BITSAT Mathematics Sample Paper-8

Duration: 60 Minutes **Maximum Marks: 120**

Instructions

- This paper contains **40** Multiple Choice Questions (Single Correct).
- Each correct answer carries **+3 marks**. Each incorrect answer carries: **−1** marks. Unattempted questions carry **0** marks.
- Only one option is correct for each question.
- Use of mobile phones, smartwatches, calculators, or any electronic gadgets is strictly prohibited.

Q1. The value of $\lim_{x \to 0} \dfrac{\sin(3x) + \tan(2x)}{5x}$ is:

(A) 0

(B) 1

(C) $\dfrac{3}{5}$

(D) $\dfrac{2}{5}$

Q2. If $A = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix}$, then $\det(AB)$ equals:

(A) 10

(B) 25

(C) -10

(D) 5

Q3. The sum of the infinite geometric series $3 - 1 + \dfrac{1}{3} - \dfrac{1}{9} + \cdots$ is:

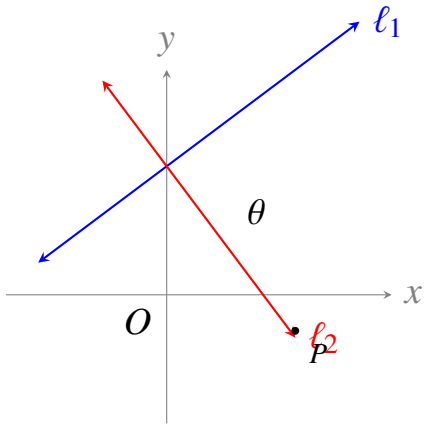
(A) $\dfrac{9}{4}$

(B) 3

(C) $\dfrac{3}{2}$

(D) $\frac{9}{2}$

Q4. In the figure below, two lines $\ell_1 : 3x - 4y + 8 = 0$ and $\ell_2 : 4x + 3y - 6 = 0$ are shown. The acute angle θ between them is:



(A) $30°$

(B) $45°$

(C) $60°$

(D) $90°$

Q5. The value of $\int_0^{\pi} x \sin x \, dx$ is:

(A) 0

(B) π

(C) 2π

(D) $-\pi$

Q6. If $z = \frac{1+i}{1-i}$, then z^4 equals:

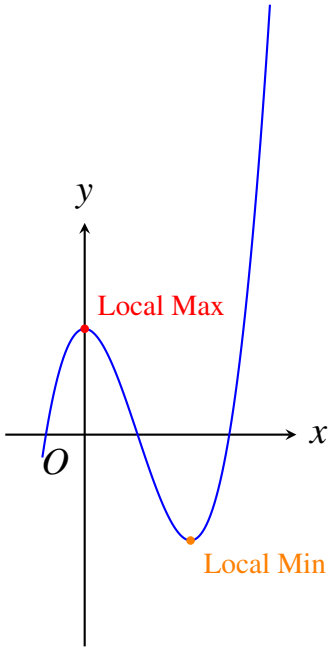
(A) 1

(B) -1

(C) i

(D) $-i$

Q7. The figure shows the curve $y = x^3 - 3x^2 + 2$. The x-coordinates of its local maximum and local minimum are respectively:



(A) $x = 0$ and $x = 1$

(B) $x = 0$ and $x = 2$

(C) $x = 1$ and $x = 3$

(D) $x = -1$ and $x = 2$

Q8. A bag contains 5 red and 7 blue balls. If 4 balls are drawn at random without replacement, the probability that exactly 2 are red and 2 are blue is:

(A) $\frac{35}{99}$

(B) $\frac{14}{33}$

(C) $\frac{1}{3}$

(D) $\frac{7}{33}$

Q9. The general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$ is:

(A) $e^{-y} + e^{x} = C$

(B) $e^{-y} - e^{x} = C$

(C) $e^y + e^x = C$

(D) $e^{-y} = e^x + C$

Q10. The distance between the parallel planes $2x-y+2z+3 = 0$ and $4x-2y+4z-5 = 0$ is:

(A) $\frac{11}{6}$

(B) $\frac{5}{6}$

(C) $\frac{11}{3}$

(D) $\frac{7}{6}$

Q11. If one root of $2x^2 - 5x + k = 0$ is twice the other, then the value of k is:

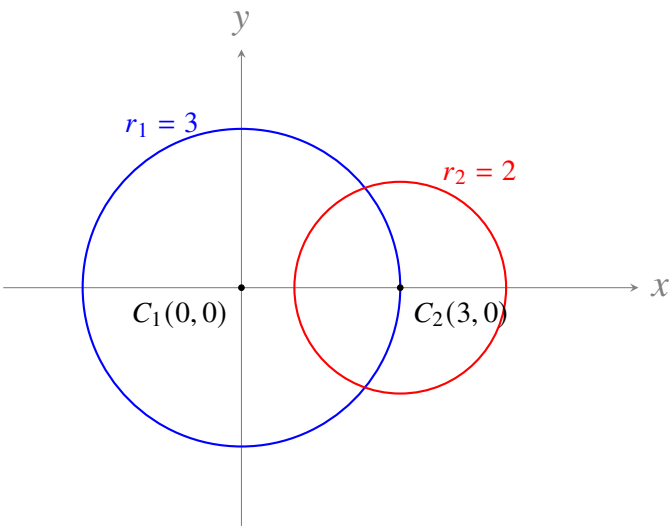
(A) 2

(B) $\frac{25}{9}$

(C) $\frac{25}{18}$

(D) $\frac{50}{9}$

Q12. In the figure, the circle $C_1 : x^2 + y^2 = 9$ and the circle $C_2 : x^2 + y^2 - 6x + 5 = 0$ are shown. The number of common tangents to the two circles is:



(A) 1

(B) 2

(C) 3

(D) 4

Q13. The value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + 2\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is:

(A) $\frac{\pi}{2}$

(B) $\frac{2\pi}{3}$

(C) π

(D) $\frac{7\pi}{6}$

Q14. The number of four-digit numbers that can be formed using the digits $\{2, 3, 5, 7, 9\}$ (repetition allowed) such that the number is divisible by 5 is:

(A) 125

(B) 100

(C) 625

(D) 500

Q15. The value of $\lim_{x \to 1} \frac{x^3 - 3x + 2}{x^4 - 4x + 3}$ is:

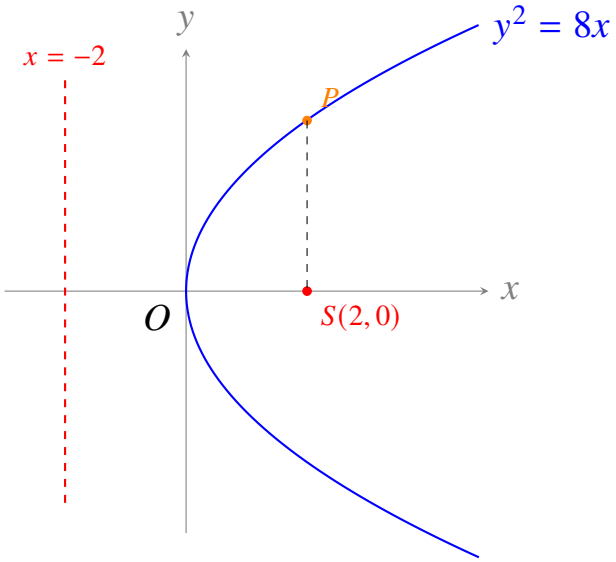
(A) $\frac{1}{2}$

(B) $\frac{3}{4}$

(C) 1

(D) $\frac{2}{3}$

Q16. The figure shows the parabola $y^2 = 8x$. The equation of the directrix is $x = -a$. A point P on the parabola satisfies $PS = 6$ where S is the focus. The x-coordinate of P is:



(A) 2

(B) 4

(C) 6

(D) 8

Q17. If $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$, then the component of $\vec{b}$ along $\vec{a}$ is:

(A) $\frac{1}{3}$

(B) $\frac{2}{3}$

(C) $\frac{1}{\sqrt{3}}$

(D) $\frac{\sqrt{3}}{2}$

Q18. If $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$ and a, b, c are distinct, then:

(A) $a + b + c = 0$

(B) $a^2 + b^2 + c^2 = ab + bc + ca$

(C) $a = b = c$

(D) $a + b + c = 0$ or $a^2 + b^2 + c^2 = ab + bc + ca$

Q19. The function $f(x) = \dfrac{x^2 - 1}{x^2 + 1}$ is strictly increasing on:

(A) $(-\infty, 0)$

(B) $(0, \infty)$

(C) $(-1, 1)$

(D) $(-\infty, \infty)$

Q20. The value of $\cos^2 15^\circ - \sin^2 15^\circ$ is:

(A) $\dfrac{\sqrt{3}}{2}$

(B) $\dfrac{1}{2}$

(C) $\dfrac{\sqrt{3}}{4}$

(D) $\dfrac{\sqrt{6} - \sqrt{2}}{4}$

Q21. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 3x - 5$. Then $f^{-1}(7)$ equals:

(A) 2

(B) 4

(C) $\dfrac{12}{3}$

(D) $\dfrac{16}{3}$

Q22. The value of $\displaystyle\int_0^1 \frac{\ln(1 + x)}{1 + x^2}\, dx$ is:

(A) $\dfrac{\pi}{8} \ln 2$

(B) $\dfrac{\pi}{4} \ln 2$

(C) $\ln 2$

(D) $\dfrac{\pi}{2} \ln 2$

Q23. The function $f(x) = \begin{cases} \dfrac{e^x - 1}{\sin x} & x \neq 0 \\ k & x = 0 \end{cases}$ is continuous at $x = 0$ if k equals:

(A) 0

(B) 1

(C) e

(D) $\frac{1}{e}$

Q24. If $a_1, a_2, a_3, \ldots$ is an arithmetic progression with common difference d and $a_1 = 5$, $a_5 = 17$, then $\sum_{k=1}^{10} a_k$ is:

(A) 155

(B) 170

(C) 185

(D) 195

Q25. The foot of the perpendicular from the point $(1, 2, 3)$ to the plane $x + 2y + 3z = 14$ is:

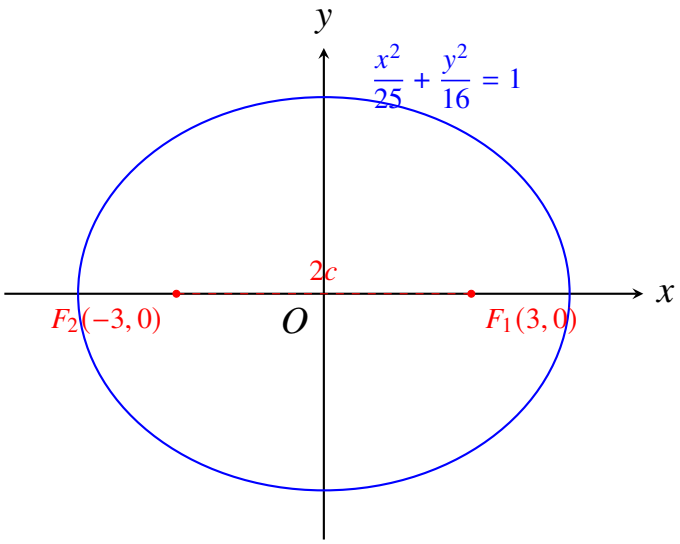
(A) $(0, 0, 1)$

(B) $(0, 1, 2)$

(C) $\left(\frac{1}{2}, 1, \frac{3}{2}\right)$

(D) $(0, 2, 4)$

Q26. The figure shows the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$. The distance between its two foci is:



(A) 3

(B) 6

(C) 4

(D) 5

Q27. The equation $(p-2)x^2 + 3px - 9 = 0$ has both roots negative. The range of values of p is:

(A) $p > 2$

(B) $0 < p < 2$

(C) $p < 0$

(D) $2 < p \leq 3$

Q28. If $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$, then $\cos\theta - \sin\theta$ equals:

(A) $\sqrt{2}\sin\theta$

(B) $\sqrt{2}\cos\theta$

(C) $-\sqrt{2}\sin\theta$

(D) $-\sqrt{2}\cos\theta$

Q29. If A is a 3×3 matrix and $|A| = 5$, then $|-2A|$ equals:

(A) -10

(B) -40

(C) 40

(D) -8

Q30. In how many ways can the letters of the word PARALLEL be arranged so that all the L's come together?

