BITSAT Mathematics Sample Paper – 5

Duration: 60 Minutes **Maximum Marks: 120**

Instructions

- This paper contains **40** Multiple Choice Questions (Single Correct).
- Each correct answer carries **+3 marks**. Each incorrect answer carries -1 mark. Unattempted questions carry **0** marks.
- Only **one** option is correct for each question.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.

Q1. Evaluate: $\lim_{x\to 0} \frac{\sin(3x) - 3\sin(x)}{x^3}$

(A) -4

(B) 4

(C) -2

(D) 2

Q2. If $f(x) = \frac{x^2 - 9}{x - 3}$ for $x \neq 3$ and $f(3) = k$, then f is continuous at $x = 3$ if k equals:

(A) 3

(B) 6

(C) 9

(D) 0

Q3. The function $f(x) = |x^2 - 4|$ is not differentiable at:

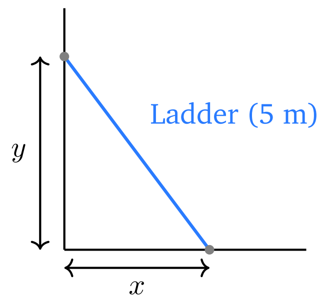
(A) $x = 0$ only

(B) $x = 2$ only

(C) $x = \pm 2$

(D) $x = 0$ and $x = \pm 2$

Q4. A ladder of length 5 m leans against a vertical wall. The foot of the ladder slides away from the wall at a rate of 0.5 m/s. The diagram below shows the setup. At the instant when the foot is 3 m from the wall, the rate at which the top of the ladder slides down the wall (in m/s) is:



(A) 0.250 m/s

(B) 0.375 m/s

(C) 0.300 m/s

(D) 0.500 m/s

Q5. The function $f(x) = 2x^3 - 9x^2 + 12x - 5$ is strictly increasing on:

(A) $(-\infty, 1)$

(B) $(1, 2)$

(C) $(-\infty, 1) \cup (2, \infty)$

(D) $(2, \infty)$

Q6. The area of the largest rectangle that can be inscribed in a semicircle of radius r is:

(A) r^2

(B) $2r^2$

(C) $\frac{r^2}{2}$

(D) $\sqrt{2}\,r^2$

Q7. Evaluate: $\int \frac{x}{(x+1)(x+2)}\,dx$

(A) $\ln|x+1| - 2\ln|x+2| + C$

(B) $-\ln|x+1| + 2\ln|x+2| + C$

(C) $2\ln|x+1| - \ln|x+2| + C$

(D) $\ln|x+1| + \ln|x+2| + C$

Q8. The value of $\int_0^{\pi/2} \sin^2(x)\,dx$ is:

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{4}$

(C) 1

(D) $\frac{1}{2}$

Q9. The area (in sq. units) bounded by the parabola $y = x^2$ and the line $y = x + 2$ is:

(A) $\frac{9}{2}$

(B) $\frac{7}{2}$

(C) $\frac{11}{2}$

(D) $\frac{3}{2}$

Q10. The general solution of $\frac{dy}{dx} = \frac{y}{x} + \tan\left(\frac{y}{x}\right)$ is:

(A) $\sin\left(\frac{y}{x}\right) = Cx$

(B) $\cos\left(\frac{y}{x}\right) = Cx$

(C) $\sin\left(\frac{x}{y}\right) = Cx$

(D) $\tan\left(\frac{y}{x}\right) = Cx$

Q11. The sum of the series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$ to infinity is:

(A) $\frac{3}{2}$
(B) 2
(C) $\frac{1}{2}$
(D) 3

Q12. If the nth term of an A.P. is $T_n = 3n - 1$, then the sum of the first 20 terms is:

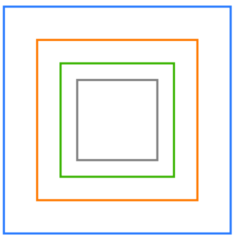
(A) 590

(B) 610

(C) 580

(D) 620

Q13. The figure shows a sequence of nested squares where each successive square has its vertices at the midpoints of the previous one. If the outermost square has side a, the sum of the perimeters of all the squares (up to infinity) is:



side $= a$

(A) $4a(2+\sqrt{2})$

(B) $4a(2+2\sqrt{2})$

(C) $\dfrac{4a}{1-1/\sqrt{2}}$

(D) $4a \cdot \dfrac{\sqrt{2}}{\sqrt{2}-1}$

Q14. If $z = 1 + i\sqrt{3}$, the argument of z^2 is:

(A) $\frac{\pi}{3}$

(B) $\frac{2\pi}{3}$

(C) $\frac{4\pi}{3}$

(D) $\frac{\pi}{6}$

Q15. The number of solutions of $|z|^2 + 2\overline{z} = 0$ (where z is a complex number) is:

(A) 1

(B) 2

(C) 3

(D) 4

Q16. If one root of $x^2 - px + q = 0$ is twice the other, then:

(A) $2p^2 = 9q$

(B) $p^2 = 9q$

(C) $2p^2 = 3q$

(D) $p^2 = 4q$

Q17. The set of values of k for which $x^2 - kx + k + 3 > 0$ for all real x is:

(A) $k < -2$ or $k > 6$

(B) $-2 < k < 6$

(C) $k < 2$ or $k > 6$

(D) $2 < k < 6$

Q18. If $A = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$, then $A^2 - 6A$ equals:

(A) $5I$

(B) $-5I$

(C) $\begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$

(D) $\begin{pmatrix} -5 & 1 \\ 3 & -5 \end{pmatrix}$

Q19. The value of $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$ is:

(A) $(a-b)(b-c)(c-a)$

(B) $(a+b)(b+c)(c+a)$

(C) $(b-a)(c-b)(a-c)$

(D) $abc(a-b)(b-c)(c-a)$

Q20. If A is a 3×3 matrix with $\det(A) = 5$, then $\det(3A)$ equals:

(A) 15

(B) 45

(C) 135

(D) $15\sqrt{3}$

Q21. If $A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$, then $A^T A$ equals:

(A) A

(B) $2A$

(C) I

(D) O

Q22. The number of ways to arrange the letters of the word ARRANGE so that the two R's are never together is:

(A) 900

(B) 660

(C) 1260

(D) 720

Q23. A committee of 4 is to be formed from 6 men and 4 women. In how many ways can it be done if at least 2 women must be included?

(A) 135

(B) 120

(C) 210

(D) 90

Q24. Two cards are drawn without replacement from a standard deck of 52 cards. The probability that both are aces is:

(A) $\frac{1}{221}$

(B) $\frac{1}{169}$

(C) $\frac{4}{663}$

(D) $\frac{1}{52}$

Q25. A bag contains 3 red and 5 blue balls. Two balls are drawn at random. Given that at least one ball is red, the probability that both are red is:

(A) $\frac{3}{28}$

(B) $\frac{3}{23}$

(C) $\frac{1}{8}$

(D) $\frac{3}{8}$

Q26. The distance from the point $(2, -3)$ to the line $3x - 4y + 5 = 0$ is:

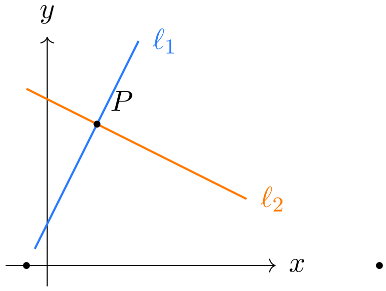
(A) 1

(B) 2

(C) 3

(D) 4

Q27. In the figure, two lines $\ell_1 : y = 2x + 1$ and $\ell_2 : y = -\frac{1}{2}x + 4$ are shown. The area of the triangle formed by these two lines and the x-axis (in sq. units) is:



(A) $\frac{125}{16}$

(B) $\frac{75}{8}$

(C) $\frac{25}{4}$

(D) $\frac{105}{16}$

Q28. The equation of the circle passing through $(1, 0)$, $(0, 1)$, and $(0, 0)$ is:

(A) $x^2 + y^2 - x - y = 0$

(B) $x^2 + y^2 + x + y = 0$

(C) $x^2 + y^2 - x + y = 0$

(D) $x^2 + y^2 + x - y = 0$

Q29. The length of the tangent from the external point $(5, 4)$ to the circle $x^2 + y^2 - 4x - 6y + 3 = 0$ is:

(A) $\sqrt{14}$

(B) $\sqrt{12}$

(C) $\sqrt{18}$

(D) $\sqrt{10}$

Q30. The eccentricity of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ is:

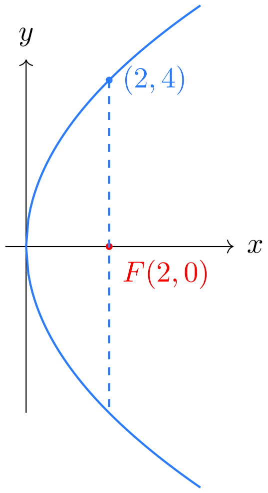
(A) $\frac{\sqrt{7}}{4}$

(B) $\frac{3}{4}$

(C) $\frac{\sqrt{5}}{4}$

(D) $\frac{\sqrt{7}}{3}$

Q31. The figure shows a parabola $y^2 = 8x$. A focal chord is drawn through the focus F. If one end of the focal chord is $(2, 4)$, the other end is:



