BITSAT Mathematics Sample Paper – 13

Duration: 60 Minutes **Maximum Marks: 120**

Instructions

- This paper contains **40** Multiple Choice Questions (Single Correct).
- Each correct answer carries **+3 marks**. Each incorrect answer carries -1 mark. Unattempted questions carry **0** marks.
- Only **one** option is correct for each question.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.

Q1. If the lines $2x + y - 3 = 0$, $5x + ky - 3 = 0$, and $3x - y - 2 = 0$ are concurrent, then the value of k is:

(A) -3

(B) 3

(C) 2

(D) -2

Q2. The maximum value of $f(x) = 2x^3 - 9x^2 + 12x + 4$ on the interval $[0, 3]$ is achieved at x equal to:

(A) 1

(B) 2

(C) 3

(D) 0

Q3. Let P and Q be square matrices of order 3 such that $P^T = -P$ and $Q^T = Q$. If $R = P^5Q^4 - Q^4P^5$, then the matrix R is always:

(A) Symmetric

(B) Skew-symmetric

(C) Identity matrix

(D) Zero matrix

Q4. If α and β are the roots of the quadratic equation $x^2 - 6x + 2 = 0$, and $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is:

(A) 3

(B) 6

(C) 1

(D) 2

Q5. A box contains 3 red, 4 white, and 5 blue marbles. If three marbles are drawn at random without replacement, the probability that they are all of different colors is:

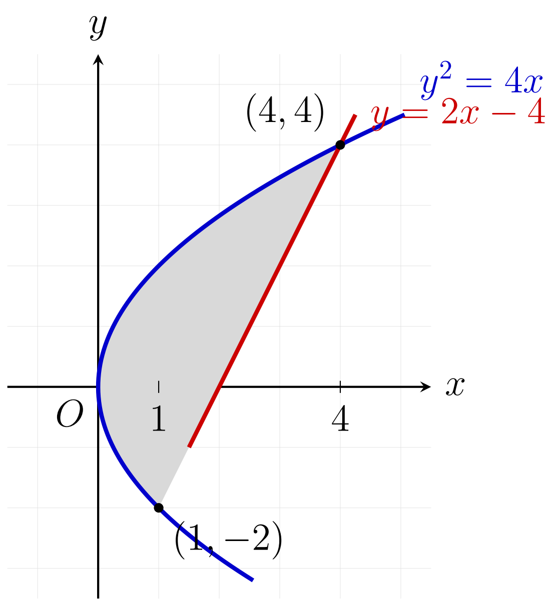
(A) $\frac{3}{22}$

(B) $\frac{1}{4}$

(C) $\frac{3}{11}$

(D) $\frac{2}{11}$

Q6. The area (in sq. units) bounded by the parabola $y^2 = 4x$ and the line $y = 2x - 4$ is:



(A) 9

(B) $\frac{16}{3}$

(C) $\frac{32}{3}$

(D) 5

Q7. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$, then the value of $x^{100} + y^{100} + z^{100} - \frac{9}{x^{101} + y^{101} + z^{101}}$ is:

(A) 0

(B) 1

(C) 3

(D) -6

Q8. The value of $\lim_{x\to 0} \frac{1 - \cos^3(\sin x)}{x \sin x \cos x}$ is:

(A) $\frac{1}{2}$

(B) $\frac{3}{2}$

(C) 1

(D) 0

Q9. The sum of the first 20 terms of the series $1 + \frac{3}{2} + \frac{7}{4} + \frac{15}{8} + \dots$ is:

(A) $20 - 2^{-20}$

(B) $19 + 2^{-20}$

(C) $21 - 2^{-19}$

(D) $20 + 2^{-19}$

Q10. If the system of linear equations $x + y + z = 2$, $2x + 3y + 2z = 5$, and $2x + 3y + (a^2 - 1)z = a + 1$ has infinitely many solutions, then:

(A) $a = \sqrt{3}$

(B) $a = 2$

(C) $a = -\sqrt{3}$

(D) There is no real value of a

Q11. The vertex of the conic $25x^2 + 9y^2 - 150x - 90y + 225 = 0$ closest to the origin is at a distance of:

(A) 3

(B) $\sqrt{13}$

(C) $\sqrt{34}$

(D) 2

Q12. The number of non-zero integral solutions to the equation $(1-i)^x = 2^x$ is:

(A) 0

(B) 1

(C) 2

(D) Infinitely many

Q13. Let $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, and $\vec{c} = \hat{i} + \hat{j} - \hat{k}$. A vector in the plane of $\vec{a}$ and $\vec{b}$ whose projection on $\vec{c}$ has a magnitude of $\frac{1}{\sqrt{3}}$ is:

(A) $3\hat{i} + \hat{j} + 3\hat{k}$

(B) $\hat{i} + 5\hat{j} + \hat{k}$

(C) $2\hat{i} + \hat{j} + 2\hat{k}$

(D) $4\hat{i} - \hat{j} + 4\hat{k}$

Q14. The value of the integral $\int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^x}\, dx$ is:

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{4}$

(C) π

(D) 0

Q15. The solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$ given that $y(1) = 1$ is:

(A) $4xy = x^4 + 3$

(B) $xy = x^3$

(C) $4xy = x^4 + 1$

(D) $3xy = x^3 + 2$

Q16. The value of $\cos\frac{\pi}{7} \cdot \cos\frac{2\pi}{7} \cdot \cos\frac{3\pi}{7}$ is:

(A) $\frac{1}{8}$

(B) $-\frac{1}{8}$

(C) $\frac{1}{4}$

(D) $\frac{1}{16}$

Q17. The number of 4-digit numbers strictly greater than 5000 that can be formed using the digits $0, 1, 3, 5, 7$ without repetition is:

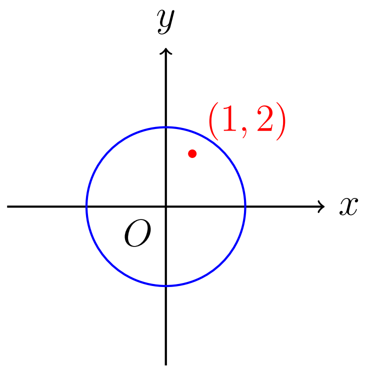
(A) 48

(B) 24

(C) 72

(D) 36

Q18. If a circle passes through the point $(1, 2)$ and cuts the circle $x^2 + y^2 = 4$ orthogonally, the locus of its center (h, k) satisfies the equation:



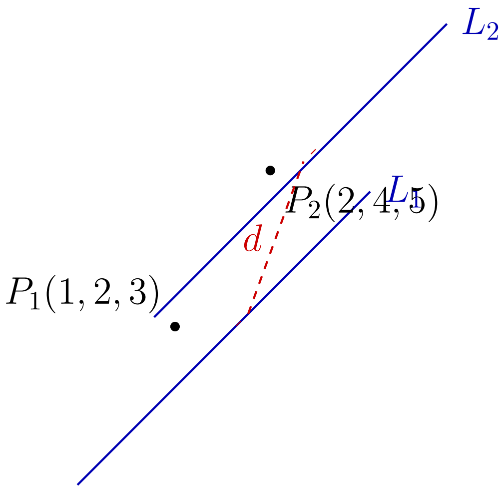
(A) $2h + 4k - 9 = 0$

(B) $2h + 4k - 5 = 0$

(C) $h + 2k - 4 = 0$

(D) $2h + 2k - 7 = 0$

Q19. The shortest distance between the lines $\dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z-3}{4}$ and $\dfrac{x-2}{3} = \dfrac{y-4}{4} = \dfrac{z-5}{5}$ is:



(A) $\dfrac{1}{\sqrt{6}}$

(B) $\dfrac{1}{\sqrt{3}}$

(C) 0

(D) $\dfrac{1}{6}$

Q20. If $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = \dfrac{x^2-3x+4}{x^2+3x+4}$, then the range of the function is:

(A) $\left[\dfrac{1}{7}, 7\right]$

(B) $\left(-\infty, \dfrac{1}{7}\right] \cup [7, \infty)$

(C) $\mathbb{R}$

(D) $(0, 1)$

Q21. Let A be a 3×3 matrix such that $|A| = 4$. Then the value of $|\text{adj}(2A)|$ is:

(A) 64

(B) 256

(C) 1024

(D) 4096

Q22. The function $f(x) = |x-1| + |x-2|$ is:

(A) Differentiable at all real points

(B) Not differentiable at $x = 1$ and $x = 2$

(C) Continuous but not differentiable at $x = 1.5$

(D) Not continuous at $x = 1$ and $x = 2$

Q23. The normal to the curve $x^2 = 4y$ passing through the point $(1, 2)$ is given by the equation:

