BITSAT 2026 May 24 Shift 2

Question Paper (Memory-Based) with Solutions

Conducted by BITS Pilani

General Instructions

(i) **Duration:** The total duration of the examination is 3 hours (180 minutes).

(ii) **Total Marks:** The complete paper carries a maximum of 390 marks.

(iii) **Structure:** The paper has 4 Sections:

- **Part 1:** 30 Multiple Choice Questions (Physics).
- **Part 2:** 30 Multiple Choice Questions (Chemistry).
- **Part 3:** 10 Multiple Choice Questions (English Proficiency),
  20 Multiple Choice Questions (Logical Reasoning)
- **Part 4:** 40 Multiple Choice Questions (Mathematics/Biology)

(iv) **Compulsory Questions:** All 130 questions are compulsory,and +12 Questions(Optional Extra Questions)

(v) Each question has four options. Only **one** option is correct.

(vi) **Correct Answer:** +3 marks.

(vii) **Incorrect Answer:** −1 (Negative marking).

(viii) **Unanswered/Marked for Review:** 0 marks.

PHYSICS

1. Two point charges $+q$ and $-q$ are held fixed at a distance d apart. The net electric potential at a point midway between the two charges and its net field are?

(A) $E = 0 \quad V = 0$

(B) $E \neq 0 \quad V \neq 0$

(C) $E = 0 \quad V \neq 0$

(D) $E \neq 0 \quad V = 0$

Correct Answer: (D) $E \neq 0 \quad V = 0$

Solution:

Concept: Electric potential (V) is a scalar quantity, so the total potential at a point is the simple algebraic sum of the potentials due to individual charges.

$$V_{\text{net}} = \sum V_i = \frac{kq_i}{r_i}$$

Electric field (E) is a vector quantity, so the net electric field is the vector sum of individual fields, taking both magnitude and direction into account.

$$\vec{E}_{\text{net}} = \vec{E}_1 + \vec{E}_2 + \dots$$

Step 1: Calculating the net electric potential (V) at the midway point.

Let the distance of the midway point from both charges be $r = \frac{d}{2}$.

$$V_{\text{net}} = V_+ + V_- = \frac{kq}{\frac{d}{2}} + \frac{k(-q)}{\frac{d}{2}} = \frac{2kq}{d} - \frac{2kq}{d} = 0$$

Step 2: Calculating the net electric field (E) at the midway point.

The electric field due to the positive charge ($+q$) points away from it (towards the right). The electric field due to the negative charge ($-q$) points towards it (also towards the right).

Since both field vectors point in the same direction, their magnitudes add up:

$$E_{\text{net}} = E_+ + E_- = \frac{kq}{\left(\frac{d}{2}\right)^2} + \frac{kq}{\left(\frac{d}{2}\right)^2} = \frac{4kq}{d^2} + \frac{4kq}{d^2} = \frac{8kq}{d^2} \neq 0$$

Quick Tip: For a dipole system, at any point on the perpendicular bisector (including the midpoint), the net electric potential is always zero, while the net electric field is non-zero and parallel to the dipole axis.

---

2. A charge Q is placed at the center of an uncharged conducting spherical shell of inner radius

R_1 and outer radius R_2. The surface charge density on the outer surface of the shell is:

(A) $\frac{Q}{(4\pi R_1^2)}$
(B) $\frac{Q}{(4\pi R_2^2)}$
(C) $\frac{-Q}{(4\pi R_1^2)}$
(D) Zero

Correct Answer: (B) $\frac{Q}{(4\pi R_2^2)}$

Solution:

Concept: Inside a conductor under electrostatic equilibrium, the electric field is zero everywhere. When a charge is placed inside the cavity of a conducting shell, electrostatic induction occurs:

- An equal and opposite charge is induced on the inner surface of the shell.
- An equal and similar charge is induced on the outer surface of the shell to maintain its initial net charge state.

The surface charge density (σ) is given by the formula:

$$\sigma = \frac{\text{Charge on the surface}}{\text{Surface area}} = \frac{q}{4\pi R^2}$$

Step 1: Determining the induced charges on the inner and outer surfaces.

