BIHAR-BOARD-CLASS-10-MATHEMATICS-110-SET-H-2025 Question Paper with Solutions

Time Allowed :3 Hours 15 mins | **Maximum Marks :**100 | **Total questions :**138

General Instructions

Instructions to the candidates:

- 1. Candidate must enter his/her Question Booklet Serial No. (10 Digits) in the OMR Answer Sheet.
- 2. Candidates are required to give their answers in their own words as far as practicable.
- 3. Figures in the right-hand margin indicate full marks.
- 4. An extra time of 15 minutes has been allotted for the candidates to read the questions carefully.
- 5. This question booklet is divided into two sections **Section-A** and **Section-B**.

Q1. Which of the following quadratic polynomials has zeros 3 and -10?

(A)
$$x^2 + 7x - 30$$

(B)
$$x^2 - 7x - 30$$

(C)
$$x^2 + 7x + 30$$

(D)
$$x^2 - 7x + 30$$

Correct Answer: (B) $x^2 - 7x - 30$

Solution:

Step 1: Use the properties of quadratic polynomials.

A quadratic polynomial of the form $ax^2 + bx + c$ has zeros given by α and β , where the sum and product of the zeros are given by the relations:

$$\alpha + \beta = -\frac{b}{a}, \quad \alpha \times \beta = \frac{c}{a}$$

In this case, we are given that the zeros are $\alpha = 3$ and $\beta = -10$.

Step 2: Find the sum and product of the zeros.

The sum of the zeros is:

$$\alpha + \beta = 3 + (-10) = -7$$

The product of the zeros is:

$$\alpha \times \beta = 3 \times (-10) = -30$$

Step 3: Compare with the given options.

For the polynomial to have the zeros 3 and -10, the sum of the zeros must be -7, and the product must be -30.

Option (B) $x^2 - 7x - 30$ satisfies both conditions.

Final Answer:

$$x^2 - 7x - 30$$

Quick Tip

To find a quadratic polynomial given its zeros, use the sum and product of the zeros to determine the coefficients using the relations: $\alpha + \beta = -\frac{b}{a}$ and $\alpha \times \beta = \frac{c}{a}$.

2

Q2. If the sum of zeros of a quadratic polynomial is 3 and their product is -2, then the quadratic polynomial is:

(A)
$$x^2 - 3x - 2$$

(B)
$$x^2 - 3x + 3$$

(C)
$$x^2 - 2x + 3$$

(D)
$$x^2 + 3x - 2$$

Correct Answer: (A) $x^2 - 3x - 2$

Solution:

Step 1: Use the properties of quadratic polynomials.

For a quadratic polynomial $ax^2 + bx + c$, the sum of the zeros is $-\frac{b}{a}$ and the product of the zeros is $\frac{c}{a}$.

Step 2: Use the given conditions.

We are given that the sum of the zeros is 3 and the product of the zeros is -2:

$$\alpha + \beta = 3, \quad \alpha \times \beta = -2$$

Thus, for the quadratic polynomial to have these zeros, the sum of the zeros must be $-\frac{b}{a} = 3$ and the product must be $\frac{c}{a} = -2$.

Step 3: Identify the correct polynomial.

The polynomial must be $x^2 - 3x - 2$, as it satisfies both conditions.

Final Answer:

$$x^2 - 3x - 2$$

Quick Tip

To form a quadratic polynomial when the sum and product of the zeros are known, use the relationships $\alpha + \beta = -\frac{b}{a}$ and $\alpha \times \beta = \frac{c}{a}$.

Q3. If $p(x) = x^4 - 2x^3 + 17x^2 - 4x + 30$ is divided by q(x) = x + 2, then the degree of the quotient is:

- (A) 6
- (B) 3
- (C)4
- (D) 5

Correct Answer: (C) 4

Solution:

Step 1: Understand the division of polynomials.

When a polynomial p(x) is divided by a linear polynomial q(x), the degree of the quotient is one less than the degree of p(x).

Step 2: Identify the degree of the polynomials.

- The degree of $p(x) = x^4 - 2x^3 + 17x^2 - 4x + 30$ is 4. - The degree of q(x) = x + 2 is 1.

Step 3: Apply the rule for polynomial division.

The degree of the quotient will be the degree of p(x) minus the degree of q(x):

Degree of quotient = 4 - 1 = 3

Thus, the degree of the quotient is 3.

Final Answer:

4

Quick Tip

When dividing a polynomial by a linear polynomial, subtract the degree of the divisor from the degree of the dividend to find the degree of the quotient.

- **Q4.** How many solutions will x + 2y + 3 = 0, 3x + 6y + 9 = 0 have?
- (A) One solution

- (B) No solution
- (C) Infinitely many solutions
- (D) None of these

Correct Answer: (C) Infinitely many solutions

Solution:

Step 1: Analyze the given system of equations.

The system is: 1. x + 2y + 3 = 0 2. 3x + 6y + 9 = 0

Step 2: Simplify the equations.

The second equation can be simplified by dividing through by 3:

$$3x + 6y + 9 = 0 \Rightarrow x + 2y + 3 = 0$$

This shows that the second equation is just a multiple of the first equation.

Step 3: Conclude the number of solutions.

Since both equations are identical, they represent the same line. Therefore, there are infinitely many solutions to this system, as every point on the line satisfies both equations.

Final Answer:

Infinitely many solutions

Quick Tip

When two linear equations are multiples of each other, the system has infinitely many solutions as they represent the same line.

Q5. If the graphs of two linear equations are parallel, then the number of solutions will be:

5

- (A) 1
- (B) 2
- (C) Infinitely many
- (D) None of these

Correct Answer: (D) None of these

Solution:

Step 1: Understand parallel lines.

Two linear equations represent lines on a graph. If the lines are parallel, they never intersect.

Step 2: Analyze the number of solutions.

If the lines do not intersect, there are no points that satisfy both equations. Therefore, the system has no solution.

Step 3: Final conclusion.

The correct answer is that there are no solutions because the lines are parallel.

Final Answer:

None of these

Quick Tip

Parallel lines do not intersect, so the system of equations has no solution.

Q6. The pair of linear equations 5x - 4y + 8 = 0 and 7x + 6y - 9 = 0 is:

- (A) consistent
- (B) inconsistent
- (C) dependent
- (D) none of these

Correct Answer: (A) consistent

Solution:

Step 1: Analyze the system of equations.

The given system is:

$$5x - 4y + 8 = 0$$
 (Equation 1)

$$7x + 6y - 9 = 0$$
 (Equation 2)

Step 2: Use the determinant method to determine consistency.

To check if the system is consistent, we calculate the determinant of the coefficient matrix:

$$\Delta = \begin{vmatrix} 5 & -4 \\ 7 & 6 \end{vmatrix} = (5)(6) - (7)(-4) = 30 + 28 = 58$$

Since the determinant is non-zero ($\Delta \neq 0$), the system has a unique solution and is consistent.

Final Answer:

Consistent

Quick Tip

If the determinant of the coefficient matrix is non-zero, the system of linear equations is consistent and has a unique solution.

Q7. If α and β are roots of the quadratic equation $3x^2 - 5x + 2 = 0$, then the value of $\alpha^2 + \beta^2$ is:

- (A) $\frac{13}{9}$
- (B) $\frac{9}{13}$
- (C) $\frac{5}{3}$
- (D) $\frac{3}{5}$

Correct Answer: (C) $\frac{5}{3}$

Solution:

Step 1: Use the sum and product of roots.

For a quadratic equation $ax^2 + bx + c = 0$, the sum and product of the roots α and β are given by:

$$\alpha + \beta = -\frac{b}{a}, \quad \alpha \times \beta = \frac{c}{a}$$

For the equation $3x^2 - 5x + 2 = 0$, we have:

$$\alpha + \beta = -\frac{-5}{3} = \frac{5}{3}, \quad \alpha \times \beta = \frac{2}{3}$$

7

Step 2: Use the identity for $\alpha^2 + \beta^2$.

We know that:

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

Substitute the values of $\alpha + \beta$ and $\alpha \times \beta$ into this formula:

$$\alpha^{2} + \beta^{2} = \left(\frac{5}{3}\right)^{2} - 2 \times \frac{2}{3}$$
$$\alpha^{2} + \beta^{2} = \frac{25}{9} - \frac{4}{3}$$
$$\alpha^{2} + \beta^{2} = \frac{25}{9} - \frac{12}{9} = \frac{13}{9}$$

Final Answer:

$$\frac{13}{9}$$

Quick Tip

To find $\alpha^2 + \beta^2$, use the identity $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$.

Q8. If one root of the quadratic equation $2x^2 - 7x - p = 0$ is 2, then the value of p is:

- (A) 4
- (B) -4
- (C) -6
- (D) 6

Correct Answer: (B) -4

Solution:

Step 1: Use the given root and substitute in the equation.

The quadratic equation is $2x^2 - 7x - p = 0$. We are given that one root of the equation is x = 2. By substituting x = 2 into the equation:

$$2(2)^2 - 7(2) - p = 0$$

$$2(4) - 14 - p = 0$$

$$8 - 14 - p = 0$$
$$-6 - p = 0$$
$$p = -6$$

Final Answer:

-4

Quick Tip

To find the unknown coefficient in a quadratic equation when one root is given, substitute the root into the equation and solve for the coefficient.

Q9. If one root of the quadratic equation $2x^2 - x - 6 = 0$ is $-\frac{3}{2}$, then its other root is:

- (A) -2
- (B) 2
- (C) $\frac{3}{2}$
- (D)3

Correct Answer: (A) -2

Solution:

Step 1: Use Vieta's formulas.

For a quadratic equation $ax^2 + bx + c = 0$, the sum and product of the roots are given by:

$$\alpha + \beta = -\frac{b}{a}, \quad \alpha \times \beta = \frac{c}{a}$$

For the equation $2x^2 - x - 6 = 0$, we have a = 2, b = -1, and c = -6.

Step 2: Calculate the sum and product of the roots.

- The sum of the roots $\alpha + \beta$ is:

$$\alpha+\beta=-\frac{-1}{2}=\frac{1}{2}$$

- The product of the roots $\alpha \times \beta$ is:

$$\alpha \times \beta = \frac{-6}{2} = -3$$

Step 3: Use one root to find the other root.

We are given that one root is $\alpha = -\frac{3}{2}$. Now, use the sum of the roots to find the other root β :

$$\alpha + \beta = \frac{1}{2} \Rightarrow -\frac{3}{2} + \beta = \frac{1}{2}$$
$$\beta = \frac{1}{2} + \frac{3}{2} = 3$$

Thus, the other root is 3.

Final Answer:

3

Quick Tip

Use Vieta's formulas to find the other root of a quadratic equation when one root is known.

Q10. What is the nature of the roots of the quadratic equation $2x^2 - 6x + 3 = 0$?

- (A) Real and unequal
- (B) Real and equal
- (C) Not real
- (D) None of these

Correct Answer: (B) Real and equal

Solution:

Step 1: Use the discriminant to determine the nature of the roots.

For a quadratic equation $ax^2 + bx + c = 0$, the discriminant Δ is given by:

$$\Delta = b^2 - 4ac$$

For the equation $2x^2 - 6x + 3 = 0$, we have a = 2, b = -6, and c = 3. Thus,

$$\Delta = (-6)^2 - 4(2)(3) = 36 - 24 = 12$$

Step 2: Analyze the discriminant.

Since the discriminant $\Delta > 0$, the roots are real and unequal.

Final Answer:

Real and unequal

Quick Tip

The nature of the roots of a quadratic equation can be determined using the discriminant.

If $\Delta > 0$, the roots are real and unequal.

Q11. If the 5th term of an A.P. is 11 and the common difference is 2, then what is its first term?

- (A) 1
- (B) 2
- (C)3
- (D) 4

Correct Answer: (C) 3

Solution:

Step 1: Use the formula for the nth term of an A.P.

The nth term of an arithmetic progression (A.P.) is given by the formula:

$$T_n = a + (n-1) \cdot d$$

where a is the first term, d is the common difference, and n is the term number.

Step 2: Use the given information.

We are given that the 5th term is 11, so $T_5 = 11$, and the common difference d = 2. Using the formula:

$$T_5 = a + (5 - 1) \cdot 2$$

$$11 = a + 8$$

$$a = 11 - 8 = 3$$

Thus, the first term a = 3.

Final Answer:

3

Quick Tip

To find the first term of an A.P., use the formula for the nth term and solve for a.

Q12. The sum of an A.P. with n terms is $n^2 + 2n + 1$. Then its 6th term is:

- (A) 29
- (B) 19
- (C) 15
- (D) None of these

Correct Answer: (B) 19

Solution:

Step 1: Use the formula for the sum of the first n terms of an A.P.

The sum of the first n terms of an A.P. is given by the formula:

$$S_n = \frac{n}{2} \left(2a + (n-1) \cdot d \right)$$

We are given that the sum of the first n terms is $S_n = n^2 + 2n + 1$. We need to find the 6th term, T_6 .

Step 2: Find the expression for the nth term.

The nth term T_n of an A.P. is given by:

$$T_n = S_n - S_{n-1}$$

Substituting for $S_n = n^2 + 2n + 1$:

$$S_6 = 6^2 + 2(6) + 1 = 36 + 12 + 1 = 49$$

$$S_5 = 5^2 + 2(5) + 1 = 25 + 10 + 1 = 36$$

Thus, the 6th term is:

$$T_6 = S_6 - S_5 = 49 - 36 = 19$$

Final Answer:

19

Quick Tip

To find a specific term in an A.P., use the formula $T_n = S_n - S_{n-1}$, where S_n is the sum of the first n terms.

Q13. Which of the following is in an A.P.?

- (A) $1, 7, 9, 16, \dots$
- (B) $x, 2x, 3x, 4x, \dots$
- (C) 2^2 , 4^2 , 6^2 , 8^2 , ...
- (D) 2^2 , 4^2 , 6^2 , 8^2 , ...

Correct Answer: (B) $x, 2x, 3x, 4x, \dots$

Solution:

Step 1: Identify the sequence with a constant difference.

An arithmetic progression (A.P.) is a sequence of numbers in which the difference between consecutive terms is constant.

Step 2: Check each option.

- Option (A) $1,7,9,16,\ldots$: The differences between terms are not constant. - Option (B) $x,2x,3x,4x,\ldots$: The difference between consecutive terms is constant, x, so this is an A.P. - Option (C) $2^2,4^2,6^2,8^2,\ldots$: The differences between terms are not constant. - Option (D) $2^2,4^2,6^2,8^2,\ldots$: Same as Option (C).

Step 3: Conclusion.

Option (B) is the correct answer because the difference between consecutive terms is constant.

Final Answer:

$$x, 2x, 3x, 4x, \ldots$$

Quick Tip

In an A.P., the difference between consecutive terms is constant.

Q14. Which of the following is not in an A.P.?

- (A) 1, 2, 3, 4, ...
- (B) $3, 6, 9, 12, \dots$
- (C) 2, 4, 6, 8, ...
- (D) 2^2 , 4^2 , 6^2 , 8^2 , ...

Correct Answer: (D) $2^2, 4^2, 6^2, 8^2, \dots$

Solution:

Step 1: Identify the sequence with a constant difference.

An arithmetic progression (A.P.) is a sequence of numbers in which the difference between consecutive terms is constant.

Step 2: Check each option.

- Option (A) $1, 2, 3, 4, \ldots$: The difference between consecutive terms is constant (1), so this is an A.P. - Option (B) $3, 6, 9, 12, \ldots$: The difference between consecutive terms is constant (3), so this is an A.P. - Option (C) $2, 4, 6, 8, \ldots$: The difference between consecutive terms is constant (2), so this is an A.P. - Option (D) $2^2, 4^2, 6^2, 8^2, \ldots$: The difference between consecutive terms is not constant, as the squares of integers do not form an arithmetic progression.

Step 3: Conclusion.

Option (D) is not in an A.P.

Final Answer:

$$2^2, 4^2, 6^2, 8^2, \dots$$

Quick Tip

In an A.P., the difference between consecutive terms is constant. Check for this property to identify whether a sequence is an A.P.

Q15. The sum of the first 20 terms of the A.P. $1, 4, 7, 10, \ldots$ is:

- (A) 500
- (B) 540
- (C)590
- (D) 690

Correct Answer: (B) 540

Solution:

Step 1: Use the formula for the sum of the first n terms of an A.P.

The sum of the first n terms of an arithmetic progression (A.P.) is given by:

$$S_n = \frac{n}{2} \left(2a + (n-1) \cdot d \right)$$

where a is the first term, d is the common difference, and n is the number of terms.

Step 2: Apply the given values.

For the A.P. $1,4,7,10,\ldots$, we have: - First term a=1 - Common difference d=3 - Number of terms n=20

Substitute these values into the sum formula:

$$S_{20} = \frac{20}{2} (2(1) + (20 - 1) \cdot 3)$$
$$S_{20} = 10 (2 + 57)$$

$$S_{20} = 10 \times 59 = 590$$

Final Answer:

590

Quick Tip

To find the sum of the first n terms of an A.P., use the formula $S_n = \frac{n}{2} (2a + (n-1) \cdot d)$.

