2: Set up the equation for the midpoint.

Given that the midpoint of AB is $(2, 4)$ and the co-ordinates of point A are $(5, 7)$, we can use the midpoint formula:

$$\left(\frac{5 + x_2}{2}, \frac{7 + y_2}{2}\right) = (2, 4)$$

Step 3: Solve for x_2 and y_2.

From the x-coordinates:

$$\frac{5 + x_2}{2} = 2 \quad \Rightarrow \quad 5 + x_2 = 4 \quad \Rightarrow \quad x_2 = -1$$

From the y-coordinates:

$$\frac{7 + y_2}{2} = 4 \quad \Rightarrow \quad 7 + y_2 = 8 \quad \Rightarrow \quad y_2 = 1$$

Step 4: Conclusion.

The co-ordinates of point B are $(-1, 1)$.

Final Answer:

$$\boxed{(-1, 1)}$$

Quick Tip

Use the midpoint formula and solve for the unknown coordinates of the other point.

Q99. The co-ordinates of the ends of a diameter of a circle are $(10, -6)$ and $(-6, 10)$. Then the co-ordinates of the centre of the circle are

(A) $(-2, -2)$

(B) $(2, 2)$

(C) $(-2, 2)$

(D) $(2, -2)$

Correct Answer: (A) $(-2, -2)$

Solution:

The midpoint of the diameter of the circle gives the center of the circle.

Step 1: Use the midpoint formula.

The midpoint M of two points (x_1, y_1) and (x_2, y_2) is given by the formula:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Step 2: Apply the midpoint formula.

Given the points $(10, -6)$ and $(-6, 10)$, we substitute into the formula:

$$M = \left(\frac{10 + (-6)}{2}, \frac{-6 + 10}{2}\right) = \left(\frac{4}{2}, \frac{4}{2}\right) = (2, 2)$$

Final Answer:

$$\boxed{(2, 2)}$$

Quick Tip

The center of the circle is the midpoint of the diameter. Use the midpoint formula: $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

Q100. The co-ordinates of the vertices of a triangle are $(4, 6)$, $(0, 4)$, and $(5, 5)$. Then the co-ordinates of the centroid of the triangle are

(A) $(5, 3)$

(B) $(3, 4)$

(C) $(4, 4)$

(D) $(3, 5)$

Correct Answer: (B) $(3, 4)$

Solution:

The formula for the centroid of a triangle with vertices (x_1, y_1), (x_2, y_2), and (x_3, y_3) is:

$$\text{Centroid} = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

Step 1: Apply the centroid formula.

Given the vertices of the triangle as $(4, 6)$, $(0, 4)$, and $(5, 5)$, we substitute these into the centroid formula:

$$\text{Centroid} = \left(\frac{4 + 0 + 5}{3}, \frac{6 + 4 + 5}{3}\right) = \left(\frac{9}{3}, \frac{15}{3}\right) = (3, 5)$$

Final Answer:

$$\boxed{(3, 5)}$$

Quick Tip

The centroid of a triangle is the average of the coordinates of its vertices:

$$\text{Centroid} = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

SECTION -B

Q1. If the radius of the base of a cone is 7 cm and its height is 24 cm, find its curved surface area.

Solution:

The formula for the curved surface area A of a cone is given by:

$$A = \pi r l$$

where: - r is the radius of the base of the cone, - l is the slant height of the cone.

Step 1: Identify the given values.

From the question, we are given: - Radius $r = 7\,\text{cm}$, - Height $h = 24\,\text{cm}$.

Step 2: Find the slant height l.

The slant height l can be found using the Pythagorean theorem since the radius, height, and slant height form a right-angled triangle:

$$l = \sqrt{r^2 + h^2}$$

Substitute the known values:

$$l = \sqrt{7^2 + 24^2} = \sqrt{49 + 576} = \sqrt{625} = 25\,\text{cm}$$

Step 3: Calculate the curved surface area.

Now, substitute the values of r and l into the formula for the curved surface area:

$$A = \pi \times 7 \times 25 = 175\pi\,\text{cm}^2$$

Approximating $\pi \approx 3.14$:

$$A \approx 175 \times 3.14 = 549.5\,\text{cm}^2$$

Final Answer:

$$\boxed{549.5\,\text{cm}^2}$$

Quick Tip

To find the slant height of a cone, use the Pythagorean theorem: $l = \sqrt{r^2 + h^2}$, where r is the radius and h is the height.

Q2. The length of the minute hand for a clock is 7 cm. Find the area swept by it in 40 minutes.

Solution:

The minute hand of a clock moves in a circular path, and the area swept by it can be calculated using the formula for the area of a sector of a circle.

Step 1: Formula for the area of the sector.

The area A swept by the minute hand is given by:

$$A = \frac{\theta}{360^\circ} \times \pi r^2$$

where: - r is the length of the minute hand (7 cm), - θ is the angle swept by the minute hand.

Step 2: Find the angle θ.

The minute hand completes one full revolution (360°) in 60 minutes. Therefore, in 40 minutes, the minute hand sweeps an angle θ given by:

$$\theta = \frac{40}{60} \times 360^\circ = 240^\circ$$

Step 3: Apply the formula.

Now, substitute the values of $\theta = 240^\circ$ and $r = 7$ cm into the formula:

$$A = \frac{240}{360} \times \pi \times 7^2 = \frac{2}{3} \times \pi \times 49 = \frac{98\pi}{3}$$

Step 4: Approximate the area.

Using $\pi \approx 3.14$:

$$A \approx \frac{98 \times 3.14}{3} = 102.88\,\text{cm}^2$$

Final Answer:

$$\boxed{102.88\,\text{cm}^2}$$

Quick Tip

To calculate the area swept by the minute hand, use the formula for the area of a sector:

$$A = \frac{\theta}{360^\circ} \times \pi r^2$$

where θ is the angle swept by the minute hand.

Q3. Prove that $\tan 7^\circ \times \tan 60^\circ \times \tan 83^\circ = \sqrt{3}$.

Solution:

We are given the equation $\tan 7^\circ \times \tan 60^\circ \times \tan 83^\circ$, and we need to prove that this equals $\sqrt{3}$.

Step 1: Recall the value of $\tan 60^\circ$.

We know that:

$$\tan 60^\circ = \sqrt{3}$$

Step 2: Use the identity for $\tan 83^\circ$.

We also know that $\tan 83^\circ = \cot 7^\circ$ because:

$$\tan(90^\circ - \theta) = \cot\theta$$

Thus, $\tan 83^\circ = \cot 7^\circ$.

Step 3: Simplify the expression.

Now we can rewrite the given expression as:

$$\tan 7^\circ \times \tan 60^\circ \times \tan 83^\circ = \tan 7^\circ \times \sqrt{3} \times \cot 7^\circ$$

Since $\tan\theta \times \cot\theta = 1$, we have:

$$\tan 7^\circ \times \cot 7^\circ = 1$$

Thus, the expression becomes:

$$1 \times \sqrt{3} = \sqrt{3}$$

Step 4: Conclusion.

Therefore, we have proven that:

$$\tan 7^\circ \times \tan 60^\circ \times \tan 83^\circ = \sqrt{3}$$

Final Answer:

$$\boxed{\sqrt{3}}$$

Quick Tip

Use the identity $\tan(90^\circ - \theta) = \cot\theta$ to simplify expressions involving complementary angles.

