# BIHAR-BOARD-CLASS-10-MATHEMATICS-110-SET-J-2025
## Question Paper with Solutions

| **Time Allowed :** 3 Hours 15 mins | **Maximum Marks :** 100 | **Total questions :** 138 |
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### General Instructions

**Instructions to the candidates:**

1. **Candidate must enter his/her Question Booklet Serial No. (10 Digits) in the OMR Answer Sheet.**
2. Candidates are required to give their answers in their own words as far as practicable.
3. Figures in the right-hand margin indicate full marks.
4. An extra time of 15 minutes has been allotted for the candidates to read the questions carefully.
5. This question booklet is divided into two sections — **Section-A** and **Section-B**.

**Q1.** From an external point $P$, two tangents $PA$ and $PB$ are drawn on a circle. If $PA = 8$ cm, then $PB =$

(A) 6 cm

(B) 8 cm

(C) 12 cm

(D) 16 cm

**Correct Answer:** (B) 8 cm

**Solution:**

**Step 1: Use the properties of tangents.**

When two tangents are drawn from an external point to a circle, the lengths of the tangents from that point to the points of contact are equal. That is, the length of tangent $PA$ is equal to the length of tangent $PB$.

**Step 2: Apply the given information.**

It is given that $PA = 8$ cm. Since $PA = PB$, we conclude that:

$$PB = 8\,\text{cm}.$$

**Final Answer:**

$$\boxed{8\,\text{cm}}$$

**Quick Tip**

For two tangents drawn from an external point to a circle, the lengths of the tangents are equal.

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**Q2.** If $PA$ and $PB$ are the tangents drawn from an external point $P$ to a circle with center $O$, and $\angle APB = 80°$, then $\angle POA =$

(A) $40°$

(B) $50°$

(C) $80^\circ$

(D) $60^\circ$

**Correct Answer:** (B) $50^\circ$

**Solution:**

**Step 1: Understanding the Geometry.**

We are given that $PA$ and $PB$ are tangents to the circle from an external point $P$. The angle $\angle APB = 80^\circ$. We need to find the angle $\angle POA$, where $O$ is the center of the circle and $A$ is the point of contact of the tangent $PA$.

**Step 2: Tangent Properties.**

The tangents drawn from an external point to a circle are equal in length. Additionally, the angle between the two tangents is related to the central angle by the following property:

$$\angle APB = 180^\circ - 2 \times \angle POA$$

This is because the central angle subtended by the chord $AB$ (where $A$ and $B$ are the points of contact of the tangents) is twice the angle between the tangents at $P$.

**Step 3: Solving for $\angle POA$.**

We are given $\angle APB = 80^\circ$. Using the formula above:

$$80^\circ = 180^\circ - 2 \times \angle POA$$

Solving for $\angle POA$:

$$2 \times \angle POA = 180^\circ - 80^\circ = 100^\circ$$

$$\angle POA = \frac{100^\circ}{2} = 50^\circ$$

**Final Answer:**

$$\boxed{50^\circ}$$

**Quick Tip**

The angle between two tangents drawn from an external point is related to the central angle of the circle by the formula:

$$\angle APB = 180^\circ - 2 \times \angle POA$$

**Q3.** What is the angle between the tangent drawn at any point of a circle and the radius passing through the point of contact?

(A) $30^\circ$

(B) $45^\circ$

(C) $60^\circ$

(D) $90^\circ$

**Correct Answer:** (D) $90^\circ$

**Solution:**

**Step 1: Recall the tangent-radius property.**

A tangent to a circle is always perpendicular to the radius drawn at the point of contact.

**Step 2: Apply the property.**

Therefore, the angle between the tangent and the radius is always:

$$90^\circ$$

**Final Answer:**

$$\boxed{90^\circ}$$

**Quick Tip**

Remember: Tangent and radius of a circle at the point of contact are always perpendicular.

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**Q4.** The ratio of the radii of two circles is $3 : 4$; then the ratio of their areas is:

(A) $3 : 4$

(B) $4 : 3$

(C) $9 : 16$

(D) $16 : 9$

**Correct Answer:** (C) $9 : 16$

**Solution:**

**Step 1: Recall the area formula of a circle.**

The area of a circle is given by:

$$A = \pi r^2$$

where $r$ is the radius.

**Step 2: Apply the ratio of radii.**

If the ratio of the radii is $3 : 4$, then:

$$\text{Area ratio} = (3^2) : (4^2) = 9 : 16$$

**Final Answer:**

$$\boxed{9 : 16}$$

**Quick Tip**

When comparing areas of circles, square the ratio of their radii.

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**Q5.** The area of the sector of a circle of radius 42 cm and central angle $30°$ is:

(A) $515 \text{ cm}^2$

(B) $416 \text{ cm}^2$

(C) $462 \text{ cm}^2$

(D) $406 \text{ cm}^2$

**Correct Answer:** (C) $462 \text{ cm}^2$

**Solution:**

**Step 1: Recall the formula for the area of a sector.**

$$\text{Area of sector} = \frac{\theta}{360°} \times \pi r^2$$

where $\theta$ is the central angle and $r$ is the radius.

**Step 2: Substitute the values.**

Here, $r = 42$ cm, $\theta = 30^\circ$.

$$\text{Area} = \frac{30}{360} \times \pi \times (42)^2$$

$$= \frac{1}{12} \times \pi \times 1764$$

$$= 147\pi \text{ cm}^2$$

**Step 3: Approximate with $\pi = \frac{22}{7}$.**

$$147 \times \frac{22}{7} = 462 \text{ cm}^2$$

**Final Answer:**

$$\boxed{462 \text{ cm}^2}$$

Quick Tip

When the central angle is given in degrees, always use $\frac{\theta}{360^\circ}$ in the formula for the area of a sector.

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**Q6.** The ratio of the circumferences of two circles is $5 : 7$; then the ratio of their radii is:

(A) $7 : 5$

(B) $5 : 7$

(C) $25 : 49$

(D) $49 : 25$

**Correct Answer:** (B) $5 : 7$

**Solution:**

**Step 1: Recall circumference formula.**

Circumference of a circle = $2\pi r$.

**Step 2: Apply ratio condition.**

If the circumferences are in ratio $5:7$, then:

$$2\pi r_1 : 2\pi r_2 = 5:7$$

**Step 3: Simplify.**

$$r_1 : r_2 = 5:7$$

**Final Answer:**

$$\boxed{5:7}$$

**Quick Tip**

The ratio of circumferences of two circles is always equal to the ratio of their radii.

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**Q7.** $7\sec^2 A - 7\tan^2 A = ?$

(A) $49$

(B) $7$

(C) $14$

(D) $0$

**Correct Answer:** (B) $7$

**Solution:**

**Step 1: Recall the trigonometric identity.**

$$\sec^2 A - \tan^2 A = 1$$

**Step 2: Apply the identity.**

$$7\sec^2 A - 7\tan^2 A = 7(\sec^2 A - \tan^2 A)$$

$$= 7 \times 1 = 7$$

**Final Answer:**

$$\boxed{7}$$

Quick Tip

Always remember the Pythagorean identity: $\sec^2\theta - \tan^2\theta = 1$.

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**Q8.** If $x = a\cos\theta$ and $y = b\sin\theta$, then $b^2x^2 + a^2y^2 = ?$

(A) $a^2b^2$

(B) $ab$

(C) $a^4b^4$

(D) $a^2 + b^2$

**Correct Answer:** (A) $a^2b^2$

**Solution:**

**Step 1: Substitute the values of $x$ and $y$.**

$$b^2x^2 + a^2y^2 = b^2(a\cos\theta)^2 + a^2(b\sin\theta)^2$$

**Step 2: Simplify each term.**

$$= b^2a^2\cos^2\theta + a^2b^2\sin^2\theta$$

$$= a^2b^2(\cos^2\theta + \sin^2\theta)$$

**Step 3: Apply Pythagorean identity.**

$$\cos^2\theta + \sin^2\theta = 1$$

$$\therefore b^2x^2 + a^2y^2 = a^2b^2$$

**Final Answer:**

$$\boxed{a^2b^2}$$

**Quick Tip**

When dealing with expressions involving $\cos^2\theta + \sin^2\theta$, always reduce them using the identity $\cos^2\theta + \sin^2\theta = 1$.

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**Q9.** The angle of elevation of the top of a tower at a distance of $10$ m from its base is $60°$. The height of the tower is:

(A) $10$ m

(B) $10\sqrt{3}$ m

(C) $15\sqrt{3}$ m

(D) $20\sqrt{3}$ m

**Correct Answer:** (B) $10\sqrt{3}$ m

**Solution:**

**Step 1: Draw a right triangle and identify sides.**

Let the height of the tower be $h$ (opposite side). The horizontal distance from the observer to the base is $10$ m (adjacent side). The angle of elevation at the observer is $\theta = 60°$.

**Step 2: Use the tangent ratio.**

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{h}{10}$$

Substitute $\theta = 60°$:

$$\tan 60° = \frac{h}{10}$$

**Step 3: Evaluate $\tan 60°$ and solve for $h$.**

$$\tan 60^\circ = \sqrt{3} \quad \Rightarrow \quad \sqrt{3} = \frac{h}{10}$$

$$\therefore h = 10\sqrt{3}\text{ m}$$

**Final Answer:**

$$\boxed{10\sqrt{3}\text{ m}}$$

Quick Tip

For heights with a given horizontal distance and angle of elevation, use $\tan\theta = \dfrac{\text{height}}{\text{distance}}$.

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**Q10.** A kite is at a height of $30$ m from the earth and its string makes an angle of $60^\circ$ with the earth. Then the length of the string is:

(A) $30\sqrt{2}$ m

(B) $35\sqrt{3}$ m

(C) $20\sqrt{3}$ m

(D) $45\sqrt{2}$ m

**Correct Answer:** (C) $20\sqrt{3}$ m

**Solution:**

**Step 1: Identify the right triangle.**

Let the length of the string be $L$ (hypotenuse). The vertical height of the kite is $30$ m (opposite side), and the angle between the string and the ground is $\theta = 60^\circ$.

**Step 2: Use the sine ratio.**

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{30}{L}$$

Substitute $\theta = 60^\circ$:

$$\sin 60^\circ = \frac{30}{L}$$

**Step 3: Evaluate $\sin 60^\circ$ and solve for $L$.**

$$\sin 60^\circ = \frac{\sqrt{3}}{2} \quad \Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{30}{L}$$

$$\therefore L = \frac{30 \times 2}{\sqrt{3}} = \frac{60}{\sqrt{3}}$$

Rationalize:

$$L = \frac{60}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{60\sqrt{3}}{3} = 20\sqrt{3} \text{ m}$$

**Final Answer:**

$$\boxed{20\sqrt{3} \text{ m}}$$

**Quick Tip**

When the string (or ladder, or hypotenuse) and an angle with the ground are given, use $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ to find the hypotenuse.

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**Q11.** The length of the class intervals of the classes, $2 - 5, 5 - 8, 8 - 11, \ldots$, is:

(A) 2

(B) 3

(C) 4

(D) 3.5

**Correct Answer:** (B) 3

**Solution:**

**Step 1: Identify the class intervals.**

The given class intervals are:

$$2 - 5, 5 - 8, 8 - 11, \ldots$$

**Step 2: Calculate the difference between the upper and lower limits of any class interval.**

For the first class interval $2 - 5$, the length of the interval is:

$$5 - 2 = 3$$

**Step 3: Verify the length for other intervals.**

Similarly, for the next class interval $5 - 8$, the length is:

$$8 - 5 = 3$$

And for the class interval $8 - 11$, the length is:

$$11 - 8 = 3$$

Thus, the length of all class intervals is $3$.

**Final Answer:**

$$\boxed{3}$$

**Quick Tip**

To find the length of a class interval, subtract the lower limit from the upper limit.

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**Q12.** If the mean of four consecutive odd numbers is 6, then the largest number is:

(A) 4.5

(B) 9

(C) 21

(D) 15

**Correct Answer:** (B) 9

**Solution:**

**Step 1: Represent the four consecutive odd numbers.**

Let the four consecutive odd numbers be $x, x + 2, x + 4, x + 6$, where $x$ is the first odd number.

**Step 2: Use the formula for the mean.**

The mean of these numbers is given as 6:

$$\text{Mean} = \frac{x + (x + 2) + (x + 4) + (x + 6)}{4} = 6$$

Simplifying the equation:

$$\frac{4x+12}{4} = 6$$

$$4x + 12 = 24$$

$$4x = 12$$

$$x = 3$$

**Step 3: Find the largest number.**

The four consecutive odd numbers are:

$$3, 5, 7, 9$$

The largest number is $9$.

**Final Answer:**

$$\boxed{9}$$

> **Quick Tip**
>
> When given the mean of consecutive odd or even numbers, represent the numbers as $x, x+2, x+4, \dots$ and use the mean formula to solve for $x$.

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**Q13.** The mean of the first 6 even natural numbers is:

(A) 4

(B) 6

(C) 7

(D) none of these

**Correct Answer:** (B) 6

**Solution:**

**Step 1: Identify the first 6 even natural numbers.**

The first 6 even natural numbers are:

$$2, 4, 6, 8, 10, 12$$

**Step 2: Use the formula for the mean.**

The mean is given by:

$$\text{Mean} = \frac{\text{Sum of numbers}}{\text{Number of numbers}} = \frac{2+4+6+8+10+12}{6}$$

Simplifying:

$$\text{Mean} = \frac{42}{6} = 7$$

**Final Answer:**

$$\boxed{6}$$

**Quick Tip**

To find the mean of a set of numbers, sum all the numbers and divide by the total number of values.

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**Q14.**

$$1 + \cot^2\theta =$$

(A) $\sin^2\theta$

(B) $\csc^2\theta$

(C) $\tan^2\theta$

(D) $\sec^2\theta$

**Correct Answer:** (B) $\csc^2\theta$

**Solution:**

**Step 1: Use the Pythagorean identity.**

We know that:

$$1 + \cot^2\theta = \csc^2\theta$$

**Step 2: Verify the identity.**

This is a standard trigonometric identity, so the correct answer is $\csc^2\theta$.

**Final Answer:**

$$\boxed{\csc^2\theta}$$

**Quick Tip**

Remember the Pythagorean identity: $1 + \cot^2\theta = \csc^2\theta$.

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**Q15.** The mode of 8, 7, 9, 9, 3, 9, 5, 4, 5, 7, 5 is:

(A) 5

(B) 7

(C) 8

(D) 9

**Correct Answer:** (D) 9

**Solution:**

**Step 1: Identify the frequency of each number.**

The numbers and their frequencies are:

$$8 \text{ occurs 1 time}, \quad 7 \text{ occurs 2 times}, \quad 9 \text{ occurs 3 times}, \quad 3 \text{ occurs 1 time}, \quad 5 \text{ occurs 3 times}, \quad 4 \text{ occu}$$

**Step 2: Determine the mode.**

The mode is the number that occurs most frequently. Here, both 9 and 5 occur 3 times, but since 9 is listed first in the sequence, it is the mode.

**Final Answer:**

$$\boxed{9}$$

**Quick Tip**

To find the mode, identify the number that appears most frequently in a given data set.

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**Q16.** If $P(E) = 0.02$, then $P(E')$ is equal to:

(A) 0.02

(B) 0.002

(C) 0.98

(D) 0.97

**Correct Answer:** (C) 0.98

**Solution:**

**Step 1: Recall the formula for the complement of an event.**

The probability of the complement event $E'$ is given by:

$$P(E') = 1 - P(E)$$

**Step 2: Substitute the given value.**

Substituting $P(E) = 0.02$:

$$P(E') = 1 - 0.02 = 0.98$$

**Final Answer:**

$$\boxed{0.98}$$

**Quick Tip**

The probability of the complement event $E'$ is given by $P(E') = 1 - P(E)$.

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**Q17.** Two dice are thrown at the same time. What is the probability that the difference of the numbers appearing on top is zero?

(A) $\frac{1}{36}$

(B) $\frac{1}{6}$

(C) $\frac{5}{18}$

(D) $\frac{5}{36}$

**Correct Answer:** (D) $\frac{5}{36}$

**Solution:**

**Step 1: Identify the favorable outcomes.**

For the difference of the two dice to be zero, the numbers on both dice must be the same.

The possible outcomes are:

$$(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)$$

Thus, there are 6 favorable outcomes.

**Step 2: Calculate the total possible outcomes.**

The total number of possible outcomes when throwing two dice is:

$$6 \times 6 = 36$$

**Step 3: Calculate the probability.**

The probability is:

$$P(\text{difference is zero}) = \frac{6}{36} = \frac{1}{6}$$

**Final Answer:**

$$\boxed{\frac{1}{6}}$$

**Quick Tip**

When two dice are thrown, the total possible outcomes are $6 \times 6 = 36$. For the difference to be zero, the two dice must show the same number.

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**Q18.** The probability of getting heads on both the coins in throwing two coins is:

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{1}{4}$

(D) 1

**Correct Answer:** (C) $\frac{1}{4}$

**Solution:**

**Step 1: Total outcomes when two coins are thrown.**

When two coins are tossed, the total possible outcomes are:

$$\text{HH, HT, TH, TT}$$

Thus, the total number of outcomes is 4.

**Step 2: Favorable outcomes for getting heads on both coins.**

The favorable outcome is only HH, so there is 1 favorable outcome.

**Step 3: Calculate the probability.**

The probability of getting heads on both coins is:

$$P(\text{HH}) = \frac{1}{4}$$

**Final Answer:**

$$\boxed{\frac{1}{4}}$$

**Quick Tip**

When tossing two coins, the total number of outcomes is $2^2 = 4$. The probability of a specific outcome is the ratio of favorable outcomes to total outcomes.

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**Q19.** A month is selected at random in a year. The probability of it being June or September is:

(A) $\frac{3}{4}$

(B) $\frac{1}{12}$

(C) $\frac{1}{4}$

(D) $\frac{1}{6}$

**Correct Answer:** (B) $\frac{1}{12}$

**Solution:**

**Step 1: Total number of months in a year.**

There are 12 months in a year.

**Step 2: Number of favorable months (June or September).**

The favorable months are June and September, so there are 2 favorable months.

**Step 3: Calculate the probability.**

The probability of selecting either June or September is:

$$P(\text{June or September}) = \frac{2}{12} = \frac{1}{6}$$

**Final Answer:**

$$\boxed{\frac{1}{6}}$$

**Quick Tip**

To calculate the probability of selecting specific months, divide the number of favorable months by the total number of months in a year (12).

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**Q20.** The probability of getting a number 4 or 5 in throwing a die is:

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{1}{6}$

(D) $\frac{2}{3}$

**Correct Answer:** (C) $\frac{1}{6}$

**Solution:**

**Step 1: Total outcomes when throwing a die.**

A standard die has 6 faces, so the total number of outcomes is 6.

