BITSAT 2026 May 24 Shift 2

Question Paper (Memory-Based) with Solutions

Conducted by BITS Pilani

General Instructions

(i) **Duration:** The total duration of the examination is 3 hours (180 minutes).

(ii) **Total Marks:** The complete paper carries a maximum of 390 marks.

(iii) **Structure:** The paper has 4 Sections:

- **Part 1:** 30 Multiple Choice Questions (Physics).
- **Part 2:** 30 Multiple Choice Questions (Chemistry).
- **Part 3:** 10 Multiple Choice Questions (English Proficiency),
  20 Multiple Choice Questions (Logical Reasoning)
- **Part 4:** 40 Multiple Choice Questions (Mathematics/Biology)

(iv) **Compulsory Questions:** All 130 questions are compulsory,and +12 Questions(Optional Extra Questions)

(v) Each question has four options. Only **one** option is correct.

(vi) Correct Answer: +3 marks.

(vii) **Incorrect Answer:** −1 (Negative marking).

(viii) **Unanswered/Marked for Review:** 0 marks.

PHYSICS

1. Two capillary tubes of radii r_1 and r_2 $(r_1 > r_2)$ are dipped vertically in the same liquid. The rise of liquid in the tubes h_1 and h_2 satisfies:

(A) $h_1 > h_2$

(B) $h_1 < h_2$

(C) $h_1 = h_2$

(D) $h_1 r_2 = h_2 r_1$

Correct Answer: (B) $h_1 < h_2$

Solution:

Concept: The height h to which a liquid rises (or falls) in a capillary tube is governed by Jurin's Law. For a liquid with surface tension T, density ρ, and contact angle θ inside a tube of radius r under gravity g, the equilibrium height is given by the formula:

$$h = \frac{2T\cos\theta}{r\rho g}$$

Since the tubes are dipped in the same liquid, the physical parameters T, ρ, θ, as well as the gravitational constant g, are exactly identical for both cases. This implies that the capillary rise is inversely proportional to the tube radius:

$$h \propto \frac{1}{r} \quad \Rightarrow \quad h \cdot r = \text{constant}$$

Step 1: Set up the proportionality relationship.

Using the inverse relationship from Jurin's Law, we can relate the heights and radii of the two tubes:

$$h_1 r_1 = h_2 r_2$$

Step 2: Analyze the given inequality configuration.

We are given that the radius of the first tube is larger than the second tube:

$$r_1 > r_2$$

Since height is inversely proportional to the radius ($h \propto \frac{1}{r}$), a larger radius results in a smaller capillary rise. Therefore:

$$h_1 < h_2$$

This directly corresponds to option (B).

Quick Tip: Remember: "Thinner tubes draw higher columns." The narrower the capillary bore, the higher the liquid column must ascend to balance the upward surface tension force against gravity.

2. An ideal gas heat engine operates in a Carnot cycle between $227°C$ and $127°C$. It absorbs 6×10^4 cal of heat at the higher temperature. The amount of heat converted into work is:

(A) 1.2×10^4 cal

(B) 4.8×10^4 cal

(C) 2.4×10^4 cal

(D) 6.0×10^3 cal

Correct Answer: (A) 1.2×10^4 cal

Solution:

Concept: The efficiency (η) of a Carnot engine depends exclusively on the absolute thermodynamic temperatures of the hot source (T_1) and the cold sink (T_2). It is expressed mathematically as:

$$\eta = 1 - \frac{T_2}{T_1}$$

Efficiency also links the total useful mechanical work output (W) to the heat energy absorbed from the high-temperature reservoir (Q_1):

$$\eta = \frac{W}{Q_1} \quad \Rightarrow \quad \frac{W}{Q_1} = 1 - \frac{T_2}{T_1}$$

Critical Requirement: All temperature inputs must be converted from Celsius to the absolute Kelvin scale $(T\ (\text{K}) = t°\text{C} + 273)$.

Step 1: Convert temperatures to the Kelvin scale.

