BITSAT Physics Sample Paper – 17

Duration: 40 Minutes **Maximum Marks: 90**

Instructions

- This paper contains **30** Multiple Choice Questions (Single Correct).
- Each correct answer carries **+3 marks**. Each incorrect answer carries -1 mark. Unattempted questions carry **0** marks.
- Only **one** option is correct for each question.
- Use of mobile phones, smartwatches, or any electronic gadgets is strictly prohibited.

Q1. An experiment measures the acceleration due to gravity g using a simple pendulum. The length of the pendulum is measured as $L = (100 \pm 0.1)$ cm and the time for 100 oscillations is measured as $T = (200 \pm 2)$ s using a stopwatch of resolution 0.1 s. What is the maximum percentage error in the determination of g?

(A) 0.2%

(B) 1.1%

(C) 2.1%

(D) 3.0%

Q2. A block of mass m is placed on a rough horizontal surface with coefficient of static friction $\mu_s = 0.6$. A time-dependent horizontal force $F = kt$ (where k is a positive constant) begins to act on the block at $t = 0$. Which graph qualitatively represents the variation of the friction force f acting on the block as a function of time t?

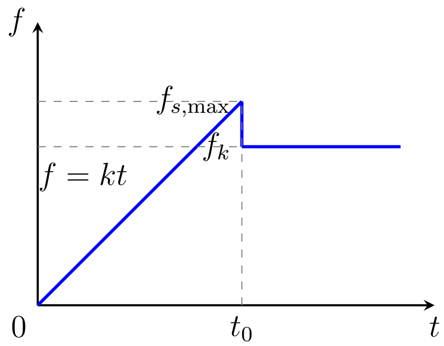


Figure 1: Qualitative variation of friction force with time.

(A) A straight line passing through the origin with a positive slope for all time t.

(B) A straight line with a positive slope up to a certain time, followed by a constant horizontal line at a slightly lower value.

(C) A constant horizontal line from the beginning.

(D) A curve that increases exponentially and then drops sharply to zero.

Q3. A small body of mass m slides down from the top of a smooth hemisphere of radius R fixed on a horizontal floor. At what vertical distance below the top of the hemisphere will the body lose contact with the surface?

(A) $\frac{R}{3}$

(B) $\frac{R}{2}$

(C) $\frac{2R}{3}$

(D) $\frac{R}{4}$

Q4. A potential energy function for a particle moving along the x-axis is given by $U(x) = \frac{a}{x^2} - \frac{b}{x}$, where a and b are positive constants. The particle executes small oscillations about its stable equilibrium position. The angular frequency ω of these oscillations for a particle of mass m is:

(A) $\frac{b^2}{4a}\sqrt{\frac{1}{2ma}}$

(B) $\frac{b^2}{2a}\sqrt{\frac{1}{ma}}$

(C) $\frac{b^4}{8a^3m}$

(D) $\frac{b^2}{a}\sqrt{\frac{1}{2ma}}$

Q5. A uniform solid sphere of mass M and radius R is rolling without slipping on a horizontal surface with a translational velocity v. It then rolls up a rough inclined plane of inclination angle θ. If the friction is sufficient to prevent slipping throughout the motion, the maximum vertical height h reached by the sphere is:

(A) $\frac{v^2}{2g}$

(B) $\frac{7v^2}{10g}$

(C) $\frac{3v^2}{4g}$

(D) $\frac{5v^2}{7g}$

Q6. A thin uniform rod of mass M and length L is bent at its midpoint to form a 90° angle. What is the moment of inertia of this bent rod about an axis passing through the vertex of the angle and perpendicular to the plane containing the two segments?

(A) $\frac{1}{3}ML^2$

(B) $\frac{1}{12}ML^2$

(C) $\frac{1}{24}ML^2$

(D) $\frac{1}{6}ML^2$

Q7. Two satellites of masses m and $4m$ are orbiting a planet in circular orbits of radii $2R$ and R respectively. The ratio of their kinetic energies, K_1/K_2, is:

(A) $1 : 8$

(B) $1 : 4$

(C) $1 : 2$

(D) $1 : 1$

Q8. A large open tank filled with water has a small hole of area A at its bottom and another small hole of area $2A$ at a height h from the bottom.

