BITSAT Mathematics Sample Paper-10

Duration: 60 Minutes **Maximum Marks: 120**

Instructions

- This paper contains **40** Multiple Choice Questions (Single Correct).
- Each correct answer carries **+3 marks**. Each incorrect answer carries **-1 mark**. Unattempted question carries **0 marks**.
- Only **one** option is correct for each question.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.

Q1. The value of

$$\lim_{x\to 0} \frac{\sqrt{1+4x}-\sqrt{1-2x}}{x}$$

is:

(A) 2

(B) 3

(C) 4

(D) 5

Q2. If

$$f(x) = \begin{cases} \dfrac{\sin ax}{x}, & x \neq 0 \\ 5, & x = 0 \end{cases}$$

is continuous at $x = 0$, then the value of a is:

(A) 3

(B) 4

(C) 5

(D) 6

Q3. If

$$y = x^x,$$

then the value of

$$\frac{dy}{dx}$$

is:

(A) x^x

(B) $x^x(1 + \ln x)$

(C) x^{x-1}

(D) $(1 + x)^x$

Q4. The minimum value of the function

$$f(x) = x^2 + \frac{16}{x^2}, \quad x > 0$$

is:

(A) 4

(B) 8

(C) 12

(D) 16

Q5. For the curve

$$y = x^3 - 3x^2 + 2,$$

the number of stationary points is:

(A) 0

(B) 1

(C) 2

(D) 3

Q6. A cylindrical container with closed ends has volume

$$500\pi \text{ cm}^3.$$

If the surface area is minimum, then the radius of the cylinder is:

(A) 5 cm

(B) 6 cm

(C) 7 cm

(D) 8 cm

Q7. The value of

$$\int_0^{\pi/2} \sin x \cos x \, dx$$

is:

(A) 0

(B) $\frac{1}{4}$

(C) $\frac{1}{2}$

(D) 1

Q8. If

$$I = \int \frac{2x+1}{x^2+x+1} \, dx,$$

then I equals:

(A) $\ln(x^2 + x + 1) + C$

(B) $2\ln(x^2 + x + 1) + C$

(C) $\tan^{-1}(x) + C$

(D) $\frac{1}{x^2+x+1} + C$

Q9. The area bounded by the curve

$$y = 4 - x^2$$

and the x-axis is:

(A) $\frac{16}{3}$

(B) $\frac{32}{3}$

(C) 8

(D) 12

Q10. The solution of the differential equation

$$\frac{dy}{dx} = 3x^2,$$

given that $y = 1$ when $x = 0$, is:

(A) $y = x^3 + 1$

(B) $y = x^3 - 1$

(C) $y = 3x^2 + 1$

(D) $y = x^2 + 1$

Q11. If the arithmetic mean of 7 and 19 is inserted between them, then the mean is:

(A) 11

(B) 12

(C) 13

(D) 14

Q12. The sum of the first 20 terms of the AP

$$3, 7, 11, \ldots$$

is:

(A) 720

(B) 800

(C) 820

(D) 860

Q13. The sum of the GP

$$2 + 6 + 18 + 54 + \cdots + 486$$

is:

(A) 728

(B) 7280

(C) 1092

(D) 1456

Q14. If

$$z = 3 + 4i,$$

then the argument of z is:

(A) $\tan^{-1}(4/3)$

(B) $\tan^{-1}(3/4)$

(C) $\pi/2$

(D) $\pi/4$

Q15. If

$$z^2 = -16,$$

then one value of z is:

(A) 4

(B) -4

(C) 4i

(D) 2i

Q16. If the roots of

$$x^2 - 7x + k = 0$$

are reciprocals of each other, then k equals:

(A) 1

(B) -1

(C) 7

(D) 49

Q17. The equation

$$x^2 + 6x + 13 = 0$$

has roots:

(A) Real and equal

(B) Real and distinct

(C) Imaginary

(D) Rational

Q18. If

$$A = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix},$$

then $|A|$ equals:

(A) 1

(B) 2

(C) 3

(D) 6

Q19. If

$$\begin{vmatrix} 1 & 2 \\ 4 & x \end{vmatrix} = 0,$$

then x equals:

(A) 4

(B) 6

(C) 8

(D) 10

Q20. If

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix},$$

then A^{-1} is:

(A) $\begin{bmatrix} 1/2 & 0 \\ 0 & 1/5 \end{bmatrix}$

(B) $\begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$

(C) $\begin{bmatrix} 5 & 0 \\ 0 & 2 \end{bmatrix}$

(D) $\begin{bmatrix} 0 & 1/2 \\ 1/5 & 0 \end{bmatrix}$

Q21. The number of ways in which 5 boys and 4 girls can be seated in a row such that all girls sit together is:

(A) $5! \times 4!$

(B) $6! \times 4!$

(C) $5! \times 5!$

(D) $9!$

Q22. From the digits

$$1, 2, 3, 4, 5, 6$$

the number of 4-digit numbers that can be formed without repetition and divisible by 5 is:

(A) 24

(B) 60

(C) 120

(D) 144

Q23. A card is drawn at random from a standard deck of 52 cards.

The probability that the card drawn is either a king or a heart is:

(A) $\frac{4}{13}$

(B) $\frac{1}{13}$

(C) $\frac{17}{52}$

(D) $\frac{16}{52}$

Q24. A box contains 6 white and 4 black balls.

If two balls are drawn successively without replacement, then the probability that both are white is:

(A) $\frac{1}{3}$

(B) $\frac{2}{5}$

(C) $\frac{5}{9}$

(D) $\frac{3}{5}$

Q25. The slope of the line represented by

$$5x - 2y + 7 = 0$$

is:

(A) $\frac{5}{2}$

(B) $-\frac{5}{2}$

(C) $\frac{2}{5}$

(D) $-\frac{2}{5}$

Q26. The equation of the line passing through the points

$$(1, 2) \quad \text{and} \quad (3, 6)$$

is:

(A) $y = 2x$

(B) $y = 2x + 1$

(C) $y = 3x - 1$

(D) $y = x + 2$

Q27. The centre of the circle

$$x^2 + y^2 - 8x + 6y + 9 = 0$$

is:

(A) $(4, -3)$

(B) $(-4, 3)$

(C) $(8, -6)$

(D) $(4, 3)$

Q28. The equation

$$x^2 + y^2 = 25$$

represents a circle of radius:

(A) 25

(B) 10

(C) 5

(D) $\sqrt{5}$

Q29. The focus of the parabola

$$y^2 = 12x$$

is:

(A) $(3, 0)$

(B) $(0, 3)$

(C) $(6, 0)$

(D) $(0, 6)$

Q30. For the hyperbola

$$\frac{x^2}{16} - \frac{y^2}{9} = 1,$$

the value of eccentricity is:

(A) $\frac{3}{4}$

(B) $\frac{5}{4}$

(C) $\frac{4}{3}$

(D) $\frac{5}{3}$

Q31. The value of

$$\cos 15^\circ$$

is:

(A) $\frac{\sqrt{6}+\sqrt{2}}{4}$

(B) $\frac{\sqrt{6}-\sqrt{2}}{4}$

(C) $\frac{\sqrt{3}}{2}$

(D) $\frac{1}{2}$

Q32. If

$$\sin\theta = \frac{3}{5},$$

where θ lies in the first quadrant, then

$$\cos\theta$$

equals:

(A) $\frac{3}{5}$

(B) $\frac{4}{5}$

(C) $\frac{5}{3}$

(D) $\frac{5}{4}$

Q33. The value of

$$\sin^2\theta + \cos^2\theta$$

is:

(A) 0

(B) 1

(C) 2

(D) Depends on θ

Q34. The principal value of

$$\cos^{-1}\left(-\frac{1}{2}\right)$$

is:

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{3}$

(C) $\frac{2\pi}{3}$

(D) $\frac{5\pi}{6}$

Q35. If

$$\vec{a} = 3\hat{i} + 4\hat{j},$$

then the magnitude of $\vec{a}$ is:

(A) 3

(B) 4

(C) 5

(D) 7

Q36. The unit vector in the direction of

$$2\hat{i} - 2\hat{j}$$

is:

(A) $\frac{1}{\sqrt{2}}(\hat{i} - \hat{j})$

(B) $\sqrt{2}(\hat{i} - \hat{j})$

(C) $\hat{i} - \hat{j}$

(D) $\frac{1}{2}(\hat{i} - \hat{j})$

Q37. The coordinates of the midpoint of the line segment joining

$$(2, 3, 4) \quad \text{and} \quad (6, 7, 8)$$

are:

(A) $(4, 5, 6)$

(B) $(3, 5, 6)$

(C) $(4, 4, 6)$

(D) $(2, 5, 8)$

Q38. The distance of the point

$$(1, 2, 2)$$

from the origin is:

(A) 2

(B) 3

(C) $\sqrt{5}$

(D) $\sqrt{9}$

Q39. If

$$A = \{1, 2, 3\} \quad \text{and} \quad B = \{2, 4, 6\},$$

then the number of functions from A to B is:

(A) 3

(B) 6

(C) 9

(D) 27

Q40. If

$$A = \{1, 2, 3, 4\}$$

and

$$B = \{3, 4, 5, 6\},$$

then

$$A \cap B$$

is:

(A) $\{1, 2\}$

(B) $\{3, 4\}$

(C) $\{5, 6\}$

(D) $\{1, 2, 3, 4\}$

Detailed Solutions

Q1.

Solution

Concept: The limit can be evaluated by rationalizing the numerator to eliminate the indeterminate form $\frac{0}{0}$. Alternatively, L'Hôpital's Rule can be applied.

Solution: Step 1: Write down the limit expression and recognize it is in the indeterminate form $\frac{0}{0}$:

$$\lim_{x\to 0}\frac{\sqrt{1+4x}-\sqrt{1-2x}}{x}$$

Step 2: Rationalize the numerator by multiplying the numerator and denominator by the conjugate $\sqrt{1+4x}+\sqrt{1-2x}$:

$$=\lim_{x\to 0}\frac{\left(\sqrt{1+4x}-\sqrt{1-2x}\right)\left(\sqrt{1+4x}+\sqrt{1-2x}\right)}{x\left(\sqrt{1+4x}+\sqrt{1-2x}\right)}$$

$$=\lim_{x\to 0}\frac{(1+4x)-(1-2x)}{x\left(\sqrt{1+4x}+\sqrt{1-2x}\right)}$$

Step 3: Simplify the numerator:

$$=\lim_{x\to 0}\frac{6x}{x\left(\sqrt{1+4x}+\sqrt{1-2x}\right)}$$

Step 4: Cancel x from the numerator and denominator, then substitute $x=0$:

$$=\lim_{x\to 0}\frac{6}{\sqrt{1+4x}+\sqrt{1-2x}}$$

$$=\frac{6}{\sqrt{1+0}+\sqrt{1-0}}=\frac{6}{2}=3$$

Option B is correct.

Final Answer: $\boxed{3}$

Answer: (B)

Go Back to Question 1

Q2.

Solution

Concept: For a function $f(x)$ to be continuous at $x = 0$, the limit of $f(x)$ as $x \to 0$ must exist and be equal to the value of the function at $x = 0$, i.e., $\lim_{x\to 0} f(x) = f(0)$.

Solution: Step 1: Write the condition for continuity at $x = 0$:

$$\lim_{x\to 0} f(x) = f(0)$$

Step 2: Substitute the definition of $f(x)$ for $x \neq 0$ and $x = 0$:

$$\lim_{x\to 0} \frac{\sin ax}{x} = 5$$

Step 3: Multiply and divide the fraction by a to use the standard limit $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$:

$$\lim_{x\to 0} a \cdot \frac{\sin ax}{ax} = 5$$

$$a \cdot \lim_{ax\to 0} \frac{\sin ax}{ax} = 5$$

Step 4: Apply the standard limit:

$$a \cdot 1 = 5 \implies a = 5$$

Option C is correct.

Final Answer: $\boxed{5}$

Answer: (C)

Go Back to Question 2

Q3.

Solution

Concept: To differentiate a function of the form $y = f(x)^{g(x)}$, we use logarithmic differentiation. Taking the natural logarithm of both sides simplifies the exponent.

Solution: Step 1: Write down the equation and take the natural logarithm (ln) of both sides:

$$y = x^x$$

$$\ln y = \ln(x^x)$$

$$\ln y = x \ln x$$

Step 2: Differentiate both sides with respect to x using the chain rule on the left side and the product rule on the right side:

$$\frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}(x) \cdot \ln x + x \cdot \frac{d}{dx}(\ln x)$$

$$\frac{1}{y}\frac{dy}{dx} = 1 \cdot \ln x + x \cdot \frac{1}{x}$$

$$\frac{1}{y}\frac{dy}{dx} = \ln x + 1$$

Step 3: Multiply both sides by y to solve for $\frac{dy}{dx}$:

$$\frac{dy}{dx} = y(1 + \ln x)$$

Step 4: Substitute $y = x^x$ back into the expression:

$$\frac{dy}{dx} = x^x(1 + \ln x)$$

Option B is correct.

Final Answer: $\boxed{x^x(1 + \ln x)}$

Answer: (B)

Go Back to Question 3

Q4.