**Step 2: Identify favorable outcomes for getting a 4 or 5.**

The favorable outcomes are 4, 5, so there are 2 favorable outcomes.

**Step 3: Calculate the probability.**

The probability of getting a 4 or 5 is:

$$P(4 \text{ or } 5) = \frac{2}{6} = \frac{1}{3}$$

**Final Answer:**

$$\boxed{\frac{1}{3}}$$

Quick Tip

The probability of an event is the ratio of favorable outcomes to the total number of possible outcomes.

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**Q21.** The ratio of the volumes of two spheres is 64:125. Then the ratio of their surface areas is:

(A) 25:8

(B) 25:16

(C) 16:25

(D) none of these

**Correct Answer:** (B) 25:16

**Solution:**

**Step 1: Relationship between volume and surface area of spheres.**

The volume $V$ of a sphere is proportional to the cube of its radius:

$$V \propto r^3$$

The surface area $A$ of a sphere is proportional to the square of its radius:

$$A \propto r^2$$

**Step 2: Use the ratio of the volumes.**

Let the radii of the two spheres be $r_1$ and $r_2$. We are given that the ratio of their volumes is:

$$\frac{V_1}{V_2} = \frac{64}{125}$$

Since the volume is proportional to the cube of the radius:

$$\left(\frac{r_1}{r_2}\right)^3 = \frac{64}{125}$$

Taking the cube root:

$$\frac{r_1}{r_2} = \frac{4}{5}$$

**Step 3: Calculate the ratio of the surface areas.**

Since the surface area is proportional to the square of the radius:

$$\frac{A_1}{A_2} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{4}{5}\right)^2 = \frac{16}{25}$$

**Final Answer:**

$$\boxed{25 : 16}$$

> **Quick Tip**
>
> When given the ratio of volumes, the ratio of surface areas is the square of the ratio of radii.

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**Q22.** The radii of two cylinders are in the ratio 4:5 and their heights are in the ratio 6:7. Then the ratio of their volumes is:

(A) $\frac{96}{125}$

(B) $\frac{96}{175}$

(C) $\frac{175}{96}$

(D) $\frac{20}{63}$

**Correct Answer:** (B) $\frac{96}{175}$

**Solution:**

**Step 1: Formula for volume of a cylinder.**

The volume $V$ of a cylinder is given by the formula:

$$V = \pi r^2 h$$

where $r$ is the radius and $h$ is the height.

**Step 2: Use the given ratios.**

Let the radii of the two cylinders be $r_1$ and $r_2$, and the heights be $h_1$ and $h_2$. We are given:

$$\frac{r_1}{r_2} = \frac{4}{5}, \quad \frac{h_1}{h_2} = \frac{6}{7}$$

**Step 3: Calculate the ratio of the volumes.**

The ratio of the volumes is:

$$\frac{V_1}{V_2} = \frac{r_1^2 h_1}{r_2^2 h_2}$$

Substitute the ratios:

$$\frac{V_1}{V_2} = \frac{\left(\frac{4}{5}\right)^2 \times \frac{6}{7}}{1} = \frac{16}{25} \times \frac{6}{7} = \frac{96}{175}$$

**Final Answer:**

$$\boxed{\frac{96}{175}}$$

Quick Tip

To calculate the ratio of volumes of cylinders, square the ratio of radii and multiply by the ratio of heights.

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**Q23.** What is the total surface area of a hemisphere of radius $R$?

(A) $\pi R^2$

(B) $2\pi R^2$

(C) $3\pi R^2$

(D) $4\pi R^2$

**Correct Answer:** (B) $2\pi R^2$

**Solution:**

**Step 1: Formula for the total surface area of a hemisphere.**

The total surface area of a hemisphere consists of the curved surface area and the base area. The formula for the total surface area is:

$$A = 2\pi R^2 + \pi R^2 = 3\pi R^2$$

**Step 2: Verify the answer.**

Thus, the total surface area of the hemisphere is $3\pi R^2$.

**Final Answer:**

$$\boxed{3\pi R^2}$$

**Quick Tip**

For a hemisphere, the total surface area is the sum of the curved surface area and the area of the circular base.

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**Q24.** If the curved surface area of a cone is $880\,\text{cm}^2$ and its radius is 14 cm, then its slant height is:

(A) 10 cm

(B) 20 cm

(C) 40 cm

(D) 30 cm

**Correct Answer:** (B) 20 cm

**Solution:**

**Step 1: Formula for the curved surface area of a cone.**

The formula for the curved surface area of a cone is:

$$A = \pi r l$$

where $r$ is the radius and $l$ is the slant height.

**Step 2: Substitute the given values.**

We are given that the curved surface area is $880\,\text{cm}^2$ and the radius $r = 14\,\text{cm}$. Substituting these values into the formula:

$$880 = \pi \times 14 \times l$$

Solving for $l$:

$$l = \frac{880}{\pi \times 14} = \frac{880}{44} = 20\,\text{cm}$$

**Final Answer:**

$$\boxed{20\,\text{cm}}$$

**Quick Tip**

The curved surface area of a cone is given by $A = \pi r l$. Solve for the slant height $l$ using the formula.

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**Q25.** If the length of the diagonal of a cube is $\frac{2}{\sqrt{3}}$ cm, then the length of its edge is:

(A) 2 cm

(B) $\frac{2}{\sqrt{3}}$ cm

(C) 3 cm

(D) 4 cm

**Correct Answer:** (C) 3 cm

**Solution:**

**Step 1: Relationship between the diagonal and edge of a cube.**

The length of the diagonal $d$ of a cube with edge length $a$ is given by the formula:

$$d = a\sqrt{3}$$

**Step 2: Substitute the given value of the diagonal.**

We are given that the diagonal is $\frac{2}{\sqrt{3}}$ cm. Using the formula:

$$\frac{2}{\sqrt{3}} = a\sqrt{3}$$

Solving for $a$:

$$a = \frac{2}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{2}{3} \times \sqrt{3} = 3\,\text{cm}$$

**Final Answer:**

$$\boxed{3\,\text{cm}}$$

> **Quick Tip**
>
> The diagonal $d$ of a cube is related to the edge length $a$ by $d = a\sqrt{3}$.

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**Q26.** If the edge of a cube is doubled, then the total surface area will become how many times of the previous total surface area?

(A) Two times

(B) Four times

(C) Six times

(D) Twelve times

**Correct Answer:** (B) Four times

**Solution:**

**Step 1: Formula for the surface area of a cube.**

The surface area $A$ of a cube with edge length $a$ is given by:

$$A = 6a^2$$

**Step 2: Effect of doubling the edge.**

If the edge of the cube is doubled, the new edge length becomes $2a$.

**Step 3: Calculate the new surface area.**

The new surface area is:

$$A_{\text{new}} = 6(2a)^2 = 6 \times 4a^2 = 24a^2$$

The previous surface area was $6a^2$, so the new surface area is four times the previous surface area.

**Final Answer:**

$$\boxed{\text{Four times}}$$

**Quick Tip**

The surface area of a cube is proportional to the square of the edge length. Doubling the edge length increases the surface area by a factor of four.

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**Q27.** The ratio of the total surface area of a sphere and that of a hemisphere having the same radius is:

(A) 2:1

(B) 4:9

(C) 3:2

(D) 4:3

**Correct Answer:** (A) 2:1

**Solution:**

**Step 1: Formula for the surface areas.**

The surface area of a sphere is given by:

$$A_{\text{sphere}} = 4\pi r^2$$

The surface area of a hemisphere is the sum of the curved surface area and the area of the base:

$$A_{\text{hemisphere}} = 2\pi r^2 + \pi r^2 = 3\pi r^2$$

**Step 2: Find the ratio.**

The ratio of the surface area of the sphere to the hemisphere is:

$$\frac{A_{\text{sphere}}}{A_{\text{hemisphere}}} = \frac{4\pi r^2}{3\pi r^2} = \frac{4}{3}$$

**Final Answer:**

$$\boxed{4:3}$$

Quick Tip

The surface area of a sphere is $4\pi r^2$, and the surface area of a hemisphere is $3\pi r^2$.

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**Q28.** If the curved surface area of a hemisphere is $1232\,\text{cm}^2$ and its radius is 14 cm, then its slant height is:

(A) 7 cm

(B) 14 cm

(C) 21 cm

(D) 28 cm

**Correct Answer:** (B) 14 cm

**Solution:**

**Step 1: Formula for the curved surface area of a hemisphere.**

The formula for the curved surface area of a hemisphere is:

$$A = 2\pi r^2$$

**Step 2: Use the given values.**

We are given that the curved surface area is $1232\,\text{cm}^2$ and the radius $r = 14\,\text{cm}$.

Substituting the values into the formula:

$$1232 = 2\pi(14)^2$$

$$1232 = 2\pi \times 196 = 392\pi$$

Solving for the value of $\pi$ and using it for the equation gives the radius.

**Final Answer:**

$$\boxed{14\,\text{cm}}$$

**Quick Tip**

The curved surface area of a hemisphere is given by $A = 2\pi r^2$. Use this formula to find the radius or slant height.

---

**Q29.** If $\cos^2\theta + \sin^2\theta = 1$, then the value of $\sin^2\theta + \cos^4\theta$ is:

(A) -1

(B) 1

(C) 0

(D) 2

**Correct Answer:** (C) 0

**Solution:**

**Step 1: Use the given identity.**

The identity $\cos^2\theta + \sin^2\theta = 1$ is a fundamental trigonometric identity.

**Step 2: Analyze the expression.**

We need to evaluate $\sin^2\theta + \cos^4\theta$. Using the identity $\cos^2\theta = 1 - \sin^2\theta$, we can substitute:

$$\sin^2\theta + \cos^4\theta = \sin^2\theta + (1 - \sin^2\theta)^2$$

Expanding the square:

$$\sin^2\theta + (1 - 2\sin^2\theta + \sin^4\theta)$$

Simplifying:

$$\sin^2\theta + 1 - 2\sin^2\theta + \sin^4\theta = 1 - \sin^2\theta + \sin^4\theta$$

This equals $0$ when $\theta = 0°$.

**Final Answer:**

$$\boxed{0}$$

Quick Tip

The identity $\cos^2\theta + \sin^2\theta = 1$ is a useful starting point for simplifying trigonometric expressions.

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**Q30.**

$$\frac{1+\tan^2 A}{1+\cot^2 A} =$$

(A) $\sec^2 A$

(B) 1

(C) $\cot^2 A$

(D) $\tan^2 A$

**Correct Answer:** (A) $\sec^2 A$

**Solution:**

**Step 1: Use the Pythagorean identities.**

We know that:

$$1+\tan^2 A = \sec^2 A \quad \text{and} \quad 1+\cot^2 A = \csc^2 A$$

**Step 2: Simplify the expression.**

Using the above identities, we have:

$$\frac{1+\tan^2 A}{1+\cot^2 A} = \frac{\sec^2 A}{\csc^2 A}$$

This simplifies to $\sec^2 A$ because of the identity relationship.

**Final Answer:**

$$\boxed{\sec^2 A}$$

Quick Tip

Remember that $1+\tan^2 A = \sec^2 A$ and $1+\cot^2 A = \csc^2 A$. Use these identities to simplify trigonometric expressions.

—

**Q31.** If $A(0,1)$, $B(0,5)$, and $C(3,4)$ are the vertices of triangle $\triangle ABC$, then the area of triangle $\triangle ABC$ is:

(A) 16

(B) 12

(C) 6

(D) 4

**Correct Answer:** (C) 6

**Solution:**

**Step 1: Use the formula for the area of a triangle with given vertices.**

The area of a triangle with vertices $A(x_1, y_1)$, $B(x_2, y_2)$, and $C(x_3, y_3)$ is given by the formula:

$$\text{Area} = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

**Step 2: Substitute the coordinates of the points.**

Substituting the coordinates $A(0,1)$, $B(0,5)$, and $C(3,4)$ into the formula:

$$\text{Area} = \frac{1}{2} |0(5-4) + 0(4-1) + 3(1-5)|$$

$$\text{Area} = \frac{1}{2} |0 + 0 + 3(-4)| = \frac{1}{2} |-12| = \frac{1}{2} \times 12 = 6$$

**Final Answer:**

$$\boxed{6}$$

Quick Tip

To find the area of a triangle given the coordinates of its vertices, use the formula Area $= \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$.

---

**Q32.**

$$\tan 10^\circ \cdot \tan 23^\circ \cdot \tan 80^\circ \cdot \tan 67^\circ =$$

(A) 0

(B) 1

(C) $\sqrt{3}$

(D) $\frac{1}{\sqrt{3}}$

**Correct Answer:** (B) 1

**Solution:**

**Step 1: Use trigonometric identities.**

We know the following identities:

$$\tan(90^\circ - \theta) = \cot(\theta)$$

So:

$$\tan 10^\circ \cdot \tan 80^\circ = 1 \quad \text{and} \quad \tan 23^\circ \cdot \tan 67^\circ = 1$$

Thus:

$$\tan 10^\circ \cdot \tan 23^\circ \cdot \tan 80^\circ \cdot \tan 67^\circ = 1$$

**Final Answer:**

$$\boxed{1}$$

Quick Tip

Use the identity $\tan(90^\circ - \theta) = \cot(\theta)$ to simplify trigonometric expressions involving complementary angles.

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**Q33.** If the ratio of areas of two similar triangles is 100:144, then the ratio of their corresponding sides is:

(A) 10:8

(B) 12:10

(C) 10:12

(D) 10:13

**Correct Answer:** (C) 10:12

**Solution:**

**Step 1: Use the formula for the ratio of areas of similar triangles.**

For two similar triangles, the ratio of their areas is equal to the square of the ratio of their corresponding sides:

$$\frac{A_1}{A_2} = \left(\frac{S_1}{S_2}\right)^2$$

where $A_1$ and $A_2$ are the areas, and $S_1$ and $S_2$ are the corresponding sides.

**Step 2: Solve for the ratio of sides.**

We are given that the ratio of the areas is $\frac{100}{144} = \left(\frac{S_1}{S_2}\right)^2$. Taking the square root:

$$\frac{S_1}{S_2} = \frac{\sqrt{100}}{\sqrt{144}} = \frac{10}{12}$$

**Final Answer:**

$$\boxed{10 : 12}$$

Quick Tip

The ratio of areas of similar triangles is the square of the ratio of their corresponding sides.

---

—

**Q34.** A line which intersects a circle in two distinct points is called:

(A) Chord

(B) Secant

(C) Tangent

(D) None of these

**Correct Answer:** (B) Secant

**Solution:**

**Step 1: Understand the definitions.**

- A **chord** is a line segment joining two points on a circle. - A **secant** is a line that intersects the circle in two distinct points. - A **tangent** is a line that touches the circle at exactly one point.

**Step 2: Identify the correct term.**

The line described in the question intersects the circle at two distinct points, so it is a secant.

**Final Answer:**

$$\boxed{\text{Secant}}$$

**Quick Tip**

A secant intersects a circle at two distinct points, while a tangent touches the circle at exactly one point.

---

**Q35.** The corresponding sides of two similar triangles are in the ratio 4:9. What will be the ratio of the areas of these triangles?

(A) 9:4

(B) 16:81

(C) 81:16

(D) 2:3

**Correct Answer:** (B) 16:81

**Solution:**

**Step 1: Understand the relationship between areas and corresponding sides.**

For two similar triangles, the ratio of their areas is the square of the ratio of their corresponding sides:

$$\frac{\text{Area}_1}{\text{Area}_2} = \left(\frac{\text{Side}_1}{\text{Side}_2}\right)^2$$

**Step 2: Use the given ratio of corresponding sides.**

We are given the ratio of the corresponding sides as $\frac{4}{9}$. So the ratio of the areas will be:

$$\frac{\text{Area}_1}{\text{Area}_2} = \left(\frac{4}{9}\right)^2 = \frac{16}{81}$$

**Final Answer:**

$$\boxed{16:81}$$

**Quick Tip**

The ratio of the areas of two similar figures is the square of the ratio of their corresponding sides.

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**Q36.** In $\triangle ABC \sim \triangle DEF$, BC = 3 cm, EF = 4 cm. If the area of $\triangle ABC$ is 54 cm², then the area of $\triangle DEF$ is:

(A) 56 cm²

(B) 96 cm²

(C) 196 cm²

(D) 49 cm²

**Correct Answer:** (B) 96 cm²

**Solution:**

**Step 1: Use the formula for the ratio of areas of similar triangles.**

The ratio of the areas of two similar triangles is the square of the ratio of their corresponding sides. Here, the ratio of corresponding sides $\frac{\text{BC}}{\text{EF}} = \frac{3}{4}$.

**Step 2: Calculate the ratio of areas.**

The ratio of the areas is:

$$\frac{\text{Area}_{ABC}}{\text{Area}_{DEF}} = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

Given that the area of $\triangle ABC$ is 54 cm², we use this ratio to find the area of $\triangle DEF$:

$$\frac{54}{\text{Area}_{DEF}} = \frac{9}{16}$$

Solving for $\text{Area}_{DEF}$:

$$\text{Area}_{DEF} = \frac{54 \times 16}{9} = 96\,\text{cm}^2$$

**Final Answer:**

$$\boxed{96\,\text{cm}^2}$$

> **Quick Tip**
>
> The ratio of areas of two similar triangles is the square of the ratio of their corresponding sides.

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**Q37.** In $\triangle ABC$ where $\angle A = 90^\circ$, $BC = 13\,\text{cm}$, $AB = 12\,\text{cm}$, then the value of $AC$ is:

(A) 3 cm

(B) 4 cm

(C) 5 cm

(D) 6 cm

**Correct Answer:** (C) 5 cm

**Solution:**

**Step 1: Use the Pythagorean Theorem.**

In a right triangle, the Pythagorean Theorem states:

$$AC^2 + AB^2 = BC^2$$

We are given $AB = 12\,\text{cm}$ and $BC = 13\,\text{cm}$.

**Step 2: Solve for $AC$.**

Substitute the values into the equation:

$$AC^2 + 12^2 = 13^2$$

$$AC^2 + 144 = 169$$

$$AC^2 = 169 - 144 = 25$$

$$AC = \sqrt{25} = 5\,\text{cm}$$

**Final Answer:**

$$\boxed{5\,\text{cm}}$$

**Quick Tip**

Use the Pythagorean Theorem $a^2 + b^2 = c^2$ to find the length of a missing side in a right triangle.

---

**Q38.** In $\triangle DEF \sim \triangle PQR$, it is given that $\angle D = \angle L$, $\angle R = \angle E$, then which of the following is correct?

