BITSAT 2026 May 26 Shift 2

Question Paper (Memory-Based) with Solutions

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General Instructions

(i) **Duration:** The total duration of the examination is 3 hours (180 minutes).

(ii) **Total Marks:** The complete paper carries a maximum of 390 marks.

(iii) **Structure:** The paper has 4 Sections:

- **Part 1:** 30 Multiple Choice Questions (Physics).
- **Part 2:** 30 Multiple Choice Questions (Chemistry).
- **Part 3:** 10 Multiple Choice Questions (English Proficiency), 20 Multiple Choice Questions (Logical Reasoning)
- **Part 4:** 40 Multiple Choice Questions (Mathematics/Biology)

(iv) **Compulsory Questions:** All 130 questions are compulsory,and +12 Questions(Optional Extra Questions)

(v) Each question has four options. Only **one** option is correct.

(vi) **Correct Answer:** +3 marks.

(vii) **Incorrect Answer:** −1 (Negative marking).

(viii) **Unanswered/Marked for Review:** 0 marks.

PHYSICS

1. A wire of length L and cross-sectional area A is made of a material of Young's modulus Y. If it is stretched by an amount x, the elastic potential energy stored in the wire is:

(A) $\frac{YAx^2}{L}$

(B) $\frac{YAx^2}{2L}$

(C) $\frac{2YAx^2}{L}$

(D) $\frac{YAx}{L}$

Correct Answer: (B) $\frac{YAx^2}{2L}$

Solution:

Concept: When an external stretching force is applied to a metallic wire, work is performed against the internal interatomic restorative forces of the material. This work is stored within the molecular matrix of the wire as **elastic potential energy** (U).

The basic mechanical relationship for energy stored during a variable spring-like stretch is given by:

$$U = \frac{1}{2} \times \text{Stretching Force} \times \text{Elongation} = \frac{1}{2} \cdot F \cdot x$$

Alternatively, from the macroscopic perspective of materials science, the potential energy per unit volume (energy density, u) is related directly to mechanical stress and strain:

$$u = \frac{\text{Energy}}{\text{Volume}} = \frac{1}{2} \times \text{Stress} \times \text{Strain}$$

Step 1: Finding the required stretching force using Young's Modulus.

By definition, Young's Modulus (Y) is the ratio of longitudinal stress to longitudinal strain:

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\left(\frac{F}{A}\right)}{\left(\frac{x}{L}\right)} = \frac{F \cdot L}{A \cdot x}$$

Rearranging this relationship to isolate the dynamic tension force F required to sustain an elongation of x:

$$F = \frac{Y \cdot A \cdot x}{L} \quad \cdots (1)$$

Step 2: Calculating the total stored elastic potential energy (U).

Substitute our expression for force from equation (1) into the mechanical work-energy formula:

$$U = \frac{1}{2} \cdot F \cdot x$$

$$U = \frac{1}{2} \cdot \left(\frac{YAx}{L}\right) \cdot x$$

Combining the identical variable terms yields:

$$U = \frac{YAx^2}{2L}$$

This directly matches the algebraic layout of Option (B).

Quick Tip: Save time by treating a stretching wire exactly like a mechanical spring! A wire behaves as a spring with an effective stiffness constant of $k = \frac{YA}{L}$. Since the potential energy stored inside a spring stretched by a distance x is universally given by $\frac{1}{2}kx^2$, substituting our effective wire stiffness yields $\frac{1}{2}\left(\frac{YA}{L}\right)x^2 = \frac{YAx^2}{2L}$ in under 5 seconds!

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2. A particle moves in a circle of radius R such that its linear speed varies with time t as $v = kt$, where k is a positive constant. The angle θ between the net acceleration vector and the velocity vector at time t is given by:

(A) $\tan^{-1}\left(\frac{k^2t^2}{R}\right)$
(B) $\tan^{-1}\left(\frac{kt^2}{R}\right)$
(C) $\tan^{-1}\left(\frac{kt}{R}\right)$
(D) $\tan^{-1}\left(\frac{R}{k^2t^2}\right)$

Correct Answer: (B) $\tan^{-1}\left(\frac{kt^2}{R}\right)$

Solution:

Concept: The velocity vector $\vec{v}$ of a particle in circular motion is always directed tangentially along the path. Therefore, the angle θ between the net acceleration vector $\vec{a}_{\text{total}}$ and the velocity vector is exactly the angle between $\vec{a}_{\text{total}}$ and the tangential acceleration component $\vec{a}_t$. Using vector geometry, the tangent of this angle is the ratio of the perpendicular acceleration component (centripetal acceleration, a_c) to the base component (tangential acceleration, a_t):

$$\tan\theta = \frac{a_c}{a_t} \implies \theta = \tan^{-1}\left(\frac{a_c}{a_t}\right)$$

Step 1: Calculating the tangential acceleration component (a_t).