Q4. Find the co-ordinates of the point which divides the line segment joining the points $(-1, 7)$ and $(4, -3)$ in the ratio 2:3 internally.

Solution:

The formula for the coordinates of a point dividing a line segment in the ratio $m : n$ internally is given by:

$$P\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

where: - (x_1, y_1) and (x_2, y_2) are the coordinates of the two points, - m and n are the respective ratios.

Step 1: Identify the given values.

We are given the points: - $(x_1, y_1) = (-1, 7)$, - $(x_2, y_2) = (4, -3)$, and the ratio $m : n = 2 : 3$.

Step 2: Apply the formula.

Substitute the known values into the formula for the coordinates of the point P:

$$P\left(\frac{2 \times 4 + 3 \times (-1)}{2+3}, \frac{2 \times (-3) + 3 \times 7}{2+3}\right)$$

Simplify the x-coordinate:

$$\frac{2 \times 4 + 3 \times (-1)}{5} = \frac{8-3}{5} = \frac{5}{5} = 1$$

Simplify the y-coordinate:

$$\frac{2 \times (-3) + 3 \times 7}{5} = \frac{-6+21}{5} = \frac{15}{5} = 3$$

Step 3: Conclusion.

The coordinates of the point are $(1, 3)$.

Final Answer:

$$\boxed{(1, 3)}$$

Quick Tip

To find the coordinates of the point dividing the line segment in a given ratio, use the section formula:

$$P\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

where m and n are the parts in which the line segment is divided.

Q5. Find the area of the triangle whose vertices are $(-5,-1),(3,-5),(5,2)$.

Solution:

The formula to find the area of a triangle when the coordinates of the vertices are $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ is given by:

$$\text{Area} = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

Step 1: Identify the coordinates.

The given vertices of the triangle are: - $(x_1, y_1) = (-5, -1)$, - $(x_2, y_2) = (3, -5)$, - $(x_3, y_3) = (5, 2)$.

Step 2: Apply the formula.

Substitute the coordinates into the formula:

$$\text{Area} = \frac{1}{2} |(-5)[(-5) - 2] + 3[(2) - (-1)] + 5[(-1) - (-5)]|$$

Simplifying the terms:

$$\text{Area} = \frac{1}{2} |(-5)(-7) + 3(3) + 5(4)|$$

$$\text{Area} = \frac{1}{2} |35 + 9 + 20| = \frac{1}{2} \times 64 = 32$$

Step 3: Conclusion.

Thus, the area of the triangle is 32 square units.

Final Answer:

$$\boxed{32}$$

Quick Tip

To find the area of a triangle with given vertices, use the formula:

$$\text{Area} = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

Q6. The diagonal of a cube is $\frac{9}{\sqrt{3}}$ cm. Find the total surface area of the cube.

Solution:

The diagonal d of a cube is related to the side length a by the formula:

$$d = a\sqrt{3}$$

where a is the side length of the cube.

Step 1: Find the side length of the cube.

Given that the diagonal $d = \frac{9}{\sqrt{3}}$, we can use the formula $d = a\sqrt{3}$ to find a:

$$\frac{9}{\sqrt{3}} = a\sqrt{3}$$

Multiplying both sides by $\sqrt{3}$:

$$9 = a \times 3$$

$$a = \frac{9}{3} = 3\,\text{cm}$$

Step 2: Find the total surface area.

The total surface area A of a cube is given by the formula:

$$A = 6a^2$$

Substitute $a = 3$ cm into the formula:

$$A = 6(3)^2 = 6 \times 9 = 54\,\text{cm}^2$$

Step 3: Conclusion.

Thus, the total surface area of the cube is $54\,\text{cm}^2$.

Final Answer:

$$\boxed{54\,\text{cm}^2}$$

Quick Tip

To find the total surface area of a cube, first find the side length using the diagonal formula $d = a\sqrt{3}$, then use the formula for surface area $A = 6a^2$.

Q7. Using the quadratic formula, find the roots of the equation $2x^2 - 2\sqrt{2}x + 1 = 0$.

Solution:

The quadratic formula for solving the equation $ax^2 + bx + c = 0$ is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Step 1: Identify the coefficients.

For the equation $2x^2 - 2\sqrt{2}x + 1 = 0$, the coefficients are: - $a = 2$, - $b = -2\sqrt{2}$, - $c = 1$.

Step 2: Apply the quadratic formula.

Substitute the values of a, b, and c into the quadratic formula:

$$x = \frac{-(-2\sqrt{2}) \pm \sqrt{(-2\sqrt{2})^2 - 4(2)(1)}}{2(2)}$$

Simplify the terms:

$$x = \frac{2\sqrt{2} \pm \sqrt{8 - 8}}{4}$$

$$x = \frac{2\sqrt{2} \pm \sqrt{0}}{4}$$

$$x = \frac{2\sqrt{2} \pm 0}{4}$$

$$x = \frac{2\sqrt{2}}{4}$$

$$x = \frac{\sqrt{2}}{2}$$

Step 3: Conclusion.

Thus, the root of the equation is:

$$x = \frac{\sqrt{2}}{2}$$

Final Answer:

$$\boxed{\frac{\sqrt{2}}{2}}$$

Quick Tip

The quadratic formula is a powerful tool for solving any quadratic equation. Always check the discriminant $\Delta = b^2 - 4ac$ to determine the nature of the roots.

Q8. Find the sum of $3 + 11 + 19 + \cdots + 67$.

Solution:

The given sequence is an arithmetic progression (AP), where: - The first term $a = 3$, - The common difference $d = 11 - 3 = 8$, - The last term $l = 67$.

The formula for the sum of the first n terms of an AP is:

$$S_n = \frac{n}{2} \times (a + l)$$

where: - n is the number of terms, - a is the first term, - l is the last term.

Step 1: Find the number of terms n.

The n-th term of an AP is given by:

$$l = a + (n - 1) \times d$$

Substitute the known values:

$$67 = 3 + (n - 1) \times 8$$

Simplify:

$$67 - 3 = (n - 1) \times 8$$

$$64 = (n - 1) \times 8$$

$$n - 1 = \frac{64}{8} = 8$$

$$n = 9$$

Step 2: Calculate the sum S_n.

Now, substitute the values of $n = 9$, $a = 3$, and $l = 67$ into the sum formula:

$$S_9 = \frac{9}{2} \times (3 + 67) = \frac{9}{2} \times 70 = 9 \times 35 = 315$$

Step 3: Conclusion.

Thus, the sum of the sequence is:

$$S_9 = 315$$

Final Answer:

$$\boxed{315}$$

Quick Tip

To find the sum of an arithmetic sequence, first determine the number of terms using the formula for the n-th term, then apply the sum formula.

Q9. If the 5th and 9th terms of an A.P. are 43 and 79 respectively, find the A.P.

Solution:

The general formula for the n-th term of an arithmetic progression (A.P.) is:

$$T_n = a + (n - 1)d$$

where: - a is the first term, - d is the common difference, - n is the term number.

Step 1: Use the given values.

For the 5th term:

$$T_5 = a + (5 - 1)d = a + 4d = 43 \quad \text{(equation 1)}$$

For the 9th term:

$$T_9 = a + (9 - 1)d = a + 8d = 79 \quad \text{(equation 2)}$$

Step 2: Solve the system of equations.

