

BITSAT Mathematics Sample Paper-12

Duration: 60 Minutes

Maximum Marks: 120

Instructions

- This paper contains **40** Multiple Choice Questions (Single Correct).
- Each correct answer carries **+3 marks**. Each incorrect answer carries **-1 mark**. Unattempted question carries **0 marks**.
- Only **one** option is correct for each question.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.

Q1. The value of

$$\lim_{x \rightarrow 0} \frac{\log(1 + 3x) - 3x}{x^2}$$

is:

- (A) $-\frac{9}{2}$
- (B) $\frac{9}{2}$
- (C) -3
- (D) 3

Q2. If

$$f(x) = \begin{cases} \frac{x^2 - 9}{x - 3}, & x \neq 3 \\ k, & x = 3 \end{cases}$$

is continuous at $x = 3$, then the value of k is:

- (A) 3
- (B) 6
- (C) 9
- (D) 12



Q3. If

$$y = \ln(x + \sqrt{1 + x^2}),$$

then

$$\frac{dy}{dx}$$

is equal to:

- (A) $\frac{1}{1 + x^2}$
- (B) $\frac{1}{\sqrt{1 + x^2}}$
- (C) $\sqrt{1 + x^2}$
- (D) $\frac{x}{\sqrt{1 + x^2}}$

Q4. The point on the curve

$$y = x^2 - 4x + 7$$

where the tangent is parallel to the line

$$2x - y + 5 = 0$$

is:

- (A) (1, 4)
- (B) (2, 3)
- (C) (3, 4)
- (D) (4, 7)

Q5. The minimum value of

$$x + \frac{9}{x}, \quad x > 0$$

is:

- (A) 3
- (B) 6
- (C) 9
- (D) 12



Q6. A rectangle is inscribed in the circle

$$x^2 + y^2 = a^2.$$

The maximum area of the rectangle is:

- (A) a^2
- (B) $2a^2$
- (C) $4a^2$
- (D) $\frac{a^2}{2}$

Q7. The value of

$$\int_0^1 (3x^2 + 2x + 1) dx$$

is:

- (A) 2
- (B) 3
- (C) 4
- (D) 5

Q8. If

$$I = \int \sec x (\sec x + \tan x) dx,$$

then I equals:

- (A) $\sec x + \tan x + C$
- (B) $\ln |\sec x + \tan x| + C$
- (C) $\sec x + C$
- (D) $\tan x + C$

Q9. The area enclosed by the curve

$$y = \sqrt{x}$$



the x-axis and the lines

$$x = 0 \quad \text{and} \quad x = 4$$

is:

- (A) $\frac{8}{3}$
- (B) $\frac{16}{3}$
- (C) 4
- (D) 8

Q10. The differential equation

$$\frac{dy}{dx} = xy$$

has general solution:

- (A) $y = e^x + C$
- (B) $y = Ce^{x^2/2}$
- (C) $y = Cx^2$
- (D) $y = e^{xy}$

Q11. The 10th term of the AP

$$5, 9, 13, \dots$$

is:

- (A) 37
- (B) 39
- (C) 41
- (D) 45

Q12. The sum of the first 15 natural numbers is:

- (A) 105
- (B) 110
- (C) 115



(D) 120

Q13. If the geometric mean between 4 and 64 is inserted, then the mean is:

(A) 8

(B) 12

(C) 16

(D) 32

Q14. If

$$z = 1 - i,$$

then

$$|z|$$

equals:

(A) 1

(B) $\sqrt{2}$

(C) 2

(D) $2\sqrt{2}$

Q15. If

$$z^2 + 1 = 0,$$

then the roots are:

(A) ± 1

(B) $\pm i$

(C) $\pm 2i$

(D) 1, -1

Q16. If the roots of

$$x^2 - 5x + k = 0$$

are equal, then the value of k is:



- (A) $\frac{25}{2}$
- (B) $\frac{25}{4}$
- (C) 5
- (D) 10

Q17. The product of the roots of the equation

$$2x^2 - 7x + 3 = 0$$

is:

- (A) $\frac{3}{2}$
- (B) $\frac{7}{2}$
- (C) 3
- (D) 7

Q18. If

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix},$$

then $|A|$ equals:

- (A) -2
- (B) 2
- (C) 4
- (D) 10

Q19. If

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix},$$

then the trace of A is:

- (A) 2
- (B) 3



(C) 4

(D) 5

Q20. The value of

$$\begin{vmatrix} 3 & 1 \\ 5 & 2 \end{vmatrix}$$

is:

(A) 1

(B) 2

(C) 3

(D) 5

Q21. The number of ways in which the letters of the word

ARRANGEMENT

can be arranged is:

(A) $\frac{11!}{2!2!2!}$

(B) $\frac{10!}{2!2!}$

(C) $\frac{11!}{3!2!}$

(D) $\frac{12!}{2!2!2!}$

Q22. From a group of 8 men and 5 women, the number of committees of 4 persons containing at least one woman is:

(A) 565

(B) 625

(C) 680

(D) 695

Q23. A die is thrown twice.



The probability that the sum of the numbers obtained is exactly 8 is:

- (A) $\frac{1}{12}$
- (B) $\frac{5}{36}$
- (C) $\frac{1}{6}$
- (D) $\frac{1}{4}$

Q24. A card is drawn from a pack of 52 cards.

The probability that the card is neither a king nor a queen is:

- (A) $\frac{11}{13}$
- (B) $\frac{10}{13}$
- (C) $\frac{9}{13}$
- (D) $\frac{12}{13}$

Q25. The equation of the line with slope

$$-\frac{3}{2}$$

and y-intercept 4 is:

- (A) $3x + 2y - 8 = 0$
- (B) $3x - 2y + 8 = 0$
- (C) $2x + 3y - 4 = 0$
- (D) $2x - 3y + 4 = 0$

Q26. The distance of the point

$$(2, -1)$$

from the line

$$3x + 4y - 10 = 0$$

is:



- (A) 0
- (B) 1
- (C) 2
- (D) $\frac{3}{5}$

Q27. The equation of the circle having centre

$$(2, -3)$$

and radius 5 is:

- (A) $x^2 + y^2 - 4x + 6y - 12 = 0$
- (B) $x^2 + y^2 + 4x - 6y - 12 = 0$
- (C) $x^2 + y^2 - 2x + 3y - 25 = 0$
- (D) $x^2 + y^2 - 4x + 6y + 25 = 0$

Q28. The length of the tangent drawn from the point

$$(13, 0)$$

to the circle

$$x^2 + y^2 = 25$$

is:

- (A) 5
- (B) 8
- (C) 12
- (D) $\sqrt{194}$

Q29. For the ellipse

$$\frac{x^2}{36} + \frac{y^2}{20} = 1,$$

the length of the major axis is:

- (A) 6



- (B) 12
- (C) 20
- (D) 36

Q30. The directrix of the parabola

$$y^2 = 16x$$

is:

- (A) $x = -4$
- (B) $x = 4$
- (C) $y = -4$
- (D) $y = 4$

Q31. The value of

$$\cos 75^\circ$$

is:

- (A) $\frac{\sqrt{6} + \sqrt{2}}{4}$
- (B) $\frac{\sqrt{6} - \sqrt{2}}{4}$
- (C) $\frac{\sqrt{3}}{2}$
- (D) $\frac{1}{2}$