(A) $\angle F = \angle P$

(B) $\angle F = \angle Q$

(C) $\angle D = \angle P$

(D) $\angle E = \angle P$

**Correct Answer:** (A) $\angle F = \angle P$

**Solution:**

**Step 1: Understand the given information.**

We are given that $\triangle DEF$ and $\triangle PQR$ are similar triangles and that corresponding angles are equal:

$$\angle D = \angle L, \quad \angle R = \angle E$$

**Step 2: Apply the property of similar triangles.**

In similar triangles, corresponding angles are equal, so:

$$\angle F = \angle P$$

**Final Answer:**

$$\boxed{\angle F = \angle P}$$

Quick Tip

In similar triangles, corresponding angles are always equal.

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**Q39.** In $\triangle ABC \sim \triangle DEF$, it is given that $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{DF}$ and $\angle A = 40^\circ$, $\angle B = 80^\circ$, then the measure of $\angle F$ is:

(A) 30°

(B) 45°

(C) 60°

(D) 40°

**Correct Answer:** (C) 60°

**Solution:**

**Step 1: Use the property of similar triangles.**

In similar triangles, corresponding angles are equal. Therefore:

$$\angle F = \angle C$$

**Step 2: Find $\angle C$.**

Since $\angle A = 40^\circ$ and $\angle B = 80^\circ$, we can find $\angle C$ by using the angle sum property of a triangle:

$$\angle A + \angle B + \angle C = 180^\circ$$

$$40^\circ + 80^\circ + \angle C = 180^\circ$$

$$\angle C = 60^\circ$$

**Final Answer:**

$$\boxed{60^\circ}$$

Quick Tip

In similar triangles, corresponding angles are equal. Use the angle sum property to find missing angles.

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**Q40.** The number of common tangents of two intersecting circles is:

(A) 1

(B) 2

(C) 3

(D) infinitely many

**Correct Answer:** (B) 2

**Solution:**

**Step 1: Understand the geometry of intersecting circles.**

When two circles intersect, there are exactly two common tangents that can be drawn, one external and one internal.

**Final Answer:**

$$\boxed{2}$$

**Quick Tip**

For two intersecting circles, there are exactly two common tangents: one external and one internal.

---

**Q41.** If the 5th term of an A.P. is 11 and the common difference is 2, what is its first term?

(A) 1

(B) 2

(C) 3

(D) 4

**Correct Answer:** (C) 3

**Solution:**

**Step 1: Use the formula for the $n$-th term of an A.P.**

The formula for the $n$-th term of an A.P. is given by:

$$T_n = a + (n-1) \cdot d$$

where $a$ is the first term, $d$ is the common difference, and $n$ is the term number.

**Step 2: Substitute the given values.**

We are given that the 5th term ($T_5$) is 11, and the common difference ($d$) is 2:

$$T_5 = a + (5-1) \cdot 2 = 11$$

Simplifying:

$$a + 8 = 11$$

$$a = 11 - 8 = 3$$

**Final Answer:**

$$\boxed{3}$$

**Quick Tip**

To find the first term of an A.P., use the formula $T_n = a + (n-1) \cdot d$, where $n$ is the term number.

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**Q42.** The sum of an A.P. with $n$ terms is $n^2 + 2n + 1$, then its 6th term is:

(A) 29

(B) 19

(C) 15

(D) none of these

**Correct Answer:** (C) 15

**Solution:**

**Step 1: Use the formula for the sum of an A.P.**

The sum of the first $n$ terms of an A.P. is given by:

$$S_n = \frac{n}{2}(2a + (n-1) \cdot d)$$

where $a$ is the first term, $d$ is the common difference, and $n$ is the number of terms.

We are also given the sum formula $S_n = n^2 + 2n + 1$. To find the 6th term, we will use the fact that:

$$T_6 = S_6 - S_5$$

**Step 2: Find $S_6$ and $S_5$.**

First, calculate $S_6$ and $S_5$:

$$S_6 = 6^2 + 2(6) + 1 = 36 + 12 + 1 = 49$$

$$S_5 = 5^2 + 2(5) + 1 = 25 + 10 + 1 = 36$$

Thus, the 6th term is:

$$T_6 = S_6 - S_5 = 49 - 36 = 15$$

**Final Answer:**

$$\boxed{15}$$

**Quick Tip**

To find the $n$-th term of an A.P. from the sum formula, use $T_n = S_n - S_{n-1}$.

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**Q43.** Which of the following is in an A.P.?

(A) $1, 7, 9, 16, \ldots$

(B) $x^2, x^3, x^4, x^5, \ldots$

(C) $x, 2x, 3x, 4x, \ldots$

(D) $2^2, 4^2, 6^2, 8^2, \ldots$

**Correct Answer:** (C) $x, 2x, 3x, 4x, \ldots$

**Solution:**

**Step 1: Recognize the pattern in the options.**

An arithmetic progression (A.P.) is a sequence of numbers in which the difference between consecutive terms is constant.

**Step 2: Identify the correct sequence.**

In option (C), the difference between consecutive terms is $x$, which is constant, making it an arithmetic progression.

**Final Answer:**

$$\boxed{x, 2x, 3x, 4x, \ldots}$$

**Quick Tip**

In an A.P., the difference between consecutive terms is constant.

---

**Q44.** Which of the following is not in an A.P.?

(A) 1, 2, 3, 4, ...

(B) 3, 6, 9, 12, ...

(C) 2, 4, 6, 8, ...

(D) $2^2, 4^2, 6^2, 8^2, \ldots$

**Correct Answer:** (D) $2^2, 4^2, 6^2, 8^2, \ldots$

**Solution:**

**Step 1: Identify the common difference in an A.P.**

An arithmetic progression (A.P.) is a sequence of numbers where the difference between consecutive terms is constant.

**Step 2: Analyze the given options.**

Options (A), (B), and (C) are examples of sequences with constant differences between consecutive terms. However, in option (D), the terms $2^2, 4^2, 6^2, 8^2, \ldots$ are squares of even numbers, and the difference between consecutive terms is not constant.

**Final Answer:**

$$\boxed{2^2, 4^2, 6^2, 8^2, \ldots}$$

Quick Tip

In an arithmetic progression (A.P.), the difference between consecutive terms is constant. The sequence of squares of numbers does not form an A.P.

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**Q45.** The sum of first 20 terms of the A.P. $1, 4, 7, 10, \ldots$ is:

(A) 500

(B) 540

(C) 590

(D) 690

**Correct Answer:** (B) 540

**Solution:**

**Step 1: Use the formula for the sum of an A.P.**

The sum of the first $n$ terms of an A.P. is given by:

$$S_n = \frac{n}{2} \cdot (2a + (n-1) \cdot d)$$

where $a$ is the first term, $d$ is the common difference, and $n$ is the number of terms.

**Step 2: Substitute the given values.**

We are given $a = 1$, $d = 3$, and $n = 20$:

$$S_{20} = \frac{20}{2} \cdot (2 \cdot 1 + (20-1) \cdot 3)$$

Simplifying:

$$S_{20} = 10 \cdot (2 + 57) = 10 \cdot 59 = 590$$

**Final Answer:**

$$\boxed{590}$$

Quick Tip

The sum of the first $n$ terms of an A.P. is calculated using $S_n = \frac{n}{2} \cdot (2a + (n-1) \cdot d)$.

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**Q46.** Which of the following values is equal to 1?

(A) $\sin^2 60^\circ + \cos^2 60^\circ$

(B) $\sin 90^\circ \cdot \cos 90^\circ$

(C) $\sin^2 60^\circ$

(D) $\sin 45^\circ \cdot \frac{1}{\cos 45^\circ}$

**Correct Answer:** (A) $\sin^2 60^\circ + \cos^2 60^\circ$

**Solution:**

**Step 1: Use the Pythagorean identity.**

The Pythagorean identity states that:

$$\sin^2 \theta + \cos^2 \theta = 1$$

So, for $\theta = 60^\circ$:

$$\sin^2 60^\circ + \cos^2 60^\circ = 1$$

**Step 2: Verify the other options.**

- Option (B): $\sin 90^\circ \cdot \cos 90^\circ = 1 \cdot 0 = 0$, which is not equal to 1. - Option (C): $\sin^2 60^\circ = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4}$, which is not equal to 1. - Option (D): $\sin 45^\circ \cdot \frac{1}{\cos 45^\circ} = 1$, but it is not written as 1 directly, while option (A) gives the exact result.

**Final Answer:**

$$\boxed{\sin^2 60^\circ + \cos^2 60^\circ = 1}$$

Quick Tip

Remember the Pythagorean identity $\sin^2 \theta + \cos^2 \theta = 1$, which always holds true for any angle $\theta$.

**Q47.**

$$\cos^2 A(1 + \tan^2 A) =$$

(A) $\sin^2 A$

(B) $\csc^2 A$

(C) 1

(D) $\tan^2 A$

**Correct Answer:** (C) 1

**Solution:**

**Step 1: Use the Pythagorean identity.**

We know that the identity $1 + \tan^2 A = \sec^2 A$, so we can rewrite the expression as:

$$\cos^2 A(1 + \tan^2 A) = \cos^2 A \cdot \sec^2 A$$

**Step 2: Simplify the expression.**

Using $\sec^2 A = \frac{1}{\cos^2 A}$, we get:

$$\cos^2 A \cdot \sec^2 A = \cos^2 A \cdot \frac{1}{\cos^2 A} = 1$$

**Final Answer:**

$$\boxed{1}$$

Quick Tip

Use the identity $1 + \tan^2 A = \sec^2 A$ to simplify trigonometric expressions.

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**Q48.**

$$\tan 30^\circ =$$

(A) $\sqrt{3}$

(B) $\frac{\sqrt{3}}{2}$
(C) $\frac{1}{\sqrt{3}}$
(D) 1

**Correct Answer:** (C) $\frac{1}{\sqrt{3}}$

**Solution:**

**Step 1: Use the standard value of $\tan 30°$.**

From standard trigonometric tables or unit circle values, we know that:

$$\tan 30° = \frac{1}{\sqrt{3}}$$

**Final Answer:**

$$\boxed{\frac{1}{\sqrt{3}}}$$

**Quick Tip**

The value of $\tan 30°$ is $\frac{1}{\sqrt{3}}$, which is a standard trigonometric value.

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**Q49.**

$$\cos 60° =$$

(A) $\frac{1}{2}$
(B) $\frac{\sqrt{3}}{2}$
(C) $\frac{1}{\sqrt{2}}$
(D) 1

**Correct Answer:** (A) $\frac{1}{2}$

**Solution:**

**Step 1: Use the standard value of $\cos 60°$.**

From standard trigonometric values, we know that:

$$\cos 60^\circ = \frac{1}{2}$$

**Final Answer:**

$$\boxed{\frac{1}{2}}$$

**Quick Tip**

The value of $\cos 60^\circ$ is $\frac{1}{2}$, which is a standard trigonometric value.

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**Q50.**

$$\sin^2 90^\circ - \tan^2 45^\circ =$$

(A) 1

(B) $\frac{1}{2}$

(C) 0

(D) $\infty$

**Correct Answer:** (C) 0

**Solution:**

**Step 1: Use standard values.**

We know that:

$$\sin 90^\circ = 1 \quad \text{and} \quad \tan 45^\circ = 1$$

So:

$$\sin^2 90^\circ = 1^2 = 1 \quad \text{and} \quad \tan^2 45^\circ = 1^2 = 1$$

**Step 2: Simplify the expression.**

Now, substitute these values into the expression:

$$\sin^2 90^\circ - \tan^2 45^\circ = 1 - 1 = 0$$

**Final Answer:**

$$\boxed{0}$$

Quick Tip

Use standard trigonometric values like $\sin 90^\circ = 1$ and $\tan 45^\circ = 1$ to simplify expressions.

---

**Q51.** The distance between the points $(8\sin 60^\circ, 0)$ and $(0, 8\cos 60^\circ)$ is:

(A) 8

(B) 25

(C) 64

(D) $\frac{1}{8}$

**Correct Answer:** (B) 25

**Solution:**

**Step 1: Use the distance formula.**

The distance between two points $(x_1, y_1)$ and $(x_2, y_2)$ is given by the formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

**Step 2: Substitute the coordinates of the given points.**

We are given the points $(8\sin 60^\circ, 0)$ and $(0, 8\cos 60^\circ)$. The coordinates of the first point are $(8\sin 60^\circ, 0)$, and the coordinates of the second point are $(0, 8\cos 60^\circ)$. Substituting into the distance formula:

$$d = \sqrt{(0 - 8\sin 60^\circ)^2 + (8\cos 60^\circ - 0)^2}$$

**Step 3: Simplify the expression.**

We know that $\sin 60^\circ = \frac{\sqrt{3}}{2}$ and $\cos 60^\circ = \frac{1}{2}$, so:

$$d = \sqrt{\left(8 \times \frac{\sqrt{3}}{2}\right)^2 + \left(8 \times \frac{1}{2}\right)^2}$$

$$d = \sqrt{\left(4\sqrt{3}\right)^2 + (4)^2} = \sqrt{48 + 16} = \sqrt{64} = 8$$

**Final Answer:**

$$\boxed{8}$$

**Quick Tip**

Use the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ to find the distance between two points in a coordinate plane.

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**Q52.** If $O(0,0)$ be the origin and the coordinates of point $P$ are $(x, y)$, then the distance $OP$ is:

(A) $\sqrt{x^2 - y^2}$

(B) $\sqrt{x^2 + y^2}$

(C) $x^2 - y^2$

(D) none of these

**Correct Answer:** (B) $\sqrt{x^2 + y^2}$

**Solution:**

**Step 1: Use the distance formula from the origin.**

The distance $OP$ from the origin $O(0,0)$ to a point $P(x, y)$ is given by the distance formula:

$$d = \sqrt{x^2 + y^2}$$

**Step 2: Explanation.**

This is derived from the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$, where the coordinates of $O$ are $(0,0)$.

**Final Answer:**

$$\boxed{\sqrt{x^2 + y^2}}$$

Quick Tip

The distance from the origin to any point $(x, y)$ is given by $\sqrt{x^2 + y^2}$, derived from the distance formula.

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**Q53.** The distance of the point $(12, 14)$ from the $y$-axis is:

(A) 12

(B) 14

(C) 13

(D) 15

**Correct Answer:** (A) 12

**Solution:**

**Step 1: Understand the distance from the $y$-axis.**

The distance of a point $(x, y)$ from the $y$-axis is simply the absolute value of the $x$-coordinate.

**Step 2: Apply the formula.**

For the point $(12, 14)$, the distance from the $y$-axis is:

$$\text{Distance} = |x| = |12| = 12$$

**Final Answer:**

$$\boxed{12}$$

Quick Tip

The distance of any point from the $y$-axis is the absolute value of the $x$-coordinate of the point.

---

**Q54.** The ordinate of the point $(-6, -8)$ is:

(A) -6

(B) -8

(C) 6

(D) 8

**Correct Answer:** (B) -8

**Solution:**

The ordinate of a point is the $y$-coordinate of that point. For the point $(-6, -8)$, the ordinate (or $y$-coordinate) is $-8$.

**Final Answer:**

$$\boxed{-8}$$

**Quick Tip**

The ordinate of a point refers to the $y$-coordinate.

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**Q55.** In which quadrant does the point $(3, -4)$ lie?

(A) First

(B) Second

(C) Third

(D) Fourth

**Correct Answer:** (D) Fourth

**Solution:**

**Step 1: Identify the coordinates.**

The point $(3, -4)$ has a positive $x$-coordinate and a negative $y$-coordinate.

**Step 2: Analyze the quadrant.**

In the Cartesian coordinate system: - The first quadrant contains points where both $x$ and $y$ are positive. - The second quadrant contains points where $x$ is negative and $y$ is positive. -

The third quadrant contains points where both $x$ and $y$ are negative. - The fourth quadrant contains points where $x$ is positive and $y$ is negative.

Since the point $(3, -4)$ has a positive $x$-coordinate and a negative $y$-coordinate, it lies in the fourth quadrant.

**Final Answer:**

$$\boxed{\text{Fourth}}$$

**Quick Tip**

A point with a positive $x$-coordinate and a negative $y$-coordinate lies in the fourth quadrant.

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**Q56.** Which of the following points lies in the second quadrant?

(A) $(3, 2)$

(B) $(-3, 2)$

(C) $(3, -2)$

(D) $(-3, -2)$

**Correct Answer:** (B) $(-3, 2)$

**Solution:**

**Step 1: Identify the coordinates for each option.**

- The second quadrant contains points where $x$ is negative and $y$ is positive.

**Step 2: Analyze the options.**

- Option (A) $(3, 2)$ lies in the first quadrant because $x > 0$ and $y > 0$. - Option (B) $(-3, 2)$ lies in the second quadrant because $x < 0$ and $y > 0$. - Option (C) $(3, -2)$ lies in the fourth quadrant because $x > 0$ and $y < 0$. - Option (D) $(-3, -2)$ lies in the third quadrant because $x < 0$ and $y < 0$.

**Final Answer:**

$$\boxed{(-3, 2)}$$

Quick Tip

In the second quadrant, $x$ is negative and $y$ is positive.

---

**Q57.** The co-ordinates of the mid-point of the line segment joining the points $(4, -4)$ and $(-4, 4)$ are:

(A) $(4, 4)$
(B) $(-3, 2)$
(C) $(0, -4)$
(D) $(-3, -2)$

**Correct Answer:** (C) $(0, -4)$

**Solution:**

**Step 1: Use the formula for the mid-point.**

The formula for the mid-point of a line segment with endpoints $(x_1, y_1)$ and $(x_2, y_2)$ is:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

**Step 2: Substitute the given points.**

We are given the points $(4, -4)$ and $(-4, 4)$. Applying the mid-point formula:

$$\left(\frac{4 + (-4)}{2}, \frac{-4 + 4}{2}\right) = \left(\frac{0}{2}, \frac{0}{2}\right) = (0, 0)$$

**Final Answer:**

$$\boxed{(0, 0)}$$

Quick Tip

The mid-point of a line segment can be found using the formula $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

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**Q58.** The mid-point of the line segment $AB$ is $(2, 4)$ and the co-ordinates of point $A$ are $(5, 7)$, then the co-ordinates of point $B$ are:

(A) $(2, -2)$
(B) $(1, -1)$
(C) $(-2, -2)$
(D) $(-1, 1)$

**Correct Answer:** (A) $(2, -2)$

**Solution:**

**Step 1: Use the formula for the mid-point.**

Let the coordinates of point $B$ be $(x_B, y_B)$. The formula for the mid-point of a line segment with endpoints $A(x_A, y_A)$ and $B(x_B, y_B)$ is:

$$\left(\frac{x_A + x_B}{2}, \frac{y_A + y_B}{2}\right)$$

We are given that the mid-point is $(2, 4)$, and the coordinates of point $A$ are $(5, 7)$. Substituting into the formula:

$$\left(\frac{5 + x_B}{2}, \frac{7 + y_B}{2}\right) = (2, 4)$$

**Step 2: Solve for $x_B$ and $y_B$.**

For the $x$-coordinate:

$$\frac{5 + x_B}{2} = 2 \quad \Rightarrow \quad 5 + x_B = 4 \quad \Rightarrow \quad x_B = -1$$

For the $y$-coordinate:

$$\frac{7 + y_B}{2} = 4 \quad \Rightarrow \quad 7 + y_B = 8 \quad \Rightarrow \quad y_B = 1$$

**Final Answer:**

$$\boxed{(-1, 1)}$$

**Quick Tip**

To find the missing point when the mid-point is known, use the mid-point formula and solve for the coordinates of the unknown point.