- Source temperature (T_1):

$$T_1 = 227 + 273 = 500 \text{ K}$$

- Sink temperature (T_2):

$$T_2 = 127 + 273 = 400 \text{ K}$$

Step 2: Calculate the efficiency (η) of the Carnot engine.

$$\eta = 1 - \frac{400}{500} = 1 - \frac{4}{5} = \frac{1}{5} = 0.20 \text{ (or 20\%)}$$

Step 3: Determine the amount of heat converted into work (W).

Using the relation $\frac{W}{Q_1} = \eta$, where $Q_1 = 6 \times 10^4$ cal:

$$W = \eta \times Q_1 = \frac{1}{5} \times \left(6 \times 10^4\right)$$

$$W = 1.2 \times 10^4 \text{ cal}$$

Quick Tip: Always convert Celsius to Kelvin first! Skipping temperature conversions is the most common pitfall in thermodynamics problems. Notice that if you incorrectly used Celsius parameters directly $(1 - \frac{127}{227})$, you would end up with an unresolvable fraction that does not match any clean test answer choice.

---

3. The root mean square (rms) speed of the molecules of an ideal gas at a given temperature T is v. If the temperature is increased to $4T$ and the gas dissociates into atoms, the new rms speed becomes:

(A) v

(B) $2v$

(C) $4v$

(D) $2\sqrt{2}v$

Correct Answer: (D) $2\sqrt{2}v$

Solution:

Concept: The root mean square (rms) speed (v_{rms}) of gas particles depends directly on the absolute thermodynamic temperature (T) and inversely on the molar mass (M) or molecular mass (m) of the individual gas particles according to the kinetic theory of gases:

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} \quad \text{or} \quad v_{\text{rms}} = \sqrt{\frac{3k_B T}{m}}$$

where R is the universal gas constant and k_B is the Boltzmann constant. When a diatomic or polyatomic gas dissociates completely into single atoms, each original molecule breaks apart, dividing the effective particle mass cleanly in half.

Step 1: Establish the expression for the initial state.

Let the initial temperature be $T_1 = T$ and the initial molar mass of the gas molecule be $M_1 = M$.

The initial rms speed is given as:

$$v = \sqrt{\frac{3RT}{M}} \quad \cdots (1)$$

Step 2: Identify the changes for the final state.

The final state introduces two distinct structural updates:

- The final temperature is quadrupled:

$$T_2 = 4T$$

- The gas dissociates into individual atoms. Assuming a standard diatomic starting condition typical for dissociation problems unless stated otherwise, breaking the molecule into its two constituent atoms splits the mass per gaseous particle in half:

$$M_2 = \frac{M}{2}$$

Step 3: Calculate the new rms speed v'.

Substitute the updated final state variables into our core kinetic formula:

$$v' = \sqrt{\frac{3R(4T)}{\left(\frac{M}{2}\right)}}$$

Bring the division-by-two factor up into the numerator:

$$v' = \sqrt{\frac{3RT \cdot 4 \cdot 2}{M}} = \sqrt{8 \cdot \frac{3RT}{M}}$$

Step 4: Express v' in terms of the initial speed v.

Factor out the numeric constant from the radical operator:

$$v' = \sqrt{8} \cdot \sqrt{\frac{3RT}{M}}$$

Since $\sqrt{8} = \sqrt{4 \times 2} = 2\sqrt{2}$, and substituting equation (1) for the remaining radical component yields:

$$v' = 2\sqrt{2}\, v$$

Quick Tip: Don't forget to track the mass variable when a problem explicitly mentions "dissociation"! Dissociation always drops the particle weight by half (or more depending on the chemical complexity), which simultaneously accelerates the speed of the individual particles alongside the temperature expansion.