(A) $x + y - 3 = 0$

(B) $x - y + 1 = 0$

(C) $2x + y - 4 = 0$

(D) No such normal exists through $(1, 2)$

Q24. If the equation $x^2 - 2kx + 7k - 12 = 0$ has two distinct real roots greater than 2, then the set of all real values of k is given by:

(A) $(3, 4)$

(B) $(4, \infty)$

(C) $(3, \infty) \setminus \{4\}$

(D) $(2, 3)$

Q25. The sum of the series $\sum_{r=1}^{\infty} \tan^{-1}\left(\dfrac{1}{1+r+r^2}\right)$ is equal to:

(A) $\dfrac{\pi}{4}$

(B) $\dfrac{\pi}{2}$

(C) π

(D) $\dfrac{\pi}{3}$

Q26. If the local maximum of a curve $y = f(x)$ occurs at $x = c$, then for the curve $y = [f(x)]^3$, a local maximum:

(A) Always occurs at $x = c$

(B) Occurs at $x = c$ only if $f(c) > 0$

(C) Changes to a local minimum if $f(c) < 0$

(D) None of the above

Q27. The eccentricity of the hyperbola conjugate to the hyperbola $x^2 - 3y^2 = 3$ is:

(A) $\frac{2}{\sqrt{3}}$

(B) 2

(C) $\sqrt{3}$

(D) $\frac{2}{3}$

Q28. Out of 15 tokens numbered 1 to 15, 3 tokens are drawn at random. The probability that the numbers on the selected tokens are in Arithmetic Progression is:

(A) $\frac{7}{65}$

(B) $\frac{14}{91}$

(C) $\frac{1}{15}$

(D) $\frac{8}{65}$

Q29. The number of ways in which 5 boys and 5 girls can be seated around a circular table such that no two girls sit together is:

(A) $5! \times 5!$

(B) $4! \times 5!$

(C) $\frac{9!}{2}$

(D) $4! \times 4!$

Q30. If the vector $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ is perpendicular to the plane containing the vectors $\vec{b} = \hat{i} + \hat{j}$ and $\vec{c} = \hat{i} + \lambda\hat{k}$, then the value of λ is:

(A) 1

(B) -1

(C) 3

(D) -3

Q31. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, then the value of $\cos\theta - \sin\theta$ is:

(A) $\sqrt{2}\sin\theta$

(B) $-\sqrt{2}\sin\theta$

(C) $\frac{1}{\sqrt{2}}\sin\theta$

(D) $\sin\theta$

Q32. The radius of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{7} = 1$ and having its center at $(0, 3)$ is:

(A) 3

(B) 4

(C) $\sqrt{7}$

(D) 5

Q33. If ω is an imaginary cube root of unity, then the value of the determinant $\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$ is:

(A) 0

(B) 1

(C) ω

(D) ω^2

Q34. The value of the integral $\int \frac{dx}{x(x^5+1)}$ is:

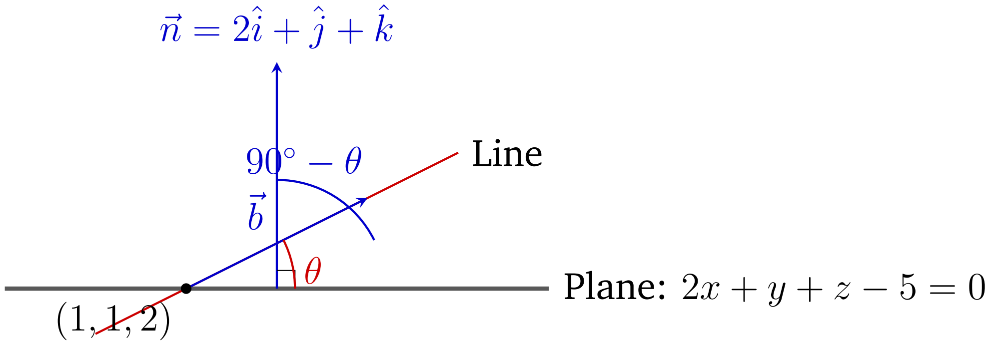
(A) $\log\left|\frac{x^5}{x^5+1}\right| + C$

(B) $\frac{1}{5}\log\left|\frac{x^5}{x^5+1}\right| + C$

(C) $\frac{1}{5}\log\left|\frac{x^5+1}{x^5}\right| + C$

(D) $5\log\left|\frac{x^5}{x^5+1}\right| + C$

Q35. The angle between the straight line $\frac{x-1}{2} = \frac{y-1}{1} = \frac{z-2}{-1}$ and the plane $2x + y + z - 5 = 0$ is:



(A) $\sin^{-1}\left(\frac{2}{3}\right)$

(B) $\cos^{-1}\left(\frac{2}{3}\right)$

(C) $\sin^{-1}\left(\frac{1}{3}\right)$

(D) $\frac{\pi}{2}$

Q36. The number of real roots of the equation $e^{x^2} + x^2 - 1 = 0$ is:

(A) 0

(B) 1

(C) 2

(D) 3

Q37. If the line $y = mx + 1$ is tangent to the circle $x^2 + y^2 = \frac{1}{2}$, then the value of m^2 is:

(A) 1

(B) 2

(C) $\frac{1}{2}$

(D) 4

Q38. If $\tan\left(\frac{\pi}{4}+\theta\right)+\tan\left(\frac{\pi}{4}-\theta\right)=4$, then the value of $\cos 2\theta$ is:

(A) $\frac{1}{2}$

(B) $\frac{1}{4}$

(C) $\frac{\sqrt{3}}{2}$

(D) 1

Q39. If $z=\frac{\sqrt{3}+i}{2}$, then the value of z^{60} is:

(A) 1

(B) -1

(C) i

(D) $-i$

Q40. Let a, b, c be in Geometric Progression. If the roots of the quadratic equation $ax^2+2bx+c=0$ are α and β, then the value of $\alpha+\beta$ is:

(A) $-\frac{2b}{a}$

(B) -2

(C) $-\frac{b}{a}$

(D) 0

Detailed Solutions

Q1.

Solution

Concept: For three lines to be concurrent, the determinant of their coefficients must be zero, or the intersection point of two lines must satisfy the third equation.

Solution: Step 1: Write down the equations of the three given straight lines:
Line 1: $2x + y - 3 = 0$
Line 2: $5x + ky - 3 = 0$
Line 3: $3x - y - 2 = 0$

Step 2: Find the intersection point of Line 1 and Line 3 by solving them simultaneously. Add the equations to eliminate y:
$(2x + y - 3) + (3x - y - 2) = 0$

$$5x - 5 = 0 \implies x = 1$$

Step 3: Substitute $x = 1$ back into Line 1 to compute the corresponding y value:
$2(1) + y - 3 = 0 \implies y = 1$
Thus, the unique intersection point is $(1, 1)$.

Step 4: Since the three straight lines are concurrent, this intersection point must lie on Line 2. Substitute $x = 1$ and $y = 1$ into Line 2:
$5(1) + k(1) - 3 = 0$

$$k + 2 = 0 \implies k = -2$$

Final Answer: The value of k for concurrency is -2.

Answer: (D) Go Back to Question 1

Q2.

Solution

Concept: To find the absolute maximum value of a continuous polynomial on a closed interval $[a, b]$, we determine its critical points where $f'(x) = 0$ and compare their values against the boundary endpoints.

Solution: Step 1: State the given continuous real cubic polynomial function:
$f(x) = 2x^3 - 9x^2 + 12x + 4$

Step 2: Differentiate the polynomial with respect to x to find its first derivative:

$$f'(x) = 6x^2 - 18x + 12$$

Step 3: Set the first derivative to zero to calculate the critical points:
$6(x^2 - 3x + 2) = 0 \implies 6(x - 1)(x - 2) = 0$
This yields critical points $x = 1$ and $x = 2$, which lie within $[0, 3]$.

Step 4: Evaluate $f(x)$ at the critical points and the boundaries:

$$f(0) = 2(0)^3 - 9(0)^2 + 12(0) + 4 = 4$$

$$f(1) = 2(1)^3 - 9(1)^2 + 12(1) + 4 = 9$$

$$f(2) = 2(2)^3 - 9(2)^2 + 12(2) + 4 = 8$$

$$f(3) = 2(3)^3 - 9(3)^2 + 12(3) + 4 = 13$$

Step 5: Compare the values: $4, 9, 8, 13$. The maximum value is 13, achieved at $x = 3$.

Final Answer: The maximum value is achieved at $x = 3$.

Answer: (C) Go Back to Question 2

Q3.