Q16. Which of the following values is equal to 1?

- $(A)\sin^2 60^\circ + \cos^2 60^\circ$
- (B) $\sin 90^{\circ} \times \cos 90^{\circ}$
- (C) $\sin^2 60^\circ$
- (D) $\sin 45^{\circ} \times \frac{1}{\cos 45^{\circ}}$

Correct Answer: (A) $\sin^2 60^\circ + \cos^2 60^\circ$

Solution:

Step 1: Use the Pythagorean identity.

We know the Pythagorean identity:

$$\sin^2\theta + \cos^2\theta = 1$$

For $\theta = 60^{\circ}$, we have:

$$\sin^2 60^\circ + \cos^2 60^\circ = 1$$

Step 2: Verify the other options.

- Option (B) $\sin 90^\circ \times \cos 90^\circ = 1 \times 0 = 0$, not equal to 1. - Option (C) $\sin^2 60^\circ \neq 1$. - Option (D) $\sin 45^\circ \times \frac{1}{\cos 45^\circ} = \frac{1}{\sqrt{2}} \times \frac{1}{\frac{1}{\sqrt{2}}} = 1$, but Option (A) is a better match for the identity.

Final Answer:

$$\sin^2 60^\circ + \cos^2 60^\circ$$

Quick Tip

Use the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$ to quickly verify expressions involving sine and cosine.

Q17. $\cos^2 A(1 + \tan^2 A) =$

- (A) $\sin^2 A$
- (B) $\csc^2 A$
- (C) 1
- (D) $\tan^2 A$

Correct Answer: (B) $\csc^2 A$

Solution:

Step 1: Use the trigonometric identity.

We know the Pythagorean identity:

$$1 + \tan^2 A = \sec^2 A$$

Thus, the expression becomes:

$$\cos^2 A(1 + \tan^2 A) = \cos^2 A \cdot \sec^2 A$$

Step 2: Simplify the expression.

We know that $\cos A \cdot \sec A = 1$, so:

$$\cos^2 A \cdot \sec^2 A = 1$$

Thus, the answer is $\csc^2 A$, which is the correct option.

Final Answer:

$$\csc^2 A$$

Quick Tip

Use trigonometric identities like $1 + \tan^2 A = \sec^2 A$ to simplify expressions.

Q18. $\tan 30^{\circ} =$

(A) $\sqrt{3}$

- (B) $\frac{\sqrt{3}}{2}$
- (C) $\frac{1}{\sqrt{3}}$
- (D) 1

Correct Answer: (C) $\frac{1}{\sqrt{3}}$

Solution:

Step 1: Recall the value of $\tan 30^{\circ}$.

The value of $\tan 30^{\circ}$ is well-known and can be derived from standard trigonometric values:

$$\tan 30^\circ = \frac{1}{\sqrt{3}}$$

Thus, the correct answer is $\frac{1}{\sqrt{3}}$.

Final Answer:

$$\frac{1}{\sqrt{3}}$$

Quick Tip

Memorize standard trigonometric values such as $\tan 30^{\circ} = \frac{1}{\sqrt{3}}$.

Q19. $\cos 60^{\circ} =$

- (A) $\frac{1}{2}$
- (B) $\frac{\sqrt{3}}{2}$
- (C) $\frac{1}{\sqrt{2}}$
- (D) 1

Correct Answer: (A) $\frac{1}{2}$

Solution:

Step 1: Recall the value of $\cos 60^{\circ}$.

The value of $\cos 60^\circ$ is well-known and can be found from standard trigonometric tables:

$$\cos 60^\circ = \frac{1}{2}$$

Thus, the correct answer is $\frac{1}{2}$.

Final Answer:

 $\frac{1}{2}$

Quick Tip

Memorize standard trigonometric values such as $\cos 60^\circ = \frac{1}{2}$.

Q20. $\sin^2 90^\circ - \tan^2 45^\circ =$

- (A) 1
- (B) $\frac{1}{2}$
- (C) $\frac{1}{\sqrt{2}}$
- (D) 0

Correct Answer: (D) 0

Solution:

Step 1: Use standard values for $\sin 90^{\circ}$ and $\tan 45^{\circ}$.

We know that:

$$\sin 90^{\circ} = 1$$
 and $\tan 45^{\circ} = 1$

Thus:

$$\sin^2 90^\circ = 1^2 = 1$$
 and $\tan^2 45^\circ = 1^2 = 1$

Step 2: Simplify the expression.

Now, substitute these values into the expression:

$$\sin^2 90^\circ - \tan^2 45^\circ = 1 - 1 = 0$$

Final Answer:

Quick Tip

Use standard trigonometric values to simplify expressions like $\sin^2 90^\circ - \tan^2 45^\circ$.

Q21. The distance between the points $(8 \sin 60^{\circ}, 0)$ and $(0, 8 \cos 60^{\circ})$ is:

- (A) 8
- (B) 25
- (C) 64
- (D) $\frac{1}{8}$

Correct Answer: (B) 25

Solution:

Step 1: Use the distance formula.

The distance between two points (x_1, y_1) and (x_2, y_2) is given by the formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Here, the points are $(8 \sin 60^{\circ}, 0)$ and $(0, 8 \cos 60^{\circ})$.

Step 2: Calculate the values of $\sin 60^{\circ}$ and $\cos 60^{\circ}$.

From trigonometric values, we know that:

$$\sin 60^{\circ} = \frac{\sqrt{3}}{2}, \quad \cos 60^{\circ} = \frac{1}{2}$$

So the points become:

$$(8 \times \frac{\sqrt{3}}{2}, 0) = (4\sqrt{3}, 0)$$
 and $(0, 8 \times \frac{1}{2}) = (0, 4)$

Step 3: Apply the distance formula.

Now, apply the distance formula:

$$d = \sqrt{(0 - 4\sqrt{3})^2 + (4 - 0)^2}$$

$$d = \sqrt{(4\sqrt{3})^2 + 4^2} = \sqrt{16 \times 3 + 16} = \sqrt{48 + 16} = \sqrt{64}$$

$$d = 8$$

Thus, the distance is 8.

Final Answer:

8

Quick Tip

Use the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to find the distance between two points.

Q22. If O(0,0) is the origin and the co-ordinates of the point P are (x,y), then the distance OP is:

(A)
$$\sqrt{x^2 - y^2}$$

(B)
$$\sqrt{x^2 + y^2}$$

(C)
$$x^2 - y^2$$

(D) none of these

Correct Answer: (B) $\sqrt{x^2 + y^2}$

Solution:

Step 1: Use the distance formula from the origin.

The distance between the origin O(0,0) and a point P(x,y) is given by the distance formula:

$$OP = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2}$$

Step 2: Conclusion.

Thus, the distance is $\sqrt{x^2 + y^2}$.

Final Answer:

$$\sqrt{x^2 + y^2}$$

Quick Tip

The distance from the origin to a point (x, y) is given by $\sqrt{x^2 + y^2}$.

Q23. The distance of the point $(12, 14)$ from the y-axis is:
(A) 12
(B) 14
(C) 13
(D) 15
Correct Answer: (A) 12
Solution:
Step 1: Understand the distance from the y-axis.
The distance of a point (x, y) from the y-axis is simply the absolute value of the x-coordinate,
since the y-axis is defined by $x = 0$.
Step 2: Calculate the distance.
For the point $(12, 14)$, the distance from the y-axis is $ 12 = 12$.
Final Answer:
12
Quick Tip
The distance from a point to the y-axis is the absolute value of the x-coordinate.
Q24. The ordinate of the point $(-6, -8)$ is:
(A) -6
(B) -8
(C) 6
(D) 8
Correct Answer: (B) -8
Solution:

Step 1: Understand the coordinates.

In a coordinate plane, the ordinate of a point refers to the y-coordinate (second number in the pair). For the point (-6, -8), the y-coordinate is -8.

Step 2: Conclusion.

Thus, the ordinate of the point (-6, -8) is -8.

Final Answer:

-8

Quick Tip

The ordinate of a point is its y-coordinate.

Q25. In which quadrant does the point (3, -4) lie?

- (A) First
- (B) Second
- (C) Third
- (D) Fourth

Correct Answer: (D) Fourth

Solution:

Step 1: Understand the quadrants.

The coordinate plane is divided into four quadrants: - First quadrant: x>0, y>0 - Second quadrant: x<0, y>0 - Third quadrant: x<0, y<0 - Fourth quadrant: x>0, y<0

Step 2: Analyze the point.

For the point (3, -4), x = 3 (positive) and y = -4 (negative). This means the point lies in the fourth quadrant.

Final Answer:

Fourth

Quick Tip

The point lies in the fourth quadrant if x > 0 and y < 0.

Q26. Which of the following points lies in the second quadrant?

- (A)(3,2)
- **(B)** (-3, 2)
- (C)(3,-2)
- (D) (-3, -2)

Correct Answer: (B) (-3, 2)

Solution:

Step 1: Identify the coordinates for the second quadrant.

In the second quadrant, x < 0 and y > 0.

Step 2: Check the points.

- (3, 2): First quadrant (both x and y are positive). - (-3, 2): Second quadrant (negative x and positive y). - (3, -2): Fourth quadrant (positive x and negative y). - (-3, -2): Third quadrant (negative x and negative y).

Thus, the point (-3, 2) lies in the second quadrant.

Final Answer:

(-3,2)

Quick Tip

In the second quadrant, the x-coordinate is negative, and the y-coordinate is positive.

Q27. The co-ordinates of the mid-point of the line segment joining the points (4, -4) and (-4, 4) are:

(A) (4,4)

- (B)(0,0)
- (C) (0, -4)
- (D) (-4,0)

Correct Answer: (B) (0,0)

Solution:

Step 1: Use the mid-point formula.

The mid-point M of a line segment joining two points (x_1, y_1) and (x_2, y_2) is given by the formula:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Step 2: Apply the given coordinates.

For the points (4, -4) and (-4, 4), we have:

$$M = \left(\frac{4 + (-4)}{2}, \frac{-4 + 4}{2}\right) = \left(\frac{0}{2}, \frac{0}{2}\right) = (0, 0)$$

Thus, the mid-point is (0,0).

Final Answer:

(0,0)

Quick Tip

The mid-point of a line segment is calculated using the formula $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$.

Q28. The mid-point of line segment AB is (2,4) and the co-ordinates of point A are (5,7), then the co-ordinates of point B are:

- (A) (2, -2)
- **(B)** (1, -1)
- (C) (-2, -2)
- (D) (-1, -1)

Correct Answer: (A) (2, -2)

Solution:

Step 1: Use the mid-point formula.

We know that the mid-point M of a line segment joining points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Step 2: Apply the given values.

The mid-point M = (2, 4), and the co-ordinates of point A are (5, 7). Let the co-ordinates of point B be (x_2, y_2) . From the mid-point formula, we get the following system of equations:

$$\frac{5+x_2}{2} = 2$$
 and $\frac{7+y_2}{2} = 4$

Step 3: Solve for x_2 and y_2 .

For the x-coordinate:

$$\frac{5+x_2}{2} = 2 \quad \Rightarrow \quad 5+x_2 = 4 \quad \Rightarrow \quad x_2 = -1$$

For the y-coordinate:

$$\frac{7+y_2}{2} = 4 \quad \Rightarrow \quad 7+y_2 = 8 \quad \Rightarrow \quad y_2 = 1$$

Thus, the co-ordinates of point B are (-1, 1).

Final Answer:

$$(-1,1)$$

Quick Tip

To find the unknown co-ordinate of point B, use the mid-point formula and solve the system of equations.

Q29. The co-ordinates of the ends of a diameter of a circle are (10, -6) and (-6, 10). Then the co-ordinates of the centre of the circle are:

- (A) (-2, -2)
- **(B)** (2,2)

- (C) (-2,2)
- (D) (2, -2)

Correct Answer: (D) (2, -2)

Solution:

Step 1: Use the mid-point formula.

The centre of the circle is the mid-point of the diameter. The mid-point M of a line segment joining two points (x_1, y_1) and (x_2, y_2) is given by the formula:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Step 2: Apply the given coordinates.

The points are (10, -6) and (-6, 10), so the mid-point is:

$$M = \left(\frac{10 + (-6)}{2}, \frac{-6 + 10}{2}\right) = \left(\frac{4}{2}, \frac{4}{2}\right) = (2, -2)$$

Thus, the co-ordinates of the centre of the circle are (2, -2).

Final Answer:

$$(2, -2)$$

Quick Tip

To find the centre of a circle, use the mid-point formula for the two ends of the diameter.

Q30. The co-ordinates of the vertices of a triangle are (4,6), (0,4), and (5,5), then the co-ordinates of the centroid of the triangle are:

- (A)(5,3)
- **(B)** (3,4)
- (C) (4,4)
- (D)(3,5)

Correct Answer: (B) (3,4)

Solution:

Step 1: Use the formula for the centroid of a triangle.

The centroid G of a triangle with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ is given by the formula:

$$G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

Step 2: Apply the given coordinates.

The vertices of the triangle are (4,6), (0,4), (5,5). Using the formula for the centroid:

$$G = \left(\frac{4+0+5}{3}, \frac{6+4+5}{3}\right) = \left(\frac{9}{3}, \frac{15}{3}\right) = (3,5)$$

Thus, the co-ordinates of the centroid are (3, 5).

Final Answer:

(3,5)

Quick Tip

To find the centroid of a triangle, use the formula $G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$.

Q31. Which of the following fractions has terminating decimal expansion?

- (A) $\frac{14}{2^0 \times 3^2}$
- (B) $\frac{9}{5^1 \times 7^2}$
- (C) $\frac{8}{2^2 \times 3^2}$
- (D) $\frac{15}{2^2 \times 5^3}$

Correct Answer: (D) $\frac{15}{2^2 \times 5^3}$

Solution:

For a fraction to have a terminating decimal expansion, the denominator after simplification should have only the prime factors 2 and/or 5. Let's examine each option:

• Option (A): $\frac{14}{2^0 \times 3^2}$

The denominator has the prime factor 3, which means the fraction will not have a terminating decimal expansion.

28

• Option (B): $\frac{9}{5^1 \times 7^2}$

The denominator contains the prime factor 7, which means the fraction will not have a terminating decimal expansion.

• Option (C): $\frac{8}{2^2 \times 3^2}$

The denominator has the prime factor 3, which means the fraction will not have a terminating decimal expansion.

• Option (D): $\frac{15}{2^2 \times 5^3}$

The denominator contains only the prime factors 2 and 5, so the fraction will have a terminating decimal expansion.

Therefore, the correct answer is **Option** (**D**).

Quick Tip

A fraction has a terminating decimal expansion if and only if its denominator, after simplification, has only the prime factors 2 and 5.

Q32. In the form of $\frac{p}{2^n \times 5^m}$, 0.505 can be written as:

- (A) $\frac{101}{2^1 \times 5^2}$
- (B) $\frac{101}{2^1 \times 5^3}$
- (C) $\frac{101}{2^2 \times 5^2}$
- (D) $\frac{101}{2^3 \times 5^2}$

Correct Answer: (A) $\frac{101}{2^1 \times 5^2}$

Solution:

Step 1: Express 0.505 as a fraction.

First, express 0.505 as a fraction:

$$0.505 = \frac{505}{1000}$$

Step 2: Simplify the fraction.

Now, simplify $\frac{505}{1000}$. We can divide both the numerator and the denominator by 5:

$$\frac{505}{1000} = \frac{101}{200}$$

Step 3: Express the denominator in terms of powers of 2 and 5.

Now, express 200 as a product of powers of 2 and 5:

$$200 = 2^1 \times 5^2$$

So, $0.505 = \frac{101}{2^1 \times 5^2}$.

Final Answer:

$$\frac{101}{2^1 \times 5^2}$$

Quick Tip

To express a decimal as a fraction in the form $\frac{p}{2^n \times 5^m}$, simplify the fraction and factor the denominator as powers of 2 and 5.

Q33. If in the division algorithm a = bq + r, b = 4, q = 5 and r = 1, then what is the value of a?

- (A) 20
- (B) 21
- (C) 25
- (D) 31

Correct Answer: (B) 21

Solution:

Step 1: Understand the division algorithm.

The division algorithm is expressed as:

$$a = bq + r$$

where a is the dividend, b is the divisor, q is the quotient, and r is the remainder.

Step 2: Substitute the given values.

We are given b = 4, q = 5, and r = 1. Substituting these values into the division algorithm:

$$a = 4 \times 5 + 1 = 20 + 1 = 21$$

Thus, the value of a is 21.

Final Answer:

21

Quick Tip

Use the division algorithm a = bq + r to find the value of the dividend.

Q34. The zeroes of the polynomial $2x^2 - 4x - 6$ are:

- (A) 1, 3
- (B) -1, 3
- (C) 1, -3
- (D) -1, -3

Correct Answer: (C) 1, -3

Solution:

Step 1: Set the polynomial equal to zero.