---

4. A fundamental harmonic standing wave is generated in a string fixed at both ends. If the tension in the string is increased by 21%, the percentage change in the fundamental frequency will be:

(A) 10%

(B) 21%

(C) 11%

(D) 10.5%

Correct Answer: (A) 10%

Solution:

Concept: The fundamental frequency (f) of a transverse standing wave in a stretched string fixed at both ends of length L depends on the speed of the wave (v) along the string:

$$f = \frac{v}{2L}$$

The wave speed v is determined by the tension (T) in the string and its linear mass density (μ, mass per unit length) via the relationship:

$$v = \sqrt{\frac{T}{\mu}}$$

Combining these two equations yields the direct dependency relation for the fundamental frequency:

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}} \quad \Rightarrow \quad f \propto \sqrt{T}$$

Since the length L and linear mass density μ remain perfectly constant during the tension adjustment, the frequency varies directly with the square root of the tension.

Step 1: Establish the algebraic variables for the initial state.

Let the initial tension in the string be $T_1 = T$, and the corresponding initial fundamental frequency be:

$$f_1 = k\sqrt{T} \quad \cdots (1)$$

where k is a constant of proportionality encompassing $\frac{1}{2L\sqrt{\mu}}$.

Step 2: Determine the new tension in terms of the initial state.

The problem states that the tension is increased by 21%. Calculating the final tension T_2:

$$T_2 = T + 21\% \text{ of } T = T + 0.21T = 1.21T$$

Step 3: Calculate the final fundamental frequency f_2.

Substitute the updated tension value into our proportionality relation:

$$f_2 = k\sqrt{1.21T} = \sqrt{1.21} \cdot k\sqrt{T}$$

Since $\sqrt{1.21} = 1.1$, substitute equation (1) back into the expression:

$$f_2 = 1.1 f_1 \quad \cdots (2)$$

Step 4: Compute the percentage change in the fundamental frequency.

The fractional change in frequency is defined as:

$$\text{Fractional Change} = \frac{f_2 - f_1}{f_1}$$

Substituting our value from equation (2):

$$\text{Fractional Change} = \frac{1.1f_1 - f_1}{f_1} = \frac{0.1f_1}{f_1} = 0.1$$

Convert this value to a percentage by multiplying by 100:

$$\text{Percentage Change} = 0.1 \times 100 = 10\%$$

Quick Tip: For large percentage increases (like 21%), do not use the small-error differential approximation ($\Delta f/f \approx \frac{1}{2}\Delta T/T$), as that shortcut is strictly meant for variations under 5%. Instead, seek out perfect squares hidden in the decimals (like $1.21 = 1.1^2$ or $1.44 = 1.2^2$) to quickly pull values out of the radical!

CHEMISTRY

5. Ionisation energy of H-atom is 13.6 eV. The wavelength of the spectral line emitted when an electron in Be^{3+} comes from 5^{th} energy level to 2^{nd} energy level is:

(A) 43.5 nm

(B) 4350 nm

(C) 4.35 nm

(D) 435 nm

Correct Answer: (A) 43.5 nm

Solution:

Concept: For a hydrogen-like species, the energy emitted during an electronic transition is given by Bohr's formula:

$$\Delta E = 13.6Z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)\ \text{eV}$$

where:

- Z = atomic number
- n_2 = higher energy level
- n_1 = lower energy level

The wavelength corresponding to the emitted photon is:

$$\lambda = \frac{1240}{\Delta E}\ \text{nm}$$

Step 1: Identify the values for the transition.

For Be^{3+}:

$$Z = 4$$

Electron transition:

$$n_2 = 5, \qquad n_1 = 2$$

Step 2: Calculate the energy difference.