From equation 1:

$$a + 4d = 43$$

From equation 2:

$$a + 8d = 79$$

Subtract equation 1 from equation 2:

$$(a + 8d) - (a + 4d) = 79 - 43$$

$$4d = 36$$

$$d = 9$$

Step 3: Find the value of a.

Substitute $d = 9$ into equation 1:

$$a + 4(9) = 43$$

$$a + 36 = 43$$

$$a = 7$$

Step 4: Write the general form of the A.P.

The first term $a = 7$ and the common difference $d = 9$, so the general form of the A.P. is:

$$7, 16, 25, 34, 43, 52, 61, 70, 79, \ldots$$

Final Answer:

$$\boxed{7, 16, 25, 34, 43, 52, 61, 70, 79, \ldots}$$

Quick Tip

In an A.P., the difference between any two consecutive terms is constant and is called the common difference d. Use the formula for the n-th term to solve for missing values.

Q10. Find two consecutive positive integers, the sum of whose squares is 365.

Solution:

Let the two consecutive integers be x and $x + 1$.

Step 1: Set up the equation for the sum of squares.

The sum of their squares is:

$$x^2 + (x + 1)^2 = 365$$

Expanding the terms:

$$x^2 + (x^2 + 2x + 1) = 365$$

$$2x^2 + 2x + 1 = 365$$

Subtract 365 from both sides:

$$2x^2 + 2x - 364 = 0$$

Divide the entire equation by 2:

$$x^2 + x - 182 = 0$$

Step 2: Solve the quadratic equation.

The equation is a quadratic equation of the form $ax^2 + bx + c = 0$, where $a = 1$, $b = 1$, and $c = -182$. We will solve it using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substitute the values:

$$x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-182)}}{2(1)}$$
$$x = \frac{-1 \pm \sqrt{1 + 728}}{2}$$
$$x = \frac{-1 \pm \sqrt{729}}{2}$$
$$x = \frac{-1 \pm 27}{2}$$

Step 3: Find the values of x.

$$x = \frac{-1 + 27}{2} = \frac{26}{2} = 13$$

or

$$x = \frac{-1 - 27}{2} = \frac{-28}{2} = -14$$

Since we are looking for positive integers, we take $x = 13$.

Step 4: Find the two integers.

The two consecutive integers are 13 and 14.

Final Answer:

$$\boxed{13 \text{ and } 14}$$

Quick Tip

When given a problem about consecutive integers, let the first integer be x and the second $x + 1$. Set up an equation based on the given conditions and solve it using the quadratic formula or factoring.

Q11. The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Write the equation for this statement.

Solution:

Let the two numbers be x and y, where x is the smaller number and y is the larger number.

Step 1: Write the equation for the difference of squares.

The difference of squares is given by:

$$x^2 - y^2 = 180$$

This can be factored as:

$$(x - y)(x + y) = 180 \quad \text{(equation 1)}$$

Step 2: Write the equation for the square of the smaller number.

The square of the smaller number is 8 times the larger number:

$$x^2 = 8y \quad \text{(equation 2)}$$

Step 3: Solve the system of equations.

From equation 2:

$$x^2 = 8y$$

Substitute this into equation 1:

$$(8y - y)(8y + y) = 180$$

Simplify:

$$7y \times 9y = 180$$

$$63y^2 = 180$$

$$y^2 = \frac{180}{63} = \frac{60}{21} = \frac{20}{7}$$

$$y = \sqrt{\frac{20}{7}}$$

Step 4: Conclusion.

The system of equations is solved as shown above. The value of y can be simplified further as needed.

Final Answer:

$$\boxed{y = \sqrt{\frac{20}{7}}}$$

Quick Tip

In problems involving the difference of squares, factor the equation to make the solution easier. When given conditions about squares, set up the appropriate equations and solve them step by step.

Q12. In triangle POR, two points S and T are on the sides PQ and PR respectively, such that

$$\frac{PS}{PQ} = \frac{PT}{PR} \quad \text{and} \quad \angle PST = \angle PRT,$$

prove that $\triangle POR$ is an isosceles triangle.

Solution:

Given that in $\triangle POR$: - $\frac{PS}{PQ} = \frac{PT}{PR}$, - $\angle PST = \angle PRT$.

Step 1: Apply the properties of similar triangles.

From the given ratio $\frac{PS}{PQ} = \frac{PT}{PR}$, we can conclude that $\triangle PST \sim \triangle PRT$ by the basic proportionality theorem (Thales' theorem).

Step 2: Use the condition $\angle PST = \angle PRT$.

Since $\triangle PST \sim \triangle PRT$, and the angles $\angle PST$ and $\angle PRT$ are equal, it implies that the corresponding angles of the two triangles are equal.

Step 3: Prove $\triangle POR$ is isosceles.

Since $\triangle PST \sim \triangle PRT$, we can deduce that the two triangles are proportional and the corresponding sides are equal. This implies that:

$$PQ = PR.$$

Therefore, $\triangle POR$ is an isosceles triangle with $PQ = PR$.

Final Answer:

$$\boxed{\triangle POR \text{ is an isosceles triangle.}}$$

Quick Tip

In geometry, when two triangles are similar and their corresponding angles are equal, the sides opposite those angles are also proportional. This helps in proving the properties of isosceles triangles.

Q13. $AB = AC$ in triangle $\triangle ABC$. E is a point on side CB produced of an isosceles triangle $\triangle ABC$ with $AB = AC$. If $AD \perp BC$ and $EF \perp AC$, prove that $\triangle ABD \cong \triangle ECF$.

Solution:

Given: - $AB = AC$ in $\triangle ABC$, - $AD \perp BC$ and $EF \perp AC$.

Step 1: Prove that $\triangle ABD$ and $\triangle ECF$ are right triangles.

Since $AD \perp BC$, $\triangle ABD$ is a right triangle at D. Similarly, since $EF \perp AC$, $\triangle ECF$ is a right triangle at F.

Step 2: Apply the Hypotenuse-Leg (HL) Theorem.

Since $AB = AC$ (given), $AD = EF$ (both are perpendiculars from the same type of isosceles triangle), and $\angle ABD = \angle ECF = 90^\circ$, we can apply the Hypotenuse-Leg theorem (HL theorem), which states that two right triangles are congruent if their hypotenuses and one pair of corresponding legs are equal.

Step 3: Conclude congruency.

Thus, by the HL theorem:

$$\triangle ABD \cong \triangle ECF.$$

Final Answer:

$$\boxed{\triangle ABD \cong \triangle ECF.}$$

Quick Tip

The Hypotenuse-Leg (HL) theorem can be used to prove the congruency of right triangles when the hypotenuse and one leg are equal.

Q14. In triangle $\triangle ABC$, sides AB and BC and median AD of $\triangle ABC$ are respectively proportional to sides PQ and PR and median PM of another triangle $\triangle PQR$. Prove that $\triangle ABC \sim \triangle PQR$.

Solution:

Given: - $\frac{AB}{PQ} = \frac{BC}{PR}$, - $\frac{AD}{PM}$ (the median ratio).

Step 1: Apply the criteria for similarity of triangles.

To prove that two triangles are similar, we need to show that: - The corresponding sides are proportional, and - The corresponding angles are equal.

We are already given that the sides of $\triangle ABC$ are proportional to the sides of $\triangle PQR$.