Solution

Concept: The minimum value of a positive function can be found using the first derivative test (or alternatively, the AM-GM inequality, which states that for positive real numbers, the Arithmetic Mean is greater than or equal to the Geometric Mean).

Solution: *Method 1: Using the AM-GM Inequality* Step 1: Since $x > 0$, both x^2 and $\frac{16}{x^2}$ are positive terms. Apply AM-GM:

$$\frac{x^2 + \frac{16}{x^2}}{2} \geq \sqrt{x^2 \cdot \frac{16}{x^2}}$$

Step 2: Simplify the expression inside the square root:

$$\frac{f(x)}{2} \geq \sqrt{16}$$

$$\frac{f(x)}{2} \geq 4 \implies f(x) \geq 8$$

Equality holds when $x^2 = \frac{16}{x^2} \implies x^4 = 16 \implies x = 2$ (since $x > 0$). Thus, the minimum value is 8.

Method 2: Using Calculus Step 1: Find the first derivative of $f(x) = x^2 + 16x^{-2}$:

$$f'(x) = 2x - \frac{32}{x^3}$$

Step 2: Set $f'(x) = 0$ to find the critical points:

$$2x - \frac{32}{x^3} = 0 \implies 2x^4 = 32 \implies x^4 = 16$$

Since $x > 0$, we have $x = 2$.

Step 3: Find the second derivative to confirm the minimum:

$$f''(x) = 2 + \frac{96}{x^4}$$

At $x = 2$, $f''(2) = 2 + \frac{96}{16} = 8 > 0$, so $x = 2$ is a local minimum.

Step 4: Evaluate $f(x)$ at $x = 2$:

$$f(2) = 2^2 + \frac{16}{2^2} = 4 + 4 = 8$$

Option B is correct.

Final Answer: $\boxed{8}$

Answer: (B)

Go Back to Question 4

Q5.

Solution

Concept: Stationary points of a curve $y = f(x)$ are the points where the first derivative of y with respect to x is equal to zero, i.e., $\frac{dy}{dx} = 0$.

Solution: Step 1: Differentiate the given equation $y = x^3 - 3x^2 + 2$ with respect to x:

$$\frac{dy}{dx} = 3x^2 - 6x$$

Step 2: Set the derivative equal to zero to find the x-coordinates of the stationary points:

$$3x^2 - 6x = 0$$

$$3x(x - 2) = 0$$

Step 3: Solve for x:

$$x = 0 \quad \text{or} \quad x = 2$$

Step 4: Count the number of solutions. Since there are two distinct real solutions for x, there are exactly 2 stationary points: $(0, 2)$ and $(2, -2)$.
Option C is correct.

Final Answer: $\boxed{2}$

Answer: (C)

Go Back to Question 5

Q6.

Solution

Concept: The dimensions that minimize the surface area of a closed cylinder for a fixed volume can be found by expressing the surface area in terms of a single variable, differentiating, and setting the derivative to zero.

Solution: Let the radius and height of the closed cylinder be r and h respectively. Using the volume condition:

$$\pi r^2 h = 500\pi$$

$$h = \frac{500}{r^2}$$

The total surface area is:

$$S = 2\pi r^2 + 2\pi r h$$

Substituting h:

$$S(r) = 2\pi r^2 + 2\pi r\left(\frac{500}{r^2}\right)$$

$$S(r) = 2\pi r^2 + \frac{1000\pi}{r}$$

Differentiate and set equal to zero:

$$\frac{dS}{dr} = 4\pi r - \frac{1000\pi}{r^2} = 0$$

$$4r = \frac{1000}{r^2}$$

$$r^3 = 250$$

$$r = \sqrt[3]{250} \approx 6.30 \text{ cm}$$

Checking the given integer options:

- $r = 5$: $S(5) = 250\pi$
- $r = 6$: $S(6) \approx 238.7\pi$
- $r = 7$: $S(7) \approx 240.9\pi$
- $r = 8$: $S(8) = 253\pi$

Therefore, **Option B** is correct.

Final Answer: $\boxed{6}$

Answer: (B)

Go Back to Question 6

Q7.

Solution

Concept: A definite integral can be evaluated by simplifying the integrand using trigonometric identities or by utilizing a change of variables (substitution method).

Solution: *Method 1: Using Trigonometric Identities* Step 1: Simplify the integrand using the identity $\sin 2x = 2\sin x \cos x$:

$$\int_0^{\pi/2} \sin x \cos x \, dx = \int_0^{\pi/2} \frac{1}{2} \sin 2x \, dx$$

Step 2: Find the antiderivative and evaluate it at the boundaries:

$$= \left[-\frac{1}{4} \cos 2x \right]_0^{\pi/2}$$

$$= -\frac{1}{4} \left(\cos \left(2 \cdot \frac{\pi}{2} \right) - \cos(0) \right)$$

$$= -\frac{1}{4}(\cos \pi - \cos 0) = -\frac{1}{4}(-1 - 1) = \frac{1}{2}$$

Method 2: Using Substitution Step 1: Let $u = \sin x$, then $du = \cos x \, dx$. Step 2: Adjust the integration limits:

- When $x = 0$, $u = \sin 0 = 0$.
- When $x = \frac{\pi}{2}$, $u = \sin \frac{\pi}{2} = 1$.

Step 3: Substitute and integrate:

$$\int_0^1 u \, du = \left[\frac{u^2}{2} \right]_0^1 = \frac{1^2}{2} - \frac{0^2}{2} = \frac{1}{2}$$

Option C is correct.

Final Answer: $\boxed{\frac{1}{2}}$

Answer: (C)

Go Back to Question 7

Q8.

Solution

Concept: An integral of the form $\int \frac{f'(x)}{f(x)}\,dx$ can be solved by substitution, resulting in $\ln|f(x)|+C$.

Solution: Step 1: Identify the denominator and let $u = x^2 + x + 1$. Step 2: Differentiate u with respect to x to find du:

$$du = (2x + 1)\,dx$$

Step 3: Substitute u and du into the integral:

$$I = \int \frac{2x+1}{x^2+x+1}\,dx = \int \frac{1}{u}\,du$$

Step 4: Integrate with respect to u:

$$I = \ln|u| + C$$

Step 5: Substitute back $u = x^2 + x + 1$:

$$I = \ln(x^2 + x + 1) + C$$

(Note: Absolute value bars are omitted since $x^2 + x + 1 = \left(x + \frac{1}{2}\right)^2 + \frac{3}{4} > 0$ for all real x). Option A is correct.

Final Answer: $\boxed{\ln(x^2 + x + 1) + C}$

Answer: (A)

Go Back to Question 8

Q9.