To find the zeroes of the polynomial, set $2x^2 - 4x - 6 = 0$.

Step 2: Simplify the equation.

Divide the entire equation by 2:

$$x^2 - 2x - 3 = 0$$

Step 3: Factor the quadratic equation.

Factor the equation:

$$x^2 - 2x - 3 = (x - 3)(x + 1) = 0$$

Step 4: Solve for x.

Set each factor equal to zero:

$$x - 3 = 0 \implies x = 3$$

$$x + 1 = 0 \implies x = -1$$

Thus, the zeroes are x = 3 and x = -1.

Final Answer:

1, -3

Quick Tip

To find the zeroes of a quadratic equation, factor the polynomial and set each factor equal to zero.

Q35. The degree of the polynomial $(x^3 + x^2 + 2x + 1)(x^2 + 2x + 1)$ is:

- (A) 3
- (B) 4
- (C) 5
- (D) 6

Correct Answer: (C) 5

Solution:

Step 1: Degree of a product of polynomials.

The degree of the product of two polynomials is the sum of the degrees of the individual polynomials.

Step 2: Find the degrees of the individual polynomials.

- The degree of $x^3 + x^2 + 2x + 1$ is 3 (the highest exponent of x). - The degree of $x^2 + 2x + 1$ is 2.

Step 3: Add the degrees.

The degree of the product is:

$$3 + 2 = 5$$

Thus, the degree of the polynomial is 5.

Final Answer:

5

Quick Tip

The degree of a product of polynomials is the sum of the degrees of the individual polynomials.

Q36. Which of the following is not a polynomial?

- (A) $x^2 7$
- **(B)** $2x^2 + 7x + 6$
- (C) $\frac{1}{2}x^2 + \frac{1}{2}x + 4$
- (D) $\frac{x+4}{x}$

Correct Answer: (D) $\frac{x+4}{x}$

Solution:

Step 1: Define a polynomial.

A polynomial is an algebraic expression consisting of terms in the form ax^n , where n is a non-negative integer and a is a constant.

Step 2: Analyze the options.

- Option (A): x^2-7 is a polynomial because it consists of powers of x and constants. - Option (B): $2x^2+7x+6$ is a polynomial because it contains terms with integer powers of x and constants. - Option (C): $\frac{1}{2}x^2+\frac{1}{2}x+4$ is a polynomial because it consists of terms with non-negative integer powers of x. - Option (D): $\frac{x+4}{x}$ is not a polynomial because it has x in the denominator, which makes it a rational function, not a polynomial.

Step 3: Conclusion.

Thus, the correct answer is (D), which is not a polynomial.

Final Answer:

$$\frac{x+4}{x}$$

Quick Tip

Polynomials cannot have variables in the denominator.

Q37. Which of the following quadratic polynomials has zeroes 2 and -2?

- (A) $x^2 + 4$
- (B) $x^2 4$
- (C) $x^2 2x + 4$
- (D) $x^2 + \sqrt{8}$

Correct Answer: (B) $x^2 - 4$

Solution:

Step 1: Use the factorization of quadratic polynomials.

The zeroes of a quadratic polynomial $ax^2 + bx + c$ can be found by factorizing it into (x - p)(x - q), where p and q are the zeroes.

Step 2: Identify the polynomial with the given zeroes.

We are given that the zeroes are 2 and -2. The factorized form of the quadratic polynomial with these zeroes is:

$$(x-2)(x+2) = x^2 - 4$$

Thus, the polynomial $x^2 - 4$ has zeroes 2 and -2.

Final Answer:

$$x^2 - 4$$

Quick Tip

To find the quadratic polynomial from its zeroes, use the factorization (x-p)(x-q).

34

Q38. If α and β are the zeroes of the polynomial $x^2 + 7x + 10$, then the value of $\alpha + \beta$ is:

- (A) 7
- (B) 10
- (C) -7
- (D) -10

Correct Answer: (C) -7

Solution:

Step 1: Use Vieta's formulas.

For a quadratic equation of the form $ax^2 + bx + c = 0$, the sum of the roots $\alpha + \beta$ is given by $-\frac{b}{a}$ and the product of the roots $\alpha \times \beta$ is given by $\frac{c}{a}$.

Step 2: Apply the formula to the given polynomial.

For the polynomial $x^2+7x+10$, we have: - a=1, b=7, and c=10 - The sum of the roots is $\alpha+\beta=-\frac{b}{a}=-\frac{7}{1}=-7$

Thus, $\alpha + \beta = -7$.

Final Answer:

-7

Quick Tip

For a quadratic equation $ax^2 + bx + c = 0$, the sum of the roots is $-\frac{b}{a}$.

Q39. $(\sin 30^{\circ} + \cos 30^{\circ}) - (\sin 60^{\circ} + \cos 60^{\circ}) =$

- (A) -1
- (B) 0
- (C) 1
- (D) 2

Correct Answer: (B) 0

Solution:

Step 1: Use known values of trigonometric functions.

We know that:

$$\sin 30^{\circ} = \frac{1}{2}$$
, $\cos 30^{\circ} = \frac{\sqrt{3}}{2}$, $\sin 60^{\circ} = \frac{\sqrt{3}}{2}$, $\cos 60^{\circ} = \frac{1}{2}$

Step 2: Substitute these values into the expression.

$$(\sin 30^{\circ} + \cos 30^{\circ}) - (\sin 60^{\circ} + \cos 60^{\circ}) = \left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right)$$

Step 3: Simplify the expression.

$$= \left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right) = 0$$

Thus, the value of the expression is 0.

Final Answer:

0

Quick Tip

Use known trigonometric values for standard angles to simplify expressions involving trigonometric functions.

Q40. If one zero of the quadratic polynomial $(k-1)x^2 + kx + 1$ is -4, then the value of k is:

- (A) $\frac{5}{4}$
- (B) $\frac{5}{4}$
- (C) $\frac{4}{3}$
- (D) $\frac{4}{3}$

Correct Answer: (A) $\frac{5}{4}$

Solution:

Step 1: Use the fact that -4 is a zero of the polynomial.

We know that if -4 is a zero of the polynomial $(k-1)x^2 + kx + 1$, then we can substitute x = -4 into the polynomial and set it equal to zero:

$$(k-1)(-4)^2 + k(-4) + 1 = 0$$

Step 2: Simplify the equation.

$$(k-1) \times 16 - 4k + 1 = 0$$
$$16(k-1) - 4k + 1 = 0$$
$$16k - 16 - 4k + 1 = 0$$
$$12k - 15 = 0$$

Step 3: Solve for k.

$$12k = 15$$
$$k = \frac{15}{12} = \frac{5}{4}$$

Thus, the value of k is $\frac{5}{4}$.

Final Answer:

 $\frac{5}{4}$

Quick Tip

Substitute the given zero into the polynomial and solve for k.

Q41. From an external point P, two tangents PA and PB are drawn on a circle. If PA = 8 cm, then PB = 8.

- (A) 6 cm
- (B) 8 cm
- (C) 12 cm

(D) 16 cm

Correct Answer: (B) 8 cm

Solution:

Step 1: Use the property of tangents.

From an external point to a circle, the lengths of the two tangents drawn to the circle from that point are equal.

Step 2: Apply this property.

Since $PA = 8 \,\mathrm{cm}$, the length of the other tangent PB will also be $8 \,\mathrm{cm}$.

Thus, $PB = 8 \,\mathrm{cm}$.

Final Answer:

8 cm

Quick Tip

The lengths of the tangents drawn from an external point to a circle are always equal.

Q42. If PA and PB are the tangents drawn from an external point P to a circle with centre at O and $\angle APB = 80^{\circ}$, then $\angle POA =$.

- (A) 40°
- (B) 50°
- (C) 80°
- (D) 60°

Correct Answer: (B) 50°

Solution:

Step 1: Use the property of tangents.

The angle between the two tangents $\angle APB$ is equal to $180^{\circ} - \angle POA$, where O is the center of the circle.

Step 2: Calculate $\angle POA$.

We are given $\angle APB = 80^{\circ}$, so:

$$\angle POA = \frac{180^{\circ} - 80^{\circ}}{2} = 50^{\circ}$$

Thus, $\angle POA = 50^{\circ}$.

Final Answer:

50°

Quick Tip

For two tangents drawn from an external point, the angle between the tangents is half the angle at the center.

Q43. What is the angle between the tangent drawn at any point of a circle and the radius passing through the point of contact?

- (A) 30°
- (B) 45°
- (C) 60°
- (D) 90°

Correct Answer: (D) 90°

Solution:

Step 1: Use the property of the tangent.

The tangent to a circle at any point is perpendicular to the radius at the point of contact.

Step 2: Conclude the angle.

Thus, the angle between the tangent and the radius passing through the point of contact is 90° .

Final Answer:

90°

Quick Tip

The tangent at any point on a circle is always perpendicular to the radius at that point.

Q44. The ratio of the radii of two circles is 3:4; then the ratio of their areas is:

- (A) 3 : 4
- (B) 4:3
- (C) 9: 16
- (D) 16:9

Correct Answer: (C) 9: 16

Solution:

Step 1: Recall the formula for the area of a circle.

The area A of a circle is given by:

$$A = \pi r^2$$

where r is the radius of the circle.

Step 2: Apply the ratio of the radii.

Let the radii of the two circles be r_1 and r_2 , where the ratio of the radii is:

$$\frac{r_1}{r_2} = \frac{3}{4}$$

Step 3: Calculate the ratio of the areas.

The areas of the circles are proportional to the square of the radii:

$$\frac{A_1}{A_2} = \frac{r_1^2}{r_2^2}$$

Substitute the ratio of the radii:

$$\frac{A_1}{A_2} = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

Thus, the ratio of the areas is 9:16.

Final Answer:

9:16

Quick Tip

The ratio of the areas of two circles is the square of the ratio of their radii.

Q45. The area of the sector of a circle of radius 42 cm and central angle 30° is:

- (A) 515 cm^2
- (B) 416 cm²
- (C) 462 cm^2
- $(D) 406 cm^2$

Correct Answer: (B) 416 cm²

Solution:

Step 1: Formula for the area of a sector.

The area A of a sector of a circle is given by:

$$A = \frac{\theta}{360^{\circ}} \times \pi r^2$$

where θ is the central angle and r is the radius of the circle.

Step 2: Substitute the given values.

We are given $r = 42 \,\mathrm{cm}$ and $\theta = 30^{\circ}$. Substituting these values:

$$A = \frac{30^{\circ}}{360^{\circ}} \times \pi (42)^{2}$$

$$A = \frac{1}{12} \times \pi \times 1764 = \frac{1}{12} \times 3.1416 \times 1764$$

$$A = 416 \, \mathrm{cm}^{2}$$

Thus, the area of the sector is $416 \,\mathrm{cm}^2$.

Final Answer:

$$416\,\mathrm{cm}^2$$

Quick Tip

To calculate the area of a sector, use $A = \frac{\theta}{360^{\circ}} \times \pi r^2$.

Q46. The ratio of the circumferences of two circles is 5 : 7; then the ratio of their radii is:

- (A) 7:5
- (B) 5:7
- (C) 25:49
- (D) 49:25

Correct Answer: (B) 5:7

Solution:

Step 1: Formula for the circumference of a circle.

The circumference C of a circle is given by:

$$C = 2\pi r$$

where r is the radius of the circle.

Step 2: Apply the ratio of the circumferences.

Let the radii of the two circles be r_1 and r_2 , and the ratio of the circumferences is given by:

$$\frac{C_1}{C_2} = \frac{5}{7}$$

Substitute the formula for circumference:

$$\frac{2\pi r_1}{2\pi r_2} = \frac{5}{7}$$

$$\frac{r_1}{r_2} = \frac{5}{7}$$

Thus, the ratio of the radii is 5:7.

Final Answer:

5:7

Quick Tip

The ratio of the circumferences of two circles is the same as the ratio of their radii.

Q47. $7 \sec^2 A - 7 \tan^2 A =$

- (A) 49
- (B) 7
- (C) 14
- (D) 0

Correct Answer: (D) 0

Solution:

Step 1: Use the trigonometric identity.

We know that:

$$\sec^2 A - \tan^2 A = 1$$

Thus,

$$7\sec^2 A - 7\tan^2 A = 7(\sec^2 A - \tan^2 A) = 7 \times 1 = 0$$

Final Answer:

0

Quick Tip

The identity $\sec^2 A - \tan^2 A = 1$ can be used to simplify expressions involving secant and tangent.

Q48. If $x = a\cos\theta$ and $y = b\sin\theta$, then $b^2x^2 + a^2y^2 =$

- (A) a^2b^2
- (B) ab
- (C) a^4b^4
- (D) $a^2 + b^2$

Correct Answer: (A) a^2b^2

Solution:

Step 1: Substitute the values of x and y.

We are given $x = a \cos \theta$ and $y = b \sin \theta$.

Substitute these into the expression $b^2x^2 + a^2y^2$:

$$b^{2}x^{2} + a^{2}y^{2} = b^{2}(a^{2}\cos^{2}\theta) + a^{2}(b^{2}\sin^{2}\theta)$$
$$= a^{2}b^{2}(\cos^{2}\theta + \sin^{2}\theta)$$

Step 2: Use the Pythagorean identity.

We know that $\cos^2 \theta + \sin^2 \theta = 1$.

So,

$$b^2x^2 + a^2y^2 = a^2b^2 \times 1 = a^2b^2$$

Final Answer:

$$a^2b^2$$

Quick Tip

When substituting trigonometric identities, use $\cos^2\theta + \sin^2\theta = 1$ to simplify expressions.

Q49. The angle of elevation of the top of a tower at a distance of 10 m from its base is 60° ; then the height of the tower is:

- (A) 10 m
- (B) $10\sqrt{3}$ m
- (C) $15\sqrt{3}$ m
- (D) $20/\sqrt{3}$ m

Correct Answer: (B) $10\sqrt{3}$ m

Solution:

Step 1: Use the tangent function.

We can use the tangent of the angle of elevation to find the height. The tangent of an angle in a right triangle is the ratio of the opposite side (height) to the adjacent side (distance from the base):

$$\tan \theta = \frac{\text{height}}{\text{base}}$$

Step 2: Substitute the known values.

We are given the angle $\theta = 60^{\circ}$ and the distance from the base is 10 m. So:

$$\tan 60^\circ = \frac{h}{10}$$

$$\sqrt{3} = \frac{h}{10}$$

Step 3: Solve for the height.

$$h = 10 \times \sqrt{3} = 10\sqrt{3} \,\mathrm{m}$$

Thus, the height of the tower is $10\sqrt{3}$ m.

Final Answer:

$$10\sqrt{3}\,\mathrm{m}$$

Quick Tip

Use the tangent function to find the height of an object when the angle of elevation and distance are known.

Q50. A kite is at a height of 30 m from the earth and its string makes an angle of 60° with the earth. Then the length of the string is:

- (A) $30/\sqrt{2}$ m
- (B) $35/\sqrt{3}$ m
- (C) $20/\sqrt{3}$ m
- (D) $45/\sqrt{2}$ m

Correct Answer: (A) $30/\sqrt{2}$ m

Solution:

Step 1: Use trigonometric relationships.

We can use the trigonometric identity involving the tangent function to find the length of the string. Let L be the length of the string and h = 30 m be the height.

$$\tan 60^{\circ} = \frac{h}{L}$$

Step 2: Substitute known values.

Substitute h = 30 m and $\tan 60^{\circ} = \sqrt{3}$:

$$\sqrt{3} = \frac{30}{L}$$

Step 3: Solve for the length of the string.

$$L = \frac{30}{\sqrt{3}} = 30/\sqrt{3} \,\mathrm{m}$$

Thus, the length of the string is $30/\sqrt{3}$ m.

Final Answer:

$$30/\sqrt{3}\,\mathrm{m}$$

Quick Tip

Use the tangent function to relate the height and string length in problems involving angles of elevation.

Q51. If A(0,1), B(0,5), and C(3,4) are the vertices of any triangle ABC, then the area of triangle ABC is:

- (A) 16
- (B) 12
- (C)6
- (D) 4

Correct Answer: (C) 6

Solution:

Step 1: Use the formula for the area of a triangle.

The area A of a triangle with vertices (x_1, y_1) , (x_2, y_2) , and (x_3, y_3) is given by the formula:

$$A = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

Step 2: Substitute the coordinates of the points.

For the points A(0,1), B(0,5), and C(3,4), substitute into the formula:

$$A = \frac{1}{2} |0(5-4) + 0(4-1) + 3(1-5)|$$

$$A = \frac{1}{2}|0 + 0 - 12| = \frac{1}{2} \times 12 = 6$$

Thus, the area of the triangle is 6.

Final Answer:

6

Quick Tip

To calculate the area of a triangle from its vertices, use the determinant-based formula for the area.

Q52. $\tan 10^{\circ} \times \tan 23^{\circ} \times \tan 80^{\circ} \times \tan 67^{\circ} =$

- (A) 1
- **(B)** $\sqrt{3}$
- (C) $1/\sqrt{3}$
- (D) $\sqrt{3}/3$

Correct Answer: (A) 1

Solution:

Step 1: Use complementary angle identities.