The initial net charge on the conducting spherical shell is zero (uncharged). When a charge $+Q$ is placed at its center:

$$\text{Induced charge on the inner surface (at radius } R_1) = -Q$$

To keep the net charge of the shell zero, a balancing positive charge must appear on its exterior boundary:

$$\text{Induced charge on the outer surface (at radius } R_2) = +Q$$

Step 2: Calculating the surface charge density on the outer surface.

The surface area of the outer sphere with radius R_2 is $4\pi R_2^2$. Substituting the outer charge and surface area into the density formula gives:

$$\sigma_{\text{outer}} = \frac{\text{Charge on outer surface}}{\text{Outer surface area}} = \frac{Q}{4\pi R_2^2}$$

Quick Tip: By Gauss's Law, the total charge enclosed within any Gaussian surface drawn inside the conducting material must be zero, forcing the inner surface charge to be completely opposite to the central cavity charge.

---

3. A parallel plate capacitor is charged and then disconnected from the charging battery. If the plates are now pulled further apart using insulating handles:

(A) The charge increases

(B) The voltage decreases

(C) The capacitance increases

(D) The electrostatic energy stored increases

Correct Answer: (D) The electrostatic energy stored increases

Solution:

Concept: When a capacitor is disconnected from its charging battery, the isolated system can no longer exchange charge with any source. Therefore, the charge Q remains constant. The capacitance C, potential difference V, and electrostatic energy stored U depend on the geometric configuration formulas:

- Capacitance: $C = \frac{\varepsilon_0 A}{d}$
- Potential Difference: $V = \frac{Q}{C}$
- Electrostatic Energy: $U = \frac{Q^2}{2C}$

Step 1: Analyzing the effect of pulling the plates further apart.

When the plates are pulled further apart, the separation distance d between them increases. From the capacitance relation:

$$C \propto \frac{1}{d} \quad \Rightarrow \quad \text{As } d \text{ increases, } C \text{ decreases.}$$

Hence, Option (C) is incorrect.

Step 2: Evaluating the change in voltage and stored energy.

Since the capacitor is disconnected from the battery, the charge Q must stay constant ($Q =$ constant). Hence, Option (A) is incorrect. Using the potential difference relationship:

$$V = \frac{Q}{C} \quad \Rightarrow \quad \text{Since } C \text{ decreases, } V \text{ must increase.}$$

Hence, Option (B) is incorrect.

Now, evaluating the stored electrostatic energy U:

$$U = \frac{Q^2}{2C} \quad \Rightarrow \quad \text{Since } Q \text{ is constant and } C \text{ decreases, } U \text{ must increase.}$$

This additional stored energy comes directly from the mechanical work done by an external agent against the attractive electrostatic forces between the oppositely charged plates.

Quick Tip: Remember the golden rule for capacitor transformations: Battery disconnected $\Rightarrow Q$ remains constant. Battery remains connected $\Rightarrow V$ remains constant.

---

4. The magnetic field at the center of a circular current-carrying loop of radius R is B_0. The magnetic field at an axial point at a distance $x = R$ from the center of the loop is:

(A) $B_0/2$

(B) $B_0/2\sqrt{2}$

(C) $B_0/4$

(D) $B_0\sqrt{2}$

Correct Answer: (B) $B_0/2\sqrt{2}$

Solution:

Concept: The magnetic field due to a circular loop carrying a current I depends heavily on the evaluation point.

- At the center of the loop, the magnetic field is given by:

$$B_{\text{center}} = \frac{\mu_0 I}{2R}$$

- At any axial point at a distance x from the center, the magnetic field formula is:

$$B_{\text{axial}} = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}$$

Step 1: Expressing the given magnetic field at the center.

We are given that the magnetic field at the center is B_0. Therefore:

$$B_0 = \frac{\mu_0 I}{2R} \quad \cdots (1)$$

Step 2: Calculating the magnetic field at the axial distance $x = R$.