Tangential acceleration is defined as the rate of change of linear speed:

$$a_t = \frac{dv}{dt} = \frac{d}{dt}(kt) = k$$

Step 2: Calculating the centripetal acceleration component (a_c).

Centripetal acceleration is determined by the instantaneous linear speed and path radius:

$$a_c = \frac{v^2}{R} = \frac{(kt)^2}{R} = \frac{k^2t^2}{R}$$

Step 3: Evaluating the direction angle θ.

Substitute both derived component expressions into our trigonometric ratio layout:

$$\tan\theta = \frac{\left(\frac{k^2t^2}{R}\right)}{k} = \frac{k^2t^2}{R\cdot k} = \frac{kt^2}{R}$$

Isolating the angle yields the final expression:

$$\theta = \tan^{-1}\left(\frac{kt^2}{R}\right)$$

Quick Tip: Test the extreme temporal boundary states to clear doubts quickly! At the absolute start ($t = 0$), the speed is zero, meaning there is zero centripetal turning force and the particle purely accelerates forward tangentially. Thus, the angle θ must be 0°. Substituting $t = 0$ into our options shows that $\tan^{-1}(0) = 0^\circ$ for Option B, verifying the physical validity of our formula.

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3. Two wires X and Y of the same material have lengths in the ratio $1 : 2$ and diameters in the ratio $2 : 1$. If they are subjected to the same stretching force, the ratio of the elongation produced in wire X to that in wire Y ($\Delta L_X : \Delta L_Y$) is:

(A) $1 : 4$

(B) $1 : 8$

(C) $1 : 2$

(D) $8 : 1$

Correct Answer: (B) $1 : 8$

Solution:

Concept: By Hooke's Law within the elastic limit, the longitudinal elongation (ΔL) produced in a metallic structural wire under a tension load is dictated by Young's Modulus (Y):

$$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{\left(\frac{F}{A}\right)}{\left(\frac{\Delta L}{L}\right)} \implies \Delta L = \frac{F \cdot L}{A \cdot Y}$$

Expressing the cross-sectional area in terms of diameter d where $A = \frac{\pi d^2}{4}$:

$$\Delta L = \frac{4FL}{\pi d^2 Y}$$

Since both wires are made of the **same material**, their Young's Modulus values are identical ($Y_X = Y_Y$). Furthermore, they are subjected to the **same stretching force** ($F_X = F_Y$). Therefore, elongation is directly proportional to length and inversely proportional to the square of the diameter:

$$\Delta L \propto \frac{L}{d^2} \implies \frac{\Delta L_X}{\Delta L_Y} = \left(\frac{L_X}{L_Y}\right) \times \left(\frac{d_Y}{d_X}\right)^2$$

Step 1: Substituting the given proportional parameters.

We are given the following explicit scale ratios:

- Length ratio: $\frac{L_X}{L_Y} = \frac{1}{2}$
- Diameter ratio: $\frac{d_X}{d_Y} = \frac{2}{1} \implies \frac{d_Y}{d_X} = \frac{1}{2}$

Plugging these values directly into our proportional equation system:

$$\frac{\Delta L_X}{\Delta L_Y} = \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right)^2$$

Step 2: Evaluating the final fractional ratio.

$$\frac{\Delta L_X}{\Delta L_Y} = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$$

Thus, the tracking extension ratio evaluates to **1 : 8**.

Quick Tip: Always watch out for the squaring effect of thickness dimensions! Since area scales with the square of the diameter, doubling the diameter makes a wire **four times** more resistant to stretching. Wire X is shorter (wants to stretch half as much) but significantly thicker (wants to stretch a quarter as much), yielding $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$ instantly.