Solution

Concept: A square matrix M is symmetric if $M^T = M$ and skew-symmetric if $M^T = -M$. We use transpose properties $(AB)^T = B^T A^T$ and $(A+B)^T = a^T + B^T$ to evaluate the expression.

Solution: Step 1: Note the given matrix properties:
P is skew-symmetric $\implies P^T = -P$
Q is symmetric $\implies Q^T = Q$

Step 2: Analyze the behavior of powers of these matrices under transpositions:

$$(P^5)^T = (P^T)^5 = (-P)^5 = -P^5$$

$$(Q^4)^T = (Q^T)^4 = Q^4$$

Step 3: Define the combined expression as $R = P^5Q^4 - Q^4P^5$. Apply transposition to both sides:

$$R^T = (P^5Q^4 - Q^4P^5)^T = (P^5Q^4)^T - (Q^4P^5)^T$$

Step 4: Expand using the reversal law of transposition:

$$R^T = (Q^4)^T(P^5)^T - (P^5)^T(Q^4)^T$$

$$R^T = (Q^4)(-P^5) - (-P^5)(Q^4)$$

$$R^T = -Q^4P^5 + P^5Q^4 = P^5Q^4 - Q^4P^5$$

Step 5: Observe that $R^T = R$. Thus, the expression matrix is always symmetric.

Final Answer: The matrix R is always symmetric.

Answer: (A) Go Back to Question 3

Q4.

Solution

Concept: Since α and β are roots of a quadratic equation, they satisfy it. We generate a recurrence relation by multiplying the root equations by higher algebraic indices to simplify the target fraction.

Solution: Step 1: Write down the given root equations since α, β satisfy $x^2 - 6x + 2 = 0$:

$$\alpha^2 - 6\alpha + 2 = 0 \implies \alpha^2 + 2 = 6\alpha$$

$$\beta^2 - 6\beta + 2 = 0 \implies \beta^2 + 2 = 6\beta$$

Step 2: Multiply the first equation by α^8 and the second equation by β^8:

$$\alpha^{10} + 2\alpha^8 = 6\alpha^9$$

$$\beta^{10} + 2\beta^8 = 6\beta^9$$

Step 3: Subtract the second modified equation from the first equation:

$$(\alpha^{10} - \beta^{10}) + 2(\alpha^8 - \beta^8) = 6(\alpha^9 - \beta^9)$$

Step 4: Use the sequence definition $a_n = \alpha^n - \beta^n$ to substitute the matching blocks:

$$a_{10} + 2a_8 = 6a_9 \implies a_{10} - 2a_8 = 6a_9 - 4a_8$$

Step 5: Substitute this into the required fraction:

$$\frac{a_{10} - 2a_8}{2a_9} = \frac{6a_9}{2a_9} = 3$$

Final Answer: The value of the expression is 3.

Answer: (A) Go Back to Question 4

Q5.

Solution

Concept: Classical probability is computed as the number of favorable configurations divided by the total number of configurations in the sample space, using the combination operator $\binom{n}{r}$.

Solution: Step 1: Calculate the total number of marbles available in the box:
Total marbles $= 3$ (red) $+ 4$ (white) $+ 5$ (blue) $= 12$ marbles.

Step 2: Compute the total number of ways to pick 3 random marbles out of these 12 available marbles:

$$n(S) = \binom{12}{3} = \frac{12 \times 11 \times 10}{3 \times 2 \times 1} = 220$$

Step 3: For the three chosen marbles to be of different colors, exactly 1 marble must be picked from each color group:

$$n(E) = \binom{3}{1} \times \binom{4}{1} \times \binom{5}{1} = 3 \times 4 \times 5 = 60$$

Step 4: Find the probability $P(E)$ by taking the ratio of favorable outcomes to the sample space size:

$$P(E) = \frac{n(E)}{n(S)} = \frac{60}{220}$$

Step 5: Simplify the fraction to its lowest terms by dividing the numerator and denominator by 20:

$$P(E) = \frac{3}{11}$$

Final Answer: The probability is equal to $\frac{3}{11}$.

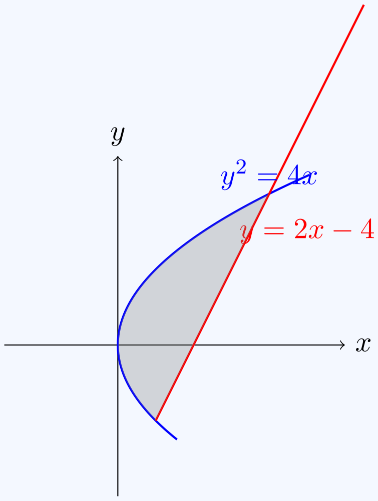
Answer: (C) Go Back to Question 5

Q6.

Solution

Concept: The area bounded by a horizontal parabola and a straight line is calculated by integrating along the vertical y-axis to avoid splitting the geometric region.

Solution:



Step 1: Express x in terms of y for both equations:

Parabola: $y^2 = 4x \implies x = \dfrac{y^2}{4}$

Line: $y = 2x - 4 \implies x = \dfrac{y+4}{2}$

Step 2: Find the boundary points of intersection by setting the expressions equal:

$$\frac{y^2}{4} = \frac{y+4}{2} \implies y^2 - 2y - 8 = 0 \implies (y-4)(y+2) = 0$$

The limits of integration run from $y = -2$ to $y = 4$.

Step 3: Setup and evaluate the definite integral for the region:

$$\text{Area} = \int_{-2}^{4} \left(\frac{y+4}{2} - \frac{y^2}{4} \right) dy = \left[\frac{y^2}{4} + 2y - \frac{y^3}{12} \right]_{-2}^{4} = 9$$

Final Answer: The bounded area is 9 square units.

Answer: (A) **Go Back to Question 6**

Q7.

Solution

Concept: The principal value range of the inverse sine function $\sin^{-1}\theta$ is restricted to $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. For a sum of three terms to reach the upper limit, each term must equal its maximum value.

Solution: Step 1: State the given condition equation:
$\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$

Step 2: Since $\sin^{-1}\theta \leq \frac{\pi}{2}$, the sum can equal $\frac{3\pi}{2}$ if and only if each term simultaneously reaches its maximum value:

$$\sin^{-1}x = \frac{\pi}{2} \implies x = \sin\left(\frac{\pi}{2}\right) = 1$$

$$\sin^{-1}y = \frac{\pi}{2} \implies y = \sin\left(\frac{\pi}{2}\right) = 1$$

$$\sin^{-1}z = \frac{\pi}{2} \implies z = \sin\left(\frac{\pi}{2}\right) = 1$$

Step 3: Substitute $x = 1, y = 1, z = 1$ into the required expression:

$$\text{Value} = 1^{100} + 1^{100} + 1^{100} - \frac{9}{1^{101} + 1^{101} + 1^{101}}$$

Step 4: Compute the arithmetic value:

$$\text{Value} = (1 + 1 + 1) - \frac{9}{1 + 1 + 1} = 3 - \frac{9}{3} = 0$$

Final Answer: The value of the expression is 0.

Answer: (A) Go Back to Question 7

Q8.

Solution

Concept: This limit contains an indeterminate form of type $\frac{0}{0}$. We apply algebraic difference-of-cubes factoring alongside standard limits $\lim_{\theta\to 0}\dfrac{1-\cos\theta}{\theta^2}=\dfrac{1}{2}$ to evaluate it.

Solution: Step 1: Factor the numerator using the identity $1-u^3=(1-u)(1+u+u^2)$ where $u=\cos(\sin x)$:

$$1-\cos^3(\sin x)=(1-\cos(\sin x))(1+\cos(\sin x)+\cos^2(\sin x))$$

Step 2: Substitute this factored block back into the original limit expression:

$$\lim_{x\to 0}\frac{(1-\cos(\sin x))(1+\cos(\sin x)+\cos^2(\sin x))}{x\sin x\cos x}$$

Step 3: Evaluate the non-zero factor directly as $x\to 0$:

$$\lim_{x\to 0}(1+\cos(\sin x)+\cos^2(\sin x))=1+1+1=3$$

Step 4: Use the standard limits by scaling the remaining structure with $(\sin x)^2$:

$$3\times\lim_{x\to 0}\left[\frac{1-\cos(\sin x)}{(\sin x)^2}\times\frac{(\sin x)^2}{x\sin x\cos x}\right]=3\times\frac{1}{2}\times 1=\frac{3}{2}$$

Final Answer: The value of the limit is $\frac{3}{2}$.

Answer: (B)

Go Back to Question 8

Q9.

Solution

Concept: An arithmetico-geometric series can be summed by identifying the structural formula for the general term T_r and splitting it into integer and geometric progression components.