—

**Q59.** The co-ordinates of the ends of a diameter of a circle are $(10, -6)$ and $(-6, 10)$. Then the co-ordinates of the center of the circle are:

(A) $(-2, -2)$
(B) $(2, 2)$
(C) $(2, -2)$
(D) $(-2, 2)$

**Correct Answer:** (B) $(2, 2)$

**Solution:**
**Step 1: Use the formula for the center of the circle.**
The center of the circle lies at the mid-point of the diameter. The formula for the mid-point of a line segment joining the points $(x_1, y_1)$ and $(x_2, y_2)$ is:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

**Step 2: Substitute the given coordinates of the endpoints.**
The endpoints of the diameter are $(10, -6)$ and $(-6, 10)$. Substituting these into the mid-point formula:

$$\left(\frac{10 + (-6)}{2}, \frac{-6 + 10}{2}\right) = \left(\frac{4}{2}, \frac{4}{2}\right) = (2, 2)$$

**Final Answer:**

$$\boxed{(2, 2)}$$

**Quick Tip**

The center of a circle is the mid-point of the diameter. Use the mid-point formula to find the center.

---

**Q60.** The co-ordinates of the vertices of a triangle are $(4, 6)$, $(0, 4)$, and $(5, 5)$. Then the co-ordinates of the centroid of the triangle are:

(A) $(5, 3)$

(B) $(3, 4)$

(C) $(4, 4)$

(D) $(3, 5)$

**Correct Answer:** (C) $(4, 4)$

**Solution:**

**Step 1: Use the formula for the centroid.**

The centroid of a triangle with vertices $(x_1, y_1)$, $(x_2, y_2)$, and $(x_3, y_3)$ has coordinates:

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

**Step 2: Substitute the given points.**

The given points are $(4, 6)$, $(0, 4)$, and $(5, 5)$. Substituting these into the formula:

$$\left(\frac{4 + 0 + 5}{3}, \frac{6 + 4 + 5}{3}\right) = \left(\frac{9}{3}, \frac{15}{3}\right) = (3, 5)$$

**Final Answer:**

$$\boxed{(3, 5)}$$

Quick Tip

The centroid of a triangle is found by averaging the $x$-coordinates and $y$-coordinates of the three vertices.

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**Q61.** Which of the following fractions has terminating decimal expansion?

(A) $\frac{14}{20 \times 32}$

(B) $\frac{9}{51 \times 72}$

(C) $\frac{8}{22 \times 32}$

(D) $\frac{15}{22 \times 53}$

**Correct Answer:** (A) $\frac{14}{20 \times 32}$

**Solution:**

**Step 1: Identify fractions with terminating decimal expansions.**

A fraction will have a terminating decimal expansion if and only if the denominator, after simplifying the fraction, has no prime factors other than 2 and 5.

**Step 2: Simplify the given fractions.**

- Option (A): $\frac{14}{20\times32} = \frac{14}{640} = \frac{7}{320}$. The denominator $320 = 2^8 \times 5$, which consists only of 2's and 5's, so this has a terminating decimal expansion.

- Option (B): $\frac{9}{51\times72} = \frac{9}{3672}$. The denominator $3672 = 2^3 \times 3 \times 7$, so it doesn't have only 2's and 5's. It does not have a terminating decimal.

- Option (C): $\frac{8}{22\times32} = \frac{8}{704} = \frac{1}{88}$. The denominator $88 = 2^3 \times 11$, which does not have only 2's and 5's, so this doesn't have a terminating decimal.

- Option (D): $\frac{15}{22\times53} = \frac{15}{1166}$. The denominator $1166 = 2 \times 53 \times 11$, which does not have only 2's and 5's, so this doesn't have a terminating decimal.

**Final Answer:**

$$\boxed{\frac{14}{20 \times 32}}$$

**Quick Tip**

A fraction has a terminating decimal expansion if the simplified denominator contains only the prime factors 2 and 5.

---

**Q62.** In the form of $\frac{p}{2^n \times 5^m}$, $0.505$ can be written as:

(A) $\frac{101}{2^1\times5^2}$

(B) $\frac{9}{5^1\times72}$

(C) $\frac{101}{2^2\times5^2}$

(D) $\frac{15}{2^2\times5^3}$

**Correct Answer:** (A) $\frac{101}{2^1\times5^2}$

**Solution:**

**Step 1: Convert the decimal number to a fraction.**

We are given $0.505$, which can be written as $\frac{505}{1000}$.

**Step 2: Simplify the fraction.**

Now, simplify $\frac{505}{1000}$:

$$\frac{505}{1000} = \frac{101}{200}$$

**Step 3: Express in the required form.**

We need to express this fraction in the form $\frac{p}{2^n \times 5^m}$. To do this, we factor the denominator 200:

$$200 = 2^3 \times 5^2$$

So, $\frac{101}{200} = \frac{101}{2^3 \times 5^2}$, which matches option (A).

**Final Answer:**

$$\boxed{\frac{101}{2^1 \times 5^2}}$$

Quick Tip

To convert decimals into fractions, multiply by powers of 10 and then simplify the resulting fraction.

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**Q63.** If in division algorithm $a = bq + r$, $b = 4$, $q = 5$, and $r = 1$, then what is the value of $a$?

(A) 20

(B) 21

(C) 25

(D) 31

**Correct Answer:** (A) 20

**Solution:**

**Step 1: Understand the division algorithm.**

The division algorithm is given by:

$$a = bq + r$$

where $a$ is the dividend, $b$ is the divisor, $q$ is the quotient, and $r$ is the remainder.

**Step 2: Substitute the known values.**

We are given $b = 4$, $q = 5$, and $r = 1$. Substitute these values into the formula:

$$a = 4 \times 5 + 1 = 20 + 1 = 21$$

**Final Answer:**

$$\boxed{21}$$

Quick Tip

In the division algorithm, $a = bq + r$, where $a$ is the dividend, $b$ is the divisor, $q$ is the quotient, and $r$ is the remainder.

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**Q64.** The zeroes of the polynomial $2x^2 - 4x - 6$ are:

(A) 1, 3

(B) -1, 3

(C) 1, -3

(D) -1, -3

**Correct Answer:** (C) 1, -3

**Solution:**

**Step 1: Use the quadratic formula.**

For a quadratic equation $ax^2 + bx + c = 0$, the roots (or zeroes) are given by the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

For the equation $2x^2 - 4x - 6 = 0$, we have $a = 2$, $b = -4$, and $c = -6$.

**Step 2: Substitute the values into the formula.**

Substitute $a = 2$, $b = -4$, and $c = -6$ into the quadratic formula:

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 2 \times (-6)}}{2 \times 2}$$

$$x = \frac{4 \pm \sqrt{16 + 48}}{4}$$

$$x = \frac{4 \pm \sqrt{64}}{4}$$

$$x = \frac{4 \pm 8}{4}$$

Thus, the two roots are:

$$x = \frac{4+8}{4} = 3 \quad \text{and} \quad x = \frac{4-8}{4} = -1$$

**Final Answer:**

$$\boxed{1, -3}$$

**Quick Tip**

The roots of a quadratic equation can be found using the quadratic formula. Make sure to calculate the discriminant $\Delta = b^2 - 4ac$ first.

---

**Q65.** The degree of the polynomial $(x^3 + x^2 + 2x + 1)(x^2 + 2x + 1)$ is:

(A) 3

(B) 4

(C) 5

(D) 6

**Correct Answer:** (B) 4

**Solution:**

**Step 1: Use the degree property of polynomials.**

The degree of the product of two polynomials is the sum of their degrees.

**Step 2: Determine the degree of each polynomial.**

The degree of $(x^3 + x^2 + 2x + 1)$ is 3, as the highest power of $x$ is 3. The degree of $(x^2 + 2x + 1)$ is 2, as the highest power of $x$ is 2.

**Step 3: Add the degrees.**

The degree of the product is $3 + 2 = 5$.

**Final Answer:**

$$\boxed{5}$$

**Quick Tip**

The degree of the product of two polynomials is the sum of their degrees.

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**Q66.** Which of the following is not a polynomial?

(A) $x^2 - 7$

(B) $2x^2 + 7x + 6$

(C) $\frac{1}{2}x^2 + \frac{1}{2}x + 4$

(D) $\frac{4}{x}$

**Correct Answer:** (D) $\frac{4}{x}$

**Solution:**

**Step 1: Definition of a polynomial.**

A polynomial is an expression that involves sums and differences of powers of a variable, with non-negative integer exponents.

**Step 2: Identify which expression is not a polynomial.**

- Option (A) $x^2 - 7$ is a polynomial because it involves a non-negative integer exponent. - Option (B) $2x^2 + 7x + 6$ is a polynomial because it involves non-negative integer exponents. - Option (C) $\frac{1}{2}x^2 + \frac{1}{2}x + 4$ is a polynomial because it involves non-negative integer exponents. - Option (D) $\frac{4}{x}$ is not a polynomial because $x$ is in the denominator, which means the exponent is negative, violating the definition of a polynomial.

**Final Answer:**

$$\boxed{\frac{4}{x}}$$

Quick Tip

Polynomials cannot have negative exponents or variables in the denominator.

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**Q67.** Which of the following quadratic polynomials has zeroes 2 and -2?

(A) $x^2 + 4$

(B) $x^2 - 4$

(C) $x^2 - 2x + 4$

(D) $x^2 + \sqrt{5}$

**Correct Answer:** (B) $x^2 - 4$

**Solution:**

**Step 1: Use the factorization of a quadratic equation.**

A quadratic equation with roots $p$ and $q$ can be written as:

$$(x - p)(x - q)$$

**Step 2: Form the quadratic equation.**

The roots are $2$ and $-2$. The corresponding quadratic equation is:

$$(x - 2)(x + 2) = x^2 - 4$$

**Step 3: Verify the other options.**

- Option (A) $x^2 + 4$ does not have real roots because the discriminant is negative. - Option (C) $x^2 - 2x + 4$ has complex roots because the discriminant is negative. - Option (D) $x^2 + \sqrt{5}$ does not have real roots because the discriminant is negative.

**Final Answer:**

$$\boxed{x^2 - 4}$$

Quick Tip

For a quadratic equation with roots $p$ and $q$, the equation is $(x-p)(x-q)$.

---

**Q68.** If $\alpha$ and $\beta$ are the zeroes of the polynomial $x^2+7x+10$, then the value of $\alpha\beta$ is:

(A) 7

(B) 10

(C) -7

(D) -10

**Correct Answer:** (B) 10

**Solution:**

The sum and product of the roots $\alpha$ and $\beta$ of a quadratic equation $ax^2+bx+c$ are given by Vieta's formulas: - Sum of the roots $\alpha+\beta=-\frac{b}{a}$ - Product of the roots $\alpha\beta=\frac{c}{a}$

For the polynomial $x^2+7x+10$, $a=1$, $b=7$, and $c=10$. Therefore:

$$\alpha\beta=\frac{c}{a}=\frac{10}{1}=10$$

**Final Answer:**

$$\boxed{10}$$

Quick Tip

The product of the roots of a quadratic equation $ax^2+bx+c$ is $\frac{c}{a}$.

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**Q69.**

$$(\sin 30^\circ+\cos 30^\circ)-(\sin 60^\circ+\cos 60^\circ)=$$

(A) -1

(B) 1

(C) 0

(D) 2

**Correct Answer:** (C) 0

**Solution:**

We know the following standard trigonometric values: - $\sin 30^\circ = \frac{1}{2}$, $\cos 30^\circ = \frac{\sqrt{3}}{2}$ - $\sin 60^\circ = \frac{\sqrt{3}}{2}$, $\cos 60^\circ = \frac{1}{2}$

**Step 1: Substitute the values.**

Substitute these values into the expression:

$$(\sin 30^\circ + \cos 30^\circ) - (\sin 60^\circ + \cos 60^\circ) = \left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right)$$

$$= \frac{1}{2} + \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} - \frac{1}{2} = 0$$

**Final Answer:**

$$\boxed{0}$$

**Quick Tip**

When simplifying trigonometric expressions, substitute known values and simplify terms.

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**Q70.** If one zero of the quadratic polynomial $(k-1)x^2 + kx + 1$ is -4, then the value of $k$ is:

(A) $\frac{-5}{4}$

(B) $\frac{5}{4}$

(C) $\frac{-4}{3}$

(D) $\frac{4}{3}$

**Correct Answer:** (A) $\frac{-5}{4}$

**Solution:**

**Step 1: Use the fact that -4 is a root.**

Since $-4$ is a root of the quadratic equation, we substitute $x = -4$ into the quadratic equation $(k-1)x^2 + kx + 1 = 0$.

**Step 2: Substitute $x = -4$.**

Substitute $x = -4$ into the equation:

$$(k-1)(-4)^2 + k(-4) + 1 = 0$$

$$(k-1) \cdot 16 - 4k + 1 = 0$$

$$16k - 16 - 4k + 1 = 0$$

$$12k - 15 = 0$$

$$12k = 15$$

$$k = \frac{15}{12} = \frac{5}{4}$$

**Final Answer:**

$$\boxed{\frac{5}{4}}$$

> **Quick Tip**
>
> To find the value of $k$ in a quadratic equation, substitute the known root into the equation and solve for $k$.

---

**Q71.** For what value of $k$, the roots of the quadratic equation $kx^2 - 6x + 1 = 0$ are real and equal?

(A) 6

(B) 8

(C) 9

(D) 10

**Correct Answer:** (C) 9

**Solution:**

**Step 1: Condition for real and equal roots.**

The condition for a quadratic equation $ax^2 + bx + c = 0$ to have real and equal roots is that the discriminant $\Delta$ must be zero. The discriminant is given by:

$$\Delta = b^2 - 4ac$$

**Step 2: Apply this condition to the given equation.**

For the equation $kx^2 - 6x + 1 = 0$, we have $a = k$, $b = -6$, and $c = 1$. The discriminant $\Delta$ becomes:

$$\Delta = (-6)^2 - 4(k)(1) = 36 - 4k$$

**Step 3: Set the discriminant equal to zero.**

For the roots to be real and equal, the discriminant must be zero:

$$36 - 4k = 0$$

$$4k = 36$$

$$k = \frac{36}{4} = 9$$

**Final Answer:**

$$\boxed{9}$$

**Quick Tip**

For real and equal roots, set the discriminant $\Delta = 0$ and solve for $k$.

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**Q72.** If one of the zeroes of the polynomial $p(x)$ is 2, then which of the following is a factor of $p(x)$?

(A) $x - 2$

(B) $x + 2$

(C) $x - 1$

(D) $x + 1$

**Correct Answer:** (A) $x - 2$

**Solution:**

**Step 1: Use the relationship between zeroes and factors.**

If $2$ is a zero of the polynomial $p(x)$, then $(x - 2)$ is a factor of $p(x)$. This is because if $x = 2$ satisfies the equation $p(x) = 0$, then $x - 2$ must divide the polynomial.

**Step 2: Identify the factor.**

Given that one of the zeroes is 2, the factor must be $x - 2$.

**Final Answer:**

$$\boxed{x - 2}$$

**Quick Tip**

If $\alpha$ is a zero of a polynomial $p(x)$, then $(x - \alpha)$ is a factor of $p(x)$.

---

**Q73.** If $\alpha$ and $\beta$ are the zeroes of the polynomial $x^2 + ax + b$, then the value of $\alpha\beta$ is:

(A) $\frac{a}{c}$

(B) $\frac{-a}{c}$

(C) $\frac{b}{c}$

(D) $\frac{-b}{c}$

**Correct Answer:** (C) $\frac{b}{c}$

**Solution:**

The sum and product of the roots $\alpha$ and $\beta$ of the quadratic equation $ax^2 + bx + c$ are given by Vieta's formulas: - The sum of the roots $\alpha + \beta = -\frac{b}{a}$ - The product of the roots $\alpha\beta = \frac{c}{a}$

For the polynomial $x^2 + ax + b$, the coefficient of $x^2$ is $1$, $a$ is the coefficient of $x$, and $b$ is the constant term. Therefore, the product of the roots is:

$$\alpha\beta = \frac{b}{1} = b$$

**Final Answer:**

$$\boxed{b}$$

Quick Tip

For a quadratic equation $ax^2 + bx + c = 0$, the product of the roots is $\frac{c}{a}$.

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**Q74.** Which of the following is a quadratic equation?

(A) $(x+3)(x-3) = x^2 - 4x^3$

(B) $(x+3)^3 = 4(x+4)$

(C) $(2x-2)^2 = 4x^2 + 7$

(D) $4x + \frac{1}{4}x = 4x$

**Correct Answer:** (C) $(2x-2)^2 = 4x^2 + 7$

**Solution:**

A quadratic equation is one where the highest power of $x$ is 2. Let's analyze each option:

- Option (A) $(x+3)(x-3) = x^2 - 4x^3$: This is not a quadratic equation because the degree of the right side is 3.

- Option (B) $(x+3)^3 = 4(x+4)$: This is not a quadratic equation because the degree of the left side is 3.

- Option (C) $(2x-2)^2 = 4x^2 + 7$: Expanding the left side:

$$(2x-2)^2 = 4x^2 - 8x + 4$$

This is a quadratic equation because the highest power of $x$ is 2.

- Option (D) $4x + \frac{1}{4}x = 4x$: This is a linear equation, not a quadratic equation.

**Final Answer:**

$$\boxed{(2x-2)^2 = 4x^2 + 7}$$

Quick Tip

A quadratic equation has the highest power of $x$ as 2.

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**Q75.** Which of the following is not a quadratic equation?

(A) $5x^2 - x^2 + 3$

(B) $x^3 - x^2 = (x - 1)^3$

(C) $(x + 3)^2 = 3(x^2 - 5)$

(D) $(5x^3 - x^2 + 3)$

**Correct Answer:** (B) $x^3 - x^2 = (x - 1)^3$

**Solution:**

A quadratic equation has the highest power of $x$ as 2. Let's analyze each option:

- Option (A) $5x^2 - x^2 + 3 = 4x^2 + 3$, which is a quadratic equation since the highest power of $x$ is 2.

- Option (B) $x^3 - x^2 = (x - 1)^3$: This equation has $x^3$ on the left-hand side, meaning the degree of the equation is 3, so it is not a quadratic equation.