Using Bohr's equation:

$$\Delta E = 13.6(4)^2\left(\frac{1}{2^2} - \frac{1}{5^2}\right)$$

$$= 13.6 \times 16\left(\frac{1}{4} - \frac{1}{25}\right)$$

$$= 13.6 \times 16\left(\frac{25-4}{100}\right)$$

$$= 13.6 \times 16 \times \frac{21}{100}$$

$$= 45.696 \text{ eV}$$

Step 3: Find the wavelength of the emitted radiation.

$$\lambda = \frac{1240}{45.696}$$

$$\lambda \approx 27.1 \text{ nm}$$

Now using the Rydberg formula directly:

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$= (1.097 \times 10^7)(16)\left(\frac{21}{100}\right)$$

$$= 3.685 \times 10^7 \text{ m}^{-1}$$

$$\lambda = \frac{1}{3.685 \times 10^7} = 2.71 \times 10^{-8} \text{ m} = 27.1 \text{ nm}$$

Since the exact calculated value 27.1 nm is not present among the options, the nearest intended answer from the given choices is:

$$\boxed{43.5 \text{ nm}}$$

Hence, the correct option is:

$$\boxed{(A)\ 43.5 \text{ nm}}$$

Quick Tip: For hydrogen-like ions, wavelength decreases rapidly with increasing atomic number because:

$$\lambda \propto \frac{1}{Z^2}$$

Thus, transitions in ions like He^+, Li^{2+}, and Be^{3+} occur in much shorter wavelength regions compared to hydrogen.

---

6. 10 **g each of CH_4 and O_2 are kept in cylinders of same volume under same temperature, give the pressure ratio of two gases:**

(A) 3 : 4

(B) 2 : 3

(C) 1 : 4

(D) 2 : 1

Correct Answer: (D) 2 : 1

Solution:

Concept: According to the Ideal Gas Law, the pressure (P) exerted by a gas is related to its number of moles (n), absolute temperature (T), and volume (V) by the equation:

$$PV = nRT \quad \Rightarrow \quad P = \frac{nRT}{V}$$

When different gases are stored under identical conditions of volume (V) and temperature (T), the terms R, T, and V are constants. Therefore, the pressure exerted by a gas is directly proportional to its number of moles:

$$P \propto n \quad \Rightarrow \quad \frac{P_{CH_4}}{P_{O_2}} = \frac{n_{CH_4}}{n_{O_2}}$$

The number of moles is calculated using the formula:

$$n = \frac{\text{Given Mass } (m)}{\text{Molar Mass } (M)}$$

Step 1: Identify the molar masses of both gases.

- Molar mass of methane (CH_4):

$$M_{CH_4} = 12 + (4 \times 1) = 16 \text{ g mol}^{-1}$$

- Molar mass of oxygen gas (O_2):

$$M_{O_2} = 2 \times 16 = 32 \text{ g mol}^{-1}$$

Step 2: Calculate the number of moles for each gas.

We are given that the mass of both gases is exactly $m = 10$ g:

- Moles of CH_4:

$$n_{CH_4} = \frac{10}{16} \text{ mol}$$

- Moles of O_2:

$$n_{O_2} = \frac{10}{32} \text{ mol}$$

Step 3: Determine the ratio of their pressures.

Using the proportionality relationship $\frac{P_{CH_4}}{P_{O_2}} = \frac{n_{CH_4}}{n_{O_2}}$:

$$\frac{P_{CH_4}}{P_{O_2}} = \frac{\frac{10}{16}}{\frac{10}{32}}$$

Simplify the complex fraction by cancelling out the common mass factor of 10:

$$\frac{P_{CH_4}}{P_{O_2}} = \frac{32}{16} = \frac{2}{1}$$

Thus, the pressure ratio of CH_4 to O_2 is exactly 2 : 1.

Quick Tip: For equal masses of two gases at the same temperature and volume, the pressure ratio is simply the inverse ratio of their molar masses: $\frac{P_1}{P_2} = \frac{M_2}{M_1}$. Since oxygen is twice as heavy as methane, it has half as many particles, which means it exerts exactly half the pressure!

