BITSAT 2026 May 24 Shift 1

Question Paper (Memory-Based) with Solutions

Conducted by BITS Pilani

General Instructions

(i) **Duration:** The total duration of the examination is 3 hours (180 minutes).

(ii) **Total Marks:** The complete paper carries a maximum of 390 marks.

(iii) **Structure:** The paper has 4 Sections:

- **Part 1:** 30 Multiple Choice Questions (Physics).
- **Part 2:** 30 Multiple Choice Questions (Chemistry).
- **Part 3:** 10 Multiple Choice Questions (English Proficiency), 20 Multiple Choice Questions (Logical Reasoning)
- **Part 4:** 40 Multiple Choice Questions (Mathematics/Biology)

(iv) **Compulsory Questions:** All 130 questions are compulsory,and +12 Questions(Optional Extra Questions)

(v) Each question has four options. Only **one** option is correct.

(vi) **Correct Answer:** +3 marks.

(vii) **Incorrect Answer:** −1 (Negative marking).

(viii) **Unanswered/Marked for Review:** 0 marks.

PHYSICS

1. A projectile is fired from the surface of the earth with a velocity of v_0 at an angle θ with the horizontal. If the maximum height reached is equal to its horizontal range, then the angle of projection θ is given by:

(A) $\tan^{-1}(1)$

(B) $\tan^{-1}(2)$

(C) $\tan^{-1}(4)$

(D) $\tan^{-1}(0.5)$

Correct Answer: (C) $\tan^{-1}(4)$

Solution:

Concept: For any projectile launched with an initial velocity v_0 at an angle θ with the horizontal under gravity g, the formulas for maximum height (H) and horizontal range (R) are given by:

$$H = \frac{v_0^2 \sin^2\theta}{2g}$$

$$R = \frac{v_0^2 \sin(2\theta)}{g} = \frac{2v_0^2 \sin\theta\cos\theta}{g}$$

We can establish a direct geometric relationship between the height and range by finding their ratio.

Step 1: Deriving the fundamental relation between H and R.

Dividing the maximum height equation by the horizontal range equation:

$$\frac{H}{R} = \frac{\frac{v_0^2 \sin^2\theta}{2g}}{\frac{2v_0^2 \sin\theta\cos\theta}{g}}$$

Simplifying the fractions by cancelling out the common terms v_0^2 and g:

$$\frac{H}{R} = \frac{\sin^2\theta}{2\cdot 2\sin\theta\cos\theta} = \frac{\sin\theta}{4\cos\theta} = \frac{\tan\theta}{4}$$

This gives us the standard projectile identity:

$$\tan\theta = \frac{4H}{R}$$

Step 2: Applying the problem's condition.

The problem states that the maximum height reached is equal to the horizontal range, meaning $H = R$. Substituting this condition into our identity:

$$\tan\theta = \frac{4H}{H} = 4$$

Taking the inverse tangent on both sides to solve for the angle of projection θ:

$$\theta = \tan^{-1}(4)$$

Quick Tip: Memorize the relation $\tan\theta = \frac{4H}{R}$ for quick objective problems. If $H = R$, then $\tan\theta$ is always equal to 4.

---

2. A particle moves under the action of a variable force $F = (3x^2 - 2x + 5)$ in Newtons from $x = 0$ to $x = 2$ meters. The work done by the force is:

(A) 10 J

(B) 12 J

(C) 14 J

(D) 16 J

Correct Answer: (C) 14 J

Solution:

Concept: When a force varies with position x, the work done (W) cannot be computed using simple multiplication ($F \cdot d$). Instead, it must be determined by integrating the force function over the interval of displacement:

$$W = \int_{x_i}^{x_f} F\,dx$$

We will apply standard polynomial integration rules: $\int x^n\,dx = \frac{x^{n+1}}{n+1}$ and $\int k\,dx = kx$.

Step 1: Setting up the definite integral with the given limits.