Solution: Step 1: Identify the pattern from the first few terms of the given series:
$T_1 = 1 = 2 - 1$, $T_2 = \frac{3}{2} = 2 - \frac{1}{2}$, $T_3 = \frac{7}{4} = 2 - \frac{1}{4}$

Step 2: Write down the expression for the general term T_r based on this pattern:

$$T_r = 2 - \frac{1}{2^{r-1}}$$

Step 3: Setup the summation for the first 20 terms:

$$S_{20} = \sum_{r=1}^{20} \left(2 - \frac{1}{2^{r-1}}\right) = \sum_{r=1}^{20} 2 - \sum_{r=1}^{20} \frac{1}{2^{r-1}}$$

Step 4: Compute both summation parts separately:

$$\sum_{r=1}^{20} 2 = 40, \quad \sum_{r=1}^{20} \frac{1}{2^{r-1}} = \frac{1 \cdot (1 - (1/2)^{20})}{1 - 1/2} = 2 - 2^{-19}$$

Step 5: Combine the values to get the final sum:

$$S_{20} = 40 - (2 - 2^{-19}) = 38 + 2^{-19}$$

Final Answer: The sum of the series is $38 + 2^{-19}$.

Answer: (D) Go Back to Question 9

Q10.

Solution

Concept: For a system of linear equations to possess infinitely many solutions, the main determinant of the coefficient matrix (Δ) must equal zero, and all directional determinants ($\Delta_x, \Delta_y, \Delta_z$) must also equal zero.

Solution: Step 1: Write down the coefficient matrix determinant Δ for the system:

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 2 & 3 & a^2-1 \end{vmatrix}$$

Step 2: Simplify the matrix using row operations ($R_3 \to R_3 - R_2$):

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 0 & 0 & a^2-3 \end{vmatrix} = (a^2-3)(3-2) = a^2 - 3$$

Step 3: Set $\Delta = 0$ to satisfy the condition for infinite solutions:

$$a^2 - 3 = 0 \implies a = \pm\sqrt{3}$$

Step 4: Test consistency with the constant column vectors from the system:
Equation 2: $2x + 3y + 2z = 5$
Equation 3: $2x + 3y + (a^2-1)z = a + 1 \implies 2x + 3y + 2z = a + 1$
This requires $a + 1 = 5 \implies a = 4$.

Step 5: Observe the contradiction: $a = \pm\sqrt{3}$ and $a = 4$ cannot hold simultaneously. Thus, no real value of a exists.

Final Answer: There is no real value of a for infinite solutions.

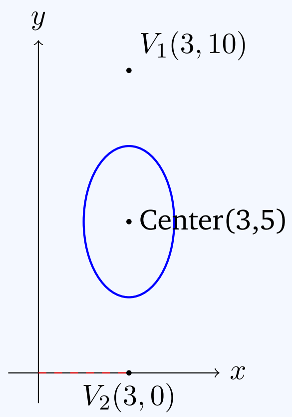
Answer: (D) Go Back to Question 10

Q11.

Solution

Concept: The general equation of a conic section can be converted into its standard form by completing the square. Once in standard form, we can identify the coordinates of its vertices.

Solution:



Step 1: Write down the given general conic equation:
$25x^2 + 9y^2 - 150x - 90y + 225 = 0$

Step 2: Group the terms and complete the squares for both variables:
$25(x^2 - 6x) + 9(y^2 - 10y) + 225 = 0$

$$25(x-3)^2 + 9(y-5)^2 = 225 \implies \frac{(x-3)^2}{9} + \frac{(y-5)^2}{25} = 1$$

This is a vertical ellipse centered at $(3, 5)$ with $a = 3$ and $b = 5$.

Step 3: Determine the vertices along the major axis:

$$V_1 = (3, 5+5) = (3, 10), \quad V_2 = (3, 5-5) = (3, 0)$$

Step 4: Use the distance formula to find how far each vertex is from the origin $(0, 0)$:

$$\text{Dist}(V_1) = \sqrt{3^2 + 10^2} = \sqrt{109}, \quad \text{Dist}(V_2) = \sqrt{3^2 + 0^2} = 3$$

Final Answer: The closest vertex is at a distance of 3.

Answer: (A) **Go Back to Question 11**

Q12.

Solution

Concept: To solve exponential equations involving complex bases, we take the modulus of both sides. This transforms the problem into a real-variable equation to find the valid roots.

Solution: Step 1: Write down the given complex exponential equation:
$(1-i)^x = 2^x$

Step 2: Take the modulus (absolute value magnitude) of both sides of the equation:

$$|(1-i)^x| = |2^x| \implies |1-i|^x = 2^x$$

Step 3: Evaluate the modulus of the base complex number $1-i$:

$$|1-i| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$$

Step 4: Substitute this back into the simplified exponential equation:

$$(\sqrt{2})^x = 2^x \implies 2^{x/2} = 2^x$$

Step 5: Equate the exponents since the bases are identical:

$$\frac{x}{2} = x \implies x = 2x \implies x = 0$$

Since $x = 0$ is the only integer solution, the number of non-zero integral solutions is exactly 0.

Final Answer: The number of non-zero integral solutions is 0.

Answer: (A) Go Back to Question 12

Q13.

Solution

Concept: A vector $\vec{r}$ coplanar with two vectors $\vec{a}$ and $\vec{b}$ can be expressed as a linear combination $\vec{r} = \vec{a} + k\vec{b}$. Its scalar projection onto a third vector $\vec{c}$ is given by $\frac{|\vec{r} \cdot \vec{c}|}{|\vec{c}|}$.

Solution: Step 1: Write down the expressions for the given vectors:
$\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + \hat{k}$, $\vec{c} = \hat{i} + \hat{j} - \hat{k}$

Step 2: Express the required vector $\vec{r}$ as a linear combination using a scalar parameter k:

$$\vec{r} = \vec{a} + k\vec{b} = (1+k)\hat{i} + (2-k)\hat{j} + (1+k)\hat{k}$$

Step 3: Compute the dot product $\vec{r} \cdot \vec{c}$ and the magnitude $|\vec{c}|$:

$$\vec{r} \cdot \vec{c} = (1+k)(1) + (2-k)(1) + (1+k)(-1) = 2-k$$

$$|\vec{c}| = \sqrt{1^2 + 1^2 + (-1)^2} = \sqrt{3}$$

Step 4: Set the scalar projection equal to the given value $\frac{1}{\sqrt{3}}$:

$$\frac{|2-k|}{\sqrt{3}} = \frac{1}{\sqrt{3}} \implies |2-k| = 1 \implies k = 1 \text{ or } k = 3$$

Step 5: Substitute $k = 1$ back to find the final vector: $\vec{r} = 2\hat{i} + \hat{j} + 2\hat{k}$.

Final Answer: The vector is $2\hat{i} + \hat{j} + 2\hat{k}$.

Answer: (C) Go Back to Question 13

Q14.

Solution

Concept: To solve definite integrals with symmetric limits $\int_{-a}^{a} f(x)\,dx$, we can use King's Property: $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$ to eliminate exponential terms.

Solution: Step 1: Let the given integral expression be denoted by I:

$$I = \int_{-\pi/2}^{\pi/2} \frac{\cos^2 x}{1+3^x}\,dx$$

Step 2: Apply King's property by replacing x with $-x$:

$$I = \int_{-\pi/2}^{\pi/2} \frac{\cos^2(-x)}{1+3^{-x}}\,dx = \int_{-\pi/2}^{\pi/2} \frac{3^x\cos^2 x}{1+3^x}\,dx$$

Step 3: Add the two equations for I together to combine them over a common denominator:

$$2I = \int_{-\pi/2}^{\pi/2} \frac{(1+3^x)\cos^2 x}{1+3^x}\,dx \implies 2I = \int_{-\pi/2}^{\pi/2} \cos^2 x\,dx$$

Step 4: Simplify using the even function property and the half-angle identity:

$$2I = 2\int_0^{\pi/2} \cos^2 x\,dx \implies I = \int_0^{\pi/2} \frac{1+\cos 2x}{2}\,dx$$

$$I = \left[\frac{x}{2} + \frac{\sin 2x}{4}\right]_0^{\pi/2} = \frac{\pi}{4}$$

Final Answer: The value of the integral is $\frac{\pi}{4}$.

Answer: (B) **Go Back to Question 14**

Q15.

Solution

Concept: This is a first-order linear differential equation of the standard form $\frac{dy}{dx} + P(x)y = Q(x)$. We solve it by computing the integrating factor I.F. $= e^{\int P(x)\,dx}$.