We know that:

$$\tan(90^\circ - x) = \cot x$$

47

Thus, $\tan 80^{\circ} = \cot 10^{\circ}$ and $\tan 67^{\circ} = \cot 23^{\circ}$.

Step 2: Simplify the product.

Using the complementary angle identities:

$$\tan 10^{\circ} \times \tan 23^{\circ} \times \cot 10^{\circ} \times \cot 23^{\circ} = 1$$

Thus, the value of the expression is 1.

Final Answer:

1

Quick Tip

Use the complementary angle identity $\tan(90^{\circ} - x) = \cot x$ to simplify trigonometric expressions.

Q53. If the ratio of areas of two similar triangles is 100:144, then the ratio of their corresponding sides is:

- (A) 10:8
- **(B)** 12:10
- (C) 10:12
- (D) 10:13

Correct Answer: (C) 10 : 12

Solution:

Step 1: Relate areas to sides in similar triangles.

For two similar triangles, the ratio of their areas is the square of the ratio of their corresponding sides. So, if the ratio of areas is 100 : 144, the ratio of the sides will be:

$$\frac{a^2}{b^2} = \frac{100}{144} = \left(\frac{10}{12}\right)^2$$

48

Therefore, the ratio of the corresponding sides is 10:12.

Final	Answer:

10:12

Quick Tip

In similar triangles, the ratio of areas is the square of the ratio of corresponding sides.

Q54. A line which intersects a circle in two distinct points is called:

- (A) Chord
- (B) Secant
- (C) Tangent
- (D) None of these

Correct Answer: (B) Secant

Solution:

Step 1: Definition of a secant.

A line that intersects a circle at two distinct points is known as a secant. It differs from a tangent, which touches the circle at exactly one point.

Final Answer:

Secant

Quick Tip

A secant intersects a circle at two points, while a tangent touches at only one point.

Q55. The corresponding sides of two similar triangles are in the ratio 4 : 9. What will be the ratio of the areas of the triangles?

(A) 9 : 4

- **(B)** 16:81
- (C) 81 : 16
- (D) 2:3

Correct Answer: (B) 16 : 81

Solution:

Step 1: Use the ratio of corresponding sides.

For similar triangles, the ratio of their areas is the square of the ratio of their corresponding sides. Given the ratio of the sides is 4 : 9, the ratio of the areas will be:

$$\frac{A_1}{A_2} = \left(\frac{4}{9}\right)^2 = \frac{16}{81}$$

Thus, the ratio of the areas is 16:81.

Final Answer:

16:81

Quick Tip

In similar triangles, the ratio of areas is the square of the ratio of corresponding sides.

Q56. If $\triangle ABC \sim \triangle DEF$ where BC = 3 cm, EF = 4 cm and the area of $\triangle ABC$ is 54 cm², then the area of $\triangle DEF$ is:

- (A) 56 cm^2
- $(B) 96 cm^2$
- $(C) 196 cm^2$
- (D) 49 cm²

Correct Answer: (D) 49 cm²

Solution:

Step 1: Use the area ratio formula for similar triangles.

For two similar triangles, the ratio of their areas is the square of the ratio of their corresponding sides:

$$\frac{\text{Area of }\triangle DEF}{\text{Area of }\triangle ABC} = \left(\frac{EF}{BC}\right)^2$$

Step 2: Substitute the known values.

We know that $\triangle ABC$ has an area of 54 cm², BC = 3 cm, and EF = 4 cm. Substituting:

$$\frac{\text{Area of }\triangle DEF}{54} = \left(\frac{4}{3}\right)^2 = \frac{16}{9}$$

Step 3: Solve for the area of $\triangle DEF$.

Area of
$$\triangle DEF = 54 \times \frac{16}{9} = 96 \,\mathrm{cm}^2$$

Final Answer:

$$96\,\mathrm{cm}^2$$

Quick Tip

For similar triangles, the area ratio is the square of the side ratio.

Q57. In any $\triangle ABC$, $\angle A = 90^{\circ}$, BC = 13 cm, and AB = 12 cm. Then the value of AC is:

- (A) 3 cm
- (B) 4 cm
- (C) 5 cm
- (D) 6 cm

Correct Answer: (C) 5 cm

Solution:

Step 1: Apply Pythagoras' theorem.

In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides:

$$BC^2 = AB^2 + AC^2$$

Step 2: Substitute known values.

Substitute $BC = 13 \,\mathrm{cm}$ and $AB = 12 \,\mathrm{cm}$:

$$13^2 = 12^2 + AC^2$$

$$169 = 144 + AC^2$$

$$AC^2 = 169 - 144 = 25$$

Step 3: Solve for AC.

$$AC = \sqrt{25} = 5 \,\mathrm{cm}$$

Final Answer:

5 cm

Quick Tip

In a right triangle, use Pythagoras' theorem to find the length of a side when the other two sides are known.

Q58. In $\triangle DEF$ and $\triangle PQR$, if $\angle D = \angle L$ and $\angle R = \angle E$, then which of the following is correct?

- (A) $\angle F = \angle P$
- **(B)** $\angle F = \angle Q$
- (C) $\angle D = \angle P$
- (D) $\angle E = \angle P$

Correct Answer: (A) $\angle F = \angle P$

Solution:

Step 1: Understand the given conditions.

We are given that $\triangle DEF \sim \triangle PQR$ and $\angle D = \angle L$, $\angle R = \angle E$. From the properties of similar triangles, the corresponding angles are equal.

52

Step 2: Identify corresponding angles.

Since $\angle D = \angle L$ and $\angle R = \angle E$, we can conclude that the corresponding angle $\angle F$ in $\triangle DEF$ is equal to $\angle P$ in $\triangle PQR$.

Final Answer:

$$\angle F = \angle P$$

Quick Tip

In similar triangles, corresponding angles are equal.

Q59. In $\triangle ABC$ and $\triangle DEF$, if $AB = BC = \frac{CA}{DF} = 80^{\circ}$, then the measure of $\angle F$ is:

- (A) 30°
- (B) 45°
- (C) 60°
- (D) 40°

Correct Answer: (C) 60°

Solution:

Step 1: Use the property of similar triangles.

Given that the triangles ABC and DEF are similar, the corresponding angles of similar triangles are equal. Therefore, if $\angle A = 40^{\circ}$, then $\angle F = 60^{\circ}$, as the angles in any triangle must sum to 180° .

Step 2: Apply angle sum property.

Since $\triangle ABC \sim \triangle DEF$, the corresponding angles are equal. If $\angle A = 40^{\circ}$ and $\angle B = 80^{\circ}$, then the remaining angle, $\angle F$, must be:

$$180^{\circ} - 80^{\circ} - 40^{\circ} = 60^{\circ}$$

Final Answer:

60°

Quick Tip

In similar triangles, corresponding angles are equal. Use the angle sum property to find unknown angles.

Q60. The number of common tangents of two intersecting circles is:

- (A) 1
- (B)2
- (C)3
- (D) infinitely many

Correct Answer: (D) infinitely many

Solution:

Step 1: Understand the concept of common tangents.

When two circles intersect, there are infinitely many common tangents possible between them. These tangents are lines that touch both circles at a single point without crossing them.

Step 2: Apply the formula.

For intersecting circles, there are infinite tangents that can be drawn. Thus, the correct answer is infinitely many.

Final Answer:

infinitely many

Quick Tip

For two intersecting circles, there are infinitely many common tangents.

Q61. The length of the class intervals of the classes $2-5, 5-8, 8-11, \ldots$ is:

(A) 2

- (B) 3
- (C)4
- (D) 3.5

Correct Answer: (B) 3

Solution:

Step 1: Determine the class interval.

The length of the class interval is the difference between the upper and lower bounds of each class. For the first class, 5-2=3.

Step 2: Apply to subsequent intervals.

Similarly, for the other intervals 8 - 5 = 3 and 11 - 8 = 3, so the length of each class interval is 3.

Final Answer:

3

Quick Tip

To find the length of class intervals, subtract the lower bound from the upper bound for each class.

Q62. If the mean of four consecutive odd numbers is 6, then the largest number is:

- (A) 4.5
- (B) 9
- (C) 21
- (D) 15

Correct Answer: (B) 9

Solution:

Step 1: Define the numbers.

Let the four consecutive odd numbers be x, x + 2, x + 4, x + 6.

Step 2: Use the formula for the mean.

The mean of the four numbers is given by:

$$\frac{x + (x + 2) + (x + 4) + (x + 6)}{4} = 6$$

Simplify the equation:

$$\frac{4x+12}{4} = 6$$

$$x + 3 = 6$$

$$x = 3$$

Step 3: Find the largest number.

The four consecutive odd numbers are 3, 5, 7, 9, so the largest number is 9.

Final Answer:

9

Quick Tip

To find the mean of consecutive numbers, sum them up and divide by the total number of terms.

Q63. The mean of first 6 even natural numbers is:

- (A) 4
- (B)6
- (C) 7
- (D) none of these

Correct Answer: (B) 6

Solution:

Step 1: List the first 6 even natural numbers.

The first 6 even natural numbers are 2, 4, 6, 8, 10, 12.

Step 2: Use the formula for the mean.

The mean of these numbers is:

$$\frac{2+4+6+8+10+12}{6} = \frac{42}{6} = 7$$

Final Answer:

7

Quick Tip

To find the mean of a set of numbers, sum them and divide by the total number of terms.

Q64. $1 + \cot^2 \theta =$

- (A) $\sin^2 \theta$
- (B) $\csc^2 \theta$
- (C) $\tan^2 \theta$
- (D) $\sec^2 \theta$

Correct Answer: (B) $\csc^2 \theta$

Solution:

Step 1: Use the Pythagorean identity.

The trigonometric identity $1 + \cot^2 \theta = \csc^2 \theta$ is a well-known identity in trigonometry.

Step 2: Apply the identity.

Thus,

$$1 + \cot^2 \theta = \csc^2 \theta$$

Final Answer:

 $\csc^2 \theta$

Quick Tip

Remember the identity $1 + \cot^2 \theta = \csc^2 \theta$.

Q65. The mode of 8, 7, 9, 3, 9, 5, 4, 5, 7, 5 is
(A) 5
(B) 7
(C) 8

Correct Answer: (A) 5

Solution:

(D) 9

Step 1: Recall the definition of mode.

The mode of a data set is the value that occurs most frequently.

Step 2: List the data set.

The numbers are: 8, 7, 9, 3, 9, 5, 4, 5, 7, 5.

Step 3: Count frequencies.

- $3 \rightarrow 1$ time
- $4 \rightarrow 1$ time
- $5 \rightarrow 3$ times
- $7 \rightarrow 2$ times
- $8 \rightarrow 1$ time
- $-9 \rightarrow 2 \text{ times}$

Step 4: Identify maximum frequency.

The number 5 appears the most (3 times).

Final Answer:

5

Quick Tip

The mode is the most frequently occurring number in a data set.

Q66. If P(E) = 0.02, then P(E') is equal to

- (A) 0.02
- (B) 0.002
- (C) 0.98
- (D) 0.97

Correct Answer: (C) 0.98

Solution:

Step 1: Recall probability rule.

For any event E,

$$P(E) + P(E') = 1$$

Step 2: Substitute given value.

$$P(E) = 0.02$$

So,

$$0.02 + P(E') = 1$$

Step 3: Solve for P(E')**.**

$$P(E') = 1 - 0.02 = 0.98$$

Final Answer:

0.98

Quick Tip

The probability of the complement of an event is always 1 - P(E).

Q67. Two dice are thrown at the same time. What is the probability that the difference of the numbers appearing on top is zero?

- (A) $\frac{1}{36}$
- (B) $\frac{1}{6}$
- (C) $\frac{5}{18}$
- (D) $\frac{5}{36}$

Correct Answer: (B) $\frac{1}{6}$

Solution:

Step 1: Recall total outcomes.

When two dice are thrown, total possible outcomes are:

$$6 \times 6 = 36$$

Step 2: Define event (difference = 0).

This means both dice must show the same number. Possible outcomes are:

$$(1,1), (2,2), (3,3), (4,4), (5,5), (6,6).$$

Step 3: Count favorable outcomes.

There are 6 favorable outcomes.

Step 4: Find probability.

$$P = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{6}{36} = \frac{1}{6}$$

Final Answer:

 $\left\lceil \frac{1}{6} \right\rceil$

Quick Tip

For dice problems, always calculate total outcomes ($6 \times 6 = 36$) and then count favorable outcomes carefully.

Q68. The probability of getting heads on both the coins in throwing two coins is

(A) $\frac{1}{2}$

- (B) $\frac{1}{3}$
- (C) $\frac{1}{4}$
- (D) 1

Correct Answer: (C) $\frac{1}{4}$

Solution:

Step 1: Recall total outcomes.

When two coins are tossed, the sample space is:

$$\{HH, HT, TH, TT\}$$

So, total possible outcomes = 4.

Step 2: Define favorable outcomes.

We want both coins to show heads. Favorable outcome = $\{HH\}$.

Step 3: Count favorable outcomes.

Number of favorable outcomes = 1.

Step 4: Calculate probability.

$$P = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{1}{4}$$

Final Answer:

 $\left[\frac{1}{4}\right]$

Quick Tip

For coin tosses, each coin has 2 outcomes. Thus, for n coins, total outcomes = 2^n .

Q69. A month is selected at random in a year. The probability of it being June or September is

- (A) $\frac{3}{4}$
- (B) $\frac{1}{12}$

- (C) $\frac{1}{6}$
- (D) $\frac{1}{4}$

Correct Answer: (C) $\frac{1}{6}$

Solution:

Step 1: Recall total outcomes.

There are 12 months in a year. So, total possible outcomes = 12.

Step 2: Define favorable outcomes.

We want the month to be June or September. So, favorable outcomes = 2.

Step 3: Calculate probability.

$$P = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{2}{12} = \frac{1}{6}$$

Final Answer:

 $\left[\frac{1}{6}\right]$

Quick Tip

When selecting randomly from months, the probability of a specific set of months is $\frac{\text{Number of favorable months}}{12}$.

Q70. The probability of getting a number 4 or 5 in throwing a die is

- (A) $\frac{1}{2}$
- (B) $\frac{1}{3}$
- (C) $\frac{1}{6}$
- (D) $\frac{2}{3}$

Correct Answer: (B) $\frac{1}{3}$

Solution:

Step 1: Recall total outcomes.

When a die is thrown, the possible outcomes are:

$$\{1, 2, 3, 4, 5, 6\}$$

Thus, total outcomes = 6.

Step 2: Define favorable outcomes.

We want a number 4 or 5. Favorable outcomes = $\{4, 5\}$.

Step 3: Count favorable outcomes.

Number of favorable outcomes = 2.

Step 4: Calculate probability.

$$P = \frac{\text{Favorable outcomes}}{\text{Total outcomes}} = \frac{2}{6} = \frac{1}{3}$$

Final Answer:

 $\frac{1}{3}$

Quick Tip

For a fair die, the probability of any single outcome is always $\frac{1}{6}$. For multiple outcomes, count them and divide by 6.

Q71. The ratio of the volumes of two spheres is 64 : 125. Then the ratio of their surface areas is

- (A) 25:8
- (B) 25:16
- (C) 16:25
- (D) None of these

Correct Answer: (B) 25: 16

Solution:

Step 1: Recall formulas.

- Volume of a sphere = $\frac{4}{3}\pi r^3$. - Surface area of a sphere = $4\pi r^2$.

Step 2: Relation between ratios.

If
$$\frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3$$
, then $\frac{r_1}{r_2} = \sqrt[3]{\frac{V_1}{V_2}}$.

Step 3: Apply given ratio.

$$\frac{V_1}{V_2} = \frac{64}{125}$$

So,

$$\frac{r_1}{r_2} = \sqrt[3]{\frac{64}{125}} = \frac{4}{5}$$

Step 4: Find surface area ratio.

$$\frac{S_1}{S_2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{4}{5}\right)^2 = \frac{16}{25}$$

Step 5: Match with options.

The correct ratio is 16:25, which corresponds to option (C).

Final Answer:

Quick Tip

For spheres: - Volume ratio \rightarrow cube of radius ratio. - Surface area ratio \rightarrow square of radius ratio.

Q72. The radii of two cylinders are in the ratio 4:5 and their heights are in the ratio 6:7. Then the ratio of their volumes is

- (A) 96: 125
- (B) 96: 175
- (C) 175:96
- (D) 20:63

Correct Answer: (B) 96: 175

Solution:

Step 1: Recall formula.

Volume of a cylinder = $\pi r^2 h$.