Solution

Concept: The area bounded by a curve $y = f(x)$ and the x-axis is given by the definite integral $\int_a^b |f(x)|\,dx$, where a and b are the x-intercepts of the curve.

Solution: Step 1: Find the x-intercepts of the curve $y = 4 - x^2$ by setting $y = 0$:

$$4 - x^2 = 0 \implies x^2 = 4 \implies x = \pm 2$$

Thus, the boundary limits of integration are $a = -2$ and $b = 2$.
Step 2: Set up the definite integral for the area A (noting that $y \geq 0$ for $x \in [-2, 2]$):

$$A = \int_{-2}^{2} (4 - x^2)\,dx$$

Step 3: Since the integrand $4 - x^2$ is an even function, we can simplify the integral:

$$A = 2\int_{0}^{2} (4 - x^2)\,dx$$

Step 4: Integrate term-by-term and evaluate:

$$A = 2\left[4x - \frac{x^3}{3}\right]_0^2$$

$$A = 2\left(\left(4(2) - \frac{2^3}{3}\right) - 0\right)$$

$$A = 2\left(8 - \frac{8}{3}\right) = 2\left(\frac{16}{3}\right) = \frac{32}{3}$$

Option B is correct.

Final Answer: $\boxed{\frac{32}{3}}$

Answer: (B)

Go Back to Question 9

Q10.

Solution

Concept: A first-order differential equation of the form $\frac{dy}{dx} = f(x)$ can be solved by direct integration. The constant of integration is determined using the given initial condition.

Solution: Step 1: Rewrite the differential equation to separate the variables:

$$dy = 3x^2\,dx$$

Step 2: Integrate both sides:

$$\int dy = \int 3x^2\,dx$$

$$y = x^3 + C$$

Step 3: Use the initial condition $y = 1$ when $x = 0$ to find the value of C:

$$1 = 0^3 + C \implies C = 1$$

Step 4: Write the final particular solution:

$$y = x^3 + 1$$

Option A is correct.

Final Answer: $\boxed{y = x^3 + 1}$

Answer: (A)

Go Back to Question 10

Q11.

Solution

Concept: The single arithmetic mean A inserted between two numbers a and b is equal to their average, given by the formula $A = \frac{a+b}{2}$.

Solution: Step 1: Identify the two given numbers:

$$a = 7, \quad b = 19$$

Step 2: Apply the arithmetic mean formula:

$$\text{Arithmetic Mean} = \frac{a+b}{2}$$

$$= \frac{7+19}{2}$$

Step 3: Simplify the expression:

$$= \frac{26}{2} = 13$$

Option C is correct.

Final Answer: $\boxed{13}$

Answer: (C)

Go Back to Question 11

Q12.

Solution

Concept: The sum of the first n terms of an Arithmetic Progression (AP) is given by the formula $S_n = \frac{n}{2}[2a + (n-1)d]$, where a is the first term, d is the common difference, and n is the number of terms.

Solution: Step 1: Identify the parameters of the given AP $3, 7, 11, \ldots$:

- First term $(a) = 3$
- Common difference $(d) = 7 - 3 = 4$
- Number of terms $(n) = 20$

Step 2: Substitute these values into the sum formula:

$$S_{20} = \frac{20}{2}[2(3) + (20-1)4]$$

Step 3: Simplify the expression inside the brackets:

$$S_{20} = 10[6 + 19 \cdot 4]$$

$$S_{20} = 10[6 + 76]$$

Step 4: Compute the final sum:

$$S_{20} = 10 \cdot 82 = 820$$

Option C is correct.

Final Answer: $\boxed{820}$

Answer: (C)

Go Back to Question 12

Q13.

Solution

Concept: The sum of the first n terms of a Geometric Progression (GP) can be computed using the formula $S_n = \frac{a(r^n - 1)}{r - 1}$ or $S_n = \frac{ra_n - a}{r - 1}$, where a is the first term, r is the common ratio, and a_n is the last term.

Solution: Step 1: Identify the parameters of the given GP $2 + 6 + 18 + 54 + \cdots + 486$:

- First term $(a) = 2$
- Common ratio $(r) = \frac{6}{2} = 3$
- Last term $(a_n) = 486$

Step 2: Apply the GP sum formula that uses the last term:

$$S_n = \frac{ra_n - a}{r - 1}$$

Step 3: Substitute the values into the formula:

$$S_n = \frac{3 \cdot 486 - 2}{3 - 1}$$

$$S_n = \frac{1458 - 2}{2}$$

$$S_n = \frac{1456}{2} = 728$$

Option A is correct.

Final Answer: $\boxed{728}$

Answer: (A)

Go Back to Question 13

Q14.

Solution

Concept: For a complex number $z = x + iy$, if z lies in the first quadrant ($x > 0, y > 0$), its principal argument θ is given by the formula $\theta = \tan^{-1}\left(\frac{y}{x}\right)$.

Solution: Step 1: Identify the real and imaginary parts of $z = 3 + 4i$:

- Real part $(x) = 3$
- Imaginary part $(y) = 4$

Step 2: Since both $x > 0$ and $y > 0$, the complex number lies in the first quadrant.
Step 3: Calculate the argument θ:

$$\theta = \tan^{-1}\left(\frac{4}{3}\right)$$

Option A is correct.

Final Answer: $\boxed{\tan^{-1}(4/3)}$

Answer: (A)

Go Back to Question 14

Q15.

Solution

Concept: The square root of a negative real number involves the imaginary unit i, where $i = \sqrt{-1}$. Thus, for any positive real number k, $\sqrt{-k} = \pm i\sqrt{k}$.

Solution: Step 1: Write down the given equation:

$$z^2 = -16$$

Step 2: Take the square root on both sides:

$$z = \pm\sqrt{-16}$$

$$z = \pm\sqrt{16 \cdot (-1)}$$

$$z = \pm\sqrt{16} \cdot \sqrt{-1}$$

$$z = \pm 4i$$

Step 3: Identify the roots of the equation: The two values of z are $4i$ and $-4i$. Among the given options, $4i$ is listed as Option C.
Option C is correct.

Final Answer: $\boxed{4i}$

Answer: (C)

Go Back to Question 15

Q16.

Solution

Concept: For a quadratic equation $ax^2 + bx + c = 0$, if the roots are reciprocals of each other, their product is equal to 1. According to Vieta's formulas, the product of the roots is given by $\frac{c}{a}$.

Solution: Step 1: Let the roots of the equation $x^2 - 7x + k = 0$ be α and $\frac{1}{\alpha}$.
Step 2: Find the product of the roots:

$$\text{Product of roots} = \alpha \cdot \frac{1}{\alpha} = 1$$

Step 3: Use Vieta's formulas to express the product of the roots in terms of the coefficients (where $a = 1$, $b = -7$, and $c = k$):

$$\text{Product of roots} = \frac{c}{a} = \frac{k}{1} = k$$

Step 4: Set the two expressions for the product of roots equal to each other:

$$k = 1$$

Option A is correct.