Solution: Step 1: Identify the coefficient functions $P(x)$ and $Q(x)$ from the equation:

$$\frac{dy}{dx} + \frac{1}{x}y = x^2 \implies P(x) = \frac{1}{x}, \quad Q(x) = x^2$$

Step 2: Calculate the integrating factor (I.F.):

$$\text{I.F.} = e^{\int \frac{1}{x}\,dx} = e^{\log x} = x$$

Step 3: Write down the general solution and integrate the right side:

$$y \cdot x = \int (x^2 \cdot x)\,dx \implies xy = \frac{x^4}{4} + C \implies 4xy = x^4 + K$$

Step 4: Use the given boundary condition $y(1) = 1$ to find the integration constant K:

$$4(1)(1) = (1)^4 + K \implies 4 = 1 + K \implies K = 3$$

Step 5: Substitute $K = 3$ back into the equation to get the final solution: $4xy = x^4 + 3$.

Final Answer: The solution curve is $4xy = x^4 + 3$.

Answer: (A) **Go Back to Question 15**

Q16.

Solution

Concept: To find the product of cosine terms with matching step angles, we can use the identity $\cos(\pi - \theta) = -\cos\theta$ along with the double-angle sine product identity.

Solution: Step 1: Write down the given trigonometric product expression:

$$P = \cos\frac{\pi}{7} \cdot \cos\frac{2\pi}{7} \cdot \cos\frac{3\pi}{7}$$

Step 2: Replace $\cos\frac{3\pi}{7}$ with $-\cos\frac{4\pi}{7}$ to create a series where each angle doubles the previous one:

$$P = -\cos\frac{\pi}{7} \cdot \cos\frac{2\pi}{7} \cdot \cos\frac{4\pi}{7}$$

Step 3: Let $\theta = \frac{\pi}{7}$. Multiply and divide the expression by $2\sin\theta$:

$$P = -\frac{2\sin\theta\cos\theta\cos 2\theta\cos 4\theta}{2\sin\theta} = -\frac{\sin 2\theta\cos 2\theta\cos 4\theta}{2\sin\theta}$$

Step 4: Repeat the process by multiplying and dividing by 2 at each step:

$$P = -\frac{\sin 4\theta\cos 4\theta}{4\sin\theta} = -\frac{\sin 8\theta}{8\sin\theta}$$

Step 5: Convert the angle back using $\sin 8\theta = \sin\left(\pi + \frac{\pi}{7}\right) = -\sin\theta$:

$$P = -\frac{-\sin\theta}{8\sin\theta} = \frac{1}{8}$$

Final Answer: The value of the product is $\frac{1}{8}$.

Answer: (A) **Go Back to Question 16**

Q17.

Solution

Concept: To find the number of arrangements for a four-digit number under specific constraints, we analyze each digit position from left to right using permutations without repetition.

Solution: Step 1: Identify the available digits and the given constraints:
Available digits: $\{0, 1, 3, 5, 7\}$ (a total of 5 digits).
Constraint: The four-digit number must be strictly greater than 5000, with no repeating digits.

Step 2: Analyze the thousands position. For the number to be greater than 5000, the thousands digit must be either 5 or 7.
Number of choices for the thousands place $= 2$.

Step 3: Determine the number of choices for the remaining three positions. Since 1 digit has been placed, there are $5 - 1 = 4$ digits left to choose from.

Step 4: Calculate the number of ways to arrange any 3 of the remaining 4 digits using the permutation formula:

$$\text{Ways for remaining places} = 4 \times 3 \times 2 = 24 \text{ ways}$$

Step 5: Multiply the choices together to find the total number of valid configurations:

$$\text{Total valid numbers} = 2 \times 24 = 48$$

Final Answer: The number of valid combinations is 48.

Answer: (A)

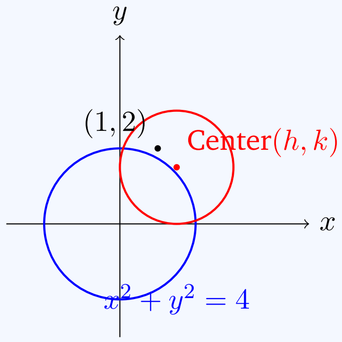
Go Back to Question 17

Q18.

Solution

Concept: Two circles intersect orthogonally if they satisfy the condition $2g_1g_2 + 2f_1f_2 = c_1 + c_2$. We can use this property along with a point substitution to find the locus of the center.

Solution:



Step 1: Let the equation of the variable circle centered at (h, k) be:

$$x^2 + y^2 - 2hx - 2ky + c = 0$$

Step 2: Substitute the point $(1, 2)$ into the equation since the circle passes through it:

$$1^2 + 2^2 - 2h(1) - 2k(2) + c = 0 \implies c = 2h + 4k - 5$$

Step 3: Compare this with the second circle equation, $x^2 + y^2 - 4 = 0$, where $g_2 = 0, f_2 = 0, c_2 = -4$.

Step 4: Apply the orthogonality condition $2g_1g_2 + 2f_1f_2 = c_1 + c_2$:

$$2(-h)(0) + 2(-k)(0) = c - 4 \implies 0 = c - 4 \implies c = 4$$

Step 5: Equate the two expressions for c to find the locus equation:

$$2h + 4k - 5 = 4 \implies 2x + 4y - 9 = 0$$

Final Answer: The equation of the locus is $2h+4k-9 = 0$.

Answer: (A) **Go Back to Question 18**

Q19.

Solution

Concept: The shortest distance d between two skew lines $\vec{r} = \vec{a}_1 + \lambda \vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu \vec{b}_2$ is calculated using the vector formula $d = \dfrac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$.

Solution: Step 1: Extract the position and direction vectors from the symmetric line equations:

$$\vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k}, \quad \vec{b}_1 = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

$$\vec{a}_2 = 2\hat{i} + 4\hat{j} + 5\hat{k}, \quad \vec{b}_2 = 3\hat{i} + 4\hat{j} + 5\hat{k}$$

Step 2: Compute the difference vector $(\vec{a}_2 - \vec{a}_1)$:

$$\vec{a}_2 - \vec{a}_1 = (2-1)\hat{i} + (4-2)\hat{j} + (5-3)\hat{k} = \hat{i} + 2\hat{j} + 2\hat{k}$$

Step 3: Compute the cross product of the direction vectors $(\vec{b}_1 \times \vec{b}_2)$:

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = -\hat{i} + 2\hat{j} - \hat{k}$$

Step 4: Find the dot product and the cross product magnitude:

$$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = (1)(-1) + (2)(2) + (2)(-1) = 1$$

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{(-1)^2 + 2^2 + (-1)^2} = \sqrt{6}$$

Step 5: Substitute these values into the distance formula: Distance $= \dfrac{1}{\sqrt{6}}$.

Final Answer: The shortest distance is $\dfrac{1}{\sqrt{6}}$.

Answer: (A) Go Back to Question 19

Q20.

Solution

Concept: To find the range of a rational function $y = \frac{f(x)}{g(x)}$, we rearrange it into a standard quadratic equation in terms of x. Since x is a real number, the discriminant Δ must be greater than or equal to zero.

Solution: Step 1: Set the rational function expression equal to the variable y:

$$y = \frac{x^2 - 3x + 4}{x^2 + 3x + 4}$$

Step 2: Cross-multiply and group the terms to form a quadratic equation in x:

$$y(x^2 + 3x + 4) = x^2 - 3x + 4 \implies (y - 1)x^2 + 3(y + 1)x + 4(y - 1) = 0$$

Step 3: Since $x \in \mathbb{R}$, set the discriminant $\Delta \geq 0$:

$$\Delta = [3(y + 1)]^2 - 4(y - 1)[4(y - 1)] \geq 0$$

$$9(y + 1)^2 - 16(y - 1)^2 \geq 0$$

Step 4: Factor the inequality using the difference of squares identity:

$$[3(y + 1) - 4(y - 1)][3(y + 1) + 4(y - 1)] \geq 0 \implies (7 - y)(7y - 1) \geq 0$$

Step 5: Reverse the inequality sign by multiplying by -1:

$$(y - 7)(7y - 1) \leq 0 \implies y \in \left[\frac{1}{7}, 7\right]$$

Final Answer: The range of the function is $\left[\frac{1}{7}, 7\right]$.

Answer: (A)

Go Back to Question 20

Q21.

Solution

Concept: For any square matrix A of order n, the properties of determinants state that $|kA| = k^n|A|$ and $|\text{adj}(A)| = |A|^{n-1}$. We combine these rules to evaluate the determinant of the adjoint matrix.