Step 2: Prove the corresponding angles are equal.

Since the medians AD and PM of $\triangle ABC$ and $\triangle PQR$ are also proportional, we can conclude that the corresponding angles $\angle A = \angle P$, $\angle B = \angle Q$, and $\angle C = \angle R$ are also equal.

Step 3: Conclude similarity.

Since both the corresponding sides are proportional and the corresponding angles are equal, by the criteria for similarity of triangles, we can conclude:

$$\triangle ABC \sim \triangle PQR.$$

Final Answer:

$$\boxed{\triangle ABC \sim \triangle PQR.}$$

Quick Tip

When two triangles have proportional sides and equal corresponding angles, they are similar by the AA (Angle-Angle) similarity criterion.

Q15. In triangle $\triangle ABC$ and $\triangle DEF$, their areas are $9\,\text{cm}^2$ and $64\,\text{cm}^2$ respectively. If $DE = 5.1\,\text{cm}$, find AB.

Solution:

Given: - Area of $\triangle ABC = 9\,\text{cm}^2$, - Area of $\triangle DEF = 64\,\text{cm}^2$, - $DE = 5.1\,\text{cm}$.

Step 1: Use the area formula for similar triangles.

Since $\triangle ABC \sim \triangle DEF$, the ratio of their areas is the square of the ratio of their corresponding sides. That is:

$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle DEF} = \left(\frac{AB}{DE}\right)^2.$$

Substitute the given areas:

$$\frac{9}{64} = \left(\frac{AB}{5.1}\right)^2.$$

Step 2: Solve for AB.

Taking the square root of both sides:

$$\frac{3}{8} = \frac{AB}{5.1}.$$

Now, solving for AB:

$$AB = \frac{3}{8} \times 5.1 = 1.9125\,\text{cm}.$$

Final Answer:

$$\boxed{AB = 1.9125\,\text{cm}}.$$

Quick Tip

The ratio of areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Q16. Prove that:

$$\frac{\sqrt{1+\cos\theta}}{\sqrt{1-\cos\theta}} = \frac{1+\cos\theta}{\sin\theta}$$

Solution:

Step 1: Start with the left-hand side (LHS).

We are given:

$$\text{LHS} = \frac{\sqrt{1+\cos\theta}}{\sqrt{1-\cos\theta}}.$$

Step 2: Multiply numerator and denominator by $\sqrt{1+\cos\theta}$.

To simplify the expression, multiply both the numerator and denominator by $\sqrt{1+\cos\theta}$:

$$\text{LHS} = \frac{\sqrt{1+\cos\theta}}{\sqrt{1-\cos\theta}} \times \frac{\sqrt{1+\cos\theta}}{\sqrt{1+\cos\theta}} = \frac{1+\cos\theta}{\sqrt{(1-\cos\theta)(1+\cos\theta)}}.$$

Step 3: Use the identity for $(a-b)(a+b) = a^2 - b^2$.

Simplifying the denominator:

$$(1-\cos\theta)(1+\cos\theta) = 1^2 - (\cos\theta)^2 = 1 - \cos^2\theta.$$

Now the expression becomes:

$$\text{LHS} = \frac{1+\cos\theta}{\sqrt{1-\cos^2\theta}}.$$

Step 4: Use the Pythagorean identity $1 - \cos^2\theta = \sin^2\theta$.

We know from the Pythagorean identity that:

$$1 - \cos^2\theta = \sin^2\theta.$$

Thus, the expression becomes:

$$\text{LHS} = \frac{1+\cos\theta}{\sqrt{\sin^2\theta}} = \frac{1+\cos\theta}{\sin\theta}.$$

Step 5: Conclusion.

We have shown that:

$$\text{LHS} = \frac{1+\cos\theta}{\sin\theta}.$$

This is the same as the right-hand side (RHS).

Final Answer:

$$\boxed{\frac{\sqrt{1+\cos\theta}}{\sqrt{1-\cos\theta}} = \frac{1+\cos\theta}{\sin\theta}}.$$

Quick Tip

When dealing with square roots in trigonometric identities, it can be helpful to multiply both the numerator and denominator by a conjugate expression to simplify the terms.

Q17. Prove that:

$$\tan 9^\circ \cdot \tan 27^\circ = \cot 63^\circ \cdot \cot 81^\circ$$

Solution:

Step 1: Express the cotangent terms in terms of tangent.

We know the identity:

$$\cot\theta = \frac{1}{\tan\theta}.$$

So, we can write:

$$\cot 63^\circ = \frac{1}{\tan 63^\circ}, \quad \cot 81^\circ = \frac{1}{\tan 81^\circ}.$$

Thus, the right-hand side becomes:

$$\cot 63^\circ \cdot \cot 81^\circ = \frac{1}{\tan 63^\circ} \cdot \frac{1}{\tan 81^\circ}.$$

Step 2: Use the identity $\tan(90^\circ - x) = \cot x$.

From the identity $\tan(90^\circ - x) = \cot x$, we have:

$$\tan 63^\circ = \cot 27^\circ \quad \text{and} \quad \tan 81^\circ = \cot 9^\circ.$$

Now substitute these values in the equation:

$$\cot 63^\circ \cdot \cot 81^\circ = \frac{1}{\tan 63^\circ} \cdot \frac{1}{\tan 81^\circ} = \frac{1}{\cot 27^\circ} \cdot \frac{1}{\cot 9^\circ} = \tan 27^\circ \cdot \tan 9^\circ.$$

Step 3: Conclusion.

We have shown that:

$$\tan 9^\circ \cdot \tan 27^\circ = \cot 63^\circ \cdot \cot 81^\circ.$$

Final Answer:

$$\boxed{\tan 9^\circ \cdot \tan 27^\circ = \cot 63^\circ \cdot \cot 81^\circ}.$$

Quick Tip

Using the identity $\tan(90^\circ - x) = \cot x$ can simplify expressions involving cotangent and tangent.

Q18. If $\cos A = \frac{4}{5}$, then find the values of $\cot A$ and $\csc A$.

Solution:

Step 1: Use the identity $\sin^2 A + \cos^2 A = 1$.

We are given:

$$\cos A = \frac{4}{5}.$$

We can use the identity $\sin^2 A + \cos^2 A = 1$ to find $\sin A$:

$$\sin^2 A = 1 - \cos^2 A = 1 - \left(\frac{4}{5}\right)^2 = 1 - \frac{16}{25} = \frac{9}{25}.$$

Thus:

$$\sin A = \frac{3}{5}.$$

Step 2: Find $\cot A$.

We know that:

$$\cot A = \frac{\cos A}{\sin A}.$$

Substitute the values of $\cos A$ and $\sin A$:

$$\cot A = \frac{\frac{4}{5}}{\frac{3}{5}} = \frac{4}{3}.$$

Step 3: Find $\csc A$.

We know that:

$$\csc A = \frac{1}{\sin A}.$$

Substitute the value of $\sin A$:

$$\csc A = \frac{1}{\frac{3}{5}} = \frac{5}{3}.$$

Final Answer:

$$\boxed{\cot A = \frac{4}{3}, \quad \csc A = \frac{5}{3}}.$$

Quick Tip

To find $\cot A$ and $\csc A$ when given $\cos A$, use the Pythagorean identity $\sin^2 A + \cos^2 A = 1$ to find $\sin A$, and then use the definitions of $\cot A$ and $\csc A$.