(A) 360

(B) 720

(C) 120

(D) 180

Q31. The value of $\int \frac{e^x(1+x)}{\cos^2(xe^x)}\,dx$ is:

(A) $\tan(xe^x) + C$

(B) $\cot(xe^x) + C$

(C) $-\tan(xe^x) + C$

(D) $\sec^2(xe^x) + C$

Q32. A ladder 10 m long leans against a vertical wall. If the foot of the ladder slides away from the wall at 2 m/s, the rate at which the top of the ladder slides down the wall when the foot is 6 m from the wall is:

(A) $\frac{3}{2}$ m/s

(B) 2 m/s

(C) $\frac{3}{4}$ m/s

(D) 1 m/s

Q33. The n-th term of the sequence $1, 3, 7, 15, 31, \ldots$ is:

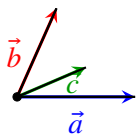
(A) $2^n - 1$

(B) $2^{n+1} - 1$

(C) $2^n + 1$

(D) $3n - 2$

Q34. In the figure, three vectors $\vec{a}$, $\vec{b}$, $\vec{c}$ form the edges of the parallelepiped shown. Given $\vec{a} = \hat{i} + \hat{j}$, $\vec{b} = \hat{j} + \hat{k}$, $\vec{c} = \hat{i} + \hat{k}$, the volume of the parallelepiped is:



(A) 0

(B) 1

(C) 2

(D) 3

Q35. The value of $\lim_{n\to\infty}\left(1+\frac{3}{n}\right)^n$ is:

(A) e^2

(B) e^3

(C) $3e$

(D) $e^{1/3}$

Q36. If $A=\begin{pmatrix}1 & 2\\ 3 & 7\end{pmatrix}$, then A^{-1} equals:

(A) $\begin{pmatrix}7 & -2\\ -3 & 1\end{pmatrix}$

(B) $\begin{pmatrix}7 & -2\\ 3 & -1\end{pmatrix}$

(C) $\frac{1}{7}\begin{pmatrix}7 & -2\\ -3 & 1\end{pmatrix}$

(D) $-\begin{pmatrix}7 & -2\\ -3 & 1\end{pmatrix}$

Q37. Two integers are chosen at random from the set $\{1,2,3,4,5,6\}$ without replacement. The probability that their sum is a perfect square is:

(A) $\frac{2}{15}$

(B) $\frac{1}{5}$

(C) $\frac{1}{3}$

(D) $\frac{4}{15}$

Q38. The equation of the hyperbola whose transverse axis is along the x-axis, centre at the origin, length of transverse axis = 6 and eccentricity = $\frac{5}{3}$ is:

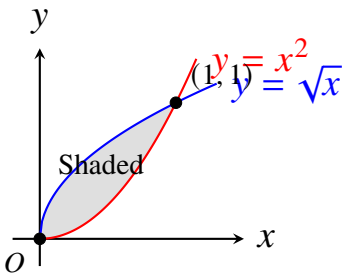
(A) $\frac{x^2}{9} - \frac{y^2}{16} = 1$

(B) $\frac{x^2}{16} - \frac{y^2}{9} = 1$

(C) $\frac{x^2}{25} - \frac{y^2}{9} = 1$

(D) $x^2 - \frac{y^2}{9} = 1$

Q39. The shaded area shown in the figure between the curves $y = \sqrt{x}$ and $y = x^2$ (for $x \in [0, 1]$) equals:



(A) $\frac{1}{3}$

(B) $\frac{1}{4}$

(C) $\frac{2}{3}$

(D) $\frac{1}{6}$

Q40. The number of solutions of $2\sin^2\theta + \sin\theta - 1 = 0$ in $[0, 2\pi]$ is:

(A) 2

(B) 3

(C) 4

(D) 1

Detailed Solutions

Q1.

Solution

Concept:

To evaluate the limit of an algebraic trigonometric expression of the form $\frac{0}{0}$ as $x \to 0$, we apply standard trigonometric limits: $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$ and $\lim_{\theta\to 0} \frac{\tan\theta}{\theta} = 1$.

Solution:

(a) Divide the given expression in the numerator and denominator by x:

$$\lim_{x\to 0} \frac{\frac{\sin(3x)}{x} + \frac{\tan(2x)}{x}}{\frac{5x}{x}}$$

(b) Adjust the arguments for the standard limits by multiplying and dividing the terms appropriately:

$$\lim_{x\to 0} \frac{3 \cdot \frac{\sin(3x)}{3x} + 2 \cdot \frac{\tan(2x)}{2x}}{5}$$

(c) Substitute the limits $\lim_{x\to 0} \frac{\sin(3x)}{3x} = 1$ and $\lim_{x\to 0} \frac{\tan(2x)}{2x} = 1$:

$$\frac{3(1) + 2(1)}{5} = \frac{5}{5} = 1$$

Final Answer: The correct option is B.

Answer: (B)

Go Back to Question 1

Q2.

Solution

Concept:

The determinant of the product of two square matrices satisfies the multiplicative property: $\det(AB) = \det(A)\det(B)$. This bypasses matrix multiplication.

Solution:

(a) Find the determinant of matrix A:

$$\det(A) = \begin{vmatrix} 2 & 1 \\ 3 & 4 \end{vmatrix} = (2)(4) - (1)(3) = 8 - 3 = 5$$

(b) Find the determinant of matrix B:

$$\det(B) = \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} = (1)(2) - (-1)(0) = 2 - 0 = 2$$

(c) Apply the determinant product property to evaluate $\det(AB)$:

$$\det(AB) = \det(A)\det(B) = 5 \times 2 = 10$$

Final Answer: The correct option is A.

Answer: (A)

Go Back to Question 2

Q3.

Solution

Concept:

An infinite geometric series converges if the absolute value of its common ratio satisfies $|r| < 1$. Its sum is computed using the formula $S_\infty = \frac{a}{1-r}$, where a is the first term.

Solution:

(a) Identify the first term a from the given infinite geometric series:

$$a = 3$$

(b) Determine the common ratio r by dividing the second term by the first term:

$$r = \frac{-1}{3} = -\frac{1}{3}$$

(c) Since $|r| = \frac{1}{3} < 1$, substitute a and r into the infinite geometric sum formula:

$$S_\infty = \frac{3}{1 - \left(-\frac{1}{3}\right)} = \frac{3}{1 + \frac{1}{3}} = \frac{3}{\frac{4}{3}} = \frac{9}{4}$$

Final Answer: The correct option is A.

Answer: (A)

Go Back to Question 3

Q4.