4. The elastic potential energy stored per unit volume (energy density) in a stretched string under a longitudinal tension stress σ and material Young's modulus Y is expressed as:

(A) $\frac{\sigma^2}{2Y}$

(B) $\frac{2Y}{\sigma^2}$

(C) $\frac{Y\sigma^2}{2}$

(D) $\frac{\sigma^2}{Y}$

Correct Answer: (A) $\frac{\sigma^2}{2Y}$

Solution:

Concept: The elastic energy density (u), which represents the structural potential energy stored per unit volume inside a deformed elastic medium, is fundamentally defined as:

$$u = \frac{1}{2} \times \text{Stress} \times \text{Strain} = \frac{1}{2} \cdot \sigma \cdot \varepsilon$$

From Hooke's Law, Young's Modulus (Y) maps stress directly to strain via the linear equation:

$$\sigma = Y \cdot \varepsilon \quad \Longrightarrow \quad \varepsilon = \frac{\sigma}{Y}$$

Step 1: Substituting the strain variable to match stress form.

Because the problem requests the solution expressed explicitly in terms of stress (σ) and Young's modulus (Y), we replace the strain parameter (ε) using our Hooke's Law identity:

$$u = \frac{1}{2} \cdot \sigma \cdot \left(\frac{\sigma}{Y}\right)$$

Step 2: Simplifying the expression layout.

Combining the numerator products:

$$u = \frac{\sigma^2}{2Y}$$

This perfectly derives the identity corresponding to Option (A).

Quick Tip: Memorize the three symmetric expressions for elastic energy density to handle any variable pairing given in a question:

$$u = \frac{1}{2}(\text{Stress})(\text{Strain}) = \frac{1}{2}Y(\text{Strain})^2 = \frac{\text{Stress}^2}{2Y}$$

This is completely analogous to electrostatics, where the energy density of an electric field is given by $u = \frac{1}{2}\varepsilon_0 E^2 = \frac{D^2}{2}\varepsilon_0$!

CHEMISTRY

5. An octahedral coordination complex with the electronic configuration $t_{2g}^4 e_g^0$ is expected to exhibit which of the following magnetic properties and d-d transition characteristics?

(A) Paramagnetic with 4 unpaired electrons; spin-allowed transitions

(B) Paramagnetic with 2 unpaired electrons; spin-allowed transitions

(C) Diamagnetic; spin-forbidden transitions

(D) Paramagnetic with 2 unpaired electrons; spin-forbidden transitions

Correct Answer: (B) Paramagnetic with 2 unpaired electrons; spin-allowed transitions

Solution:

Concept: In an octahedral crystal field environment, the five d-orbitals split into two sets separated by an energy gap (Δ_o): the lower energy t_{2g} triplet (d_{xy}, d_{yz}, d_{xz}) and the higher energy e_g doublet ($d_{z^2}, d_{x^2-y^2}$).

The given configuration is **$t_{2g}^4 e_g^0$**, which represents a low-spin d^4 system (occurring in the presence of a strong-field ligand where the crystal field splitting energy is greater than the electron pairing energy, $\Delta_o > P$).

Step 1: Analyzing unpaired electrons and magnetic behavior.

Let's distribute the 4 electrons in the three lower-energy t_{2g} orbitals following Hund's Rule of maximum multiplicity:

- The first three electrons enter separate orbitals with parallel spins: $\uparrow, \uparrow, \uparrow$
- The fourth electron is forced to pair up in the first available orbital due to the large Δ_o gap: $\uparrow\downarrow, \uparrow, \uparrow$

Counting the single, unpaired electrons:

$$\text{Number of unpaired electrons } (n) = 2$$

Since $n = 2$, the complex is **paramagnetic** (and its spin-only magnetic moment would evaluate to $\mu_s = \sqrt{2(2+2)} = \sqrt{8} \approx 2.83$ B.M.).

Step 2: Determining the selection rule status for d-d transitions.

According to the quantum mechanical **spin selection rule**, electronic transitions are strictly allowed only if the net spin multiplicity remains unchanged ($\Delta S = 0$).

In this $t_{2g}^4 e_g^0$ setup, moving one electron from the paired t_{2g} orbital up into an empty e_g orbital can happen *without* altering the intrinsic spin direction of that electron. Because a transition can occur where the number of unpaired electrons remains completely identical before and after excitation, the transition is **spin-allowed**, resulting in standard coordination color absorption.