Substitute $x = R$ into the axial magnetic field formula:

$$B_{\text{axial}} = \frac{\mu_0 I R^2}{2(R^2 + R^2)^{3/2}} = \frac{\mu_0 I R^2}{2(2R^2)^{3/2}}$$

Simplifying the denominator $(2R^2)^{3/2}$:

$$(2R^2)^{3/2} = 2^{3/2} \cdot (R^2)^{3/2} = 2\sqrt{2}R^3$$

Now, substitute this simplified expression back into the equation:

$$B_{\text{axial}} = \frac{\mu_0 I R^2}{2 \cdot 2\sqrt{2}R^3} = \frac{\mu_0 I}{2R \cdot 2\sqrt{2}}$$

Using equation (1), replace $\frac{\mu_0 I}{2R}$ with B_0:

$$B_{\text{axial}} = \frac{B_0}{2\sqrt{2}}$$

Quick Tip: For rapid calculations on a circular loop's axis, remember the ratio formula: $\frac{B_{\text{axial}}}{B_{\text{center}}} = \left(1 + \frac{x^2}{R^2}\right)^{-3/2}$. Substituting $x = R$ directly yields $(1+1)^{-3/2} = 2^{-3/2} = \frac{1}{2\sqrt{2}}$.

CHEMISTRY

5. The value of enthalpy change (ΔH) for the reaction $C_2H_5OH(l) + 3O_2(g) \rightarrow 2CO_2(g) + 3H_2O(l)$, at 27°C is -1366.5 kJ mol^{-1}. The value of internal energy change for the above reaction at this temperature will be

(A) -1371.5 kJ mol^{-1}

(B) -1369.0 kJ mol^{-1}

(C) -1364.0 kJ mol^{-1}

(D) -1361.5 kJ mol^{-1}

Correct Answer: (C) -1364.0 kJ mol^{-1}

Solution:

Concept: The relationship between enthalpy change (ΔH) and internal energy change (ΔU) for a chemical reaction at a constant temperature T is given by the formula:

$$\Delta H = \Delta U + \Delta n_g RT$$

Where:

- Δn_g = (Sum of stoichiometric coefficients of gaseous products) − (Sum of stoichiometric coefficients of gaseous reactants)
- R = Universal gas constant = 8.314 J mol^{-1} K^{-1} = 8.314×10^{-3} kJ mol^{-1} K^{-1}
- T = Absolute temperature in Kelvin

Step 1: Calculating the change in gaseous moles (Δn_g) and converting temperature to Kelvin.
Look at the physical states given in the balanced chemical equation:

$$C_2H_5OH(l) + 3O_2(g) \rightarrow 2CO_2(g) + 3H_2O(l)$$

- Gaseous products: 2 moles of $CO_2(g)$
- Gaseous reactants: 3 moles of $O_2(g)$

$$\Delta n_g = 2 - 3 = -1$$

Convert temperature to Kelvin:

$$T = 27^\circ C + 273 = 300 \text{ K}$$

Step 2: Calculating the internal energy change (ΔU).
Rearranging the main formula to solve for ΔU:

$$\Delta U = \Delta H - \Delta n_g RT$$

Substitute the given values into the equation ($\Delta H = -1366.5$ kJ mol^{-1}):

$$\Delta U = -1366.5 - \left[(-1) \times \left(8.314 \times 10^{-3}\right) \times 300\right]$$

$$\Delta U = -1366.5 + [8.314 \times 0.3]$$

$$\Delta U = -1366.5 + 2.4942 \approx -1364.0 \text{ kJ mol}^{-1}$$

Quick Tip: Always double-check the physical states of matter (l, g, s) in the equation. Liquids and solids are ignored completely when computing Δn_g. Also, keep a close eye on the units; ensure ΔH and RT are both converted to kJ before calculating.