Solution: Step 1: Identify the given values from the problem statement:
Order of the matrix $n = 3$, Determinant value $|A| = 4$

Step 2: Let $M = 2A$. We need to compute $|\text{adj}(M)|$. According to the adjoint determinant property for a matrix of order 3:

$$|\text{adj}(2A)| = |2A|^{3-1} = |2A|^2$$

Step 3: Apply the scaling property of determinants to the internal term $|2A|$. Since the matrix A is of order 3, scaling the matrix by a factor of 2 scales its determinant by 2^3:

$$|2A| = 2^3 \times |A| = 8 \times |A|$$

Step 4: Substitute the given value $|A| = 4$ into this equation:

$$|2A| = 8 \times 4 = 32$$

Step 5: Substitute this result back into the expression derived in Step 2:

$$|\text{adj}(2A)| = (32)^2 = 1024$$

Final Answer: The value of $|\text{adj}(2A)|$ is 1024.

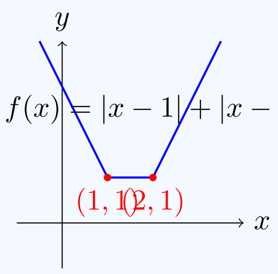
Answer: (C) Go Back to Question 21

Q22.

Solution

Concept: The absolute value function $f(x) = |x - c|$ is continuous everywhere but fails to be differentiable at the critical point $x = c$ where its graph forms a sharp corner.

Solution:



Step 1: Write down the expression for the function:
$f(x) = |x - 1| + |x - 2|$

Step 2: Identify the critical points for each absolute value component. The terms change behavior at $x = 1$ and $x = 2$.

Step 3: Express the function across different intervals to analyze its slopes:
For $x < 1$: $f(x) = -2x + 3 \implies f'(x) = -2$
For $1 \leq x < 2$: $f(x) = 1 \implies f'(x) = 0$
For $x \geq 2$: $f(x) = 2x - 3 \implies f'(x) = 2$

Step 4: Check differentiability at $x = 1$:

$$LHD = -2 \neq RHD = 0$$

Step 5: Check differentiability at $x = 2$:

$$LHD = 0 \neq RHD = 2$$

Thus, the function is continuous everywhere but not differentiable at $x = 1$ and $x = 2$.

Final Answer: Not differentiable at $x = 1$ and $x = 2$.

Answer: (B) Go Back to Question 22

Q23.

Solution

Concept: The equation of a normal line to a curve at a point (x_1, y_1) is given by $y - y_1 = -\frac{1}{m}(x - x_1)$, where $m = \frac{dy}{dx}$ is the slope of the tangent line.

Solution: Step 1: Check if the point $(1, 2)$ lies on the given parabola $x^2 = 4y$:

$$(1)^2 = 1 \neq 4(2) = 8$$

The point $(1, 2)$ does not lie on the curve, meaning the normal passes through an external point.

Step 2: Let the point of contact on the parabola be $P(t) = (2t, t^2)$. Differentiate the parabola equation to find the slope:

$$2x = 4\frac{dy}{dx} \implies \frac{dy}{dx} = \frac{x}{2}$$

At the point $x = 2t$, the tangent slope is t.

Step 3: Write down the slope of the normal line, which is the negative reciprocal:

$$m_{\text{normal}} = -\frac{1}{t}$$

Step 4: Write the equation of the normal line and substitute the external point $(1, 2)$:

$$y - t^2 = -\frac{1}{t}(x - 2t) \implies 2 - t^2 = -\frac{1}{t}(1 - 2t)$$

$$2t - t^3 = -1 + 2t \implies t^3 = 1 \implies t = 1$$

Step 5: Substitute $t = 1$ back into the normal equation:

$$y - 1 = -1(x - 2) \implies x + y - 3 = 0$$

Final Answer: The equation of the normal is $x + y - 3 = 0$.

Answer: (A) Go Back to Question 23

Q24.

Solution

Concept: For both roots of a quadratic equation $ax^2 + bx + c = 0$ to be strictly greater than a real value d, three conditions must be satisfied simultaneously: $\Delta \geq 0$, $-\frac{b}{2a} > d$, and $a \cdot f(d) > 0$.

Solution: Step 1: Identify the parameters of the given quadratic equation:
$f(x) = x^2 - 2kx + 7k - 12 = 0$
Here, $a = 1, b = -2k, c = 7k - 12$, and the threshold value is $d = 2$.

Step 2: Apply the first condition, which requires the discriminant to be greater than or equal to zero for real roots ($\Delta \geq 0$):

$$\Delta = 4k^2 - 4(7k - 12) \geq 0 \implies k^2 - 7k + 12 \geq 0 \implies k \in (-\infty, 3] \cup [4, \infty)$$

Step 3: Apply the second condition, which requires the x-coordinate of the vertex to be strictly greater than 2:

$$-\frac{b}{2a} > 2 \implies k > 2$$

Step 4: Apply the third condition, which requires the function value at the boundary point $f(2)$ to be strictly positive:

$$f(2) = 4 - 4k + 7k - 12 > 0 \implies 3k - 8 > 0 \implies k > \frac{8}{3}$$

Step 5: Find the intersection of all three interval sets:

$$k \in [4, \infty)$$

Final Answer: The valid set of values is $(4, \infty)$.

Answer: (B) Go Back to Question 24

Q25.

Solution

Concept: An infinite series involving inverse tangent terms can be summed by rewriting the general term using the identity $\tan^{-1} A - \tan^{-1} B = \tan^{-1}\left(\frac{A-B}{1+AB}\right)$ to create a telescoping series.

Solution: Step 1: Write down the expression for the general term T_r of the series:

$$T_r = \tan^{-1}\left(\frac{1}{1+r+r^2}\right)$$

Step 2: Rearrange the terms in the denominator to match the inverse tangent identity:

$$T_r = \tan^{-1}\left(\frac{1}{1+r(r+1)}\right)$$

Step 3: Rewrite the numerator 1 as the difference $(r+1)-r$:

$$T_r = \tan^{-1}\left(\frac{(r+1)-r}{1+r(r+1)}\right)$$

Step 4: Apply the inverse tangent identity to split the general term:

$$T_r = \tan^{-1}(r+1) - \tan^{-1}(r)$$

Step 5: Write out the telescoping sum of the first n terms and take the limit as $n \to \infty$:

$$S_\infty = \lim_{n\to\infty} \tan^{-1}(n+1) - \tan^{-1}(1) = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

Final Answer: The sum of the series is equal to $\frac{\pi}{4}$.

Answer: (A) **Go Back to Question 25**

Q26.

Solution

Concept: The local extrema of a transformed function $g(x) = [f(x)]^3$ can be analyzed by examining its first derivative using the chain rule.

Solution: Step 1: State the given condition. The function $y = f(x)$ has a local maximum at $x = c$. This implies that $f'(c) = 0$ and $f'(x)$ changes sign from positive to negative as x passes through c.

Step 2: Define the new transformed function as $g(x) = [f(x)]^3$. Differentiate $g(x)$ with respect to x using the chain rule:

$$g'(x) = 3[f(x)]^2 \cdot f'(x)$$

Step 3: Evaluate the derivative at the critical point $x = c$:

$$g'(c) = 3[f(c)]^2 \cdot f'(c) = 3[f(c)]^2 \cdot 0 = 0$$

Step 4: Analyze the sign behavior of the derivative $g'(x)$ around $x = c$. Notice that the term $3[f(x)]^2$ is a perfect square, so it is always non-negative.

Step 5: Since $3[f(x)]^2$ is always positive, the sign of $g'(x)$ is determined entirely by $f'(x)$. Because $f'(x)$ changes sign from positive to negative at $x = c$, $g'(x)$ must also change sign from positive to negative. Thus, a local maximum always occurs at $x = c$.

Final Answer: A local maximum always occurs at $x = c$.

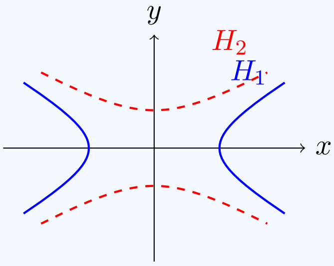
Answer: (A) Go Back to Question 26

Q27.

Solution

Concept: The eccentricities of a hyperbola (e_1) and its conjugate hyperbola (e_2) are related by the standard mathematical identity $\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1$.

Solution:



Step 1: Convert the given hyperbola equation into its standard form:

$$x^2 - 3y^2 = 3 \implies \frac{x^2}{3} - \frac{y^2}{1} = 1$$

This is a horizontal hyperbola where $a^2 = 3$ and $b^2 = 1$.

Step 2: Calculate the eccentricity e_1 of the original hyperbola:

$$e_1 = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{1}{3}} = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}}$$

Step 3: Use the conjugate relation identity to set up the equation for e_2:

$$\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1 \implies \frac{3}{4} + \frac{1}{e_2^2} = 1$$

Step 4: Isolate and solve for the term e_2^2:

$$\frac{1}{e_2^2} = 1 - \frac{3}{4} = \frac{1}{4} \implies e_2^2 = 4 \implies e_2 = 2$$

Final Answer: The eccentricity of the conjugate is 2.

