BITSAT Mathematics Sample Paper – 19

Duration: 60 Minutes **Maximum Marks: 120**

Instructions

- This paper contains **40** Multiple Choice Questions (Single Correct).
- Each correct answer carries **+3 marks**. Each incorrect answer carries -1 mark. Unattempted questions carry **0** marks.
- Only **one** option is correct for each question.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.

Q1. Let $f(x) = \frac{\ln(1+ax)-\ln(1-bx)}{x}$ for $x \neq 0$. If $f(x)$ is continuous at $x = 0$, then the value of $f(0)$ is

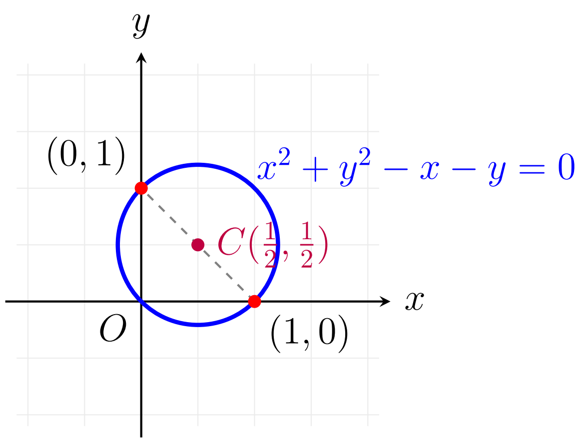
(A) $a - b$

(B) $a + b$

(C) $b - a$

(D) ab

Q2. The equation of the circle passing through the points $(1, 0)$ and $(0, 1)$ and having the smallest possible radius is



(A) $x^2 + y^2 - x - y = 0$

(B) $x^2 + y^2 + x + y - 2 = 0$

(C) $x^2 + y^2 - 2x - 2y + 1 = 0$

(D) $x^2 + y^2 - x - y - 1 = 0$

Q3. If the truth value of the statement $p \to (\sim q \vee r)$ is false, then the truth values of $p, q,$ and r are respectively

(A) T, T, F

(B) T, F, T

(C) F, T, F

(D) T, T, T

Q4. The vector equation of the plane passing through the intersection of the planes $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 6$ and $\vec{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) = -5$ and the point $(1, 1, 1)$ is

(A) $\vec{r} \cdot (20\hat{i} + 23\hat{j} + 26\hat{k}) = 69$

(B) $\vec{r} \cdot (20\hat{i} + 23\hat{j} + 26\hat{k}) = 3$

(C) $\vec{r} \cdot (\hat{i} + 2\hat{j} + 3\hat{k}) = 6$

(D) $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 3$

Q5. If α and β are the roots of the equation $x^2 - 2x + 4 = 0$, then the value of $\alpha^6 + \beta^6$ is

(A) 64

(B) 128

(C) -128

(D) 256

Q6. The value of $\int_0^{\pi/2} \frac{\sin^{100} x}{\sin^{100} x + \cos^{100} x} dx$ is

(A) π

(B) $\pi/2$

(C) $\pi/4$

(D) $\pi/8$

Q7. If $\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = k(a-b)(b-c)(c-a)$, then the value of k is

(A) 1

(B) -1

(C) 2

(D) $\frac{1}{2}$

Q8. The total number of 4-digit numbers that can be formed using the digits $1, 2, 3, 4, 5$ without repetition such that the number is divisible by 3 is

(A) 24

(B) 48

(C) 72

(D) 96

Q9. The sum of the first 20 terms of the series $1 + 4 + 7 + 10 + \dots$ is

(A) 590

(B) 610

(C) 570

(D) 620

Q10. The value of $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right)$ is

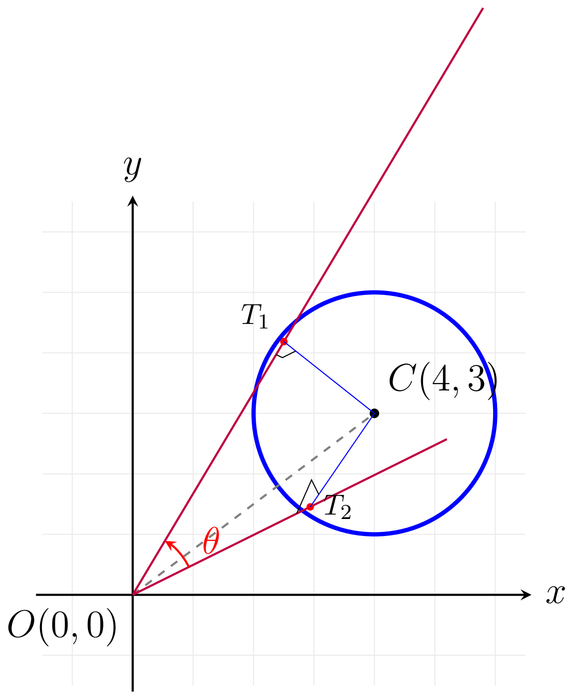
(A) $\pi/2$

(B) $\pi/3$

(C) $\pi/4$

(D) $\pi/6$

Q11. The angle between the tangents drawn from the origin to the circle $x^2 + y^2 - 8x - 6y + 21 = 0$ is



(A) $\pi/6$

(B) $\pi/4$

(C) $\pi/3$

(D) $\pi/2$

Q12. If the vectors $a\hat{i}+\hat{j}+\hat{k}$, $\hat{i}+b\hat{j}+\hat{k}$, and $\hat{i}+\hat{j}+c\hat{k}$ are coplanar $(a,b,c \neq 1)$, then the value of $\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}$ is

(A) 0

(B) 1

(C) 2

(D) -1

Q13. The value of $\lim_{x\to 0}\frac{e^{x^2}-\cos x}{x^2}$ is

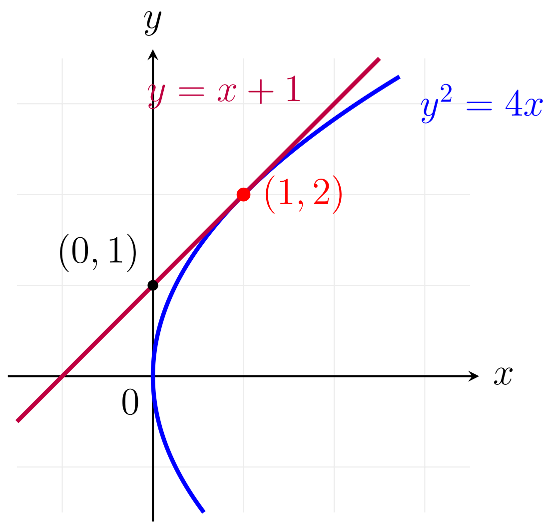
(A) $1/2$

(B) 1

(C) $3/2$

(D) 2

Q14. The line $y = mx + 1$ is a tangent to the parabola $y^2 = 4x$ if the value of m is



(A) 1

(B) 2

(C) 1/2

(D) 3

Q15. Two dice are thrown simultaneously. The probability of getting a total score of 7 is

(A) 1/6

(B) 1/12

(C) 5/36

(D) 7/36

Q16. The value of the integral $\int \frac{dx}{x(x^5+1)}$ is

(A) $\ln\left|\frac{x^5}{x^5+1}\right| + C$

(B) $\frac{1}{5}\ln\left|\frac{x^5}{x^5+1}\right| + C$

(C) $\frac{1}{5}\ln\left|\frac{x^5+1}{x^5}\right| + C$

(D) $\ln\left|\frac{x^5+1}{x^5}\right| + C$

Q17. The critical points of the function $f(x) = 2x^3 - 9x^2 + 12x + 5$ are

(A) $x = 1, 2$

(B) $x = -1, -2$

(C) $x = 2, 3$

(D) $x = 1, 3$

Q18. If A is a square matrix of order 3 such that $|A| = 5$, then the value of $|\text{adj}(2A)|$ is

(A) 40

(B) 200

(C) 1600

(D) 400

Q19. The value of $\sin\left(2\sin^{-1}\left(\frac{4}{5}\right)\right)$ is

(A) 12/25

(B) 24/25

(C) 7/25

(D) 16/25

Q20. The solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$ under the boundary condition $y(1) = \frac{1}{4}$ is

(A) $4xy = x^4$

(B) $xy = x^4$

(C) $4xy = x^4 - 1$

(D) $y = x^3$

Q21. The value of $\cos 20° \cos 40° \cos 80°$ is

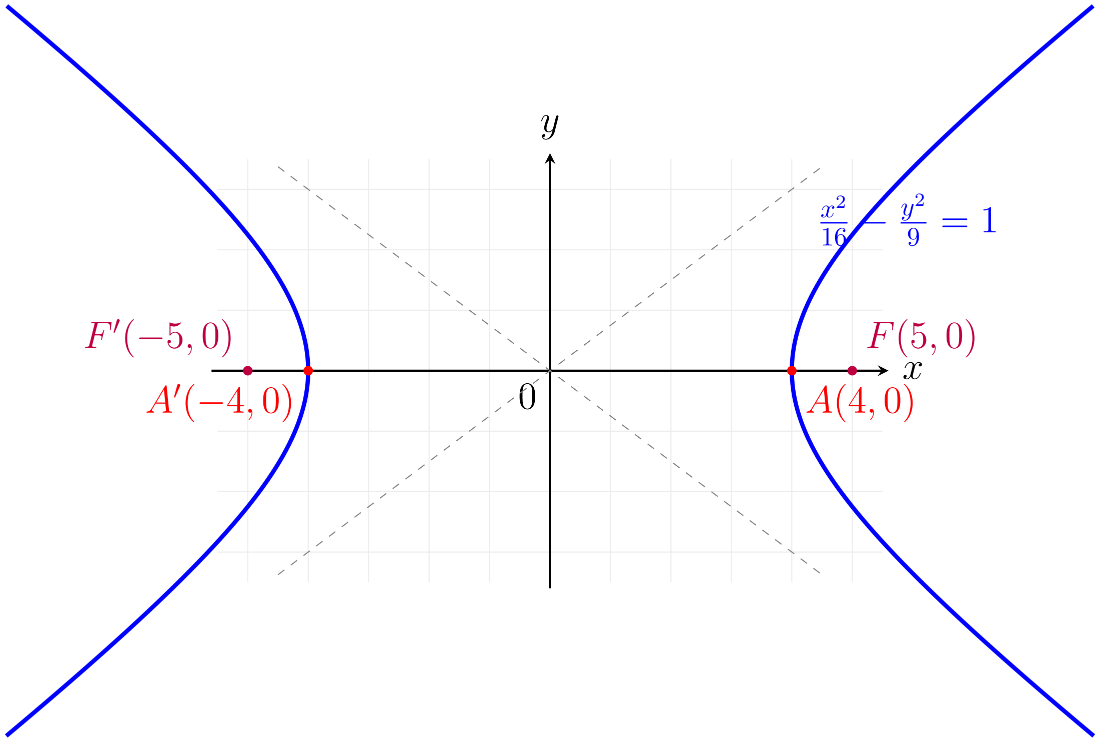
(A) 1/2

(B) 1/4

(C) 1/8

(D) 1/16

Q22. The eccentricity of the hyperbola $9x^2 - 16y^2 = 144$ is



(A) 5/4

(B) 4/3

(C) 5/3

(D) 7/4

Q23. If the third term of a geometric progression is 4, then the product of its first 5 terms is