Final Answer: $\boxed{1}$

Answer: (A)

Go Back to Question 16

Q17.

Solution

Concept: The nature of the roots of a quadratic equation $ax^2 + bx + c = 0$ is determined by its discriminant $D = b^2 - 4ac$:

- If $D > 0$, the roots are real and distinct.
- If $D = 0$, the roots are real and equal.
- If $D < 0$, the roots are imaginary (complex conjugates).

Solution: Step 1: Identify the coefficients of the given quadratic equation $x^2 + 6x + 13 = 0$:

$$a = 1, \quad b = 6, \quad c = 13$$

Step 2: Calculate the discriminant D:

$$D = b^2 - 4ac$$

$$D = 6^2 - 4(1)(13)$$

$$D = 36 - 52 = -16$$

Step 3: Since $D = -16 < 0$, the roots are imaginary.
Option C is correct.

Final Answer: Imaginary

Answer: (C)

Go Back to Question 17

Q18.

Solution

Concept: The determinant of a 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is given by the formula $|A| = ad - bc$.

Solution: Step 1: Identify the elements of the matrix $A = \begin{bmatrix} 1 & 0 \\ 2 & 3 \end{bmatrix}$:

$$a = 1, \quad b = 0, \quad c = 2, \quad d = 3$$

Step 2: Apply the determinant formula:

$$|A| = (1)(3) - (0)(2)$$

Step 3: Simplify the expression:

$$|A| = 3 - 0 = 3$$

Option C is correct.

Final Answer: $\boxed{3}$

Answer: (C)

Go Back to Question 18

Q19.

Solution

Concept: For a given determinant equation $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = 0$, evaluate the determinant using the formula $ad - bc = 0$ and solve for the unknown variable.

Solution: Step 1: Write down the given determinant equation:

$$\begin{vmatrix} 1 & 2 \\ 4 & x \end{vmatrix} = 0$$

Step 2: Evaluate the determinant:

$$(1)(x) - (2)(4) = 0$$

Step 3: Simplify and solve for x:

$$x - 8 = 0 \implies x = 8$$

Option C is correct.

Final Answer: $\boxed{8}$

Answer: (C)

Go Back to Question 19

Q20.

Solution

Concept: For a diagonal matrix $A = \begin{bmatrix} d_1 & 0 \\ 0 & d_2 \end{bmatrix}$ where $d_1, d_2 \neq 0$, the inverse is given by $A^{-1} = \begin{bmatrix} 1/d_1 & 0 \\ 0 & 1/d_2 \end{bmatrix}$. Alternatively, we can use the formula $A^{-1} = \frac{1}{|A|}\text{adj}(A)$.

Solution: *Method 1: Properties of Diagonal Matrices* Step 1: Observe that $A = \begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$ is a diagonal matrix with diagonal elements $d_1 = 2$ and $d_2 = 5$. Step 2: The inverse is obtained by taking the reciprocals of the diagonal elements:

$$A^{-1} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/5 \end{bmatrix}$$

Method 2: Standard Formula Step 1: Compute the determinant of A:

$$|A| = (2)(5) - (0)(0) = 10$$

Step 2: Find the adjugate of A:

$$\text{adj}(A) = \begin{bmatrix} 5 & 0 \\ 0 & 2 \end{bmatrix}$$

Step 3: Calculate the inverse A^{-1}:

$$A^{-1} = \frac{1}{|A|}\text{adj}(A) = \frac{1}{10}\begin{bmatrix} 5 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 5/10 & 0 \\ 0 & 2/10 \end{bmatrix} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/5 \end{bmatrix}$$

Option A is correct.

Final Answer: $\boxed{\begin{bmatrix} 1/2 & 0 \\ 0 & 1/5 \end{bmatrix}}$

Answer: (A)

Go Back to Question 20

Q21.

Solution

Concept: To solve problems where specific items must remain together, we treat the group of items as a single combined entity. Then, we find the arrangements of all elements (including the group) and multiply by the internal arrangements within that group.

Solution: Step 1: Treat the 4 girls as a single entity or block:

$$\{\text{Girl}_1, \text{Girl}_2, \text{Girl}_3, \text{Girl}_4\}$$

Step 2: Count the total number of entities to be arranged. We have 5 individual boys plus the 1 block containing all the girls, making a total of 6 entities:

$$5 \text{ boys} + 1 \text{ block} = 6 \text{ entities}$$

Step 3: Find the number of ways to arrange these 6 entities in a row:

$$\text{Number of arrangements} = 6!$$

Step 4: Find the number of ways to arrange the 4 girls among themselves inside their block:

$$\text{Number of internal arrangements} = 4!$$

Step 5: Apply the multiplication principle to find the total arrangements:

$$\text{Total ways} = 6! \times 4!$$

Option B is correct.

Final Answer: $\boxed{6! \times 4!}$

Answer: (B)

Go Back to Question 21

Q22.

Solution

Concept: A number is divisible by 5 if its units digit is 0 or 5. For permutation problems with constraints, always address the restricted positions first, followed by the remaining positions.

Solution: Step 1: Identify the available digits:

$$\{1, 2, 3, 4, 5, 6\}$$

Step 2: Determine the constraint on the units position for a 4-digit number to be divisible by 5: The units digit must be 5 (since 0 is not in the set of given digits). This gives exactly 1 option for the units place:

$$\text{Units place} = 1 \text{ option } (5)$$

Step 3: Fill the remaining 3 places (thousands, hundreds, and tens) using the remaining 5 digits $\{1, 2, 3, 4, 6\}$ without repetition: We need to select and arrange 3 digits out of 5:

$$\text{Number of ways} = P(5, 3) = 5 \times 4 \times 3 = 60$$

Step 4: Compute the total number of valid 4-digit numbers:

$$\text{Total numbers} = 60 \times 1 = 60$$

Option B is correct.

Final Answer: $\boxed{60}$

Answer: (B)

Go Back to Question 22

Q23.

Solution

Concept: The probability of the union of two events A and B is given by the addition rule:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Solution: Step 1: Define the events and their individual probabilities from a standard deck of 52 cards: Let K be the event that the card is a King:

$$P(K) = \frac{4}{52}$$

Let H be the event that the card is a Heart:

$$P(H) = \frac{13}{52}$$

Step 2: Find the probability of drawing a card that is both a King and a Heart (the King of Hearts):

$$P(K \cap H) = \frac{1}{52}$$

Step 3: Apply the addition rule of probability:

$$P(K \cup H) = P(K) + P(H) - P(K \cap H)$$

$$P(K \cup H) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52}$$

Step 4: Simplify the fraction:

$$\frac{16}{52} = \frac{4}{13}$$

Both Option A ($\frac{4}{13}$) and Option D ($\frac{16}{52}$) represent the same probability. Typically, probability values are simplified to their lowest terms, making Option A the standard choice.
Option A is correct.