The particle moves from the initial position $x_i = 0$ to the final position $x_f = 2$. Substituting the force function $F = 3x^2 - 2x + 5$:

$$W = \int_0^2 (3x^2 - 2x + 5)\,dx$$

Step 2: Integrating the expression term by term.

$$W = \left[3\left(\frac{x^3}{3}\right) - 2\left(\frac{x^2}{2}\right) + 5x\right]_0^2$$

Simplifying the coefficients:

$$W = \left[x^3 - x^2 + 5x\right]_0^2$$

Step 3: Substituting the upper and lower limits.

Substitute the upper limit ($x = 2$):

$$W_{\text{upper}} = (2)^3 - (2)^2 + 5(2) = 8 - 4 + 10 = 14$$

Substitute the lower limit ($x = 0$):

$$W_{\text{lower}} = (0)^3 - (0)^2 + 5(0) = 0$$

Subtracting the lower limit value from the upper limit value:

$$W = 14 - 0 = 14 \text{ J}$$

Quick Tip: For variable force calculations, look closely at the lower limit. If the lower limit is 0 and the expression is a simple polynomial, the entire lower limit evaluation becomes 0, saving you time during exams.

---

3. The ratio of the radii of gyration of a hollow cone and a hollow cylinder of the same mass and radius about a tangential axis parallel to their central axis is:

(A) $\sqrt{3} : \sqrt{2}$

(B) $\sqrt{3} : 2$

(C) $1 : \sqrt{2}$

(D) $\sqrt{5} : \sqrt{6}$

Correct Answer: (B) $\sqrt{3} : 2$

Solution: **Concept:** The radius of gyration k is related to moment of inertia I by:

$$I = Mk^2$$

or

$$k = \sqrt{\frac{I}{M}}$$

To find the moment of inertia about a tangential axis parallel to the central axis, we use the Parallel Axis Theorem:

$$I = I_{\text{cm}} + MR^2$$

where R is the perpendicular distance between the two axes.

Step 1: Find the radius of gyration of the hollow cone.

For a hollow cone about its central axis:

$$I_{\text{cm, cone}} = \frac{1}{2}MR^2$$

Using the Parallel Axis Theorem:

$$I_{\text{tangent, cone}} = \frac{1}{2}MR^2 + MR^2$$

$$= \frac{3}{2}MR^2$$

Now,

$$k_1 = \sqrt{\frac{I}{M}}$$

$$= \sqrt{\frac{\frac{3}{2}MR^2}{M}}$$

$$= \sqrt{\frac{3}{2}}R$$

Step 2: Find the radius of gyration of the hollow cylinder.

For a thin hollow cylinder about its central axis:

$$I_{\text{cm, cylinder}} = MR^2$$

Applying the Parallel Axis Theorem:

$$I_{\text{tangent, cylinder}} = MR^2 + MR^2$$

$$= 2MR^2$$

Hence,

$$k_2 = \sqrt{\frac{2MR^2}{M}}$$

$$= \sqrt{2}R$$

Step 3: Calculate the required ratio.

$$\frac{k_1}{k_2} = \frac{\sqrt{\frac{3}{2}}R}{\sqrt{2}R}$$

$$= \frac{\sqrt{3}}{2}$$

Therefore,

$$k_1 : k_2 = \sqrt{3} : 2$$

Hence, the correct answer is:

$$\boxed{\text{(B)}}$$

Quick Tip: For a thin hollow cylinder about its own axis:

$$I = MR^2$$

For a hollow cone about its own axis:

$$I = \frac{1}{2}MR^2$$

Always apply the Parallel Axis Theorem carefully when shifting to a tangential axis.