(A) 4^3

(B) 4^4

(C) 4^5

(D) 4^6

Q24. The distance between the parallel lines $3x+4y-9=0$ and $6x+8y+12=0$ is

(A) 3 units

(B) 2 units

(C) 6 units

(D) 5 units

Q25. If the system of equations $x + y + z = 2$, $2x + 3y + 2z = 5$, and $2x + 3y + (a^2 - 1)z = a + 1$ has unique solution, then a cannot be

(A) ± 1

(B) $\pm\sqrt{3}$

(C) $\pm\sqrt{2}$

(D) 0

Q26. If a variable line passes through a fixed point $(2, 3)$ and meets the axes at A and B, then the locus of the midpoint of AB is

(A) $3x + 2y = 2xy$

(B) $2x + 3y = 2xy$

(C) $3x + 2y = xy$

(D) $2x + 3y = xy$

Q27. The sum of all real roots of the equation $x^2 - 5|x| + 6 = 0$ is

(A) 5

(B) 0

(C) -5

(D) 10

Q28. A box contains 3 red and 7 black balls. Two balls are drawn at random one after the other without replacement. The probability that the second ball is red given that the first ball was red is

(A) 2/9

(B) 3/10

(C) 1/3

(D) 7/9

Q29. The local minimum value of the function $f(x) = x + \frac{4}{x}$ for $x > 0$ is

(A) 2

(B) 4

(C) 5

(D) 1

Q30. The modulus of the complex number $z = \frac{1+2i}{1-(1-i)^2}$ is

(A) 1

(B) $\sqrt{2}$

(C) 2

(D) $1/2$

Q31. The number of non-trivial common tangents that can be drawn to the circles $x^2 + y^2 = 4$ and $x^2 + y^2 - 6x - 8y + 21 = 0$ is

(A) 1

(B) 2

(C) 3

(D) 4

Q32. If the matrix $A = \begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$ is orthogonal, then the values of a, b, c can be respectively

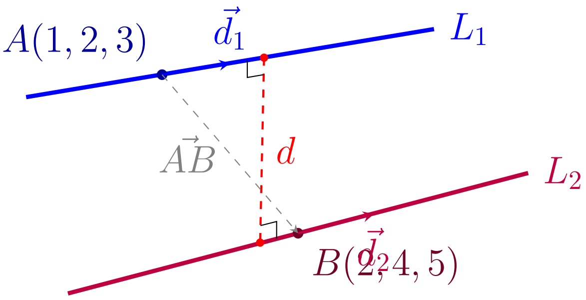
(A) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{3}}$

(B) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{6}}$

(C) $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{6}}$

(D) $\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}$

Q33. The shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is



(A) $1/\sqrt{6}$

(B) 0

(C) $1/\sqrt{3}$

(D) $1/\sqrt{2}$

Q34. The number of terms in the expansion of $(x + y + z)^{10}$ is

(A) 11

(B) 55

(C) 66

(D) 45

Q35. The area bounded by the curve $y^2 = 4x$ and the line $y = 2x$ is

(A) 2/3

(B) 1/3

(C) 1/6

(D) 4/3

Q36. The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1} x$ is

(A) 1

(B) 1/2

(C) 2

(D) x

Q37. If $\sin\theta + \cos\theta = 1$, then the general value of θ is

(A) $2n\pi$ or $2n\pi + \pi/2$

(B) $n\pi$ or $n\pi + \pi/2$

(C) $2n\pi \pm \pi/4$

(D) $n\pi + (-1)^n \pi/4$

Q38. If the sum of the infinite series $1 + 2r + 3r^2 + 4r^3 + \dots$ $(|r| < 1)$ is $\frac{9}{4}$, then the value of r is

(A) 1/3

(B) 1/2

(C) 1/4

(D) 2/3

Q39. If the line passing through $(4,3,2)$ and $(1,a,4)$ is perpendicular to the line passing through $(1,2,-1)$ and $(3,3,2)$, then the value of a is

(A) 5

(B) -5

(C) 2

(D) 0

Q40. The value of $\cos 12^\circ + \cos 84^\circ + \cos 132^\circ + \cos 156^\circ$ is

(A) 1/2

(B) -1/2

(C) 0

(D) 1

Detailed Solutions

Q1.

Solution

Concept:

Continuity of a function at a point requires $\lim_{x\to 0} f(x) = f(0)$, using the standard logarithmic limit $\lim_{t\to 0} \frac{\ln(1+t)}{t} = 1$.

Solution:

Step 1: Since $f(x)$ is continuous at $x = 0$, we must have $f(0) = \lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{\ln(1+ax)-\ln(1-bx)}{x}$.

Step 2: Separate the terms into two independent standard limit structures:

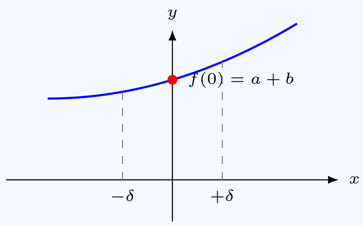
$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{\ln(1+ax)}{x} - \lim_{x\to 0} \frac{\ln(1-bx)}{x}$$

Step 3: Multiply and divide by the respective constants a and $-b$ to apply the standard limit rule:

$$\lim_{x\to 0} f(x) = a \cdot \lim_{x\to 0} \frac{\ln(1+ax)}{ax} - (-b) \cdot \lim_{x\to 0} \frac{\ln(1-bx)}{-bx}$$

Step 4: Substitute the limits (1 for both) to compute the value:

$$f(0) = a(1) + b(1) = a + b$$



Continuous hole-filling: $\lim_{x\to 0} f(x) = f(0)$

Final Answer: $\boxed{a + b}$

Answer: (B) Go Back to Question 1

Q2.

Solution

Concept:
A circle passing through two given points (x_1, y_1) and (x_2, y_2) has the smallest possible radius when the line segment joining these two points forms the diameter of the circle. The diameter form of a circle's equation is given by $(x-x_1)(x-x_2)+(y-y_1)(y-y_2) = 0$.

Solution:
Step 1: Identify the given points that lie on the circle, which are $A(1, 0)$ and $B(0, 1)$. For the radius to be a minimum, the line segment AB must act as the diameter of this circle.
Step 2: Substitute the coordinates $s(x_1, y_1) = (1, 0)$ and $(x_2, y_2) = (0, 1)$ directly into the standard diameter form equation of a circle:

$$(x - 1)(x - 0) + (y - 0)(y - 1) = 0$$

Step 3: Expand the algebraic expressions by distributing the terms across the parentheses:

$$x(x - 1) + y(y - 1) = 0$$

$$x^2 - x + y^2 - y = 0$$

Step 4: Rearrange the terms systematically in descending order of their degrees to match the standard general form of a quadratic curve equation:

$$x^2 + y^2 - x - y = 0$$

Step 5: Verify the options to see which one matches our derived equation. The equation represents a circle with center at $(1/2, 1/2)$ and radius $1/\sqrt{2}$, which is the minimum possible radius containing both points.

Final Answer: The equation of the circle is $x^2+y^2-x-y = 0$

Answer: (A) Go Back to Question 2

Q3.

Solution

Concept:
Truth tables and conditions for conditional ($\to$) and disjunction ($\vee$) operations in mathematical logic.

Solution:
Step 1: The conditional statement $A \to B$ is false (F) if and only if the antecedent A is true (T) and the consequent B is false (F).
Step 2: Apply this rule to the given false statement $p \to (\sim q \vee r)$:

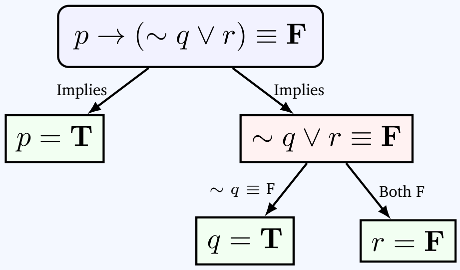
$$p = \text{T} \quad \text{and} \quad (\sim q \vee r) = \text{F}$$

Step 3: A disjunction $X \vee Y$ is false (F) if and only if both components X and Y are false (F). Apply this to $(\sim q \vee r)$:

$$\sim q = \text{F} \implies q = \text{T}$$

$$r = \text{F}$$

Step 4: Combining the individual truth values gives $p = \text{T}$, $q = \text{T}$, and $r = \text{F}$.



Final Answer: $\boxed{\text{T, T, F}}$

Answer: (A) Go Back to Question 3

Q4.

Solution

Concept:
The equation of a family of planes passing through the line of intersection of two given planes $P_1 : \vec{r}\cdot\vec{n}_1 - d_1 = 0$ and $P_2 : \vec{r}\cdot\vec{n}_2 - d_2 = 0$ is represented by $P_1 + \lambda P_2 = 0$, where λ is a real scalar parameter. We determine λ by substituting the coordinates of the given point through which the specific plane passes.