---

6. For the complete combustion of ethanol, $C_2H_5OH(l) + 3O_2(g) \rightarrow 2CO_2(g) + 3H_2O(l)$, the amount of heat produced as measured in bomb calorimeter is 1364.47 kJ mol^{-1} at 25°C. Assuming ideality the enthalpy of combustion, ΔH_C, for the reaction will be (R = 8.314 JK^{-1}mol^{-1})

(A) -1366.95 kJ mol^{-1}

(B) -1361.95 kJ mol^{-1}

(C) -1460.50 kJ mol^{-1}

(D) -1350.50 kJ mol^{-1}

Correct Answer: (A) -1366.95 kJ mol^{-1}

Solution:

Concept: A bomb calorimeter operates at a constant volume. Therefore, the heat produced or exchanged in a bomb calorimeter is equal to the internal energy change (ΔU) of the reaction. Because heat is *produced* (exothermic combustion), ΔU is negative. The relation between the enthalpy of combustion (ΔH_C) and the internal energy change (ΔU) is given by:

$$\Delta H_C = \Delta U + \Delta n_g RT$$

Where:

- Δn_g = (Sum of moles of gaseous products) − (Sum of moles of gaseous reactants)
- R = Universal gas constant = 8.314×10^{-3} kJ mol^{-1} K^{-1}

- T = Absolute temperature in Kelvin

Step 1: Determining ΔU, Δn_g, and converting temperature to Kelvin.

Given that the heat produced at constant volume is 1364.47 kJ mol^{-1}:

$$\Delta U = -1364.47 \text{ kJ mol}^{-1}$$

From the balanced chemical equation:

$$C_2H_5OH(l) + 3O_2(g) \rightarrow 2CO_2(g) + 3H_2O(l)$$

The gaseous components are 2 moles of $CO_2(g)$ and 3 moles of $O_2(g)$. Liquids are omitted.

$$\Delta n_g = 2 - 3 = -1$$

Convert the given temperature to Kelvin:

$$T = 25^\circ C + 273.15 = 298.15 \text{ K}$$

Step 2: Calculating the enthalpy of combustion (ΔH_C).

Substitute the parameters into the relation:

$$\Delta H_C = -1364.47 + \left[(-1) \times \left(8.314 \times 10^{-3}\right) \times 298.15\right]$$

$$\Delta H_C = -1364.47 - \left[\frac{8.314 \times 298.15}{1000}\right]$$

$$\Delta H_C = -1364.47 - 2.4788 \approx -1366.95 \text{ kJ mol}^{-1}$$

Quick Tip: Always remember the basic definition: Heat at constant volume $(q_v) = \Delta U$ (measured via Bomb Calorimeter). Heat at constant pressure $(q_p) = \Delta H$ (measured via Coffee-cup Calorimeter).

---

7. What is $[NH_4^+]$ in a solution that is 0.02M NH_3 and 0.01 M KOH? $[K_b(NH_3) = 1.8 \times 10^{-5}]$

(A) 3.6×10^{-5} M

(B) 1.8×10^{-5} M

(C) 0.9×10^{-5} M

(D) 7.2×10^{-5} M

Correct Answer: (A) 3.6×10^{-5} M

Solution:

Concept: This is a problem based on the **common ion effect**.

- Potassium hydroxide (KOH) is a strong base and dissociates completely in water to yield a high concentration of hydroxyl ions (OH^-).
- Ammonia (NH_3) is a weak base that partially ionizes in water to form NH_4^+ and OH^-.
- The presence of the common ion OH^- from the strong base shifts the equilibrium of the weak base backward, suppressing its dissociation even further.

Step 1: Determining total ion concentrations at equilibrium.

First, write down the complete dissociation of the strong base:

$$KOH(aq) \rightarrow K^+(aq) + OH^-(aq)$$

Since $[KOH] = 0.01$ M, the concentration of OH^- contributed by KOH is 0.01 M.