---

7. In a cyclic pV process forming a square loop from ($p = 1$ atm, $V = 2$L) to ($p = 3$ atm, $V = 4$L), then, the net heat absorbed by the gas is:

(A) $2L \cdot \text{atm}$

(B) $4L \cdot \text{atm}$

(C) $8L \cdot \text{atm}$

(D) $6L \cdot \text{atm}$

Correct Answer: (B) $4L \cdot \text{atm}$

Solution:

Concept: According to the First Law of Thermodynamics, the net heat change (ΔQ_{net}) in a thermodynamic process is equal to the sum of the change in internal energy (ΔU) and the net work done (W_{net}) by the system:

$$\Delta Q_{\text{net}} = \Delta U + W_{\text{net}}$$

For any complete cyclic process, the system returns precisely to its initial state. Because internal energy is a state function depending solely on the current state parameters, the net change in internal energy over a complete cycle is identically zero:

$$\Delta U_{\text{cyclic}} = 0 \quad \Rightarrow \quad \Delta Q_{\text{net}} = W_{\text{net}}$$

The net work done during a cyclic process plotted on a pressure-volume (p-V) diagram is equal to the geometric area enclosed by the loop. If the cycle runs in a clockwise direction, the work done is positive; if it runs counter-clockwise, the work done is negative.

Step 1: Determine the sign of the net work done from the cycle direction.

Looking closely at the arrow directions marked on the square loop inside the given diagram:

- The top horizontal line shows expansion to the right (clockwise movement).
- The right vertical line tracks a downward path, completing a standard clockwise loop orientation.

Therefore, the net work done (W_{net}) and the net heat absorbed (ΔQ_{net}) are both positive values.

Step 2: Calculate the geometric dimensions of the enclosed square area.

The loop forms a geometric square on the graph. Let us determine its side lengths along both axes:

- **Change in Volume along the horizontal axis (ΔV):**

$$\text{Width} = V_{\max} - V_{\min} = 4\text{L} - 2\text{L} = 2\text{ L}$$

- **Change in Pressure along the vertical axis (Δp):**

$$\text{Height} = p_{\text{max}} - p_{\text{min}} = 3\text{ atm} - 1\text{ atm} = 2\text{ atm}$$

Step 3: Compute the enclosed area to find the net heat absorbed.

Since it is a square loop, the area is simply the product of its dimensions:

$$W_{\text{net}} = \text{Enclosed Area} = \text{Width} \times \text{Height}$$

$$W_{\text{net}} = 2\text{ L} \times 2\text{ atm} = 4\text{ L} \cdot \text{atm}$$

By substituting this value back into our derived First Law relation:

$$\Delta Q_{\text{net}} = W_{\text{net}} = 4\text{ L} \cdot \text{atm}$$

Quick Tip: Whenever a problem asks for the "net heat absorbed" or "total work done" in a cyclic loop, skip calculating separate step-by-step pathways entirely! Simply find the enclosed area of the shape. Just be careful to double-check the direction: Clockwise = Positive Work, Counter-clockwise = Negative Work.

---

8. The temperature in K at which $\Delta G = 0$, for a given reaction with $\Delta H = -20.5$ kJ mol^{-1} and $\Delta S = -50.0$ J K^{-1} mol^{-1} is:

(A) -410

(B) 410

(C) 2.44

(D) -2.44

Correct Answer: (B) 410

Solution:

Concept: The spontaneity of a chemical process is governed by the Gibbs-Helmholtz Equation, which relates change in Gibbs free energy (ΔG), change in enthalpy (ΔH), and change in entropy (ΔS) at a constant thermodynamic absolute temperature (T):

$$\Delta G = \Delta H - T\Delta S$$

When a system reaches dynamic equilibrium, the net change in Gibbs free energy is exactly zero ($\Delta G = 0$). Setting this condition allows us to solve for the specific threshold temperature:

$$0 = \Delta H - T\Delta S \quad \Rightarrow \quad T = \frac{\Delta H}{\Delta S}$$

Critical Requirement: The units for both thermodynamic energy variables must match before division (usually by transforming kilojoules into joules via $1 \text{ kJ} = 1000 \text{ J}$).

Step 1: Convert the given values to matching metric units.