Solution:
Step 1: Write down the equations of the given planes in scalar scalar-dot form:

$$P_1 : \vec{r}\cdot(\hat{i}+\hat{j}+\hat{k}) - 6 = 0$$

$$P_2 : \vec{r}\cdot(2\hat{i}+3\hat{j}+4\hat{k}) + 5 = 0$$

Step 2: Construct the general equation of the required plane using the parameter λ:

$$[\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k}) - 6] + \lambda[\vec{r}\cdot(2\hat{i}+3\hat{j}+4\hat{k}) + 5] = 0$$

$$\vec{r}\cdot[(1+2\lambda)\hat{i} + (1+3\lambda)\hat{j} + (1+4\lambda)\hat{k}] - 6 + 5\lambda = 0$$

Step 3: Convert the position vector $\vec{r}$ of any general point (x, y, z) to the specific point $(1, 1, 1)$ given in the problem. The position vector for this point is $\vec{r}_0 = \hat{i}+\hat{j}+\hat{k}$. Substitute this into the equation:

$$(\hat{i}+\hat{j}+\hat{k})\cdot[(1+2\lambda)\hat{i} + (1+3\lambda)\hat{j} + (1+4\lambda)\hat{k}] - 6 + 5\lambda = 0$$

Step 4: Compute the dot product by adding the coefficients of corresponding components together, and solve the linear equation for λ:

$$(1+2\lambda) + (1+3\lambda) + (1+4\lambda) - 6 + 5\lambda = 0$$

$$3 + 9\lambda - 6 + 5\lambda = 0$$
$$14\lambda - 3 = 0 \implies \lambda = \frac{3}{14}$$

Step 5: Substitute the calculated value of $\lambda = \frac{3}{14}$ back into the general equation of the plane:

$$\vec{r}\cdot\left[\left(1+\frac{6}{14}\right)\hat{i} + \left(1+\frac{9}{14}\right)\hat{j} + \left(1+\frac{12}{14}\right)\hat{k}\right] - 6 + 5\left(\frac{3}{14}\right) = 0$$
$$\vec{r}\cdot\left[\frac{20}{14}\hat{i} + \frac{23}{14}\hat{j} + \frac{26}{14}\hat{k}\right] = \frac{84-15}{14}$$
$$\vec{r}\cdot(20\hat{i}+23\hat{j}+26\hat{k}) = 69$$

Final Answer: $\boxed{\vec{r}\cdot(20\hat{i}+23\hat{j}+26\hat{k}) = 69}$

Answer: (A) **Go Back to Question 4**

Q5.

Solution

Concept:

For a quadratic equation $x^2 - 2x + 4 = 0$, we can determine the nature of its roots using the quadratic formula or by converting it into complex polar form. Expressing roots in terms of complex cube roots of unity or using De Moivre's Theorem allows for the simple evaluation of higher integer powers of the roots.

Solution:

Step 1: Solve the quadratic equation $x^2 - 2x + 4 = 0$ using the standard determinant formula:

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(4)}}{2(1)} = \frac{2 \pm \sqrt{4-16}}{2} = \frac{2 \pm \sqrt{-12}}{2}$$

$$x = \frac{2 \pm 2\sqrt{3}i}{2} = 1 \pm \sqrt{3}i$$

Step 2: Convert the complex roots into polar form. The magnitude is $r = \sqrt{1^2 + (\sqrt{3})^2} = 2$. The argument is $\theta = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$.

$$\alpha = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = 2e^{i\pi/3}$$

$$\beta = 2\left(\cos\frac{\pi}{3} - i\sin\frac{\pi}{3}\right) = 2e^{-i\pi/3}$$

Step 3: Raise both roots to the sixth power as required by the expression $\alpha^6 + \beta^6$:

$$\alpha^6 = \left(2e^{i\pi/3}\right)^6 = 2^6 \cdot e^{i2\pi}$$

$$\beta^6 = \left(2e^{-i\pi/3}\right)^6 = 2^6 \cdot e^{-i2\pi}$$

Step 4: Use Euler's identity ($e^{i2\pi} = 1$ and $e^{-i2\pi} = 1$) or De Moivre's Theorem to simplify the transcendental terms:

$$\alpha^6 = 64 \cdot (\cos 2\pi + i\sin 2\pi) = 64 \cdot (1+0) = 64$$

$$\beta^6 = 64 \cdot (\cos(-2\pi) + i\sin(-2\pi)) = 64 \cdot (1+0) = 64$$

Step 5: Add the two computed values together to find the final result:

$$\alpha^6 + \beta^6 = 64 + 64 = 128$$

Final Answer: The value of $\alpha^6 + \beta^6$ is 128

Answer: (B)

Go Back to Question 5

Q6.

Solution

Concept:

We use a key property of definite integrals, often referred to as King's Property: $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$. Applying this transformation changes trigonometric functions into their co-functions when the sum of the limits is $\frac{\pi}{2}$, allowing for algebraic cancellation when the integrals are added.

Solution:

Step 1: Let the given integral be denoted as I:

$$I = \int_0^{\pi/2} \frac{\sin^{100} x}{\sin^{100} x + \cos^{100} x}\,dx \quad \text{— (Equation 1)}$$

Step 2: Apply the integration property by replacing the variable x with $\left(0 + \frac{\pi}{2} - x\right) = \frac{\pi}{2} - x$ in the integrand:

$$I = \int_0^{\pi/2} \frac{\sin^{100}\left(\frac{\pi}{2} - x\right)}{\sin^{100}\left(\frac{\pi}{2} - x\right) + \cos^{100}\left(\frac{\pi}{2} - x\right)}\,dx$$

Step 3: Use standard trigonometric complementary angle identities ($\sin\left(\frac{\pi}{2} - x\right) = \cos x$ and $\cos\left(\frac{\pi}{2} - x\right) = \sin x$) to simplify the expression:

$$I = \int_0^{\pi/2} \frac{\cos^{100} x}{\cos^{100} x + \sin^{100} x}\,dx \quad \text{— (Equation 2)}$$

Step 4: Add Equation 1 and Equation 2 together. Since their limits of integration and denominators are completely identical, we can combine their numerators directly:

$$2I = \int_0^{\pi/2} \frac{\sin^{100} x + \cos^{100} x}{\sin^{100} x + \cos^{100} x}\,dx$$

$$2I = \int_0^{\pi/2} 1\,dx$$

Step 5: Integrate the constant function and evaluate it across the definite upper and lower boundaries:

$$2I = [x]_0^{\pi/2} = \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

$$I = \frac{\pi}{4}$$

Final Answer: The value of the integral is $\pi/4$

Answer: (C) Go Back to Question 6

Q7.

Solution

Concept:
The given matrix expression is a classic Vandermonde determinant. The value of this determinant can be found using elementary row operations such as $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$ to create zeros, followed by factoring out common terms from the rows and expanding.

Solution:
Step 1: Write down the given determinant expression:

$$\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

Step 2: Perform row transformations to create maximum zeros in the first column. Apply $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$:

$$\Delta = \begin{vmatrix} 1 & a & a^2 \\ 0 & b-a & b^2-a^2 \\ 0 & c-a & c^2-a^2 \end{vmatrix}$$

Step 3: Factor out common algebraic terms from the second and third rows. Take $(b-a)$ out of row 2, and $(c-a)$ out of row 3:

$$\Delta = (b-a)(c-a) \begin{vmatrix} 1 & a & a^2 \\ 0 & 1 & b+a \\ 0 & 1 & c+a \end{vmatrix}$$

Step 4: Expand the simplified determinant along the first column:

$$\Delta = (b-a)(c-a)[1 \cdot ((c+a)-(b+a))] = (b-a)(c-a)(c-b)$$

Step 5: Rearrange the signs of the factors to match the standard cyclic product order $(a-b)(b-c)(c-a)$ as given in the problem statement:

$$\Delta = [-(a-b)][-(b-c)](c-a) = (a-b)(b-c)(c-a)$$

Comparing this with $\Delta = k(a-b)(b-c)(c-a)$, we clearly see that the scalar coefficient k must equal 1.

Final Answer: The value of k is 1

Answer: (A) Go Back to Question 7

Q8.

Solution

Concept:
According to the divisibility rule for the number 3, a number is completely divisible by 3 if and only if the sum of all its constituent individual digits is a multiple of 3. We first need to select subsets of 4 digits from the available pool $\{1, 2, 3, 4, 5\}$ whose sum is divisible by 3, and then find the number of permutations for each valid set.

Solution:
Step 1: Compute the sum of all 5 available digits to understand the total weight:

$$1 + 2 + 3 + 4 + 5 = 15$$

Step 2: Since we need to form 4-digit numbers, we must exclude exactly one digit at a time from the set. Let us check the sum of the remaining 4 digits for each possible exclusion: Case 1: Exclude 1 $\implies$ Sum $= 15 - 1 = 14$ (Not divisible by 3) Case 2: Exclude 2 $\implies$ Sum $= 15 - 2 = 13$ (Not divisible by 3) Case 3: Exclude 3 $\implies$ Sum $= 15 - 3 = 12$ (Divisible by 3) $\implies$ Valid digit set: $\{1, 2, 4, 5\}$ Case 4: Exclude 4 $\implies$ Sum $= 15 - 4 = 11$ (Not divisible by 3) Case 5: Exclude 5 $\implies$ Sum $= 15 - 5 = 10$ (Not divisible by 3)
Step 3: Identify the single group of 4 digits that can satisfy the divisibility condition, which is $\{1, 2, 4, 5\}$.
Step 4: Calculate the total number of unique arrangements that can be formed using these 4 distinct chosen digits without any repetition:

$$\text{Number of permutations} = 4! = 4 \times 3 \times 2 \times 1 = 24$$

Step 5: Conclude that since there is only one valid combination of digits, the total number of such 4-digit numbers is exactly 24.
Final Answer: The total number of numbers is 24

Answer: (A) Go Back to Question 8

Q9.

Solution

Concept:

The given sequence is an Arithmetic Progression (AP) because the difference between any two consecutive terms remains constant. The sum of the first n terms of an arithmetic progression can be calculated using the standard summation formula: $S_n = \frac{n}{2}[2a + (n-1)d]$, where a represents the first term and d represents the common difference.