Next, set up the equilibrium reaction table for the weak base NH_3:

Reaction:	$NH_3(aq) + H_2O(l)$ $\rightleftharpoons$	$NH_4^+(aq)$ + $OH^-(aq)$
Initial (M):	0.02	0 0.01
Change (M):	$-x$	$+x$ $+x$
Equilibrium (M):	$0.02 - x$	x $0.01 + x$

Since K_b is very small (1.8×10^{-5}) and dissociation is highly suppressed due to the common ion effect, we can approximate:

$$0.02 - x \approx 0.02 \quad \text{and} \quad 0.01 + x \approx 0.01$$

Step 2: Solving for the ammonium ion concentration $[NH_4^+]$.

Write the expression for the base dissociation constant (K_b):

$$K_b = \frac{[NH_4^+][OH^-]}{[NH_3]}$$

Substitute the equilibrium concentrations and the given value of K_b:

$$1.8 \times 10^{-5} = \frac{(x)(0.01)}{0.02}$$

Simplify the fraction:

$$1.8 \times 10^{-5} = x \times \frac{1}{2}$$

$$x = 1.8 \times 10^{-5} \times 2 = 3.6 \times 10^{-5} \text{ M}$$

Since $x = [NH_4^+]$:

$$[NH_4^+] = 3.6 \times 10^{-5} \text{ M}$$

Quick Tip: When a weak base is mixed with a strong base, you can find the concentration of the conjugate acid directly using a shortened shortcut formula: $[\text{Conjugate Acid}] = K_b \times \frac{[\text{Weak Base}]}{[\text{Strong Base}]}$. Here, $1.8 \times 10^{-5} \times \frac{0.02}{0.01} = 3.6 \times 10^{-5}$ M.

---

8. 1.1 mole of A mixed with 2.2 moles of B and the mixture is kept in a 1 L flask and the equilibrium, $A + 2B \rightleftharpoons 2C + D$ is reached. If at equilibrium 0.2 mole of C is formed then the value of K_c will be

(A) 0.1

(B) 0.01

(C) 0.001

(D) 1

Correct Answer: (C) 0.001

Solution:

Concept: The equilibrium constant (K_c) for a generalized reversible chemical reaction is expressed as the ratio of the product of the equilibrium molar concentrations of the products to that of the reactants, with each concentration term raised to a power equal to its stoichiometric coefficient in the balanced chemical equation. For the reaction:

$$A + 2B \rightleftharpoons 2C + D$$

The equilibrium constant expression is:

$$K_c = \frac{[C]^2[D]}{[A][B]^2}$$

Where $[X] = \frac{\text{moles of X at equilibrium}}{\text{Volume of flask in Liters}}$.

Step 1: Setting up the equilibrium table in terms of degree of dissociation (x).

Let $2x$ moles of C be formed at equilibrium based on its stoichiometric coefficient. The ICE (Initial, Change, Equilibrium) setup for the moles is as follows:

Reaction:	A	+	2B	$\rightleftharpoons$	2C	+	D
Initial (moles):	1.1		2.2		0		0
Change (moles):	$-x$		$-2x$		$+2x$		$+x$
Equilibrium (moles):	$1.1 - x$		$2.2 - 2x$		$2x$		x

Given that the number of moles of C at equilibrium is 0.2:

$$2x = 0.2 \quad \Rightarrow \quad x = 0.1$$

Step 2: Calculating equilibrium concentrations and computing K_c.

Substitute the value of $x = 0.1$ into the equilibrium expressions to find the final moles:

- Moles of A $= 1.1 - 0.1 = 1.0$ mole
- Moles of B $= 2.2 - 2(0.1) = 2.0$ moles
- Moles of C $= 0.2$ mole
- Moles of D $= 0.1$ mole

Since the volume of the container is $V = 1$ L, the active masses (molarities) are identical to the number of moles:

$$[A] = 1.0 \text{ M}, \quad [B] = 2.0 \text{ M}, \quad [C] = 0.2 \text{ M}, \quad [D] = 0.1 \text{ M}$$

Substitute these molarities into the K_c expression:

$$K_c = \frac{(0.2)^2 \times (0.1)}{(1.0) \times (2.0)^2}$$

$$K_c = \frac{0.04 \times 0.1}{1.0 \times 4.0} = \frac{0.004}{4} = 0.001$$

Quick Tip: Always verify container volume variations early. When the volume $V = 1$ L, your calculation speed increases because moles directly equal molarities. If $V \neq 1$ L, forgetting to divide individual equilibrium moles by the volume is a common pitfall that alters the final power values.