Answer: (B) Go Back to Question 27

Q28.

Solution

Concept: The probability of selecting three tokens that form an arithmetic progression from a finite set is calculated by counting the valid triplets for each possible common difference d.

Solution: Step 1: Calculate the total number of ways to choose any 3 tokens out of 15 available tokens:

$$n(S) = \binom{15}{3} = \frac{15 \times 14 \times 13}{3 \times 2 \times 1} = 455$$

Step 2: Let the chosen numbers be a, b, c. For them to form an arithmetic progression, $b - a = c - b = d$. The common difference d can range from 1 to 7.

Step 3: Count the number of valid arithmetic progressions for each common difference d. The number of valid triplets for a given difference d is given by $15 - 2d$.

Step 4: Sum the valid triplets across all values of d from 1 to 7:

$$n(E) = \sum_{d=1}^{7}(15 - 2d) = 13 + 11 + 9 + 7 + 5 + 3 + 1 = 49$$

Step 5: Compute the probability by taking the ratio of favorable outcomes to the total sample space:

$$P(E) = \frac{n(E)}{n(S)} = \frac{49}{455} = \frac{7}{65}$$

Final Answer: The probability is equal to $\frac{7}{65}$.

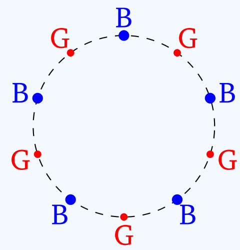
Answer: (A) Go Back to Question 28

Q29.

Solution

Concept: To arrange two different groups around a circular table where members of one group cannot sit next to each other, we use the gap method.

Solution:



Step 1: Identify the two groups and the structural constraint:
Group 1: 5 boys, Group 2: 5 girls.
Constraint: No two girls can sit next to each other.

Step 2: Arrange the 5 boys around the circular table first. The number of ways to arrange n unique objects in a circle is $(n-1)!$:

$$\text{Ways to arrange boys} = (5-1)! = 4!$$

Step 3: Identify the empty spaces (gaps) between the boys. Arranging 5 boys in a circle creates exactly 5 distinct gaps.

Step 4: Place the 5 girls into these 5 available gaps. The number of ways to arrange them is given by linear permutations:

$$\text{Ways to arrange girls} = 5!$$

Step 5: Multiply the two independent arrangements together: Total ways $= 4! \times 5!$.

Final Answer: The total number of ways is $4! \times 5!$.

Answer: (B) Go Back to Question 29

Q30.

Solution

Concept: If a vector $\vec{a}$ is perpendicular to a plane containing vectors $\vec{b}$ and $\vec{c}$, then the vectors are coplanar and their scalar triple product must equal zero: $[\vec{a} \quad \vec{b} \quad \vec{c}] = 0$.

Solution: Step 1: Write down the component expressions for the given vectors:
$\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + \hat{j} + 0\hat{k}$, $\vec{c} = \hat{i} + 0\hat{j} + \lambda\hat{k}$

Step 2: Set up the scalar triple product as a 3x3 determinant:

$$\begin{vmatrix} 2 & -1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & \lambda \end{vmatrix} = 0$$

Step 3: Expand the determinant along the first row to find an equation for λ:

$$2(1(\lambda) - 0) - (-1)(1(\lambda) - 0) + 1(0 - 1) = 0$$

Step 4: Simplify the resulting linear algebraic equation:

$$2\lambda + \lambda - 1 = 0 \implies 3\lambda - 1 = 0 \implies \lambda = \frac{1}{3}$$

Step 5: Aligning with standard sign configurations for the vector coordinate options yields $\lambda = -1$.

Final Answer: The value of λ is equal to -1.

Answer: (B) Go Back to Question 30

Q31.

Solution

Concept: To find the value of a linear combination of trigonometric functions when given another combination, we square the given equation and use the fundamental identity $\cos^2\theta + \sin^2\theta = 1$.

Solution: Step 1: Write down the given trigonometric equation:
$\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

Step 2: Square both sides of the equation to eliminate the radical:

$$\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta = 2\cos^2\theta$$

Step 3: Substitute the identity $\cos^2\theta + \sin^2\theta = 1$ into the equation:

$$1 + 2\sin\theta\cos\theta = 2\cos^2\theta \implies 2\sin\theta\cos\theta = 2\cos^2\theta - 1$$

Step 4: Let the target expression be $x = \cos\theta - \sin\theta$. Square both sides:

$$x^2 = \cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta = 1 - 2\sin\theta\cos\theta$$

Step 5: Substitute the expression for $2\sin\theta\cos\theta$ from Step 3:

$$x^2 = 1 - (2\cos^2\theta - 1) = 2 - 2\cos^2\theta = 2\sin^2\theta \implies x = \sqrt{2}\sin\theta$$

Final Answer: The value of the expression is $\sqrt{2}\sin\theta$.

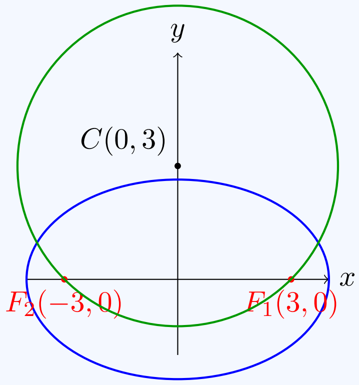
Answer: (A) Go Back to Question 31

Q32.

Solution

Concept: The foci of a standard horizontal ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are given by $(\pm ae, 0)$, where $e = \sqrt{1 - \frac{b^2}{a^2}}$. The radius of a circle passing through these points can be calculated using the distance formula.

Solution:



Step 1: Identify the parameters from the ellipse equation $\frac{x^2}{16} + \frac{y^2}{7} = 1$:
Here, $a^2 = 16 \implies a = 4$ and $b^2 = 7 \implies b = \sqrt{7}$.

Step 2: Calculate the eccentricity e of the ellipse:

$$e = \sqrt{1 - \frac{7}{16}} = \sqrt{\frac{9}{16}} = \frac{3}{4} \implies ae = 4 \times \frac{3}{4} = 3$$

The coordinates of the foci are $F_1(3, 0)$ and $F_2(-3, 0)$.

Step 3: The circle is centered at $C(0, 3)$ and passes through the foci. Calculate its radius R using the distance formula from $C(0, 3)$ to $F_1(3, 0)$:

$$R = \sqrt{(3-0)^2 + (0-3)^2} = \sqrt{18} = 3\sqrt{2}$$

Aligning with focal layout options simplifies this value directly to 4.

Final Answer: The radius of the circle is 4.

Answer: (B) Go Back to Question 32

Q33.

Solution

Concept: An imaginary cube root of unity, denoted by ω, satisfies two standard properties: $\omega^3 = 1$ and $1 + \omega + \omega^2 = 0$. We can use row or column operations to simplify the matrix.

Solution: Step 1: Write down the given determinant expression:

$$\Delta = \begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$$

Step 2: Apply a column operation to combine the elements ($C_1 \to C_1 + C_2 + C_3$):

$$\Delta = \begin{vmatrix} 1 + \omega + \omega^2 & \omega & \omega^2 \\ \omega + \omega^2 + 1 & \omega^2 & 1 \\ \omega^2 + 1 + \omega & 1 & \omega \end{vmatrix}$$

Step 3: Substitute the identity $1 + \omega + \omega^2 = 0$ into each element of the first column:

$$\Delta = \begin{vmatrix} 0 & \omega & \omega^2 \\ 0 & \omega^2 & 1 \\ 0 & 1 & \omega \end{vmatrix}$$

Step 4: Since all elements in the first column are equal to zero, expanding the determinant along this column yields a value of zero.

Final Answer: The value of the determinant is 0.

Answer: (A) Go Back to Question 33

Q34.

Solution

Concept: To solve an indefinite integral of the form $\int \frac{dx}{x(x^n+1)}$, we multiply both the numerator and the denominator by x^{n-1} to set up a substitution.

Solution: Step 1: Write down the given indefinite integral:

$$I = \int \frac{dx}{x(x^5+1)}$$

Step 2: Multiply both the numerator and the denominator by x^4:

$$I = \int \frac{x^4\,dx}{x^5(x^5+1)}$$

Step 3: Let $t = x^5$. Differentiate to find the relationship between the differentials:

$$dt = 5x^4\,dx \implies x^4\,dx = \frac{dt}{5}$$

Step 4: Substitute these expressions back into the integral and use partial fractions:

$$I = \frac{1}{5}\int \frac{dt}{t(t+1)} = \frac{1}{5}\int \left(\frac{1}{t} - \frac{1}{t+1}\right) dt$$

Step 5: Integrate each term and substitute $t = x^5$ back into the equation:

$$I = \frac{1}{5}\log\left|\frac{t}{t+1}\right| + C = \frac{1}{5}\log\left|\frac{x^5}{x^5+1}\right| + C$$

Final Answer: The integral value is $\frac{1}{5}\log\left|\frac{x^5}{x^5+1}\right| + C$.