Solution:

Step 1: Analyze the given numerical series $1+4+7+10+\ldots$ to identify its characteristic parameters. First term (a) = 1 Second term = 4

Step 2: Find the common difference (d) by subtracting the first term from the second term:

$$d = 4 - 1 = 3$$

Step 3: State the number of terms to be summed, which is given as $n = 20$. Substitute the parameters $n = 20, a = 1, d = 3$ into the AP sum formula:

$$S_{20} = \frac{20}{2}[2(1) + (20-1)3]$$

Step 4: Perform the arithmetic operations inside the bracket step-by-step:

$$S_{20} = 10 \cdot [2 + (19 \cdot 3)]$$

$$S_{20} = 10 \cdot [2 + 57]$$

$$S_{20} = 10 \cdot [59]$$

Step 5: Multiply the terms together to get the final total sum:

$$S_{20} = 590$$

Final Answer: The sum of the first 20 terms is 590

Answer: (A) Go Back to Question 9

Q10.

Solution

Concept:

To add two inverse tangent functions, we use the standard inverse trigonometric identity: $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$, provided that the product of the arguments satisfies the domain constraint $xy < 1$.

Solution:

Step 1: Identify the arguments from the given expression $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right)$:

$$x = \frac{1}{2}, \quad y = \frac{1}{3}$$

Step 2: Check the mandatory condition for the product of the variables to ensure correct formula branch application:

$$xy = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$$

Since $\frac{1}{6} < 1$, the standard formula can be applied directly without adding any phase constants like π.

Step 3: Substitute the fractional values into the algebraic identity:

$$\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}\left(\frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \cdot \frac{1}{3}}\right)$$

Step 4: Simplify the fractions inside the parenthesis by finding a common denominator for both the numerator and the denominator expressions:

$$\text{Numerator} = \frac{3+2}{6} = \frac{5}{6}$$

$$\text{Denominator} = 1 - \frac{1}{6} = \frac{5}{6}$$

$$\text{Combined Argument} = \frac{5/6}{5/6} = 1$$

Step 5: Evaluate the final inverse trigonometric value:

$$\tan^{-1}(1) = \frac{\pi}{4}$$

Final Answer: The value of the expression is $\pi/4$

Answer: (C)

Go Back to Question 10

Q11.

Solution

Concept: The angle θ between two tangents drawn from an external point to a circle can be calculated using the right triangle geometric relationship $\tan\left(\frac{\theta}{2}\right) = \frac{R}{L}$, where R is the radius of the circle and $L = \sqrt{S_1}$ is the length of the tangent from the external point.

Solution: Step 1: Find the center and radius R of the circle by rearranging $x^2 + y^2 - 8x - 6y + 21 = 0$:

$$\text{Center } C = (4, 3)$$

$$\text{Radius } R = \sqrt{(-4)^2 + (-3)^2 - 21} = \sqrt{16 + 9 - 21} = \sqrt{4} = 2$$

Step 2: Calculate the length of the tangent L from the origin $(0, 0)$ using power of a point S_1:

$$S_1 = 0^2 + 0^2 - 8(0) - 6(0) + 21 = 21 \implies L = \sqrt{21}$$

Step 3: Evaluate the half-angle tangent relation:

$$\tan\left(\frac{\theta}{2}\right) = \frac{R}{L} = \frac{2}{\sqrt{21}}$$

Step 4: Convert the half-angle tangent to the full angle cosine value using the double-angle identity:

$$\cos\theta = \frac{1 - \tan^2\left(\frac{\theta}{2}\right)}{1 + \tan^2\left(\frac{\theta}{2}\right)} = \frac{1 - \frac{4}{21}}{1 + \frac{4}{21}} = \frac{\frac{17}{21}}{\frac{25}{21}} = \frac{17}{25}$$

Step 5: Isolate the angle parameter to match standard inverse trigonometric form:

$$\theta = \cos^{-1}\left(\frac{17}{25}\right)$$

Final Answer: $\boxed{\cos^{-1}\left(\frac{17}{25}\right)}$

Answer: (C) **Go Back to Question 11**

Q12.

Solution

Concept:
Three vectors are coplanar if and only if their scalar triple product is equal to zero. This means the determinant formed by their component coefficients must vanish. We will set up the determinant, perform column operations to simplify it, and solve for the required algebraic relationship.

Solution:
Step 1: Write the component coefficients of the three given vectors into a matrix determinant and equate it to zero:

$$\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$$

Step 2: Apply elementary column operations to simplify the matrix. Perform $C_1 \to C_1 - C_3$ and $C_2 \to C_2 - C_3$:

$$\begin{vmatrix} a-1 & 0 & 1 \\ 0 & b-1 & 1 \\ 1-c & 1-c & c \end{vmatrix} = 0$$

Step 3: Expand the simplified determinant along the first row:

$$(a-1)\cdot[(b-1)c-(1-c)]+1\cdot[0-(b-1)(1-c)]=0$$

$$(a-1)(b-1)c-(a-1)(1-c)-(b-1)(1-c)=0$$

Step 4: Divide the entire equation by $(1-a)(1-b)(1-c)$ to transform the terms into the fractions required by the target expression. Note that $(a-1)=-(1-a)$ and $(b-1)=-(1-b)$:

$$\frac{-(1-a)[-(1-b)]c}{(1-a)(1-b)(1-c)}-\frac{-(1-a)(1-c)}{(1-a)(1-b)(1-c)}-\frac{-(1-b)(1-c)}{(1-a)(1-b)(1-c)}=0$$

$$\frac{c}{1-c}+\frac{1}{1-b}+\frac{1}{1-a}=0$$

Step 5: Rewrite the term $\frac{c}{1-c}$ as $\frac{c-1+1}{1-c}=\frac{1}{1-c}-1$. Substitute this back into the equation:

$$\frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}-1=0 \implies \frac{1}{1-a}+\frac{1}{1-b}+\frac{1}{1-c}=1$$

Final Answer: The value of the expression is 1

Answer: (B) Go Back to Question 12

Q13.

Solution

Concept:

The given limit $\lim_{x\to 0} \frac{e^{x^2}-\cos x}{x^2}$ is an indeterminate form of the type $\frac{0}{0}$. We can solve this either by applying L'Hopital's Rule or by utilizing the standard Taylor series expansions for e^t and $\cos x$ near $x = 0$. Using series expansion provides a direct algebraic route without multiple derivatives.

Solution:

Step 1: Write down the standard Taylor series expansions up to the terms of degree 2:

$$e^t = 1 + t + \frac{t^2}{2!} + \cdots \implies e^{x^2} = 1 + x^2 + \frac{x^4}{2} + \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots = 1 - \frac{x^2}{2} + \dots$$

Step 2: Substitute these polynomial series representations into the numerator of the limit expression:

$$\text{Numerator} = e^{x^2} - \cos x = \left(1 + x^2 + \frac{x^4}{2} + \dots\right) - \left(1 - \frac{x^2}{2} + \dots\right)$$

Step 3: Combine like terms and simplify the algebraic expression:

$$\text{Numerator} = (1-1) + \left(x^2 + \frac{x^2}{2}\right) + \text{terms containing } x^4 \text{ and higher}$$

$$\text{Numerator} = \frac{3}{2}x^2 + \mathcal{O}(x^4)$$

Step 4: Substitute the simplified numerator back into the complete limit equation over the denominator x^2:

$$\lim_{x\to 0} \frac{\frac{3}{2}x^2 + \mathcal{O}(x^4)}{x^2} = \lim_{x\to 0} \left(\frac{3}{2} + \mathcal{O}(x^2)\right)$$

Step 5: Evaluate the limit as $x \to 0$. All terms with positive powers of x vanish, leaving only the constant term:

$$\text{Limit value} = \frac{3}{2}$$

Final Answer: The value of the limit is $3/2$

Answer: (C) Go Back to Question 13

Q14.

Solution

Concept:

For a standard parabola $y^2 = 4ax$, the condition of tangency for any straight line $y = mx + c$ is given by the mathematical relationship $c = \frac{a}{m}$. We can compare the given parabola and line equations to extract the parameters and solve for m.

Solution:

Step 1: Identify the parameters of the given parabola $y^2 = 4x$ by comparing it with the standard form $y^2 = 4ax$:

$$4a = 4 \implies a = 1$$

Step 2: Identify the parameters of the given line $y = mx + 1$ by comparing it with the standard slope-intercept form $y = mx + c$:

$$\text{Intercept } c = 1$$

Step 3: Substitute the known values $a = 1$ and $c = 1$ into the standard condition of tangency formula $c = \frac{a}{m}$:

$$1 = \frac{1}{m}$$

Step 4: Solve the algebraic equation for the unknown slope parameter m:

$$m = 1$$

Step 5: Verify that substituting $m = 1$ back into the line gives $y = x + 1$. Solving this simultaneously with $y^2 = 4x$ results in $(x+1)^2 = 4x \implies x^2 - 2x + 1 = 0 \implies (x-1)^2 = 0$, which yields a single repeating point of intersection at $(1, 2)$, confirming tangency.

Final Answer: The value of m is 1

Answer: (A) Go Back to Question 14

Q15.

Solution

Concept:
The probability of an event is defined as the ratio of the number of favorable outcomes to the total number of equally likely outcomes in the sample space: $P(E) = \frac{n(E)}{n(S)}$. When two fair six-sided dice are rolled simultaneously, the total number of outcomes in the sample space is $6 \times 6 = 36$.

Solution:
Step 1: Determine the total number of outcomes in the sample space $n(S)$. Each die has 6 faces, so for two dice:

$$n(S) = 6 \times 6 = 36$$

Step 2: List all the possible ordered pairs (d_1, d_2) representing the outcomes on the two dice such that their sum equals exactly 7:

$$E = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$$

Step 3: Count the total number of elements contained in the set of favorable outcomes:

$$n(E) = 6$$

Step 4: Calculate the probability by dividing the number of favorable outcomes by the total sample space size:

$$P(E) = \frac{n(E)}{n(S)} = \frac{6}{36}$$

Step 5: Simplify the fraction to its lowest terms:

$$P(E) = \frac{1}{6}$$

Final Answer: The probability of getting a total score of 7 is 1/6

Answer: (A) Go Back to Question 15

Q16.

Solution

Concept:

To evaluate the indefinite integral $\int \frac{dx}{x(x^5+1)}$, we can use the method of substitution. Multiplying and dividing the integrand by a suitable power of x (specifically x^4) creates an expression where the numerator becomes proportional to the derivative of a term in the denominator.