MATHEMATICS

9. If $y^x = e^{y-x}$, then $\frac{dy}{dx}$ is equal to

(A) $\frac{1+\log y}{y \log y}$

(B) $\frac{(1+\log y)^2}{y \log y}$

(C) $\frac{1+\log y}{(\log y)^2}$

(D) $\frac{(1+\log y)^2}{\log y}$

Correct Answer: (D) $\frac{(1+\log y)^2}{\log y}$

Solution:

Concept: When a variable exists in the exponent of a function, we apply **logarithmic differentiation**. Taking the natural logarithm ($\log_e$ or ln) on both sides simplifies the exponents into linear products using the logarithm power rule:

$$\log(a^b) = b \log a$$

Once simplified, we can express x explicitly in terms of y to compute $\frac{dx}{dy}$, and then find the derivative using the reciprocal identity:

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$$

Step 1: Taking the natural logarithm on both sides and simplifying.

Given equation:

$$y^x = e^{y-x}$$

Taking natural log (log) on both sides:

$$\log(y^x) = \log(e^{y-x})$$

$$x \log y = (y - x) \log e$$

Since $\log e = 1$, the expression simplifies to:

$$x \log y = y - x$$

Rearranging terms to collect all x parameters on one side:

$$x \log y + x = y$$

$$x(1 + \log y) = y \quad \Rightarrow \quad x = \frac{y}{1 + \log y} \quad \cdots (1)$$

Step 2: Differentiating x with respect to y using the Quotient Rule.

Recall the quotient rule for differentiation: $\frac{d}{dy}\left(\frac{u}{v}\right) = \frac{v \cdot u' - u \cdot v'}{v^2}$. Applying this to equation (1):

$$\frac{dx}{dy} = \frac{(1 + \log y) \cdot \frac{d}{dy}(y) - y \cdot \frac{d}{dy}(1 + \log y)}{(1 + \log y)^2}$$

$$\frac{dx}{dy} = \frac{(1 + \log y) \cdot 1 - y \cdot \left(0 + \frac{1}{y}\right)}{(1 + \log y)^2}$$

$$\frac{dx}{dy} = \frac{1 + \log y - 1}{(1 + \log y)^2} = \frac{\log y}{(1 + \log y)^2}$$

Step 3: Finding $\frac{dy}{dx}$.

Taking the reciprocal to get the final derivative:

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{(1+\log y)^2}{\log y}$$

Quick Tip: Whenever an implicit function can be explicitly rewritten as $x = f(y)$, finding $\frac{dx}{dy}$ first via the quotient rule is often much less prone to algebraic errors than performing implicit differentiation with the product rule directly on $x \log y = y - x$.

---

10. The domain of the function $f(x) = \sqrt{x - \sqrt{1-x^2}}$ is

(A) $\left[-1, -\frac{1}{\sqrt{2}}\right] \cup \left[\frac{1}{\sqrt{2}}, 1\right]$
(B) $[-1, 1]$
(C) $\left(-\infty, -\frac{1}{\sqrt{2}}\right] \cup \left[\frac{1}{\sqrt{2}}, +\infty\right]$
(D) $\left[\frac{1}{\sqrt{2}}, 1\right]$

Correct Answer: (D) $\left[\frac{1}{\sqrt{2}}, 1\right]$

Solution:

Concept: For a real-valued function, the domain is the set of all real numbers x for which the function expression is mathematically well-defined. For square root functions of the form $\sqrt{g(x)}$, the expression inside the square root must be non-negative:

$$g(x) \geq 0$$

When multiple square roots are nested, these constraints must be applied systematically to all radical expressions from the inside out, and the final domain is the intersection of all resulting solution sets.