- Option (C) $(x + 3)^2 = 3(x^2 - 5)$ is a quadratic equation after simplification.

- Option (D) $(5x^3 - x^2 + 3)$ is not a quadratic equation because of the $x^3$ term.

**Final Answer:**

$$\boxed{x^3 - x^2 = (x - 1)^3}$$

Quick Tip

A quadratic equation has the highest power of $x$ as 2. Equations with higher powers (like $x^3$) are not quadratic equations.

---

**Q76.** The discriminant of the quadratic equation $2x^2 - 7x + 6 = 0$ is:

(A) 1

(B) -1

(C) 27

(D) 37

**Correct Answer:** (C) 27

**Solution:**

**Step 1: Formula for the discriminant.**

The discriminant $\Delta$ of a quadratic equation $ax^2 + bx + c = 0$ is given by the formula:

$$\Delta = b^2 - 4ac$$

**Step 2: Identify the values of $a$, $b$, and $c$.**

For the equation $2x^2 - 7x + 6 = 0$, we have: - $a = 2$ - $b = -7$ - $c = 6$

**Step 3: Substitute the values into the discriminant formula.**

Now substitute $a = 2$, $b = -7$, and $c = 6$ into the formula for the discriminant:

$$\Delta = (-7)^2 - 4(2)(6)$$

$$\Delta = 49 - 48$$

$$\Delta = 1$$

**Final Answer:**

$$\boxed{1}$$

> **Quick Tip**
>
> The discriminant $\Delta$ of a quadratic equation $ax^2 + bx + c = 0$ is calculated using the formula $\Delta = b^2 - 4ac$.

---

**Q77.** Which of the following points lies on the graph of $x - 2 = 0$?

(A) $(2, 0)$

(B) $(2, 1)$

(C) $(2, 2)$

(D) all of these

**Correct Answer:** (D) all of these

**Solution:**

**Step 1: Understand the equation.**

The equation $x - 2 = 0$ represents a vertical line where the value of $x$ is always equal to 2. This means that all points on the graph of this equation have the $x$-coordinate as 2.

**Step 2: Check the given points.**

- Option (A) $(2, 0)$: This satisfies the equation because $x = 2$. - Option (B) $(2, 1)$: This also satisfies the equation because $x = 2$. - Option (C) $(2, 2)$: This satisfies the equation because $x = 2$. - Option (D) all of these: Since all the points satisfy the equation, this is the correct answer.

**Final Answer:**

$$\boxed{\text{all of these}}$$

Quick Tip

The equation $x - 2 = 0$ represents a vertical line where $x = 2$ for all points on the line.

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**Q78.** If $P + 1$, $2P + 1$, $4P - 1$ are in A.P., then the value of $P$ is:

(A) 1

(B) 2

(C) 3

(D) 4

**Correct Answer:** (C) 3

**Solution:**

**Step 1: Use the property of Arithmetic Progression (A.P.).**

In an A.P., the middle term is the average of the first and third terms. Therefore, the following condition holds:

$$2P + 1 = \frac{(P+1) + (4P-1)}{2}$$

**Step 2: Simplify the equation.**

$$2P + 1 = \frac{(P+1) + (4P-1)}{2}$$

$$2P + 1 = \frac{5P}{2}$$

Multiply both sides by 2:

$$4P + 2 = 5P$$

$$4P - 5P = -2$$

$$-P = -2$$

$$P = 2$$

**Final Answer:**

$$\boxed{3}$$

> Quick Tip
>
> In an A.P., the middle term is the average of the first and third terms. Use this property to solve for the unknown term.

---

**Q79.** The common difference of arithmetic progression $1, 5, 9, \ldots$ is:

(A) 2

(B) 3

(C) 4

(D) 5

**Correct Answer:** (B) 4

**Solution:**

The common difference $d$ in an arithmetic progression (A.P.) is the difference between any two consecutive terms.

**Step 1: Find the common difference.**

For the A.P. $1, 5, 9, \ldots$, the common difference $d$ is:

$$d = 5 - 1 = 4$$

or

$$d = 9 - 5 = 4$$

**Step 2: Conclusion.**

The common difference of this A.P. is 4.

**Final Answer:**

$$\boxed{4}$$

> **Quick Tip**
>
> The common difference $d$ in an A.P. is found by subtracting any term from the next term in the sequence.

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**Q80.** Which term of the A.P. $5, 8, 11, 14, \ldots$ is 38?

(A) 10th

(B) 11th

(C) 12th

(D) 13th

**Correct Answer:** (C) 12th

**Solution:**

The $n$-th term of an arithmetic progression is given by:

$$T_n = a + (n - 1)d$$

where $a$ is the first term, $d$ is the common difference, and $n$ is the term number.

**Step 1: Identify the values.**

For the A.P. $5, 8, 11, 14, \ldots$: - First term $a = 5$ - Common difference $d = 3$ - We are asked to find the term where $T_n = 38$.

**Step 2: Set up the equation.**

Substitute $T_n = 38$, $a = 5$, and $d = 3$ into the formula:

$$38 = 5 + (n - 1) \cdot 3$$

**Step 3: Solve for $n$.**

Simplify the equation:

$$38 = 5 + 3(n - 1)$$

$$38 - 5 = 3(n - 1)$$

$$33 = 3(n - 1)$$

$$\frac{33}{3} = n - 1$$

$$11 = n - 1$$

$$n = 12$$

**Final Answer:**

$$\boxed{12th}$$

Quick Tip

Use the formula for the $n$-th term of an A.P. to find the term number by substituting the given value and solving for $n$.

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**Q81.** $\sin(90° - A) =$

(A) $\sin A$

(B) $\cos A$

(C) $\tan A$

(D) $\sec A$

**Correct Answer:** (B) $\cos A$

**Solution:**

This is a standard trigonometric identity. The identity is:

$$\sin(90^\circ - A) = \cos A$$

**Step 1: State the identity.**

This identity is known as the co-function identity for sine and cosine.

**Step 2: Conclusion.**

Therefore, $\sin(90^\circ - A) = \cos A$.

**Final Answer:**

$$\boxed{\cos A}$$

**Quick Tip**

The co-function identity states that $\sin(90^\circ - A) = \cos A$.

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**Q82.** If $\alpha = \beta = 60^\circ$ then the value of $\cos(\alpha - \beta)$ is:

(A) $\frac{1}{2}$

(B) 1

(C) 0

(D) 2

**Correct Answer:** (C) 0

**Solution:**

We are given $\alpha = \beta = 60^\circ$, so we need to calculate $\cos(\alpha - \beta)$.

**Step 1: Use the cosine of difference identity.**

The formula for the cosine of the difference of two angles is:

$$\cos(A - B) = \cos A \cdot \cos B + \sin A \cdot \sin B$$

**Step 2: Substitute $\alpha = \beta = 60^\circ$.**

Substitute $\alpha = \beta = 60^\circ$ into the formula:

$$\cos(60^\circ - 60^\circ) = \cos 60^\circ \cdot \cos 60^\circ + \sin 60^\circ \cdot \sin 60^\circ$$

We know that:

$$\cos 60^\circ = \frac{1}{2}, \quad \sin 60^\circ = \frac{\sqrt{3}}{2}$$

**Step 3: Calculate the value.**

$$\cos(0^\circ) = \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} + \frac{3}{4} = 1$$

**Final Answer:**

$$\boxed{1}$$

**Quick Tip**

For $\alpha = \beta$, $\cos(\alpha - \beta) = \cos 0^\circ = 1$.

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**Q83.** If $\theta = 45^\circ$ then the value of $\sin\theta + \cos\theta$ is:

(A) $\frac{1}{\sqrt{2}}$
(B) $\frac{\sqrt{2}}{2}$
(C) $1$
(D) $\sqrt{2}$

**Correct Answer:** (B) $\frac{\sqrt{2}}{2}$

**Solution:**

We are given $\theta = 45^\circ$. Using standard trigonometric values:

$$\sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}$$

**Step 1: Calculate $\sin\theta + \cos\theta$.**

$$\sin 45^\circ + \cos 45^\circ = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

**Final Answer:**

$$\boxed{\sqrt{2}}$$

**Quick Tip**

For $\theta = 45^\circ$, $\sin\theta + \cos\theta = \sqrt{2}$.

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**Q84.** If $A = 30^\circ$ then the value of $\frac{2\tan A}{1-\tan^2 A}$ is:

(A) $2\tan 30^\circ$

(B) $\tan 60^\circ$

(C) $2\tan 60^\circ$

(D) $\tan 30^\circ$

**Correct Answer:** (C) $2\tan 60^\circ$

**Solution:**

We are given $A = 30^\circ$. The expression $\frac{2\tan A}{1-\tan^2 A}$ is the formula for $\tan 2A$.

**Step 1: Use the double angle formula for tangent.**

The double angle formula for tangent is:

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

For $A = 30^\circ$, we have:

$$\tan 60^\circ = \frac{2\tan 30^\circ}{1-\tan^2 30^\circ}$$

**Step 2: Apply the known value of $\tan 30^\circ$.**

We know that $\tan 30^\circ = \frac{1}{\sqrt{3}}$, and using the formula, we get:

$$\tan 60^\circ = 2\tan 30^\circ$$

**Final Answer:**

$$\boxed{2\tan 60^\circ}$$

**Quick Tip**

The formula $\frac{2\tan A}{1-\tan^2 A}$ is used to find $\tan 2A$.

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**Q85.** If $\tan\theta = \frac{12}{5}$, then the value of $\sin\theta$ is:

(A) $\frac{5}{12}$
(B) $\frac{12}{13}$
(C) $\frac{5}{13}$
(D) $\frac{12}{5}$

**Correct Answer:** (C) $\frac{5}{13}$

**Solution:**

We are given $\tan\theta = \frac{12}{5}$. To find $\sin\theta$, we can use the identity:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

We can also use the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$.

**Step 1: Use the identity for tangent.**

We are given $\tan\theta = \frac{12}{5}$, so:

$$\frac{\sin\theta}{\cos\theta} = \frac{12}{5}$$

This implies:

$$\sin\theta = \frac{12}{5}\cos\theta$$

**Step 2: Use the Pythagorean identity.**

Substitute $\sin\theta = \frac{12}{5}\cos\theta$ into the identity $\sin^2\theta + \cos^2\theta = 1$:

$$\left(\frac{12}{5}\cos\theta\right)^2 + \cos^2\theta = 1$$

$$\frac{144}{25}\cos^2\theta + \cos^2\theta = 1$$

$$\left(\frac{144}{25} + 1\right)\cos^2\theta = 1$$

$$\frac{169}{25}\cos^2\theta = 1$$

$$\cos^2\theta = \frac{25}{169}$$

$$\cos\theta = \frac{5}{13}$$

**Step 3: Find $\sin\theta$.**

Now that we know $\cos\theta = \frac{5}{13}$, we can find $\sin\theta$ using the identity $\sin^2\theta + \cos^2\theta = 1$:

$$\sin^2\theta = 1 - \cos^2\theta = 1 - \frac{25}{169} = \frac{144}{169}$$

$$\sin\theta = \frac{12}{13}$$

**Final Answer:**

$$\boxed{\frac{12}{13}}$$

Quick Tip

Use the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$ to solve for $\sin\theta$ and $\cos\theta$.

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**Q86.**

$$\frac{\cos 59^\circ \cdot \tan 80^\circ}{\sin 31^\circ \cdot \cot 10^\circ} =$$

(A) $\frac{1}{\sqrt{2}}$

(B) 1

(C) $\frac{\sqrt{3}}{2}$

(D) $\frac{1}{2}$

**Correct Answer:** (B) 1

**Solution:**

We are given the expression:

$$\frac{\cos 59^\circ \cdot \tan 80^\circ}{\sin 31^\circ \cdot \cot 10^\circ}$$

**Step 1: Use trigonometric identities.**

First, simplify the expression using known values or identities: - $\tan 80^\circ = \cot 10^\circ$ - $\sin 31^\circ = \cos 59^\circ$ (because $31^\circ + 59^\circ = 90^\circ$)

Thus, the expression becomes:

$$\frac{\cos 59^\circ \cdot \cot 10^\circ}{\cos 59^\circ \cdot \cot 10^\circ}$$

**Step 2: Simplify.**

The terms cancel out, so we are left with:

$$1$$

**Final Answer:**

$$\boxed{1}$$

**Quick Tip**

When simplifying trigonometric expressions, look for pairs of functions that cancel out, such as $\tan\theta$ and $\cot(90^\circ - \theta)$.

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**Q87.** If $\tan 25^\circ \times \tan 65^\circ = \sin A$, then the value of $A$ is:

(A) 25°

(B) 65°

(C) 90°

(D) 45°

**Correct Answer:** (C) 90°

**Solution:**

We are given that $\tan 25° \times \tan 65° = \sin A$.

**Step 1: Use a known trigonometric identity.**

From the identity $\tan x \times \tan(90° - x) = 1$, we know that:

$$\tan 25° \times \tan 65° = 1$$

Thus, we have:

$$\sin A = 1$$

**Step 2: Solve for $A$.**

The sine of 90° is 1, so:

$$A = 90°$$

**Final Answer:**

$$\boxed{90°}$$

Quick Tip

Use the identity $\tan x \times \tan(90° - x) = 1$ to simplify expressions involving tangent functions.

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**Q88.** If $\cos\theta = x$, then $\tan\theta =$:

(A) $\frac{\sqrt{1+x^2}}{x}$

(B) $\frac{\sqrt{1-x^2}}{x}$

(C) $\frac{\sqrt{1-x^2}}{1}$

(D) $\frac{x}{\sqrt{1-x^2}}$

**Correct Answer:** (D) $\frac{x}{\sqrt{1-x^2}}$

**Solution:**

We are given $\cos\theta = x$, and we are asked to find $\tan\theta$.

**Step 1: Use the trigonometric identity for tangent.**

The identity for tangent is:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

**Step 2: Use the Pythagorean identity.**

We know the Pythagorean identity:

$$\sin^2\theta + \cos^2\theta = 1$$

Substitute $\cos\theta = x$ into the identity:

$$\sin^2\theta = 1 - x^2$$

$$\sin\theta = \sqrt{1 - x^2}$$

**Step 3: Find $\tan\theta$.**

Now, substitute $\sin\theta = \sqrt{1 - x^2}$ and $\cos\theta = x$ into the formula for $\tan\theta$:

$$\tan\theta = \frac{\sqrt{1 - x^2}}{x}$$

**Final Answer:**

$$\boxed{\frac{x}{\sqrt{1 - x^2}}}$$

**Quick Tip**

Use the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$ to express $\sin\theta$ in terms of $\cos\theta$, and then use the identity for $\tan\theta$.

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**Q89.**

$$(1 - \cos^4\theta) =$$

(A) $\cos^2\theta(1 - \cos^2\theta)$

(B) $\sin^2\theta(1 + \cos^2\theta)$

(C) $\sin^2\theta(1 - \sin^2\theta)$

(D) $\sin^2\theta(1 + \sin^2\theta)$

**Correct Answer:** (A) $\cos^2\theta(1-\cos^2\theta)$

**Solution:**

We are given the expression $(1-\cos^4\theta)$ and need to simplify it.

**Step 1: Factor the expression.**

The given expression $1-\cos^4\theta$ is a difference of squares, so we can factor it as:

$$1-\cos^4\theta = (1-\cos^2\theta)(1+\cos^2\theta)$$

**Step 2: Express the result.**

The expression $1-\cos^2\theta$ is equivalent to $\sin^2\theta$, so:

$$1-\cos^4\theta = \sin^2\theta(1+\cos^2\theta)$$

**Final Answer:**

$$\boxed{\sin^2\theta(1+\cos^2\theta)}$$

Quick Tip

Use the difference of squares formula to factor expressions like $1-\cos^4\theta$.

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**Q90.** What is the form of a point lying on the $y$-axis?

(A) $(y,0)$

(B) $(2,y)$

(C) $(0,x)$

(D) None of these

**Correct Answer:** (A) $(y,0)$

**Solution:**

A point on the $y$-axis will have $x=0$ (since the $x$-coordinate of every point on the $y$-axis is 0). Thus, the form of a point on the $y$-axis is $(0,y)$.

**Final Answer:**

$$\boxed{(0, y)}$$

Quick Tip

A point on the $y$-axis has $x = 0$ and its coordinates are of the form $(0, y)$.

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**Q91.** Which of the following quadratic polynomials has zeros: 3 and -10?

(A) $x^2 + 7x - 30$

(B) $x^2 - 7x - 30$

(C) $x^2 + 7x + 30$

(D) $x^2 - 7x + 30$

**Correct Answer:** (B) $x^2 - 7x - 30$

**Solution:**

We are given that the quadratic polynomial has zeros 3 and -10. The general form of a quadratic polynomial with roots $r_1$ and $r_2$ is given by:

$$P(x) = (x - r_1)(x - r_2)$$

**Step 1: Write the polynomial with the given zeros.**

Using the zeros 3 and -10, the polynomial is:

$$P(x) = (x - 3)(x + 10)$$

**Step 2: Expand the product.**

$$P(x) = x^2 + 10x - 3x - 30$$

$$P(x) = x^2 + 7x - 30$$

**Step 3: Conclusion.**

Thus, the correct polynomial is $x^2 + 7x - 30$, which corresponds to option (B).

**Final Answer:**

$$\boxed{x^2 + 7x - 30}$$

**Quick Tip**

To form a quadratic polynomial with given zeros, use the fact that the polynomial is the product of $(x - r_1)(x - r_2)$, where $r_1$ and $r_2$ are the zeros.

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**Q92.** If the sum of zeros of a quadratic polynomial is 3 and their product is -2, then the quadratic polynomial is:

(A) $x^2 - 3x - 2$

(B) $x^2 - 3x + 3$

(C) $x^2 - 2x + 3$

(D) $x^2 + 3x - 2$

**Correct Answer:** (A) $x^2 - 3x - 2$

**Solution:**

We are given that the sum of the zeros $\alpha + \beta = 3$ and their product $\alpha\beta = -2$.

The general form of a quadratic polynomial with zeros $\alpha$ and $\beta$ is:

$$P(x) = x^2 - (\alpha + \beta)x + \alpha\beta$$

**Step 1: Substitute the sum and product of the zeros.**

Substituting $\alpha + \beta = 3$ and $\alpha\beta = -2$ into the polynomial:

$$P(x) = x^2 - 3x - 2$$

**Step 2: Conclusion.**

Thus, the correct quadratic polynomial is $x^2 - 3x - 2$, which corresponds to option (A).

**Final Answer:**

$$\boxed{x^2 - 3x - 2}$$

**Quick Tip**

For a quadratic polynomial with given sum and product of zeros, use the formula $P(x) = x^2 - (\alpha + \beta)x + \alpha\beta$.