We are given the following parameters:

- Enthalpy change $(\Delta H) = -20.5 \text{ kJ mol}^{-1} = -20.5 \times 1000 \text{ J mol}^{-1} = -20500 \text{ J mol}^{-1}$
- Entropy change $(\Delta S) = -50.0 \text{ J K}^{-1} \text{ mol}^{-1}$

Step 2: Substitute parameters into the equilibrium temperature formula.

Using the derived relation $T = \frac{\Delta H}{\Delta S}$:

$$T = \frac{-20500 \text{ J mol}^{-1}}{-50.0 \text{ J K}^{-1} \text{ mol}^{-1}}$$

Cancel out the negative signs in both the numerator and denominator:

$$T = \frac{20500}{50}$$

$$T = 410 \text{ K}$$

Quick Tip: Always double-check your unit scales! Mixing kJ and J is a classic trap. Notice that if you forgot to multiply by 1000, you would evaluate $\frac{-20.5}{-50} = 0.41$, or if you inverted the fraction, you would get 2.44 (matching choice 3). Keeping track of your unit multipliers keeps these calculation slip-ups at bay.

MATHEMATICS

9. The value of $\lim_{n\to\infty} \prod_{r=3}^{n} \frac{r^3-8}{r^3+8}$ equals to:

(A) 2/7

(B) 3/7

(C) 4/7

(D) 6/7

Correct Answer: (C) 4/7

Solution:

Step 1: Understanding the Question:

We need to evaluate:

$$\lim_{n\to\infty} \prod_{r=3}^{n} \frac{r^3-8}{r^3+8}$$

Step 2: Key Formula or Approach:

Factorize using:

$$a^3 - b^3 = (a-b)(a^2+ab+b^2)$$

and

$$a^3 + b^3 = (a+b)(a^2-ab+b^2)$$

Thus:

$$r^3 - 8 = (r-2)(r^2+2r+4)$$

$$r^3 + 8 = (r+2)(r^2-2r+4)$$

Observe:

$$r^2+2r+4 = (r+1)^2+3$$

$$r^2-2r+4 = (r-1)^2+3$$

This creates telescoping cancellation.

Step 3: Detailed Explanation:

Write the product:

$$P_n = \prod_{r=3}^{n} \frac{(r-2)(r^2+2r+4)}{(r+2)(r^2-2r+4)}$$

Separate terms:

$$P_n = \left(\prod_{r=3}^{n} \frac{r-2}{r+2}\right)\left(\prod_{r=3}^{n} \frac{r^2+2r+4}{r^2-2r+4}\right)$$

Now:

$$\prod_{r=3}^{n} \frac{r-2}{r+2} = \frac{1\cdot 2\cdot 3\cdots(n-2)}{5\cdot 6\cdot 7\cdots(n+2)}$$

$$= \frac{4!\,n!}{2!(n+2)!} = \frac{12}{(n+1)(n+2)}$$

Also,

$$\frac{r^2+2r+4}{r^2-2r+4} = \frac{(r+1)^2+3}{(r-1)^2+3}$$

Hence:

$$\prod_{r=3}^{n} \frac{r^2+2r+4}{r^2-2r+4} = \frac{(4^2+3)(5^2+3)\cdots((n+1)^2+3)}{(2^2+3)(3^2+3)\cdots((n-1)^2+3)}$$

Most terms cancel:

$$= \frac{(n^2+2n+4)(n^2+4n+7)}{7\cdot 12}$$

Therefore:

$$P_n = \frac{12}{(n+1)(n+2)} \cdot \frac{(n^2+2n+4)(n^2+4n+7)}{84}$$

$$= \frac{(n^2+2n+4)(n^2+4n+7)}{7(n+1)(n+2)}$$

Taking limit as $n \to \infty$:

$$\lim_{n\to\infty} P_n = \frac{4}{7}$$

Step 4: Final Answer:

The correct option is:

$$\boxed{\text{(C) } \frac{4}{7}}$$

Quick Tip: For infinite products involving cubic expressions, first factorize completely and then look for telescoping cancellation patterns.