Water is poured into the tank from the top at a constant volumetric flow rate Q. If the water level in the tank remains stationary at a total height H from the bottom, then Q is equal to:

(A) $A\sqrt{2gH} + 2A\sqrt{2g(H-h)}$

(B) $A\sqrt{2gH} + 2A\sqrt{2gh}$

(C) $3A\sqrt{2gH}$

(D) $A\sqrt{2g(H-h)} + 2A\sqrt{2gH}$

Q9. One mole of an ideal monoatomic gas expands such that its volume V and temperature T relate as $VT^2 = \text{constant}$. If the temperature of the gas increases by ΔT, the amount of heat absorbed by the gas during this process is:

(A) $\frac{1}{2}R\Delta T$

(B) $R\Delta T$

(C) $\frac{3}{2}R\Delta T$

(D) $2R\Delta T$

Q10. An ideal gas is taken through a cyclic process $A \to B \to C \to A$. The process $A \to B$ is isothermal at temperature T_0, $B \to C$ is isobaric, and $C \to A$ is isochoric. If the volume at B is double the volume at A, the net work done by the gas in one complete cycle is:

(A) $RT_0\left(\ln 2 - \frac{1}{2}\right)$

(B) $RT_0 \ln 2$

(C) $RT_0\left(\ln 2 + \frac{1}{2}\right)$

(D) $\frac{1}{2}RT_0$

Q11. A Carnot engine operates between two reservoirs at temperatures $T_1 = 600$ K and $T_2 = 300$ K. If the engine absorbs 1000 J of heat from the hot reservoir in each cycle, the efficiency of the engine and the work done per cycle are respectively:

(A) 50%, 500 J

(B) 50%, 1000 J

(C) 33.3%, 333 J

(D) 66.7%, 667 J

Q12. Two identical point charges $+q$ are fixed at coordinates $(0, a)$ and $(0, -a)$ on the y-axis. Another point charge $-Q$ of mass m is placed at the origin. If $-Q$ is displaced slightly along the x-axis by a distance $x \ll a$ and released, it will execute simple harmonic motion with a time period of:

(A) $2\pi\sqrt{\frac{4\pi\varepsilon_0 ma^3}{qQ}}$

(B) $2\pi\sqrt{\frac{2\pi\varepsilon_0 ma^3}{qQ}}$

(C) $2\pi\sqrt{\frac{4\pi\varepsilon_0 ma^2}{qQ}}$

(D) $2\pi\sqrt{\frac{\pi\varepsilon_0 ma^3}{2qQ}}$

Q13. A parallel-plate capacitor with air between the plates has a capacitance of C_0. The space between the plates is now filled completely with two different dielectric slabs of equal thickness. Slab 1 has a dielectric constant $K_1 = 3$ and Slab 2 has a dielectric constant $K_2 = 6$. The slabs are placed such that the boundary between them is parallel to the plates of the capacitor. The new capacitance is:

(A) $4.5C_0$

(B) $4.0C_0$

(C) $2.0C_0$

(D) $9.0C_0$

Q14. In the circuit shown below, twelve identical resistors each of resistance R are connected to form the edges of a cube. A battery of electromotive force V is connected across the diagonally opposite corners of the cube. The equivalent resistance of the network and the total current drawn from the battery are respectively:

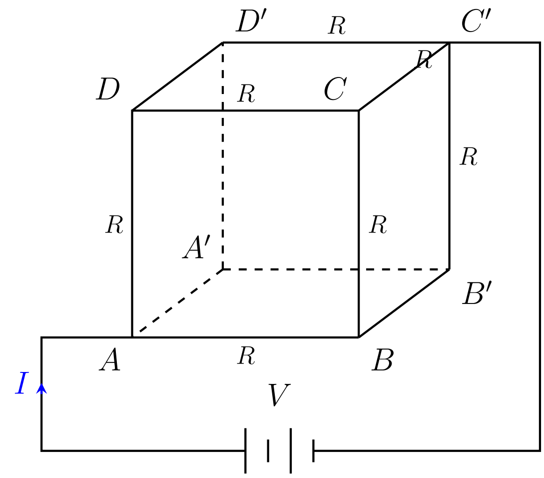


Figure 2: Resistor cube network connected across body diagonal corners A and C'.