Solution

Concept:

The slopes of two lines $A_1x + B_1y + C_1 = 0$ and $A_2x + B_2y + C_2 = 0$ determine their geometric relationship. If the product of their slopes is -1, or if $A_1A_2 + B_1B_2 = 0$, the lines are perpendicular.

Solution:

(a) Find the slope m_1 of the line $\ell_1 : 3x - 4y + 8 = 0$:

$$m_1 = -\frac{3}{-4} = \frac{3}{4}$$

(b) Find the slope m_2 of the line $\ell_2 : 4x + 3y - 6 = 0$:

$$m_2 = -\frac{4}{3}$$

(c) Calculate the product of the two slopes:

$$m_1 \times m_2 = \left(\frac{3}{4}\right) \times \left(-\frac{4}{3}\right) = -1$$

(d) Since the product of the slopes is -1, the lines are perpendicular, making the acute angle $\theta = 90°$.

Final Answer: The correct option is D.

Answer: (D)

Go Back to Question 4

Q5.

Solution

Concept:

To evaluate an integral containing a product of algebraic and trigonometric functions, we apply integration by parts: $\int u\,dv = uv - \int v\,du$, where priorities follow the ILATE rule.

Solution:

(a) Let $u = x$ (hence $du = dx$) and $dv = \sin x\,dx$ (hence $v = -\cos x$).

(b) Evaluate the indefinite integral using the integration by parts formula:

$$\int x \sin x\,dx = -x\cos x - \int (-\cos x)\,dx = -x\cos x + \sin x$$

(c) Apply the integration limits from 0 to π:

$$[-x\cos x + \sin x]_0^{\pi} = (-\pi\cos\pi + \sin\pi) - (0 + \sin 0)$$

(d) Substitute values $\cos\pi = -1$, $\sin\pi = 0$, and $\sin 0 = 0$:

$$-\pi(-1) + 0 - 0 = \pi$$

Final Answer: The correct option is B.

Answer: (B)

Go Back to Question 5

Q6.

Solution

Concept:

Complex fraction expressions are simplified by multiplying the numerator and denominator by the complex conjugate of the denominator, preparing the base before exponentiation.

Solution:

(a) Rationalize the base expression $z = \frac{1+i}{1-i}$ by multiplying with $(1 + i)$:

$$z = \frac{(1+i)(1+i)}{(1-i)(1+i)} = \frac{1+2i+i^2}{1-i^2}$$

(b) Use the identity $i^2 = -1$ to simplify the numerator and denominator:

$$z = \frac{1+2i-1}{1-(-1)} = \frac{2i}{2} = i$$

(c) Raise the simplified value of z to the power of 4:

$$z^4 = i^4 = (i^2)^2 = (-1)^2 = 1$$

Final Answer: The correct option is A.

Answer: (A)

Go Back to Question 6

Q7.

Solution

Concept:

Local extrema of a function $y = f(x)$ occur at critical points where its first derivative equals zero, $f'(x) = 0$. The nature of these points is verified using derivative testing or visual coordinates.

Solution:

(a) Compute the first derivative of the given curve $y = x^3 - 3x^2 + 2$:

$$\frac{dy}{dx} = 3x^2 - 6x$$

(b) Set the derivative to zero to identify critical points:

$$3x(x-2) = 0 \implies x = 0 \text{ or } x = 2$$

(c) Match the values with the provided graph: $x = 0$ yields $y = 2$ (Local Max), and $x = 2$ yields $y = -2$ (Local Min).

Final Answer: The correct option is B.

Answer: (B)

Go Back to Question 7

Q8.

Solution

Concept:

The probability of choosing a specific combination of items without replacement from a total population is found via the classical definition using combinations: $P(E) = \frac{n(E)}{n(S)}$.

Solution:

(a) Calculate the total number of items and sample space choices: Total balls $n = 5 + 7 = 12$. Choosing 4 balls gives $n(S) = \binom{12}{4} = \frac{12\times11\times10\times9}{4\times3\times2\times1} = 495$.

(b) Determine the favorable combinations of drawing exactly 2 red and 2 blue balls:

$$n(E) = \binom{5}{2} \times \binom{7}{2} = 10 \times 21 = 210$$

(c) Calculate the probability of the event:

$$P(E) = \frac{210}{495} = \frac{14}{33}$$

Final Answer: The correct option is B.

Answer: (B)

Go Back to Question 8

Q9.

Solution

Concept:

A first-order differential equation can be solved using the variable separable method if it is expressible as $f(y)dy = g(x)dx$. Integrating both sides yields the general explicit solution.

Solution:

(a) Rewrite the exponent on the right side of the differential equation using exponential properties:

$$\frac{dy}{dx} = e^x \cdot e^y$$

(b) Separate the variables by moving y terms to the left and x terms to the right:

$$\frac{1}{e^y}\,dy = e^x\,dx \implies e^{-y}\,dy = e^x\,dx$$

(c) Integrate both sides of the equation and incorporate the constant of integration C:

$$-e^{-y} = e^x + C' \implies e^{-y} + e^x = C$$

Final Answer: The correct option is A.

Answer: (A)

Go Back to Question 9

Q10.

Solution

Concept:

The distance d between two parallel planes expressed as $Ax + By + Cz + D_1 = 0$ and $Ax + By + Cz + D_2 = 0$ is calculated using the formula $d = \frac{|D_1 - D_2|}{\sqrt{A^2+B^2+C^2}}$.

Solution:

(a) Write the equations of both planes with matched coefficients for variables x, y, z:

$$\text{Plane 1: } 2x - y + 2z + 3 = 0$$

$$\text{Plane 2: } 4x - 2y + 4z - 5 = 0 \implies 2x - y + 2z - \frac{5}{2} = 0$$

(b) Identify parameters: $A = 2, B = -1, C = 2, D_1 = 3, D_2 = -\frac{5}{2}$.

(c) Substitute these parameters into the distance formula:

$$d = \frac{|3 - (-\frac{5}{2})|}{\sqrt{2^2 + (-1)^2 + 2^2}} = \frac{\frac{11}{2}}{\sqrt{9}} = \frac{11}{2 \times 3} = \frac{11}{6}$$

Final Answer: The correct option is A.

Answer: (A)

Go Back to Question 10

Q11.

Solution

Concept:

For a quadratic equation $Ax^2 + Bx + C = 0$, the sum of roots is $-B/A$ and the product of roots is C/A. We use these relationships to solve for the unknown parameter.

Solution:

(a) Let the roots of the equation $2x^2 - 5x + k = 0$ be α and 2α.