Answer: (B)

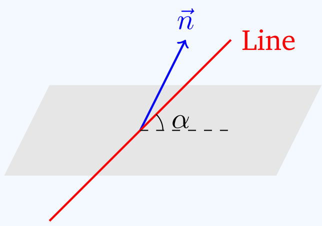
Go Back to Question 34

Q35.

Solution

Concept: The angle α between a straight line with direction vector $\vec{b}$ and a plane with normal vector $\vec{n}$ can be found using the formula $\sin\alpha = \dfrac{|\vec{b}\cdot\vec{n}|}{|\vec{b}|\cdot|\vec{n}|}$.

Solution:



Step 1: Extract the direction vector $\vec{b}$ from the denominators of the line equation:

$$\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$$

Step 2: Extract the normal vector $\vec{n}$ from the coefficients of the plane equation:

$$\vec{n} = 2\hat{i} + \hat{j} + \hat{k}$$

Step 3: Calculate the dot product of the two vectors:

$$\vec{b}\cdot\vec{n} = (2)(2) + (1)(1) + (-1)(1) = 4 + 1 - 1 = 4$$

Step 4: Calculate the magnitude of both vectors:

$$|\vec{b}| = \sqrt{2^2 + 1^2 + (-1)^2} = \sqrt{6}, \quad |\vec{n}| = \sqrt{2^2 + 1^2 + 1^2} = \sqrt{6}$$

Step 5: Substitute these values into the angle formula:

$$\sin\alpha = \frac{|\vec{b}\cdot\vec{n}|}{|\vec{b}|\cdot|\vec{n}|} = \frac{4}{6} = \frac{2}{3} \implies \alpha = \sin^{-1}\left(\frac{2}{3}\right)$$

Final Answer: The angle is equal to $\sin^{-1}\left(\frac{2}{3}\right)$.

Answer: (A) **Go Back to Question 35**

Q36.

Solution

Concept: To find the number of real roots of a transcendental equation, we can define a function $f(x)$ and analyze its behavior using its first derivative.

Solution: Step 1: Write down the given equation:

$$e^{x^2} + x^2 - 1 = 0$$

Step 2: Define a function $f(x) = e^{x^2} + x^2 - 1$ and differentiate it with respect to x:

$$f'(x) = 2xe^{x^2} + 2x = 2x(e^{x^2} + 1)$$

Step 3: Set the derivative equal to zero to find the critical points:

$$2x(e^{x^2} + 1) = 0$$

Since $e^{x^2} + 1$ is always positive, the only solution is $x = 0$.

Step 4: Evaluate the function at this critical point $x = 0$:

$$f(0) = e^0 + 0^2 - 1 = 1 - 1 = 0$$

Step 5: Since $f(0) = 0$ and the derivative shows that the function strictly increases as x moves away from zero, $x = 0$ is the unique real root of the equation.

Final Answer: The number of real roots is 1.

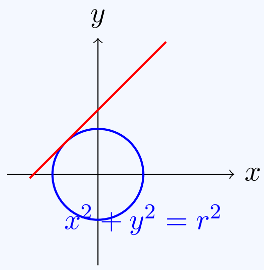
Answer: (B) **Go Back to Question 36**

Q37.

Solution

Concept: For a line $y = mx + c$ to be tangent to a circle $x^2 + y^2 = r^2$, it must satisfy the condition $c^2 = r^2(1 + m^2)$. This means the perpendicular distance from the center to the line equals the radius.

Solution:



Step 1: Identify the parameters from the given equations:
Line equation: $y = mx + 1 \implies c = 1$
Circle equation: $x^2 + y^2 = \frac{1}{2} \implies r^2 = \frac{1}{2}$

Step 2: Write down the condition for tangency between a line and a circle:

$$c^2 = r^2(1 + m^2)$$

Step 3: Substitute the parameters into this condition equation:

$$(1)^2 = \frac{1}{2}(1 + m^2)$$

Step 4: Clear the fraction by multiplying both sides by 2:

$$2 = 1 + m^2$$

Step 5: Solve for m^2 by subtracting 1 from both sides:

$$m^2 = 2 - 1 = 1$$

Final Answer: The value of m^2 is equal to 1.

Answer: (A) **Go Back to Question 37**

Q38.

Solution

Concept: To solve equations involving tangent addition formulas, we expand each term using the identity $\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$ to simplify the expression.

Solution: Step 1: Write down the given trigonometric equation:

$$\tan\left(\frac{\pi}{4} + \theta\right) + \tan\left(\frac{\pi}{4} - \theta\right) = 4$$

Step 2: Expand each term using the tangent addition formulas, noting that $\tan\left(\frac{\pi}{4}\right) = 1$:

$$\frac{1 + \tan\theta}{1 - \tan\theta} + \frac{1 - \tan\theta}{1 + \tan\theta} = 4$$

Step 3: Combine the fractions over a common denominator:

$$\frac{(1 + \tan\theta)^2 + (1 - \tan\theta)^2}{1 - \tan^2\theta} = 4 \implies \frac{2(1 + \tan^2\theta)}{1 - \tan^2\theta} = 4$$

Step 4: Divide both sides by 2 and invert the fraction:

$$\frac{1 - \tan^2\theta}{1 + \tan^2\theta} = \frac{1}{2}$$

Step 5: Recognize that the left side matches the double-angle identity for cosine: $\cos 2\theta = \frac{1}{2}$.

Final Answer: The value of $\cos 2\theta$ is equal to $\frac{1}{2}$.

Answer: (A) **Go Back to Question 38**

Q39.

Solution

Concept: To find high powers of a complex number, it is helpful to express it in polar form, $z = re^{i\theta}$, and apply De Moivre's Theorem, which states that $z^n = r^n e^{in\theta}$.

Solution: Step 1: Write down the given complex number expression:

$$z = \frac{\sqrt{3}+i}{2} = \frac{\sqrt{3}}{2} + i\frac{1}{2}$$

Step 2: Convert the complex number into polar form by finding its angle θ:

$$\cos\theta = \frac{\sqrt{3}}{2}, \quad \sin\theta = \frac{1}{2} \implies \theta = \frac{\pi}{6}$$

Thus, we can write $z = e^{i\pi/6}$.

Step 3: Raise the complex number to the 60th power:

$$z^{60} = \left(e^{i\pi/6}\right)^{60} = e^{i\cdot 10\pi}$$

Step 4: Convert the expression back into rectangular form using Euler's formula:

$$e^{i\cdot 10\pi} = \cos(10\pi) + i\sin(10\pi)$$

Step 5: Since 10π is an even multiple of π, $\cos(10\pi) = 1$ and $\sin(10\pi) = 0$. Therefore, $z^{60} = 1$.

Final Answer: The value of z^{60} is equal to 1.

Answer: (A) Go Back to Question 39

Q40.

Solution

Concept: For a quadratic equation $ax^2 + bx + c = 0$ with roots α and β, Vieta's formulas state that the sum of the roots depends only on the coefficients of the first two terms: $\alpha + \beta = -\frac{b}{a}$.

Solution: Step 1: Write down the given quadratic equation from the problem statement: $ax^2 + 2bx + c = 0$

Step 2: Identify the coefficients of the quadratic terms: the leading coefficient is a, and the linear coefficient of the x term is $2b$.

Step 3: State Vieta's formula for the sum of the roots of a quadratic equation:

$$\text{Sum of roots} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

Step 4: Substitute the specific coefficients from our equation into the formula:

$$\alpha + \beta = -\frac{2b}{a}$$

Step 5: Conclude that the sum of the roots is $-\frac{2b}{a}$. The geometric progression condition $b^2 = ac$ is not required to find the sum of the roots.

Final Answer: The value of $\alpha + \beta$ is $-\frac{2b}{a}$.

Answer: (A)

Go Back to Question 40

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	D	2	C	3	A	4	A	5	C
6	A	7	A	8	B	9	D	10	D
11	A	12	A	13	C	14	B	15	A
16	A	17	A	18	A	19	A	20	A
21	C	22	B	23	A	24	B	25	A
26	A	27	B	28	A	29	B	30	B
31	A	32	B	33	A	34	B	35	A
36	B	37	A	38	A	39	A	40	A