Q19. A ladder 7 m long makes an angle of $30°$ with the wall. Find the height of the point on the wall where the ladder touches the wall.

Solution:

Step 1: Analyze the problem geometry.

Let the length of the ladder be $L = 7\,\text{m}$. The angle made with the wall is $\theta = 30^\circ$. The height where the ladder touches the wall is the opposite side in the right triangle formed by the ladder, the wall, and the ground.

Step 2: Use trigonometry.

We can use the sine function, which relates the opposite side (height) to the hypotenuse (ladder length):

$$\sin(\theta) = \frac{\text{Opposite}}{\text{Hypotenuse}}.$$

Substitute the known values:

$$\sin(30^\circ) = \frac{h}{7}.$$

Since $\sin(30^\circ) = \frac{1}{2}$, the equation becomes:

$$\frac{1}{2} = \frac{h}{7}.$$

Step 3: Solve for h.

Multiply both sides of the equation by 7:

$$h = \frac{7}{2} = 3.5\,\text{m}.$$

Final Answer:

$$\boxed{h = 3.5\,\text{m}}.$$

Quick Tip

To find the height, use the sine function in right-angled triangles. The height corresponds to the opposite side.

Q20. In a parallelogram ABCD, if AD is extended to a point E and BE intersects CD at F, then prove that $\triangle ABE \cong \triangle CFB$.

Solution:

Step 1: Understand the properties of a parallelogram.

In a parallelogram, opposite sides are equal and parallel. Thus, $AB \parallel CD$ and $AD = BC$. Also, angles formed by parallel lines are equal.

Step 2: Identify congruent triangles.

From the given information, we know: - $AB = CB$ (since they are opposite sides of a parallelogram), - $\angle ABE = \angle CFB$ (alternate interior angles because $AB \parallel CD$), - $\angle AEB = \angle CFB$ (vertically opposite angles).

Step 3: Apply criteria for triangle congruence.

We have two triangles $\triangle ABE$ and $\triangle CFB$ with: - $AB = CB$ (by property of the parallelogram), - $\angle ABE = \angle CFB$ (alternate interior angles), - $\angle AEB = \angle CFB$ (vertically opposite angles).

By the Side-Angle-Side (SAS) criterion for congruence, we can conclude that:

$$\triangle ABE \cong \triangle CFB.$$

Final Answer:

$$\boxed{\triangle ABE \cong \triangle CFB}.$$

> Quick Tip
>
> In parallelograms, opposite sides are equal, and angles formed by parallel lines are equal. These properties help in proving triangle congruence.

Q21. ABC is an isosceles right triangle with $\angle C$ as a right angle. Prove that $AB^2 = 2AC^2$.

Solution:

Step 1: Understanding the given conditions.

We are given an isosceles right triangle, so $AB = AC$. Also, $\angle C = 90°$, which makes triangle ABC a right-angled triangle.

Step 2: Apply the Pythagorean theorem.

In any right-angled triangle, the Pythagorean theorem states that:

$$AB^2 = AC^2 + BC^2.$$

Step 3: Use the property of an isosceles triangle.

Since triangle ABC is isosceles, we have $AB = AC$. Therefore, $BC = AC$ as well. Now, substitute $BC = AC$ into the Pythagorean theorem:

$$AB^2 = AC^2 + AC^2 = 2AC^2.$$

Final Answer:

$$\boxed{AB^2 = 2AC^2}.$$

> **Quick Tip**
>
> In an isosceles right triangle, the legs are equal, and the hypotenuse is related to the legs by the Pythagorean theorem.

Q22. If $\tan\theta = \frac{5}{12}$, then find the value of $\sin\theta + \cos\theta$.

Solution:

Step 1: Use the identity for $\tan\theta$.

We are given that $\tan\theta = \frac{5}{12}$. This implies that the opposite side is 5 and the adjacent side is 12. Using the Pythagorean theorem, we can find the hypotenuse.

$$\text{Hypotenuse} = \sqrt{5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13.$$

Step 2: Find $\sin\theta$ and $\cos\theta$.

Now, we can find $\sin\theta$ and $\cos\theta$ using the definitions of sine and cosine:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{5}{13}, \quad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{12}{13}.$$

Step 3: Calculate $\sin\theta + \cos\theta$.

Now add the values of $\sin\theta$ and $\cos\theta$:

$$\sin\theta + \cos\theta = \frac{5}{13} + \frac{12}{13} = \frac{17}{13}.$$

Final Answer:

$$\boxed{\sin\theta + \cos\theta = \frac{17}{13}}.$$

Quick Tip

Use the Pythagorean theorem to find the hypotenuse when given the values of the opposite and adjacent sides in a right-angled triangle.

Q23. If $\sin 3A = \cos(A - 26^\circ)$, then find the value of A, where $3A$ is an acute angle.

Solution:

Step 1: Use the complementary angle identity.

We are given that:

$$\sin 3A = \cos(A - 26^\circ).$$

Using the identity $\sin\theta = \cos(90^\circ - \theta)$, we can rewrite $\sin 3A$ as:

$$\cos(90^\circ - 3A) = \cos(A - 26^\circ).$$

Step 2: Set the angles equal.

Since the cosines are equal, we can set the arguments equal to each other:

$$90^\circ - 3A = A - 26^\circ.$$

Step 3: Solve for A.

Now, solve for A:

$$90^\circ + 26^\circ = 4A,$$

$$116^\circ = 4A,$$

$$A = \frac{116^\circ}{4} = 29^\circ.$$

Final Answer:

$$\boxed{A = 29^\circ}.$$

Quick Tip

Use trigonometric identities to simplify and solve for angles in equations involving sine and cosine.

Q24. The sum of two numbers is 50, and one number is $\frac{7}{3}$ times the other. Find the numbers.

Solution:

Step 1: Let the two numbers be x and y.

Let the two numbers be x and y, where x is $\frac{7}{3}$ times y. So, we can write:

$$x = \frac{7}{3}y.$$

Step 2: Use the sum of the numbers.

We are also given that the sum of the two numbers is 50, so:

$$x + y = 50.$$

Substitute $x = \frac{7}{3}y$ into the equation:

$$\frac{7}{3}y + y = 50.$$

Step 3: Solve for y.

Simplify the equation:

$$\frac{7}{3}y + \frac{3}{3}y = 50,$$
$$\frac{10}{3}y = 50.$$

Multiply both sides by 3:

$$10y = 150,$$

$$y = 15.$$

Step 4: Find x.

Now substitute $y = 15$ into $x = \frac{7}{3}y$:

$$x = \frac{7}{3} \times 15 = 35.$$

Final Answer: The two numbers are $x = 35$ and $y = 15$.

Quick Tip

Let one number be a variable and express the second number in terms of the first using the given ratio. Then, solve the system of equations.

Q25. Prove that $5 - \sqrt{3}$ is an irrational number.

Solution:

Step 1: Assume the opposite.

Assume that $5 - \sqrt{3}$ is a rational number. Then it can be written as:

$$5 - \sqrt{3} = \frac{p}{q},$$

where $\frac{p}{q}$ is a rational number, and p and q are integers with no common factors (i.e., the fraction is in its simplest form).

Step 2: Solve for $\sqrt{3}$.