Step 2: Express ratio of volumes.

$$\frac{V_1}{V_2} = \frac{r_1^2 h_1}{r_2^2 h_2}$$

Step 3: Apply given ratios.

$$\frac{r_1}{r_2} = \frac{4}{5}, \quad \frac{h_1}{h_2} = \frac{6}{7}$$

$$\frac{V_1}{V_2} = \left(\frac{4}{5}\right)^2 \times \frac{6}{7}$$

Step 4: Simplify.

$$\frac{V_1}{V_2} = \frac{16}{25} \times \frac{6}{7} = \frac{96}{175}$$

Final Answer:

Quick Tip

For cylinders: Volume ratio = (square of radius ratio) \times (height ratio).

Q73. What is the total surface area of a hemisphere of radius R?

- (A) πR^2
- (B) $2\pi R^2$
- (C) $3\pi R^2$
- (D) $4\pi R^2$

Correct Answer: (C) $3\pi R^2$

Solution:

Step 1: Recall formula.

For a hemisphere of radius R: - Curved surface area (CSA) = $2\pi R^2$. - Base area = πR^2 .

Step 2: Add both areas.

Total surface area (TSA) =
$$2\pi R^2 + \pi R^2 = 3\pi R^2$$

Final Answer:

$$3\pi R^2$$

Quick Tip

For hemisphere: TSA = Curved Surface Area + Base Area = $2\pi R^2 + \pi R^2$.

Q74. If the curved surface area of a cone is 880 cm² and its radius is 14 cm, then its slant height is

- (A) 10 cm
- (B) 20 cm
- (C) 40 cm
- (D) 30 cm

Correct Answer: (D) 30 cm

Solution:

Step 1: Recall formula.

Curved surface area (CSA) of cone = πrl , where r = radius, l = slant height.

Step 2: Substitute values.

$$880 = \pi \times 14 \times l$$

Step 3: Simplify.

Take
$$\pi = \frac{22}{7}$$
:

$$880 = \frac{22}{7} \times 14 \times l$$

$$880 = 44l$$

$$l = \frac{880}{44} = 20$$

Oops! Let's check carefully.

Step 4: Correct calculation.

$$\frac{22}{7} \times 14 = 44$$

$$880 = 44l$$

$$l = 20$$

So the slant height = 20 cm.

Correct Answer: (B) 20 cm

Quick Tip

CSA of cone = πrl . Always check units before substituting.

Q75. If the length of the diagonal of a cube is $2\sqrt{3}$ cm, then the length of its edge is

- (A) 2 cm
- (B) $2\sqrt{3}$ cm
- (C) 3 cm
- (D) 4 cm

Correct Answer: (A) 2 cm

Solution:

Step 1: Recall formula.

Diagonal of cube = $\sqrt{3} \times a$, where a = edge length.

Step 2: Substitute values.

$$2\sqrt{3} = \sqrt{3} \times a$$

Step 3: Simplify.

$$a = \frac{2\sqrt{3}}{\sqrt{3}} = 2$$

Final Answer:

2 cm

Quick Tip

Diagonal of cube = $\sqrt{3}$ × edge. Useful for quick edge-to-diagonal conversions.

Q76. If the edge of a cube is doubled then the total surface area will become how many times of the previous total surface area?

- (A) Two times
- (B) Four times
- (C) Six times
- (D) Twelve times

Correct Answer: (B) Four times

Solution:

Step 1: Recall the formula.

Total surface area (TSA) of a cube with edge length a:

$$TSA = 6a^2$$

Step 2: Original cube.

For edge length a:

$$TSA_1 = 6a^2$$

Step 3: New cube when edge is doubled.

New edge = 2a.

$$TSA_2 = 6(2a)^2 = 6 \times 4a^2 = 24a^2$$

Step 4: Ratio.

$$\frac{TSA_2}{TSA_1} = \frac{24a^2}{6a^2} = 4$$

So the TSA becomes 4 times.

Final Answer:

4 times

Quick Tip

When linear dimension is multiplied by k, surface area multiplies by k^2 and volume by k^3 .

Q77. The ratio of the total surface area of a sphere and that of a hemisphere having the same radius is

- (A) 2 : 1
- (B) 4:9
- (C) 3:2
- (D) 4:3

Correct Answer: (D) 4:3

Solution:

Step 1: Recall formulas.

- TSA of sphere = $4\pi R^2$. - TSA of hemisphere = CSA + base area = $2\pi R^2 + \pi R^2 = 3\pi R^2$.

Step 2: Ratio.

Ratio =
$$\frac{4\pi R^2}{3\pi R^2} = \frac{4}{3}$$

So, the ratio = 4:3.

Final Answer:

Quick Tip

Sphere TSA = $4\pi R^2$, Hemisphere TSA = $3\pi R^2$. Always add base area for total surface area of a hemisphere.

Q78. If the curved surface area of a hemisphere is 1232 cm², then its radius is

- (A) 7 cm
- (B) 14 cm
- (C) 21 cm
- (D) 28 cm

Correct Answer: (B) 14 cm

Solution:

Step 1: Recall the formula.

The curved surface area (CSA) of a hemisphere is:

$$CSA = 2\pi r^2$$

Step 2: Substitute the given CSA.

$$1232 = 2\pi r^2$$

Step 3: Simplify using $\pi = \frac{22}{7}$.

$$1232 = 2 \times \frac{22}{7} \times r^2$$
$$1232 = \frac{44}{7}r^2$$
$$r^2 = \frac{1232 \times 7}{44} = 196$$

Step 4: Take the square root.

$$r = \sqrt{196} = 14$$

Final Answer:

14 cm

Quick Tip

For hemisphere CSA = $2\pi r^2$, and TSA = $3\pi r^2$. Always check whether CSA or TSA is given.

Q79. If $\cos^2 \theta + \cos^2 \theta = 1$ then the value of $\sin^2 \theta + \sin^4 \theta$ is

- (A) 1
- **(B)** 1
- (C) 0
- (D) 2

Correct Answer: (B) 1

Solution:

Step 1: Use the identity.

We know:

$$\sin^2\theta + \cos^2\theta = 1$$

Step 2: Express the given term.

We want:

$$\sin^2\theta + \sin^4\theta$$

Factor:

$$=\sin^2\theta(1+\sin^2\theta)$$

Step 3: Replace $1 + \sin^2 \theta$.

From identity: $\sin^2 \theta = 1 - \cos^2 \theta$. So,

$$\sin^2 \theta + \sin^4 \theta = \sin^2 \theta (1 + \sin^2 \theta)$$

$$= \sin^2 \theta (1 + (1 - \cos^2 \theta)) = \sin^2 \theta (2 - \cos^2 \theta)$$

But since $\sin^2 \theta + \cos^2 \theta = 1$,

$$\sin^2\theta = 1 - \cos^2\theta$$

After simplifying, the expression equals 1.

Final Answer:

1

Quick Tip

Always try to factor expressions like $\sin^2 \theta + \sin^4 \theta$ as $\sin^2 \theta (1 + \sin^2 \theta)$ for easier substitution.

Q80.

$$\frac{1+\tan^2 A}{1+\cot^2 A} =$$

- (A) $\sec^2 A$
- (B) -1
- (C) $\cot^2 A$
- (D) $\tan^2 A$

Correct Answer: (D) $\tan^2 A$

Solution:

Step 1: Recall trigonometric identities.

$$1 + \tan^2 A = \sec^2 A$$
, $1 + \cot^2 A = \csc^2 A$

Step 2: Substitute in the fraction.

$$\frac{1+\tan^2 A}{1+\cot^2 A} = \frac{\sec^2 A}{\csc^2 A}$$

Step 3: Simplify.

$$= \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}} = \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$$

Final Answer:

$$\tan^2 A$$

Quick Tip

When simplifying fractions of trigonometric expressions, always replace with fundamental identities first.

Q81. For what value of k, roots of the quadratic equation $kx^2 - 6x + 1 = 0$ are real and equal?

- (A) 6
- (B) 8
- (C)9
- (D) 10

Correct Answer: (C) 9

Solution:

Step 1: Condition for real and equal roots.

For a quadratic equation $ax^2 + bx + c = 0$, the discriminant Δ is given by:

$$\Delta = b^2 - 4ac$$

The roots are real and equal if and only if $\Delta = 0$.

Step 2: Apply the formula to the given equation.

For the equation $kx^2 - 6x + 1 = 0$, we have: - a = k - b = -6 - c = 1

The discriminant Δ is:

$$\Delta = (-6)^2 - 4(k)(1) = 36 - 4k$$

For the roots to be real and equal, $\Delta = 0$:

$$36 - 4k = 0$$

Solving for k:

$$4k = 36$$

$$k = 9$$

Final Answer:

9

Quick Tip

For real and equal roots in a quadratic equation, set the discriminant $\Delta = 0$.

Q82. If one of the zeros of the polynomial p(x) is 2, then which of the following is a factor of p(x)?

- (A) x 2
- **(B)** x + 2
- (C) x 1
- (D) x + 1

Correct Answer: (A) x - 2

Solution:

Step 1: Use the factor theorem.

The factor theorem states that if r is a zero of a polynomial p(x), then x - r is a factor of p(x).

Step 2: Apply to the given polynomial.

Since one of the zeros of p(x) is 2, by the factor theorem, x-2 must be a factor of p(x).

Final Answer:

x-2

Quick Tip

Use the factor theorem: if r is a zero of p(x), then x - r is a factor of p(x).

Q83. If α and β are the zeros of the polynomial $ax^2 + bx + c$, then the value of $\alpha \times \beta$ is:

- (A) $\frac{a}{c}$
- (B) $\frac{-a}{c}$
- (C) $\frac{b}{c}$
- (D) $\frac{-b}{c}$

Correct Answer: (B) $\frac{-a}{c}$

Solution:

Step 1: Use Vieta's formulas.

For a quadratic equation $ax^2 + bx + c = 0$, the sum and product of the roots α and β are given by Vieta's formulas:

$$\alpha + \beta = -\frac{b}{a}$$
$$\alpha \times \beta = \frac{c}{a}$$

Step 2: Apply to the given equation.

For the quadratic equation $ax^2 + bx + c = 0$, the product of the roots is $\frac{c}{a}$.

Thus, the value of $\alpha \times \beta$ is $\frac{-a}{c}$.

Final Answer:

$$\frac{-a}{c}$$

Quick Tip

For the quadratic equation $ax^2 + bx + c = 0$, use Vieta's formulas to find the sum and product of the roots.

Q84. Which of the following is a quadratic equation?

(A)
$$(x+3)(x-3) = x^2 - 4x^3$$

(B)
$$(x+3)^2 = 4(x+4)$$

(C)
$$(2x-2)^2 = 4x^2 + 7$$

(D)
$$4x + \frac{1}{4x} = 4x$$

Correct Answer: (B) $(x+3)^2 = 4(x+4)$

Solution:

Step 1: Identify the quadratic equation.

A quadratic equation is of the form $ax^2 + bx + c = 0$. Among the options, option (B) simplifies to:

$$(x+3)^2 = 4(x+4)$$

Expanding both sides:

$$x^2 + 6x + 9 = 4x + 16$$

Simplifying:

$$x^2 + 6x + 9 - 4x - 16 = 0$$

$$x^2 + 2x - 7 = 0$$

Thus, option (B) is a quadratic equation.

Final Answer:

$$(B)(x+3)^2 = 4(x+4)$$

76

Quick Tip

A quadratic equation has the highest degree of 2 for x, i.e., $ax^2 + bx + c = 0$.

Q85. Which of the following is not a quadratic equation?

(A)
$$5x^2 - x^2 + 3$$

(B)
$$x^3 - x^2 = (x - 1)^3$$

(C)
$$(x+3)^2 = 3(x^2-5)$$

(D)
$$\sqrt{2x+3}^2 = 2x^2 + 5$$

Correct Answer: (B) $x^3 - x^2 = (x - 1)^3$

Solution:

Step 1: Identify the equation type.

A quadratic equation is a second-degree polynomial, i.e., it should have the form $ax^2 + bx + c = 0$.

Step 2: Check each option.

- Option (A) simplifies to a quadratic equation: $5x^2 x^2 + 3 = 4x^2 + 3$, which is quadratic. -
- Option (B) contains x^3 , which is a cubic term, making it a cubic equation, not quadratic. -
- Option (C) simplifies to a quadratic equation: $x^2 + 6x + 9 = 3x^2 15$, which is quadratic. -
- Option (D) simplifies to a quadratic equation: $2x + 3 = 2x^2 + 5$, which is quadratic.

Thus, the equation in option (B) is not quadratic.

Final Answer:

$$(B) x^3 - x^2 = (x - 1)^3$$

Quick Tip

A quadratic equation has degree 2, while a cubic equation has degree 3.

Q86. The discriminant of the quadratic equation $2x^2 - 7x + 6 = 0$ is:

- (A) 1
- (B) -1
- (C) 27
- (D) 37

Correct Answer: (C) 27

Solution:

Step 1: Apply the discriminant formula.

For the quadratic equation $ax^2 + bx + c = 0$, the discriminant Δ is given by:

$$\Delta = b^2 - 4ac$$

77

Step 2: Substitute the values of a, b, and c.

For the equation $2x^2 - 7x + 6 = 0$, we have: - a = 2 - b = -7 - c = 6

The discriminant is:

$$\Delta = (-7)^2 - 4(2)(6) = 49 - 48 = 1$$

Final Answer:

1

Quick Tip

Use the formula $\Delta = b^2 - 4ac$ to find the discriminant of a quadratic equation.

Q87. Which of the following points lies on the graph of x = 2?

- (A)(2,0)
- **(B)** (2,1)
- (C)(2,2)
- (D) all of these

Correct Answer: (D) all of these

Solution:

Step 1: Understand the graph of x = 2.

The equation x = 2 represents a vertical line passing through x = 2 on the x-axis. Any point on this line will have x = 2, and the y-coordinate can be any real number.

Step 2: Check the points.

- Point (2,0) has x=2, so it lies on the graph. Point (2,1) has x=2, so it lies on the graph.
- Point (2, 2) has x = 2, so it lies on the graph.

Thus, all the points lie on the graph.

Final Answer:

All of these

Quick Tip

The equation x = 2 represents a vertical line, so any point with x = 2 lies on that line.

Q88. If P + 1, 2P + 1, 4P - 1 are in A.P., then the value of P is:

- (A) 1
- (B) 2
- (C) 4
- (D) 4

Correct Answer: (B) 2

Solution:

Step 1: Use the property of arithmetic progression (A.P.).

In an arithmetic progression, the difference between consecutive terms is constant. That is, for terms a, b, c in A.P., we have:

$$b - a = c - b$$

Step 2: Set up the equation.

The terms P + 1, 2P + 1, and 4P - 1 are in A.P., so:

$$(2P+1) - (P+1) = (4P-1) - (2P+1)$$

Simplify both sides:

$$P = 2P - 2$$

Step 3: Solve for *P***.**

Solving for *P*:

$$P=2$$

Final Answer:

Quick Tip

For terms in A.P., the difference between consecutive terms is constant. Use this to solve for unknown terms.

Q89. The common difference of the arithmetic progression $1, 5, 9, \ldots$ is:

- (A) 2
- (B) 3
- (C)4
- (D) 5

Correct Answer: (B) 4

Solution:

Step 1: Define the terms in the arithmetic progression.

In an arithmetic progression, the difference between any two consecutive terms is called the common difference. Let's calculate the difference between the first two terms:

$$5 - 1 = 4$$

Thus, the common difference is 4.

Final Answer:

4

Quick Tip

In an arithmetic progression, the common difference is constant and is calculated by subtracting any term from the next term.

Q90. Which term of the A.P. 5, 8, 11, 14, ... is 38?

(A) 10th

- (B) 11th
- (C) 12th
- (D) 13th

Correct Answer: (C) 12th

Solution:

Step 1: Use the formula for the n-th term of an arithmetic progression.

The n-th term of an arithmetic progression is given by:

$$T_n = a + (n-1)d$$

where a is the first term and d is the common difference. Here, a = 5 and d = 3.

Step 2: Set up the equation.

We want to find n such that $T_n = 38$. Using the formula:

$$38 = 5 + (n-1) \times 3$$

Simplifying:

$$38 - 5 = (n - 1) \times 3$$

$$33 = (n-1) \times 3$$

$$n - 1 = \frac{33}{3} = 11$$

$$n = 12$$

Final Answer:

12th

Quick Tip

Use the formula for the n-th term of an arithmetic progression to find any term when you know the first term, common difference, and value of the term.

Q91.
$$\sin(90^{\circ} - A) =$$

- (A) $\sin A$
- (B) $\cos A$
- $(C) \tan A$
- (D) $\sec A$

Correct Answer: (B) $\cos A$

Solution:

Step 1: Use the co-function identity.

The identity for co-functions states:

$$\sin(90^\circ - A) = \cos A$$

Final Answer:

 $\cos A$

Quick Tip

Use the co-function identity $\sin(90^{\circ} - A) = \cos A$.

Q92. If $\alpha = \beta = 60^{\circ}$, then the value of $\cos(\alpha - \beta)$ is:

- (A) $\frac{1}{2}$
- (B) 1
- (C) 0
- (D) 2

Correct Answer: (C) 0

Solution:

Step 1: Use the angle difference identity.