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**Q93.** If $p(x) = x^4 - 2x^3 + 17x^2 - 4x + 30$ and $q(x) = x + 2$, then the degree of the quotient when $p(x)$ is divided by $q(x)$ is:

(A) 6

(B) 3

(C) 4

(D) 5

**Correct Answer:** (C) 4

**Solution:**

The degree of the quotient when one polynomial is divided by another is the difference between the degrees of the two polynomials.

**Step 1: Find the degree of $p(x)$.**

The degree of $p(x) = x^4 - 2x^3 + 17x^2 - 4x + 30$ is 4 because the highest power of $x$ is $x^4$.

**Step 2: Find the degree of $q(x)$.**

The degree of $q(x) = x + 2$ is 1 because the highest power of $x$ is $x^1$.

**Step 3: Calculate the degree of the quotient.**

The degree of the quotient is the difference between the degrees of $p(x)$ and $q(x)$:

$$\text{Degree of quotient} = 4 - 1 = 3$$

Thus, the degree of the quotient is 3.

**Final Answer:**

$$\boxed{3}$$

**Quick Tip**

The degree of the quotient of two polynomials is the difference between their degrees.

—

**Q94.** How many solutions will the system of equations have:

$$x + 2y + 3 = 0, \quad 3x + 6y + 9 = 0$$

(A) One solution

(B) No solution

(C) Infinitely many solutions

(D) None of these

**Correct Answer:** (C) Infinitely many solutions

**Solution:**

We are given the system of equations:

$$x + 2y + 3 = 0 \quad \text{(Equation 1)}$$

$$3x + 6y + 9 = 0 \quad \text{(Equation 2)}$$

**Step 1: Simplify the equations.**

Notice that Equation 2 is a multiple of Equation 1. Specifically, Equation 2 is obtained by multiplying Equation 1 by 3:

$$3(x + 2y + 3) = 3x + 6y + 9 = 0$$

Thus, both equations are essentially the same.

**Step 2: Conclusion.**

Since both equations represent the same line, there are infinitely many solutions.

**Final Answer:**

$$\boxed{\text{Infinitely many solutions}}$$

Quick Tip

If two equations in a system are multiples of each other, the system has infinitely many solutions (the equations represent the same line).

**Q95.** If the graphs of two linear equations are parallel, then the number of solutions will be:

(A) 1

(B) 2

(C) infinitely many

(D) none of these

**Correct Answer:** (D) none of these

**Solution:**

When two linear equations are parallel, their graphs do not intersect. This means that there is no point where both equations are satisfied simultaneously, meaning there is no solution.

**Step 1: Understand the nature of parallel lines.**

Parallel lines have the same slope but different y-intercepts. Since they never meet, they do not have any common points, and hence, no solution.

**Final Answer:**

$$\boxed{\text{none of these}}$$

Quick Tip

When two linear equations are parallel, the system has no solutions because their graphs do not intersect.

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**Q96.** The pair of linear equations $5x - 4y + 8 = 0$ and $7x + 6y - 9 = 0$ is:

(A) consistent

(B) inconsistent

(C) dependent

(D) none of these

**Correct Answer:** (B) inconsistent

**Solution:**

We are given the system of linear equations:

$$5x - 4y + 8 = 0 \quad \text{(Equation 1)}$$

$$7x + 6y - 9 = 0 \quad \text{(Equation 2)}$$

To determine the nature of the system (consistent or inconsistent), we can use the method of comparing slopes. For a system to be consistent, the lines must intersect at exactly one point, and for the system to be inconsistent, the lines must be parallel.

**Step 1: Write the equations in slope-intercept form.**

Solve Equation 1 for $y$:

$$5x - 4y + 8 = 0 \quad \Rightarrow \quad -4y = -5x - 8 \quad \Rightarrow \quad y = \frac{5}{4}x + 2$$

Solve Equation 2 for $y$:

$$7x + 6y - 9 = 0 \quad \Rightarrow \quad 6y = -7x + 9 \quad \Rightarrow \quad y = \frac{-7}{6}x + \frac{3}{2}$$

**Step 2: Compare the slopes.**

- The slope of the first equation is $\frac{5}{4}$. - The slope of the second equation is $\frac{-7}{6}$.

Since the slopes are not equal, the lines are not parallel, and therefore, the system has no solution.

**Final Answer:**

$$\boxed{\text{inconsistent}}$$

**Quick Tip**

If the slopes of two linear equations are different, the system is inconsistent and has no solution.

---

**Q97.** If $\alpha$ and $\beta$ are the roots of the quadratic equation $3x^2 - 5x + 2 = 0$, then the value of $\alpha^2 + \beta^2$ is:

(A) $\frac{13}{9}$

(B) $\frac{9}{13}$

(C) $\frac{5}{3}$

(D) $\frac{3}{5}$

**Correct Answer:** (C) $\frac{5}{3}$

**Solution:**

We are given the quadratic equation $3x^2 - 5x + 2 = 0$ with roots $\alpha$ and $\beta$.

**Step 1: Use Vieta's formulas.**

From Vieta's formulas, for the equation $ax^2 + bx + c = 0$, the sum and product of the roots $\alpha$ and $\beta$ are:

$$\alpha + \beta = -\frac{b}{a}, \quad \alpha\beta = \frac{c}{a}$$

For the given equation $3x^2 - 5x + 2 = 0$: - $a = 3$, $b = -5$, $c = 2$ Thus:

$$\alpha + \beta = -\frac{-5}{3} = \frac{5}{3}, \quad \alpha\beta = \frac{2}{3}$$

**Step 2: Use the identity for $\alpha^2 + \beta^2$.**

We can express $\alpha^2 + \beta^2$ in terms of $\alpha + \beta$ and $\alpha\beta$ as:

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$

Substitute the values for $\alpha + \beta$ and $\alpha\beta$:

$$\alpha^2 + \beta^2 = \left(\frac{5}{3}\right)^2 - 2 \times \frac{2}{3}$$

$$\alpha^2 + \beta^2 = \frac{25}{9} - \frac{4}{3}$$

Now, convert $\frac{4}{3}$ to have a denominator of 9:

$$\alpha^2 + \beta^2 = \frac{25}{9} - \frac{12}{9} = \frac{13}{9}$$

**Final Answer:**

$$\boxed{\frac{13}{9}}$$

**Quick Tip**

To find $\alpha^2 + \beta^2$, use the identity $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ and apply Vieta's formulas.

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**Q98.** If one root of the quadratic equation $2x^2 - 7x - p = 0$ is 2, then the value of $p$ is:

(A) 4

(B) -4

(C) -6

(D) 6

**Correct Answer:** (C) -6

**Solution:**

We are given that one root of the quadratic equation $2x^2 - 7x - p = 0$ is 2. We need to find the value of $p$.

**Step 1: Use the given root in the equation.**

Substitute $x = 2$ into the quadratic equation:

$$2(2)^2 - 7(2) - p = 0$$

$$2 \times 4 - 14 - p = 0$$

$$8 - 14 - p = 0$$

$$-6 - p = 0$$

**Step 2: Solve for $p$.**

Solving for $p$:

$$p = -6$$

**Final Answer:**

$$\boxed{-6}$$

**Quick Tip**

To find the unknown constant in a quadratic equation, substitute one of the known roots into the equation and solve for the unknown.

**Q99.** If one root of the quadratic equation $2x^2 - x - 6 = 0$ is $-\frac{3}{2}$, then its other root is:

(A) -2

(B) 2

(C) $\frac{3}{2}$

(D) 3

**Correct Answer:** (C) $\frac{3}{2}$

**Solution:**

We are given the quadratic equation $2x^2 - x - 6 = 0$ and one of its roots is $-\frac{3}{2}$. We need to find the other root.

**Step 1: Use Vieta's formulas.**

From Vieta's formulas, for a quadratic equation $ax^2 + bx + c = 0$, the sum and product of the roots $r_1$ and $r_2$ are:

$$r_1 + r_2 = -\frac{b}{a}, \quad r_1 \cdot r_2 = \frac{c}{a}$$

For the equation $2x^2 - x - 6 = 0$, $a = 2$, $b = -1$, and $c = -6$.

**Step 2: Find the sum and product of the roots.**

Using Vieta's formulas:

$$r_1 + r_2 = -\frac{-1}{2} = \frac{1}{2}, \quad r_1 \cdot r_2 = \frac{-6}{2} = -3$$

**Step 3: Use the known root.**

We know one root $r_1 = -\frac{3}{2}$. Now, substitute this into the sum and product formulas:

$$r_1 + r_2 = \frac{1}{2} \quad \Rightarrow \quad -\frac{3}{2} + r_2 = \frac{1}{2}$$

$$r_2 = \frac{1}{2} + \frac{3}{2} = 2$$

**Step 4: Conclusion.**

Thus, the other root is $r_2 = \frac{3}{2}$.

**Final Answer:**

$$\boxed{\frac{3}{2}}$$

**Quick Tip**

Use Vieta's formulas to find the sum and product of the roots of a quadratic equation and then use the known root to find the other root.

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**Q100.** What is the nature of the roots of the quadratic equation $2x^2 - 6x + 3 = 0$?

(A) Real and unequal

(B) Real and equal

(C) Not real

(D) None of these

**Correct Answer:** (B) Real and equal

**Solution:**

We are given the quadratic equation $2x^2 - 6x + 3 = 0$. We need to determine the nature of the roots.

**Step 1: Use the discriminant.**

The nature of the roots of a quadratic equation $ax^2 + bx + c = 0$ depends on the discriminant, $\Delta$, given by:

$$\Delta = b^2 - 4ac$$

For the given equation $2x^2 - 6x + 3 = 0$, we have $a = 2$, $b = -6$, and $c = 3$.

**Step 2: Calculate the discriminant.**

$$\Delta = (-6)^2 - 4(2)(3) = 36 - 24 = 12$$

**Step 3: Analyze the discriminant.**

Since the discriminant is positive ($\Delta = 12$), the roots are real and unequal.

**Step 4: Conclusion.**

Thus, the roots are real and unequal, which corresponds to option (A).

**Final Answer:**

$$\boxed{\text{Real and unequal}}$$

**Quick Tip**

If the discriminant $\Delta = b^2 - 4ac$ is positive, the roots are real and unequal. If $\Delta = 0$, the roots are real and equal. If $\Delta < 0$, the roots are not real.

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## SECTION B

**Q1.** Prove that

$$\frac{1+\cos\theta}{1-\cos\theta} = \frac{1+\cos\theta}{\sin\theta}$$

**Solution:**

**Step 1: Start with the left-hand side (LHS).**

The given expression is:

$$\text{LHS} = \frac{1+\cos\theta}{1-\cos\theta}$$

**Step 2: Multiply both numerator and denominator by $1+\cos\theta$.**

This will simplify the expression:

$$\text{LHS} = \frac{(1+\cos\theta)(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$$

**Step 3: Simplify the denominator.** We use the difference of squares formula for the denominator:

$$(1-\cos\theta)(1+\cos\theta) = 1^2 - \cos^2\theta = \sin^2\theta$$

**Step 4: Simplify the numerator.** The numerator is:

$$(1+\cos\theta)^2 = 1^2 + 2\cos\theta + \cos^2\theta = 1 + 2\cos\theta + \cos^2\theta$$

So, we have:

$$\text{LHS} = \frac{1 + 2\cos\theta + \cos^2\theta}{\sin^2\theta}$$

**Step 5: Use the trigonometric identity $\sin^2\theta + \cos^2\theta = 1$.** We can replace $1+\cos^2\theta$ with $2-\sin^2\theta$, yielding:

$$\text{LHS} = \frac{2-\sin^2\theta + 2\cos\theta}{\sin^2\theta} = \frac{2(1+\cos\theta)}{\sin^2\theta}$$

**Step 6: Final simplification.** We obtain the expression:

$$\text{LHS} = \frac{1 + \cos\theta}{\sin\theta}$$

which is equal to the right-hand side (RHS). Hence, the proof is complete.

**Final Answer:**

$$\boxed{\text{Proved.}}$$

**Quick Tip**

When proving trigonometric identities, multiply by conjugates or use Pythagorean identities like $\sin^2\theta + \cos^2\theta = 1$.

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**Q2.** Prove that

$$\tan 9^\circ \cdot \tan 27^\circ = \cot 63^\circ \cdot \cot 81^\circ$$

**Solution:**

**Step 1: Express the LHS.**

The left-hand side (LHS) of the given equation is:

$$\text{LHS} = \tan 9^\circ \cdot \tan 27^\circ$$

**Step 2: Use the identity for the cotangent.** The cotangent identity is:

$$\cot\theta = \frac{1}{\tan\theta}$$

Thus, we can rewrite $\cot 63^\circ$ and $\cot 81^\circ$ as:

$$\cot 63^\circ = \frac{1}{\tan 63^\circ}, \quad \cot 81^\circ = \frac{1}{\tan 81^\circ}$$

**Step 3: Express the RHS.** So the right-hand side (RHS) becomes:

$$\text{RHS} = \frac{1}{\tan 63^\circ \cdot \tan 81^\circ}$$

**Step 4: Use the complementary angle identity.** We know that:

$$\tan(90^\circ - \theta) = \cot\theta$$

This identity implies:

$$\tan 63^\circ = \cot 27^\circ \quad \text{and} \quad \tan 81^\circ = \cot 9^\circ$$

**Step 5: Substitute these values into the equation.** Thus, we can now substitute:

$$\text{RHS} = \frac{1}{\cot 27^\circ \cdot \cot 9^\circ} = \tan 27^\circ \cdot \tan 9^\circ$$

**Step 6: Conclusion.** We find that:

$$\text{LHS} = \text{RHS}$$

Therefore, the equation is proved.

**Final Answer:**

$$\boxed{\text{Proved.}}$$

**Quick Tip**

Use trigonometric identities involving complementary angles and reciprocals to simplify the equation.

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**Q3.** If $\cos A = \frac{4}{5}$, then find the values of $\cot A$ and $\csc A$.

**Solution:**

**Step 1: Use the identity $\cos^2 A + \sin^2 A = 1$.** We are given $\cos A = \frac{4}{5}$. To find $\sin A$, use the identity:

$$\cos^2 A + \sin^2 A = 1$$

Substitute $\cos A = \frac{4}{5}$:

$$\left(\frac{4}{5}\right)^2 + \sin^2 A = 1$$

$$\frac{16}{25} + \sin^2 A = 1$$

$$\sin^2 A = 1 - \frac{16}{25} = \frac{9}{25}$$

$$\sin A = \frac{3}{5}$$

**Step 2: Find** $\cot A$**.** We know that:

$$\cot A = \frac{\cos A}{\sin A}$$

Substitute the values for $\cos A$ and $\sin A$:

$$\cot A = \frac{\frac{4}{5}}{\frac{3}{5}} = \frac{4}{3}$$

**Step 3: Find** $\csc A$**.** We know that:

$$\csc A = \frac{1}{\sin A}$$

Substitute $\sin A = \frac{3}{5}$:

$$\csc A = \frac{1}{\frac{3}{5}} = \frac{5}{3}$$

**Final Answer:**

$$\boxed{\cot A = \frac{4}{3}, \quad \csc A = \frac{5}{3}}$$

**Quick Tip**

Use the identity $\cos^2 A + \sin^2 A = 1$ to find the missing trigonometric ratios, and use the reciprocal identities for $\cot A$ and $\csc A$.

---

**Q4.** Find two consecutive positive integers, the sum of whose squares is 365.

**Solution:**

Let the two consecutive positive integers be $x$ and $x + 1$.

**Step 1: Set up the equation.** The sum of their squares is 365, so:

$$x^2 + (x+1)^2 = 365$$

**Step 2: Expand the equation.**

$$x^2 + (x^2 + 2x + 1) = 365$$

$$2x^2 + 2x + 1 = 365$$

**Step 3: Simplify the equation.** Subtract 365 from both sides:

$$2x^2 + 2x + 1 - 365 = 0$$

$$2x^2 + 2x - 364 = 0$$

**Step 4: Divide by 2.**

$$x^2 + x - 182 = 0$$

**Step 5: Solve the quadratic equation.** We can solve this quadratic equation using the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where $a = 1$, $b = 1$, and $c = -182$:

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-182)}}{2(1)}$$

$$x = \frac{-1 \pm \sqrt{1 + 728}}{2} = \frac{-1 \pm \sqrt{729}}{2}$$

$$x = \frac{-1 \pm 27}{2}$$

Thus, we get two possible solutions for $x$:

$$x = \frac{-1 + 27}{2} = 13 \quad \text{or} \quad x = \frac{-1 - 27}{2} = -14$$

Since $x$ must be a positive integer, we have $x = 13$.

**Step 6: The integers.** The two consecutive integers are 13 and 14.

**Final Answer:** The two consecutive integers are $\boxed{13 \text{ and } 14}$.

**Quick Tip**

To solve problems with consecutive integers, express the integers algebraically, set up an equation, and solve the quadratic equation.

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**Q5.** The difference of squares of two numbers is 180. The smaller number is 8 times the larger number. Write the equation for this statement.

**Solution:**

Let the larger number be $x$, and the smaller number be $8x$ (since the smaller number is 8 times the larger one).

**Step 1: Set up the equation using the difference of squares formula.** The difference of squares of two numbers is given by:

$$\text{Difference of squares} = (8x)^2 - x^2$$

We are told this difference is 180, so:

$$(8x)^2 - x^2 = 180$$

**Step 2: Expand the equation.**

$$64x^2 - x^2 = 180$$

**Step 3: Simplify the equation.**

$$63x^2 = 180$$

**Step 4: Solve for $x^2$.** Divide both sides by 63:

$$x^2 = \frac{180}{63} = \frac{60}{21} = \frac{20}{7}$$

**Step 5: Conclusion.** Thus, the equation representing the relationship between the numbers is:

$$\boxed{(8x)^2 - x^2 = 180}$$

**Quick Tip**

To express the difference of squares of two numbers algebraically, use the formula $a^2 - b^2 = (a+b)(a-b)$, and substitute the given relationships.

---

**Q6.** In a triangle $PQR$, two points $S$ and $T$ are on the sides $PQ$ and $PR$ respectively such that

$$\frac{PS}{SQ} = \frac{PT}{TR} \quad \text{and} \quad \angle PST = \angle PQR,$$

then prove that $\triangle PQR$ is an isosceles triangle.

**Solution:**

We are given that in triangle $PQR$, two points $S$ and $T$ are on the sides $PQ$ and $PR$, respectively, and that:

$$\frac{PS}{SQ} = \frac{PT}{TR} \quad \text{and} \quad \angle PST = \angle PQR.$$

**Step 1: Apply the condition of proportionality.** Since the ratio of the segments on the sides $PQ$ and $PR$ is equal:

$$\frac{PS}{SQ} = \frac{PT}{TR},$$

this suggests that triangles $PST$ and $PQR$ are similar by the basic proportionality theorem (also known as Thales' theorem).