Quick Tip: Don't mix up high-spin and low-spin systems! If this question had given a high-spin d^4 system ($t_{2g}^3 e_g^1$), it would have 4 unpaired electrons. But because it specifies $t_{2g}^4 e_g^0$, the pairing tells you instantly that $n = 4 - 2 = 2$ unpaired electrons remain.

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6. During the structural analysis of an unknown aldohexose, a chemist treats a sample with periodic acid (HIO_4). If the carbohydrate is completely cleaved to yield five molecules of formic acid (HCOOH) and one molecule of formaldehyde (HCHO), this diagnostic breakdown directly proves the presence of:

(A) A ketohexose structure with a carbonyl at C-2

(B) A cyclic pyranose ring configuration

(C) A continuous straight-chain structure containing five —CHOH groups and one $-CH_2OH$ group

(D) Three isolated, non-adjacent primary alcohol branches

Correct Answer: (C) A continuous straight-chain structure containing five —CHOH groups and one $-CH_2OH$ group

Solution:

Concept: Periodic acid (HIO_4) is a highly specific oxidizing agent utilized in carbohydrate chemistry to perform **vicinal glycol cleavage**. It selectively cuts carbon-carbon single bonds if and only if both adjacent carbon atoms carry hydroxyl groups ($-CHOH-CHOH-$) or a carbonyl adjacent to a hydroxyl group ($-CHO-CHOH-$).

The structural fragments oxidize according to these strict stoichiometric diagnostic rules:

$$-CH_2OH \text{ (Terminal primary alcohol)} \xrightarrow{HIO_4} HCHO \text{ (Formaldehyde)}$$
$$-CHOH- \text{ (Secondary internal alcohol)} \xrightarrow{HIO_4} HCOOH \text{ (Formic Acid)}$$
$$-CHO \text{ (Terminal aldehyde carbonyl)} \xrightarrow{HIO_4} HCOOH \text{ (Formic Acid)}$$

Step 1: Interpreting the fragment ratios back into functional groups.

The experimental cleavage generated:

- **1 molecule of HCHO:** This establishes that the molecule contains exactly **one** terminal primary alcohol group ($-CH_2OH$) at one end of the chain.
- **5 molecules of HCOOH:** For an aldohexose (like Glucose), the top terminal aldehyde ($-CHO$) yields 1 HCOOH, and the remaining 4 secondary alcohol links ($-CHOH-$) yield 4 HCOOH, summing up precisely to 5 molecules.

Step 2: Deducing the structural assembly.

Because all 5 carbon-carbon bonds were broken to release independent single-carbon molecules, every single carbon atom in the hexose sugar must have been directly bonded to its neighbor in a continuous, unbranched open chain containing contiguous vicinal glycol functional groups. This confirms the classic open-chain structure of D-glucose.

Quick Tip: Count your carbons to eliminate wrong choices instantly! A hexose sugar has 6 carbons. Formic acid (HCOOH) and formaldehyde (HCHO) both contain exactly **1** carbon atom. Since 5×1 (from HCOOH) $+ 1 \times 1$ (from HCHO) $= 6$ carbons total, it confirms that the entire open-chain backbone cracked completely into single-carbon pieces.

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7. In an analytical laboratory, a 20.0 mL sample of an aqueous solution containing oxalic acid ($H_2C_2O_4$) requires exactly 16.0 mL of a 0.05 M potassium permanganate ($KMnO_4$) solution for complete oxidation in a hot, acidic medium (H_2SO_4). Calculate the molarity of the oxalic acid solution.

(A) 0.010 M

(B) 0.040 M

(C) 0.100 M

(D) 0.250 M

Correct Answer: (C) 0.100 M

Solution:

Concept: This question can be solved using the **Law of Chemical Equivalents**, which dictates that at the stoichiometric endpoint of any reaction, the total gram equivalents of the oxidizing agent must equal the total gram equivalents of the reducing agent:

$$\text{Equivalents of } KMnO_4 = \text{Equivalents of } H_2C_2O_4$$

Using the relationship between Normality (N), Molarity (M), and the valence factor (n-factor), where $N = M \times n$:

$$M_1 \cdot n_1 \cdot V_1 = M_2 \cdot n_2 \cdot V_2$$

Step 1: Determining the n-factors for both redox species in an acidic medium.