4. The escape velocity from the earth is v_e. If an object is now launched from the center of Earth where a tunnel has been dug, the escape velocity from that position is:

(A) $v_e\sqrt{\frac{3}{2}}$

(B) $2v_e \overline{\sqrt{3}(C)v_e/\sqrt{2}}$

(D) $v_e\sqrt{2}$

Correct Answer: (A) $v_e\sqrt{\frac{3}{2}}$

Solution:

Concept: Escape velocity (v) from any point is the minimum speed required for a particle to escape the gravitational field of a massive body completely (reaching infinity with zero kinetic energy). It is derived using the law of conservation of mechanical energy:

$$E_i = E_f \quad \Rightarrow \quad K_i + U_i = K_f + U_f$$

At infinity, both potential energy and kinetic energy are zero ($U_f = 0, K_f = 0$). Thus:

$$\frac{1}{2}mv^2 + U_i = 0 \quad \Rightarrow \quad v = \sqrt{\frac{-2U_i}{m}}$$

Step 1: Relating the standard surface escape velocity v_e.

The gravitational potential energy of a mass m on the surface of a solid uniform sphere (Earth) of mass M and radius R is:

$$U_{\text{surface}} = -\frac{GMM}{R}$$

Substituting this into the energy balance gives the standard surface escape velocity:

$$\frac{1}{2}mv_e^2 - \frac{GMm}{R} = 0 \quad \Rightarrow \quad v_e = \sqrt{\frac{2GM}{R}} \quad \cdots(1)$$

Step 2: Finding the potential energy at the center of the Earth.

The gravitational potential V inside a uniform solid sphere at a distance r from the center is given by the formula:

$$V(r) = -\frac{GM}{2R^3}(3R^2 - r^2)$$

At the center of the Earth, $r = 0$. Substituting this value yields the potential at the center:

$$V_{\text{center}} = -\frac{3GM}{2R}$$

Therefore, the gravitational potential energy U_{center} of a mass m at the center is:

$$U_{\text{center}} = m \cdot V_{\text{center}} = -\frac{3GMm}{2R}$$

Step 3: Calculating the new escape velocity v' from the center.

Using the conservation of energy from the center to infinity:

$$\frac{1}{2}m(v')^2 + U_{\text{center}} = 0 \quad \Rightarrow \quad \frac{1}{2}m(v')^2 - \frac{3GMm}{2R} = 0$$

$$(v')^2 = \frac{3GM}{R} \quad \Rightarrow \quad v' = \sqrt{\frac{3GM}{R}} \quad \cdots (2)$$

Step 4: Expressing v' in terms of v_e.

Divide equation (2) by equation (1):

$$\frac{v'}{v_e} = \frac{\sqrt{\frac{3GM}{R}}}{\sqrt{\frac{2GM}{R}}} = \sqrt{\frac{3}{2}}$$

$$v' = v_e\sqrt{\frac{3}{2}}$$

Quick Tip: The potential at the center of a uniform solid sphere is always exactly 1.5 times (or $\frac{3}{2}$) the potential at its surface. Consequently, the square of the escape velocity from the center scales directly by this same factor.

CHEMISTRY

5. If 20 g of $CaCO_3$ is treated with 100 mL of 20% HCl solution, the amount of CO_2 produced is:

(A) 22.4 L

(B) 8.80 g

(C) 4.40 g

(D) 2.24 L

Correct Answer: (B) 8.80 g

Solution:

Concept: This problem requires a stoichiometric calculation involving a limiting reagent. First, we write the balanced chemical equation representing the reaction between calcium carbonate and hydrochloric acid:

$$CaCO_3 + 2HCl \rightarrow CaCl_2 + H_2O + CO_2$$

From the balanced equation, 1 mole of $CaCO_3$ reacts completely with 2 moles of HCl to yield 1 mole of CO_2. We must find the initial moles of each reactant to determine which one limits the production of products.

Step 1: Calculate the moles of $CaCO_3$ available.

The molar mass of $CaCO_3$ is:

$$\text{Molar Mass} = 40(\text{Ca}) + 12(\text{C}) + 3 \times 16(\text{O}) = 100 \text{ g/mol}$$

Given mass of $CaCO_3 = 20$ g:

$$\text{Moles of } CaCO_3 = \frac{20 \text{ g}}{100 \text{ g/mol}} = 0.2 \text{ moles}$$

Step 2: Calculate the moles of HCl available.