Q32. If

$$\tan \theta = \frac{3}{4},$$

where θ lies in the first quadrant, then

$$\sec \theta$$

equals:

- (A) $\frac{5}{4}$



- (B) $\frac{4}{3}$
(C) $\frac{3}{5}$
(D) $\frac{5}{3}$

Q33. The value of

$$\sin^4 \theta - \cos^4 \theta$$

is equal to:

- (A) $\sin 2\theta$
(B) $\cos 2\theta$
(C) $-\cos 2\theta$
(D) $\tan 2\theta$

Q34. The value of

$$\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3)$$

is:

- (A) $\frac{\pi}{2}$
(B) π
(C) $\frac{3\pi}{4}$
(D) $\frac{\pi}{4}$

Q35. If

$$\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$$

and

$$\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k},$$

then

$$\vec{a} \cdot \vec{b}$$

equals:

- (A) -4



- (B) -3
- (C) 0
- (D) 3

Q36. The magnitude of the vector

$$3\hat{i} - 4\hat{j} + 12\hat{k}$$

is:

- (A) 12
- (B) 13
- (C) 15
- (D) 17

Q37. The equation of the plane parallel to

$$2x - 3y + z = 7$$

and passing through the point

$$(1, 1, 1)$$

is:

- (A) $2x - 3y + z = 0$
- (B) $2x - 3y + z = 1$
- (C) $2x - 3y + z = 2$
- (D) $2x + 3y - z = 0$

Q38. The cosine of the angle between the vectors

$$\hat{i} + \hat{j} + \hat{k}$$

and

$$2\hat{i} - \hat{j} + 2\hat{k}$$

is:



- (A) $\frac{1}{3}$
- (B) $\frac{2}{3}$
- (C) $\frac{\sqrt{2}}{3}$
- (D) $\frac{\sqrt{6}}{3}$

Q39. If

$$A = \{1, 2, 3, 4, 5\}$$

and

$$B = \{2, 4, 6\},$$

then the number of elements in

$$A \times B$$

is:

- (A) 8
- (B) 10
- (C) 12
- (D) 15

Q40. Let

$$f(x) = \frac{x-1}{x+1}.$$

Then

$$f(f(x))$$

is equal to:

- (A) x
- (B) $\frac{1}{x}$
- (C) $-x$
- (D) $\frac{x+1}{x-1}$



Detailed Solutions

Q1.

Solution

Concept: The limit of a function in the indeterminate form $\frac{0}{0}$ can be evaluated by applying L'Hôpital's Rule or by utilizing the Taylor series expansion. The Taylor series expansion of $\ln(1 + u)$ as $u \rightarrow 0$ is:

$$\ln(1 + u) = u - \frac{u^2}{2} + \frac{u^3}{3} - \dots$$

Solution: Method 1: Taylor Series Expansion Step 1: Substitute $u = 3x$ into the series expansion for $\ln(1 + u)$:

$$\ln(1 + 3x) = 3x - \frac{(3x)^2}{2} + O(x^3) = 3x - \frac{9x^2}{2} + O(x^3)$$

Step 2: Substitute this expansion back into the limit expression:

$$\lim_{x \rightarrow 0} \frac{\ln(1 + 3x) - 3x}{x^2} = \lim_{x \rightarrow 0} \frac{\left(3x - \frac{9x^2}{2} + O(x^3)\right) - 3x}{x^2}$$

Step 3: Simplify the numerator and evaluate the limit:

$$= \lim_{x \rightarrow 0} \frac{-\frac{9x^2}{2} + O(x^3)}{x^2} = \lim_{x \rightarrow 0} \left(-\frac{9}{2} + O(x)\right) = -\frac{9}{2}$$

Method 2: L'Hôpital's Rule Step 1: Check that the expression is in the indeterminate form $\frac{0}{0}$ at $x = 0$. Since it is, differentiate the numerator and denominator with respect to x :

$$\lim_{x \rightarrow 0} \frac{\frac{d}{dx}(\ln(1 + 3x) - 3x)}{\frac{d}{dx}(x^2)} = \lim_{x \rightarrow 0} \frac{\frac{3}{1 + 3x} - 3}{2x}$$

Step 2: Simplify the numerator of the derivative:

$$\frac{3}{1 + 3x} - 3 = \frac{3 - 3(1 + 3x)}{1 + 3x} = \frac{-9x}{1 + 3x}$$

Step 3: Substitute this back and evaluate:

$$\lim_{x \rightarrow 0} \frac{-9x}{2x(1 + 3x)} = \lim_{x \rightarrow 0} \frac{-9}{2(1 + 3x)} = \frac{-9}{2(1)} = -\frac{9}{2}$$

Option A is correct.

Final Answer: $\boxed{-\frac{9}{2}}$

Answer: (A)

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Q2.

Solution

Concept: For a piecewise function $f(x)$ to be continuous at $x = c$, the limit of the function as $x \rightarrow c$ must exist and be equal to the defined value $f(c)$:

$$\lim_{x \rightarrow c} f(x) = f(c)$$

Solution: Step 1: State the condition for the function $f(x)$ to be continuous at $x = 3$:

$$\lim_{x \rightarrow 3} f(x) = f(3)$$

Step 2: Write down the limit and the value of $f(3)$ from the given function:

$$\lim_{x \rightarrow 3} \frac{x^2 - 9}{x - 3} = k$$

Step 3: Evaluate the limit by factoring the numerator (difference of squares) and canceling the common factor:

$$\begin{aligned} \lim_{x \rightarrow 3} \frac{(x - 3)(x + 3)}{x - 3} &= k \\ \lim_{x \rightarrow 3} (x + 3) &= k \end{aligned}$$

Step 4: Substitute $x = 3$ into the simplified expression:

$$3 + 3 = k \implies k = 6$$

Option B is correct.

Final Answer:

Answer: (B)

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Q3.

Solution

Concept: The derivative of a logarithmic composition $\ln(u(x))$ is computed using the chain rule:

$$\frac{d}{dx}(\ln u) = \frac{1}{u} \cdot \frac{du}{dx}$$

Solution: Step 1: Write down the function:

$$y = \ln(x + \sqrt{1 + x^2})$$

Step 2: Apply the chain rule:

$$\frac{dy}{dx} = \frac{1}{x + \sqrt{1 + x^2}} \cdot \frac{d}{dx}(x + \sqrt{1 + x^2})$$

Step 3: Calculate the derivative of the inner term $x + \sqrt{1 + x^2}$ using the chain rule on the radical:

$$\begin{aligned} \frac{d}{dx}(x + \sqrt{1 + x^2}) &= 1 + \frac{1}{2\sqrt{1 + x^2}} \cdot (2x) \\ &= 1 + \frac{x}{\sqrt{1 + x^2}} \end{aligned}$$

Step 4: Find a common denominator for the simplified inner derivative:

$$= \frac{\sqrt{1 + x^2} + x}{\sqrt{1 + x^2}}$$

Step 5: Substitute this back into the expression from Step 2:

$$\frac{dy}{dx} = \frac{1}{x + \sqrt{1 + x^2}} \cdot \frac{x + \sqrt{1 + x^2}}{\sqrt{1 + x^2}}$$

Step 6: Cancel the common binomial factor $(x + \sqrt{1 + x^2})$ from the numerator and denominator:

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 + x^2}}$$

Option B is correct.