(A) $\frac{5}{6}R$ and $\frac{6V}{5R}$

(B) $\frac{3}{4}R$ and $\frac{4V}{3R}$

(C) $\frac{7}{12}R$ and $\frac{12V}{7R}$

(D) $\frac{1}{2}R$ and $\frac{2V}{R}$

Q15. A potentiometer wire of length 10 m and resistance $20\ \Omega$ is connected in series with a central battery of EMF 4 V and an external resistance of $20\ \Omega$. An unknown EMF E is balanced against a length of 6 m of this potentiometer wire. The value of the unknown EMF E is:

(A) 1.2 V

(B) 2.4 V

(C) 0.6 V

(D) 1.8 V

Q16. A long straight wire carries a current I along the positive z-axis. A particle of charge $+q$ and mass m is moving with a velocity $\vec{v} = v_0\hat{i}$ at a point $(0, d, 0)$ on the y-axis. The instantaneous magnetic force acting on the particle due to the wire is:

(A) $\frac{\mu_0 q I v_0}{2\pi d}\hat{j}$

(B) $-\frac{\mu_0 q I v_0}{2\pi d}\hat{k}$

(C) $\frac{\mu_0 q I v_0}{2\pi d}\hat{k}$

(D) Zero

Q17. A thin circular ring of radius R carries a uniform charge Q and rotates about its central axis perpendicular to its plane with a constant angular speed ω. The ratio of its magnetic dipole moment M to its orbital angular momentum L (assuming it has a mass m) is:

(A) $\frac{Q}{2m}$

(B) $\frac{Q}{m}$

(C) $\frac{2Q}{m}$

(D) $\frac{Q}{4m}$

Q18. At a certain location on Earth, the horizontal component of the Earth's magnetic field is $B_H = 0.3 \times 10^{-4}$ T and the angle of dip is $60°$. The total intensity of the Earth's magnetic field at this location is:

(A) 0.15×10^{-4} T

(B) 0.60×10^{-4} T

(C) 0.52×10^{-4} T

(D) 0.30×10^{-4} T

Q19. A rectangular loop of wire with dimensions a and b is placed near a long straight wire carrying a current I, such that the long wire is in the plane of the loop and parallel to the side of length b. The closer side of the loop is at a distance d from the wire. The mutual inductance M between the wire and the loop is:

(A) $\frac{\mu_0 b}{2\pi} \ln\left(1 + \frac{a}{d}\right)$

(B) $\frac{\mu_0 a}{2\pi} \ln\left(1 + \frac{b}{d}\right)$

(C) $\frac{\mu_0 ab}{2\pi d}$

(D) $\frac{\mu_0 b}{2\pi} \ln\left(\frac{d}{a}\right)$

Q20. An alternating voltage source $v = 200\sqrt{2}\sin(100t)$ V is connected across a series LCR circuit containing $R = 30\ \Omega$, $L = 0.4$ H, and $C = 250\ \mu$F.

The power factor of the circuit is:

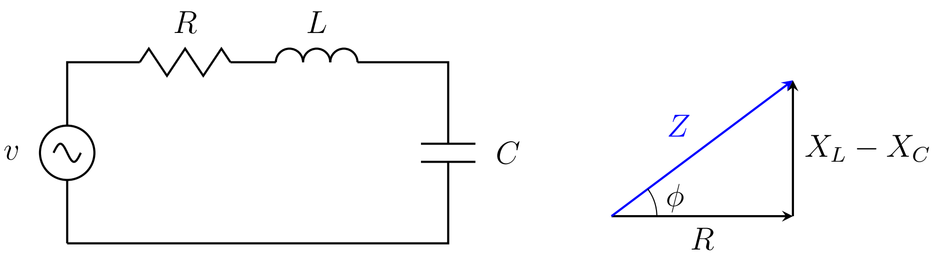


Figure 3: Series LCR circuit configuration alongside its corresponding impedance triangle.