Assuming the 20% HCl solution is specified by mass/volume percentage (% w/v), which is typical for standard aqueous reactions in introductory chemistry unless otherwise specified:

$$20\% \text{ w/v means 20 g of HCl is present in 100 mL of solution.}$$

Therefore, the mass of HCl in our 100 mL solution is exactly 20 g.

The molar mass of HCl is:

$$\text{Molar Mass} = 1(\text{H}) + 35.5(\text{Cl}) = 36.5 \text{ g/mol}$$

Calculating the moles of HCl:

$$\text{Moles of HCl} = \frac{20 \text{ g}}{36.5 \text{ g/mol}} \approx 0.548 \text{ moles}$$

Step 3: Determine the limiting reagent.

According to stoichiometry, 0.2 moles of $CaCO_3$ requires:

$$2 \times 0.2 = 0.4 \text{ moles of HCl}$$

Since we possess 0.548 moles of HCl (which is greater than 0.4 moles), HCl is present in excess. Thus, $CaCO_3$ is the **limiting reagent** and dictates the maximum amount of CO_2 formed.

Step 4: Calculate the mass of CO_2 produced.

The stoichiometric ratio shows that 1 mole of $CaCO_3$ generates 1 mole of CO_2. Therefore, 0.2 moles of $CaCO_3$ yields 0.2 moles of CO_2.

The molar mass of CO_2 is:

$$\text{Molar Mass} = 12(\text{C}) + 2 \times 16(\text{O}) = 44 \text{ g/mol}$$

Calculating the final mass:

$$\text{Mass of } CO_2 = 0.2 \text{ moles} \times 44 \text{ g/mol} = 8.80 \text{ g}$$

Quick Tip: Always double check the units given in the options! Options (2) and (3) are given in grams (g), while options (1) and (4) are in liters (L). Calculating the mass first saves you from doing an unnecessary conversion to volume.

---

6. The density (in g mL^{-1}) of a 3.60 M sulphuric acid solution that is 29% H_2SO_4 (molar mass = 98 g mol^{-1}) by mass will be:

(A) 1.64

(B) 1.88

(C) 1.22

(D) 1.45

Correct Answer: (C) 1.22

Solution:

Concept: Molarity (M) is defined as the number of moles of solute dissolved per liter (1000 mL) of solution. Mass percentage (% w/w) represents the mass of solute present in 100 g of solution.

These two concentration terms can be interconnected using the density (d) of the solution through fundamental conversion variables or a direct relational formula:

$$M = \frac{10 \times (\text{\% by mass}) \times d}{\text{Molar Mass of Solute}}$$

Step 1: Setting up the values from the given problem statement.

We are given the following values for the sulphuric acid (H_2SO_4) solution:

- Molarity (M) $= 3.60$ mol/L
- Mass percentage (% w/w) $= 29\%$
- Molar mass of solute (M_B) $= 98$ g/mol
- Density of solution $= d$ g/mL

Step 2: Substituting the parameters into the relationship formula.

Using our conversion identity to isolate the variable d:

$$3.60 = \frac{10 \times 29 \times d}{98}$$

$$3.60 = \frac{290 \times d}{98}$$

Step 3: Solving for the density d.

Rearranging the equation to isolate d:

$$d = \frac{3.60 \times 98}{290}$$

$$d = \frac{352.8}{290} \approx 1.2165 \text{ g/mL}$$

Rounding to two decimal places to match the given choices, we get:

$$d \approx 1.22 \text{ g mL}^{-1}$$

Quick Tip: Deriving concentration relation short-cuts can save immense time during competitive examinations. Memorize the identity $M = \frac{10 \cdot \% \cdot d}{M_{\text{solute}}}$ to transform multi-step mass-to-volume logic tracks into a single computational step.