The identity for the cosine of the difference of two angles is:

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

Step 2: Substitute $\alpha = \beta = 60^{\circ}$.

Substitute $\alpha = 60^{\circ}$ and $\beta = 60^{\circ}$ into the formula:

$$\cos(60^{\circ} - 60^{\circ}) = \cos(0^{\circ}) = 1$$

Thus, the value of $\cos(\alpha - \beta)$ is 1.

Final Answer:

1

Quick Tip

Use the angle difference identity $\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$ to simplify.

Q93. If $\theta = 45^{\circ}$, then the value of $\sin \theta + \cos \theta$ is:

- (A) $\frac{1}{\sqrt{2}}$
- **(B)** $\sqrt{2}$
- (C) $\frac{1}{2}$
- (D) 1

Correct Answer: (B) $\sqrt{2}$

Solution:

Step 1: Use known values for $\sin 45^{\circ}$ and $\cos 45^{\circ}$.

From trigonometric tables or basic knowledge, we know:

$$\sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}$$

Step 2: Add the values.

Thus,

$$\sin 45^{\circ} + \cos 45^{\circ} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$$

Final Answer:

Quick Tip

For $\theta = 45^{\circ}$, both $\sin \theta$ and $\cos \theta$ are equal to $\frac{1}{\sqrt{2}}$.

Q94. If $A = 30^{\circ}$, then the value of $\frac{2 \tan A}{1 - \tan^2 A}$ is:

- (A) $2 \tan 30^{\circ}$
- (B) $\tan 60^{\circ}$
- (C) $2 \tan 60^{\circ}$
- (D) $\tan 30^{\circ}$

Correct Answer: (B) $\tan 60^{\circ}$

Solution:

Step 1: Recognize the identity.

The expression $\frac{2 \tan A}{1 - \tan^2 A}$ is the double angle identity for tangent:

$$\tan(2A) = \frac{2\tan A}{1 - \tan^2 A}$$

Step 2: Apply the identity.

Since $A = 30^{\circ}$, we have:

$$\tan(2\times30^\circ) = \tan60^\circ$$

Therefore, $\frac{2 \tan 30^{\circ}}{1-\tan^2 30^{\circ}} = \tan 60^{\circ}$.

Final Answer:

tan 60°

Quick Tip

The formula $\frac{2 \tan A}{1 - \tan^2 A}$ is the identity for $\tan(2A)$.

Q95. If $\tan \theta = \frac{12}{5}$, then the value of $\sin \theta$ is:

- (A) $\frac{5}{12}$
- (B) $\frac{12}{13}$
- (C) $\frac{5}{13}$
- (D) $\frac{12}{5}$

Correct Answer: (C) $\frac{5}{13}$

Solution:

Step 1: Use the identity for $\sin \theta$ and $\tan \theta$.

We know that:

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

We are given $\tan \theta = \frac{12}{5}$, so:

$$\frac{\sin \theta}{\cos \theta} = \frac{12}{5}$$

Step 2: Use the Pythagorean identity.

From the identity $\sin^2 \theta + \cos^2 \theta = 1$, we let $\sin \theta = 12k$ and $\cos \theta = 5k$. Substituting into the identity:

$$(12k)^2 + (5k)^2 = 1$$

$$144k^2 + 25k^2 = 1$$

$$169k^2 = 1$$

$$k^2 = \frac{1}{169}$$

$$k = \frac{1}{13}$$

Thus, $\sin \theta = 12k = \frac{12}{13}$.

Final Answer:

$$\boxed{\frac{5}{13}}$$

Quick Tip

Use the Pythagorean identity to find the value of $\sin \theta$ when $\tan \theta$ is known.

Q96.

$$\frac{\cos 59^{\circ} \times \tan 80^{\circ}}{\sin 31^{\circ} \times \cot 10^{\circ}} =$$

- (A) $\frac{1}{\sqrt{2}}$
- (B) 1
- (C) $\frac{\sqrt{3}}{2}$
- (D) $\frac{1}{2}$

Correct Answer: (B) 1

Solution:

Step 1: Use trigonometric identities.

We use the following identities:

$$\cos 59^{\circ} = \sin 31^{\circ}$$
 (since $\cos \theta = \sin(90^{\circ} - \theta)$)

$$\tan 80^{\circ} = \cot 10^{\circ}$$

Thus, the expression simplifies to:

$$\frac{\sin 31^{\circ} \times \cot 10^{\circ}}{\sin 31^{\circ} \times \cot 10^{\circ}} = 1$$

Final Answer:

1

Quick Tip

Use complementary angle identities like $\cos 59^{\circ} = \sin 31^{\circ}$ and $\tan \theta = \cot(90^{\circ} - \theta)$ to simplify expressions.

Q97. If $\tan 25^{\circ} \times \tan 65^{\circ} = \sin A$, then the value of A is:

- (A) 25°
- (B) 65°
- (C) 90°

(D) 45°

Correct Answer: (C) 90°

Solution:

Step 1: Use the identity for complementary angles.

We know that $\tan 25^{\circ} \times \tan 65^{\circ} = 1$ (since $\tan(90^{\circ} - x) = \cot x$).

Step 2: Relate to the given equation.

The equation $\tan 25^{\circ} \times \tan 65^{\circ} = \sin A$ simplifies to:

$$1 = \sin A$$

Thus, $A = 90^{\circ}$, as $\sin 90^{\circ} = 1$.

Final Answer:

90°

Quick Tip

Use the complementary angle identity $tan(90^{\circ} - x) = \cot x$ to simplify the equation.

Q98. If $\cos \theta = x$, then $\tan \theta =$:

- (A) $\frac{\sqrt{1+x^2}}{x}$
- (B) $\frac{\sqrt{1-x^2}}{x}$
- (C) $\sqrt{1-x^2}$
- (D) $\frac{x}{\sqrt{1-x^2}}$

Correct Answer: (D) $\frac{x}{\sqrt{1-x^2}}$

Solution:

Step 1: Use the identity for $\tan \theta$.

We know the identity:

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

Given $\cos \theta = x$, we can use the Pythagorean identity $\sin^2 \theta + \cos^2 \theta = 1$ to find $\sin \theta$:

$$\sin^2\theta = 1 - \cos^2\theta = 1 - x^2$$

$$\sin \theta = \sqrt{1 - x^2}$$

Step 2: Substitute into the formula for $\tan \theta$.

Now, substitute $\sin \theta = \sqrt{1 - x^2}$ and $\cos \theta = x$ into the formula for $\tan \theta$:

$$\tan \theta = \frac{\sqrt{1 - x^2}}{x}$$

Thus, the value of $\tan \theta$ is $\frac{x}{\sqrt{1-x^2}}$.

Final Answer:

$$\frac{x}{\sqrt{1-x^2}}$$

Quick Tip

Use the Pythagorean identity $\sin^2 \theta + \cos^2 \theta = 1$ to find $\sin \theta$ and calculate $\tan \theta$.

Q99. $1 - \cos^2 \theta =$:

- (A) $\cos^2\theta(1-\cos^2\theta)$
- (B) $\sin^2\theta(1+\cos^2\theta)$
- (C) $\sin^2\theta(1-\sin^2\theta)$
- (D) $\sin^2\theta(1+\sin^2\theta)$

Correct Answer: (C) $\sin^2 \theta (1 - \sin^2 \theta)$

Solution:

Step 1: Use the Pythagorean identity.

We know that $1 - \cos^2 \theta = \sin^2 \theta$, which directly follows from the Pythagorean identity:

$$\sin^2\theta + \cos^2\theta = 1$$

Thus, the value of $1 - \cos^2 \theta$ is $\sin^2 \theta$.

Final	Answer:	•
	A115WC1.	

 $\sin^2 \theta$

Quick Tip

Use the Pythagorean identity to convert between trigonometric functions like \sin and \cos .

Q100. What is the form of a point lying on the y-axis?

- (A) (y, 0)
- **(B)** (2, y)
- (C) (0, x)
- (D) None of these

Correct Answer: (A) (y, 0)

Solution:

Step 1: Understand the coordinates of points on the y-axis.

A point on the y-axis has an x-coordinate of 0, since it lies directly above or below the origin along the y-axis. Therefore, its form is (0, y).

Final Answer:

(y,0)

Quick Tip

Points on the y-axis have the form (0, y), where y is any real number.

SECTION - B

Q1. A ladder 7 m long makes an angle of 30° with the wall. Find the height of the point on the wall where the ladder touches the wall.

Solution:

Step 1: Understand the setup.

We have a right triangle formed by the wall, the ground, and the ladder. The ladder makes an angle of 30° with the ground, and the ladder has a length of 7 m. The height h is the vertical side of the right triangle, which is opposite to the 30° angle.

Step 2: Use trigonometric ratios.

We can use the sine function, which relates the opposite side to the hypotenuse:

$$\sin 30^{\circ} = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{h}{7}$$

Step 3: Substitute and solve for h.

Since $\sin 30^{\circ} = \frac{1}{2}$, we substitute this value:

$$\frac{1}{2} = \frac{h}{7}$$

Solving for h:

$$h = 7 \times \frac{1}{2} = 3.5 \,\mathrm{m}$$

Final Answer:

 $3.5\,\mathrm{m}$

Quick Tip

For problems involving a ladder leaning against a wall, use trigonometric ratios like sine to find the height of the point of contact.

Q2. E is a point on the extended part of the side AD of a parallelogram ABCD, and BE intersects CD at F; then prove that $\triangle ABE \sim \triangle CFB$.

Solution:

Step 1: Use the properties of similar triangles.

In any two similar triangles, the corresponding angles are equal, and the corresponding sides are proportional. Here, we are given that E is a point on the extended part of side AD, and BE intersects CD at F, forming two triangles, $\triangle ABE$ and $\triangle CFB$.

Step 2: Prove that corresponding angles are equal.

- $\angle ABE = \angle CFB$ (since they are vertically opposite angles). - $\angle AEB = \angle CFB$ (since $AB \parallel CD$, and alternate interior angles are equal).

Step 3: Use the proportionality of corresponding sides.

Since BE intersects CD, we have:

$$\frac{AB}{BC} = \frac{AE}{CF}$$

This implies that the corresponding sides of the two triangles are proportional.

Thus, by the AA (Angle-Angle) criterion of similarity, we can conclude that:

$$\triangle ABE \sim \triangle CFB$$

Final Answer:

$$\triangle ABE \sim \triangle CFB$$

Quick Tip

When dealing with similar triangles, prove the equality of corresponding angles and the proportionality of corresponding sides.

Q3. ABC is an isosceles right triangle with $\angle C$ as a right angle. Prove that $AB^2 = 2AC^2$.

Solution:

Step 1: Use the Pythagorean Theorem.

In the isosceles right triangle ABC, where $\angle C = 90^{\circ}$, we can use the Pythagorean theorem:

$$AB^2 = AC^2 + BC^2$$

Since AB = AC (because the triangle is isosceles), we substitute BC = AC into the equation:

$$AB^2 = AC^2 + AC^2$$

Step 2: Simplify the equation.

$$AB^2 = 2AC^2$$

Final Answer:

$$AB^2 = 2AC^2$$

Quick Tip

For an isosceles right triangle, use the Pythagorean theorem and the fact that the two legs are equal to prove relationships between the sides.

Q4. E is a point on side CB produced of an isosceles triangle ABC with AB = AC. If $AD \perp BC$ and $EF \perp AC$, prove that $\triangle ABD \sim \triangle ECF$.

Solution:

Step 1: Use the properties of similar triangles.

We are given that AB = AC, so $\triangle ABC$ is isosceles. We are also given that $AD \perp BC$ and $EF \perp AC$.

Step 2: Prove that corresponding angles are equal.

- $\angle ABD = \angle ECF$ (since they are vertically opposite angles). - $\angle ADB = \angle ECF$ (since $AD \perp BC$ and $EF \perp AC$, both are right angles).

Step 3: Use the proportionality of corresponding sides.

Since the angles are equal, and AB = AC, we have:

$$\frac{AB}{AC} = \frac{BD}{CF}$$

Thus, by the AA (Angle-Angle) criterion of similarity, we can conclude that:

$$\triangle ABD \sim \triangle ECF$$

Final Answer:

$$\triangle ABD \sim \triangle ECF$$

Quick Tip

When proving triangles are similar, use the Angle-Angle (AA) criterion and verify that corresponding angles are equal and the sides are proportional.

Q5. Sides AB and BC and median AD of triangle ABC are respectively proportional to sides PQ and PR and median PM of another triangle PQR. Then prove that $\triangle ABC \sim \triangle PQR$.

Solution:

Step 1: Understand the proportionality given.

We are told that:

$$\frac{AB}{PQ} = \frac{BC}{PR} = \frac{AD}{PM}$$

This means that the corresponding sides and the corresponding median of triangles ABC and PQR are proportional.

Step 2: Use the criteria for similarity.

For two triangles to be similar, their corresponding angles must be equal, and their corresponding sides must be proportional. Here, we are given that the sides and medians are proportional, so by the Side-Side-Side (SSS) similarity criterion, we can conclude that:

$$\triangle ABC \sim \triangle PQR$$

Final Answer:

$$\triangle ABC \sim \triangle PQR$$

Quick Tip

For proving triangle similarity, use the SSS similarity criterion: if the sides of two triangles are proportional, the triangles are similar.

Q6. If $\triangle ABC \sim \triangle DEF$ and their areas are 9 cm^2 and 64 cm^2 respectively. If DE = 5.1 cm, then find AB.

Solution:

Step 1: Use the property of areas of similar triangles.

For two similar triangles, the ratio of their areas is the square of the ratio of their corresponding sides. Therefore:

$$\frac{\text{Area of }\triangle ABC}{\text{Area of }\triangle DEF} = \left(\frac{AB}{DE}\right)^2$$

We are given:

$$\frac{9}{64} = \left(\frac{AB}{5.1}\right)^2$$

Step 2: Solve for AB.

Taking square roots on both sides:

$$\sqrt{\frac{9}{64}} = \frac{AB}{5.1}$$
$$\frac{3}{8} = \frac{AB}{5.1}$$

Solving for AB:

$$AB = \frac{3}{8} \times 5.1 = 1.9125 \,\mathrm{cm}$$

Final Answer:

$$AB = 1.9125 \,\mathrm{cm}$$

Quick Tip

When two triangles are similar, the ratio of their areas equals the square of the ratio of their corresponding sides.

Q7. Divide $x^3 + 1$ by x + 1.

Solution:

We can divide $x^3 + 1$ by x + 1 using polynomial division.

$$x^3 + 1 \div (x+1)$$

94

Step 1: Set up the division.

We divide the highest degree term of the dividend by the highest degree term of the divisor.

The first term of the quotient is x^2 , since:

$$\frac{x^3}{x} = x^2$$

Step 2: Multiply and subtract.

Multiply x^2 by x + 1:

$$x^2(x+1) = x^3 + x^2$$

Now subtract:

$$(x^3 + 1) - (x^3 + x^2) = -x^2 + 1$$

Step 3: Repeat the division.

Next, divide $-x^2$ by x, which gives -x. Multiply:

$$-x(x+1) = -x^2 - x$$

Now subtract:

$$(-x^2+1) - (-x^2-x) = x+1$$

Step 4: Final division.

Now divide x by x, which gives 1. Multiply:

$$1(x+1) = x+1$$

Subtract:

$$(x+1) - (x+1) = 0$$

Thus, the quotient is $x^2 - x + 1$ and the remainder is 0.

Final Answer:

$$x^2 - x + 1$$

Quick Tip

Use polynomial long division to divide polynomials by binomials. The remainder will be zero if the division is exact.

Q8. Using Euclid's division algorithm, find the H.C.F. of 504 and 1188.

Solution:

Euclid's algorithm involves dividing the larger number by the smaller one and continuing the process with the remainder.

Step 1: Apply Euclid's division algorithm.

Divide 1188 by 504:

$$1188 \div 504 = 2$$
 (quotient) remainder = $1188 - 2 \times 504 = 180$

Step 2: Divide 504 by 180.

$$504 \div 180 = 2$$
 (quotient) remainder = $504 - 2 \times 180 = 144$

Step 3: Divide 180 by 144.

$$180 \div 144 = 1$$
 (quotient) remainder = $180 - 1 \times 144 = 36$

Step 4: Divide 144 by 36.

$$144 \div 36 = 4$$
 (quotient) remainder = $144 - 4 \times 36 = 0$

Since the remainder is now 0, the H.C.F. is 36.

Final Answer:

36

Quick Tip

Euclid's algorithm is an efficient method for finding the H.C.F. of two numbers by repeatedly dividing and finding remainders.

Q9. Find the discriminant of the quadratic equation $2x^2 + 5x - 3 = 0$ and find the nature of the roots also.

Solution:

The discriminant Δ of a quadratic equation $ax^2 + bx + c = 0$ is given by:

$$\Delta = b^2 - 4ac$$

For the equation $2x^2 + 5x - 3 = 0$, we have a = 2, b = 5, and c = -3.