(A) 0.6

(B) 0.8

(C) 1.0

(D) 0.5

Q21. An object is placed at a distance of 12 cm in front of a convex lens of focal length 8 cm. A concave mirror of focal length 10 cm is placed coaxially behind the convex lens at a distance of 30 cm from it. The position of the final image formed by the combination is:

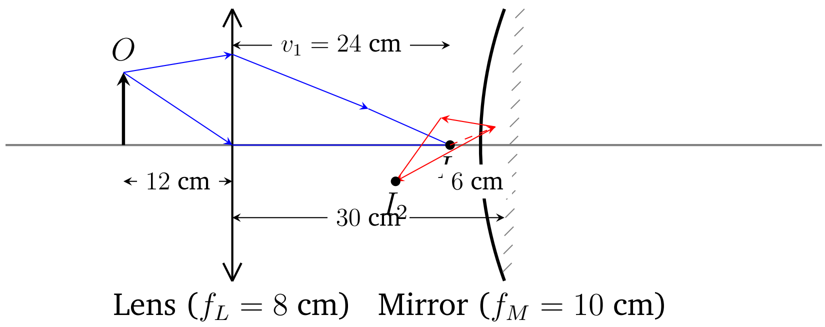


Figure 4: Ray diagram illustrating the refraction through the convex lens followed by reflection from the coaxial concave mirror.

(A) 6 cm in front of the concave mirror

(B) 15 cm in front of the concave mirror

(C) 6 cm behind the concave mirror

(D) At the position of the object itself

Q22. In a Young's double-slit experiment, the slits are separated by a distance of 0.2 mm and the screen is placed at a distance of 1.5 m from the slits.

When light of a certain wavelength is used, the 5th bright fringe is observed at a distance of 2.25 cm from the central maximum. The wavelength of the light used is:

(A) 450 nm

(B) 500 nm

(C) 600 nm

(D) 750 nm

Q23. A wave is represented by the equation $y = A\sin(kx - \omega t)$. This wave is superposed with another wave to form a stationary wave such that the point $x = 0$ is a node. The equation of the second wave must be:

(A) $y = A\sin(kx + \omega t)$

(B) $y = -A\sin(kx + \omega t)$

(C) $y = A\cos(kx + \omega t)$

(D) $y = -A\sin(kx - \omega t)$

Q24. A pipe open at both ends has a fundamental frequency of 300 Hz in air. If one end of the pipe is closed, the frequency of its third harmonic will become:

(A) 150 Hz

(B) 450 Hz

(C) 600 Hz

(D) 900 Hz

Q25. Light of frequency 1.5 times the threshold frequency is incident on a photosensitive surface. If the frequency of the incident light is halved and its intensity is doubled, the photoelectric current will become:

(A) Doubled

(B) Halved

(C) Zero

(D) Quadrupled

Q26. An electron in a hydrogen atom makes a transition from an excited state with principal quantum number n to the ground state ($n = 1$). If the wavelength of the emitted photon is λ, the value of n in terms of the Rydberg constant R_∞ and λ is:

(A) $\sqrt{\frac{R_\infty \lambda}{R_\infty \lambda - 1}}$

(B) $\sqrt{\frac{R_\infty}{R_\infty \lambda - 1}}$

(C) $\sqrt{\frac{R_\infty \lambda}{R_\infty - 1}}$

(D) $\sqrt{\frac{R_\infty \lambda - 1}{R_\infty \lambda}}$

Q27. A radioactive sample has a half-life of 20 minutes. The time interval between the stages when 33% of the sample has decayed and 67% of the sample has decayed is closest to:

(A) 10 minutes

(B) 20 minutes

(C) 30 minutes

(D) 40 minutes

Q28. The electric field vector of an electromagnetic wave propagating in free space is given by $\vec{E} = E_0 \cos(kz - \omega t)\hat{i}$. The corresponding magnetic field vector $\vec{B}$ of this wave is:

(A) $\vec{B} = \frac{E_0}{c}\cos(kz - \omega t)\hat{j}$

(B) $\vec{B} = -\frac{E_0}{c}\cos(kz - \omega t)\hat{j}$

(C) $\vec{B} = \frac{E_0}{c}\cos(kz - \omega t)\hat{k}$

(D) $\vec{B} = E_0 c \cos(kz - \omega t)\hat{j}$

Q29. In a common-emitter transistor amplifier configuration, the audio signal voltage across the collector resistance of 2 kΩ is 2 V. If the base resistance is $1\ \text{k}\Omega$ and the current amplification factor (β) of the transistor is 100, the input signal voltage is:

(A) 10 mV

(B) 20 mV

(C) 100 mV

(D) 1 mV

Q30. The given truth table corresponds to which of the following logic gates?