Step 1: Applying the condition for the inner square root.

Consider the inner square root expression $\sqrt{1-x^2}$. For this term to be defined:

$$1 - x^2 \geq 0 \quad \Rightarrow \quad x^2 \leq 1$$

Taking the square root on both sides defines our first constraint interval:

$$-1 \le x \le 1 \quad \Rightarrow \quad x \in [-1, 1] \quad \cdots (1)$$

Step 2: Applying the condition for the outer square root.

For the outer square root to be well-defined, its entire internal expression must be non-negative:

$$x - \sqrt{1-x^2} \ge 0 \quad \Rightarrow \quad x \ge \sqrt{1-x^2} \quad \cdots (2)$$

Since a principal square root value is always non-negative ($\sqrt{1-x^2} \ge 0$), equation (2) dynamically forces x to also be non-negative:

$$x \ge 0 \quad \cdots (3)$$

Combining constraint (1) and condition (3) yields a narrowed temporary boundary: $x \in [0, 1]$. Now, because both sides of equation (2) are non-negative in this interval, we can safely square both sides without introducing extraneous solution anomalies:

$$x^2 \ge 1 - x^2$$

$$2x^2 \ge 1 \quad \Rightarrow \quad x^2 \ge \frac{1}{2}$$

Since we established $x \ge 0$, taking the positive square root gives:

$$x \ge \frac{1}{\sqrt{2}} \quad \cdots (4)$$

Step 3: Finding the intersection of all constraints.

We find the final domain by taking the intersection of all structural constraint intervals:

- From inner radical: $x \in [-1, 1]$
- From outer radical: $x \in \left[\frac{1}{\sqrt{2}}, +\infty\right)$

$$\text{Domain} = [-1, 1] \cap \left[\frac{1}{\sqrt{2}}, +\infty\right) = \left[\frac{1}{\sqrt{2}}, 1\right]$$

Quick Tip: Always analyze the sign profiles before squaring an inequality. In this problem, the condition $x \geq \sqrt{1-x^2}$ immediately eliminates all negative numbers from the domain, allowing you to instantly discard options (1), (2), and (3) without completing any further algebraic calculations!

---

11. If A and B are symmetric matrices of same order such that AB + BA = X and AB − BA = Y, then $(XY)^T =$

(A) XY

(B) X^TY^T

(C) −YX

(D) $-Y^TX^T$

Correct Answer: (C) −YX

Solution:

Concept: The transpose of a matrix satisfies several key operational properties:

- Reversal law for multiplication: $(M_1M_2)^T = M_2^TM_1^T$
- Distributive law over addition/subtraction: $(M_1 \pm M_2)^T = M_1^T \pm M_2^T$
- For a symmetric matrix M, $M^T = M$
- For a skew-symmetric matrix M, $M^T = -M$

Step 1: Determining the nature of matrices X and Y using given conditions.

Since A and B are symmetric matrices, we know that:

$$A^T = A \quad \text{and} \quad B^T = B$$

Now let's find the transpose of matrix $X = AB + BA$:

$$X^T = (AB + BA)^T = (AB)^T + (BA)^T$$

Applying the reversal law of transpose:

$$X^T = B^TA^T + A^TB^T$$

Substituting $A^{\mathrm{T}} = A$ and $B^{\mathrm{T}} = B$:

$$X^{\mathrm{T}} = BA + AB = AB + BA = X$$

Thus, X is a **symmetric matrix** ($X^{\mathrm{T}} = X$).

Next, let's find the transpose of matrix $Y = AB - BA$:

$$Y^{\mathrm{T}} = (AB - BA)^{\mathrm{T}} = (AB)^{\mathrm{T}} - (BA)^{\mathrm{T}}$$

Applying the reversal law of transpose:

$$Y^{\mathrm{T}} = B^{\mathrm{T}}A^{\mathrm{T}} - A^{\mathrm{T}}B^{\mathrm{T}}$$

Substituting $A^{\mathrm{T}} = A$ and $B^{\mathrm{T}} = B$:

$$Y^{\mathrm{T}} = BA - AB = -(AB - BA) = -Y$$

Thus, Y is a **skew-symmetric matrix** ($Y^{\mathrm{T}} = -Y$).