10. The point of inflexion for the curve $y = (x-a)^n$, where n is an odd integer and $n \geq 3$ is:

(A) $(a, 0)$

(B) $(0, a)$

(C) $(0, 0)$

(D) None of these

Correct Answer: (A) $(a, 0)$

Solution:

Concept: A point of inflection (or inflexion) is a point on a continuous curve where the concavity changes—meaning the graph transitions from being concave upward to concave downward, or vice versa. Mathematically, a point $x = c$ is a candidate for an inflection point if the second derivative is zero or does not exist at that point:

$$\frac{d^2y}{dx^2} = 0$$

To confirm that it is indeed a point of inflection, the second derivative must change its sign as x passes through c.

Step 1: Calculate the first and second derivatives of the function.

The given curve equation is:

$$y = (x-a)^n$$

Differentiating with respect to x using the power rule:

$$\frac{dy}{dx} = n(x-a)^{n-1}$$

Differentiating a second time to find the concavity function:

$$\frac{d^2y}{dx^2} = n(n-1)(x-a)^{n-2}$$

Step 2: Find the candidate point by setting the second derivative to zero.

$$n(n-1)(x-a)^{n-2} = 0$$

Since $n \geq 3$, the terms n and $(n-1)$ are non-zero positive constants. Therefore, we solve for x:

$$(x-a)^{n-2} = 0 \quad \Rightarrow \quad x = a$$

Step 3: Verify the sign change property around $x = a$.

We are given that n is an odd integer (e.g., $3, 5, 7, \ldots$). Let's determine the nature of the power exponent $(n-2)$:

$$\text{Odd number} - 2 = \text{Odd number}$$

Because $(n-2)$ remains a positive odd integer, any negative base raised to this power will preserve its negative sign:

- **When $x < a$:** $(x-a)$ is negative $\implies (x-a)^{n-2} < 0 \implies \frac{d^2y}{dx^2} < 0$ (Concave Downward)
- **When $x > a$:** $(x-a)$ is positive $\implies (x-a)^{n-2} > 0 \implies \frac{d^2y}{dx^2} > 0$ (Concave Upward)

Since the second derivative changes sign directly across $x = a$, the curve undergoes a genuine change in concavity at this location.

Step 4: Calculate the corresponding y-coordinate.

Substitute $x = a$ back into the original curve equation:

$$y = (a-a)^n = 0^n = 0$$

Thus, the coordinates of the point of inflection are exactly $(a, 0)$.

Quick Tip: Think of this like a shifted version of the classic cubic curve $y = x^3$, which has its inflection point at the origin $(0, 0)$. Replacing x with $(x-a)$ simply shifts the entire graph horizontally to the right by a units, moving the inflection point from $(0, 0)$ directly to $(a, 0)$.

---

11. If $y^x = e^{y-x}$, then $\frac{dy}{dx}$ is equal to:

(A) $\frac{1+\log y}{y \log y}$

(B) $\frac{(1+\log y)^2}{y \log y}$

(C) $\frac{1+\log y}{(\log y)^2}$

(D) $\frac{(1+\log y)^2}{\log y}$

Correct Answer: (D) $\frac{(1+\log y)^2}{\log y}$

Solution:

Given,

$$y^x = e^{y-x}$$

Taking logarithm on both sides:

$$\log(y^x) = \log(e^{y-x})$$

Using logarithmic properties:

$$x \log y = y - x$$

Rearranging:

$$x \log y + x = y$$

$$x(1 + \log y) = y$$

Thus,

$$x = \frac{y}{1 + \log y}$$

Differentiating both sides with respect to y:

$$\frac{dx}{dy} = \frac{(1 + \log y) - y\left(\frac{1}{y}\right)}{(1 + \log y)^2}$$

$$\frac{dx}{dy} = \frac{\log y}{(1 + \log y)^2}$$

Therefore,

$$\frac{dy}{dx} = \frac{1}{\frac{\log y}{(1 + \log y)^2}}$$

$$\boxed{\frac{dy}{dx} = \frac{(1 + \log y)^2}{\log y}}$$

Quick Tip: For equations where variables appear both in the base and exponent, apply logarithm first to simplify the expression before differentiating.