Rearrange the equation to isolate $\sqrt{3}$:

$$\sqrt{3} = 5 - \frac{p}{q} = \frac{5q - p}{q}.$$

Step 3: Contradiction.

Since $\frac{5q-p}{q}$ is a rational number (the numerator and denominator are both integers), this implies that $\sqrt{3}$ is rational, which contradicts the fact that $\sqrt{3}$ is irrational.

Step 4: Conclusion.

Therefore, our assumption is false. Thus, $5 - \sqrt{3}$ is irrational.

Final Answer: $5 - \sqrt{3}$ is irrational.

Quick Tip

The difference between a rational number and an irrational number is always irrational. If the assumption leads to a contradiction, the original statement is true.

Q26. For what value of k are the points $(1, 1), (1, 3), (k, 3)$, and $(-1, 4)$ collinear?

Solution:

Step 1: Check the condition for collinearity.

Four points are collinear if the area of the quadrilateral formed by them is zero. We can use the area formula for a quadrilateral with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3), (x_4, y_4)$:

$$\text{Area} = \frac{1}{2} \left| x_1y_2 + x_2y_3 + x_3y_4 + x_4y_1 - (y_1x_2 + y_2x_3 + y_3x_4 + y_4x_1) \right|.$$

For the points $(1, 1), (1, 3), (k, 3), (-1, 4)$, we substitute the coordinates into the formula:

$$\text{Area} = \frac{1}{2} \left| 1 \cdot 3 + 1 \cdot 3 + k \cdot 4 + (-1) \cdot 1 - (1 \cdot 1 + 3 \cdot k + 3 \cdot (-1) + 4 \cdot 1) \right|.$$

Step 2: Simplify the expression.

Simplify both terms inside the absolute value:

$$\begin{aligned} \text{Area} &= \frac{1}{2} \left| 3 + 3 + 4k - 1 - (1 + 3k - 3 + 4) \right| \\ &= \frac{1}{2} \left| 5 + 4k - (3 + 3k) \right| \\ &= \frac{1}{2} \left| 5 + 4k - 3 - 3k \right| \\ &= \frac{1}{2} \left| 2 + k \right|. \end{aligned}$$

Step 3: Solve for k.

For the points to be collinear, the area must be zero:

$$\frac{1}{2} \left| 2 + k \right| = 0.$$

Thus, $2 + k = 0$, which gives:

$$k = -2.$$

Final Answer: The value of k for which the points are collinear is $k = -2$.

> **Quick Tip**
>
> To check collinearity, use the area formula for a quadrilateral formed by the given points. If the area is zero, the points are collinear.

Q27. Find such a point on the y-axis which is equidistant from the points $(6, 5)$ and $(-4, 3)$.

Solution:

Step 1: Let the point on the y-axis be $(0, y)$.

The point lies on the y-axis, so its x-coordinate is 0. Let the point be $(0, y)$, where y is the unknown.

Step 2: Set up the distance formula.

The distance between two points (x_1, y_1) and (x_2, y_2) is given by:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

We are given that the distances from the point $(0, y)$ to $(6, 5)$ and $(-4, 3)$ are equal. So, we use the distance formula to write the following equation:

$$\text{Distance from } (0, y) \text{ to } (6, 5) = \text{Distance from } (0, y) \text{ to } (-4, 3).$$

Step 3: Apply the distance formula.

For the distance from $(0, y)$ to $(6, 5)$:

$$\sqrt{(6 - 0)^2 + (5 - y)^2} = \sqrt{6^2 + (5 - y)^2}.$$

For the distance from $(0, y)$ to $(-4, 3)$:

$$\sqrt{(-4 - 0)^2 + (3 - y)^2} = \sqrt{(-4)^2 + (3 - y)^2}.$$

Equating both distances, we get:

$$\sqrt{36 + (5 - y)^2} = \sqrt{16 + (3 - y)^2}.$$

Step 4: Simplify the equation.

Square both sides:

$$36 + (5 - y)^2 = 16 + (3 - y)^2.$$

Expand both sides:

$$36 + (25 - 10y + y^2) = 16 + (9 - 6y + y^2).$$

Simplify:

$$36 + 25 - 10y + y^2 = 16 + 9 - 6y + y^2.$$

Cancel out the y^2 terms:

$$61 - 10y = 25 - 6y.$$

Solve for y:

$$61 - 25 = 10y - 6y,$$

$$36 = 4y,$$

$$y = 9.$$

Final Answer: The point on the y-axis that is equidistant from $(6, 5)$ and $(-4, 3)$ is $(0, 9)$.

Quick Tip

When dealing with equidistant points, set up an equation using the distance formula and solve for the unknown coordinate.

Q28. Divide $x^3 + 1$ by $x + 1$.

Solution:

Step 1: Recognize that $x^3 + 1$ is a sum of cubes.

We can factor $x^3 + 1$ as a sum of cubes:

$$x^3 + 1 = (x + 1)(x^2 - x + 1).$$

Now, we divide $(x^3 + 1)$ by $(x + 1)$:

$$\frac{x^3 + 1}{x + 1} = \frac{(x + 1)(x^2 - x + 1)}{x + 1}.$$

Step 2: Simplify the expression.

Since $x + 1$ is a common factor in both the numerator and denominator, they cancel out:

$$\frac{(x + 1)(x^2 - x + 1)}{x + 1} = x^2 - x + 1.$$

Final Answer: The result of dividing $x^3 + 1$ by $x + 1$ is $x^2 - x + 1$.

Quick Tip

For division of polynomials, if the numerator is a sum of cubes, use the formula $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$.

Q29. Using Euclid's division algorithm, find the H.C.F. of 504 and 1188.

Solution:

Step 1: Apply Euclid's Division Algorithm.

Euclid's division algorithm states that:

$$\text{H.C.F.}(a, b) = \text{H.C.F.}(b, a \mod b).$$

We start with the numbers 504 and 1188. Perform the division:

$$1188 \div 504 = 2 \quad \text{(quotient)} \quad \text{remainder} = 1188 - 2 \times 504 = 1188 - 1008 = 180.$$

So, we get the remainder 180.

Step 2: Apply the division algorithm again.

Now, use 504 and 180:

$$504 \div 180 = 2 \quad \text{(quotient)} \quad \text{remainder} = 504 - 2 \times 180 = 504 - 360 = 144.$$

Now, we have the remainder 144.

Step 3: Apply the division algorithm again.

Now, use 180 and 144:

$$180 \div 144 = 1 \quad \text{(quotient)} \quad \text{remainder} = 180 - 1 \times 144 = 180 - 144 = 36.$$

Now, we have the remainder 36.

Step 4: Apply the division algorithm again.

Now, use 144 and 36:

$$144 \div 36 = 4 \quad \text{(quotient)} \quad \text{remainder} = 144 - 4 \times 36 = 144 - 144 = 0.$$

When the remainder is 0, the divisor 36 is the H.C.F.

Final Answer: The H.C.F. of 504 and 1188 is 36.

Quick Tip

To find the H.C.F. using Euclid's algorithm, repeatedly divide the larger number by the smaller number and replace the larger number with the remainder until the remainder is 0. The divisor at that stage is the H.C.F.

Q30. Find the discriminant of the quadratic equation $2x^2 + 5x - 3 = 0$ and find the nature of the roots.

Solution:

Step 1: Identify the coefficients of the quadratic equation.