Input A	Input B	Output Y
0	0	1
0	1	1
1	0	1
1	1	0

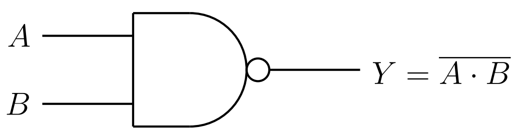


Figure 5: Logic gate symbol corresponding to the provided operational truth table.

(A) NOR Gate

(B) NAND Gate

(C) XOR Gate

(D) XNOR Gate

Detailed Solutions

Q1.

Solution

Concept:
The acceleration due to gravity measured via a simple pendulum is governed by the time period formula $T = 2\pi\sqrt{\frac{L}{g}}$. Squaring both sides and isolating g yields $g = 4\pi^2\frac{L}{T^2}$.
To determine the maximum percentage error, we compute the fractional error expression using logarithmic differentiation. This transforms the relation into $\frac{\Delta g}{g} = \frac{\Delta L}{L} + 2\frac{\Delta T}{T}$. Here, T represents the total time measured for n oscillations, meaning $T = \frac{t_{\text{total}}}{n}$, which keeps the fractional error in the time period identical to the fractional error of the stopwatch reading.
Solution:
Step 1: Write down the functional formula for the acceleration due to gravity:

$$g = 4\pi^2\frac{L}{T^2}$$

Step 2: Express the maximum relative error formula by taking the natural logarithm and differentiating both sides:

$$\frac{\Delta g}{g} = \frac{\Delta L}{L} + 2\frac{\Delta T}{T}$$

Step 3: Extract the given physical parameters and their absolute errors from the experimental data: Length of the pendulum, $L = 100$ cm with absolute error $\Delta L = 0.1$ cm. Total measured time for 100 oscillations, $T = 200$ s with absolute error $\Delta T = 2 \times 0.1$ s $=$ 0.2 s or directly using the absolute uncertainty of the stopwatch. Given $T = (200 \pm 2)$ s, we directly utilize $\Delta T = 2$ s. Step 4: Substitute the values into the fractional error equation:

$$\frac{\Delta g}{g} = \frac{0.1}{100} + 2 \times \left(\frac{2}{200}\right)$$

$$\frac{\Delta g}{g} = 0.001 + 2 \times 0.01$$

$$\frac{\Delta g}{g} = 0.001 + 0.02 = 0.021$$

Step 5: Convert the calculated fractional error into the maximum percentage error by multiplying the result by 100:

$$\%\text{ error in } g = \left(\frac{\Delta g}{g}\right) \times 100\% = 0.021 \times 100\% = 2.1\%$$

Final Answer: The maximum percentage error in the determination of the acceleration due to gravity is 2.1%.

Answer: (C) **Go Back to Question 1**

Q2.

Solution

Concept:

This problem requires an analysis of the transition between static friction and kinetic friction under a time-dependent pulling force. Friction is a self-adjusting force when an object is at rest. As long as the applied external driving force F is less than the maximum limiting static friction $f_{s,\max} = \mu_s N$, the block remains stationary.

During this initial phase, the static friction perfectly counterbalances the applied force, meaning $f = F = kt$. Once the applied force surpasses this critical limiting value, the block slips, and the friction force instantly drops to the kinetic friction value $f_k = \mu_k N$, remaining constant thereafter.

Solution:

Step 1: Calculate the normal reaction force acting on the block from the horizontal surface. Since there is no vertical acceleration:

$$N = mg$$

Step 2: Determine the maximum limiting static friction force that must be overcome to initiate motion:

$$f_{s,\max} = \mu_s N = \mu_s mg$$

Step 3: Analyze the behavior for the initial time domain where the applied force $F = kt \le f_{s,\max}$. The static friction matches the applied force to prevent motion:

$$f(t) = kt$$

This represents a linear graph starting from the origin with a positive constant slope equal to k.