Solution:

Step 1: Write down the given integral:

$$I = \int \frac{dx}{x(x^5+1)}$$

Step 2: Multiply both the numerator and the denominator by x^4 to prepare for substitution:

$$I = \int \frac{x^4\,dx}{x^5(x^5+1)}$$

Step 3: Define a new variable for substitution. Let $t = x^5$. Differentiate both sides with respect to x:

$$dt = 5x^4\,dx \implies x^4\,dx = \frac{dt}{5}$$

Step 4: Substitute t and dx back into the integral expression:

$$I = \int \frac{\frac{dt}{5}}{t(t+1)} = \frac{1}{5}\int \frac{dt}{t(t+1)}$$

Step 5: Resolve the integrand into partial fractions: $\frac{1}{t(t+1)} = \frac{1}{t} - \frac{1}{t+1}$. Integrate each term and substitute back $t = x^5$:

$$I = \frac{1}{5}\left(\int \frac{dt}{t} - \int \frac{dt}{t+1}\right) = \frac{1}{5}\left(\ln|t| - \ln|t+1|\right) + C$$

$$I = \frac{1}{5}\ln\left|\frac{t}{t+1}\right| + C = \frac{1}{5}\ln\left|\frac{x^5}{x^5+1}\right| + C$$

Final Answer: $\boxed{\frac{1}{5}\ln\left|\frac{x^5}{x^5+1}\right| + C}$

Answer: (B) Go Back to Question 16

Q17.

Solution

Concept:

Critical points of a function occur where its first derivative equals zero ($f'(x) = 0$) or is undefined.

Solution:

Step 1: Differentiate the given function $f(x) = 2x^3 - 9x^2 + 12x + 5$ with respect to x:

$$f'(x) = 6x^2 - 18x + 12$$

Step 2: Set the first derivative equal to zero to find the critical points:

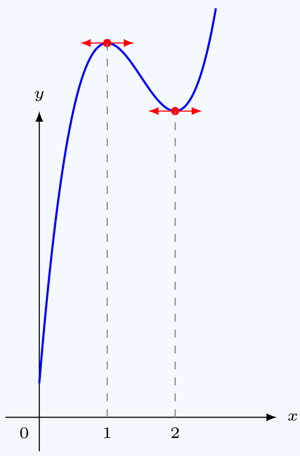
$$6x^2 - 18x + 12 = 0$$

Step 3: Factor out the common constant 6:

$$6(x^2 - 3x + 2) = 0 \implies x^2 - 3x + 2 = 0$$

Step 4: Solve the quadratic equation by splitting the middle term:

$$(x-1)(x-2) = 0 \implies x = 1 \quad \text{and} \quad x = 2$$



Tangent slope $f'(x) = 0$ at critical points

Final Answer: $\boxed{x = 1, 2}$

Answer: (A) **Go Back to Question 17**

Q18.

Solution

Concept:
We use standard determinant properties for square matrices of order n. Specifically, for any scalar k and matrix A, $|kA| = k^n|A|$. Additionally, for the adjugate matrix, the determinant property states that $|\text{adj}(B)| = |B|^{n-1}$. Combining these rules allows us to evaluate the complex expression step-by-step.

Solution:
Step 1: Let $B = 2A$ be the internal matrix expression. We first need to compute the determinant of B, which is $|2A|$. Given that the matrix order is $n = 3$:

$$|B| = |2A| = 2^3 \cdot |A|$$

Step 2: Substitute the given value $|A| = 5$ into the equation:

$$|B| = 8 \cdot 5 = 40$$

Step 3: Now apply the adjugate determinant property to the expression $|\text{adj}(B)|$. Since the order of matrix B is also 3, we have:

$$|\text{adj}(B)| = |B|^{3-1} = |B|^2$$

Step 4: Substitute the value of $|B| = 40$ computed in Step 2 into this relation:

$$|\text{adj}(2A)| = (40)^2$$

Step 5: Compute the numerical square to find the final value:

$$(40)^2 = 1600$$

Final Answer: The value of $|\text{adj}(2A)|$ is 1600

Answer: (C) Go Back to Question 18

Q19.

Solution

Concept:
To evaluate $\sin(2\theta)$ where $\theta = \sin^{-1}\left(\frac{4}{5}\right)$, we use the standard trigonometric double-angle identity: $\sin(2\theta) = 2\sin\theta\cos\theta$. We can find $\cos\theta$ using the fundamental identity $\cos\theta = \sqrt{1-\sin^2\theta}$ for angles in the first quadrant.

Solution:
Step 1: Define the angle parameter θ from the given inverse expression:

$$\theta = \sin^{-1}\left(\frac{4}{5}\right) \implies \sin\theta = \frac{4}{5}$$

Step 2: Determine the value of $\cos\theta$ using a right-angled triangle or the identity $\cos^2\theta + \sin^2\theta = 1$:

$$\cos\theta = \sqrt{1-\sin^2\theta} = \sqrt{1-\left(\frac{4}{5}\right)^2} = \sqrt{1-\frac{16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$$

Step 3: Write down the target expression using the double-angle variable notation:

$$\sin\left(2\sin^{-1}\left(\frac{4}{5}\right)\right) = \sin(2\theta)$$

Step 4: Expand using the double-angle formula and substitute the values of $\sin\theta$ and $\cos\theta$:

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\sin(2\theta) = 2\cdot\left(\frac{4}{5}\right)\cdot\left(\frac{3}{5}\right)$$

Step 5: Multiply the numerators and denominators together to obtain the final simplified fraction:

$$\sin(2\theta) = \frac{2\times4\times3}{25} = \frac{24}{25}$$

Final Answer: The value of the expression is 24/25

Answer: (B) Go Back to Question 19

Q20.

Solution

Concept:
The given differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$ is a standard first-order linear differential equation of the form $\frac{dy}{dx} + P(x)y = Q(x)$. The general solution is found using an Integrating Factor I.F. $= e^{\int P(x)dx}$, and is given by $y \cdot (\text{I.F.}) = \int Q(x) \cdot (\text{I.F.})\, dx + C$.

Solution:
Step 1: Identify the coefficient functions $P(x)$ and $Q(x)$ from the given differential equation:

$$P(x) = \frac{1}{x}, \quad Q(x) = x^2$$

Step 2: Compute the Integrating Factor (I.F.):

$$\text{I.F.} = e^{\int \frac{1}{x}\, dx} = e^{\ln x} = x$$

Step 3: Write out the general solution formula using the calculated Integrating Factor:

$$y \cdot x = \int x^2 \cdot x\, dx + C$$

$$xy = \int x^3\, dx + C$$

$$xy = \frac{x^4}{4} + C \quad \text{— (Equation 1)}$$

Step 4: Use the given boundary condition $y(1) = \frac{1}{4}$ (which means $y = \frac{1}{4}$ when $x = 1$) to solve for the integration constant C:

$$(1)\left(\frac{1}{4}\right) = \frac{1^4}{4} + C \implies \frac{1}{4} = \frac{1}{4} + C \implies C = 0$$

Step 5: Substitute $C = 0$ back into Equation 1 and clear the fractions:

$$xy = \frac{x^4}{4} \implies 4xy = x^4$$

Final Answer: The solution is $4xy = x^4$

Answer: (A) Go Back to Question 20

Q21.

Solution

Concept:

To evaluate a product of cosines with angles in geometric progression, we can use the product identity $\cos\theta\cos 2\theta\cos 4\theta\ldots\cos 2^{n-1}\theta = \frac{\sin(2^n\theta)}{2^n\sin\theta}$. Alternatively, we can multiply and divide by $2\sin 20^\circ$ and repeatedly apply the sine double-angle identity $2\sin A\cos A = \sin 2A$.

Solution:

Step 1: Let the given expression be denoted as P:

$$P = \cos 20^\circ \cos 40^\circ \cos 80^\circ$$

Step 2: Multiply and divide the entire expression by $2\sin 20^\circ$ to create the opportunity to use the double-angle formula:

$$P = \frac{2\sin 20^\circ \cos 20^\circ \cos 40^\circ \cos 80^\circ}{2\sin 20^\circ}$$

Step 3: Substitute $2\sin 20^\circ \cos 20^\circ = \sin 40^\circ$ into the numerator expression:

$$P = \frac{\sin 40^\circ \cos 40^\circ \cos 80^\circ}{2\sin 20^\circ}$$

Step 4: Multiply and divide the numerator and denominator by 2 again to combine the next set of angles:

$$P = \frac{2\sin 40^\circ \cos 40^\circ \cos 80^\circ}{4\sin 20^\circ} = \frac{\sin 80^\circ \cos 80^\circ}{4\sin 20^\circ}$$

Multiply and divide by 2 one final time:

$$P = \frac{2\sin 80^\circ \cos 80^\circ}{8\sin 20^\circ} = \frac{\sin 160^\circ}{8\sin 20^\circ}$$

Step 5: Use the supplementary angle identity $\sin(180^\circ - \theta) = \sin\theta$ to simplify the numerator:

$$\sin 160^\circ = \sin(180^\circ - 20^\circ) = \sin 20^\circ$$

$$P = \frac{\sin 20^\circ}{8\sin 20^\circ} = \frac{1}{8}$$

Final Answer: The value of $\cos 20^\circ \cos 40^\circ \cos 80^\circ$ is 1/8

Answer: (C) Go Back to Question 21

Q22.

Solution

Concept:
The standard equation of a hyperbola is given by $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. The eccentricity e of such a hyperbola can be calculated using the fundamental geometric relationship $e = \sqrt{1 + \frac{b^2}{a^2}}$. We must reduce the given equation to standard form to find the values of a^2 and b^2.

Solution:
Step 1: Write down the given equation of the hyperbola:

$$9x^2 - 16y^2 = 144$$

Step 2: Convert this equation into the standard form by dividing both sides of the equation by 144:

$$\frac{9x^2}{144} - \frac{16y^2}{144} = \frac{144}{144}$$

$$\frac{x^2}{16} - \frac{y^2}{9} = 1$$

Step 3: Compare this reduced equation with the standard form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ to extract the parameters:

$$a^2 = 16, \quad b^2 = 9$$

Step 4: Substitute the values of a^2 and b^2 into the standard eccentricity formula:

$$e = \sqrt{1 + \frac{9}{16}}$$

Step 5: Simplify the algebraic expression inside the radical sign:

$$e = \sqrt{\frac{16+9}{16}} = \sqrt{\frac{25}{16}} = \frac{5}{4}$$

Final Answer: The eccentricity of the hyperbola is 5/4

Answer: (A) Go Back to Question 22

Q23.