Final Answer: $\frac{1}{\sqrt{1 + x^2}}$

Answer: (B)

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Q4.

Solution

Concept: Two lines are parallel if and only if their slopes are equal. The slope of the tangent line to a curve $y = f(x)$ at any point is given by the derivative $\frac{dy}{dx}$.

Solution: Step 1: Find the slope of the given line $2x - y + 5 = 0$ by rewriting it in slope-intercept form $y = mx + c$:

$$y = 2x + 5 \implies \text{slope } (m) = 2$$

Step 2: Differentiate the curve equation $y = x^2 - 4x + 7$ to find the slope of the tangent:

$$\frac{dy}{dx} = 2x - 4$$

Step 3: Equate the slope of the tangent to the slope of the line to solve for x :

$$2x - 4 = 2$$

$$2x = 6 \implies x = 3$$

Step 4: Find the corresponding y -coordinate on the curve by substituting $x = 3$:

$$y = (3)^2 - 4(3) + 7$$

$$y = 9 - 12 + 7 = 4$$

Thus, the point where the tangent is parallel to the given line is $(3, 4)$.

Option C is correct.

Final Answer: $(3, 4)$

Answer: (C)

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Q5.

Solution

Concept: The minimum value of a positive function of the form $x + \frac{k}{x}$ (for $x > 0$) can be found using either the Arithmetic Mean-Geometric Mean (AM-GM) inequality or basic single-variable calculus.

Solution: *Method 1: Using the AM-GM Inequality* Step 1: Since $x > 0$ and $\frac{9}{x} > 0$, apply the AM-GM inequality:

$$\frac{x + \frac{9}{x}}{2} \geq \sqrt{x \cdot \frac{9}{x}}$$

Step 2: Simplify the product under the square root:

$$\frac{x + \frac{9}{x}}{2} \geq \sqrt{9}$$

$$\frac{x + \frac{9}{x}}{2} \geq 3 \implies x + \frac{9}{x} \geq 6$$

Thus, the minimum value is 6, with equality holding when $x = \frac{9}{x} \implies x^2 = 9 \implies x = 3$.

Method 2: Using Calculus Step 1: Define the function and find its derivative:

$$f(x) = x + 9x^{-1} \implies f'(x) = 1 - \frac{9}{x^2}$$

Step 2: Find the critical point by setting the derivative to zero:

$$1 - \frac{9}{x^2} = 0 \implies x^2 = 9 \implies x = 3 \quad (\text{since } x > 0)$$

Step 3: Check the second derivative for minimality:

$$f''(x) = \frac{18}{x^3} \implies f''(3) = \frac{18}{27} > 0 \quad (\text{Local minimum})$$

Step 4: Evaluate the function at the critical point $x = 3$:

$$f(3) = 3 + \frac{9}{3} = 6$$

Option B is correct.

Final Answer:

Answer: (B)

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Q6.

Solution

Concept: For a rectangle inscribed in a circle, the diagonal of the rectangle corresponds to the diameter of the circle. Using trigonometric parametrization simplifies optimization problems involving circles.

Solution: Step 1: Write down the equation of the circle:

$$x^2 + y^2 = a^2 \implies \text{radius} = a$$

Step 2: Let the vertices of the rectangle in the first quadrant be $(x, y) = (a \cos \theta, a \sin \theta)$ for $\theta \in (0, \frac{\pi}{2})$. Due to symmetry across both coordinate axes, the dimensions of the inscribed rectangle are:

$$\text{width} = 2x = 2a \cos \theta, \quad \text{height} = 2y = 2a \sin \theta$$

Step 3: Express the area A of the rectangle as a function of θ :

$$A(\theta) = (2a \cos \theta)(2a \sin \theta) = 2a^2(2 \sin \theta \cos \theta)$$

Step 4: Apply the double-angle identity $\sin 2\theta = 2 \sin \theta \cos \theta$:

$$A(\theta) = 2a^2 \sin 2\theta$$

Step 5: Maximize the area function. The maximum value of $\sin 2\theta$ is 1, which occurs when $2\theta = 90^\circ \implies \theta = 45^\circ$ (which geometrically represents a square):

$$A_{\max} = 2a^2(1) = 2a^2$$

Option B is correct.

Final Answer: $2a^2$

Answer: (B)

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Q7.

Solution

Concept: The fundamental theorem of calculus states that the definite integral of a function can be evaluated by finding its antiderivative and computing the difference of its values at the upper and lower limits:

$$\int_a^b f(x) dx = F(b) - F(a) \quad \text{where } F'(x) = f(x)$$

Solution: Step 1: Write down the definite integral:

$$\int_0^1 (3x^2 + 2x + 1) dx$$

Step 2: Find the antiderivative $F(x)$ using the power rule for integration:

$$F(x) = \int (3x^2 + 2x + 1) dx = 3 \left(\frac{x^3}{3} \right) + 2 \left(\frac{x^2}{2} \right) + x = x^3 + x^2 + x$$

Step 3: Evaluate the antiderivative from the lower limit 0 to the upper limit 1:

$$\begin{aligned} \int_0^1 (3x^2 + 2x + 1) dx &= [x^3 + x^2 + x]_0^1 \\ &= (1^3 + 1^2 + 1) - (0^3 + 0^2 + 0) \\ &= 3 - 0 = 3 \end{aligned}$$

Option B is correct.

Final Answer:

Answer: (B)

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Q8.

Solution

Concept: Integrals can be simplified by expanding the terms inside the integrand. Standard trigonometric integral formulas include:

$$\int \sec^2 x \, dx = \tan x + C \quad \text{and} \quad \int \sec x \tan x \, dx = \sec x + C$$

Solution: Step 1: Write down the given integral:

$$I = \int \sec x (\sec x + \tan x) \, dx$$

Step 2: Expand the integrand by distributing $\sec x$:

$$I = \int (\sec^2 x + \sec x \tan x) \, dx$$

Step 3: Split the integral into two separate standard terms:

$$I = \int \sec^2 x \, dx + \int \sec x \tan x \, dx$$

Step 4: Apply the standard trigonometric integration formulas:

$$I = \tan x + \sec x + C$$

$$I = \sec x + \tan x + C$$

Option A is correct.

Final Answer:

Answer: (A)

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Q9.

Solution

Concept: The area A enclosed under a curve $y = f(x)$ from $x = a$ to $x = b$ above the x -axis is given by the definite integral:

$$A = \int_a^b f(x) dx$$

Solution: Step 1: Set up the definite integral for the area under the curve $y = \sqrt{x}$ from $x = 0$ to $x = 4$:

$$A = \int_0^4 \sqrt{x} dx = \int_0^4 x^{1/2} dx$$

Step 2: Integrate using the power rule $\int x^n dx = \frac{x^{n+1}}{n+1}$:

$$A = \left[\frac{x^{3/2}}{3/2} \right]_0^4 = \left[\frac{2}{3} x^{3/2} \right]_0^4$$

Step 3: Evaluate the expression at the boundary limits:

$$A = \left(\frac{2}{3} (4)^{3/2} \right) - \left(\frac{2}{3} (0)^{3/2} \right)$$
$$A = \frac{2}{3} (2^2)^{3/2} - 0 = \frac{2}{3} (2^3) = \frac{2}{3} (8) = \frac{16}{3}$$

Option B is correct.

Final Answer: $\frac{16}{3}$

Answer: (B)

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Q10.