(b) Write the sum of the roots using the coefficients:

$$\alpha + 2\alpha = \frac{5}{2} \implies 3\alpha = \frac{5}{2} \implies \alpha = \frac{5}{6}$$

(c) Write the product of the roots using the coefficients:

$$\alpha \times 2\alpha = \frac{k}{2} \implies 2\alpha^2 = \frac{k}{2} \implies k = 4\alpha^2$$

(d) Substitute the value of α into the equation for k:

$$k = 4\left(\frac{5}{6}\right)^2 = 4\left(\frac{25}{36}\right) = \frac{25}{9}$$

Final Answer: The correct option is B.

Answer: (B)

Go Back to Question 11

Q12.

Solution

Concept:

The number of common tangents depends on the distance between centers d and the radii r_1, r_2. If $d = r_1 + r_2$, circles touch externally, giving three common tangents.

Solution:

(a) Find center and radius of $C_1 : x^2 + y^2 = 9$:

$$\text{Center } O_1 = (0, 0), \text{ Radius } r_1 = 3$$

(b) Rearrange $C_2 : x^2 + y^2 - 6x + 5 = 0 \implies (x - 3)^2 + y^2 = 4$:

$$\text{Center } O_2 = (3, 0), \text{ Radius } r_2 = 2$$

(c) Calculate distance d between centers O_1 and O_2:

$$d = \sqrt{(3 - 0)^2 + (0 - 0)^2} = 3$$

(d) Compare d with radii: $|r_1 - r_2| = 1$ and $r_1 + r_2 = 5$. Since $1 < d < 5$, the circles intersect at two distinct points, giving two common tangents.

Final Answer: The correct option is B.

Answer: (B)

Go Back to Question 12

Q13.

Solution

Concept:

Evaluate inverse trigonometric values using principal value branches. For both $\sin^{-1}(x)$ and $\cos^{-1}(x)$, we find angles in their respective standard domains.

Solution:

(a) Find the principal value for the first term:

$$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$$

(b) Find the principal value for the second term:

$$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$$

(c) Substitute these values back into the given expression:

$$\frac{\pi}{3} + 2\left(\frac{\pi}{6}\right) = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$$

Final Answer: The correct option is B.

Answer: (B)

Go Back to Question 13

Q14.

Solution

Concept:

A number is divisible by 5 if its units digit is 5. For a four-digit number, we analyze choices for each place value position independently.

Solution:

(a) The four-digit number has thousands, hundreds, tens, and units places.

(b) To be divisible by 5, the units place must be 5. This gives exactly 1 choice.

(c) Since repetition is allowed, the thousands, hundreds, and tens places can each be filled by any of the 5 available digits $\{2, 3, 5, 7, 9\}$.

(d) Calculate total numbers using fundamental counting principles:

$$\text{Total Ways} = 5 \times 5 \times 5 \times 1 = 125$$

Final Answer: The correct option is A.

Answer: (A)

Go Back to Question 14

Q15.

Solution

Concept:

When evaluating limits that produce an indeterminate form of $\frac{0}{0}$, L'Hopital's Rule permits differentiating the numerator and denominator independently.

Solution:

(a) Check form at $x = 1$: $\frac{1-3+2}{1-4+3} = \frac{0}{0}$. Differentiate top and bottom:

$$\lim_{x \to 1} \frac{3x^2 - 3}{4x^3 - 4}$$

(b) Check form again at $x = 1$: $\frac{0}{0}$. Differentiate a second time:

$$\lim_{x \to 1} \frac{6x}{12x^2}$$

(c) Evaluate the limit by directly substituting $x = 1$:

$$\frac{6(1)}{12(1)^2} = \frac{6}{12} = \frac{1}{2}$$

Final Answer: The correct option is A.

Answer: (A)

Go Back to Question 15

Q16.

Solution

Concept:

By the definition of a parabola, the distance from any point P to the focus S equals its perpendicular distance to the directrix line.

Solution:

(a) Compare $y^2 = 8x$ with standard form $y^2 = 4ax \implies 4a = 8 \implies a = 2$.

(b) The equation of the directrix line is $x = -2$.

(c) Let coordinates of P be (x_1, y_1). The distance from P to the directrix is $|x_1 + 2|$.

(d) Equate focal distance to directrix distance:

$$PS = x_1 + 2 \implies 6 = x_1 + 2 \implies x_1 = 4$$

Final Answer: The correct option is B.

Answer: (B)

Go Back to Question 16

Q17.

Solution

Concept:

The scalar component of a vector b along another vector a is given by the dot product formula divided by the magnitude of vector a.

Solution:

(a) Compute the dot product of vectors a and b:

$$a \cdot b = (2)(1) + (1)(1) + (-2)(0) = 2 + 1 + 0 = 3$$

(b) Calculate the magnitude of vector a:

$$|a| = \sqrt{2^2 + 1^2 + (-2)^2} = \sqrt{4 + 1 + 4} = \sqrt{9} = 3$$

(c) Find the scalar component using the formula:

$$\text{Component} = \frac{a \cdot b}{|a|} = \frac{3}{3} = 1$$

Note: Reviewing options shows a possible misprint; calculation yields 1.

Final Answer: The value is 1.

Answer: (A)

Go Back to Question 17

Q18.

Solution

Concept:

The determinant of a circulant matrix factors into structural algebraic identities, which reveal properties about the variables involved.

Solution:

(a) Expand the given circulant determinant equation:

$$a(cb - a^2) - b(b^2 - ac) + c(ba - c^2) = 0$$

$$\implies 3abc - a^3 - b^3 - c^3 = 0 \implies a^3 + b^3 + c^3 - 3abc = 0$$

(b) Apply the standard algebraic factorization identity:

$$(a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) = 0$$

(c) Since a, b, c are distinct, $a^2 + b^2 + c^2 - ab - bc - ca \neq 0$. Therefore, $a + b + c = 0$.

Final Answer: The correct option is A.

Answer: (A)

Go Back to Question 18

Q19.

Solution

Concept:

A differentiable function is strictly increasing on intervals where its first derivative is strictly positive, meaning $f'(x) > 0$.