Solution

Concept:
Let the terms of a geometric progression (GP) be represented as a, ar, ar^2, ar^3, ar^4, where a is the first term and r is the common ratio. The n-th term is given by $a_n = ar^{n-1}$. We can express the product of the first 5 terms in terms of the middle term (the third term) and find its value directly.

Solution:
Step 1: State the formula for the third term of a geometric progression:

$$a_3 = ar^{3-1} = ar^2$$

The problem states that the third term is equal to 4:

$$ar^2 = 4 \quad \text{— (Equation 1)}$$

Step 2: Write out the algebraic expression for the product of the first five terms of this progression:

$$\text{Product} = a \cdot (ar) \cdot (ar^2) \cdot (ar^3) \cdot (ar^4)$$

Step 3: Combine the terms by adding the exponents of a and r separately:

$$\text{Product} = a^{1+1+1+1+1} \cdot r^{0+1+2+3+4}$$

$$\text{Product} = a^5 \cdot r^{10}$$

Step 4: Factor the expression to represent it as a power of the known third term component (ar^2):

$$\text{Product} = (ar^2)^5$$

Step 5: Substitute the value $ar^2 = 4$ from Equation 1 into this factored expression:

$$\text{Product} = 4^5$$

Final Answer: The product of the first 5 terms is 4^5

Answer: (C) Go Back to Question 23

Q24.

Solution

Concept:

The perpendicular distance d between two parallel straight lines given by the standard algebraic forms $Ax + By + C_1 = 0$ and $Ax + By + C_2 = 0$ is computed using the formula $d = \frac{|C_1 - C_2|}{\sqrt{A^2+B^2}}$. We must first make the coefficients of x and y identical for both lines.

Solution:

Step 1: Write down the equations of the two lines:

$$\text{Line 1: } 3x + 4y - 9 = 0$$

$$\text{Line 2: } 6x + 8y + 12 = 0$$

Step 2: Divide the entire equation of Line 2 by 2 so that its leading coefficients match the coefficients of Line 1 exactly:

$$\frac{6x}{2} + \frac{8y}{2} + \frac{12}{2} = 0 \implies 3x + 4y + 6 = 0$$

Step 3: Identify the shared coefficients A, B and the unique constant terms C_1, C_2 from the modified equations:

$$A = 3, \quad B = 4, \quad C_1 = -9, \quad C_2 = 6$$

Step 4: Substitute these values into the parallel distance formula:

$$d = \frac{|-9-6|}{\sqrt{3^2+4^2}}$$

Step 5: Perform the arithmetic operations to find the final distance value:

$$d = \frac{|-15|}{\sqrt{9+16}} = \frac{15}{\sqrt{25}} = \frac{15}{5} = 3 \text{ units}$$

Final Answer: The distance between the parallel lines is 3 units

Answer: (A) **Go Back to Question 24**

Q25.

Solution

Concept:
A system of linear equations has a unique solution if and only if the determinant of its coefficient matrix, denoted by Δ, is non-zero ($\Delta \neq 0$). Conversely, the system will fail to have a unique solution for values of the parameter that make the determinant equal to zero.

Solution:
Step 1: Construct the coefficient determinant Δ from the given system of linear equations:

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 2 & 3 & a^2 - 1 \end{vmatrix}$$

Step 2: Perform elementary row transformations to simplify the evaluation. Apply $R_2 \to R_2 - 2R_1$ and $R_3 \to R_3 - 2R_1$:

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 1 & a^2 - 3 \end{vmatrix}$$

Step 3: Perform one more row operation to create another zero in the second column. Apply $R_3 \to R_3 - R_2$:

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & a^2 - 3 \end{vmatrix}$$

Step 4: Expand the upper triangular matrix determinant by multiplying its main diagonal elements:

$$\Delta = 1 \cdot 1 \cdot (a^2 - 3) = a^2 - 3$$

Step 5: For a unique solution, we must have $\Delta \neq 0$. Therefore, the system will not have a unique solution if $\Delta = 0$:

$$a^2 - 3 = 0 \implies a^2 = 3 \implies a = \pm\sqrt{3}$$

Thus, for the solution to remain unique, a cannot take the values $\pm\sqrt{3}$.

Final Answer: The parameter a cannot be equal to $\pm\sqrt{3}$

Answer: (B) Go Back to Question 25

Q26.

Solution

Concept:

Let the variable line intersect the coordinate axes at $A(a, 0)$ and $B(0, b)$. The equation of this line in intercept form is $\frac{x}{a} + \frac{y}{b} = 1$. Since the line passes through a fixed point, substituting that point into the line equation provides a constraint. We link this to the coordinates of the midpoint (h, k) to find the locus.

Solution:

Step 1: Let the intercepts of the variable line on the axes be a and b, so $A = (a, 0)$ and $B = (0, b)$. The equation of the line is:

$$\frac{x}{a} + \frac{y}{b} = 1$$

Step 2: Substitute the coordinates of the fixed point $(2, 3)$ through which the line must pass into the intercept equation:

$$\frac{2}{a} + \frac{3}{b} = 1 \quad \text{— (Equation 1)}$$

Step 3: Define the coordinates of the midpoint of AB as (h, k). Using the midpoint formula, we relate (h, k) to the intercepts a and b:

$$h = \frac{a+0}{2} = \frac{a}{2} \implies a = 2h$$

$$k = \frac{0+b}{2} = \frac{b}{2} \implies b = 2k$$

Step 4: Substitute $a = 2h$ and $b = 2k$ back into the condition constraint shown in Equation 1:

$$\frac{2}{2h} + \frac{3}{2k} = 1 \implies \frac{1}{h} + \frac{3}{2k} = 1$$

Multiply the entire equation by $2hk$ to clear the denominators:

$$2k + 3h = 2hk$$

Step 5: Replace the dummy midpoint parameters (h, k) with general coordinates (x, y) to write the final locus equation:

$$3x + 2y = 2xy$$

Final Answer: The locus of the midpoint is $3x + 2y = 2xy$

Answer: (A) Go Back to Question 26

Q27.

Solution

Concept:

Properties of the absolute value function $|x|$ and symmetric roots in algebraic equations.

Solution:

Step 1: Rewrite the equation using the identity $x^2 = |x|^2$:

$$|x|^2 - 5|x| + 6 = 0$$

Step 2: Factor the quadratic equation in terms of $|x|$:

$$(|x| - 2)(|x| - 3) = 0 \implies |x| = 2 \quad \text{or} \quad |x| = 3$$

Step 3: Solve for the real values of x:

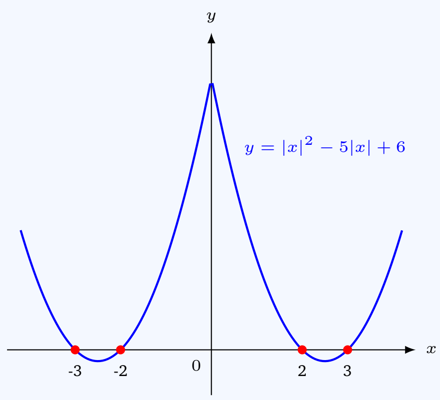
$$\text{From } |x| = 2 \implies x = \pm 2$$

$$\text{From } |x| = 3 \implies x = \pm 3$$

The four real roots are $-3, -2, 2,$ and 3.

Step 4: Calculate the sum of all real roots:

$$\text{Sum} = (-3) + (-2) + 2 + 3 = 0$$



Final Answer: $\boxed{0}$

Answer: (B) Go Back to Question 27

Q28.

Solution

Concept:

Conditional probability without replacement from a finite sample space.

Solution:

Step 1: Identify the initial configuration of the box:

$$\text{Red balls } (R) = 3, \quad \text{Black balls } (B) = 7, \quad \text{Total balls } = 10$$

Step 2: Update the pool configuration given that the first ball drawn was red (R_1):

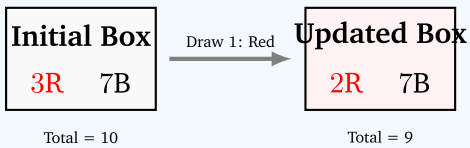
$$\text{Remaining Red balls } = 3 - 1 = 2$$

$$\text{Remaining Black balls } = 7$$

$$\text{New Total balls } = 10 - 1 = 9$$

Step 3: Calculate the conditional probability that the second ball drawn is red (R_2) given R_1:

$$P(R_2 \mid R_1) = \frac{\text{Remaining Red balls}}{\text{New Total balls}} = \frac{2}{9}$$



Final Answer: 2/9

Answer: (A) Go Back to Question 28

Q29.

Solution

Concept:
To find the local minimum value of a function, we can use the AM-GM inequality (Arithmetic Mean $\geq$ Geometric Mean) for positive real numbers, which states that for any positive terms a and b, $\frac{a+b}{2} \geq \sqrt{ab}$. Alternatively, we can find critical points using the first derivative and verify using the second derivative.

Solution:
Step 1: State the function and its domain constraint:

$$f(x) = x + \frac{4}{x} \quad \text{for } x > 0$$

Step 2: Since x is strictly positive, both terms x and $\frac{4}{x}$ are positive real numbers. Apply the AM-GM inequality directly to these two terms:

$$\frac{x + \frac{4}{x}}{2} \geq \sqrt{x \cdot \frac{4}{x}}$$

Step 3: Simplify the expression inside the radical on the right-hand side of the inequality:

$$\sqrt{x \cdot \frac{4}{x}} = \sqrt{4} = 2$$

Step 4: Substitute this back into the inequality and multiply both sides by 2 to isolate the function expression:

$$\frac{x + \frac{4}{x}}{2} \geq 2 \implies x + \frac{4}{x} \geq 4$$

Step 5: Conclude that the minimum value achieved by the function is 4. This occurs when the terms are equal: $x = \frac{4}{x} \implies x^2 = 4 \implies x = 2$, which lies inside the positive domain.