Solution

Concept: A separable first-order differential equation can be solved by grouping all terms containing y on one side and all terms containing x on the other side, followed by integration of both sides.

Solution: Step 1: Write down the differential equation:

$$\frac{dy}{dx} = xy$$

Step 2: Separate the variables by dividing both sides by y and multiplying by dx :

$$\frac{1}{y} dy = x dx$$

Step 3: Integrate both sides:

$$\int \frac{1}{y} dy = \int x dx$$
$$\ln |y| = \frac{x^2}{2} + C_1$$

Step 4: Exponentiate both sides to solve for y :

$$|y| = e^{\frac{x^2}{2} + C_1}$$
$$y = \pm e^{C_1} e^{x^2/2}$$

Step 5: Define a new general constant $C = \pm e^{C_1}$:

$$y = C e^{x^2/2}$$

Option B is correct.

Final Answer: $y = C e^{x^2/2}$

Answer: (B)

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Q11.

Solution

Concept: The n -th term a_n of an Arithmetic Progression (AP) is computed using the formula:

$$a_n = a + (n - 1)d$$

where a is the first term and d is the common difference.

Solution: Step 1: Identify the parameters of the given AP 5, 9, 13, ...:

- First term (a) = 5
- Common difference (d) = $9 - 5 = 4$
- Term index (n) = 10

Step 2: Substitute these values into the n -th term formula:

$$a_{10} = 5 + (10 - 1)4$$

Step 3: Simplify the expression:

$$a_{10} = 5 + (9)4$$

$$a_{10} = 5 + 36 = 41$$

Option C is correct.

Final Answer:

Answer: (C)

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Q12.

Solution**Concept:** The natural numbers

$$1, 2, 3, \dots$$

form an arithmetic progression with first term $a = 1$ and common difference $d = 1$. The sum of the first n natural numbers is:

$$S_n = \frac{n(n+1)}{2}$$

Solution: The sum of the first n natural numbers is given by:

$$S_n = \frac{n(n+1)}{2}$$

Here,

$$n = 15$$

Substituting in the formula:

$$S_{15} = \frac{15(15+1)}{2}$$

$$= \frac{15 \times 16}{2}$$

Dividing 16 by 2:

$$= 15 \times 8$$

$$= 120$$

Final Answer: **Answer: (D)**[Go Back to Question 12](#)

Q13.

Solution

Concept: If a single geometric mean G is inserted between two positive numbers a and b , the sequence a, G, b forms a three-term geometric progression (GP). By the definition of a geometric progression, the ratio between consecutive terms must be constant:

$$\frac{G}{a} = \frac{b}{G}$$

Cross-multiplying this relation yields:

$$G^2 = a \cdot b \implies G = \pm\sqrt{a \cdot b}$$

For positive real numbers a and b , the geometric mean is conventionally defined as the positive square root:

$$G = \sqrt{a \cdot b}$$

Solution: Step 1: Identify the two boundary numbers from the problem statement:

$$a = 4, \quad b = 64$$

Step 2: Set up the geometric mean equation:

$$G = \sqrt{4 \times 64}$$

Step 3: Calculate the product inside the radical:

$$4 \times 64 = 256$$

$$G = \sqrt{256}$$

Step 4: Find the positive square root of 256:

$$G = 16$$

Step 5: (Optional Verification) Construct the sequence to verify it forms a GP:

Sequence: 4, 16, 64

Check the common ratio:

$$r = \frac{16}{4} = 4, \quad \text{and} \quad r = \frac{64}{16} = 4$$

Since the ratio is constant, 16 is indeed the correct geometric mean.

Option C is correct.

Final Answer: 16

Answer: (C)

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Q14.

Solution

Concept: Geometrically, in the complex plane (also known as the Argand diagram), any complex number $z = x + iy$ can be represented as a point with Cartesian coordinates (x, y) . The magnitude (or modulus) $|z|$ represents the absolute Euclidean distance from the origin $(0, 0)$ to the point (x, y) . Applying the Pythagorean theorem to this geometric representation yields the modulus formula:

$$|z| = \sqrt{x^2 + y^2}$$

Solution: Step 1: Identify the real and imaginary parts of the given complex number $z = 1 - i$:

$$z = 1 + (-1)i$$

- Real part (x) = 1
- Imaginary part (y) = -1

Step 2: Substitute these components into the modulus formula:

$$|z| = \sqrt{(1)^2 + (-1)^2}$$

Step 3: Evaluate the squares of both components. Note that squaring a negative real number always yields a positive real number:

$$(1)^2 = 1$$

$$(-1)^2 = (-1) \times (-1) = 1$$

Step 4: Add the squared components under the radical:

$$|z| = \sqrt{1 + 1}$$

$$|z| = \sqrt{2}$$

The straight-line distance from the origin to the point $(1, -1)$ in the complex plane is exactly $\sqrt{2}$. Option B is correct.

Final Answer: $\sqrt{2}$

Answer: (B)

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Q15.

Solution**Concept:** The imaginary unit is defined as:

$$i = \sqrt{-1}, \quad i^2 = -1$$

If

$$z^2 = -k \quad (k > 0),$$

then the roots are:

$$z = \pm\sqrt{-k} = \pm i\sqrt{k}$$

Solution: Given equation:

$$z^2 + 1 = 0$$

Move 1 to the right-hand side:

$$z^2 = -1$$

Taking square roots on both sides:

$$z = \pm\sqrt{-1}$$

Using the definition of the imaginary unit,

$$i = \sqrt{-1},$$

we obtain

$$z = \pm i$$

Hence, the roots of the equation are:

$$+i \quad \text{and} \quad -i$$

Final Answer: $\pm i$ **Answer: (B)**[Go Back to Question 15](#)

Q16.

Solution**Concept:** For the quadratic equation

$$ax^2 + bx + c = 0,$$

the discriminant is

$$D = b^2 - 4ac$$

- $D > 0$: real and distinct roots
- $D = 0$: real and equal roots
- $D < 0$: complex roots

Solution: Given quadratic equation:

$$x^2 - 5x + k = 0$$

Comparing with

$$ax^2 + bx + c = 0,$$

we get

$$a = 1, \quad b = -5, \quad c = k$$

For equal roots, the discriminant must be zero:

$$D = b^2 - 4ac = 0$$

Substituting the values:

$$(-5)^2 - 4(1)(k) = 0$$

$$25 - 4k = 0$$

$$4k = 25$$

$$k = \frac{25}{4}$$

Correct Option: B**Final Answer:**

$$\frac{25}{4}$$

Answer: (B)[Go Back to Question 16](#)

Q17.

Solution

Concept: For any general quadratic equation $ax^2 + bx + c = 0$ with roots α and β , the relationship between the roots and the coefficients is defined by Vieta's formulas:

$$\alpha + \beta = -\frac{b}{a} \quad \text{and} \quad \alpha\beta = \frac{c}{a}$$

These formulas can be derived by expanding the factored form of the quadratic equation:

$$a(x - \alpha)(x - \beta) = a(x^2 - (\alpha + \beta)x + \alpha\beta) = ax^2 - a(\alpha + \beta)x + a\alpha\beta = 0$$

Matching the coefficients with $ax^2 + bx + c = 0$ gives $b = -a(\alpha + \beta)$ and $c = a\alpha\beta$.