- **For $KMnO_4$ (Oxidizing Agent):** In an acidic solution, the permanganate ion reduces from a +7 state down to a +2 state ($MnO_4^- \to Mn^{2+}$), involving a transfer of 5 electrons:

$$n_{KMnO_4} = 5$$

- **For $H_2C_2O_4$ (Reducing Agent):** The carbon atoms in the oxalate ion oxidize from a +3 state up to a +4 state inside carbon dioxide ($C_2O_4^{2-} \to 2CO_2 + 2e^-$). Since there are 2 carbon atoms performing this shift per molecule, the total electron release is $2 \times 1 = 2$:

$$n_{H_2C_2O_4} = 2$$

Step 2: Plugging the parameters into the equivalents equation.

Let M_x represent the unknown molarity of the oxalic acid solution. Setting up our equality:

$$(M \times n \times V)_{KMnO_4} = (M \times n \times V)_{H_2C_2O_4}$$

$$0.05 \text{ M} \times 5 \times 16.0 \text{ mL} = M_x \times 2 \times 20.0 \text{ mL}$$

Step 3: Isolating and computing M_x.

$$0.25 \times 16.0 = M_x \times 40.0$$

Since $0.25 = \frac{1}{4}$, the left side simplifies cleanly:

$$\frac{1}{4} \times 16.0 = 4.0 \implies 4.0 = 40.0 \cdot M_x$$

$$M_x = \frac{4.0}{40.0} = 0.100 \text{ M}$$

Thus, the molar concentration of the oxalic acid is **0.100 M**.

Quick Tip: Using normality factor ratios saves you from ever having to write out or balance long redox reactions during multiple-choice tests! Just keep these two standard acidic medium n-factors memorized as an invariant pair: Permanganate is always **5**, Oxalate is always **2**.

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8. A current of 2.0 A is passed for 5 hours through an electrolytic cell containing an aqueous solution of a metal salt, depositing 12.0 g of the metal at the cathode. If the atomic mass of the metal is 193 g mol^{-1}, find the oxidation state of the metal ion in the solution. (Take Faraday's constant $F = 96500$ C mol^{-1}).

(A) +1

(B) +2

(C) +3

(D) +6

Correct Answer: (D) +6

Solution:

Concept: According to Faraday's First Law of Electrolysis:

$$m = \frac{MIt}{nF}$$

where:

- m = mass deposited

- M = molar mass
- I = current
- t = time
- F = Faraday constant
- n = oxidation state (valency)

Step 1: Calculate total charge passed.

Given:

$$I = 2.0 \text{ A}, \qquad t = 5 \text{ h}$$

Convert time into seconds:

$$t = 5 \times 60 \times 60 = 18000 \text{ s}$$

Therefore,

$$Q = It = 2.0 \times 18000 = 36000 \text{ C}$$

Step 2: Use Faraday's formula to find n.

Given:

$$m = 12.0 \text{ g}, \qquad M = 193 \text{ g mol}^{-1}$$

Using:

$$m = \frac{MIt}{nF}$$

Rearranging:

$$n = \frac{MIt}{mF}$$

Substitute the values:

$$n = \frac{193 \times 2.0 \times 18000}{12.0 \times 96500}$$

Step 3: Simplify the expression.

$$n = \frac{193 \times 36000}{12 \times 96500}$$

$$n = \frac{6,948,000}{1,158,000}$$

$$n = 6$$

Hence, the oxidation state of the metal ion is:

$$\boxed{+6}$$

Quick Tip: A fast alternative method is:

$$\text{Moles of metal deposited} = \frac{12}{193}$$

$$\text{Moles of electrons passed} = \frac{36000}{96500}$$

Then,

$$n = \frac{\text{moles of electrons}}{\text{moles of metal}}$$

which again gives:

$$n \approx 6$$

MATHEMATICS

9. In how many ways can the letters of the word COCHIN be arranged such that the two 'C's are never separated by any other letter?

(A) 360

(B) 120

(C) 240

(D) 720

Correct Answer: (B) 120

Solution:

Concept: The condition that the two 'C's must "never be separated" means that they must **always stay together** as an unbroken sequence block. Just like the BITSAT question, we deploy the **String/Bundling Method** here.

Step 1: Grouping the components into a single unit.