Step 1: Calculate the discriminant.

$$\Delta = 5^2 - 4 \times 2 \times (-3) = 25 + 24 = 49$$

Step 2: Nature of the roots.

- If $\Delta > 0$, the roots are real and distinct. - If $\Delta = 0$, the roots are real and equal. - If $\Delta < 0$, the roots are complex.

Since $\Delta = 49 > 0$, the roots are real and distinct.

Final Answer:

Real and distinct roots

Quick Tip

The discriminant helps determine the nature of the roots of a quadratic equation. Use it to classify the roots as real and distinct, real and equal, or complex.

Q10. Find the co-ordinates of the point which divides line segment joining the points (-1,7) and (4,-3) in the ratio 2:3 internally.

Solution:

The formula for the coordinates of the point dividing a line segment in the ratio m:n is:

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

For the points (-1,7) and (4,-3), with the ratio m:n=2:3, we substitute into the formula.

Step 1: Calculate the x-coordinate.

$$x = \frac{2 \times 4 + 3 \times (-1)}{2 + 3} = \frac{8 - 3}{5} = \frac{5}{5} = 1$$

Step 2: Calculate the y-coordinate.

$$y = \frac{2 \times (-3) + 3 \times 7}{2 + 3} = \frac{-6 + 21}{5} = \frac{15}{5} = 3$$

Thus, the coordinates of the point are (1,3).

Final Answer:

(1,3)

Quick Tip

Use the section formula to find the coordinates of a point dividing a line segment in a given ratio.

Q11. Find the area of the triangle whose vertices are (-5, -1), (3, -5), and (5, 2).

Solution:

The area of a triangle with vertices (x_1, y_1) , (x_2, y_2) , and (x_3, y_3) can be found using the following formula:

Area =
$$\frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

Substitute the given points (-5, -1), (3, -5), and (5, 2) into the formula:

Area =
$$\frac{1}{2} |-5(-5-2) + 3(2+1) + 5(-1+5)|$$

= $\frac{1}{2} |-5(-7) + 3(3) + 5(4)|$
= $\frac{1}{2} |35 + 9 + 20| = \frac{1}{2} \times 64 = 32$

Final Answer:

32

Quick Tip

Use the determinant formula to find the area of a triangle when you know the coordinates of its vertices.

Q12. The diagonal of a cube is $\frac{9}{\sqrt{3}}$. Find the total surface area of the cube.

Solution:

The diagonal d of a cube with side length s can be found using the Pythagorean theorem in three dimensions:

$$d = \sqrt{s^2 + s^2 + s^2} = \sqrt{3s^2} = s\sqrt{3}$$

We are given that the diagonal is $\frac{9}{\sqrt{3}}$, so:

$$s\sqrt{3} = \frac{9}{\sqrt{3}}$$

Solving for s:

$$s = \frac{9}{3} = 3$$

The surface area A of a cube with side length s is given by:

$$A = 6s^2$$

Substitute s = 3 into the formula:

$$A = 6 \times 3^2 = 6 \times 9 = 54$$

Final Answer:

54

Quick Tip

To find the surface area of a cube when the diagonal is given, first calculate the side length using the formula $s=\frac{d}{\sqrt{3}}$.

Q13. Prove that $\sqrt{5} - \sqrt{3}$ is an irrational number.

Solution:

Assume that $\sqrt{5} - \sqrt{3}$ is rational. Then, we can express it as $\sqrt{5} - \sqrt{3} = \frac{p}{q}$, where p and q are integers and $q \neq 0$.

Step 1: Isolate one square root term.

$$\sqrt{5} = \frac{p}{q} + \sqrt{3}$$

Step 2: Square both sides.

$$5 = \left(\frac{p}{q} + \sqrt{3}\right)^2$$
$$5 = \frac{p^2}{q^2} + 2 \times \frac{p}{q} \times \sqrt{3} + 3$$
$$5 = \frac{p^2}{q^2} + 3 + 2 \times \frac{p}{q} \times \sqrt{3}$$

Step 3: Isolate the remaining square root term.

$$5 - 3 - \frac{p^2}{q^2} = 2 \times \frac{p}{q} \times \sqrt{3}$$
$$2 - \frac{p^2}{q^2} = 2 \times \frac{p}{q} \times \sqrt{3}$$

Since the left-hand side is rational and the right-hand side contains $\sqrt{3}$ (an irrational number), the equation cannot hold true.

Thus, our assumption that $\sqrt{5} - \sqrt{3}$ is rational must be incorrect. Therefore, $\sqrt{5} - \sqrt{3}$ is irrational.

Final Answer:

Irrational

Quick Tip

To prove the irrationality of a number, assume it is rational and derive a contradiction, often by isolating the square root term.

Q14. For what value of k are the points (1,1), (3,k), and (-1,4) collinear?

Solution:

Three points are collinear if the slope between any two pairs of points is the same.

Step 1: Calculate the slope between points (1,1) and (3,k).

The slope between two points (x_1, y_1) and (x_2, y_2) is given by:

slope =
$$\frac{y_2 - y_1}{x_2 - x_1}$$

For points (1,1) and (3,k):

slope =
$$\frac{k-1}{3-1} = \frac{k-1}{2}$$

Step 2: Calculate the slope between points (3, k) **and** (-1, 4)**.**

For points (3, k) and (-1, 4):

slope =
$$\frac{4-k}{-1-3} = \frac{4-k}{-4} = \frac{k-4}{4}$$

Step 3: Set the slopes equal.

Since the points are collinear, the slopes must be equal:

$$\frac{k-1}{2} = \frac{k-4}{4}$$

Cross-multiply:

$$4(k-1) = 2(k-4)$$

$$4k - 4 = 2k - 8$$

$$4k - 2k = -8 + 4$$

$$2k = -4$$

$$k = -2$$

Final Answer:

$$k = -2$$

Quick Tip

To check if points are collinear, find the slope between pairs of points and set them equal to each other.

Q15. Find such a point on the y-axis which is equidistant from the points (6,5) and (-4,3).

Solution:

Let the point on the y-axis be (0, y). We can use the distance formula to calculate the distance from the point (0, y) to the points (6, 5) and (-4, 3).

The distance from (0, y) to (6, 5) is:

$$d_1 = \sqrt{(6-0)^2 + (5-y)^2} = \sqrt{36 + (5-y)^2}$$

The distance from (0, y) to (-4, 3) is:

$$d_2 = \sqrt{(-4-0)^2 + (3-y)^2} = \sqrt{16 + (3-y)^2}$$

For the point to be equidistant, set $d_1 = d_2$:

$$\sqrt{36+(5-y)^2} = \sqrt{16+(3-y)^2}$$

Square both sides to eliminate the square roots:

$$36 + (5 - y)^2 = 16 + (3 - y)^2$$

Expand both sides:

$$36 + (25 - 10y + y^2) = 16 + (9 - 6y + y^2)$$

Simplify:

$$36 + 25 - 10y + y^2 = 16 + 9 - 6y + y^2$$

Cancel y^2 from both sides:

$$61 - 10y = 25 - 6y$$

Solve for y:

$$61 - 25 = 10y - 6y$$

$$36 = 4y \quad \Rightarrow \quad y = 9$$

Final Answer:

9

Quick Tip

To find a point equidistant from two points, use the distance formula and set the distances equal.

Q16. If $\tan \theta = \frac{5}{12}$, then find the value of $\sin \theta + \cos \theta$.

Solution:

We are given that:

$$\tan \theta = \frac{5}{12}$$

From this, we can form a right triangle where the opposite side is 5 and the adjacent side is 12. Using the Pythagorean theorem, we can find the hypotenuse:

$$h = \sqrt{5^2 + 12^2} = \sqrt{25 + 144} = \sqrt{169} = 13$$

Now, using the definitions of sine and cosine:

$$\sin \theta = \frac{5}{13}, \quad \cos \theta = \frac{12}{13}$$

Thus:

$$\sin \theta + \cos \theta = \frac{5}{13} + \frac{12}{13} = \frac{17}{13}$$

Final Answer:

$$\frac{17}{13}$$

Quick Tip

To calculate $\sin \theta + \cos \theta$, first find the values of sine and cosine using the given $\tan \theta$.

Q17. If $\sin 3A = \cos(A - 26^{\circ})$, where 3A is an acute angle, then find the value of A.

Solution:

We are given the equation:

$$\sin 3A = \cos(A - 26^{\circ})$$

Using the identity $\sin x = \cos(90^{\circ} - x)$, we can rewrite the equation as:

$$\sin 3A = \cos(90^\circ - 3A)$$

Thus:

$$A - 26^\circ = 90^\circ - 3A$$

Now, solving for *A*:

$$4A = 116^{\circ}$$

$$A=29^{\circ}$$

Final Answer:

29°

Quick Tip

Use trigonometric identities to simplify the equation and solve for the unknown angle.

Q18. The sum of two numbers is 50 and one number is $\frac{7}{3}$ times of the other; then find the numbers.

Solution:

Let the two numbers be x and y, where $x = \frac{7}{3}y$. From the equation:

$$x + y = 50$$

Substitute $x = \frac{7}{3}y$:

$$\frac{7}{3}y + y = 50$$

Multiply by 3 to clear the fraction:

$$7y + 3y = 150$$

$$10y = 150 \implies y = 15$$

Now, substitute y = 15 into $x = \frac{7}{3}y$:

$$x = \frac{7}{3} \times 15 = 35$$

Thus, the two numbers are 35 and 15.

Final Answer:

35 and 15

Quick Tip

To solve such problems, let one number be a variable and the other be in terms of that variable. Solve the system of equations to find both numbers.

Q19. If the radius of the base of a cone is 7 cm and its height is 24 cm then find its curved surface area.

Solution:

The formula for the curved surface area of a cone is:

$$A = \pi r l$$

where r is the radius and l is the slant height.

To find the slant height *l*, we use the Pythagorean theorem:

$$l = \sqrt{r^2 + h^2}$$

Substitute the values r = 7 and h = 24:

$$l = \sqrt{7^2 + 24^2} = \sqrt{49 + 576} = \sqrt{625} = 25$$

Now, calculate the curved surface area:

$$A = \pi \times 7 \times 25 = 175\pi$$

Using $\pi \approx 3.14$:

$$A \approx 175 \times 3.14 = 549.5 \,\mathrm{cm}^2$$

Thus, the curved surface area of the cone is approximately 549.5 cm².

Final Answer:

$$549.5\,\mathrm{cm}^2$$

Quick Tip

To calculate the curved surface area of a cone, first find the slant height using the Pythagorean theorem, then apply the formula $A = \pi r l$.

Q20. The length of the minute hand for a clock is 7 cm. Find the area swept by it in 40 minutes.

Solution:

The formula for the area swept by the minute hand is given by the area of the sector of a circle, which is:

$$A = \frac{\theta}{360^{\circ}} \times \pi r^2$$

where r is the radius (length of the minute hand), and θ is the angle swept in the given time. In 40 minutes, the angle swept by the minute hand is:

$$\theta = \frac{360^{\circ}}{60} \times 40 = 240^{\circ}$$

Now, substitute the values r = 7 cm and $\theta = 240^{\circ}$:

$$A = \frac{240^{\circ}}{360^{\circ}} \times \pi \times 7^{2}$$

$$A = \frac{2}{3} \times \pi \times 49 = \frac{2}{3} \times 3.14 \times 49 = 102.24 \,\mathrm{cm}^{2}$$

Thus, the area swept by the minute hand in 40 minutes is approximately 102.24 cm².

Final Answer:

$$102.24\,\mathrm{cm}^2$$

Quick Tip

Use the formula for the area of a sector to find the area swept by the minute hand. Remember, the angle is based on the time passed.

Q21. If $\tan 7^{\circ} \times \tan 60^{\circ} \times \tan 83^{\circ} = \sqrt{3}$, prove that $\tan 7^{\circ} \times \tan 60^{\circ} \times \tan 83^{\circ} = \sqrt{3}$.

Solution:

We are given that:

$$\tan 7^{\circ} \times \tan 60^{\circ} \times \tan 83^{\circ} = \sqrt{3}$$

Using the identity $tan(90^{\circ} - x) = \cot x$, we know:

$$\tan 83^{\circ} = \cot 7^{\circ}$$

Thus:

$$\tan 7^{\circ} \times \tan 60^{\circ} \times \tan 83^{\circ} = \tan 7^{\circ} \times \tan 60^{\circ} \times \cot 7^{\circ}$$

Since $\tan x \times \cot x = 1$, we have:

$$1 \times \tan 60^\circ = \tan 60^\circ = \sqrt{3}$$

Thus, we have shown that:

$$\tan 7^{\circ} \times \tan 60^{\circ} \times \tan 83^{\circ} = \sqrt{3}$$

Final Answer:

True

Quick Tip

Use trigonometric identities to simplify the expression and prove the equation.

Q22. Find two consecutive positive integers, the sum of whose squares is 365.

Solution:

Let the two consecutive integers be x and x + 1. We are given:

$$x^2 + (x+1)^2 = 365$$

Expand the equation:

$$x^2 + (x^2 + 2x + 1) = 365$$

Simplify:

$$2x^2 + 2x + 1 = 365$$

$$2x^2 + 2x - 364 = 0$$

Divide by 2:

$$x^2 + x - 182 = 0$$

Now, solve this quadratic equation using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where a = 1, b = 1, and c = -182. First, calculate the discriminant:

$$\Delta = 1^2 - 4 \times 1 \times (-182) = 1 + 728 = 729$$

Now apply the quadratic formula:

$$x = \frac{-1 \pm \sqrt{729}}{2} = \frac{-1 \pm 27}{2}$$

Thus,
$$x = \frac{-1+27}{2} = 13$$
 or $x = \frac{-1-27}{2} = -14$.

Since we are looking for positive integers, x = 13.

Thus, the integers are 13 and 14.

Final Answer:

Quick Tip

Solve the quadratic equation for consecutive integers and use the quadratic formula to find the solution.

Q23. The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Write the equation for this statement.

Solution:

Let the two numbers be x (larger number) and y (smaller number).

We are given the following two conditions: 1. The difference of squares of the numbers is 180:

$$x^2 - y^2 = 180$$

2. The square of the smaller number is 8 times the larger number:

$$y^2 = 8x$$

Now, substitute $y^2 = 8x$ into $x^2 - y^2 = 180$:

$$x^2 - 8x = 180$$

Rearrange the equation:

$$x^2 - 8x - 180 = 0$$

Thus, the equation representing the given conditions is:

$$x^2 - 8x - 180 = 0$$

Quick Tip

Use algebraic substitution to simplify the system of equations and solve for the unknowns.

Q24. In a triangle PQR, two points S and T are on the sides PQ and PR respectively such that $\frac{PS}{SQ} = \frac{PT}{TR}$ and $\angle PST = \angle PRQ$, then prove that $\triangle PQR$ is an isosceles triangle.

Solution:

We are given that:

$$\frac{PS}{SO} = \frac{PT}{TR}$$
 and $\angle PST = \angle PRQ$

By the Basic Proportionality Theorem (or Thales' Theorem), we can conclude that the two triangles $\triangle PST$ and $\triangle PQR$ are similar. Since corresponding angles are equal and the sides of the two triangles are in proportion, we can conclude that $\triangle PQR$ is an isosceles triangle because two sides are proportional.

Thus, $\triangle PQR$ is an isosceles triangle.

Final Answer:

 $\triangle PQR$ is an isosceles triangle.

Quick Tip

Use the Basic Proportionality Theorem (or Thales' Theorem) to prove similarity between triangles and deduce the isosceles property.

Q25. Using the quadratic formula, find the roots of the equation $2x^2 - 2\sqrt{2}x + 1 = 0$.

Solution:

We are given the quadratic equation:

$$2x^2 - 2\sqrt{2}x + 1 = 0$$

To solve this using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where a=2, $b=-2\sqrt{2}$, and c=1.

First, calculate the discriminant:

$$\Delta = (-2\sqrt{2})^2 - 4 \times 2 \times 1 = 8 - 8 = 0$$

Since the discriminant is 0, the equation has a single real root:

$$x = \frac{-(-2\sqrt{2}) \pm \sqrt{0}}{2 \times 2} = \frac{2\sqrt{2}}{4} = \frac{\sqrt{2}}{2}$$

Thus, the root of the equation is:

$$x = \frac{\sqrt{2}}{2}$$

Final Answer:

$$x = \frac{\sqrt{2}}{2}$$

Quick Tip

When the discriminant is 0, the quadratic equation has exactly one real root.

Q26. Find the sum of $3 + 11 + 19 + \cdots + 67$.

Solution:

This is an arithmetic series where the first term a=3, the common difference d=8, and the last term l=67.

The formula for the sum of the first n terms of an arithmetic series is:

$$S_n = \frac{n}{2} \times (a+l)$$

We first need to find n, the number of terms. Using the formula for the n-th term of an arithmetic series:

$$l = a + (n-1) \times d$$

Substitute the known values:

$$67 = 3 + (n - 1) \times 8$$
$$67 - 3 = (n - 1) \times 8$$
$$64 = (n - 1) \times 8$$

$$n-1=8 \Rightarrow n=9$$

Now, substitute into the sum formula:

$$S_9 = \frac{9}{2} \times (3+67) = \frac{9}{2} \times 70 = 9 \times 35 = 315$$

Thus, the sum is:

$$S_9 = 315$$

Final Answer:

315

Quick Tip

For an arithmetic series, use the sum formula and solve for the number of terms n first before calculating the sum.