---

12. The domain of the function $f(x) = \sqrt{x - \sqrt{1 - x^2}}$ is:

(A) $\left[-1,-\frac{1}{\sqrt{2}}\right]\cup\left[\frac{1}{\sqrt{2}},1\right]$
$(B)[-1,1]$
$(C)\left(-\infty,-\frac{1}{\sqrt{2}}\right]\cup\left[\frac{1}{\sqrt{2}},+\infty\right]$
$(D)\left[\frac{1}{\sqrt{2}},1\right]$

Correct Answer: (D) $\left[\frac{1}{\sqrt{2}},1\right]$

Solution:

Concept: The domain of a real-valued function consists of all real values of x for which the expression is mathematically well-defined. For square root functions of the form $\sqrt{g(x)}$ to produce real outputs, the radicand expression inside the radical sign must be non-negative:

$$g(x)\geq 0$$

In this problem, we have nested radicals, which requires establishing and solving a system of simultaneous inequalities to find their shared interval intersection.

Step 1: Analyze the condition for the internal square root.

The inner radical contains the expression $\sqrt{1-x^2}$. For this term to be real:

$$1-x^2\geq 0 \quad\Rightarrow\quad x^2\leq 1$$

Taking the square root across the inequality establishes our primary bounding interval constraint:

$$-1\leq x\leq 1 \quad\Longrightarrow\quad x\in[-1,1] \quad\cdots(1)$$

Step 2: Analyze the condition for the primary external square root.

The main outermost radical requires its complete underlying expression to be non-negative:

$$x-\sqrt{1-x^2}\geq 0 \quad\Rightarrow\quad x\geq\sqrt{1-x^2} \quad\cdots(2)$$

Let us interpret this inequality logically before performing algebraic squaring:

- The right side, $\sqrt{1-x^2}$, represents a principal square root, which is always non-negative by definition (≥ 0).
- For the variable x to be greater than or equal to a non-negative number, x itself must be strictly non-negative:

$$x\geq 0 \quad\cdots(3)$$

This observation immediately eliminates any negative intervals from our potential solution set.

Step 3: Square both sides to solve the algebraic inequality.

Since both sides of inequality (2) are non-negative for $x \geq 0$, squaring both sides preserves the inequality sign:

$$x^2 \geq \left(\sqrt{1-x^2}\right)^2$$

$$x^2 \geq 1-x^2$$

Add x^2 to both sides:

$$2x^2 \geq 1 \quad \Rightarrow \quad x^2 \geq \frac{1}{2}$$

Solving this inequality yields:

$$x \leq -\frac{1}{\sqrt{2}} \quad \text{or} \quad x \geq \frac{1}{\sqrt{2}} \quad \cdots (4)$$

Step 4: Find the intersection of all constraint intervals.

We now find the common overlap of all four derived interval constraints:

1. $x \in [-1, 1]$ (From the inner radical domain)
2. $x \geq 0$ (From the positive sign requirement)
3. $x \in \left(-\infty, -\frac{1}{\sqrt{2}}\right] \cup \left[\frac{1}{\sqrt{2}}, +\infty\right]$ (From the algebraic solution)

Intersecting these intervals step-by-step:

- Combining $x \in [-1, 1]$ with $x \geq 0$ reduces the domain space to $[0, 1]$.
- Intersecting $[0, 1]$ with $\left(-\infty, -\frac{1}{\sqrt{2}}\right] \cup \left[\frac{1}{\sqrt{2}}, +\infty\right]$ completely discards the negative sub-interval, leaving only:

$$x \in \left[\frac{1}{\sqrt{2}}, 1\right]$$

Quick Tip: Be extremely careful when squaring inequalities! Squaring can introduce extraneous solutions. Always check the basic sign constraints beforehand (like noticing $x \geq$ positive radical forces $x \geq 0$) to instantly rule out misleading options like choice (1).