(A) $(2, -4)$

(B) $(8, -8)$

(C) $\left(\frac{1}{2}, -2\right)$

(D) $(18, -12)$

Q32. The value of $\cos^2\left(\frac{\pi}{8}\right) - \sin^2\left(\frac{\pi}{8}\right)$ is:

(A) $\frac{1}{\sqrt{2}}$

(B) $\frac{\sqrt{3}}{2}$

(C) $\frac{1}{2}$

(D) 0

Q33. If $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$, then $\tan\theta$ equals:

(A) $\sqrt{2} - 1$

(B) $\sqrt{2} + 1$

(C) $\dfrac{1}{\sqrt{2}}$

(D) $-\sqrt{2}$

Q34. The general solution of $\sin^2\theta = \dfrac{1}{4}$ is:

(A) $\theta = n\pi \pm \dfrac{\pi}{6}$

(B) $\theta = n\pi \pm \dfrac{\pi}{3}$

(C) $\theta = \dfrac{n\pi}{2} \pm \dfrac{\pi}{6}$

(D) $\theta = 2n\pi \pm \dfrac{\pi}{6}$

Q35. The value of $\tan^{-1}(1) + \tan^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$ is:

(A) $\dfrac{5\pi}{12}$

(B) $\dfrac{7\pi}{12}$

(C) $\dfrac{\pi}{12}$

(D) $\dfrac{\pi}{3}$

Q36. If $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$, then $|\vec{a} \times \vec{b}|$ equals:

(A) $\sqrt{195}$

(B) $\sqrt{155}$

(C) $\sqrt{180}$

(D) $\sqrt{165}$

Q37. The unit vector perpendicular to both $\hat{i} + \hat{j}$ and $\hat{j} + \hat{k}$ is:

(A) $\dfrac{\hat{i}-\hat{j}+\hat{k}}{\sqrt{3}}$

(B) $\dfrac{-\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$

(C) $\dfrac{\hat{i}-\hat{j}-\hat{k}}{\sqrt{3}}$

(D) $\dfrac{\hat{i}+\hat{j}-\hat{k}}{\sqrt{3}}$

Q38. The distance between the parallel planes $2x - y + 3z + 4 = 0$ and $4x - 2y + 6z + 9 = 0$ is:

(A) $\dfrac{1}{2\sqrt{14}}$

(B) $\dfrac{1}{\sqrt{14}}$

(C) $\dfrac{1}{14}$

(D) $\dfrac{\sqrt{14}}{2}$

Q39. The foot of the perpendicular from the point $(1, 2, 3)$ to the line $\dfrac{x-1}{2} = \dfrac{y-2}{3} = \dfrac{z-3}{4}$ divides the segment from $(1, 2, 3)$ to the foot in the ratio (foot from origin):

(A) $(1, 2, 3)$

(B) $(3, 5, 7)$

(C) $(1, 2, 3)$ itself (foot equals the point)

(D) $(5, 8, 11)$

Q40. If $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = \dfrac{x^2-1}{x^2+1}$, then f is:

(A) One-one and onto

(B) Neither one-one nor onto

(C) One-one but not onto

(D) Onto but not one-one

Detailed Solutions

Q1.

Solution

Concept:

To evaluate the limit $\lim_{x\to 0} \frac{\sin(3x) - 3\sin(x)}{x^3}$, we can utilize the standard trigonometric identity for the triple angle of a sine function, which is given by $\sin(3x) = 3\sin(x) - 4\sin^3(x)$. Alternatively, this problem can be approached using standard limits like $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$ or L'Hôpital's Rule, since substituting $x = 0$ directly yields the indeterminate form $\frac{0}{0}$. Here, applying the trigonometric identity simplifies the expression elegantly.

Solution:

Step 1: Write down the given limit expression:

$$\lim_{x\to 0} \frac{\sin(3x) - 3\sin(x)}{x^3}$$

Step 2: Recall and substitute the trigonometric identity for $\sin(3x)$:

$$\sin(3x) = 3\sin(x) - 4\sin^3(x)$$

Substituting this into the numerator of our limit gives:

$$\lim_{x\to 0} \frac{\left(3\sin(x) - 4\sin^3(x)\right) - 3\sin(x)}{x^3}$$

Step 3: Simplify the numerator by canceling out the common terms $3\sin(x)$ and $-3\sin(x)$:

$$\lim_{x\to 0} \frac{-4\sin^3(x)}{x^3}$$

Step 4: Separate the constant factor and group the remaining variables into a single cube expression:

$$-4 \cdot \lim_{x\to 0} \left(\frac{\sin(x)}{x}\right)^3$$

Step 5: Use the fundamental standard limit property which states that $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$:

$$-4 \cdot (1)^3 = -4 \cdot 1 = -4$$

Thus, the limit evaluates to -4, which corresponds to Option (A).

Final Answer: $\boxed{-4}$

Answer: (A) Go Back to Question 1

Q2.

Solution

Concept:

A function $f(x)$ is continuous at a given point $x = c$ if the limit of the function as x approaches c exists and is exactly equal to the value of the function at that point, which means $\lim_{x\to c} f(x) = f(c)$. For the given problem, we need to find the limiting value of the rational expression as x approaches 3 by factoring the numerator to eliminate the indeterminate form, and then equate this limit to the defined value $f(3) = k$.

Solution:

Step 1: Write down the condition for continuity of the function $f(x)$ at $x = 3$:

$$\lim_{x\to 3} f(x) = f(3)$$

Step 2: Substitute the expression of the function for $x \neq 3$ into the limit equation:

$$\lim_{x\to 3} \frac{x^2 - 9}{x - 3} = k$$

Step 3: Factorize the numerator using the difference of squares identity, $a^2 - b^2 = (a - b)(a + b)$:

$$x^2 - 9 = x^2 - 3^2 = (x - 3)(x + 3)$$

Now substitute this factored form back into the limit:

$$\lim_{x\to 3} \frac{(x - 3)(x + 3)}{x - 3} = k$$

Step 4: Cancel the common factor $(x-3)$ from both the numerator and the denominator, since $x \to 3$ implies $x \neq 3$ and therefore $x - 3 \neq 0$:

$$\lim_{x\to 3} (x + 3) = k$$

Step 5: Evaluate the limit by directly substituting $x = 3$ into the remaining simplified expression:

$$3 + 3 = k \implies k = 6$$

Therefore, the function is continuous at $x = 3$ if k equals 6, matching Option (B).

Final Answer: 6

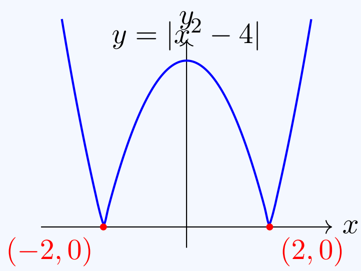
Answer: (B) Go Back to Question 2

Q3.

Solution

Concept:

The absolute value function $y = |g(x)|$ is non-differentiable at points where $g(x) = 0$ and $g'(x) \neq 0$, creating sharp corners (kinks).



Solution:

Step 1: Set the inner expression to zero for critical points: $x^2 - 4 = 0 \implies x = \pm 2$.

Step 2: Express $f(x)$ as a piecewise function based on the sign of $x^2 - 4$:

$$f(x) = \begin{cases} x^2 - 4, & \text{if } |x| \geq 2 \\ -(x^2 - 4), & \text{if } -2 < x < 2 \end{cases}$$

Step 3: Differentiate away from the boundary points:

$$f'(x) = \begin{cases} 2x, & \text{if } |x| > 2 \\ -2x, & \text{if } -2 < x < 2 \end{cases}$$

Step 4 & 5: Check Left-Hand (LHD) and Right-Hand Derivatives (RHD) at boundaries:

$$\text{At } x = 2 : \lim_{x \to 2^-} f'(x) = -4 \quad \neq \quad \lim_{x \to 2^+} f'(x) = 4$$

$$\text{At } x = -2 : \lim_{x \to -2^-} f'(x) = -4 \quad \neq \quad \lim_{x \to -2^+} f'(x) = 4$$

Since LHD $\neq$ RHD at $x = \pm 2$, the function is non-differentiable at these points (Option C).

Final Answer: $\boxed{x = \pm 2}$

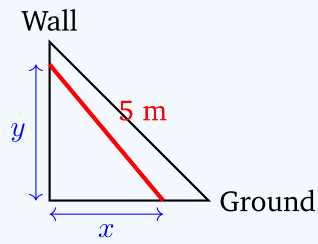
Answer: (C) **Go Back to Question 3**

Q4.