7. The degeneracy of a hydrogen atom whose energy equals $-R_H/16$ is:

(A) 8

(B) 9

(C) 16

(D) 10

Correct Answer: (C) 16

Solution:

Concept: In quantum mechanics, degeneracy refers to the total number of distinct quantum states (orbitals) that share the exact same energy level. For a single-electron species like the hydrogen atom, the energy of a shell depends solely on its principal quantum number n according to Bohr's model:

$$E_n = -\frac{R_H}{n^2}$$

For a given principal quantum number n, the total number of atomic orbitals (ignoring electron spin unless specified) is given by the mathematical summation of angular momentum states, which simplifies to n^2.

Step 1: Determining the principal quantum number n.

We are given that the energy of the hydrogen atom is:

$$E = -\frac{R_H}{16}$$

Comparing this to the standard energy formula $E_n = -\frac{R_H}{n^2}$:

$$n^2 = 16 \quad \Rightarrow \quad n = 4$$

This indicates that the electron resides in the fourth major energy shell ($n = 4$).

Step 2: Calculating the degeneracy of the shell.

The degeneracy (number of orbitals) for a single-electron system for a shell n is calculated using the formula:

$$\text{Degeneracy} = n^2$$

Substituting $n = 4$ into the relation:

$$\text{Degeneracy} = 4^2 = 16$$

Alternatively, we can verify this by summing the subshells for $n = 4$:

- For $l = 0$ (s-subshell): 1 orbital ($4s$)
- For $l = 1$ (p-subshell): 3 orbitals ($4p_x, 4p_y, 4p_z$)
- For $l = 2$ (d-subshell): 5 orbitals ($4d_{xy}, 4d_{yz}, 4d_{xz}, 4d_{x^2-y^2}, 4d_{z^2}$)
- For $l = 3$ (f-subshell): 7 orbitals

$$\text{Total Orbitals} = 1 + 3 + 5 + 7 = 16$$

Quick Tip: For hydrogenic (one-electron) species like H, He^+, and Li^{2+}, all subshells within a major shell are degenerate, meaning total degeneracy is simply n^2. Note that if a problem explicitly mentions "including spin", you must multiply the result by 2, making it $2n^2$.

---

8. If the de-Broglie wavelength of a particle of mass (m) is 100 times its velocity, then its value in terms of its mass (m) and Planck's constant (h) is:

(A) $\frac{1}{10}\sqrt{\frac{m}{h}}$
(B) $10\sqrt{\frac{h}{m}}$
(C) $\frac{1}{10}\sqrt{\frac{h}{m}}$
(D) $10\sqrt{\frac{m}{h}}$

Correct Answer: (B) $10\sqrt{\frac{h}{m}}$

Solution:

Concept: According to the de-Broglie hypothesis, every moving particle exhibits a dual wave-particle nature. The matter wavelength (λ) associated with a particle depends inversely on its linear momentum ($p = mv$) and is expressed by the fundamental formula:

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

where h is Planck's constant, m is the mass of the particle, and v is its velocity.

Step 1: Translating the given condition into an algebraic equation.

The problem states that the de-Broglie wavelength (λ) is equal to 100 times its velocity (v):

$$\lambda = 100v \quad \cdots (1)$$

Step 2: Equating the two expressions for λ to eliminate velocity.

Substitute the standard de-Broglie wavelength expression into equation (1):

$$\frac{h}{mv} = 100v$$

Rearranging terms to isolate v^2:

$$v^2 = \frac{h}{100m}$$

Taking the square root on both sides gives the velocity of the particle:

$$v = \sqrt{\frac{h}{100m}} = \frac{1}{10}\sqrt{\frac{h}{m}} \quad \cdots (2)$$

Step 3: Calculating the wavelength λ in terms of m and h.

Substitute the value of velocity from equation (2) back into the primary problem condition equation (1):

$$\lambda = 100\left(\frac{1}{10}\sqrt{\frac{h}{m}}\right)$$

Simplifying the expression:

$$\lambda = 10\sqrt{\frac{h}{m}}$$

Quick Tip: When combining equations, isolate the variable you want to eliminate (v) first, then substitute it directly. Keep constants like numbers outside the radical symbol until the final step to keep your algebra clean and fast.