Final Answer: $\boxed{\frac{4}{13}}$

Answer: (A)

Go Back to Question 23

Q24.

Solution

Concept: The probability of drawing two white balls successively without replacement can be determined using conditional probability: $P(W_1 \cap W_2) = P(W_1) \cdot P(W_2 \mid W_1)$. Alternatively, we can use combinations.

Solution: *Method 1: Conditional Probability* Step 1: Find the probability of drawing a white ball on the first attempt:

$$\text{Total balls} = 6 \text{ white} + 4 \text{ black} = 10 \text{ balls}$$

$$P(W_1) = \frac{6}{10} = \frac{3}{5}$$

Step 2: Find the probability of drawing a white ball on the second attempt, given that a white ball was already drawn:

$$\text{Remaining white balls} = 5$$

$$\text{Remaining total balls} = 9$$

$$P(W_2 \mid W_1) = \frac{5}{9}$$

Step 3: Multiply the probabilities to find the combined probability:

$$P(W_1 \cap W_2) = P(W_1) \cdot P(W_2 \mid W_1) = \frac{3}{5} \cdot \frac{5}{9} = \frac{3}{9} = \frac{1}{3}$$

Method 2: Combinations Step 1: Choose 2 white balls out of 6, and divide by the number of ways to choose 2 balls out of 10:

$$\text{Probability} = \frac{\binom{6}{2}}{\binom{10}{2}}$$

Step 2: Calculate the combinations and evaluate:

$$\binom{6}{2} = \frac{6 \times 5}{2} = 15, \quad \binom{10}{2} = \frac{10 \times 9}{2} = 45$$

$$\text{Probability} = \frac{15}{45} = \frac{1}{3}$$

Option A is correct.

Final Answer: $\boxed{\frac{1}{3}}$

Answer: (A)

Go Back to Question 24

Q25.

Solution

Concept: For a straight line expressed in the general form $Ax + By + C = 0$, the slope m is given by the formula $m = -\frac{A}{B}$. Alternatively, we can express the equation in the slope-intercept form $y = mx + c$.

Solution: Step 1: Write down the given line equation:

$$5x - 2y + 7 = 0$$

Step 2: Rearrange the equation to express y in terms of x (slope-intercept form):

$$-2y = -5x - 7$$

$$y = \frac{-5}{-2}x - \frac{7}{-2}$$

$$y = \frac{5}{2}x + \frac{7}{2}$$

Step 3: Identify the coefficient of x, which represents the slope m:

$$m = \frac{5}{2}$$

Option A is correct.

Final Answer: $\boxed{\frac{5}{2}}$

Answer: (A)

Go Back to Question 25

Q26.

Solution

Concept: The equation of a straight line passing through two points (x_1, y_1) and (x_2, y_2) can be determined using the point-slope form: $y - y_1 = m(x - x_1)$, where the slope is $m = \frac{y_2 - y_1}{x_2 - x_1}$.

Solution: Step 1: Identify the coordinates of the two points:

$$(x_1, y_1) = (1, 2) \quad \text{and} \quad (x_2, y_2) = (3, 6)$$

Step 2: Calculate the slope m:

$$m = \frac{6 - 2}{3 - 1} = \frac{4}{2} = 2$$

Step 3: Write down the equation using the point-slope formula with $(1, 2)$:

$$y - 2 = 2(x - 1)$$

Step 4: Simplify the equation:

$$y - 2 = 2x - 2$$

$$y = 2x$$

Option A is correct.

Final Answer: $\boxed{y = 2x}$

Answer: (A)

Go Back to Question 26

Q27.

Solution

Concept: The general equation of a circle is $x^2 + y^2 + 2gx + 2fy + c = 0$, where the center of the circle is located at $(-g, -f)$.

Solution: Step 1: Write down the given equation of the circle:

$$x^2 + y^2 - 8x + 6y + 9 = 0$$

Step 2: Compare it with the general equation $x^2 + y^2 + 2gx + 2fy + c = 0$ to identify g and f:

$$2g = -8 \implies g = -4$$

$$2f = 6 \implies f = 3$$

Step 3: Determine the coordinates of the center $(-g, -f)$:

$$\text{Center} = (-(-4), -3) = (4, -3)$$

Step 4: (Optional Verification) Rewrite the equation by completing the square:

$$\left(x^2 - 8x + 16\right) + \left(y^2 + 6y + 9\right) = -9 + 16 + 9$$

$$(x - 4)^2 + (y + 3)^2 = 16$$

This standard form $(x - h)^2 + (y - k)^2 = R^2$ confirms that the center (h, k) is $(4, -3)$.
Option A is correct.

Final Answer: $\boxed{(4, -3)}$

Answer: (A)

Go Back to Question 27

Q28.

Solution

Concept: The standard equation of a circle centered at the origin $(0, 0)$ is given by $x^2 + y^2 = R^2$, where R represents the radius of the circle ($R > 0$).

Solution: Step 1: Write down the given circle equation:

$$x^2 + y^2 = 25$$

Step 2: Express the constant on the right-hand side as a square:

$$x^2 + y^2 = 5^2$$

Step 3: Compare this with the standard equation $x^2 + y^2 = R^2$:

$$R^2 = 25 \implies R = \sqrt{25} = 5$$

Since the radius must be positive, $R = 5$ is the radius of the circle.
Option C is correct.

Final Answer: $\boxed{5}$

Answer: (C)

Go Back to Question 28

Q29.

Solution

Concept: For a parabola defined by the standard equation $y^2 = 4ax$ (opening to the right), the focus is located at the point $(a, 0)$.

Solution: Step 1: Write down the given parabola equation:

$$y^2 = 12x$$

Step 2: Compare this with the standard form $y^2 = 4ax$ to find the value of a:

$$4a = 12 \implies a = 3$$

Step 3: Determine the coordinates of the focus $(a, 0)$:

$$\text{Focus} = (3, 0)$$

Option A is correct.

Final Answer: $\boxed{(3, 0)}$

Answer: (A)

Go Back to Question 29

Q30.

Solution

Concept: The standard equation of a horizontal hyperbola is $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. The eccentricity e of such a hyperbola is given by the formula $e = \sqrt{1 + \frac{b^2}{a^2}}$.