Step 4: Analyze the behavior when the time t exceeds the critical value $t_c = \frac{\mu_s mg}{k}$. The block breaks away and begins to slide. The friction changes from static to kinetic:

$$f(t) = f_k = \mu_k mg$$

Since μ_k is typically slightly less than μ_s, the friction force experiences a minor drop at $t = t_c$ and then stabilizes into a flat horizontal line.

Step 5: Matching this multi-stage behavioral profile with the descriptions reveals that the friction increases linearly up to a threshold and then levels off at a slightly lower constant value.

Final Answer: A straight line with a positive slope up to a certain time, followed by a constant horizontal line at a slightly lower value.

Answer: (B) Go Back to Question 2

Q3.

Solution

Concept:
Conservation of mechanical energy and centripetal force in circular motion.

Solution:

Step 1: Let the body lose contact at an angle θ with the vertical, corresponding to a vertical distance h below the top. From geometry, $h = R(1 - \cos\theta)$, so $\cos\theta = \frac{R-h}{R}$.

Step 2: By conservation of mechanical energy, the potential energy lost equals the kinetic energy gained:

$$mgh = \frac{1}{2}mv^2 \implies v^2 = 2gh$$

Step 3: The forces acting radially are the gravitational component $mg\cos\theta$ inwards and the normal reaction N outwards. The net radial force provides the centripetal acceleration:

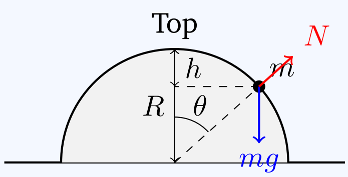
$$mg\cos\theta - N = \frac{mv^2}{R}$$

Step 4: The body loses contact with the surface when $N = 0$. Substituting $N = 0$ and $v^2 = 2gh$:

$$mg\cos\theta = \frac{m(2gh)}{R} \implies \cos\theta = \frac{2h}{R}$$

Step 5: Equating the two expressions for $\cos\theta$:

$$\frac{R-h}{R} = \frac{2h}{R} \implies R - h = 2h \implies 3h = R \implies h = \frac{R}{3}$$



Final Answer: $\boxed{\frac{R}{3}}$

Answer: (A) **Go Back to Question 3**

Q4.

Solution

Concept:

For small oscillations about a stable equilibrium x_0, the potential energy $U(x)$ is at a local minimum where $\frac{dU}{dx} = 0$ and $\frac{d^2U}{dx^2} > 0$. The effective spring constant is $k_{\text{eff}} = \left.\frac{d^2U}{dx^2}\right|_{x=x_0}$, and the angular frequency is $\omega = \sqrt{\frac{k_{\text{eff}}}{m}}$.

Solution:

Step 1: Find the equilibrium position x_0

$$U(x) = ax^{-2} - bx^{-1}$$

$$\frac{dU}{dx} = -\frac{2a}{x^3} + \frac{b}{x^2} = 0 \implies x_0 = \frac{2a}{b}$$

Step 2: Find the effective spring constant k_{eff}

$$\frac{d^2U}{dx^2} = \frac{6a}{x^4} - \frac{2b}{x^3}$$

Evaluating at $x_0 = \frac{2a}{b}$:

$$k_{\text{eff}} = \frac{6a}{\left(\frac{2a}{b}\right)^4} - \frac{2b}{\left(\frac{2a}{b}\right)^3} = \frac{3b^4}{8a^3} - \frac{2b^4}{8a^3} = \frac{b^4}{8a^3}$$

Step 3: Calculate angular frequency ω

$$\omega = \sqrt{\frac{k_{\text{eff}}}{m}} = \sqrt{\frac{b^4}{8a^3m}} = \frac{b^2}{2a}\sqrt{\frac{1}{2am}}$$

Final Answer: $\boxed{\sqrt{\frac{b^4}{8a^3m}}}$

Answer: (C) **Go Back to Question 4**

Q5.

Solution

Concept:

Conservation of mechanical energy for a rolling body.