The quadratic equation is given by:

$$2x^2 + 5x - 3 = 0.$$

The standard form of a quadratic equation is $ax^2 + bx + c = 0$. Here,

$$a = 2, \quad b = 5, \quad c = -3.$$

Step 2: Use the discriminant formula.

The discriminant Δ of a quadratic equation $ax^2 + bx + c = 0$ is given by:

$$\Delta = b^2 - 4ac.$$

Substitute the values of a, b, and c:

$$\Delta = (5)^2 - 4 \times 2 \times (-3) = 25 + 24 = 49.$$

Step 3: Find the nature of the roots.

The nature of the roots depends on the value of the discriminant: - If $\Delta > 0$, the roots are real and distinct. - If $\Delta = 0$, the roots are real and equal. - If $\Delta < 0$, the roots are complex.

Since $\Delta = 49 > 0$, the roots are real and distinct.

Final Answer: The discriminant of the quadratic equation $2x^2 + 5x - 3 = 0$ is 49, and the roots are real and distinct.

Quick Tip

The discriminant helps determine the nature of the roots. For real and distinct roots, $\Delta > 0$; for real and equal roots, $\Delta = 0$; and for complex roots, $\Delta < 0$.

Q31. Draw the graphs of the pair of linear equations $x + 3y - 6 = 0$ and $2x - 3y - 12 = 0$ and solve them.

Solution:

We are given the system of linear equations:

$$x + 3y - 6 = 0 \tag{1}$$

$$2x - 3y - 12 = 0 \tag{2}$$

Step 1: Solve the system of equations.

We can solve these equations using the substitution or elimination method. Let's use the substitution method.

From equation (1):

$$x + 3y = 6 \quad \Rightarrow \quad x = 6 - 3y.$$

Now, substitute this expression for x into equation (2):

$$2(6 - 3y) - 3y - 12 = 0.$$

Simplify the equation:

$$12 - 6y - 3y - 12 = 0 \quad \Rightarrow \quad -9y = 0 \quad \Rightarrow \quad y = 0.$$

Step 2: Substitute $y = 0$ back into equation (1) to find x.

Substitute $y = 0$ into $x = 6 - 3y$:

$$x = 6 - 3(0) = 6.$$

Step 3: Conclusion of the solution.

The solution of the system of equations is $x = 6$ and $y = 0$, i.e., the point of intersection of the two lines is $(6, 0)$.

Graphical Solution:

We can graph the lines $x + 3y - 6 = 0$ and $2x - 3y - 12 = 0$ to visualize the solution.

The equation $x + 3y - 6 = 0$ can be written as:

$$y = \frac{6 - x}{3}.$$

For plotting the graph, choose some values of x, calculate the corresponding values of y, and plot the points.

Similarly, for the equation $2x - 3y - 12 = 0$, write it as:

$$y = \frac{2x - 12}{3}.$$

Again, choose values of x, calculate the corresponding y-coordinates, and plot the points.

The two lines will intersect at the point $(6, 0)$.

Final Answer: The solution to the system of equations is $x = 6$ and $y = 0$, and the point of intersection is $(6, 0)$.

> **Quick Tip**
>
> To solve a system of linear equations, you can use substitution or elimination methods. In this case, we used substitution and found that the solution is the point of intersection of the two lines.

Q32. If one angle of a triangle is equal to one angle of the other triangle and the sides included between these angles are proportional, then prove that the triangles are similar.

Solution:

We are given two triangles $\triangle ABC$ and $\triangle DEF$ where: - $\angle A = \angle D$ (the angles are equal), - $\frac{AB}{DE} = \frac{BC}{EF}$ (the sides are proportional).

We need to prove that $\triangle ABC \sim \triangle DEF$, i.e., the triangles are similar.

Step 1: Use the AA criterion for similarity.

The Angle-Angle (AA) criterion states that if two angles of one triangle are equal to two angles of another triangle, then the triangles are similar. In our case, we are given that $\angle A = \angle D$.

Step 2: Apply the proportionality condition.

We are also given that the sides included between the equal angles are proportional, i.e.,

$$\frac{AB}{DE} = \frac{BC}{EF}$$

Step 3: Conclude similarity.

Since $\angle A = \angle D$ and the sides AB and DE, BC and EF are proportional, by the AA criterion for similarity, we can conclude that:

$$\triangle ABC \sim \triangle DEF$$

Final Answer: Thus, $\triangle ABC$ is similar to $\triangle DEF$.

Final Answer:

$$\boxed{\triangle ABC \sim \triangle DEF}$$

Quick Tip

When two triangles have two equal angles and the sides between those angles are proportional, the triangles are similar by the AA criterion.

Q33. A two-digit number is four times the sum of its digits and twice the product of its digits. Find the number.

Solution:

Let the two-digit number be $10a + b$, where: - a is the tens digit, - b is the ones digit.

Step 1: Translate the conditions into equations.

We are given that: 1. The number is four times the sum of its digits:

$$10a + b = 4(a + b) \quad \text{(Equation 1)}$$

2. The number is twice the product of its digits:

$$10a + b = 2ab \quad \text{(Equation 2)}$$

Step 2: Solve the first equation for b.

From Equation 1:

$$10a + b = 4a + 4b$$

Simplify:

$$10a - 4a = 4b - b$$

$$6a = 3b$$

$$2a = b \quad \text{(Equation 3)}$$

Step 3: Substitute Equation 3 into Equation 2.

Substitute $b = 2a$ into Equation 2:

$$10a + 2a = 2a \times 2a$$

Simplify:

$$12a = 4a^2$$

$$4a^2 - 12a = 0$$

Factor:

$$4a(a - 3) = 0$$

Thus, $a = 0$ or $a = 3$.

Since $a = 0$ is not valid for a two-digit number, we have $a = 3$.

Step 4: Find b.

Substitute $a = 3$ into Equation 3:

$$b = 2a = 2 \times 3 = 6$$

Step 5: Find the number.

The number is $10a + b = 10 \times 3 + 6 = 36$.

Final Answer: Thus, the number is 36.

Final Answer:

$$\boxed{36}$$

Quick Tip

To solve for a two-digit number with conditions on its digits, translate the conditions into algebraic equations and solve the system.

Q34. Draw a line segment of length 7.6 cm and divide it in the ratio 5:8. Measure both parts.

Solution:

We are given that the total length of the line segment is 7.6 cm, and we are asked to divide it in the ratio 5:8.

Step 1: Find the total number of parts.

The total number of parts is $5 + 8 = 13$ parts.

Step 2: Find the length of each part.

The length of each part is:

$$\frac{7.6}{13} = 0.5846\,\text{cm}$$

Step 3: Find the lengths of the two parts.

The first part is $5 \times 0.5846 = 2.923\,\text{cm}$, and the second part is $8 \times 0.5846 = 4.6768\,\text{cm}$.

Final Answer: The lengths of the two parts are 2.923 cm and 4.677 cm.

Final Answer:

$$\boxed{2.923\,\text{cm and } 4.677\,\text{cm}}$$

> **Quick Tip**
>
> To divide a line segment in a given ratio, find the total number of parts, calculate the length of each part, and multiply by the number of parts for each segment.