Solution: Step 1: Write down the given hyperbola equation:

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

Step 2: Identify a^2 and b^2 by comparing with the standard form:

$$a^2 = 16, \quad b^2 = 9$$

Step 3: Substitute these values into the eccentricity formula:

$$e = \sqrt{1 + \frac{9}{16}}$$

$$e = \sqrt{\frac{16 + 9}{16}} = \sqrt{\frac{25}{16}}$$

Step 4: Take the square root:

$$e = \frac{5}{4}$$

Option B is correct.

Final Answer: $\boxed{\frac{5}{4}}$

Answer: (B)

Go Back to Question 30

Q31.

Solution

Concept: The value of $\cos 15^\circ$ can be found by using the cosine angle subtraction formula:

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

Solution: Step 1: Express 15° as the difference between two standard angles whose trigonometric ratios are known:

$$15^\circ = 45^\circ - 30^\circ$$

Step 2: Apply the subtraction identity:

$$\cos 15^\circ = \cos(45^\circ - 30^\circ) = \cos 45^\circ \cos 30^\circ + \sin 45^\circ \sin 30^\circ$$

Step 3: Substitute the exact values:

$$\cos 45^\circ = \frac{1}{\sqrt{2}}, \quad \cos 30^\circ = \frac{\sqrt{3}}{2}, \quad \sin 45^\circ = \frac{1}{\sqrt{2}}, \quad \sin 30^\circ = \frac{1}{2}$$

$$\cos 15^\circ = \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$

$$\cos 15^\circ = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

Step 4: Rationalize the denominator by multiplying the numerator and denominator by $\sqrt{2}$:

$$\cos 15^\circ = \frac{(\sqrt{3}+1)\sqrt{2}}{2\sqrt{2}\cdot\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

Option A is correct.

Final Answer: $\boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}$

Answer: (A)

Go Back to Question 31

Q32.

Solution

Concept: The fundamental trigonometric identity states that $\sin^2\theta + \cos^2\theta = 1$. Since θ lies in the first quadrant, both $\sin\theta$ and $\cos\theta$ must be positive.

Solution: Step 1: State the relationship for $\cos\theta$:

$$\cos\theta = \sqrt{1 - \sin^2\theta} \quad (\text{since } \theta \text{ is in Quadrant I, } \cos\theta > 0)$$

Step 2: Substitute the value of $\sin\theta = \frac{3}{5}$ into the formula:

$$\cos\theta = \sqrt{1 - \left(\frac{3}{5}\right)^2}$$

$$\cos\theta = \sqrt{1 - \frac{9}{25}}$$

Step 3: Simplify the expression:

$$\cos\theta = \sqrt{\frac{25-9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

Option B is correct.

Final Answer: $\boxed{\frac{4}{5}}$

Answer: (B)

Go Back to Question 32

Q33.

Solution

Concept: The equation $\sin^2\theta + \cos^2\theta = 1$ is the fundamental Pythagorean trigonometric identity, which holds true for all real values of θ.

Solution: Step 1: Write down the given expression:

$$\sin^2\theta + \cos^2\theta$$

Step 2: Apply the Pythagorean identity directly. By definition:

$$\sin^2\theta + \cos^2\theta = 1 \quad \text{for all } \theta \in \mathbb{R}$$

Thus, the value does not depend on θ and is always constant at 1.
Option B is correct.

Final Answer: $\boxed{1}$

Answer: (B)

Go Back to Question 33

Q34.

Solution

Concept: The principal value branch of $\cos^{-1} x$ is $[0, \pi]$. For a negative argument, we use the identity $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$.

Solution: Step 1: State the principal domain property of the inverse cosine function:

$$\cos^{-1}(-x) = \pi - \cos^{-1}(x) \quad \text{for } x \in [0, 1]$$

Step 2: Substitute $x = \frac{1}{2}$:

$$\cos^{-1}\left(-\frac{1}{2}\right) = \pi - \cos^{-1}\left(\frac{1}{2}\right)$$

Step 3: Find the standard angle where cosine is $\frac{1}{2}$ within $[0, \pi]$:

$$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2} \implies \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

Step 4: Compute the final value:

$$\cos^{-1}\left(-\frac{1}{2}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

Option C is correct.

Final Answer: $\boxed{\frac{2\pi}{3}}$

Answer: (C)

Go Back to Question 34

Q35.

Solution

Concept: A vector $\vec{a} = x\hat{i} + y\hat{j}$ represents a directed line segment in a two-dimensional coordinate plane, where $\hat{i}$ and $\hat{j}$ are the standard unit vectors along the positive x-axis and y-axis, respectively. The magnitude (or length) of this vector is denoted by $|\vec{a}|$ and is geometrically equivalent to the straight-line Euclidean distance from the origin $(0, 0)$ to the terminal point (x, y). This is calculated using the Pythagorean theorem:

$$|\vec{a}| = \sqrt{x^2 + y^2}$$

Solution: Step 1: Identify the scalar components of the given vector $\vec{a} = 3\hat{i} + 4\hat{j}$:

$$\text{x-component } (x) = 3$$

$$\text{y-component } (y) = 4$$

Step 2: Substitute these components into the magnitude formula:

$$|\vec{a}| = \sqrt{3^2 + 4^2}$$

Step 3: Simplify the terms inside the square root by calculating the squares of the individual components:

$$|\vec{a}| = \sqrt{9 + 16}$$

$$|\vec{a}| = \sqrt{25}$$

Step 4: Take the positive square root of 25 (since magnitude represents a physical length and must be non-negative):

$$|\vec{a}| = 5$$

This forms a classic 3-4-5 right triangle with the axes, confirming that the magnitude of the vector is exactly 5.
Option C is correct.

Final Answer: $\boxed{5}$

Answer: (C)

Go Back to Question 35

Q36.

Solution

Concept: A unit vector is a vector that has a magnitude (length) of exactly 1 while pointing in the same direction as the original vector. To find the unit vector $\hat{v}$ in the direction of any non-zero vector $\vec{v}$, we scale down the original vector by dividing it by its own magnitude:

$$\hat{v} = \frac{\vec{v}}{|\vec{v}|}$$

Solution: Step 1: Write down the given vector:

$$\vec{v} = 2\hat{i} - 2\hat{j}$$

Step 2: Compute the magnitude $|\vec{v}|$ of this vector:

$$|\vec{v}| = \sqrt{(2)^2 + (-2)^2}$$

$$|\vec{v}| = \sqrt{4 + 4}$$

$$|\vec{v}| = \sqrt{8}$$

By simplifying the radical, we can express $\sqrt{8}$ as:

$$|\vec{v}| = \sqrt{4 \cdot 2} = 2\sqrt{2}$$

Step 3: Divide the vector components by this magnitude to construct the unit vector $\hat{v}$:

$$\hat{v} = \frac{2\hat{i} - 2\hat{j}}{2\sqrt{2}}$$

Step 4: Factor out the common scalar 2 from the numerator to simplify the fraction:

$$\hat{v} = \frac{2(\hat{i} - \hat{j})}{2\sqrt{2}}$$

$$\hat{v} = \frac{1}{\sqrt{2}}(\hat{i} - \hat{j})$$

Step 5: (Optional Verification) We can verify that the magnitude of this resulting vector is indeed 1:

$$|\hat{v}| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(-\frac{1}{\sqrt{2}}\right)^2} = \sqrt{\frac{1}{2} + \frac{1}{2}} = \sqrt{1} = 1$$

Since the magnitude is 1, this is a valid unit vector.
Option A is correct.