MATHEMATICS

9. The probability that a certain electronic component fails when first used is 0.10. If it does not fail immediately, the probability that it lasts for one year is 0.99. The probability that a new component will last for one year is:

(A) 0.99

(B) 0.871

(C) 0.891

(D) 0.762

Correct Answer: (C) 0.891

Solution:

Concept: This problem can be effectively solved using conditional probability and the multiplication rule of probability. Let us define two events:

- A: The component does not fail immediately (it works when first used).
- B: The component lasts for one year.

For a new component to last for one year, it must satisfy two sequential criteria: it must first survive its initial usage, and then it must continue running for the remainder of the year. The intersection probability is given by:

$$P(A \cap B) = P(A) \times P(B \mid A)$$

Step 1: Calculate the probability that the component survives the initial use, $P(A)$.

We are given that the probability of immediate failure is 0.10. Since survival and immediate failure are complementary events:

$$P(A) = 1 - P(\text{Immediate Failure}) = 1 - 0.10 = 0.90$$

Step 2: Identify the conditional probability, $P(B \mid A)$.

The problem statement notes: "*If it does not fail immediately, the probability that it lasts for one year is 0.99.*" This is a direct statement of conditional probability given that event A has occurred:

$$P(B \mid A) = 0.99$$

Step 3: Compute the total probability that a new component lasts for one year.

Applying the multiplication rule to find the joint probability $P(A \cap B)$:

$$P(\text{Lasts for one year}) = P(A) \times P(B \mid A)$$

$$P(\text{Lasts for one year}) = 0.90 \times 0.99 = 0.891$$

Quick Tip: Think of this problem like a path on a probability tree diagram. To reach the final destination ("lasts 1 year"), you must traverse the first branch ("survives immediate use", 0.90) and then multiply it by the next branch down that path ("survives 1 year given it worked initially", 0.99).

---

10. The shortest distance between the lines

$$x = y + 2 = 6z - 6$$

and

$$x + 1 = 2y = -12z$$

is:

(A) $\frac{1}{2}$

(B) 2

(C) 1

(D) $\frac{3}{2}$

Correct Answer: (B) 2

Solution:

Concept: The shortest distance between two skew lines

$$\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$$

and

$$\vec{r} = \vec{a}_2 + \mu \vec{b}_2$$

is given by:

$$d = \frac{\left|(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)\right|}{|\vec{b}_1 \times \vec{b}_2|}$$

Step 1: Convert the first line into symmetric form.

Given:

$$x = y + 2 = 6z - 6$$

Let the common value be t.

Then:

$$x = t, \qquad y = t - 2, \qquad z = \frac{t+6}{6}$$

Hence:

$$\frac{x-0}{1} = \frac{y+2}{1} = \frac{z-1}{1/6}$$

Multiplying direction ratios by 6:

$$\frac{x}{6} = \frac{y+2}{6} = \frac{z-1}{1}$$

Therefore,

$$\vec{a}_1 = (0, -2, 1)$$

and

$$\vec{b}_1 = (6, 6, 1)$$

Step 2: Convert the second line into symmetric form.

Given:

$$x + 1 = 2y = -12z$$

Let the common value be s.

Then:

$$x = s - 1, \qquad y = \frac{s}{2}, \qquad z = -\frac{s}{12}$$

Thus,

$$\frac{x+1}{1} = \frac{y}{1/2} = \frac{z}{-1/12}$$

Multiplying direction ratios by 12:

$$\frac{x+1}{12} = \frac{y}{6} = \frac{z}{-1}$$

Therefore,

$$\vec{a}_2 = (-1, 0, 0)$$

and

$$\vec{b}_2 = (12, 6, -1)$$

Step 3: Find the required vector quantities.