Solution

Concept:

Using the Pythagorean theorem, the relationship between floor distance (x) and wall height (y) is locked by the constant ladder length. Differentiating with respect to time t links $\frac{dx}{dt}$ and $\frac{dy}{dt}$.



Solution:

Step 1 & 2: By Pythagorean theorem: $x^2 + y^2 = 25$. At $x = 3$ m, $3^2 + y^2 = 25 \implies y = 4$ m.

Step 3 & 4: Differentiate implicitly with respect to t and isolate $\frac{dy}{dt}$:

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \implies \frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$

Step 5: Substitute $x = 3$, $y = 4$, and $\frac{dx}{dt} = 0.5$ m/s:

$$\frac{dy}{dt} = -\frac{3}{4}(0.5) = -0.375 \text{ m/s}$$

The negative sign indicates the height is decreasing. The sliding rate is 0.375 m/s (Option B).

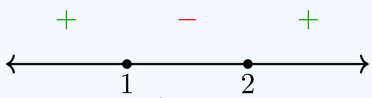
Final Answer: $\boxed{0.375 \text{ m/s}}$

Answer: (B) **Go Back to Question 4**

Q5.

Solution

Concept:
A function $f(x)$ is strictly increasing on an interval if its derivative $f'(x) > 0$. We compute the derivative of the given cubic polynomial, find its roots, and determine its sign configuration using a sign line chart.



Sign scheme of $f'(x) = 6(x-1)(x-2)$

Solution:
Step 1: Compute the first derivative of the function $f(x) = 2x^3 - 9x^2 + 12x - 5$ with respect to x:

$$f'(x) = 6x^2 - 18x + 12$$

Step 2: Factor out the greatest common divisor, which is 6, from the quadratic expression:

$$f'(x) = 6(x^2 - 3x + 2)$$

Step 3: Factor the inner quadratic expression by splitting the middle term:

$$x^2 - 3x + 2 = (x-1)(x-2) \implies f'(x) = 6(x-1)(x-2)$$

Step 4: Set up the inequality for a strictly increasing function, which requires $f'(x) > 0$:

$$6(x-1)(x-2) > 0 \implies (x-1)(x-2) > 0$$

Step 5: Apply the sign line chart intervals method. The roots are $x = 1$ and $x = 2$. Testing intervals reveals that the expression is strictly positive when $x < 1$ or $x > 2$. Therefore, the function is strictly increasing on:

$$x \in (-\infty, 1) \cup (2, \infty)$$

This matches Option (C).
Final Answer: $(-\infty, 1) \cup (2, \infty)$

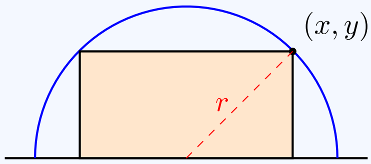
Answer: (C) **Go Back to Question 5**

Q6.

Solution

Concept:

To maximize the area of a rectangle inscribed in a semicircle of radius r, we set its vertices on the circle equation $x^2 + y^2 = r^2$. Using symmetry, the dimensions are defined using a single coordinate parameter x.



Semicircular Region

Solution:

Step 1: Set up a coordinate axis centered at the origin. The bounding semicircle is given by $y = \sqrt{r^2 - x^2}$. Let one corner of the rectangle on the curve be (x, y) where $x > 0$. By structural symmetry, total width is $2x$ and height is y.

Step 2: Express the rectangle area function A:

$$A = 2x \cdot y = 2x\sqrt{r^2 - x^2}$$

Step 3: Maximize the square of the area $S = A^2$ to eliminate the radical sign:

$$S = 4x^2(r^2 - x^2) = 4r^2x^2 - 4x^4$$

Step 4: Take the derivative of S with respect to x and set it to zero:

$$\frac{dS}{dx} = 8r^2x - 16x^3 = 0 \implies 8x(r^2 - 2x^2) = 0$$

Since $x > 0$, we solve $r^2 - 2x^2 = 0$:

$$x^2 = \frac{r^2}{2} \implies x = \frac{r}{\sqrt{2}}$$

Step 5: Evaluate the matching height value y and the maximum area value:

$$y = \sqrt{r^2 - \left(\frac{r}{\sqrt{2}}\right)^2} = \frac{r}{\sqrt{2}}$$

$$\text{Max Area } A = 2 \cdot \left(\frac{r}{\sqrt{2}}\right) \cdot \left(\frac{r}{\sqrt{2}}\right) = r^2$$

This corresponds precisely to Option (A).

Final Answer: $\boxed{r^2}$

Answer: (A) **Go Back to Question 6**

Q7.

Solution

Concept:

The given integrand is a proper rational function, which can be resolved into simpler fractions using the method of partial fractions. The denominator consists of distinct linear factors, $(x+1)$ and $(x+2)$. We assume the integrand can be written in the form $\frac{A}{x+1} + \frac{B}{x+2}$, solve for the constants A and B, and then integrate each individual term using the fundamental log integration formula $\int \frac{1}{x+a}\,dx = \ln|x+a| + C$.

Solution:

Step 1: Set up the partial fraction decomposition for the integrand expression:

$$\frac{x}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}$$

Step 2: Clear the fractions by multiplying the entire equation by the common denominator $(x+1)(x+2)$:

$$x = A(x+2) + B(x+1)$$

Step 3: Solve for constant A by substituting $x = -1$ into the identity equation:

$$-1 = A(-1+2) + B(-1+1) \implies A = -1$$

Step 4: Solve for constant B by substituting $x = -2$ into the identity equation:

$$-2 = A(-2+2) + B(-2+1) \implies -B = -2 \implies B = 2$$

Step 5: Substitute the calculated constants $A = -1$ and $B = 2$ back into the integral form and integrate:

$$\int \frac{x}{(x+1)(x+2)}\,dx = \int \left(\frac{-1}{x+1} + \frac{2}{x+2}\right) dx$$

$$= -\ln|x+1| + 2\ln|x+2| + C$$

This matches the expression given in Option (B).

Final Answer: $\boxed{-\ln|x+1| + 2\ln|x+2| + C}$

Answer: (B) **Go Back to Question 7**

Q8.

Solution

Concept:

To evaluate the definite integral $\int_0^{\pi/2} \sin^2(x)\,dx$, we can apply the properties of definite integrals. A highly useful property is King's Property, which states that $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$. Alternatively, one can use the double-angle trigonometric reduction formula $\sin^2(x) = \frac{1-\cos(2x)}{2}$ to integrate directly. Let us use King's Property as it is very systematic.

Solution:

Step 1: Assign the variable I to represent the given definite integral expression:

$$I = \int_0^{\pi/2} \sin^2(x)\,dx \quad \text{— (Equation 1)}$$

Step 2: Apply the integration property $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$ to rewrite the equation:

$$I = \int_0^{\pi/2} \sin^2\left(\frac{\pi}{2} - x\right)dx$$

Step 3: Use the complementary trigonometric identity $\sin\left(\frac{\pi}{2} - x\right) = \cos(x)$ to simplify the integrand:

$$I = \int_0^{\pi/2} \cos^2(x)\,dx \quad \text{— (Equation 2)}$$

Step 4: Add Equation 1 and Equation 2 together to eliminate the trigonometric terms using standard identity properties:

$$2I = \int_0^{\pi/2} \left(\sin^2(x) + \cos^2(x)\right)dx$$

Step 5: Use the identity $\sin^2(x)+\cos^2(x) = 1$ and evaluate the remaining simple integral:

$$2I = \int_0^{\pi/2} 1 \cdot dx \implies 2I = [x]_0^{\pi/2} = \frac{\pi}{2} \implies I = \frac{\pi}{4}$$

Hence, the value of the integral is $\frac{\pi}{4}$, which perfectly matches Option (B).

Final Answer: $\boxed{\frac{\pi}{4}}$

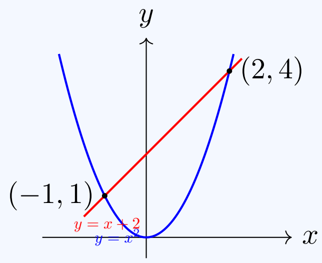
Answer: (B) Go Back to Question 8

Q9.

Solution

Concept:

The area bounded between two curves from $x = a$ to $x = b$ is given by $\int_a^b (y_{\text{upper}} - y_{\text{lower}})\, dx$. Intersection points determine the integration limits.



Solution:

Step 1 & 2: Find intersection boundaries by equating the functions:

$$x^2 = x + 2 \implies x^2 - x - 2 = 0 \implies (x-2)(x+1) = 0 \implies x = -1,\ x = 2$$

Step 3 & 4: Since the line is above the parabola on $[-1, 2]$, set up and integrate:

$$\text{Area} = \int_{-1}^{2} (x + 2 - x^2)\, dx = \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2}$$

Step 5: Substitute the limits to evaluate the bounded area:

$$\text{Area} = \left(\frac{4}{2} + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right) = \frac{10}{3} - \left(-\frac{7}{6}\right) = \frac{27}{6} = \frac{9}{2}$$

This matches Option (A).