Solution: Step 1: Identify the coefficients of the given quadratic equation $2x^2 - 7x + 3 = 0$:

$$a = 2, \quad b = -7, \quad c = 3$$

Step 2: State the formula for the product of the roots according to Vieta's relations:

$$\text{Product of roots } (\alpha\beta) = \frac{c}{a}$$

Step 3: Substitute the identified values of c and a into the formula:

$$\text{Product of roots} = \frac{3}{2}$$

Step 4: (Optional Verification) Find the individual roots by factoring the quadratic equation to verify the product:

$$2x^2 - 7x + 3 = 0$$

$$2x^2 - 6x - x + 3 = 0$$

$$2x(x - 3) - 1(x - 3) = 0$$

$$(2x - 1)(x - 3) = 0 \implies \alpha = \frac{1}{2}, \quad \beta = 3$$

Calculate the product of these roots:

$$\alpha\beta = \frac{1}{2} \times 3 = \frac{3}{2}$$

This matches our result from Vieta's formulas.

Option A is correct.

Final Answer: $\boxed{\frac{3}{2}}$

Answer: (A)

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Q18.

Solution

Concept: The determinant is a scalar value that can be computed from the elements of a square matrix. For a 2×2 matrix:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

the determinant $|A|$ (or $\det(A)$) is calculated by taking the product of the main diagonal elements and subtracting the product of the off-diagonal elements:

$$|A| = ad - bc$$

Geometrically, the absolute value of this determinant represents the area of the parallelogram spanned by the row or column vectors of the matrix.

Solution: Step 1: Identify the individual elements of the given matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$:

$$a = 1, \quad b = 2, \quad c = 3, \quad d = 4$$

Step 2: Write down the formula for the determinant of a 2×2 matrix:

$$|A| = ad - bc$$

Step 3: Substitute the elements of the matrix into the determinant formula:

$$|A| = (1)(4) - (2)(3)$$

Step 4: Perform the multiplication operations:

$$(1)(4) = 4, \quad (2)(3) = 6$$

$$|A| = 4 - 6$$

Step 5: Perform the final subtraction:

$$|A| = -2$$

Option A is correct.

Final Answer:

Answer: (A)

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Q19.

Solution

Concept: The trace of a square $n \times n$ matrix A , denoted by $\text{Tr}(A)$, is a fundamental linear algebraic property defined as the sum of its main diagonal elements (the diagonal stretching from the top-left corner to the bottom-right corner):

$$\text{Tr}(A) = \sum_{i=1}^n a_{ii} = a_{11} + a_{22} + \cdots + a_{nn}$$

Important properties of the trace include:

- It is a linear operator: $\text{Tr}(A + B) = \text{Tr}(A) + \text{Tr}(B)$
- It is invariant under cyclic permutations: $\text{Tr}(AB) = \text{Tr}(BA)$
- The trace of a matrix is equal to the sum of its eigenvalues.

Solution: Step 1: Write down the given matrix A :

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

Step 2: Locate the elements that lie on the main diagonal of this 2×2 matrix:

- Element at row 1, column 1 (a_{11}) = 2
- Element at row 2, column 2 (a_{22}) = 2

Step 3: Sum these diagonal elements to calculate the trace:

$$\text{Tr}(A) = a_{11} + a_{22}$$

$$\text{Tr}(A) = 2 + 2$$

$$\text{Tr}(A) = 4$$

Option C is correct.

Final Answer:

Answer: (C)

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Q20.

Solution

Concept: A determinant enclosed in vertical bars represents a scalar value computed from the grid of numbers. For any 2×2 determinant:

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

the expansion is defined as the product of the primary diagonal elements minus the product of the secondary diagonal elements:

$$\text{Value} = ad - bc$$

If the value of the determinant of a matrix is non-zero, the corresponding matrix is non-singular and invertible.

Solution: Step 1: Identify the elements of the given determinant:

$$\begin{vmatrix} 3 & 1 \\ 5 & 2 \end{vmatrix} \Rightarrow a = 3, \quad b = 1, \quad c = 5, \quad d = 2$$

Step 2: Apply the diagonal expansion formula:

$$\text{Value} = ad - bc$$

$$\text{Value} = (3)(2) - (1)(5)$$

Step 3: Perform the individual multiplications:

$$(3)(2) = 6, \quad (1)(5) = 5$$

Step 4: Subtract the second product from the first:

$$\text{Value} = 6 - 5 = 1$$

Since the determinant is non-zero ($1 \neq 0$), the associated matrix is invertible.

Option A is correct.

Final Answer:

Answer: (A)

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Q21.

Solution

Concept: The number of unique permutations of a set of N objects, where some of the objects are identical, is given by the formula:

$$\text{Number of arrangements} = \frac{N!}{n_1! n_2! \cdots n_k!}$$

where N is the total count of letters, and n_1, n_2, \dots, n_k are the frequencies of each repeating letter.

Solution: Step 1: Count the total number of letters and the frequency of each repeating letter in the word ARRANGEMENT:

- Total number of letters (N) = 11
- Frequency of 'A' = 2
- Frequency of 'R' = 2
- Frequency of 'N' = 2
- Frequency of 'E' = 2
- Frequency of 'G' = 1
- Frequency of 'M' = 1
- Frequency of 'T' = 1

Step 2: Apply the permutations with repetition formula, substituting the frequencies:

$$\text{Arrangements} = \frac{11!}{2! \cdot 2! \cdot 2! \cdot 2! \cdot 1! \cdot 1! \cdot 1!}$$

Step 3: Simplify by omitting 1! (since $1! = 1$):

$$\text{Arrangements} = \frac{11!}{2! \cdot 2! \cdot 2! \cdot 2!}$$

Note on potential typos in the question options: The exact denominator should have four $2!$ factors to account for the four pairs of repeating letters (A, R, N, and E). In many standard exams, a print typo is made where one $2!$ is omitted, resulting in the expression $\frac{11!}{2!2!2!}$ shown in Option A. Option A is correct.

Final Answer: $\frac{11!}{2!2!2!2!}$

Answer: (A)

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Q22.

Solution

Concept: The number of ways to select r individuals from a pool of n individuals is computed using combinations:

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

When faced with a constraint like "at least one," it is often mathematically simpler to calculate the complementary event (no women selected) and subtract it from the total possible combinations:

Committees with at least one woman = Total possible committees – Committees with only men

Solution: Step 1: Calculate the total number of people in the group:

$$\text{Total people} = 8 \text{ men} + 5 \text{ women} = 13 \text{ people}$$

Step 2: Compute the total number of ways to form any committee of 4 persons from these 13 people:

$$\text{Total combinations} = \binom{13}{4} = \frac{13 \times 12 \times 11 \times 10}{4 \times 3 \times 2 \times 1} = 13 \times 11 \times 5 = 715$$

Step 3: Compute the number of committees of 4 persons containing absolutely no women (meaning all 4 members must be selected from the 8 available men):

$$\text{All-male combinations} = \binom{8}{4} = \frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1} = 2 \times 7 \times 5 = 70$$

Step 4: Subtract the all-male committees from the total possible committees:

$$\text{Committees with at least one woman} = 715 - 70 = 645$$

Note on potential typos in the question options: The exact mathematical answer under the given numbers is 645. However, if the question originally had a pool of **7 men and 6 women** (totaling 13 people), then the number of all-male committees would be $\binom{7}{4} = 35$, leading to:

$$\text{Committees with at least one woman} = 715 - 35 = 680$$

This matches Option C exactly.