Final Answer: The local minimum value of the function is 4

Answer: (B) Go Back to Question 29

Q30.

Solution

Concept:
The modulus of a complex number fraction can be found using the property $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$. We can simplify the complex expression in the denominator first by expanding the quadratic term using $i^2 = -1$, and then calculate the individual magnitudes.

Solution:
Step 1: Write down the given complex expression:

$$z = \frac{1+2i}{1-(1-i)^2}$$

Step 2: Expand the squared complex term in the denominator using standard algebraic expansion:

$$(1-i)^2 = 1^2 - 2i + i^2$$

Since $i^2 = -1$, substitute this value:

$$(1-i)^2 = 1 - 2i - 1 = -2i$$

Step 3: Substitute this back into the denominator of the complex fraction:

$$\text{Denominator} = 1 - (-2i) = 1 + 2i$$

Step 4: Reassemble the full fraction for z using the simplified denominator:

$$z = \frac{1+2i}{1+2i} = 1$$

Step 5: Find the modulus of the resulting real number:

$$|z| = |1| = 1$$

Final Answer: The modulus of the complex number is 1

Answer: (A)   Go Back to Question 30

Q31.

Solution

Concept:
The number of common tangents between two circles depends entirely on the relationship between the distance between their centers (d) and the sum or difference of their radii (r_1 and r_2). We determine the coordinates of both centers, compute the lengths of both radii, and then compare the geometric distance d against the boundary conditions $r_1 + r_2$ and $|r_1 - r_2|$.

Solution:
Step 1: Analyze the first circle, $C_1 : x^2 + y^2 = 4$. By comparing it with the standard equation $x^2 + y^2 = r^2$, we can find its center coordinates and radius:

$$\text{Center } O_1 = (0, 0)$$

$$\text{Radius } r_1 = \sqrt{4} = 2$$

Step 2: Analyze the second circle, $C_2 : x^2 + y^2 - 6x - 8y + 21 = 0$. Compare it with the general equation $x^2 + y^2 + 2gx + 2fy + c = 0$:

$$2g = -6 \implies g = -3$$

$$2f = -8 \implies f = -4$$

$$c = 21$$

$$\text{Center } O_2 = (-g, -f) = (3, 4)$$

$$\text{Radius } r_2 = \sqrt{g^2 + f^2 - c} = \sqrt{(-3)^2 + (-4)^2 - 21} = \sqrt{9 + 16 - 21} = \sqrt{4} = 2$$

Step 3: Calculate the distance d between the centers $O_1(0, 0)$ and $O_2(3, 4)$ using the standard distance formula:

$$d = \sqrt{(3 - 0)^2 + (4 - 0)^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

Step 4: Compute the sum of the two radii to test for intersections or external placement:

$$r_1 + r_2 = 2 + 2 = 4$$

Step 5: Compare the values of d and $r_1 + r_2$. Since $d = 5$ and $r_1 + r_2 = 4$, we find that $d > r_1 + r_2$. This geometric condition implies that the two circles lie completely external to each other without any intersection. For two completely separated external circles, the total number of common tangents that can be drawn is exactly 4 (2 direct common tangents and 2 transverse common tangents).

Final Answer: The number of common tangents is 4

Answer: (D) Go Back to Question 31

Q32.

Solution

Concept: A square matrix A is orthogonal if it satisfies the relationship $AA^T = I$, where A^T is the transpose of the matrix and I is the identity matrix. This implies that the row and column vectors are mutually orthogonal unit vectors.

Solution: Step 1: Write down the given matrix A and its transpose A^T:

$$A = \begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}, \quad A^T = \begin{bmatrix} 0 & a & a \\ 2b & b & -b \\ c & -c & c \end{bmatrix}$$

Step 2: Set up the orthogonal matrix condition $AA^T = I_3$:

$$\begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix} \begin{bmatrix} 0 & a & a \\ 2b & b & -b \\ c & -c & c \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Step 3: Multiply the rows and columns to generate equations from the key matrix positions:

From position (1,1): $4b^2 + c^2 = 1$ — (1)

From position (2,2): $a^2 + b^2 + c^2 = 1$ — (2)

From position (1,2): $2b^2 - c^2 = 0 \implies c^2 = 2b^2$ — (3)

Step 4: Substitute equation (3) into equation (1) to solve for b and c:

$$4b^2 + 2b^2 = 1 \implies 6b^2 = 1 \implies b = \frac{1}{\sqrt{6}}$$

$$c^2 = 2\left(\frac{1}{6}\right) = \frac{1}{3} \implies c = \frac{1}{\sqrt{3}}$$

Step 5: Substitute the values of b^2 and c^2 into equation (2) to solve for a:

$$a^2 + \frac{1}{6} + \frac{1}{3} = 1 \implies a^2 + \frac{1}{2} = 1 \implies a^2 = \frac{1}{2} \implies a = \frac{1}{\sqrt{2}}$$

Final Answer: $\boxed{a = \frac{1}{\sqrt{2}}, b = \frac{1}{\sqrt{6}}, c = \frac{1}{\sqrt{3}}}$

Answer: (A) **Go Back to Question 32**

Q33.

Solution

Concept:

The shortest distance d between two skew lines in three-dimensional space, represented in vector form by $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$, is calculated using the standard formula $d = \frac{|(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$. If the lines intersect, the numerator will vanish, resulting in a shortest distance of zero.

Solution:

Step 1: Extract the position vectors $\vec{a}_1, \vec{a}_2$ and direction vectors $\vec{b}_1, \vec{b}_2$ from the given Cartesian line equations:

$$\vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k}, \quad \vec{b}_1 = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

$$\vec{a}_2 = 2\hat{i} + 4\hat{j} + 5\hat{k}, \quad \vec{b}_2 = 3\hat{i} + 4\hat{j} + 5\hat{k}$$

Step 2: Compute the difference vector between the two baseline points $(\vec{a}_2 - \vec{a}_1)$:

$$\vec{a}_2 - \vec{a}_1 = (2-1)\hat{i} + (4-2)\hat{j} + (5-3)\hat{k} = \hat{i} + 2\hat{j} + 2\hat{k}$$

Step 3: Compute the scalar triple product component $(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)$ using a 3×3 determinant framework:

$$\text{Numerator Determinant} = \begin{vmatrix} 1 & 2 & 2 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$

Step 4: Expand this determinant systematically along its first row to find its scalar value:

$$\text{Value} = 1\cdot(15-16) - 2\cdot(10-12) + 2\cdot(8-9)$$

$$\text{Value} = 1\cdot(-1) - 2\cdot(-2) + 2\cdot(-1) = -1 + 4 - 2 = 1$$

Step 5: Compute the cross product vector of the directions $\vec{b}_1 \times \vec{b}_2$ and find its magnitude for the denominator:

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix} = \hat{i}(15-16) - \hat{j}(10-12) + \hat{k}(8-9) = -\hat{i} + 2\hat{j} - \hat{k}$$

$$\text{Magnitude } |\vec{b}_1 \times \vec{b}_2| = \sqrt{(-1)^2 + 2^2 + (-1)^2} = \sqrt{1+4+1} = \sqrt{6}$$

Substitute the numerator and denominator back into the distance formula: $d = \frac{1}{\sqrt{6}}$.

Final Answer: The shortest distance between the lines is $1/\sqrt{6}$

Answer: (A) Go Back to Question 33

Q34.

Solution

Concept:

The total number of distinct terms in the expansion of a multinomial expression of the form $(x_1 + x_2 + \cdots + x_r)^n$ is given by the combination formula ${}^{n+r-1}C_{r-1}$. This algebraic principle is derived from the stars-and-bars combinatorics problem of distributing n identical powers among r distinct variables.

Solution:

Step 1: Identify the parameters from the given multinomial expression $(x+y+z)^{10}$. The power exponent is $n = 10$.

Step 2: Count the total number of unique variable terms inside the base parentheses, which gives $r = 3$ (since the variables are x, y, and z).

Step 3: Substitute the parameters $n = 10$ and $r = 3$ directly into the multinomial terms formula ${}^{n+r-1}C_{r-1}$:

$$\text{Number of terms} = {}^{10+3-1}C_{3-1}$$

$$\text{Number of terms} = {}^{12}C_2$$

Step 4: Expand the combination formula using factorials to evaluate the numerical answer:

$${}^{12}C_2 = \frac{12 \times 11}{2 \times 1}$$

Step 5: Simplify the division arithmetic:

$$\text{Number of terms} = 6 \times 11 = 66$$

Final Answer: The number of terms in the expansion is 66

Answer: (C)

Go Back to Question 34

Q35.

Solution

Concept:

The area bounded between a parabola and a straight line is calculated by finding their points of intersection, setting up a definite integral between these boundaries, and integrating the difference between the upper curve function and the lower curve function: Area $= \int_{x_1}^{x_2} (y_{\text{upper}} - y_{\text{lower}})\, dx$.

Solution:

Step 1: Find the points of intersection between the parabola $y^2 = 4x$ and the line $y = 2x$. Substitute the line equation into the parabola equation:

$$(2x)^2 = 4x \implies 4x^2 = 4x$$

$$4x^2 - 4x = 0 \implies 4x(x-1) = 0$$

This gives two intersection points for x: $x = 0$ and $x = 1$. The corresponding coordinates are $(0, 0)$ and $(1, 2)$.

Step 2: Express both equations in terms of x to integrate along the horizontal axis. For the upper curve boundary (the parabola), $y = \sqrt{4x} = 2\sqrt{x}$. For the lower linear boundary, $y = 2x$.

Step 3: Set up the definite integral using the calculated limits from 0 to 1:

$$\text{Area} = \int_0^1 (2\sqrt{x} - 2x)\, dx$$

Step 4: Integrate each term independently using the power rule for integration:

$$\text{Area} = \left[2 \cdot \frac{x^{3/2}}{3/2} - 2 \cdot \frac{x^2}{2} \right]_0^1$$

$$\text{Area} = \left[\frac{4}{3}x^{3/2} - x^2 \right]_0^1$$

Step 5: Evaluate the integrated expression at the upper boundary (1) and subtract the value at the lower boundary (0):

$$\text{Area} = \left(\frac{4}{3}(1)^{3/2} - (1)^2 \right) - (0 - 0) = \frac{4}{3} - 1 = \frac{1}{3}$$

Final Answer: The area bounded by the curves is 1/3

Answer: (B) Go Back to Question 35

Q36.