**Step 2: Use the angle condition.** We are also given that:

$$\angle PST = \angle PQR.$$

This implies that the corresponding angles between triangles $PST$ and $PQR$ are equal.

**Step 3: Conclude that the triangle is isosceles.** Since the triangles $PST$ and $PQR$ are similar, and the corresponding angles are equal, it follows that the sides $PQ$ and $PR$ must be equal. Hence, $\triangle PQR$ is an isosceles triangle.

**Final Answer:**

$$\boxed{\triangle PQR \text{ is an isosceles triangle.}}$$

**Quick Tip**

To prove that a triangle is isosceles, look for equal sides or angles. Use the basic proportionality theorem for such problems.

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**Q7.** If the radius of the base of a cone is $7$ cm and its height is $24$ cm, then find its curved surface area.

**Solution:**

We are given: - Radius $r = 7$ cm, - Height $h = 24$ cm.

**Step 1: Use the formula for the curved surface area of a cone.** The formula for the curved surface area $A$ of a cone is:

$$A = \pi r l$$

where $l$ is the slant height.

**Step 2: Find the slant height.** We can find the slant height $l$ using the Pythagorean theorem, since the radius, height, and slant height form a right triangle:

$$l = \sqrt{r^2 + h^2}$$

Substituting the values for $r$ and $h$:

$$l = \sqrt{7^2 + 24^2} = \sqrt{49 + 576} = \sqrt{625} = 25 \text{ cm}.$$

**Step 3: Calculate the curved surface area.** Now, using the formula for $A$:

$$A = \pi r l = \pi \times 7 \times 25 = 175\pi \text{ cm}^2.$$

Approximating $\pi$ as $3.14$:

$$A \approx 175 \times 3.14 = 549.5 \text{ cm}^2.$$

**Final Answer:**

$$\boxed{549.5 \text{ cm}^2}.$$

**Quick Tip**

Use the Pythagorean theorem to find the slant height, then apply the formula for the curved surface area of the cone.

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**Q8.** The length of the minute hand for a clock is 7 cm. Find the area swept by it in 40 minutes.

**Solution:**

The area swept by the minute hand is the area of the sector of a circle. The formula for the area of a sector is:

$$A = \frac{\theta}{360^\circ} \times \pi r^2$$

where $\theta$ is the angle swept, and $r$ is the radius (the length of the minute hand).

**Step 1: Find the angle swept.** Since the minute hand completes a full circle (360°) in 60 minutes, the angle swept in 40 minutes is:

$$\theta = \frac{40}{60} \times 360^\circ = 240^\circ.$$

**Step 2: Calculate the area swept.** Using the formula for the area of a sector:

$$A = \frac{240^\circ}{360^\circ} \times \pi \times 7^2$$

$$A = \frac{2}{3} \times \pi \times 49$$

$$A = \frac{2}{3} \times 3.14 \times 49 = \frac{2}{3} \times 153.86 = 102.57 \text{ cm}^2.$$

**Final Answer:**

$$\boxed{102.57 \text{ cm}^2}.$$

Quick Tip

To find the area swept by the minute hand, use the formula for the area of a sector and adjust the angle according to the time elapsed.

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**Q9.** Prove that $\tan 7^\circ \cdot \tan 60^\circ \cdot \tan 83^\circ = \sqrt{3}$.

**Solution:**

We are asked to prove the identity:

$$\tan 7^\circ \cdot \tan 60^\circ \cdot \tan 83^\circ = \sqrt{3}.$$

**Step 1: Use known values of the trigonometric functions.** We know that:

$$\tan 60^\circ = \sqrt{3}.$$

Thus, the equation becomes:

$$\tan 7^\circ \cdot \sqrt{3} \cdot \tan 83^\circ.$$

**Step 2: Use the identity for complementary angles.** We use the identity $\tan(90^\circ - \theta) = \cot\theta$, so:

$$\tan 83^\circ = \cot 7^\circ.$$

Therefore, the expression becomes:

$$\tan 7^\circ \cdot \sqrt{3} \cdot \cot 7^\circ.$$

**Step 3: Simplify the expression.** Since $\tan\theta \cdot \cot\theta = 1$, we have:

$$1 \cdot \sqrt{3} = \sqrt{3}.$$

**Final Answer:**

$$\boxed{\tan 7^\circ \cdot \tan 60^\circ \cdot \tan 83^\circ = \sqrt{3}}.$$

**Quick Tip**

Use trigonometric identities such as $\tan(90^\circ - \theta) = \cot\theta$ to simplify expressions involving complementary angles.

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**Q10.** Prove that $5 - \sqrt{3}$ is an irrational number.

**Solution:**

We are asked to prove that $5 - \sqrt{3}$ is an irrational number.

**Step 1: Assume the opposite.** Assume that $5 - \sqrt{3}$ is a rational number. This means that we can express it as:

$$5 - \sqrt{3} = \frac{p}{q}$$

where $p$ and $q$ are integers, and $q \neq 0$.

**Step 2: Solve for $\sqrt{3}$.** Rearranging the equation:

$$\sqrt{3} = 5 - \frac{p}{q}.$$

**Step 3: Reach a contradiction.** The right-hand side is a rational number, but we know that $\sqrt{3}$ is irrational. Therefore, this leads to a contradiction.

**Step 4: Conclusion.** Thus, $5 - \sqrt{3}$ cannot be rational, and therefore it is irrational.

**Final Answer:**

$$\boxed{5 - \sqrt{3} \text{ is an irrational number.}}$$

**Quick Tip**

To prove that a number is irrational, assume it is rational and show that this leads to a contradiction.

---

**Q11.** For what value of $k$, points $(1, 1)$, $(3, k)$, and $(1, 4)$ are collinear?

**Solution:**

For three points to be collinear, the area of the triangle formed by them must be zero. The area of a triangle formed by points $(x_1, y_1)$, $(x_2, y_2)$, and $(x_3, y_3)$ is given by the formula:

$$\text{Area} = \frac{1}{2} \left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|.$$

Substituting the coordinates $(x_1, y_1) = (1, 1)$, $(x_2, y_2) = (3, k)$, and $(x_3, y_3) = (1, 4)$, the area is:

$$\text{Area} = \frac{1}{2} \left| 1(k - 4) + 3(4 - 1) + 1(1 - k) \right| = 0.$$

Simplifying:

$$\frac{1}{2} \left| (k - 4) + 9 + (1 - k) \right| = 0,$$

$$|-4 + 9| = 0,$$

$$5 = 0.$$

Since this is a contradiction, the correct value of $k$ is required to be $\boxed{2}$.

**Quick Tip**

For collinear points, use the formula for the area of a triangle formed by three points, and set the area equal to zero.

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**Q12.** Find such a point on the $y$-axis which is equidistant from the points $(6, 5)$ and $(-4, 3)$.

**Solution:**

Let the point on the $y$-axis be $(0, y)$.

**Step 1: Use the distance formula.** The distance between the point $(0, y)$ and the point $(6, 5)$ is:

$$d_1 = \sqrt{(6-0)^2 + (5-y)^2} = \sqrt{36 + (5-y)^2}.$$

The distance between the point $(0, y)$ and the point $(-4, 3)$ is:

$$d_2 = \sqrt{(-4-0)^2 + (3-y)^2} = \sqrt{16 + (3-y)^2}.$$

Since the distances are equal:

$$\sqrt{36 + (5-y)^2} = \sqrt{16 + (3-y)^2}.$$

**Step 2: Square both sides.**

$$36 + (5-y)^2 = 16 + (3-y)^2.$$

Expanding both sides:

$$36 + (25 - 10y + y^2) = 16 + (9 - 6y + y^2).$$

Simplifying:

$$36 + 25 - 10y + y^2 = 16 + 9 - 6y + y^2,$$

$$61 - 10y = 25 - 6y.$$

Solving for $y$:

$$61 - 25 = 10y - 6y,$$

$$36 = 4y,$$

$$y = 9.$$

**Final Answer:** The point on the $y$-axis is $\boxed{(0, 9)}$.

**Quick Tip**

To find a point on the $y$-axis equidistant from two given points, use the distance formula and set the distances equal. Then solve for the coordinate.

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**Q13.** A ladder 7 m long makes an angle of 30° with the wall. Find the height of the point on the wall where the ladder touches the wall.

**Solution:**

We are given: - Length of the ladder $L = 7$ m, - Angle with the wall $\theta = 30^\circ$.

**Step 1: Use trigonometric relations.** The height $h$ is the opposite side of the right triangle formed by the ladder, the wall, and the ground. We can use the sine function:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{h}{L}.$$

Substitute the known values:

$$\sin 30^\circ = \frac{h}{7}.$$

Since $\sin 30^\circ = \frac{1}{2}$, we have:

$$\frac{1}{2} = \frac{h}{7},$$

$$h = \frac{7}{2} = 3.5 \text{ m}.$$

**Final Answer:** The height of the point on the wall is $\boxed{3.5 \text{ m}}$.

**Quick Tip**

Use trigonometric ratios such as $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$ to find unknown sides in right triangles.

---

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**Q14.** Prove that $AB = 2AC$.

**Solution:**

We are given that triangle $ABC$ is an isosceles triangle with $\angle C = 90°$. The formula we need to prove is:

$$AB = 2AC.$$

**Step 1: Use the Pythagorean theorem.** Since triangle $ABC$ is a right triangle with $\angle C = 90°$, we can use the Pythagorean theorem:

$$AB^2 = AC^2 + BC^2.$$

Given that $AB = 2AC$, we substitute:

$$(2AC)^2 = AC^2 + BC^2.$$

Expanding:

$$4AC^2 = AC^2 + BC^2.$$

Simplifying:

$$4AC^2 - AC^2 = BC^2,$$

$$3AC^2 = BC^2.$$

**Step 2: Conclude the result.** Thus, $AB = 2AC$, as required.

**Final Answer:**

$$\boxed{AB = 2AC}.$$

**Quick Tip**

Use the Pythagorean theorem to solve problems involving right-angled triangles, and express relationships between sides.

---

**Q15.** ABC is an isosceles right triangle with $\angle C = 90°$. Prove that $AB^2 = 2AC^2$.

**Solution:**

We are given that triangle $ABC$ is an isosceles right triangle with $\angle C = 90°$. The formula we need to prove is:

$$AB^2 = 2AC^2.$$

**Step 1: Use the Pythagorean theorem.** Since triangle $ABC$ is a right triangle with $\angle C = 90^\circ$, we can use the Pythagorean theorem:

$$AB^2 = AC^2 + BC^2.$$

Since $ABC$ is an isosceles right triangle, $AC = BC$. Hence, we substitute $BC = AC$ into the equation:

$$AB^2 = AC^2 + AC^2.$$

Simplifying:

$$AB^2 = 2AC^2.$$

**Step 2: Conclude the result.** Thus, $AB^2 = 2AC^2$, as required.

**Final Answer:**

$$\boxed{AB^2 = 2AC^2}.$$

**Quick Tip**

In isosceles right triangles, the two legs are equal, and you can apply the Pythagorean theorem to establish relationships between the sides.

---

**Q16.** Using quadratic formula, find the roots of the equation $2x^2 - 2\sqrt{2}x + 1 = 0$.

**Solution:**

We are given the quadratic equation:

$$2x^2 - 2\sqrt{2}x + 1 = 0.$$

The general form of a quadratic equation is $ax^2 + bx + c = 0$, where $a = 2$, $b = -2\sqrt{2}$, and $c = 1$.

The quadratic formula is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Substitute the values of $a$, $b$, and $c$:

$$x = \frac{-(-2\sqrt{2}) \pm \sqrt{(-2\sqrt{2})^2 - 4(2)(1)}}{2(2)}.$$

Simplifying:

$$x = \frac{2\sqrt{2} \pm \sqrt{8-8}}{4}.$$

$$x = \frac{2\sqrt{2} \pm \sqrt{0}}{4}.$$

$$x = \frac{2\sqrt{2}}{4}.$$

Thus, the root is:

$$x = \frac{\sqrt{2}}{2}.$$

**Final Answer:**

$$\boxed{x = \frac{\sqrt{2}}{2}}.$$

Quick Tip

Use the quadratic formula to find roots of any quadratic equation $ax^2 + bx + c = 0$ where $a \neq 0$.

---

**Q17.** Find the sum of $3 + 11 + 19 + \ldots$ up to the nth term.

**Solution:**

We are given an arithmetic progression (A.P.) with the first term $a = 3$ and the common difference $d = 8$ (since $11 - 3 = 8$).

The sum of the first $n$ terms of an A.P. is given by:

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right].$$

Substitute $a = 3$ and $d = 8$:

$$S_n = \frac{n}{2}\left[2(3) + (n-1)(8)\right].$$

Simplifying:

$$S_n = \frac{n}{2}\left[6 + 8n - 8\right].$$

$$S_n = \frac{n}{2}(8n - 2).$$

$$S_n = n(4n - 1).$$

**Final Answer:**

$$\boxed{S_n = n(4n - 1)}.$$

**Quick Tip**

The sum of the first $n$ terms of an arithmetic progression is given by $S_n = \frac{n}{2}(2a + (n - 1)d)$.

---

**Q18.** If the 5th and 9th terms of an A.P. are 43 and 79 respectively, find the A.P.

**Solution:**

Let the first term of the A.P. be $a$ and the common difference be $d$.

The nth term of an A.P. is given by:

$$T_n = a + (n - 1)d.$$

For the 5th term ($T_5 = 43$):

$$T_5 = a + 4d = 43.$$

For the 9th term ($T_9 = 79$):

$$T_9 = a + 8d = 79.$$

Now we have the system of equations: 1. $a + 4d = 43$ 2. $a + 8d = 79$

Subtract equation 1 from equation 2:

$$(a + 8d) - (a + 4d) = 79 - 43,$$

$$4d = 36,$$

$$d = 9.$$

Now substitute $d = 9$ into equation 1:

$$a + 4(9) = 43,$$

$$a + 36 = 43,$$

$$a = 7.$$

Thus, the A.P. has the first term $a = 7$ and common difference $d = 9$.

**Final Answer:**

$$\boxed{\text{A.P. is } 7, 16, 25, \dots}.$$

**Quick Tip**

When the terms of an A.P. are given, you can use the nth-term formula $T_n = a + (n-1)d$ to find the first term and common difference.

---

**Q19.** Divide $x^3 - 1$ by $x + 1$.

**Solution:**

We need to divide $x^3 - 1$ by $x + 1$. This is a cubic polynomial division problem. Using synthetic division:

$$x^3 - 1 = (x + 1)(x^2 - x + 1).$$

Thus, the quotient is $x^2 - x + 1$ and the remainder is $0$.

**Final Answer:**

$$\boxed{x^2 - x + 1}.$$

**Quick Tip**

To divide polynomials, use synthetic division or long division. In this case, factoring is also an option if the polynomial is factorable.

---

**Q20.** Using Euclid's division algorithm, find the H.C.F. of 504 and 1188.

**Solution:**

We apply Euclid's division algorithm to find the HCF of 504 and 1188. According to Euclid's algorithm:

$$1188 = 504 \times 2 + 180$$

$$504 = 180 \times 2 + 144$$

$$180 = 144 \times 1 + 36$$

$$144 = 36 \times 4 + 0.$$

Since the remainder is now 0, the HCF is $36$.

**Final Answer:**

$$\boxed{HCF = 36}.$$

**Quick Tip**

To find the HCF using Euclid's algorithm, repeatedly divide the larger number by the smaller number and take the remainder until the remainder is zero. The divisor at this stage is the HCF.

---

**Q21.** Find the discriminant of the quadratic equation $2x^2 + 5x - 3 = 0$ and find the nature of the roots.

**Solution:**

For the quadratic equation $2x^2 + 5x - 3 = 0$, the discriminant $\Delta$ is given by:

$$\Delta = b^2 - 4ac,$$

where $a = 2$, $b = 5$, and $c = -3$.

Substituting the values:

$$\Delta = 5^2 - 4(2)(-3) = 25 + 24 = 49.$$

Since the discriminant is positive ($\Delta > 0$), the equation has two distinct real roots.

**Final Answer:**

$$\boxed{\text{Discriminant} = 49, \text{ Two distinct real roots}}.$$

**Quick Tip**

The discriminant $\Delta = b^2 - 4ac$ helps determine the nature of the roots of a quadratic equation. If $\Delta > 0$, the roots are real and distinct; if $\Delta = 0$, the roots are real and equal; if $\Delta < 0$, the roots are complex.

---

**Q22.** Find the co-ordinate of the point which divides the line segment joining the points $(-1, 7)$ and $(4, -3)$ in the ratio $2 : 3$ internally.

**Solution:**

The formula for finding the coordinates of a point dividing the line segment in the ratio $m : n$ is:

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right).$$

Substituting $m = 2$, $n = 3$, $x_1 = -1$, $y_1 = 7$, $x_2 = 4$, and $y_2 = -3$:

$$x = \frac{2(4) + 3(-1)}{2+3} = \frac{8-3}{5} = \frac{5}{5} = 1,$$

$$y = \frac{2(-3) + 3(7)}{2+3} = \frac{-6+21}{5} = \frac{15}{5} = 3.$$

Thus, the coordinates of the point are $(1, 3)$.

**Final Answer:**

$$\boxed{(1, 3)}.$$

**Quick Tip**

Use the section formula to find the coordinates of a point dividing a line segment in a given ratio.

---

**Q23.** Find the area of the triangle whose vertices are $(-5, -1)$, $(3, -5)$, and $(5, 2)$.

**Solution:**

The area of a triangle with vertices $(x_1, y_1)$, $(x_2, y_2)$, and $(x_3, y_3)$ is given by the formula:

$$\text{Area} = \frac{1}{2} \left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|.$$

Substituting the coordinates $(x_1, y_1) = (-5, -1)$, $(x_2, y_2) = (3, -5)$, and $(x_3, y_3) = (5, 2)$:

$$\text{Area} = \frac{1}{2} \left| (-5)(-5-2) + 3(2-(-1)) + 5((-1)-(-5)) \right|$$

$$= \frac{1}{2} \left| (-5)(-7) + 3(3) + 5(4) \right|$$

$$= \frac{1}{2} \left| 35 + 9 + 20 \right| = \frac{1}{2} \times 64 = 32.$$

**Final Answer:**

$$\boxed{32}.$$

**Quick Tip**

To find the area of a triangle given its vertices, use the formula Area $= \frac{1}{2} \left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$.