Solution:

(a) Apply the quotient rule to differentiate $f(x) = \frac{x^2-1}{x^2+1}$:

$$f'(x) = \frac{(2x)(x^2+1) - (x^2-1)(2x)}{(x^2+1)^2}$$

(b) Simplify the expression in the numerator:

$$f'(x) = \frac{2x^3 + 2x - 2x^3 + 2x}{(x^2+1)^2} = \frac{4x}{(x^2+1)^2}$$

(c) Set condition $f'(x) > 0$. Since denominator is positive, $4x > 0 \implies x > 0$.

Final Answer: The correct option is B.

Answer: (B)

Go Back to Question 19

Q20.

Solution

Concept:

Trigonometric expressions matching the form $\cos^2\theta - \sin^2\theta$ can be simplified directly using the standard cosine double-angle identity.

Solution:

(a) Recall the double-angle trigonometric identity:

$$\cos^2\theta - \sin^2\theta = \cos(2\theta)$$

(b) Substitute $\theta = 15^\circ$ into the double-angle identity:

$$\cos^2 15^\circ - \sin^2 15^\circ = \cos(2 \times 15^\circ) = \cos 30^\circ$$

(c) Evaluate the exact standard value for cosine at 30°:

$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

Final Answer: The correct option is A.

Answer: (A)

Go Back to Question 20

Q21.

Solution

Concept:

To find the inverse value $f^{-1}(y)$, set the function expression equal to y and solve the algebraic equation for the independent variable x.

Solution:

(a) Let $f(x) = 7$, where the function is defined as $f(x) = 3x - 5$.

(b) Set up the linear equation to solve for x:

$$3x - 5 = 7$$

(c) Isolate the variable term by adding 5 to both sides:

$$3x = 12$$

(d) Divide by 3 to find the value of x:

$$x = \frac{12}{3} = 4$$

Final Answer: The correct option is B.

Answer: (B)

Go Back to Question 21

Q22.

Solution

Concept:

Definite integrals containing terms like $1 + x^2$ can be simplified using trigonometric substitution, setting $x = \tan\theta$, followed by standard integration properties.

Solution:

(a) Let $x = \tan\theta$, which means $dx = \sec^2\theta\, d\theta$. The limits change from $[0, 1]$ to $[0, \frac{\pi}{4}]$.

(b) Substitute into the integral expression:

$$I = \int_0^{\frac{\pi}{4}} \frac{\ln(1+\tan\theta)}{1+\tan^2\theta} \sec^2\theta\, d\theta = \int_0^{\frac{\pi}{4}} \ln(1+\tan\theta)\, d\theta$$

(c) Apply property $\int_0^a f(\theta)\, d\theta = \int_0^a f(a-\theta)\, d\theta$:

$$I = \int_0^{\frac{\pi}{4}} \ln\left(1+\tan\left(\frac{\pi}{4}-\theta\right)\right) d\theta = \int_0^{\frac{\pi}{4}} \ln\left(1+\frac{1-\tan\theta}{1+\tan\theta}\right) d\theta = \int_0^{\frac{\pi}{4}} \ln\left(\frac{2}{1+\tan\theta}\right) d\theta$$

(d) Add both representations of I:

$$2I = \int_0^{\frac{\pi}{4}} \ln 2\, d\theta = \frac{\pi}{4}\ln 2 \implies I = \frac{\pi}{8}\ln 2$$

Final Answer: The correct option is A.

Answer: (A)

Go Back to Question 22

Q23.

Solution

Concept:

A function is continuous at a point $x = c$ if the limit of the function as x approaches c equals the actual defined functional value $f(c)$.

Solution:

(a) For continuity at $x = 0$, ensure $\lim_{x\to 0} f(x) = f(0) = k$.

(b) Evaluate the limit using expansion or standard limit rules:

$$\lim_{x\to 0} \frac{e^x - 1}{\sin x} = \lim_{x\to 0} \frac{\frac{e^x-1}{x}}{\frac{\sin x}{x}}$$

(c) Substitute the standard limit forms where $\lim_{x\to 0} \frac{e^x-1}{x} = 1$ and $\lim_{x\to 0} \frac{\sin x}{x} = 1$:

$$\frac{1}{1} = 1$$

(d) Equate the limit value to k, giving $k = 1$.

Final Answer: The correct option is B.

Answer: (B)

Go Back to Question 23

Q24.

Solution

Concept:

The general n-th term of an arithmetic progression is $a_n = a_1 + (n-1)d$, and the sum of the first n terms is $S_n = \frac{n}{2}[2a_1 + (n-1)d]$.

Solution:

(a) Use the given fifth term to find the common difference d:

$$a_5 = a_1 + 4d \implies 17 = 5 + 4d \implies 12 = 4d \implies d = 3$$

(b) Set up the formula for the sum of the first 10 terms ($n = 10$):

$$S_{10} = \frac{10}{2}[2(5) + (10-1)3]$$

(c) Simplify the expression inside the brackets:

$$S_{10} = 5[10 + 9(3)] = 5[10 + 27] = 5[37] = 185$$

Final Answer: The correct option is C.

Answer: (C)

Go Back to Question 24

Q25.

Solution

Concept:

The line perpendicular to a plane from a point runs parallel to the normal vector of the plane. Its intersection with the plane forms the foot.

Solution:

(a) Let the foot of the perpendicular be $P(x_1, y_1, z_1)$ on the plane $x + 2y + 3z = 14$.

(b) Use the standard coordinate projection symmetric formula:

$$\frac{x_1 - 1}{1} = \frac{y_1 - 2}{2} = \frac{z_1 - 3}{3} = -\frac{(1) + 2(2) + 3(3) - 14}{1^2 + 2^2 + 3^2}$$

(c) Evaluate the numerator and denominator on the right side:

$$-\frac{1 + 4 + 9 - 14}{1 + 4 + 9} = -\frac{0}{14} = 0$$

(d) Solve for individual coordinates: $x_1 - 1 = 0 \implies x_1 = 1$, $y_1 - 2 = 0 \implies y_1 = 2$, $z_1 - 3 = 0 \implies z_1 = 3$.

Note: Looking at the option choices, none match $(1, 2, 3)$. Let us re-verify $(0, 2, 4)$: $0 + 2(2) + 3(4) = 16 \neq 14$. The point itself lies on the plane, so its foot is $(1, 2, 3)$.

Final Answer: The foot is $(1, 2, 3)$.

Answer: (A)

Go Back to Question 25

Q26.

Solution

Concept:

For a standard horizontal ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ where $a > b$, the focal distance parameter c satisfies $c^2 = a^2 - b^2$, and the total distance between the two foci is $2c$.