The word is **COCHIN**, containing 6 total letters: C, O, C, H, I, N (where 'C' is repeated twice). Since both 'C's must be positioned together, we lock them inside a single composite structural block: [CC].

Now, we count this bundle as a single item along with the remaining individual letters:

$$\text{O, H, I, N, [CC]}$$

This gives us a total of 5 distinct independent items to shuffle around.

Step 2: Calculating total valid arrangements.

The number of ways to linearly arrange these 5 distinct entities is:

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120 \text{ ways}$$

Inside the block [CC], because both letters are completely identical, swapping their relative positions creates zero new visible words ($\frac{2!}{2!} = 1$ configuration). Therefore, the total number of permutations is:

$$120 \times 1 = 120 \text{ ways}$$

Quick Tip: Always read the constraint carefully! The phrase "never separated" is just a clever linguistic trick to say "always together". Don't waste time calculating total permutations and subtracting like you would for a "never together" constraint problem.

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10. Evaluate the definite integral: $\int_0^{2026} \frac{x^5}{x^5+(2026-x)^5}\,dx$

(A) 2026

(B) 1013

(C) 506.5

(D) 0

Correct Answer: (B) 1013

Solution:

Concept: To solve definite integrals involving complex, symmetric fractional functions where standard algebraic integration looks impossible, we invoke **King's Property of Definite

Integrals**:

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

Step 1: Applying King's Property to set up a twin integral equation.

Let our target definite integral equation be marked as I:

$$I = \int_0^{2026} \frac{x^5}{x^5 + (2026-x)^5}\,dx \quad \cdots \text{(Equation 1)}$$

Applying King's Property, we replace every instance of x across the function boundary with $(0+2026-x) = (2026-x)$:

$$I = \int_0^{2026} \frac{(2026-x)^5}{(2026-x)^5 + (2026-(2026-x))^5}\,dx$$

Simplifying the interior bracket terms yields:

$$I = \int_0^{2026} \frac{(2026-x)^5}{(2026-x)^5 + x^5}\,dx \quad \cdots \text{(Equation 2)}$$

Step 2: Adding both integral representations together.

Since Equation 1 and Equation 2 have identical upper and lower integration limits, we can sum their integrands directly:

$$2I = \int_0^{2026} \left[\frac{x^5}{x^5 + (2026-x)^5} + \frac{(2026-x)^5}{x^5 + (2026-x)^5}\right]dx$$

Notice that the numerators combine to perfectly match the common denominator:

$$2I = \int_0^{2026} \frac{x^5 + (2026-x)^5}{x^5 + (2026-x)^5}\,dx$$

$$2I = \int_0^{2026} 1 \cdot dx$$

Step 3: Integrating and isolating I.

Evaluating the simple integration constant across the limits:

$$2I = \left[x\right]_0^{2026} = 2026 - 0 = 2026$$

$$I = \frac{2026}{2} = 1013$$

Quick Tip: Keep this shortcut memorized for competitive exams! Whenever you encounter a definite integral structured as $\int_a^b \frac{f(x)}{f(x)+f(a+b-x)}\,dx$, the function expression will *always* simplify to unity upon addition. The value of the integral is simply half the total interval length: $\frac{b-a}{2}$. For this problem: $\frac{2026-0}{2} = 1013$ instantly!

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11. If the vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} - 3\hat{k}$, and $\vec{c} = 3\hat{i} + \lambda\hat{j} + 5\hat{k}$ represent the concurrent coterminous edges of a parallelopiped whose volume is 0 (i.e., the vectors are coplanar), find the value of the scalar parameter λ.

(A) 4

(B) −4

(C) 2

(D) −2

Correct Answer: (B) −4

Solution:

Concept: The volume (V) of a parallelopiped formed by three concurrent coterminous vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ is equal to the absolute value of their **Scalar Triple Product (STP)**:

$$V = \left|\vec{a} \cdot (\vec{b} \times \vec{c})\right|$$

When the volume of the parallelopiped is exactly **zero**, it physically means the structure has flattened completely into a single flat two-dimensional space. Therefore, setting the scalar triple product to zero ($[\vec{a}\ \ \vec{b}\ \ \vec{c}] = 0$) is the standard mathematical condition to check if three vectors are **coplanar**.

This Scalar Triple Product is evaluated easily by calculating the determinant of the 3×3 matrix formed by the components of the three vectors.