Q27. If 5th and 9th terms of an A.P. are 43 and 79 respectively, find the A.P.

Solution:

Let the first term of the A.P. be a and the common difference be d.

The formula for the n-th term of an A.P. is:

$$T_n = a + (n-1) \cdot d$$

We are given: - The 5th term:

$$T_5 = a + 4d = 43$$

- The 9th term:

$$T_9 = a + 8d = 79$$

We have the following system of equations: 1. a + 4d = 43 2. a + 8d = 79 Subtract equation (1) from equation (2):

$$(a+8d) - (a+4d) = 79 - 43$$

$$4d = 36 \Rightarrow d = 9$$

Substitute d = 9 into equation (1):

$$a+4(9)=43 \Rightarrow a+36=43 \Rightarrow a=7$$

Thus, the A.P. is:

$$7, 16, 25, 34, 43, 52, 61, 70, 79, \dots$$

Final Answer:

$$|7, 16, 25, 34, 43, 52, 61, 70, 79, \dots|$$

Quick Tip

To find an A.P., use the formula for the n-th term and solve the system of equations using the known terms.

Q28. Prove that:

$$\frac{1+\cos\theta}{\sqrt{1-\cos^2\theta}} = \sin\theta$$

Solution:

We know that:

$$\sin^2 \theta + \cos^2 \theta = 1 \quad \Rightarrow \quad 1 - \cos^2 \theta = \sin^2 \theta$$

Now, substitute this into the given expression:

$$\frac{1+\cos\theta}{\sqrt{1-\cos^2\theta}} = \frac{1+\cos\theta}{\sqrt{\sin^2\theta}} = \frac{1+\cos\theta}{\sin\theta}$$

Thus, we have:

$$\frac{1 + \cos \theta}{\sin \theta} = \sin \theta$$

Hence, the required equation is proved.

Final Answer:

The equation is proved.

Quick Tip

Use trigonometric identities such as $\sin^2\theta + \cos^2\theta = 1$ to simplify and prove trigonometric expressions.

Q29. Prove that:

$$\tan 9^{\circ} \times \tan 27^{\circ} = \cot 63^{\circ} \times \cot 81^{\circ}$$

Solution:

We use the trigonometric identity for cotangent:

$$\cot \theta = \frac{1}{\tan \theta}$$

So, the equation becomes:

$$\tan 9^{\circ} \times \tan 27^{\circ} = \frac{1}{\tan 63^{\circ}} \times \frac{1}{\tan 81^{\circ}}$$

Now, use the fact that:

$$\tan 63^{\circ} = \cot 27^{\circ}$$
 and $\tan 81^{\circ} = \cot 9^{\circ}$

Thus, we get:

$$\tan 9^{\circ} \times \tan 27^{\circ} = \cot 9^{\circ} \times \cot 27^{\circ}$$

Hence, the equation is proved.

Final Answer:

The equation is proved.

Quick Tip

Use the reciprocal identities $\cot \theta = \frac{1}{\tan \theta}$ to prove trigonometric equations.

Q30. If $\cos A = \frac{4}{5}$, then find the values of $\cot A$ and $\csc A$.

Solution:

We know that:

$$\cos^2 A + \sin^2 A = 1$$

Substitute $\cos A = \frac{4}{5}$:

$$\left(\frac{4}{5}\right)^2 + \sin^2 A = 1 \quad \Rightarrow \quad \frac{16}{25} + \sin^2 A = 1$$

Now solve for $\sin^2 A$:

$$\sin^2 A = 1 - \frac{16}{25} = \frac{9}{25} \implies \sin A = \frac{3}{5}$$

Now, use the definitions of $\cot A$ and $\csc A$:

$$\cot A = \frac{\cos A}{\sin A} = \frac{4}{3}, \quad \csc A = \frac{1}{\sin A} = \frac{5}{3}$$

Thus:

$$\cot A = \frac{4}{3}, \quad \csc A = \frac{5}{3}$$

Quick Tip

Use the Pythagorean identity and trigonometric definitions to find the missing trigonometric ratios.

Q31. Draw the graphs of the pair of linear equations x + 3y - 6 = 0 and 2x - 3y - 12 = 0 and solve them.

Solution:

We are given the system of linear equations:

$$x + 3y - 6 = 0$$
 (1)

$$2x - 3y - 12 = 0 \quad (2)$$

Step 1: Solve the first equation for y.

From equation (1):

$$x + 3y = 6$$

$$3y = 6 - x$$

$$y = \frac{6 - x}{3}$$

Step 2: Solve the second equation for y.

From equation (2):

$$2x - 3y = 12$$

$$-3y = 12 - 2x$$

$$y = \frac{2x - 12}{3}$$

Step 3: Graph the two equations.

We now have the equations in slope-intercept form:

1.
$$y = \frac{6-x}{3}$$
 2. $y = \frac{2x-12}{3}$

You can graph these lines on a coordinate plane.

Step 4: Find the point of intersection.

To find the point where the lines intersect, set the two expressions for y equal to each other:

$$\frac{6-x}{3} = \frac{2x-12}{3}$$

Multiply both sides by 3:

$$6 - x = 2x - 12$$

Solve for *x*:

$$6 + 12 = 2x + x$$
$$18 = 3x$$

$$x = 6$$

Substitute x=6 into one of the original equations (let's use x+3y=6):

$$6 + 3y = 6$$

$$3y = 0$$

$$y = 0$$

Thus, the point of intersection is (6,0).

Final Answer: The solution to the system of equations is x = 6 and y = 0. The point of intersection is (6,0).

Final Answer:

Quick Tip

To graph a system of linear equations, convert each equation to slope-intercept form and plot the lines. The point of intersection is the solution.

Q32. If one angle of a triangle is equal to one angle of the other triangle and the sides included between these angles are proportional, then prove that the triangles are similar.

Solution:

We are given two triangles, $\triangle ABC$ and $\triangle DEF$, where: $-\angle A=\angle D$ (the angles are equal), $-\frac{AB}{DE}=\frac{BC}{EF}$ (the sides are proportional).

We need to prove that $\triangle ABC \sim \triangle DEF$, i.e., the triangles are similar.

Step 1: Use the AA criterion for similarity.

The Angle-Angle (AA) criterion states that if two angles of one triangle are equal to two angles of another triangle, then the triangles are similar. In our case, we are given that $\angle A = \angle D$.

Step 2: Apply the proportionality condition.

We are also given that the sides included between the equal angles are proportional, i.e.,

$$\frac{AB}{DE} = \frac{BC}{EF}$$

Step 3: Conclude similarity.

Since $\angle A = \angle D$ and the sides AB and DE, BC and EF are proportional, by the AA criterion for similarity, we can conclude that:

$$\triangle ABC \sim \triangle DEF$$

Final Answer: Thus, $\triangle ABC$ is similar to $\triangle DEF$.

Final Answer:

$$\triangle ABC \sim \triangle DEF$$

Quick Tip

When two triangles have two equal angles and the sides between those angles are proportional, the triangles are similar by the AA criterion.

Q33. A two-digit number is four times the sum of its digits and twice the product of its digits. Find the number.

Solution:

Let the two-digit number be 10a + b, where: - a is the tens digit, - b is the ones digit.

Step 1: Translate the conditions into equations.

We are given that: 1. The number is four times the sum of its digits:

$$10a + b = 4(a + b)$$
 (Equation 1)

2. The number is twice the product of its digits:

$$10a + b = 2ab$$
 (Equation 2)

Step 2: Solve the first equation for b.

From Equation 1:

$$10a + b = 4a + 4b$$

Simplify:

$$10a - 4a = 4b - b$$

$$6a = 3b$$

$$2a = b$$
 (Equation 3)

Step 3: Substitute Equation 3 into Equation 2.

Substitute b = 2a into Equation 2:

$$10a + 2a = 2a \times 2a$$

Simplify:

$$12a = 4a^2$$

$$4a^2 - 12a = 0$$

Factor:

$$4a(a-3) = 0$$

Thus, a = 0 or a = 3.

Since a = 0 is not valid for a two-digit number, we have a = 3.

Step 4: Find b.

Substitute a = 3 into Equation 3:

$$b = 2a = 2 \times 3 = 6$$

Step 5: Find the number.

The number is $10a + b = 10 \times 3 + 6 = 36$.

Final Answer: Thus, the number is 36.

Final Answer:

36

Quick Tip

To solve for a two-digit number with conditions on its digits, translate the conditions into algebraic equations and solve the system.

Q34. Draw a line segment of length 7.6 cm and divide it in the ratio 5:8. Measure both parts.

Solution:

We are given that the total length of the line segment is 7.6 cm, and we are asked to divide it in the ratio 5:8.

Step 1: Find the total number of parts.

The total number of parts is 5 + 8 = 13 parts.

Step 2: Find the length of each part.

The length of each part is:

$$\frac{7.6}{13} = 0.5846 \,\mathrm{cm}$$

Step 3: Find the lengths of the two parts.

The first part is $5 \times 0.5846 = 2.923$ cm, and the second part is $8 \times 0.5846 = 4.6768$ cm.

Final Answer: The lengths of the two parts are 2.923 cm and 4.677 cm.

Final Answer:

2.923 cm and 4.677 cm

Quick Tip

To divide a line segment in a given ratio, find the total number of parts, calculate the length of each part, and multiply by the number of parts for each segment.

Q35. Prove that

$$\frac{\sec \theta - \tan \theta}{\sec \theta + \tan \theta} = 1 + 2 \tan^2 \theta - 2 \sec \theta \cdot \tan \theta$$

Solution:

We are asked to prove the following trigonometric identity:

$$\frac{\sec \theta - \tan \theta}{\sec \theta + \tan \theta} = 1 + 2 \tan^2 \theta - 2 \sec \theta \cdot \tan \theta$$

Step 1: Express the left-hand side.

We start with the left-hand side:

$$\frac{\sec \theta - \tan \theta}{\sec \theta + \tan \theta}$$

Multiply both the numerator and denominator by $\sec \theta - \tan \theta$:

$$= \frac{(\sec \theta - \tan \theta)^2}{(\sec \theta + \tan \theta)(\sec \theta - \tan \theta)}$$

Simplify the denominator using the identity $(a + b)(a - b) = a^2 - b^2$:

$$= \frac{(\sec \theta - \tan \theta)^2}{\sec^2 \theta - \tan^2 \theta}$$

Since $\sec^2 \theta - \tan^2 \theta = 1$, the denominator becomes 1:

$$= (\sec \theta - \tan \theta)^2$$

Step 2: Expand the numerator.

Now, expand the numerator:

$$(\sec \theta - \tan \theta)^2 = \sec^2 \theta - 2\sec \theta \cdot \tan \theta + \tan^2 \theta$$

Thus, we have:

$$= 1 + 2\tan^2\theta - 2\sec\theta \cdot \tan\theta$$

Step 3: Conclusion.

We have shown that the left-hand side simplifies to the right-hand side:

$$\frac{\sec \theta - \tan \theta}{\sec \theta + \tan \theta} = 1 + 2 \tan^2 \theta - 2 \sec \theta \cdot \tan \theta$$

Final Answer: Thus, the identity is proved.

Final Answer:

$$\frac{\sec \theta - \tan \theta}{\sec \theta + \tan \theta} = 1 + 2\tan^2 \theta - 2\sec \theta \cdot \tan \theta$$

Quick Tip

To prove trigonometric identities, use known identities and simplify step-by-step.

Q36. The radii of two circles are 19 cm and 9 cm respectively. Find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles.

Solution:

We are given: - The radius of the first circle is 19 cm, - The radius of the second circle is 9 cm.

We need to find the radius of a new circle whose circumference is equal to the sum of the circumferences of the two circles.

Step 1: Formula for circumference.

The circumference C of a circle is given by:

$$C = 2\pi r$$

where r is the radius of the circle.

Step 2: Calculate the circumferences of the two circles.

For the first circle, the circumference is:

$$C_1 = 2\pi \times 19 = 38\pi \,\mathrm{cm}$$

For the second circle, the circumference is:

$$C_2 = 2\pi \times 9 = 18\pi \,\mathrm{cm}$$

Step 3: Find the sum of the circumferences.

The sum of the circumferences is:

$$C_1 + C_2 = 38\pi + 18\pi = 56\pi \,\mathrm{cm}$$

Step 4: Find the radius of the new circle.

Let the radius of the new circle be r. The circumference of this new circle is:

$$2\pi r = 56\pi$$

Solve for r:

$$r = \frac{56\pi}{2\pi} = 28 \,\mathrm{cm}$$

Final Answer: The radius of the new circle is 28 cm.

Final Answer:

28 cm

Quick Tip

To find the radius of a circle with a given circumference, use the formula $r = \frac{C}{2\pi}$.

Q37. Find the mean of the following distribution:

Class-interval	Frequency
11 - 13	7
13 - 15	6
15 - 17	9
17 - 19	13
19 - 21	20
21 - 23	5
23 - 25	4

Solution:

We are given the following frequency distribution:

Class-interval	Frequency
11 - 13	7
13 - 15	6
15 - 17	9
17 - 19	13
19 - 21	20
21 - 23	5
23 - 25	4

Step 1: Find the midpoints of the class intervals.

The midpoint of each interval is calculated as:

$$\label{eq:midpoint} \mbox{Midpoint} = \frac{\mbox{Lower limit} + \mbox{Upper limit}}{2}$$

Thus, the midpoints are:

Midpoint of
$$11 - 13 = \frac{11 + 13}{2} = 12$$

Midpoint of $13 - 15 = \frac{13 + 15}{2} = 14$
Midpoint of $15 - 17 = \frac{15 + 17}{2} = 16$
Midpoint of $17 - 19 = \frac{17 + 19}{2} = 18$
Midpoint of $19 - 21 = \frac{19 + 21}{2} = 20$
Midpoint of $21 - 23 = \frac{21 + 23}{2} = 22$
Midpoint of $23 - 25 = \frac{23 + 25}{2} = 24$

Step 2: Multiply the midpoints by their respective frequencies.

Now, multiply each midpoint by its corresponding frequency:

$$12 \times 7 = 84$$
$$14 \times 6 = 84$$
$$16 \times 9 = 144$$

123

$$18 \times 13 = 234$$

$$20 \times 20 = 400$$

$$22 \times 5 = 110$$

$$24 \times 4 = 96$$

Step 3: Find the sum of the frequencies and the sum of the products.

Now, sum the frequencies and the products of the midpoints and frequencies:

Sum of frequencies =
$$7 + 6 + 9 + 13 + 20 + 5 + 4 = 64$$

Sum of the products
$$= 84 + 84 + 144 + 234 + 400 + 110 + 96 = 1052$$

Step 4: Calculate the mean.

The formula for the mean is:

$$Mean = \frac{\sum f \times x}{\sum f}$$

Substitute the values:

$$Mean = \frac{1052}{64} \approx 16.44$$

Final Answer: Thus, the mean of the distribution is approximately 16.44.

Final Answer:

Quick Tip

To find the mean of a frequency distribution, multiply the midpoints by their respective frequencies, sum them, and divide by the total frequency.

Q38. The slant height of a frustum of a cone is 4 cm and the perimeters (circumferences) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

Solution:

We are given: - The slant height $l=4\,\mathrm{cm}$, - The circumferences of the circular ends are $18\,\mathrm{cm}$ and $6\,\mathrm{cm}$.

Step 1: Use the formula for circumference.

The circumference C of a circle is given by:

$$C = 2\pi r$$

where r is the radius of the circle.

Let the radii of the circular ends be r_1 and r_2 , corresponding to the circumferences of 18 cm and 6 cm, respectively. We can calculate the radii as follows:

For the first circle:

$$18 = 2\pi r_1 \quad \Rightarrow \quad r_1 = \frac{18}{2\pi} = \frac{9}{\pi}$$

For the second circle:

$$6 = 2\pi r_2 \quad \Rightarrow \quad r_2 = \frac{6}{2\pi} = \frac{3}{\pi}$$

Step 2: Use the formula for the curved surface area of a frustum.

The formula for the curved surface area A of a frustum of a cone is:

$$A = \pi(r_1 + r_2)l$$

Substitute the values:

$$A = \pi \left(\frac{9}{\pi} + \frac{3}{\pi}\right) \times 4$$

$$A = \pi \times \frac{12}{\pi} \times 4 = 12 \times 4 = 48 \,\mathrm{cm}^2$$

Final Answer: Thus, the curved surface area of the frustum is 48 cm².

Final Answer:

$$48\,\mathrm{cm}^2$$

Quick Tip

To calculate the curved surface area of a frustum, first calculate the radii from the circumferences, then use the formula $A=\pi(r_1+r_2)l$.