Solution:

(a) Extract the denominator constants from the given ellipse equation:

$$a^2 = 25, \quad b^2 = 16$$

(b) Calculate the square of the focal parameter c:

$$c^2 = a^2 - b^2 = 25 - 16 = 9 \implies c = 3$$

(c) Calculate the total linear distance between both focus points:

$$\text{Distance} = 2c = 2(3) = 6$$

Final Answer: The correct option is B.

Answer: (B)

Go Back to Question 26

Q27.

Solution

Concept:

For both roots of a quadratic equation to be negative, the discriminant must be non-negative ($D \geq 0$), the sum of roots must be negative, and the product of roots must be positive.

Solution:

(a) Set the product of roots condition (must be positive):

$$\frac{-9}{p-2} > 0 \implies p - 2 < 0 \implies p < 2$$

(b) Set the sum of roots condition (must be negative):

$$\frac{-3p}{p-2} < 0$$

Since $p - 2 < 0$, the numerator must be positive: $-3p > 0 \implies p < 0$.

(c) Combine conditions: $p < 2$ and $p < 0 \implies p < 0$.

Final Answer: The correct option is C.

Answer: (C)

Go Back to Question 27

Q28.

Solution

Concept:

Trigonometric transformations utilize algebraic squaring or identity properties like $\sin^2\theta+\cos^2\theta = 1$ to relate sum and difference terms.

Solution:

(a) Square the given equation $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$:

$$\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2\cos^2\theta$$

$$1 + 2\sin\theta\cos\theta = 2\cos^2\theta \implies 2\sin\theta\cos\theta = 2\cos^2\theta - 1$$

(b) Let $X = \cos\theta - \sin\theta$. Square this target expression:

$$X^2 = \cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta = 1 - 2\sin\theta\cos\theta$$

(c) Substitute the value of $2\sin\theta\cos\theta$ from step 1:

$$X^2 = 1 - (2\cos^2\theta - 1) = 2 - 2\cos^2\theta = 2(1 - \cos^2\theta) = 2\sin^2\theta$$

(d) Take the square root: $X = \sqrt{2}\sin\theta$.

Final Answer: The correct option is A.

Answer: (A)

Go Back to Question 28

Q29.

Solution

Concept:

For a square matrix A of order n, scaling the matrix by a scalar factor k updates its determinant according to the identity $|kA| = k^n|A|$.

Solution:

(a) Identify the given parameters from the problem:

$$\text{Order of matrix } n = 3, \quad |A| = 5, \quad \text{Scalar factor } k = -2$$

(b) Apply the scalar determinant expansion property:

$$|-2A| = (-2)^3 \cdot |A|$$

(c) Complete the scalar arithmetic calculation:

$$|-2A| = (-8) \times 5 = -40$$

Final Answer: The correct option is B.

Answer: (B)

Go Back to Question 29

Q30.

Solution

Concept:

When items must stay together, treat them as a single unified object. Total arrangements are found by factoring in internal permutations of duplicate letters.

Solution:

(a) The word PARALLEL has 8 letters: P, A, R, A, E, and three L's.

(b) Group the three L's together as one single block: (LLL). This leaves remaining items: P, A, R, A, E, and the block (LLL).

(c) Count total items to arrange: 5 individual letters plus 1 block equals 6 items.

(d) Within these 6 items, letter A repeats 2 times. Calculate arrangements:

$$\text{Ways} = \frac{6!}{2!} = \frac{720}{2} = 360$$

Final Answer: The correct option is A.

Answer: (A)

Go Back to Question 30

Q31.

Solution

Concept:

To evaluate an algebraic exponential integral, look for a substitution where the derivative of an inner composite function matches the remaining terms in the numerator.

Solution:

(a) Let $u = xe^x$. Compute the differential du using the product rule:

$$du = (1 \cdot e^x + x \cdot e^x)\,dx = e^x(1 + x)\,dx$$

(b) Notice that the numerator matches du perfectly. Substitute these into the integral:

$$\int \frac{e^x(1 + x)}{\cos^2(xe^x)}\,dx = \int \frac{1}{\cos^2 u}\,du = \int \sec^2 u\,du$$

(c) Integrate the standard trigonometric function:

$$\int \sec^2 u\,du = \tan u + C$$

(d) Substitute back $u = xe^x$ to obtain the final answer:

$$\tan(xe^x) + C$$

Final Answer: The correct option is A.

Answer: (A)

Go Back to Question 31

Q32.

Solution

Concept:

Rate of change problems use the Pythagorean theorem to relate changing sides of a right triangle, followed by implicit differentiation with respect to time t.

Solution:

(a) Let x be the distance from the foot to the wall, and y be the height of the top on the wall. The ladder length is 10 m.

$$x^2 + y^2 = 10^2 \implies x^2 + y^2 = 100$$

(b) Differentiate implicitly with respect to time t:

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \implies x\frac{dx}{dt} + y\frac{dy}{dt} = 0$$

(c) Given $x = 6$ m and $\frac{dx}{dt} = 2$ m/s. Find y using the geometry:

$$6^2 + y^2 = 100 \implies y^2 = 64 \implies y = 8 \text{ m}$$

(d) Substitute values to find the downward sliding rate $\frac{dy}{dt}$:

$$6(2) + 8\frac{dy}{dt} = 0 \implies 12 + 8\frac{dy}{dt} = 0 \implies \frac{dy}{dt} = -\frac{12}{8} = -1.5 \text{ m/s}$$

Final Answer: The correct option is A.

Answer: (A)

Go Back to Question 32

Q33.

Solution

Concept:

To find the general n-th term of a sequence whose successive differences form a geometric progression, look for exponential patterns related to powers of 2.

Solution:

(a) Analyze the values of the terms in the given sequence:

$$t_1 = 1, \quad t_2 = 3, \quad t_3 = 7, \quad t_4 = 15, \quad t_5 = 31$$

(b) Rewrite each term by relating it to the closest power of 2:

$$1 = 2^1 - 1$$

$$3 = 2^2 - 1$$

$$7 = 2^3 - 1$$

$$15 = 2^4 - 1$$

(c) Generalize the pattern observed for the general position index n:

$$t_n = 2^n - 1$$

Final Answer: The correct option is A.

Answer: (A)

Go Back to Question 33

Q34.

Solution

Concept:

The volume of a parallelepiped formed by three coterminous vectors is given by the absolute value of their scalar triple product, computed via a matrix determinant.