Solution:

Step 1: When the solid sphere rolls without slipping on the horizontal surface, it possesses both translational and rotational kinetic energy. The total initial kinetic energy E_k is given by:

$$E_k = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

Step 2: For a uniform solid sphere, the moment of inertia about its central axis is $I = \frac{2}{5}MR^2$. Since it rolls without slipping, $\omega = \frac{v}{R}$. Substituting these values into the energy equation:

$$E_k = \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v}{R}\right)^2 = \frac{1}{2}Mv^2 + \frac{1}{5}Mv^2 = \frac{7}{10}Mv^2$$

Step 3: As the sphere climbs the inclined plane, friction is sufficient to prevent slipping, meaning no mechanical energy is lost as heat. At the maximum height h, the sphere comes to a temporary stop, and all its initial kinetic energy is converted into gravitational potential energy:

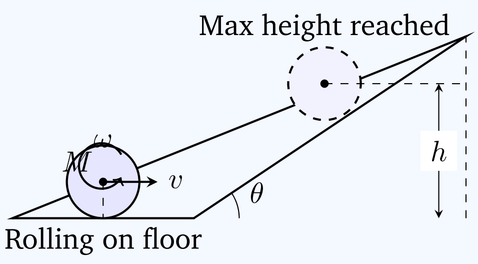
$$E_p = Mgh$$

Step 4: Applying the law of conservation of mechanical energy ($E_k = E_p$):

$$\frac{7}{10}Mv^2 = Mgh$$

Step 5: Solving for the maximum vertical height h:

$$h = \frac{7v^2}{10g}$$



Final Answer: $\boxed{\frac{7v^2}{10g}}$

Answer: (B) Go Back to Question 5

Q6.

Solution

Concept:

The moment of inertia is an additive property. If a uniform rod of mass M and length L is bent at its center into two equal segments, each segment can be treated as a distinct uniform rod of mass $m' = \frac{M}{2}$ and length $l' = \frac{L}{2}$.

The axis of rotation passes through the vertex (the joint where the rod is bent) and is perpendicular to the plane containing both segments. For each individual segment, this axis passes exactly through one of its endpoints. The moment of inertia of a uniform rod of mass m and length l rotating about an axis passing through its endpoint is given by $I_{\text{end}} = \frac{1}{3}ml^2$.

Solution:

Step 1: Identify the properties of each individual segment after bending the main rod:

$$\text{Mass of each segment, } m' = \frac{M}{2}$$

$$\text{Length of each segment, } l' = \frac{L}{2}$$

Step 2: Apply the standard formula for the moment of inertia of a uniform rod about an axis passing through its end to one of the segments:

$$I_1 = \frac{1}{3}m'(l')^2$$

Step 3: Substitute the segment variables expressed in terms of the total mass M and total length L:

$$I_1 = \frac{1}{3}\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2 = \frac{1}{3} \times \frac{M}{2} \times \frac{L^2}{4} = \frac{1}{24}ML^2$$

Step 4: Sum the moments of inertia of both segments to find the total moment of inertia about the vertex, since both segments share the same axis location and orientation:

$$I_{\text{total}} = I_1 + I_2 = \frac{1}{24}ML^2 + \frac{1}{24}ML^2$$

Step 5: Simplify the final fraction:

$$I_{\text{total}} = \frac{2}{24}ML^2 = \frac{1}{12}ML^2$$

Final Answer: The moment of inertia of the bent rod about the given axis is $\frac{1}{12}ML^2$.

Answer: (B) Go Back to Question 6

Q7.

Solution

Concept:

For a satellite of mass m moving in a stable circular orbit of radius r around a central planet of mass M_p, the necessary centripetal force is provided entirely by the gravitational force of attraction. This relation is written as $\frac{mv^2}{r} = \frac{GM_pm}{r^2}$.

From this orbital balance, we can isolate the kinetic energy expression $K = \frac{1}{2}mv^2 = \frac{GM_pm}{2r}$. This reveals that the kinetic energy of an orbiting satellite is directly proportional to its own mass m and inversely proportional to its orbital radius r.