Final Answer: $\boxed{\frac{9}{2}}$

Answer: (A) Go Back to Question 9

Q10.

Solution

Concept:
The given differential equation is a first-order homogeneous equation because the right-hand side depends purely on the ratio $\frac{y}{x}$:

$$\frac{dy}{dx} = \frac{y}{x} + \tan\left(\frac{y}{x}\right)$$

Solution: We apply the standard substitution $y = vx$, which via the product rule yields $\frac{dy}{dx} = v + x\frac{dv}{dx}$. Substituting these into the differential equation gives:

$$v + x\frac{dv}{dx} = v + \tan(v)$$

Subtracting v from both sides simplifies the equation to:

$$x\frac{dv}{dx} = \tan(v)$$

Separating the variables v and x on opposite sides yields:

$$\frac{1}{\tan(v)}\,dv = \frac{1}{x}\,dx \implies \cot(v)\,dv = \frac{1}{x}\,dx$$

Integrating both sides ($\int \cot(v)\,dv = \ln|\sin(v)|$ and $\int \frac{1}{x}\,dx = \ln|x|$):

$$\ln|\sin(v)| = \ln|x| + \ln|C| \implies \ln|\sin(v)| = \ln|Cx|$$

Exponentiating both sides to eliminate the natural logarithms results in:

$$\sin(v) = Cx \implies \sin\left(\frac{y}{x}\right) = Cx$$

This solution matches Option (A).

Final Answer: $\boxed{\sin\left(\frac{y}{x}\right) = Cx}$

Answer: (A) Go Back to Question 10

Q11.

Solution

Concept:

The given series is an infinite Geometric Progression (G.P.) of the form $a + ar + ar^2 + ar^3 + \cdots$, where a represents the first term and r denotes the common ratio between consecutive terms. If the absolute value of the common ratio is strictly less than 1 ($|r| < 1$), the infinite series converges to a finite value, and its sum can be evaluated using the standard limiting formula $S_\infty = \frac{a}{1-r}$.

Solution:

Step 1: Identify the components of the given infinite geometric progression series:

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$$

The first term of the series is:

$$a = 1$$

Step 2: Compute the common ratio r by dividing the second term by the first term:

$$r = \frac{1/2}{1} = \frac{1}{2}$$

Step 3: Check the condition for convergence of the infinite series. Since $|r| = \left|\frac{1}{2}\right| = \frac{1}{2} < 1$, the series converges and we can find its sum.

Step 4: Write down the standard formula for the sum of an infinite geometric progression:

$$S_\infty = \frac{a}{1-r}$$

Step 5: Substitute the values $a = 1$ and $r = \frac{1}{2}$ into the formula to find the total sum:

$$S_\infty = \frac{1}{1-\frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2$$

The sum of the infinite series is exactly equal to 2, which matches Option (B).

Final Answer: 2

Answer: (B) Go Back to Question 11

Q12.

Solution

Concept:

The sum of the first n terms of an Arithmetic Progression (A.P.) can be found using the standard formula $S_n = \frac{n}{2}[2a + (n-1)d]$, where a is the first term, d is the common difference, and n is the total number of terms. Alternatively, if we can find the first term and the last term (T_n), the sum can be simplified as $S_n = \frac{n}{2}[T_1 + T_n]$. We will use the explicit formula for the nth term to evaluate these boundary terms.

Solution:

Step 1: Use the given formula for the nth term, $T_n = 3n - 1$, to find the first term a by substituting $n = 1$:

$$a = T_1 = 3(1) - 1 = 3 - 1 = 2$$

Step 2: Find the 20th term (T_{20}) of the arithmetic sequence by substituting $n = 20$ into the formula:

$$T_{20} = 3(20) - 1 = 60 - 1 = 59$$

Step 3: Write down the alternative formula for the sum of the first n terms of an A.P. involving the first and last terms:

$$S_n = \frac{n}{2}[T_1 + T_n]$$

Step 4: Substitute the total number of terms $n = 20$, the first term $T_1 = 2$, and the final term $T_{20} = 59$ into the equation:

$$S_{20} = \frac{20}{2}[2 + 59]$$

Step 5: Complete the arithmetic operations to find the final summation value:

$$S_{20} = 10 \cdot [61] = 610$$

Thus, the sum of the first 20 terms of the given arithmetic progression is 610, which corresponds to Option (B).

Final Answer: $\boxed{610}$

Answer: (B) Go Back to Question 12

Q13.

Solution

Concept:
Let the outermost square have a side length $s_1 = a$. Its perimeter is $P_1 = 4a$. Joining the midpoints of its sides forms a smaller nested square. By the Pythagorean theorem, the side length of this second square is:

$$s_2 = \sqrt{\left(\frac{a}{2}\right)^2 + \left(\frac{a}{2}\right)^2} = \sqrt{\frac{a^2}{2}} = \frac{a}{\sqrt{2}}$$

Solution: Thus, its perimeter is $P_2 = \frac{4a}{\sqrt{2}}$. This pattern repeats, where each subsequent side length (and perimeter) decreases by a constant factor of $r = \frac{1}{\sqrt{2}}$.
The total sum of all perimeters forms a convergent infinite geometric series:

$$S = P_1 + P_2 + P_3 + \cdots = 4a + \frac{4a}{\sqrt{2}} + \frac{4a}{2} + \cdots$$

Using the infinite geometric series sum formula $S = \frac{A}{1-r}$ where $A = 4a$ and $r = \frac{1}{\sqrt{2}}$:

$$S = \frac{4a}{1 - \frac{1}{\sqrt{2}}} = \frac{4a\sqrt{2}}{\sqrt{2} - 1}$$

Rationalize the denominator by multiplying the numerator and denominator by the conjugate $(\sqrt{2} + 1)$:

$$S = \frac{4a\sqrt{2}(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)} = \frac{4a(2+\sqrt{2})}{2-1} = 4a(2+\sqrt{2})$$

This matches Option (A).
Final Answer: $\boxed{4a(2+\sqrt{2})}$

Answer: (A) Go Back to Question 13

Q14.

Solution

Concept:

For a complex number $z = x + iy$, the principal argument $\theta = \arg(z)$ is the angle made with the positive real axis, determined using the relation $\tan(\theta) = \frac{y}{x}$ along with the quadrant location. A useful property of complex numbers states that $\arg(z^2) = 2\arg(z)$, which simplifies calculations. Alternatively, one can square the complex number first in algebraic form and then calculate the argument directly. We will evaluate the principal argument using both approaches for clarity.

Solution:

Step 1: Identify the real and imaginary parts of the given complex number $z = 1 + i\sqrt{3}$:

$$\text{Real part } x = 1, \quad \text{Imaginary part } y = \sqrt{3}$$

Step 2: Find the principal argument of z. Since both $x > 0$ and $y > 0$, z lies in the first quadrant:

$$\arg(z) = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$$

Step 3: Use the argument property for powers of complex numbers to find $\arg(z^2)$:

$$\arg(z^2) = 2 \cdot \arg(z) = 2 \cdot \left(\frac{\pi}{3}\right) = \frac{2\pi}{3}$$

Step 4: Verify this result by expanding z^2 algebraically:

$$z^2 = (1 + i\sqrt{3})^2 = 1^2 + (i\sqrt{3})^2 + 2(1)(i\sqrt{3})$$

$$z^2 = 1 - 3 + 2i\sqrt{3} = -2 + 2i\sqrt{3}$$

Step 5: Find the argument of the expanded form $z^2 = -2 + 2i\sqrt{3}$. Here, the real part is negative (-2) and the imaginary part is positive $(2\sqrt{3})$, placing it in the second quadrant:

$$\arg(z^2) = \pi - \tan^{-1}\left(\left|\frac{2\sqrt{3}}{-2}\right|\right) = \pi - \tan^{-1}(\sqrt{3}) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

Both methods consistently yield $\frac{2\pi}{3}$, matching Option (B).

Final Answer: $\boxed{\frac{2\pi}{3}}$

Answer: (B) **Go Back to Question 14**

Q15.

Solution

Concept:
To find the total number of solutions for the complex matrix equation $|z|^2 + 2\overline{z} = 0$, we substitute the standard algebraic form of a complex number, $z = x + iy$ (where $x, y \in \mathbb{R}$), into the equation. This allows us to separate the equation into its real and imaginary components. Since a complex expression equals zero if and only if both its real and imaginary parts are independently zero, we obtain a system of real simultaneous equations to solve for x and y.