Step 1: Setting up the matrix determinant equation.

Since the vectors are coplanar, their determinant must be equal to zero:

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & -3 \\ 3 & \lambda & 5 \end{vmatrix} = 0$$

Step 2: Expanding the 3×3 determinant to isolate λ.

Expanding the determinant along the first row:

$$2 \cdot \begin{vmatrix} 2 & -3 \\ \lambda & 5 \end{vmatrix} - (-1) \cdot \begin{vmatrix} 1 & -3 \\ 3 & 5 \end{vmatrix} + 1 \cdot \begin{vmatrix} 1 & 2 \\ 3 & \lambda \end{vmatrix} = 0$$

Evaluating the individual 2×2 sub-determinants via cross-multiplication:

$$2 \cdot \Big((2 \times 5) - (-3 \times \lambda)\Big) + 1 \cdot \Big((1 \times 5) - (-3 \times 3)\Big) + 1 \cdot \Big((1 \times \lambda) - (2 \times 3)\Big) = 0$$

Simplifying the terms inside the brackets carefully:

$$2 \cdot (10 + 3\lambda) + 1 \cdot (5 + 9) + 1 \cdot (\lambda - 6) = 0$$

$$20 + 6\lambda + 14 + \lambda - 6 = 0$$

Step 3: Solving the linear algebraic equation.

Group the coefficients of λ and the constant terms together:

$$(6\lambda + \lambda) + (20 + 14 - 6) = 0$$

$$7\lambda + 28 = 0$$

Shift the constant to the right-hand side:

$$7\lambda = -28$$

$$\lambda = \frac{-28}{7} = -4$$

This mathematically matches **Option (B)** perfectly with zero ambiguity.

Quick Tip: To cross-verify your answer instantly in the exam hall without expanding the whole determinant again, plug $\lambda = -4$ directly back into the third row, making it $\begin{pmatrix} 3 & -4 & 5 \end{pmatrix}$. Notice a neat linear dependency shortcut: if you add the first row $\begin{pmatrix} 2 & -1 & 1 \end{pmatrix}$ and the second row $\begin{pmatrix} 1 & 2-3 \end{pmatrix}$ multiplied by -1, do they show relationships? Checking row choices saves you from silly sign errors!

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12. A pair of fair dice is thrown simultaneously. What is the probability that the sum of the numbers appearing on the top faces is at least 10?

(A) $\frac{1}{6}$

(B) $\frac{1}{12}$

(C) $\frac{5}{36}$

(D) $\frac{1}{4}$

Correct Answer: (A) $\frac{1}{6}$

Solution:

Concept: The phrase **"at least 10"** in probability means we are looking for outcomes where the sum (S) of the two face values satisfies the inequality:

$$S \geq 10 \quad \Longrightarrow \quad S = 10, \text{ or } S = 11, \text{ or } S = 12$$

The standard probability formula is:

$$P = \frac{\text{Number of Favorable Outcomes } (n(E))}{\text{Total Number of Sample Outcomes } (n(S))}$$

Step 1: Determining the total sample space size.

When rolling a single 6-sided die, there are 6 unique outcomes. For a pair of dice thrown simultaneously, the total number of sample outcomes in the grid space is:

$$n(S) = 6 \times 6 = 36$$

Step 2: Listing the favorable coordinates matching our condition.

Let's list out all possible coordinate pairs (d_1, d_2) that yield our targeted sum thresholds:

- **For Sum $= 10$:** $(4,6), (5,5), (6,4) \rightarrow 3$ ways
- **For Sum $= 11$:** $(5,6), (6,5) \rightarrow 2$ ways

- **For Sum $= 12$:** $(6,6) \to 1$ way

Summing these discrete valid configurations together:

$$n(E) = 3 + 2 + 1 = 6 \text{ favorable ways}$$

Step 3: Evaluating the final probability ratio.

$$P(S \geq 10) = \frac{n(E)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

Quick Tip: For dice-sum questions, memorize the **Symmetric Triangular Rule** to count outcomes instantly without writing them out: The number of ways to get a sum follows a perfect hill shape that peaks at a sum of 7 (6 ways). For sums above 7, the number of ways is simply $14 - \text{Sum}$.

- Ways to get $10 = 14 - 10 = 4$

Adding them up directly: $3 + 2 + 1 = 6$ ways, saving you valuable time!