Step 2: Evaluating the expression $(XY)^{\mathrm{T}}$.

Using the reversal law of transpose on the product expression $(XY)^{\mathrm{T}}$:

$$(XY)^{\mathrm{T}} = Y^{\mathrm{T}}X^{\mathrm{T}}$$

Substitute the results derived in Step 1 ($X^{\mathrm{T}} = X$ and $Y^{\mathrm{T}} = -Y$):

$$(XY)^{\mathrm{T}} = (-Y)(X) = -YX$$

Quick Tip: For any two symmetric matrices A and B of the same order, the expression $(AB + BA)$ is always symmetric, while the expression $(AB - BA)$ is always skew-symmetric. Remembering this property cuts your problem-solving time in half!

---

12. Let $\mathbf{A} = \{1, 2, 3, 4, 5\}$ and R be a relation defined by $\mathbf{R} = \{(x, y) : x, y \in \mathbf{A}, x + y = 5\}$. Then, R is

(A) reflexive and symmetric but not transitive

(B) an equivalence relation

(C) neither reflexive nor transitive

(D) neither reflexive nor symmetric but transitive

Correct Answer: (C) neither reflexive nor transitive

Solution:

Concept: A relation R on a non-empty set A can be categorized into various properties based on its ordered pairs (x, y):

- **Reflexive:** If $(a, a) \in R$ for every element $a \in A$.
- **Symmetric:** If $(a, b) \in R \implies (b, a) \in R$ for all $a, b \in A$.
- **Transitive:** If $(a, b) \in R$ and $(b, c) \in R \implies (a, c) \in R$ for all $a, b, c \in A$.

Step 1: Writing the relation R in roster form.

The condition for the ordered pairs is $x + y = 5$, where both elements belong to set $A = \{1, 2, 3, 4, 5\}$. Let's trace the matching pairs:

- If $x = 1 \implies y = 4$ (since $1 + 4 = 5$)
- If $x = 2 \implies y = 3$ (since $2 + 3 = 5$)
- If $x = 3 \implies y = 2$ (since $3 + 2 = 5$)
- If $x = 4 \implies y = 1$ (since $4 + 1 = 5$)
- If $x = 5 \implies y = 0$ (not possible because $0 \notin A$)

Thus, in roster form, the relation is:

$$R = \{(1, 4), (2, 3), (3, 2), (4, 1)\}$$

Step 2: Testing for Reflexive, Symmetric, and Transitive properties.

1. Reflexivity check: For R to be reflexive, every element $a \in A$ must relate to itself, i.e., $(1, 1), (2, 2), \cdots \in R$. Since $(1, 1) \notin R$ (as $1 + 1 = 2 \neq 5$), **R is not reflexive**.

2. Symmetry check: We observe the pairs:

- $(1, 4) \in R \implies (4, 1) \in R$
- $(2, 3) \in R \implies (3, 2) \in R$

Since $(x, y) \in R \implies (y, x) \in R$ holds true for all pairs, **R is symmetric**.

3. Transitivity check: For R to be transitive, if $(a, b) \in R$ and $(b, c) \in R$, then (a, c) must also belong to R. Let's select test pairs from our set:

$$(1, 4) \in R \quad \text{and} \quad (4, 1) \in R$$

Here, $a = 1, b = 4, c = 1$. For transitivity, $(a, c) = (1, 1)$ must belong to R. However, as seen previously, $(1, 1) \notin R$. Hence, **R is not transitive**.

Combining these observations, R is symmetric but neither reflexive nor transitive.

Quick Tip: When checking properties on small discrete sets, look for the counter-example first. For transitivity, always check paired cycles like (a, b) and (b, a). If their combined reflexive identity (a, a) is missing from the list, you can instantly conclude that the relation is not transitive.