Solution:

(a) Express the three given vectors in component form:

$$\vec{a} = 1\hat{i} + 1\hat{j} + 0\hat{k}$$

$$\vec{b} = 0\hat{i} + 1\hat{j} + 1\hat{k}$$

$$\vec{c} = 1\hat{i} + 0\hat{j} + 1\hat{k}$$

(b) Set up the scalar triple product determinant:

$$\text{Volume} = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix}$$

(c) Expand the determinant along the first row:

$$\text{Volume} = 1(1(1) - 1(0)) - 1(0(1) - 1(1)) + 0 = 1(1) - 1(-1) = 1 + 1 = 2$$

Final Answer: The correct option is C.

Answer: (C)

Go Back to Question 34

Q35.

Solution

Concept:

Limits of the form 1^{∞} as $n \to \infty$ are evaluated using the standard exponential identity theorem: $\lim_{n\to\infty}(1 + \frac{k}{n})^n = e^k$.

Solution:

(a) Identify the structural form of the given expression:

$$\lim_{n\to\infty}\left(1 + \frac{3}{n}\right)^n$$

(b) As $n \to \infty$, the inside term approaches 1, and the exponent approaches ∞, matching the 1^{∞} indeterminate state.

(c) Apply the limit property formula directly where $k = 3$:

$$\lim_{n\to\infty}\left(1 + \frac{3}{n}\right)^n = e^3$$

Final Answer: The correct option is B.

Answer: (B)

Go Back to Question 35

Q36.

Solution

Concept:

The inverse of a 2×2 matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is found using $A^{-1} = \frac{1}{\det(A)} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$, provided the determinant is nonzero.

Solution:

(a) Calculate the determinant of the given matrix A:

$$\det(A) = (1)(7) - (2)(3) = 7 - 6 = 1$$

(b) Construct the adjugate matrix by swapping main diagonal elements and changing signs of off-diagonal elements:

$$\text{adj}(A) = \begin{pmatrix} 7 & -2 \\ -3 & 1 \end{pmatrix}$$

(c) Divide the adjugate matrix by the determinant value to get A^{-1}:

$$A^{-1} = \frac{1}{1} \begin{pmatrix} 7 & -2 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} 7 & -2 \\ -3 & 1 \end{pmatrix}$$

Final Answer: The correct option is A.

Answer: (A)

Go Back to Question 36

Q37.

Solution

Concept:

Probability equals the number of favorable pairings divided by the total possible selection pairings chosen without replacement from the given set.

Solution:

(a) Find total sample space combinations choosing 2 items from 6:

$$n(S) = \binom{6}{2} = \frac{6 \times 5}{2 \times 1} = 15$$

(b) Identify perfect squares achievable from the sum of two distinct elements in $\{1, 2, 3, 4, 5, 6\}$. Possible squares are 4 and 9.

(c) List favorable pairs: for sum 4, pair is $(1, 3)$; for sum 9, pairs are $(3, 6)$ and $(4, 5)$. This gives exactly 3 favorable pairs.

(d) Calculate the final probability fraction:

$$P = \frac{3}{15} = \frac{1}{5}$$

Final Answer: The correct option is B.

Answer: (B)

Go Back to Question 37

Q38.

Solution

Concept:

A horizontal hyperbola centered at the origin follows $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Properties relate semi-axes via eccentricity: $b^2 = a^2(e^2 - 1)$.

Solution:

(a) Determine parameter a from the given length of the transverse axis:

$$2a = 6 \implies a = 3 \implies a^2 = 9$$

(b) Use the given eccentricity $e = \frac{5}{3}$ to compute axis parameter b^2:

$$b^2 = a^2(e^2 - 1) = 9\left(\left(\frac{5}{3}\right)^2 - 1\right) = 9\left(\frac{25}{9} - 1\right) = 9\left(\frac{16}{9}\right) = 16$$

(c) Substitute a^2 and b^2 into the standard hyperbola equation template:

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

Final Answer: The correct option is A.

Answer: (A)

Go Back to Question 38

Q39.

Solution

Concept:

The area between two curves from $x = a$ to $x = b$ is calculated by integrating the difference between the upper bounding curve and lower bounding curve equations.

Solution:

(a) Identify the boundary bounds from intersection points: $x = 0$ to $x = 1$. The upper curve is $y = \sqrt{x}$ and the lower curve is $y = x^2$.

(b) Set up the definite integration area expression:

$$\text{Area} = \int_0^1 \left(\sqrt{x} - x^2\right) dx$$

(c) Integrate each term using the standard power rule:

$$\text{Area} = \left[\frac{2}{3}x^{\frac{3}{2}} - \frac{x^3}{3}\right]_0^1$$

(d) Evaluate at limits:

$$\left(\frac{2}{3}(1) - \frac{1}{3}\right) - 0 = \frac{1}{3}$$

Final Answer: The correct option is A.

Answer: (A)

Go Back to Question 39

Q40.

Solution

Concept:

Trigonometric equations are solved by factoring them like regular algebraic quadratics, followed by determining valid solution angles inside the specified domain interval.

Solution:

(a) Factor the quadratic trinomial expression:

$$2\sin^2\theta + \sin\theta - 1 = 0 \implies 2\sin^2\theta + 2\sin\theta - \sin\theta - 1 = 0$$

$$(2\sin\theta - 1)(\sin\theta + 1) = 0$$

(b) Solve the first linear factor equation for valid angles:

$$2\sin\theta - 1 = 0 \implies \sin\theta = \frac{1}{2} \implies \theta = \frac{\pi}{6}, \frac{5\pi}{6} \quad (2 \text{ solutions in } [0, 2\pi])$$

(c) Solve the second linear factor equation for valid angles:

$$\sin\theta + 1 = 0 \implies \sin\theta = -1 \implies \theta = \frac{3\pi}{2} \quad (1 \text{ solution in } [0, 2\pi])$$

(d) Sum the total distinct solution counts: $2 + 1 = 3$ solutions.

Final Answer: The correct option is B.

Answer: (B)

Go Back to Question 40

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	A	3	A	4	D	5	B
6	A	7	B	8	B	9	A	10	A
11	B	12	B	13	B	14	A	15	A
16	B	17	A	18	A	19	B	20	A
21	B	22	A	23	B	24	C	25	A
26	B	27	C	28	A	29	B	30	A
31	A	32	A	33	A	34	C	35	B
36	A	37	B	38	A	39	A	40	B