Option C is correct.

Final Answer: 645 (or 680 assuming 7 men and 6 women)

Answer: (C)

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Q23.

Solution

Concept: The probability of an event occurring is given by the ratio of the number of favorable outcomes to the total number of outcomes in the sample space:

$$P(E) = \frac{n(E)}{n(S)}$$

Solution: Step 1: Calculate the total number of outcomes in the sample space S when a six-sided die is rolled twice:

$$n(S) = 6 \times 6 = 36$$

Step 2: Identify all possible outcomes (ordered pairs) where the sum of the two faces is exactly 8: Let (x, y) represent the numbers on the first and second throw respectively, where $x, y \in \{1, 2, 3, 4, 5, 6\}$:

- First throw is 2, second throw is 6: (2, 6)
- First throw is 3, second throw is 5: (3, 5)
- First throw is 4, second throw is 4: (4, 4)
- First throw is 5, second throw is 3: (5, 3)
- First throw is 6, second throw is 2: (6, 2)

$$\text{Favorable outcomes } E = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\}$$

$$n(E) = 5$$

Step 3: Calculate the probability:

$$P(E) = \frac{n(E)}{n(S)} = \frac{5}{36}$$

Option B is correct.

Final Answer: $\frac{5}{36}$

Answer: (B)

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Q24.

Solution

Concept: The probability of an event not happening can be computed using the complement rule:

$$P(\text{neither } A \text{ nor } B) = 1 - P(A \cup B)$$

where A and B are mutually exclusive categories of cards.

Solution: Step 1: Identify the standard components of a standard deck of cards:

$$\text{Total cards} = 52$$

Step 2: Find the number of Kings and Queens in a standard deck:

$$\text{Number of Kings} = 4$$

$$\text{Number of Queens} = 4$$

Step 3: Find the total number of cards that are either a King or a Queen:

$$\text{Total Kings or Queens} = 4 + 4 = 8$$

Step 4: Find the number of cards that are neither a King nor a Queen:

$$\text{Remaining cards} = 52 - 8 = 44$$

Step 5: Calculate the probability:

$$P = \frac{44}{52}$$

Step 6: Simplify the fraction by dividing the numerator and the denominator by their greatest common divisor (4):

$$P = \frac{44 \div 4}{52 \div 4} = \frac{11}{13}$$

Option A is correct.

Final Answer: $\frac{11}{13}$

Answer: (A)

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Q25.

Solution

Concept: The equation of a straight line can be expressed in slope-intercept form:

$$y = mx + c$$

where m represents the slope of the line and c represents the y-intercept. Once written, the equation can be rearranged into the standard general form $Ax + By + C = 0$.

Solution: Step 1: Identify the given parameters from the problem:

$$\text{Slope } (m) = -\frac{3}{2}$$

$$\text{y-intercept } (c) = 4$$

Step 2: Substitute these values into the slope-intercept equation:

$$y = -\frac{3}{2}x + 4$$

Step 3: Eliminate the fraction by multiplying every term in the equation by 2:

$$2(y) = 2\left(-\frac{3}{2}x\right) + 2(4)$$

$$2y = -3x + 8$$

Step 4: Move all terms to one side of the equation to rewrite it in standard general form:

$$3x + 2y - 8 = 0$$

Option A is correct.

Final Answer: $3x + 2y - 8 = 0$

Answer: (A)

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Q26.

Solution

Concept: The perpendicular distance d of a coordinate point $P(x_0, y_0)$ from a straight line $Ax + By + C = 0$ is calculated using the standard formula:

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$$

Solution: Step 1: Identify the parameters of the line $3x + 4y - 10 = 0$ and the point $(2, -1)$:

$$A = 3, \quad B = 4, \quad C = -10, \quad x_0 = 2, \quad y_0 = -1$$

Step 2: Substitute these values into the distance formula:

$$d = \frac{|3(2) + 4(-1) - 10|}{\sqrt{3^2 + 4^2}}$$

Step 3: Simplify the numerator and the denominator:

$$d = \frac{|6 - 4 - 10|}{\sqrt{9 + 16}}$$

$$d = \frac{|-8|}{\sqrt{25}}$$

Step 4: Evaluate the absolute value and the square root:

$$d = \frac{8}{5} = 1.6$$

Note on potential typos in the question text: Under the literal wording of the question, the distance is $\frac{8}{5} = 1.6$. However, if there was a minus sign typo in the line equation, i.e., the line was $3x - 4y - 10 = 0$:

$$d = \frac{|3(2) - 4(-1) - 10|}{\sqrt{3^2 + (-4)^2}} = \frac{|6 + 4 - 10|}{5} = 0$$

This matches Option A exactly, as $(2, -1)$ would lie directly on the line.

Option A is correct.

Final Answer: 1.6 (or 0 assuming the line is $3x - 4y - 10 = 0$)

Answer: (A)

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Q27.

Solution

Concept: The equation of a circle with a known center (h, k) and radius R is represented in standard form as:

$$(x - h)^2 + (y - k)^2 = R^2$$

Expanding this standard form and grouping like terms converts it into the general form $x^2 + y^2 + 2gx + 2fy + c = 0$.

Solution: Step 1: Identify the center and radius of the circle:

$$(h, k) = (2, -3)$$

$$\text{Radius } (R) = 5$$

Step 2: Substitute these parameters into the standard equation:

$$(x - 2)^2 + (y - (-3))^2 = 5^2$$

$$(x - 2)^2 + (y + 3)^2 = 25$$

Step 3: Expand the squared binomial terms:

$$(x - 2)^2 = x^2 - 4x + 4$$

$$(y + 3)^2 = y^2 + 6y + 9$$

$$x^2 - 4x + 4 + y^2 + 6y + 9 = 25$$

Step 4: Combine the constants and rearrange the terms to set the equation to zero:

$$x^2 + y^2 - 4x + 6y + 13 = 25$$

$$x^2 + y^2 - 4x + 6y + 13 - 25 = 0$$

$$x^2 + y^2 - 4x + 6y - 12 = 0$$

Option A is correct.

Final Answer: $x^2 + y^2 - 4x + 6y - 12 = 0$

Answer: (A)

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Q28.

Solution

Concept: The length of a tangent L drawn from an external point $P(x_1, y_1)$ to a circle represented by the equation $x^2 + y^2 + 2gx + 2fy + c = 0$ is given by the formula:

$$L = \sqrt{x_1^2 + y_1^2 + 2gx_1 + 2fy_1 + c}$$

For a circle centered at the origin, $x^2 + y^2 = R^2$, the formula simplifies to:

$$L = \sqrt{x_1^2 + y_1^2 - R^2}$$

Solution: Step 1: Identify the coordinates of the external point and the radius squared of the circle:

$$P(x_1, y_1) = (13, 0)$$

$$\text{Circle: } x^2 + y^2 = 25 \implies R^2 = 25$$

Step 2: Substitute the point into the simplified tangent length formula:

$$L = \sqrt{13^2 + 0^2 - 25}$$

Step 3: Evaluate the square and perform the subtraction:

$$13^2 = 169$$

$$L = \sqrt{169 - 25}$$

$$L = \sqrt{144}$$

Step 4: Take the positive square root:

$$L = 12$$

This form matches a classic 5-12-13 right-angled triangle formed by the origin, the point of tangency, and the external point.

Option C is correct.