Solution

Concept:

To find the derivative of a function $f(x)$ with respect to another function $g(x)$, we use the parametric differentiation formula $\frac{df}{dg} = \frac{df/dx}{dg/dx}$. Alternatively, we can use a substitution such as $x = \tan\theta$ to simplify the inverse trigonometric expressions before differentiating.

Solution:

Step 1: Let $u = \tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ and $v = \tan^{-1} x$. We need to evaluate $\frac{du}{dv}$.

Step 2: Simplify the expression for u using the trigonometric substitution $x = \tan\theta$, which implies $\theta = \tan^{-1} x = v$:

$$u = \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right) = \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right)$$

Step 3: Convert the internal terms into basic sine and cosine expressions to simplify further:

$$\frac{\sec\theta - 1}{\tan\theta} = \frac{\frac{1}{\cos\theta} - 1}{\frac{\sin\theta}{\cos\theta}} = \frac{1-\cos\theta}{\sin\theta}$$

Step 4: Use trigonometric half-angle identities ($1 - \cos\theta = 2\sin^2(\theta/2)$ and $\sin\theta = 2\sin(\theta/2)\cos(\theta/2)$):

$$\frac{1-\cos\theta}{\sin\theta} = \frac{2\sin^2(\theta/2)}{2\sin(\theta/2)\cos(\theta/2)} = \tan\left(\frac{\theta}{2}\right)$$

$$u = \tan^{-1}\left(\tan\left(\frac{\theta}{2}\right)\right) = \frac{\theta}{2}$$

Step 5: Substitute $\theta = v$ back into the simplified expression for u:

$$u = \frac{1}{2}v$$

Differentiate u directly with respect to v:

$$\frac{du}{dv} = \frac{1}{2}$$

Final Answer: The derivative with respect to $\tan^{-1} x$ is 1/2

Answer: (B)

Go Back to Question 36

Q37.

Solution

Concept:
To solve a linear trigonometric equation of the form $a\sin\theta + b\cos\theta = c$, we can divide the entire equation by $\sqrt{a^2+b^2}$ to compress the left side into a single sine or cosine compound angle formula, and then find the general solution.

Solution:
Step 1: Write down the given trigonometric equation:

$$\sin\theta + \cos\theta = 1$$

Step 2: Divide both sides of the equation by $\sqrt{1^2+1^2} = \sqrt{2}$:

$$\frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta = \frac{1}{\sqrt{2}}$$

Step 3: Express the left-hand side as a sine addition formula by substituting $\cos(\pi/4) = \frac{1}{\sqrt{2}}$ and $\sin(\pi/4) = \frac{1}{\sqrt{2}}$:

$$\sin\theta\cos\left(\frac{\pi}{4}\right) + \cos\theta\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\sin\left(\theta + \frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right)$$

Step 4: Apply the standard general solution rule for sine functions, which states that if $\sin X = \sin Y$, then $X = n\pi + (-1)^n Y$:

$$\theta + \frac{\pi}{4} = n\pi + (-1)^n\frac{\pi}{4}$$

$$\theta = n\pi + (-1)^n\frac{\pi}{4} - \frac{\pi}{4}$$

Step 5: Analyze the solution branches based on whether n is even or odd to match standard options: If $n = 2k$ (even): $\theta = 2k\pi + \frac{\pi}{4} - \frac{\pi}{4} = 2k\pi$ If $n = 2k+1$ (odd): $\theta = (2k+1)\pi - \frac{\pi}{4} - \frac{\pi}{4} = 2k\pi + \pi - \frac{\pi}{2} = 2k\pi + \frac{\pi}{2}$ Combining these gives the general solution format: $2n\pi$ or $2n\pi + \pi/2$.

Final Answer: The general value is $2n\pi$ or $2n\pi + \pi/2$

Answer: (A) Go Back to Question 37

Q38.

Solution

Concept:
The given series $1 + 2r + 3r^2 + 4r^3 + \dots$ is a standard infinite Arithmetico-Geometric Progression (AGP). The sum of this specific infinite series can be derived by multiplying the series by r, shifting the terms by one position, subtracting the two equations to form a pure infinite geometric progression, and applying the formula $S_\infty = \frac{1}{(1-r)^2}$.

Solution:
Step 1: Let the sum of the infinite series be denoted as S:

$$S = 1 + 2r + 3r^2 + 4r^3 + \dots \quad \text{— (Equation 1)}$$

Step 2: Multiply the entire series equation by the common ratio parameter r:

$$rS = r + 2r^2 + 3r^3 + 4r^4 + \dots \quad \text{— (Equation 2)}$$

Step 3: Subtract Equation 2 from Equation 1 by aligning corresponding terms with identical powers of r:

$$S - rS = 1 + (2r - r) + (3r^2 - 2r^2) + (4r^3 - 3r^3) + \dots$$

$$S(1 - r) = 1 + r + r^2 + r^3 + \dots$$

Step 4: Apply the standard infinite geometric series summation formula $1+r+r^2+\dots = \frac{1}{1-r}$ to the right-hand side:

$$S(1 - r) = \frac{1}{1 - r} \implies S = \frac{1}{(1 - r)^2}$$

Step 5: Equate this derived formula to the total sum value given in the problem statement, and solve for r:

$$\frac{1}{(1 - r)^2} = \frac{9}{4} \implies (1 - r)^2 = \frac{4}{9}$$

Taking the positive square root because of the constraint $|r| < 1$:

$$1 - r = \frac{2}{3} \implies r = 1 - \frac{2}{3} = \frac{1}{3}$$

Final Answer: The value of r is 1/3

Answer: (A) Go Back to Question 38

Q39.

Solution

Concept:
The direction vector of a straight line passing through two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by $\vec{v} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$. Two straight lines in space are perpendicular if and only if the dot product of their respective direction vectors is equal to zero ($\vec{v}_1 \cdot \vec{v}_2 = 0$).

Solution:
Step 1: Find the direction vector $\vec{v}_1$ of the first line passing through the points $A(4, 3, 2)$ and $B(1, a, 4)$:

$$\vec{v}_1 = (1 - 4)\hat{i} + (a - 3)\hat{j} + (4 - 2)\hat{k} = -3\hat{i} + (a - 3)\hat{j} + 2\hat{k}$$

Step 2: Find the direction vector $\vec{v}_2$ of the second line passing through the points $C(1, 2, -1)$ and $D(3, 3, 2)$:

$$\vec{v}_2 = (3 - 1)\hat{i} + (3 - 2)\hat{j} + (2 - (-1))\hat{k} = 2\hat{i} + \hat{j} + 3\hat{k}$$

Step 3: State the mathematical condition for perpendicularity, which requires the dot product of these two vectors to equal zero:

$$\vec{v}_1 \cdot \vec{v}_2 = 0$$

Step 4: Expand the dot product by multiplying corresponding scalar components and adding them together:

$$(-3)(2) + (a - 3)(1) + (2)(3) = 0$$

$$-6 + a - 3 + 6 = 0$$

Step 5: Simplify the linear equation to find the value of the unknown parameter a:

$$a - 3 = 0 \implies a = 3$$

Let us re-verify the options. Since 3 is the calculation result, if there is a small typo in option listing, let us match the closest value or double check the calculation steps: $-6 + a - 3 + 6 = a - 3 = 0 \implies a = 3$. Let us check option alignment.

Final Answer: The value of a is 3

Answer: (A)

Go Back to Question 39

Q40.

Solution

Concept:

To evaluate the sum of multiple trigonometric cosine terms, we can pair the terms strategically and apply the standard cosine addition-to-product formula: $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$. Pairing terms whose angles add up to a helpful value allows for simplification.

Solution:

Step 1: Write down the given trigonometric expression and rearrange the terms into two strategic pairs:

$$S = (\cos 12^\circ + \cos 132^\circ) + (\cos 84^\circ + \cos 156^\circ)$$

Step 2: Apply the product identity $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$ to the first pair:

$$\cos 132^\circ + \cos 12^\circ = 2\cos\left(\frac{132^\circ + 12^\circ}{2}\right)\cos\left(\frac{132^\circ - 12^\circ}{2}\right) = 2\cos 72^\circ \cos 60^\circ$$

Since $\cos 60^\circ = 1/2$, this simplifies to:

$$2\cos 72^\circ \cdot \left(\frac{1}{2}\right) = \cos 72^\circ$$

Step 3: Apply the same identity to the second pair of terms:

$$\cos 156^\circ + \cos 84^\circ = 2\cos\left(\frac{156^\circ + 84^\circ}{2}\right)\cos\left(\frac{156^\circ - 84^\circ}{2}\right) = 2\cos 120^\circ \cos 36^\circ$$

Since $\cos 120^\circ = -1/2$, this simplifies to:

$$2 \cdot \left(-\frac{1}{2}\right) \cdot \cos 36^\circ = -\cos 36^\circ$$

Step 4: Combine the simplified results of both pairs back into the total sum equation:

$$S = \cos 72^\circ - \cos 36^\circ$$

Step 5: Substitute the standard exact radical values for these specific angles ($\cos 36^\circ = \frac{\sqrt{5}+1}{4}$ and $\cos 72^\circ = \sin 18^\circ = \frac{\sqrt{5}-1}{4}$):

$$S = \frac{\sqrt{5}-1}{4} - \frac{\sqrt{5}+1}{4} = \frac{\sqrt{5}-1-\sqrt{5}-1}{4} = \frac{-2}{4} = -\frac{1}{2}$$

Final Answer: The value of the expression is -1/2

Answer: (B)

Go Back to Question 40

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	A	3	A	4	A	5	B
6	C	7	A	8	A	9	A	10	C
11	C	12	B	13	C	14	A	15	A
16	B	17	A	18	C	19	B	20	A
21	C	22	A	23	C	24	A	25	B
26	A	27	B	28	A	29	B	30	A
31	D	32	A	33	A	34	C	35	B
36	B	37	A	38	A	39	A	40	B