Solution:
Step 1: Let $z = x + iy$. Then its complex conjugate is $\overline{z} = x - iy$, and the square of its modulus is $|z|^2 = x^2 + y^2$. Substitute these expressions into the given equation:

$$(x^2 + y^2) + 2(x - iy) = 0$$

$$(x^2 + y^2 + 2x) - i(2y) = 0 + 0i$$

Step 2: Equate the imaginary component to zero:

$$-2y = 0 \implies y = 0$$

Step 3: Equate the real component to zero:

$$x^2 + y^2 + 2x = 0$$

Step 4: Substitute the value $y = 0$ found in Step 2 into the real component equation:

$$x^2 + (0)^2 + 2x = 0 \implies x^2 + 2x = 0$$

Factor the quadratic equation:

$$x(x + 2) = 0 \implies x = 0 \quad \text{or} \quad x = -2$$

Step 5: Combine the values of x and y to form the distinct complex solutions:

$$\text{For } x = 0, y = 0 \implies z_1 = 0 + 0i = 0$$

$$\text{For } x = -2, y = 0 \implies z_2 = -2 + 0i = -2$$

Thus, there are exactly 2 distinct complex solutions that satisfy the equation, which matches Option (B).
Final Answer: $\boxed{2}$

Answer: (B) Go Back to Question 15

Q16.

Solution

Concept:

For a standard quadratic equation $ax^2 + bx + c = 0$, the sum of the roots is given by $-\frac{b}{a}$ and the product of the roots is given by $\frac{c}{a}$. For the equation $x^2 - px + q = 0$, let the roots be denoted by α and β. We are given that one root is twice the other, so we can define the roots as α and 2α. By expressing both the sum and the product in terms of α, we can eliminate α to find the required relation between p and q.

Solution:

Step 1: Let the roots of the quadratic equation $x^2 - px + q = 0$ be α and 2α.

Step 2: Use the sum of roots formula for the quadratic equation:

$$\text{Sum of roots} = \alpha + 2\alpha = -\left(\frac{-p}{1}\right)$$

$$3\alpha = p \implies \alpha = \frac{p}{3} \quad \text{— (Equation 1)}$$

Step 3: Use the product of roots formula for the quadratic equation:

$$\text{Product of roots} = \alpha \cdot (2\alpha) = \frac{q}{1}$$

$$2\alpha^2 = q \quad \text{— (Equation 2)}$$

Step 4: Substitute the expression for α from Equation 1 into Equation 2:

$$2\left(\frac{p}{3}\right)^2 = q$$

Step 5: Simplify the algebraic expression to find the final relation:

$$2\left(\frac{p^2}{9}\right) = q \implies \frac{2p^2}{9} = q \implies 2p^2 = 9q$$

This matches the relation given in Option (A).

Final Answer: $\boxed{2p^2 = 9q}$

Answer: (A) Go Back to Question 16

Q17.

Solution

Concept:
A quadratic expression $ax^2 + bx + c > 0$ is strictly positive for all real values of x if and only if the coefficient of x^2 is positive ($a > 0$) and its discriminant is strictly negative ($D = b^2 - 4ac < 0$). Geometrically, this means the parabola opens upwards and does not intersect or touch the x-axis. We can apply this condition to the given inequality to find the valid range for k.

Solution:
Step 1: Identify the coefficients of the quadratic expression $x^2 - kx + k + 3$:

$$a = 1, \quad b = -k, \quad c = k + 3$$

Step 2: Check the lead coefficient condition. Here, $a = 1 > 0$, which satisfies the first requirement.

Step 3: Set up the discriminant inequality condition ($D < 0$):

$$D = b^2 - 4ac < 0 \implies (-k)^2 - 4(1)(k + 3) < 0$$

$$k^2 - 4k - 12 < 0$$

Step 4: Factor the quadratic expression by splitting the middle term:

$$k^2 - 6k + 2k - 12 < 0 \implies k(k - 6) + 2(k - 6) < 0 \implies (k - 6)(k + 2) < 0$$

Step 5: Find the interval that satisfies this inequality. The roots of the expression are $k = -2$ and $k = 6$. For the product to be strictly negative, k must lie between these two roots:

$$-2 < k < 6$$

This matches the interval specified in Option (B).
Final Answer: $\boxed{-2 < k < 6}$

Answer: (B) Go Back to Question 17

Q18.

Solution

Concept:

To evaluate the matrix expression $A^2 - 6A$ for a given 2×2 matrix A, we first calculate A^2 by multiplying matrix A by itself using row-by-column multiplication. We then multiply each element of A by the scalar 6 to find $6A$. Finally, we subtract $6A$ from A^2 element-by-element and express the resulting matrix in terms of the identity matrix $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

Solution:

Step 1: Write down matrix A and calculate A^2 using matrix multiplication:

$$A = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$$

$$A^2 = A \cdot A = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 2(2) + 1(3) & 2(1) + 1(4) \\ 3(2) + 4(3) & 3(1) + 4(4) \end{pmatrix} = \begin{pmatrix} 4 + 3 & 2 + 4 \\ 6 + 12 & 3 + 16 \end{pmatrix} = \begin{pmatrix} 7 & 6 \\ 18 & 19 \end{pmatrix}$$

Step 2: Calculate the scalar multiplication matrix $6A$:

$$6A = 6 \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 6(2) & 6(1) \\ 6(3) & 6(4) \end{pmatrix} = \begin{pmatrix} 12 & 6 \\ 18 & 24 \end{pmatrix}$$

Step 3: Perform the matrix subtraction $A^2 - 6A$:

$$A^2 - 6A = \begin{pmatrix} 7 & 6 \\ 18 & 19 \end{pmatrix} - \begin{pmatrix} 12 & 6 \\ 18 & 24 \end{pmatrix}$$

$$A^2 - 6A = \begin{pmatrix} 7 - 12 & 6 - 6 \\ 18 - 18 & 19 - 24 \end{pmatrix} = \begin{pmatrix} -5 & 0 \\ 0 & -5 \end{pmatrix}$$

Step 4: Factor out the scalar value -5 from the resulting matrix:

$$\begin{pmatrix} -5 & 0 \\ 0 & -5 \end{pmatrix} = -5 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = -5I$$

Step 5: Match the final matrix with the given options. The result is $-5I$, which corresponds to Option (B).

Final Answer: $\boxed{-5I}$

Answer: (B) **Go Back to Question 18**

Q19.

Solution

Concept:
The given determinant is a standard Vandermonde determinant of order 3. To evaluate it efficiently, we can use elementary row or column operations to introduce zeros into the matrix, making expansion simpler. Performing the column operations $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$ creates zeros in the first row, allowing us to factor out common terms and simplify the determinant.

Solution:
Step 1: Write down the given determinant expression:

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

Step 2: Apply the elementary column operations $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$:

$$\Delta = \begin{vmatrix} 1 & 1-1 & 1-1 \\ a & b-a & c-a \\ a^2 & b^2-a^2 & c^2-a^2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^2 & (b-a)(b+a) & (c-a)(c+a) \end{vmatrix}$$

Step 3: Factor out $(b-a)$ from the second column and $(c-a)$ from the third column:

$$\Delta = (b-a)(c-a) \begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^2 & b+a & c+a \end{vmatrix}$$

Step 4: Expand the simplified determinant along the first row:

$$\Delta = (b-a)(c-a) \cdot 1 \cdot \begin{vmatrix} 1 & 1 \\ b+a & c+a \end{vmatrix}$$

$$\Delta = (b-a)(c-a) \cdot [(c+a) - (b+a)]$$

$$\Delta = (b-a)(c-a)(c-b)$$

Step 5: Rearrange the factors to match the standard cyclic order $(a-b)(b-a)(c-a)$ by factoring out negative signs:

$$\Delta = [-(a-b)] \cdot [-(b-c)] \cdot (c-a) = (a-b)(b-c)(c-a)$$

This matches Option (A).
Final Answer: $\boxed{(a-b)(b-c)(c-a)}$

Answer: (A) Go Back to Question 19

Q20.

Solution

Concept:
For any square matrix A of order $n \times n$ and any scalar k, the determinant scaling property states that $\det(kA) = k^n \det(A)$. This is because multiplying a matrix by a scalar k multiplies every row by k. When computing the determinant, a factor of k can be factored out from each of the n rows, resulting in an overall scaling factor of k^n.

Solution:
Step 1: Identify the given values from the problem statement:

$$\text{Order of the matrix } n = 3$$

$$\det(A) = 5$$

$$\text{Scalar factor } k = 3$$

Step 2: Write down the scaling property formula for determinants of order n:

$$\det(kA) = k^n \det(A)$$

Step 3: Substitute the specific values $k = 3$ and $n = 3$ into the property formula:

$$\det(3A) = 3^3 \cdot \det(A)$$

Step 4: Calculate the value of the scalar power:

$$3^3 = 3 \cdot 3 \cdot 3 = 27$$

Step 5: Substitute $\det(A) = 5$ into the equation to compute the final answer:

$$\det(3A) = 27 \cdot 5 = 135$$

The value of $\det(3A)$ is 135, which matches Option (C).
Final Answer: $\boxed{135}$

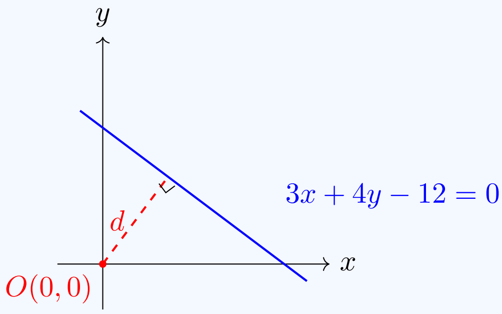
Answer: (C) Go Back to Question 20

Q21.