First,

$$\vec{a}_2 - \vec{a}_1 = (-1 - 0,\ 0 - (-2),\ 0 - 1)$$

$$= (-1, 2, -1)$$

Now compute:

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 6 & 1 \\ 12 & 6 & -1 \end{vmatrix}$$

$$= \hat{i}(6(-1) - 1(6)) - \hat{j}(6(-1) - 1(12)) + \hat{k}(6 \cdot 6 - 6 \cdot 12)$$

$$= \hat{i}(-12) - \hat{j}(-18) + \hat{k}(-36)$$

$$= -12\hat{i} + 18\hat{j} - 36\hat{k}$$

Its magnitude is:

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{(-12)^2 + 18^2 + (-36)^2}$$

$$= \sqrt{144 + 324 + 1296}$$

$$= \sqrt{1764} = 42$$

Step 4: Compute the scalar triple product.

$$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)$$

$$= (-1)(-12) + (2)(18) + (-1)(-36)$$

$$= 12 + 36 + 36$$

$$= 84$$

Therefore,

$$d = \frac{84}{42} = 2$$

Hence, the shortest distance is:

$$\boxed{2}$$

Therefore, the correct answer is:

$$\boxed{(B)}$$

Quick Tip: For shortest distance problems involving skew lines:

$$d = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$$

Always convert symmetric equations carefully before extracting direction ratios.

---

11. If $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$, then the value of $|\hat{i} \times (\vec{a} \times \hat{i})|^2 + |\hat{j} \times (\vec{a} \times \hat{j})|^2 + |\hat{k} \times (\vec{a} \times \hat{k})|^2$ is equal to:

(A) 17

(B) 18

(C) 19

(D) 20

Correct Answer: (B) 18

Solution:

Concept: This question can be simplified using the Vector Triple Product vector identity:

$$\vec{x} \times (\vec{y} \times \vec{z}) = (\vec{x} \cdot \vec{z})\vec{y} - (\vec{x} \cdot \vec{y})\vec{z}$$

Let us assume a general vector $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$. We can apply this triple product property to each term individually where $\vec{x}$ and $\vec{z}$ are replaced by the standard unit orthogonal vectors $\hat{i}$, $\hat{j}$, or $\hat{k}$.

Step 1: Simplifying the first vector triple product component.

Applying the rule to $\hat{i} \times (\vec{a} \times \hat{i})$:

$$\hat{i} \times (\vec{a} \times \hat{i}) = (\hat{i} \cdot \hat{i})\vec{a} - (\hat{i} \cdot \vec{a})\hat{i}$$

Since $\hat{i} \cdot \hat{i} = 1$ and $\hat{i} \cdot \vec{a} = x$ (the scalar x-component of vector $\vec{a}$):

$$\hat{i} \times (\vec{a} \times \hat{i}) = \vec{a} - x\hat{i} = (x\hat{i} + y\hat{j} + z\hat{k}) - x\hat{i} = y\hat{j} + z\hat{k}$$

Finding the squared magnitude of this resulting component vector:

$$|\hat{i} \times (\vec{a} \times \hat{i})|^2 = |y\hat{j} + z\hat{k}|^2 = y^2 + z^2$$

Step 2: Extrapolating the result for the $\hat{j}$ and $\hat{k}$ components.

By symmetric property distribution, executing the same vector cross operations for the $\hat{j}$ and $\hat{k}$ structures yields:

$$|\hat{j} \times (\vec{a} \times \hat{j})|^2 = |x\hat{i} + z\hat{k}|^2 = x^2 + z^2$$

$$|\hat{k} \times (\vec{a} \times \hat{k})|^2 = |x\hat{i} + y\hat{j}|^2 = x^2 + y^2$$

Step 3: Summing the three expressions together.

Add the three squared magnitude evaluations together to express the total value:

$$\text{Total Sum} = (y^2 + z^2) + (x^2 + z^2) + (x^2 + y^2) = 2(x^2 + y^2 + z^2) = 2|\vec{a}|^2$$

Step 4: Substituting the numerical components of the given vector $\vec{a}$.