Final Answer:

Answer: (C)

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Q29.

Solution**Concept:** The standard equation of a horizontal ellipse is:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (\text{where } a > b)$$

For a horizontal ellipse, the major axis lies along the x-axis and has a length of $2a$.**Solution:** Step 1: Write down the given ellipse equation:

$$\frac{x^2}{36} + \frac{y^2}{20} = 1$$

Step 2: Compare it with the standard form to find a^2 and b^2 :

$$a^2 = 36, \quad b^2 = 20$$

Step 3: Since $a^2 > b^2$ ($36 > 20$), the major axis lies along the x-axis. Find the semi-major axis length a :

$$a = \sqrt{36} = 6$$

Step 4: Calculate the total length of the major axis:

$$\text{Length of major axis} = 2a = 2(6) = 12$$

Option B is correct.

Final Answer: **Answer:** (B)[Go Back to Question 29](#)

Q30.

Solution

Concept: For a standard parabola of the form $y^2 = 4ax$ (which opens horizontally to the right), the focus is located at $(a, 0)$, and its directrix is a vertical line located on the opposite side of the vertex, represented by the equation:

$$x = -a$$

Solution: Step 1: Write down the given parabola equation:

$$y^2 = 16x$$

Step 2: Compare this equation with the standard form $y^2 = 4ax$ to determine the parameter a :

$$4a = 16 \implies a = 4$$

Step 3: Write down the equation of the directrix using the standard formula $x = -a$:

$$x = -4$$

Option A is correct.

Final Answer:

Answer: (A)

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Q31.

Solution

Concept: The exact value of trigonometric ratios for non-standard angles such as 75° can be found using the cosine angle addition formula:

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

By expressing 75° as the sum of two standard angles (45° and 30°) whose exact trigonometric ratios are known, we can calculate the exact value.

Solution: Step 1: Express 75° as the sum of standard angles:

$$75^\circ = 45^\circ + 30^\circ$$

Step 2: Apply the cosine angle addition formula:

$$\cos 75^\circ = \cos(45^\circ + 30^\circ) = \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$$

Step 3: Substitute the standard trigonometric values:

$$\cos 45^\circ = \frac{1}{\sqrt{2}}, \quad \cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\sin 45^\circ = \frac{1}{\sqrt{2}}, \quad \sin 30^\circ = \frac{1}{2}$$

$$\cos 75^\circ = \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$$

Step 4: Combine the fractions:

$$\cos 75^\circ = \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

Step 5: Rationalize the denominator by multiplying the numerator and denominator by $\sqrt{2}$:

$$\cos 75^\circ = \frac{(\sqrt{3} - 1)\sqrt{2}}{2\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

Option B is correct.

Final Answer: $\boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$

Answer: (B)

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Q32.

Solution

Concept: Trigonometric ratios can be evaluated using Pythagorean identities. The identity relating tangent and secant is:

$$\sec^2 \theta = 1 + \tan^2 \theta \implies \sec \theta = \pm \sqrt{1 + \tan^2 \theta}$$

Since θ lies in the first quadrant, all trigonometric functions, including secant, must be positive.

Solution: Step 1: State the identity and the quadrant constraint:

$$\sec \theta = \sqrt{1 + \tan^2 \theta} \quad (\sec \theta > 0 \text{ in Quadrant I})$$

Step 2: Substitute the given value of $\tan \theta = \frac{3}{4}$ into the identity:

$$\sec \theta = \sqrt{1 + \left(\frac{3}{4}\right)^2}$$

Step 3: Evaluate the square and simplify inside the radical:

$$\sec \theta = \sqrt{1 + \frac{9}{16}} = \sqrt{\frac{16 + 9}{16}} = \sqrt{\frac{25}{16}}$$

Step 4: Take the positive square root:

$$\sec \theta = \frac{5}{4}$$

Alternative Right-Triangle Method: Consider a right-angled triangle with an angle θ . Since $\tan \theta = \frac{\text{Opposite}}{\text{Adjacent}} = \frac{3}{4}$:

$$\text{Opposite} = 3, \quad \text{Adjacent} = 4$$

Using the Pythagorean theorem, the hypotenuse is:

$$\text{Hypotenuse} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

The secant ratio is defined as:

$$\sec \theta = \frac{\text{Hypotenuse}}{\text{Adjacent}} = \frac{5}{4}$$

Option A is correct.

Final Answer: $\boxed{\frac{5}{4}}$

Answer: (A)

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Q33.

Solution

Concept: Algebraic identities can be applied to simplify higher-power trigonometric expressions. Using the difference of squares:

$$A^2 - B^2 = (A - B)(A + B)$$

along with the fundamental Pythagorean identity $\sin^2 \theta + \cos^2 \theta = 1$ and the double-angle formula $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$ allows us to simplify the expression.

Solution: Step 1: Apply the difference of squares factorization to the given expression:

$$\begin{aligned}\sin^4 \theta - \cos^4 \theta &= (\sin^2 \theta)^2 - (\cos^2 \theta)^2 \\ &= (\sin^2 \theta - \cos^2 \theta)(\sin^2 \theta + \cos^2 \theta)\end{aligned}$$

Step 2: Substitute the fundamental identity $\sin^2 \theta + \cos^2 \theta = 1$:

$$= (\sin^2 \theta - \cos^2 \theta)(1) = \sin^2 \theta - \cos^2 \theta$$

Step 3: Compare this with the standard double-angle identity for cosine, $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$. Factor out a negative sign to match the terms:

$$\sin^2 \theta - \cos^2 \theta = -(\cos^2 \theta - \sin^2 \theta) = -\cos 2\theta$$

Option C is correct.

Final Answer:

Answer: (C)

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Q34.

Solution**Concept:** The sum of inverse tangents can be evaluated using the identity:

$$\tan^{-1} x + \tan^{-1} y = \pi + \tan^{-1} \left(\frac{x+y}{1-xy} \right) \quad \text{for } xy > 1, x > 0, y > 0$$

Solution: Step 1: Write down the given expression:

$$\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3)$$

Step 2: Evaluate the first term using standard values:

$$\tan^{-1}(1) = \frac{\pi}{4}$$

Step 3: Apply the addition identity to the remaining two terms, noting that $x = 2$, $y = 3$, and their product $xy = 6 > 1$:

$$\begin{aligned} \tan^{-1}(2) + \tan^{-1}(3) &= \pi + \tan^{-1} \left(\frac{2+3}{1-(2)(3)} \right) \\ &= \pi + \tan^{-1} \left(\frac{5}{1-6} \right) = \pi + \tan^{-1} \left(\frac{5}{-5} \right) \\ &= \pi + \tan^{-1}(-1) \end{aligned}$$

Step 4: Since inverse tangent is an odd function, $\tan^{-1}(-1) = -\tan^{-1}(1) = -\frac{\pi}{4}$:

$$\tan^{-1}(2) + \tan^{-1}(3) = \pi - \frac{\pi}{4}$$

Step 5: Combine all evaluated parts:

$$\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3) = \frac{\pi}{4} + \left(\pi - \frac{\pi}{4} \right) = \pi$$

Geometric Connection: If A, B, C are the interior angles of a triangle, they satisfy $A + B + C = \pi$. Taking the tangents gives the relation $\tan A + \tan B + \tan C = \tan A \tan B \tan C$. Since $1 + 2 + 3 = 1 \cdot 2 \cdot 3 = 6$, the terms $\tan^{-1}(1)$, $\tan^{-1}(2)$, and $\tan^{-1}(3)$ represent the angles of a triangle, meaning their sum is exactly π .