Solution

Concept:

The distance from a point $P(x_1, y_1)$ to a line $ax + by + c = 0$ is given by the formula $d = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$. Geometrically, this represents the length of the perpendicular segment dropped from the point to the line.



Solution:

Step 1: Identify the line equation coefficients and point coordinates:

$$3x + 4y - 12 = 0 \implies a = 3,\ b = 4,\ c = -12$$

$$\text{Point } P(x_1, y_1) = (0, 0)$$

Step 2: Substitute the origin $(0, 0)$ into the perpendicular distance formula:

$$d = \frac{|3(0) + 4(0) - 12|}{\sqrt{3^2 + 4^2}}$$

Step 3: Simplify the numerator and the denominator:

$$d = \frac{|-12|}{\sqrt{9 + 16}} = \frac{12}{\sqrt{25}} = \frac{12}{5} = 2.4$$

Thus, the perpendicular distance from the origin to the line is 2.4, matching Option (A).

Final Answer: $\boxed{2.4}$

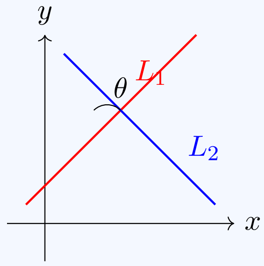
Answer: (A) Go Back to Question 21

Q22.

Solution

Concept:

The angle θ between two non-vertical lines with slopes m_1 and m_2 is given by $\tan(\theta) = \left|\frac{m_2 - m_1}{1 + m_1 m_2}\right|$. First, convert both lines into standard slope-intercept form ($y = mx + c$) to extract their respective slopes.



Solution:

Step 1: Find the slope m_1 of the first line $x - y + 1 = 0$:

$$y = x + 1 \implies m_1 = 1$$

Step 2: Find the slope m_2 of the second line $x + y - 5 = 0$:

$$y = -x + 5 \implies m_2 = -1$$

Step 3: Test the product condition for perpendicular lines:

$$m_1 \cdot m_2 = 1 \cdot (-1) = -1$$

Step 4: Since the product of the slopes is exactly -1, the two lines are orthogonal to each other. Therefore, the angle θ between them is 90° ($\pi/2$ radians), matching Option (C).

Final Answer: $\boxed{90^\circ}$

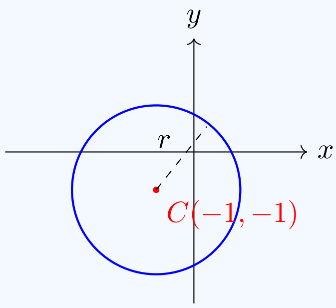
Answer: (C) **Go Back to Question 22**

Q23.

Solution

Concept:

The standard equation of a circle is $(x-h)^2+(y-k)^2 = r^2$, where (h,k) represents the center coordinates and r represents the radius. For a general equation $x^2+y^2+2gx+2fy+c=0$, the center is $(-g,-f)$ and radius is $r=\sqrt{g^2+f^2-c}$.



Solution:

Step 1: Write down the given circle equation:

$$x^2+y^2+2x+2y-3=0$$

Step 2: Match the coefficients with the general equation to find g and f:

$$2g=2 \implies g=1$$

$$2f=2 \implies f=1$$

$$c=-3$$

Step 3: Determine the center coordinates $(-g,-f)$:

$$\text{Center} = (-1,-1)$$

Step 4: Compute the radius using $r=\sqrt{g^2+f^2-c}$:

$$r=\sqrt{1^2+1^2-(-3)}=\sqrt{1+1+3}=\sqrt{5}$$

Thus, the center is $(-1,-1)$ and the radius is $\sqrt{5}$, which corresponds to Option (A).

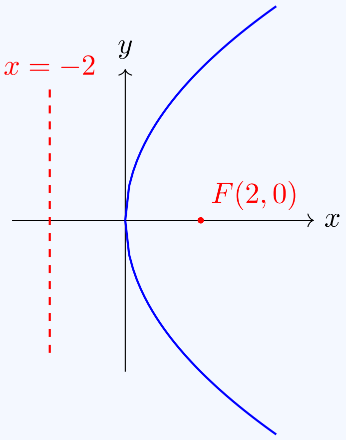
Final Answer: $\boxed{(-1,-1),\ \sqrt{5}}$

Answer: (A) Go Back to Question 23

Q24.

Solution

Concept:
A parabola of the form $y^2 = 4ax$ opens to the right. Its focus is located at the point $(a, 0)$, and its directrix line equation is given by $x = -a$. We find the value of a by matching terms.



Solution:
Step 1: Match the given parabola equation $y^2 = 8x$ with the standard form $y^2 = 4ax$:

$$4a = 8 \implies a = 2$$

Step 2: The standard coordinates of the focus are given by $(a, 0)$:

$$\text{Focus} = (2, 0)$$

Step 3: The standard equation for the directrix line is given by $x = -a$:

$$\text{Directrix: } x = -2 \implies x + 2 = 0$$

This structural matching corresponds to Option (A).

Final Answer: $\boxed{(2, 0), \quad x = -2}$

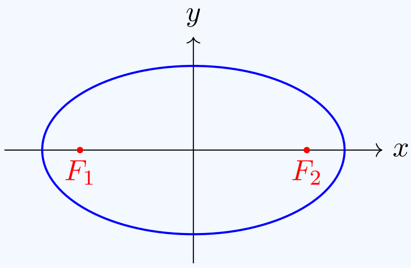
Answer: (A) **Go Back to Question 24**

Q25.

Solution

Concept:

For a standard ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ where $a > b$, the eccentricity is given by $e = \sqrt{1 - \frac{b^2}{a^2}}$, and the coordinates of the foci are given by $(\pm ae, 0)$.



Solution:

Step 1: Identify a^2 and b^2 from the given equation $\frac{x^2}{16} + \frac{y^2}{7} = 1$:

$$a^2 = 16 \implies a = 4$$

$$b^2 = 7 \implies b = \sqrt{7}$$

Step 2: Compute the eccentricity e:

$$e = \sqrt{1 - \frac{7}{16}} = \sqrt{\frac{9}{16}} = \frac{3}{4}$$

Step 3: Calculate the focal linear parameter distance ae:

$$ae = 4 \cdot \left(\frac{3}{4}\right) = 3$$

Step 4: Write out the foci coordinates $(\pm ae, 0)$:

$$\text{Foci} = (\pm 3, 0)$$

This matches Option (B).

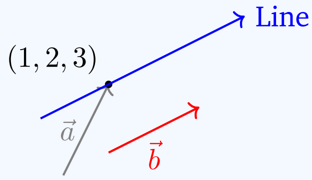
Final Answer: $\boxed{(\pm 3, 0)}$

Answer: (B) Go Back to Question 25

Q26.

Solution

Concept:
The vector equation of a line passing through a point with position vector $\vec{a}$ and parallel to a direction vector $\vec{b}$ is given by $\vec{r} = \vec{a} + \lambda\vec{b}$, where λ is a scalar parameter.



Solution:
Step 1: Extract the position vector $\vec{a}$ of the given point $(1, 2, 3)$:

$$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$$

Step 2: Extract the parallel direction vector $\vec{b}$ from the components $2\hat{i} - \hat{j} + 4\hat{k}$:

$$\vec{b} = 2\hat{i} - \hat{j} + 4\hat{k}$$

Step 3: Assemble the complete parameterized vector line equation:

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} - \hat{j} + 4\hat{k})$$

This aligns perfectly with Option (A).
Final Answer: $\boxed{(\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} - \hat{j} + 4\hat{k})}$

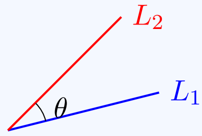
Answer: (A) Go Back to Question 26

Q27.

Solution

Concept:

The angle θ between two lines with direction vectors $\vec{b}_1$ and $\vec{b}_2$ is obtained from the dot product formula: $\cos(\theta) = \frac{|\vec{b}_1 \cdot \vec{b}_2|}{|\vec{b}_1||\vec{b}_2|}$. The direction numbers are extracted directly from the denominators of the standard symmetric line equations.