Q35. Prove that

$$\frac{\sec\theta - \tan\theta}{\sec\theta + \tan\theta} = 1 + 2\tan^2\theta - 2\sec\theta \cdot \tan\theta$$

Solution:

We are asked to prove the following trigonometric identity:

$$\frac{\sec\theta - \tan\theta}{\sec\theta + \tan\theta} = 1 + 2\tan^2\theta - 2\sec\theta \cdot \tan\theta$$

Step 1: Express the left-hand side.

We start with the left-hand side:

$$\frac{\sec\theta - \tan\theta}{\sec\theta + \tan\theta}$$

Multiply both the numerator and denominator by $\sec\theta - \tan\theta$:

$$= \frac{(\sec\theta - \tan\theta)^2}{(\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}$$

Simplify the denominator using the identity $(a+b)(a-b) = a^2 - b^2$:

$$= \frac{(\sec\theta - \tan\theta)^2}{\sec^2\theta - \tan^2\theta}$$

Since $\sec^2\theta - \tan^2\theta = 1$, the denominator becomes 1:

$$= (\sec\theta - \tan\theta)^2$$

Step 2: Expand the numerator.

Now, expand the numerator:

$$(\sec\theta - \tan\theta)^2 = \sec^2\theta - 2\sec\theta\cdot\tan\theta + \tan^2\theta$$

Thus, we have:

$$= 1 + 2\tan^2\theta - 2\sec\theta\cdot\tan\theta$$

Step 3: Conclusion.

We have shown that the left-hand side simplifies to the right-hand side:

$$\frac{\sec\theta - \tan\theta}{\sec\theta + \tan\theta} = 1 + 2\tan^2\theta - 2\sec\theta\cdot\tan\theta$$

Final Answer: Thus, the identity is proved.

Final Answer:

$$\boxed{\frac{\sec\theta - \tan\theta}{\sec\theta + \tan\theta} = 1 + 2\tan^2\theta - 2\sec\theta\cdot\tan\theta}$$

Quick Tip

To prove trigonometric identities, use known identities and simplify step-by-step.

Q36. The radii of two circles are 19 cm and 9 cm respectively. Find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles.

Solution:

We are given: - The radius of the first circle is 19 cm, - The radius of the second circle is 9 cm.

We need to find the radius of a new circle whose circumference is equal to the sum of the circumferences of the two circles.

Step 1: Formula for circumference.

The circumference C of a circle is given by:

$$C = 2\pi r$$

where r is the radius of the circle.

Step 2: Calculate the circumferences of the two circles.

For the first circle, the circumference is:

$$C_1 = 2\pi \times 19 = 38\pi \, \text{cm}$$

For the second circle, the circumference is:

$$C_2 = 2\pi \times 9 = 18\pi \, \text{cm}$$

Step 3: Find the sum of the circumferences.

The sum of the circumferences is:

$$C_1 + C_2 = 38\pi + 18\pi = 56\pi \, \text{cm}$$

Step 4: Find the radius of the new circle.

Let the radius of the new circle be r. The circumference of this new circle is:

$$2\pi r = 56\pi$$

Solve for r:

$$r = \frac{56\pi}{2\pi} = 28 \, \text{cm}$$

Final Answer: The radius of the new circle is 28 cm.

Final Answer:

$$\boxed{28\,\text{cm}}$$

Quick Tip

To find the radius of a circle with a given circumference, use the formula $r = \frac{C}{2\pi}$.

Q37. Find the mean of the following distribution:

Class-interval	Frequency
$11-13$	7
$13-15$	6
$15-17$	9
$17-19$	13
$19-21$	20
$21-23$	5
$23-25$	4

Solution:

We are given the following frequency distribution:

Class-interval	Frequency
$11-13$	7
$13-15$	6
$15-17$	9
$17-19$	13
$19-21$	20
$21-23$	5
$23-25$	4

Step 1: Find the midpoints of the class intervals.

The midpoint of each interval is calculated as:

$$\text{Midpoint} = \frac{\text{Lower limit} + \text{Upper limit}}{2}$$

Thus, the midpoints are:

$$\text{Midpoint of } 11 - 13 = \frac{11 + 13}{2} = 12$$
$$\text{Midpoint of } 13 - 15 = \frac{13 + 15}{2} = 14$$
$$\text{Midpoint of } 15 - 17 = \frac{15 + 17}{2} = 16$$
$$\text{Midpoint of } 17 - 19 = \frac{17 + 19}{2} = 18$$
$$\text{Midpoint of } 19 - 21 = \frac{19 + 21}{2} = 20$$
$$\text{Midpoint of } 21 - 23 = \frac{21 + 23}{2} = 22$$
$$\text{Midpoint of } 23 - 25 = \frac{23 + 25}{2} = 24$$

Step 2: Multiply the midpoints by their respective frequencies.

Now, multiply each midpoint by its corresponding frequency:

$$12 \times 7 = 84$$

$$14 \times 6 = 84$$

$$16 \times 9 = 144$$

$$18 \times 13 = 234$$

$$20 \times 20 = 400$$

$$22 \times 5 = 110$$

$$24 \times 4 = 96$$

Step 3: Find the sum of the frequencies and the sum of the products.

Now, sum the frequencies and the products of the midpoints and frequencies:

$$\text{Sum of frequencies} = 7 + 6 + 9 + 13 + 20 + 5 + 4 = 64$$

$$\text{Sum of the products} = 84 + 84 + 144 + 234 + 400 + 110 + 96 = 1052$$

Step 4: Calculate the mean.

The formula for the mean is:

$$\text{Mean} = \frac{\sum f \times x}{\sum f}$$

Substitute the values:

$$\text{Mean} = \frac{1052}{64} \approx 16.44$$

Final Answer: Thus, the mean of the distribution is approximately 16.44.

Final Answer:

$$\boxed{16.44}$$

Quick Tip

To find the mean of a frequency distribution, multiply the midpoints by their respective frequencies, sum them, and divide by the total frequency.

Q38. The slant height of a frustum of a cone is 4 cm and the perimeters (circumferences) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

Solution:

We are given: - The slant height $l = 4\,\text{cm}$, - The circumferences of the circular ends are $18\,\text{cm}$ and $6\,\text{cm}$.

Step 1: Use the formula for circumference.

The circumference C of a circle is given by:

$$C = 2\pi r$$

where r is the radius of the circle.

Let the radii of the circular ends be r_1 and r_2, corresponding to the circumferences of 18 cm and 6 cm, respectively. We can calculate the radii as follows:
For the first circle:

$$18 = 2\pi r_1 \quad \Rightarrow \quad r_1 = \frac{18}{2\pi} = \frac{9}{\pi}$$

For the second circle:

$$6 = 2\pi r_2 \quad \Rightarrow \quad r_2 = \frac{6}{2\pi} = \frac{3}{\pi}$$

Step 2: Use the formula for the curved surface area of a frustum.
The formula for the curved surface area A of a frustum of a cone is:

$$A = \pi(r_1 + r_2)l$$

Substitute the values:

$$A = \pi\left(\frac{9}{\pi} + \frac{3}{\pi}\right) \times 4$$

$$A = \pi \times \frac{12}{\pi} \times 4 = 12 \times 4 = 48\,\text{cm}^2$$

Final Answer: Thus, the curved surface area of the frustum is 48 cm².

Final Answer:

$$\boxed{48\,\text{cm}^2}$$

Quick Tip

To calculate the curved surface area of a frustum, first calculate the radii from the circumferences, then use the formula $A = \pi(r_1 + r_2)l$.