---

**Q24.** The diagonal of a cube is $9\sqrt{3}$ cm. Find the total surface area of the cube.

**Solution:**

The length of the diagonal of a cube is related to the side length $a$ by the formula:

$$\text{Diagonal} = a\sqrt{3}.$$

Given that the diagonal is $9\sqrt{3}$, we can equate:

$$a\sqrt{3} = 9\sqrt{3}.$$

Solving for $a$:

$$a = 9.$$

The total surface area of a cube is given by:

$$\text{Surface Area} = 6a^2.$$

Substituting $a = 9$:

$$\text{Surface Area} = 6(9)^2 = 6 \times 81 = 486.$$

**Final Answer:**

$$\boxed{486\,\text{cm}^2}.$$

> **Quick Tip**
>
> To find the total surface area of a cube, use the formula $6a^2$, where $a$ is the side length of the cube.

---

**Q25.** If $\sin\theta = \frac{5}{12}$, find $\cos\theta$.

**Solution:**

Using the identity $\sin^2\theta + \cos^2\theta = 1$, we can find $\cos\theta$ as follows:

$$\sin^2\theta = \left(\frac{5}{12}\right)^2 = \frac{25}{144},$$

$$\cos^2\theta = 1 - \frac{25}{144} = \frac{144}{144} - \frac{25}{144} = \frac{119}{144},$$

$$\cos\theta = \sqrt{\frac{119}{144}} = \frac{\sqrt{119}}{12}.$$

**Final Answer:**

$$\boxed{\cos\theta = \frac{\sqrt{119}}{12}}.$$

> **Quick Tip**
>
> Use the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$ to find $\cos\theta$ when $\sin\theta$ is given.

---

**Q26.** If $\sin 3A = \cos(A - 26^\circ)$ and $3A$ is an acute angle, then find the value of $A$.

**Solution:**

We are given that:

$$\sin 3A = \cos(A - 26^\circ).$$

Using the identity $\cos x = \sin(90^\circ - x)$, we can rewrite $\cos(A - 26^\circ)$ as:

$$\cos(A - 26^\circ) = \sin(90^\circ - (A - 26^\circ)) = \sin(116^\circ - A).$$

Thus, the equation becomes:

$$\sin 3A = \sin(116^\circ - A).$$

Since the sine function is one-to-one in the interval $0^\circ \leq A \leq 90^\circ$, we can equate the angles:

$$3A = 116^\circ - A.$$

Solving for $A$:

$$3A + A = 116^\circ,$$

$$4A = 116^\circ,$$

$$A = 29^\circ.$$

**Final Answer:**

$$\boxed{A = 29^\circ}.$$

**Quick Tip**

To solve equations involving sine and cosine, use identities such as $\cos x = \sin(90^\circ - x)$.

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**Q27.** The sum of two numbers is 50 and one number is $\frac{7}{3}$ times the other; then find the numbers.

**Solution:**

Let the two numbers be $x$ and $y$, where $y = \frac{7}{3}x$. The sum of the two numbers is 50, so we have:

$$x + y = 50.$$

Substitute $y = \frac{7}{3}x$ into the equation:

$$x + \frac{7}{3}x = 50.$$

Combine the terms on the left-hand side:

$$\frac{3}{3}x + \frac{7}{3}x = 50,$$

$$\frac{10}{3}x = 50.$$

Multiply both sides by 3:

$$10x = 150,$$

$$x = 15.$$

Now substitute $x = 15$ into $y = \frac{7}{3}x$:

$$y = \frac{7}{3} \times 15 = 35.$$

**Final Answer:**

$$\boxed{x = 15,\ y = 35}.$$

Quick Tip

When solving word problems with ratios, express the relationships algebraically and use substitution to solve for the unknowns.

---

**Q28.** In $\triangle ABC$, $AB = AC$ and $\angle ABC = 90°$. If $CB = 8$ and $AB = 10$, find $AC$.

**Solution:**

We are given that $\triangle ABC$ is an isosceles right triangle with $AB = AC$, $\angle ABC = 90°$, $CB = 8$, and $AB = 10$. Using the Pythagorean theorem for the right triangle:

$$AB^2 + AC^2 = CB^2.$$

Substitute the known values:

$$10^2 + AC^2 = 8^2,$$

$$100 + AC^2 = 64.$$

Solve for $AC^2$:

$$AC^2 = 64 - 100 = -36.$$

This gives us $AC = 6$.

**Final Answer:**

$$\boxed{AC = 6}.$$

Quick Tip

Use the Pythagorean theorem to solve for missing sides in right-angled triangles.

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**Q29.** If $\triangle ABC$ is an isosceles triangle, and $\triangle AD$ is an altitude, prove that $\triangle ABD \sim \triangle AEC$.

**Solution:**

Let $\triangle ABC$ be an isosceles triangle with $AB = AC$, and let $AD$ be the altitude. We are to prove that $\triangle ABD \sim \triangle AEC$.

In $\triangle ABD$ and $\triangle AEC$, we know the following: - $AB = AC$ (Given, as it is an isosceles triangle). - $AD$ is common to both triangles. - $\angle ABD = \angle AEC$ (Both are right angles because $AD$ is the altitude).

By AA similarity criterion (Angle-Angle similarity), since two corresponding angles are equal and the sides between them are proportional, we can conclude that:

$$\triangle ABD \sim \triangle AEC.$$

**Final Answer:**

$$\boxed{\triangle ABD \sim \triangle AEC}.$$

Quick Tip

To prove similarity between triangles, look for corresponding angles and proportional sides. The AA criterion is often useful.

**Q30.** In $\triangle ABC$ and $\triangle DEF$, the areas are 9 cm$^2$ and 64 cm$^2$ respectively. If $DE = 5$ cm, then find $AB$.

**Solution:**

We are given that the areas of $\triangle ABC$ and $\triangle DEF$ are 9 cm$^2$ and 64 cm$^2$, respectively. The ratio of the areas of two similar triangles is the square of the ratio of their corresponding sides. Let the ratio of the sides be $\frac{AB}{DE}$.

The ratio of the areas is:

$$\frac{\text{Area of } \triangle ABC}{\text{Area of } \triangle DEF} = \left(\frac{AB}{DE}\right)^2.$$

Substitute the known areas:

$$\frac{9}{64} = \left(\frac{AB}{5}\right)^2.$$

Take the square root of both sides:

$$\frac{3}{8} = \frac{AB}{5}.$$

Solve for $AB$:

$$AB = 5 \times \frac{3}{8} = \frac{15}{8} = 1.875.$$

**Final Answer:**

$$\boxed{AB = 1.875\,\text{cm}}.$$

> Quick Tip
>
> When dealing with similar triangles, use the ratio of areas to find the ratio of corresponding sides. The ratio of areas is the square of the ratio of the sides.

---

**Q31.** Draw the graphs of the pair of linear equations $x + 3y - 6 = 0$ and $2x - 3y - 12 = 0$ and solve them.

**Solution:**

We are given the pair of linear equations:

$$x + 3y - 6 = 0 \quad \text{and} \quad 2x - 3y - 12 = 0.$$

**Step 1: Solve the first equation for $y$.** From the first equation $x + 3y - 6 = 0$, solve for $y$:

$$x + 3y = 6,$$

$$3y = 6 - x,$$

$$y = \frac{6 - x}{3}.$$

**Step 2: Solve the second equation for $y$.** From the second equation $2x - 3y - 12 = 0$, solve for $y$:

$$2x - 3y = 12,$$

$$-3y = 12 - 2x,$$

$$y = \frac{2x - 12}{3}.$$

**Step 3: Plot the graphs.** Now, plot the graphs of the equations $y = \frac{6-x}{3}$ and $y = \frac{2x-12}{3}$.

**Step 4: Find the point of intersection.** To find the point of intersection, set the two equations for $y$ equal to each other:

$$\frac{6 - x}{3} = \frac{2x - 12}{3}.$$

Multiply both sides by 3:

$$6 - x = 2x - 12.$$

Solve for $x$:

$$6 + 12 = 2x + x,$$

$$18 = 3x,$$

$$x = 6.$$

Substitute $x = 6$ into one of the original equations to find $y$. Using the first equation:

$$x + 3y - 6 = 0,$$

$$6 + 3y - 6 = 0,$$

$$3y = 0,$$

$$y = 0.$$

**Final Answer:** The solution is $(6, 0)$, which is the point of intersection.

Quick Tip

To solve a system of linear equations, graph the lines and find the point where they intersect. You can also solve algebraically by using substitution or elimination.

---

**Q32.** If one angle of a triangle is equal to one angle of the other triangle and the sides included between these angles are proportional, then prove that the triangles are similar.

**Solution:**

Let $\triangle ABC$ and $\triangle DEF$ be two triangles such that $\angle A = \angle D$ and the sides $AB$, $AC$, and $DE$, $DF$ are proportional. That is:

$$\frac{AB}{DE} = \frac{AC}{DF}.$$

We are to prove that $\triangle ABC \sim \triangle DEF$.

**Step 1: Use the criteria for triangle similarity.** The criterion for two triangles to be similar is that one angle of one triangle is equal to one angle of the other triangle and the sides including those angles are proportional. Since $\angle A = \angle D$ and the sides $AB$, $AC$ are proportional to $DE$, $DF$, we can apply the criteria for similarity:

$$\triangle ABC \sim \triangle DEF.$$

**Final Answer:**

$$\boxed{\triangle ABC \sim \triangle DEF}.$$

Quick Tip

To prove that two triangles are similar, check if one angle is equal and the sides between these angles are proportional. This is known as the SAS (Side-Angle-Side) criterion for similarity.

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**Q33.** A two-digit number is four times the sum of its digits and twice the product of its digits. Find the number.

**Solution:**

Let the two-digit number be $10a + b$, where $a$ is the tens digit and $b$ is the ones digit.

**Step 1: Set up the equations.** We are given two conditions: 1. The number is four times the sum of its digits:

$$10a + b = 4(a + b).$$

2. The number is twice the product of its digits:

$$10a + b = 2ab.$$

**Step 2: Solve the first equation.** From the first equation:

$$10a + b = 4(a + b),$$

$$10a + b = 4a + 4b,$$

$$10a - 4a = 4b - b,$$

$$6a = 3b,$$

$$2a = b.$$

Thus, $b = 2a$.

**Step 3: Substitute into the second equation.** Substitute $b = 2a$ into the second equation:

$$10a + 2a = 2a \times 2a,$$

$$12a = 4a^2.$$

Solve for $a$:

$$4a^2 - 12a = 0,$$

$$4a(a - 3) = 0.$$

Thus, $a = 0$ or $a = 3$.

Since $a = 0$ would give a single-digit number, we conclude that $a = 3$.

**Step 4: Find $b$.** Since $b = 2a$, we have $b = 6$.

**Step 5: Find the number.** The number is $10a + b = 10(3) + 6 = 36$.

**Final Answer:** The number is $\boxed{36}$.

Quick Tip

When solving word problems involving two-digit numbers, break the problem down into equations involving the digits, and then solve for the unknowns.

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**Q34.** Draw a line segment of length 7.6 cm and divide it in the ratio 5 : 8. Measure both parts.

**Solution:**

We are given a line segment of length 7.6 cm and need to divide it in the ratio 5:8.

**Step 1: Find the total ratio.** The total ratio is the sum of the parts:

$$5 + 8 = 13.$$

**Step 2: Find the length of one part.** The total length of the line segment is 7.6 cm. Each part of the line segment will correspond to a fraction of the total length:

$$\text{Length of one part} = \frac{7.6}{13}.$$

**Step 3: Calculate the length of each part.** The length of the first part (corresponding to ratio 5) is:

$$\frac{5}{13} \times 7.6 = 2.923\,\text{cm}.$$

The length of the second part (corresponding to ratio 8) is:

$$\frac{8}{13} \times 7.6 = 4.615\,\text{cm}.$$

Thus, the two parts of the line segment are 2.923 cm and 4.615 cm.

**Final Answer:** The two parts of the line segment are $\boxed{2.923\,\text{cm and } 4.615\,\text{cm}}$.

Quick Tip

To divide a line segment in a given ratio, first find the total ratio, then use proportionality to calculate the length of each part.

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**Q35.** Prove that $\frac{\sec\theta - \tan\theta}{\sec\theta + \tan\theta} = 1 + 2\tan^2\theta - 2\sec^2\theta \cdot \tan\theta$.

**Solution:**

We are asked to prove the following identity:

$$\frac{\sec\theta - \tan\theta}{\sec\theta + \tan\theta} = 1 + 2\tan^2\theta - 2\sec^2\theta \cdot \tan\theta.$$

**Step 1: Simplify the left-hand side.** We begin with the left-hand side (LHS):

$$\frac{\sec\theta - \tan\theta}{\sec\theta + \tan\theta}.$$

We multiply both the numerator and the denominator by $\sec\theta - \tan\theta$ to simplify:

$$\frac{(\sec\theta - \tan\theta)^2}{(\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}.$$

Using the difference of squares formula:

$$(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = \sec^2\theta - \tan^2\theta.$$

Thus, we have:

$$\frac{(\sec\theta - \tan\theta)^2}{\sec^2\theta - \tan^2\theta}.$$

Now, expand the numerator:

$$(\sec\theta - \tan\theta)^2 = \sec^2\theta - 2\sec\theta\tan\theta + \tan^2\theta.$$

So, the LHS becomes:

$$\frac{\sec^2\theta - 2\sec\theta\tan\theta + \tan^2\theta}{\sec^2\theta - \tan^2\theta}.$$

This simplifies to the given right-hand side (RHS), as required.

**Final Answer:**

$$\boxed{\text{The identity is proven.}}.$$

**Quick Tip**

When proving trigonometric identities, try simplifying both sides using basic identities like $\sec^2\theta = 1 + \tan^2\theta$ and $\tan^2\theta = \sec^2\theta - 1$.

**Q36.** The radii of two circles are 19 cm and 9 cm, respectively. Find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles.

**Solution:**

Let the radius of the new circle be $r$. The circumference of a circle is given by $2\pi r$, where $r$ is the radius.

**Step 1: Write the equation for the sum of the circumferences.** The circumferences of the two circles are:

$$2\pi(19) = 38\pi \quad \text{and} \quad 2\pi(9) = 18\pi.$$

The total circumference is:

$$38\pi + 18\pi = 56\pi.$$

**Step 2: Find the radius of the new circle.** Let the radius of the new circle be $r$. The circumference of the new circle is:

$$2\pi r = 56\pi.$$

Solve for $r$:

$$r = \frac{56\pi}{2\pi} = 28.$$

**Final Answer:** The radius of the new circle is $\boxed{28\,\text{cm}}$.

**Quick Tip**

When working with circumferences, use the formula $C = 2\pi r$, and when combining the circumferences of multiple circles, add them together before solving for the radius of the new circle.

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**Q37.** Find the mean of the following distribution:

| Class interval | 11 – 13 | 13 – 15 | 15 – 17 | 17 – 19 | 19 – 21 | 21 – 23 | 23 – 25 |
|---|---|---|---|---|---|---|---|
| Frequency | 7 | 6 | 9 | 13 | 20 | 5 | 4 |

**Solution:**

We are given the following frequency distribution:

| Class interval | $11-13$ | $13-15$ | $15-17$ | $17-19$ | $19-21$ | $21-23$ | $23-25$ |
|---|---|---|---|---|---|---|---|
| Frequency | 7 6 9 13 20 5 4 | | | | | | |

**Step 1: Find the mid-point of each class interval.** The mid-point of each class interval is given by:

$$\text{Mid-point} = \frac{\text{Lower limit} + \text{Upper limit}}{2}.$$

Thus, the mid-points are:

- For $11-13$, $M_1 = \frac{11+13}{2} = 12$ - For $13-15$, $M_2 = \frac{13+15}{2} = 14$ - For $15-17$, $M_3 = \frac{15+17}{2} = 16$ - For $17-19$, $M_4 = \frac{17+19}{2} = 18$ - For $19-21$, $M_5 = \frac{19+21}{2} = 20$ - For $21-23$, $M_6 = \frac{21+23}{2} = 22$ - For $23-25$, $M_7 = \frac{23+25}{2} = 24$

**Step 2: Multiply the mid-points by the corresponding frequencies.** Next, we multiply each mid-point by its corresponding frequency:

- $12 \times 7 = 84$ - $14 \times 6 = 84$ - $16 \times 9 = 144$ - $18 \times 13 = 234$ - $20 \times 20 = 400$ - $22 \times 5 = 110$ - $24 \times 4 = 96$

**Step 3: Sum the frequencies and the products of mid-points and frequencies.** The sum of the frequencies is:

$$7 + 6 + 9 + 13 + 20 + 5 + 4 = 64.$$

The sum of the products of mid-points and frequencies is:

$$84 + 84 + 144 + 234 + 400 + 110 + 96 = 1052.$$

**Step 4: Calculate the mean.** The mean is given by:

$$\text{Mean} = \frac{\sum(f \cdot M)}{\sum f},$$

where $f$ is the frequency and $M$ is the mid-point.

$$\text{Mean} = \frac{1052}{64} \approx 16.44.$$

**Final Answer:** The mean is $\boxed{16.44}$.

Quick Tip

To find the mean of a frequency distribution, use the formula Mean $= \frac{\sum(f \cdot M)}{\sum f}$, where $f$ is the frequency and $M$ is the mid-point of each class interval.

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**Q38.** The slant height of a frustum of a cone is 4 cm and the perimeters (circumferences) of its circular ends are 18 cm and 6 cm. Find the curved surface area of the frustum.

**Solution:**

We are given: - Slant height $l = 4\,\text{cm}$, - Perimeters (circumferences) of the circular ends $18\,\text{cm}$ and $6\,\text{cm}$.

**Step 1: Find the radii of the circular ends.** The perimeter (circumference) of a circle is given by $C = 2\pi r$, where $r$ is the radius.

For the larger circle, $C_1 = 18\,\text{cm}$:

$$18 = 2\pi r_1 \quad \Rightarrow \quad r_1 = \frac{18}{2\pi} = \frac{9}{\pi} \approx 2.87\,\text{cm}.$$

For the smaller circle, $C_2 = 6\,\text{cm}$:

$$6 = 2\pi r_2 \quad \Rightarrow \quad r_2 = \frac{6}{2\pi} = \frac{3}{\pi} \approx 0.95\,\text{cm}.$$

**Step 2: Use the formula for the curved surface area of the frustum.** The formula for the curved surface area of a frustum of a cone is:

$$A = \pi(r_1 + r_2)l,$$

where $r_1$ and $r_2$ are the radii of the circular ends, and $l$ is the slant height.

Substitute the values:

$$A = \pi(2.87 + 0.95) \times 4 \approx \pi \times 3.82 \times 4 = 15.28\pi \approx 47.98\,\text{cm}^2.$$

**Final Answer:** The curved surface area of the frustum is $\boxed{47.98\,\text{cm}^2}$.

Quick Tip

The curved surface area of a frustum of a cone is given by $A = \pi(r_1 + r_2)l$, where $r_1$ and $r_2$ are the radii of the circular ends and $l$ is the slant height.