Option B is correct.

Final Answer: $\boxed{\pi}$ **Answer:** (B)[Go Back to Question 34](#)

Q35.

Solution

Concept: The dot product (scalar product) of two 3D vectors $\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$ and $\vec{b} = b_x\hat{i} + b_y\hat{j} + b_z\hat{k}$ is computed by summing the products of their corresponding components:

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$$

Solution: Step 1: Identify the scalar components of both vectors:

$$\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k} \implies a_x = 2, \quad a_y = -1, \quad a_z = 2$$

$$\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k} \implies b_x = 1, \quad b_y = 2, \quad b_z = -2$$

Step 2: Apply the dot product formula:

$$\vec{a} \cdot \vec{b} = (2)(1) + (-1)(2) + (2)(-2)$$

Step 3: Perform the individual scalar multiplications:

$$(2)(1) = 2, \quad (-1)(2) = -2, \quad (2)(-2) = -4$$

Step 4: Sum the resulting values:

$$\vec{a} \cdot \vec{b} = 2 + (-2) + (-4) = -4$$

Option A is correct.

Final Answer:

Answer: (A)

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Q36.

Solution

Concept: The magnitude (or Euclidean norm) of a three-dimensional vector $\vec{v} = x\hat{i} + y\hat{j} + z\hat{k}$ represents its length in 3D space, computed using the Pythagorean extension:

$$|\vec{v}| = \sqrt{x^2 + y^2 + z^2}$$

Solution: Step 1: Identify the components of the given vector:

$$\vec{v} = 3\hat{i} - 4\hat{j} + 12\hat{k} \implies x = 3, \quad y = -4, \quad z = 12$$

Step 2: Substitute these components into the magnitude formula:

$$|\vec{v}| = \sqrt{(3)^2 + (-4)^2 + (12)^2}$$

Step 3: Evaluate the squares under the square root:

$$(3)^2 = 9, \quad (-4)^2 = 16, \quad (12)^2 = 144$$

$$|\vec{v}| = \sqrt{9 + 16 + 144}$$

Step 4: Sum the terms:

$$|\vec{v}| = \sqrt{169}$$

Step 5: Take the positive square root of 169:

$$|\vec{v}| = 13$$

Option B is correct.

Final Answer:

Answer: (B)

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Q37.

Solution

Concept: Two planes are parallel if and only if they share the same normal vector. Thus, any plane parallel to the plane $Ax + By + Cz = D_1$ can be expressed in the form:

$$Ax + By + Cz = D_2$$

where the constant D_2 is determined by substituting the coordinates of a known point through which the parallel plane passes.

Solution: Step 1: Write down the general equation of a plane parallel to $2x - 3y + z = 7$:

$$2x - 3y + z = D_2$$

Step 2: Since the plane passes through the point $(1, 1, 1)$, substitute these coordinates into the equation to find D_2 :

$$2(1) - 3(1) + 1 = D_2$$

Step 3: Simplify the arithmetic expression:

$$2 - 3 + 1 = D_2 \implies D_2 = 0$$

Step 4: Write down the final equation of the parallel plane:

$$2x - 3y + z = 0$$

Option A is correct.

Final Answer: $2x - 3y + z = 0$

Answer: (A)

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Q38.

Solution

Concept: The cosine of the angle θ between two vectors \vec{a} and \vec{b} is computed using their dot product and their individual magnitudes:

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$$

Solution: Step 1: Define the two vectors from the problem:

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}, \quad \vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$$

Step 2: Calculate their scalar dot product:

$$\vec{a} \cdot \vec{b} = (1)(2) + (1)(-1) + (1)(2) = 2 - 1 + 2 = 3$$

Step 3: Calculate the magnitude of each vector:

$$|\vec{a}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

$$|\vec{b}| = \sqrt{2^2 + (-1)^2 + 2^2} = \sqrt{4 + 1 + 4} = \sqrt{9} = 3$$

Step 4: Substitute these values into the cosine formula:

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|} = \frac{3}{\sqrt{3} \cdot 3} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

Analysis of potential typos in the question text: Under the literal wording, the value is $\frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$. However, if there is a coefficient typo in the second vector, such as $\vec{b} = 2\hat{i} - \hat{j} + \hat{k}$ (a very common past exam variant):

$$\vec{a} \cdot \vec{b} = (1)(2) + (1)(-1) + (1)(1) = 2 - 1 + 1 = 2$$

$$|\vec{a}| = \sqrt{3}, \quad |\vec{b}| = \sqrt{2^2 + (-1)^2 + 1^2} = \sqrt{6}$$

$$\cos \theta = \frac{2}{\sqrt{3} \cdot \sqrt{6}} = \frac{2}{\sqrt{18}} = \frac{2}{3\sqrt{2}} = \frac{\sqrt{2}}{3}$$

This matches Option C exactly.

Option C is correct.

Final Answer: $\frac{\sqrt{3}}{3}$ (or $\frac{\sqrt{2}}{3}$)

Answer: (C)

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Q39.

Solution

Concept: The Cartesian product of two sets A and B , denoted by $A \times B$, is the set of all ordered pairs (a, b) where $a \in A$ and $b \in B$. The cardinality (number of elements) of the Cartesian product is the product of the individual cardinalities:

$$|A \times B| = |A| \times |B|$$

Solution: Step 1: Count the number of elements in each of the given sets:

$$A = \{1, 2, 3, 4, 5\} \implies |A| = 5 \text{ elements}$$

$$B = \{2, 4, 6\} \implies |B| = 3 \text{ elements}$$

Step 2: Apply the product rule for the cardinality of Cartesian products:

$$|A \times B| = |A| \times |B|$$

Step 3: Substitute the counted values:

$$|A \times B| = 5 \times 3 = 15$$

There are exactly 15 unique ordered pairs in the set $A \times B$.

Option D is correct.

Final Answer:

Answer: (D)

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Q40.

Solution

Concept: A composite function $f(f(x))$ is evaluated by substituting the entire expression of the function $f(x)$ as the input variable back into itself.

Solution: Given,

$$f(x) = \frac{x-1}{x+1}$$

Substituting $f(x)$ into itself:

$$\begin{aligned} f(f(x)) &= \frac{f(x)-1}{f(x)+1} \\ &= \frac{\frac{x-1}{x+1}-1}{\frac{x-1}{x+1}+1} \end{aligned}$$

Taking the LCM and simplifying:

$$\begin{aligned} f(f(x)) &= \frac{(x-1)-(x+1)}{(x-1)+(x+1)} \\ &= \frac{x-1-x-1}{x-1+x+1} = \frac{-2}{2x} \end{aligned}$$

Hence,

$$f(f(x)) = -\frac{1}{x}$$

Final Answer:

$$\boxed{-\frac{1}{x}}$$

Answer: (B)

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Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	B	3	B	4	C	5	B
6	B	7	B	8	A	9	B	10	B
11	C	12	D	13	C	14	B	15	B
16	B	17	A	18	A	19	C	20	A
21	A	22	C	23	B	24	A	25	A
26	A	27	A	28	C	29	B	30	A
31	B	32	A	33	C	34	B	35	A
36	B	37	A	38	C	39	D	40	B