We are given $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$, which means $x = 2$, $y = 1$, and $z = 2$.

$$|\vec{a}|^2 = x^2 + y^2 + z^2 = (2)^2 + (1)^2 + (2)^2 = 4 + 1 + 4 = 9$$

Substituting this value back into our total sum equation:

$$\text{Total Value} = 2(9) = 18$$

Quick Tip: Memorize this standard vector identity for objective tests: For any general vector $\vec{a}$, the expansion identity $\sum |\hat{i} \times (\vec{a} \times \hat{i})|^2$ always simplifies cleanly to $2|\vec{a}|^2$.

---

12. The line $y = mx$ bisects the area enclosed by the lines $x = 0$, $y = 0$, $x = \frac{3}{2}$ and the curve $y = 1 + 4x - x^2$. Then, the value of m is:

(A) 13/6

(B) 13/2

(C) 13/5

(D) 13/7

Correct Answer: (A) 13/6

Solution:

Concept: The area under a curve $y = f(x)$ bounded by the vertical lines $x = a$ and $x = b$ above the x-axis ($y = 0$) is calculated using a definite integral:

$$\text{Total Area } (A) = \int_a^b f(x)\,dx$$

When a straight line passing through the origin, $y = mx$, bisects this bounded space over the same horizontal interval, it splits the total area into two equal parts. The area under the line forms a right-angled trapezoid over the interval $[0, b]$, whose area must equal $\frac{A}{2}$.

Step 1: Calculate the total enclosed area A.

The boundaries are given by the y-axis ($x = 0$), the x-axis ($y = 0$), the vertical line $x = \frac{3}{2}$, and the downward-opening parabola $y = 1 + 4x - x^2$. Let us evaluate the definite integral from 0 to $\frac{3}{2}$:

$$A = \int_0^{3/2} (1 + 4x - x^2)\,dx$$

Integrating term-by-term:

$$A = \left[x + 2x^2 - \frac{x^3}{3} \right]_0^{3/2}$$

Substituting the upper limit $x = \frac{3}{2}$ (the lower limit at $x = 0$ evaluates cleanly to 0):

$$A = \left(\frac{3}{2}\right) + 2\left(\frac{3}{2}\right)^2 - \frac{1}{3}\left(\frac{3}{2}\right)^3$$

$$A = \frac{3}{2} + 2\left(\frac{9}{4}\right) - \frac{1}{3}\left(\frac{27}{8}\right) = \frac{3}{2} + \frac{9}{2} - \frac{9}{8}$$

Combine the first two fractions:

$$A = 6 - \frac{9}{8} = \frac{48 - 9}{8} = \frac{39}{8}$$

Step 2: Set up the mathematical condition for bisection.

Since the line $y = mx$ bisects this area, the area bounded under the line $y = mx$ from $x = 0$ to $x = \frac{3}{2}$ must be exactly equal to half of the total area ($\frac{A}{2}$):

$$\text{Area under the line } (A_{\text{line}}) = \frac{1}{2} \times \frac{39}{8} = \frac{39}{16}$$

Step 3: Integrate under the line to solve for m.

The region under the line is a simple linear function:

$$A_{\text{line}} = \int_0^{3/2} mx \, dx = \left[\frac{mx^2}{2} \right]_0^{3/2}$$

Substituting the limits:

$$A_{\text{line}} = \frac{m}{2}\left(\frac{3}{2}\right)^2 = \frac{m}{2}\left(\frac{9}{4}\right) = \frac{9m}{8}$$

Equating this area calculation to our value from Step 2:

$$\frac{9m}{8} = \frac{39}{16}$$

Isolating m by multiplying both sides by $\frac{8}{9}$:

$$m = \frac{39}{16} \times \frac{8}{9}$$

Simplifying the fractions by factoring out common terms (8 with 16, and 3 with 39 and 9):

$$m = \frac{13}{2} \times \frac{1}{3} = \frac{13}{6}$$

Quick Tip: Instead of integrating the linear function $y = mx$ with calculus, save valuable time by treating it as a simple right-angled triangle where Base $= b$ and Height $= m \cdot b$. The geometric triangle area formula gives: Area $= \frac{1}{2} \cdot b \cdot (mb) = \frac{mb^2}{2}$.