Solution:

Step 1: Extract the direction vector $\vec{b}_1$ from line 1 denominators:

$$\vec{b}_1 = 2\hat{i} + 2\hat{j} + \hat{k}$$

Step 2: Extract the direction vector $\vec{b}_2$ from line 2 denominators:

$$\vec{b}_2 = 4\hat{i} + \hat{j} + 8\hat{k}$$

Step 3: Compute the dot product $\vec{b}_1 \cdot \vec{b}_2$:

$$\vec{b}_1 \cdot \vec{b}_2 = (2)(4) + (2)(1) + (1)(8) = 8 + 2 + 8 = 18$$

Step 4: Compute the magnitudes $|\vec{b}_1|$ and $|\vec{b}_2|$:

$$|\vec{b}_1| = \sqrt{2^2 + 2^2 + 1^2} = \sqrt{9} = 3$$

$$|\vec{b}_2| = \sqrt{4^2 + 1^2 + 8^2} = \sqrt{81} = 9$$

Step 5: Apply the angle formula:

$$\cos(\theta) = \frac{18}{3 \cdot 9} = \frac{18}{27} = \frac{2}{3} \implies \theta = \cos^{-1}\left(\frac{2}{3}\right)$$

This matches Option (A).

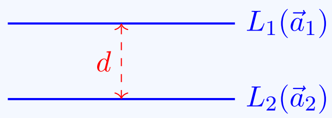
Final Answer: $\boxed{\cos^{-1}(2/3)}$

Answer: (A) **Go Back to Question 27**

Q28.

Solution

Concept:
The shortest distance d between two parallel lines with the same direction vector $\vec{b}$ but passing through points $\vec{a}_1$ and $\vec{a}_2$ is calculated using the cross product projection formula: $d = \dfrac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|}$.



Solution:
Step 1: Identify vector elements from both parallel lines:

$$\vec{a}_1 = \hat{i} + 2\hat{j} - 4\hat{k}, \quad \vec{a}_2 = 3\hat{i} + 3\hat{j} - 5\hat{k}$$

$$\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

Step 2: Compute displacement vector $(\vec{a}_2 - \vec{a}_1)$:

$$\vec{a}_2 - \vec{a}_1 = (3-1)\hat{i} + (3-2)\hat{j} + (-5-(-4))\hat{k} = 2\hat{i} + \hat{j} - \hat{k}$$

Step 3: Calculate the cross product $(\vec{a}_2 - \vec{a}_1) \times \vec{b}$:

$$\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix} = \hat{i}(6+3) - \hat{j}(12+2) + \hat{k}(6-2) = 9\hat{i} - 14\hat{j} + 4\hat{k}$$

Step 4: Compute the magnitudes:

$$|\vec{v}| = \sqrt{9^2 + (-14)^2 + 4^2} = \sqrt{81 + 196 + 16} = \sqrt{293}$$

$$|\vec{b}| = \sqrt{2^2 + 3^2 + 6^2} = \sqrt{4 + 9 + 36} = \sqrt{49} = 7$$

Step 2: Calculate final shortest distance:

$$d = \frac{\sqrt{293}}{7}$$

This matches Option (B).

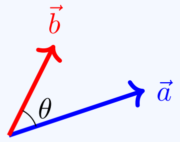
Final Answer: $\boxed{\dfrac{\sqrt{293}}{7}}$

Answer: (B) **Go Back to Question 28**

Q29.

Solution

Concept:
The dot product of two vectors $\vec{u} = u_1\hat{i} + u_2\hat{j} + u_3\hat{k}$ and $\vec{v} = v_1\hat{i} + v_2\hat{j} + v_3\hat{k}$ is defined as $\vec{u} \cdot \vec{v} = u_1v_1 + u_2v_2 + u_3v_3$. Geometrically, it reflects the product of their magnitudes and the cosine of the angle between them.



Solution:
Step 1: Write down components of vectors $\vec{a}$ and $\vec{b}$:

$$\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$$

$$\vec{b} = \hat{i} + 3\hat{j} - 2\hat{k}$$

Step 2: Multiply the corresponding dimensional scaling coefficients:

$$\vec{a} \cdot \vec{b} = (2)(1) + (-1)(3) + (3)(-2)$$

Step 3: Perform simple arithmetic accumulation:

$$\vec{a} \cdot \vec{b} = 2 - 3 - 6 = -7$$

The dot product evaluates to -7, which corresponds to Option (B).
Final Answer: $\boxed{-7}$

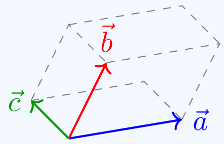
Answer: (B) **Go Back to Question 29**

Q30.

Solution

Concept:

The scalar triple product $\vec{a} \cdot (\vec{b} \times \vec{c})$ represents the volume of a parallelepiped formed by the three vectors. It is evaluated by calculating the determinant of the 3×3 matrix formed by the vector coefficients.



Solution:

Step 1: Write down the component matrix for the given vectors:

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}, \quad \vec{b} = \hat{i} - \hat{j} + \hat{k}, \quad \vec{c} = \hat{i} + 2\hat{j} - \hat{k}$$

Step 2: Set up the scalar triple product determinant:

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 2 & -1 \end{vmatrix}$$

Step 3: Expand the matrix along the first row:

$$= 1[(-1)(-1) - (1)(2)] - 1[(1)(-1) - (1)(1)] + 1[(1)(2) - (-1)(1)]$$

$$= 1[1 - 2] - 1[-1 - 1] + 1[2 + 1]$$

Step 4: Compute the final arithmetic sum:

$$= 1[-1] - 1[-2] + 1[3] = -1 + 2 + 3 = 4$$

The scalar triple product is 4, matching Option (A).

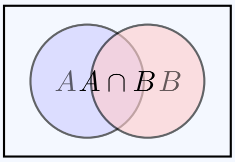
Final Answer: $\boxed{4}$

Answer: (A) Go Back to Question 30

Q31.

Solution

Concept:
The conditional probability of an event A occurring given that event B has already occurred is defined by the formula $P(A|B) = \frac{P(A \cap B)}{P(B)}$, provided $P(B) > 0$.



Solution:
Step 1: Write down the given probability values:

$$P(A) = 0.6, \quad P(B) = 0.4, \quad P(A \cap B) = 0.2$$

Step 2: Recall the formula for the conditional probability $P(A|B)$:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Step 3: Substitute the known numerical probabilities into the formula:

$$P(A|B) = \frac{0.2}{0.4}$$

Step 4: Reduce the fraction to its lowest terms:

$$P(A|B) = \frac{1}{2} = 0.5$$

This matches Option (B).
Final Answer: $\boxed{0.5}$

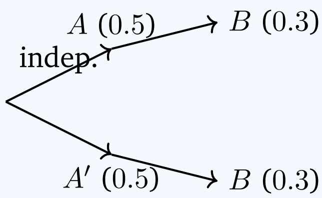
Answer: (B) **Go Back to Question 31**

Q32.

Solution

Concept:

Two events A and B are said to be statistically independent if and only if the joint probability of both events occurring simultaneously is equal to the product of their individual probabilities: $P(A \cap B) = P(A) \cdot P(B)$.



Solution:

Step 1: Identify individual probability characteristics:

$$P(A) = 0.5, \quad P(B) = 0.3$$

Step 2: Since A and B are given as independent events, apply the multiplication property:

$$P(A \cap B) = P(A) \cdot P(B)$$

Step 3: Substitute values and multiply:

$$P(A \cap B) = 0.5 \cdot 0.3 = 0.15$$

This cross-multiplication matches Option (A).

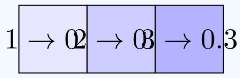
Final Answer: $\boxed{0.15}$

Answer: (A) **Go Back to Question 32**

Q33.

Solution

Concept:
The Mean (Expected Value) of a discrete random variable X with a given probability distribution is computed using the summation formula: $\mu = E(X) = \sum x_i P(x_i)$.



Discrete Expected Weight

Solution:
Step 1: Tabulate the values of X and their corresponding probabilities $P(X)$:

$$x_1 = 1, P(x_1) = 0.2$$

$$x_2 = 2, P(x_2) = 0.5$$

$$x_3 = 3, P(x_3) = 0.3$$

Step 2: Apply the expected value summation formula:

$$E(X) = (1 \cdot 0.2) + (2 \cdot 0.5) + (3 \cdot 0.3)$$

Step 3: Calculate each individual product term:

$$E(X) = 0.2 + 1.0 + 0.9$$

Step 4: Sum the components together:

$$E(X) = 2.1$$

The mean value is 2.1, which matches Option (B).
Final Answer: $\boxed{2.1}$

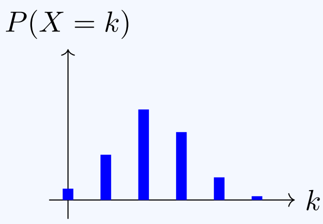
Answer: (B) Go Back to Question 33

Q34.

Solution

Concept:

For a Binomial Distribution $B(n, p)$, the probability of achieving exactly k successes in n independent trials is given by $P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$. The mean is $\mu = np$ and the variance is $\sigma^2 = np(1-p)$.