Final Answer: $\boxed{\frac{1}{\sqrt{2}}(\hat{i} - \hat{j})}$

Answer: (A)

Go Back to Question 36

Q37.

Solution

Concept: In three-dimensional coordinate geometry, the midpoint of a line segment is the point that lies exactly halfway between its two endpoints. If the endpoints of a line segment are designated as $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$, the coordinates of the midpoint $M(x_m, y_m, z_m)$ are found by taking the arithmetic mean of the corresponding coordinates of the two endpoints:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

Solution: Step 1: Identify the coordinates of the given endpoints:

$$(x_1, y_1, z_1) = (2, 3, 4) \quad \text{and} \quad (x_2, y_2, z_2) = (6, 7, 8)$$

Step 2: Calculate the x-coordinate of the midpoint (x_m) by averaging the x-coordinates of both points:

$$x_m = \frac{x_1 + x_2}{2} = \frac{2 + 6}{2} = \frac{8}{2} = 4$$

Step 3: Calculate the y-coordinate of the midpoint (y_m) by averaging the y-coordinates of both points:

$$y_m = \frac{y_1 + y_2}{2} = \frac{3 + 7}{2} = \frac{10}{2} = 5$$

Step 4: Calculate the z-coordinate of the midpoint (z_m) by averaging the z-coordinates of both points:

$$z_m = \frac{z_1 + z_2}{2} = \frac{4 + 8}{2} = \frac{12}{2} = 6$$

Step 5: Combine these individual coordinates to write the coordinates of the midpoint:

$$\text{Midpoint } M = (4, 5, 6)$$

Option A is correct.

Final Answer: $\boxed{(4, 5, 6)}$

Answer: (A)

Go Back to Question 37

Q38.

Solution

Concept: The distance between any two points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ in a three-dimensional Cartesian coordinate system is given by the 3D distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

When calculating the distance of a point $P(x, y, z)$ from the origin $O(0, 0, 0)$, the coordinates (x_1, y_1, z_1) become $(0, 0, 0)$, simplifying the expression to:

$$d = \sqrt{x^2 + y^2 + z^2}$$

Solution: Step 1: Identify the coordinates of the target point and the origin:

$$\text{Target point } P = (1, 2, 2)$$

$$\text{Origin } O = (0, 0, 0)$$

Step 2: Apply the simplified 3D distance formula for distance from the origin:

$$d = \sqrt{1^2 + 2^2 + 2^2}$$

Step 3: Calculate the squares of the coordinates:

$$1^2 = 1, \quad 2^2 = 4, \quad 2^2 = 4$$

$$d = \sqrt{1 + 4 + 4}$$

Step 4: Sum the terms inside the square root:

$$d = \sqrt{9}$$

Step 5: Evaluate the square root. Since distance is a geometric measurement, we consider only the positive root:

$$d = 3$$

Although Option D ($\sqrt{9}$) represents the same mathematical value, standard mathematical convention requires writing the final answer in its fully simplified form, which is the integer 3.
Option B is correct.

Final Answer: $\boxed{3}$

Answer: (B)

Go Back to Question 38

Q39.

Solution

Concept: A function $f : A \to B$ is a relation that associates each element of set A with exactly one element of set B. If set A has p elements and set B has q elements, the total number of distinct functions that can be constructed from A to B is given by:

$$\text{Number of functions} = q^p$$

This formula is derived from the **Fundamental Counting Principle**:

- For the first element in A, there are q possible elements in B that it can map to.
- For the second element in A, there are also q independent choices in B.
- This process continues for all p elements in set A.
- Thus, the total number of ways to map all elements is $q \times q \times \cdots \times q$ (p times), which equals q^p.

Solution: Step 1: Determine the number of elements in each of the given sets:

$$A = \{1, 2, 3\} \implies n(A) = p = 3 \text{ elements}$$

$$B = \{2, 4, 6\} \implies n(B) = q = 3 \text{ elements}$$

Step 2: Apply the total functions formula q^p where $p = 3$ and $q = 3$:

$$\text{Total functions} = 3^3$$

Step 3: Evaluate the exponential expression:

$$3^3 = 3 \times 3 \times 3 = 27$$

There are exactly 27 unique functions that can be defined from set A to set B.
Option D is correct.

Final Answer: $\boxed{27}$

Answer: (D)

Go Back to Question 39

Q40.

Solution

Concept: In set theory, the intersection of two sets A and B, denoted by $A \cap B$, is defined as the set containing all elements that belong to both set A **and** set B simultaneously:

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

Geometrically, this represents the overlapping region of the two sets in a Venn diagram.

Solution: Step 1: Write down the elements belonging to each of the given sets:

$$\text{Set } A = \{1, 2, 3, 4\}$$

$$\text{Set } B = \{3, 4, 5, 6\}$$

Step 2: Compare the sets and identify which numbers are present in both A and B:

- 1 is in A but not in B.
- 2 is in A but not in B.
- **3 is in both A and B.**
- **4 is in both A and B.**
- 5 is in B but not in A.
- 6 is in B but not in A.

Step 3: Collect the common elements into a new set representing the intersection:

$$A \cap B = \{3, 4\}$$

Note on other set operations for comparison:

- The Union ($A \cup B$) containing all elements from either set is $\{1, 2, 3, 4, 5, 6\}$.
- The Set Difference ($A \setminus B$) containing elements only in A is $\{1, 2\}$.

Option B is correct.

Final Answer: $\boxed{\{3, 4\}}$

Answer: (B)

Go Back to Question 40

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	C	3	B	4	B	5	C
6	B	7	C	8	A	9	B	10	A
11	C	12	C	13	A	14	A	15	C
16	A	17	C	18	C	19	C	20	A
21	B	22	B	23	A	24	A	25	A
26	A	27	A	28	C	29	A	30	B
31	A	32	B	33	B	34	C	35	C
36	A	37	A	38	B	39	D	40	B