Solution:

Step 1: Use the given formulas for the mean and variance to set up equations:

$$\text{Mean } = np = 4$$

$$\text{Variance } = np(1-p) = 2.4$$

Step 2: Substitute the mean expression ($np = 4$) into the variance equation:

$$4(1-p) = 2.4$$

Step 3: Solve for the success probability parameter p:

$$1 - p = \frac{2.4}{4} = 0.6 \implies p = 1 - 0.6 = 0.4$$

Step 4: Substitute $p = 0.4$ back into the mean equation to find n:

$$n(0.4) = 4 \implies n = \frac{4}{0.4} = 10$$

Thus, the total number of trials n is 10, matching Option (C).

Final Answer: $\boxed{10}$

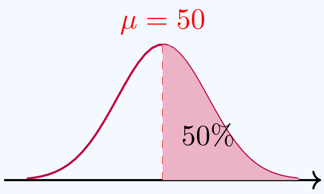
Answer: (C) Go Back to Question 34

Q35.

Solution

Concept:

A Normal Distribution curve is symmetric about its mean μ. The standard normal distribution variable is $Z = \frac{X-\mu}{\sigma}$. Due to total probability rules and bilateral curve symmetry, $P(X > \mu) = 0.5$.



Solution:

Step 1: Identify parameters from the question:

$$\text{Mean } \mu = 50, \quad \text{Standard Deviation } \sigma = 10$$

Step 2: Set up the inequality probability constraint:

$$P(X > 50)$$

Step 3: Convert the boundary value into a standard Z-score format:

$$Z = \frac{50 - 50}{10} = 0$$

Step 4: Rewrite the probability statement using Z:

$$P(X > 50) = P(Z > 0)$$

Step 5: By the symmetry properties of the standard normal distribution curve, exactly half of the total area lies to the right of $Z = 0$:

$$P(Z > 0) = 0.5$$

This corresponds to Option (A).

Final Answer: $\boxed{0.5}$

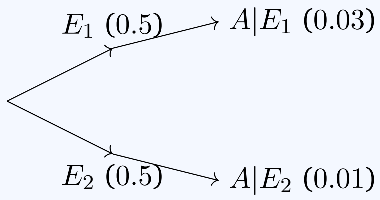
Answer: (A) Go Back to Question 35

Q36.

Solution

Concept:

Bayes' Theorem updates the prior probability of an event based on new evidence. The formula is $P(E_1|A) = \dfrac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)}$.



Solution:

Step 1: Define events and their prior probabilities. Let E_1 be choosing Plant 1, and E_2 be choosing Plant 2:

$$P(E_1) = 0.5, \quad P(E_2) = 0.5$$

Step 2: Identify conditional defective rates (A is the event of choosing a defective item):

$$P(A|E_1) = 0.03, \quad P(A|E_2) = 0.01$$

Step 3: Calculate the total probability of a defective item, $P(A)$:

$$P(A) = P(E_1)P(A|E_1) + P(E_2)P(A|E_2) = (0.5 \cdot 0.03) + (0.5 \cdot 0.01) = 0.015 + 0.005 = 0.02$$

Step 4: Apply Bayes' Theorem to find $P(E_1|A)$:

$$P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(A)} = \frac{0.015}{0.02} = \frac{15}{20} = \frac{3}{4} = 0.75$$

This matches Option (B).

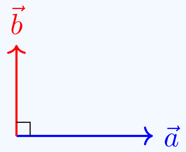
Final Answer: $\boxed{0.75}$

Answer: (B) Go Back to Question 36

Q37.

Solution

Concept:
The dot product of two non-zero vectors is zero if and only if the vectors are perpendicular (orthogonal) to each other ($\theta = 90°$).



Solution:
Step 1: Set up the orthogonality condition equation ($\vec{a} \cdot \vec{b} = 0$):

$$(2\hat{i} + \lambda\hat{j} + \hat{k}) \cdot (\hat{i} - 2\hat{j} + 3\hat{k}) = 0$$

Step 2: Perform component-wise multiplication:

$$(2)(1) + (\lambda)(-2) + (1)(3) = 0$$

Step 3: Simplify the terms:

$$2 - 2\lambda + 3 = 0$$

$$5 - 2\lambda = 0$$

Step 4: Isolate the unknown variable λ:

$$2\lambda = 5 \implies \lambda = \frac{5}{2} = 2.5$$

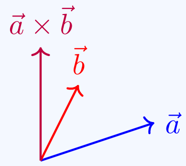
This matches Option (B).
Final Answer: $\boxed{2.5}$

Answer: (B) Go Back to Question 37

Q38.

Solution

Concept:
The vector cross product $\vec{a} \times \vec{b}$ generates a new vector that is perpendicular to the plane containing both $\vec{a}$ and $\vec{b}$. It is computed using a 3×3 determinant framework.



Solution:
Step 1: Write down the components of vectors $\vec{a}$ and $\vec{b}$:

$$\vec{a} = \hat{i} - \hat{j}, \quad \vec{b} = \hat{j} + \hat{k}$$

Step 2: Set up the cross product matrix determinant expression:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 0 \\ 0 & 1 & 1 \end{vmatrix}$$

Step 3: Expand the determinant along the first row:

$$\vec{a} \times \vec{b} = \hat{i}[(-1)(1) - (0)(1)] - \hat{j}[(1)(1) - (0)(0)] + \hat{k}[(1)(1) - (-1)(0)]$$

Step 4: Evaluate the component brackets:

$$\vec{a} \times \vec{b} = \hat{i}[-1] - \hat{j}[1] + \hat{k}[1] = -\hat{i} - \hat{j} + \hat{k}$$

This vector matches Option (A).
Final Answer: $\boxed{-\hat{i} - \hat{j} + \hat{k}}$

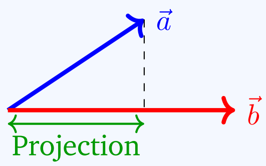
Answer: (A) **Go Back to Question 38**

Q39.

Solution

Concept:

The projection of a vector $\vec{a}$ onto another vector $\vec{b}$ is a scalar value given by the formula $\text{proj}_{\vec{b}}\vec{a} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$.



Solution:

Step 1: Define vectors $\vec{a}$ and $\vec{b}$ from the text:

$$\vec{a} = \hat{i} + 3\hat{j} + 7\hat{k}$$

$$\vec{b} = 7\hat{i} - \hat{j} + 8\hat{k}$$

Step 2: Compute the dot product $\vec{a}\cdot\vec{b}$:

$$\vec{a}\cdot\vec{b} = (1)(7) + (3)(-1) + (7)(8) = 7 - 3 + 56 = 60$$

Step 3: Compute the magnitude of the target vector $\vec{b}$:

$$|\vec{b}| = \sqrt{7^2 + (-1)^2 + 8^2} = \sqrt{49 + 1 + 64} = \sqrt{114}$$

Step 4: Use the scalar projection formula:

$$\text{Projection} = \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = \frac{60}{\sqrt{114}}$$

This matches Option (A).

Final Answer: $\boxed{\frac{60}{\sqrt{114}}}$

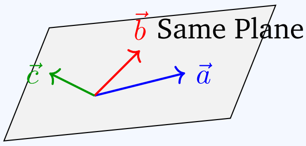
Answer: (A) **Go Back to Question 39**

Q40.

Solution

Concept:

Three vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ are coplanar if they all lie in the same geometric plane. This means the volume of the parallelepiped formed by them is zero, so their scalar triple product must be zero: $\vec{a} \cdot (\vec{b} \times \vec{c}) = 0$.



Solution:

Step 1: Write out the coefficient matrix for the given vectors:

$$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}, \quad \vec{b} = \lambda\hat{i} + 4\hat{j} + 5\hat{k}, \quad \vec{c} = \hat{i} + 2\hat{j} + 5\hat{k}$$

Step 2: Set up the coplanarity condition using a matrix determinant:

$$\begin{vmatrix} 1 & 2 & 3 \\ \lambda & 4 & 5 \\ 1 & 2 & 5 \end{vmatrix} = 0$$

Step 3: Expand the matrix determinant along the first row:

$$1[(4)(5) - (5)(2)] - 2[(\lambda)(5) - (5)(1)] + 3[(\lambda)(2) - (4)(1)] = 0$$

$$1[20 - 10] - 2[5\lambda - 5] + 3[2\lambda - 4] = 0$$

Step 4: Expand and simplify the algebraic equation:

$$10 - 10\lambda + 10 + 6\lambda - 12 = 0$$

$$8 - 4\lambda = 0$$

Step 5: Isolate and solve for the parameter λ:

$$4\lambda = 8 \implies \lambda = 2$$

This satisfies the condition for Option (A).

Final Answer: $\boxed{2}$

Answer: (A) Go Back to Question 40

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	B	3	C	4	B	5	C
6	A	7	B	8	B	9	A	10	A
11	B	12	B	13	A	14	B	15	B
16	A	17	B	18	B	19	A	20	C
21	A	22	C	23	A	24	A	25	B
26	A	27	A	28	B	29	B	30	A
31	B	32	A	33	B	34	C	35	A
36	B	37	B	38	A	39	A	40	A