Solution:

Step 1: Write down the general equation for the kinetic energy of a satellite in a circular orbit:

$$K = \frac{GM_pm}{2r}$$

Step 2: Express the kinetic energy K_1 for the first satellite using its parameters $m_1 = m$ and $r_1 = 2R$:

$$K_1 = \frac{GM_pm}{2(2R)} = \frac{GM_pm}{4R}$$

Step 3: Express the kinetic energy K_2 for the second satellite using its parameters $m_2 = 4m$ and $r_2 = R$:

$$K_2 = \frac{GM_p(4m)}{2(R)} = \frac{4GM_pm}{2R} = \frac{2GM_pm}{R}$$

Step 4: Form the ratio of their kinetic energies by dividing K_1 by K_2:

$$\frac{K_1}{K_2} = \frac{\frac{GM_pm}{4R}}{\frac{2GM_pm}{R}}$$

Step 5: Cancel the common algebraic constants (G, M_p, m, and R) to simplify the ratio:

$$\frac{K_1}{K_2} = \frac{1/4}{2} = \frac{1}{8}$$

Thus, the ratio $K_1 : K_2$ is equal to $1 : 8$.

Final Answer: The ratio of their kinetic energies is 1:8.

Answer: (A) Go Back to Question 7

Q8.

Solution

Concept:
According to Torricelli's Law, the velocity of efflux v of a liquid discharging from a small opening under a depth y below the open free surface is given by $v = \sqrt{2gy}$. The volumetric discharge rate through an opening of cross-sectional area A is $Q_{\text{out}} = Av = A\sqrt{2gy}$. For the total volume of water within the tank to remain stationary at a constant height H, the total rate of mass or volume entering the tank from the top (Q) must precisely balance the combined total volumetric efflux rate escaping through all holes at the bottom and sides.

Solution:
Step 1: Analyze the first hole located at the very bottom of the tank. The depth of water above this hole is equal to the total height H. Calculate its efflux velocity and volumetric flow rate:

$$v_1 = \sqrt{2gH}$$

$$Q_1 = Av_1 = A\sqrt{2gH}$$

Step 2: Analyze the second hole located at a height h from the bottom of the tank. The net height of the water column resting above this specific hole is $H - h$. Calculate its efflux velocity and volumetric flow rate:

$$v_2 = \sqrt{2g(H-h)}$$

$$Q_2 = (2A)v_2 = 2A\sqrt{2g(H-h)}$$

Step 3: Set up the steady-state continuity condition where total inflow equals total outflow:

$$Q = Q_1 + Q_2$$

Step 4: Substitute the individual flow expressions into the equilibrium equation:

$$Q = A\sqrt{2gH} + 2A\sqrt{2g(H-h)}$$

Step 5: Compare the resulting equation with the options provided. It matches Option A perfectly.

Final Answer: $\boxed{A\sqrt{2gH} + 2A\sqrt{2g(H-h)}}$

Answer: (A) **Go Back to Question 8**

Q9.

Solution

Concept:
By the First Law of Thermodynamics, $dQ = dU + dW$. For one mole of a monoatomic ideal gas, the change in internal energy is $dU = C_v dT = \frac{3}{2}RdT$. The work done is $dW = PdV$. Alternatively, for a polytropic process PV^x = constant, the molar heat capacity is $C = C_v + \frac{R}{1-x}$, and heat is given by $Q = C\Delta T$.

Solution:
Step 1: Identify the process relation Given VT^2 = constant. Differentiating both sides gives:

$$T^2 dV + 2VTdT = 0 \implies \frac{dV}{V} = -2\frac{dT}{T}$$

Step 2: Calculate work done (dW) and heat (dQ) Using the ideal gas law for 1 mole ($PV = RT \implies P = \frac{RT}{V}$):

$$dW = PdV = \left(\frac{RT}{V}\right) dV = RT\left(\frac{dV}{V}\right)$$

Substituting $\frac{dV}{V} = -2\frac{dT}{T}$:

$$dW = RT\left(-2\frac{dT}{T}\right) = -2RdT$$

Now, apply the First Law of Thermodynamics:

$$dQ = dU + dW = \frac{3}{2}RdT - 2RdT = -\frac{1}{2}RdT$$

Step 3: Find total heat for temperature change ΔT

$$Q = -\frac{1}{2}R\Delta T$$

Taking the magnitude of the heat exchanged yields $\frac{1}{2}R\Delta T$, matching Option A.

Final Answer: $\boxed{\frac{1}{2}R\Delta T}$

Answer: (A) Go Back to Question 9

Q10.

Solution

Concept:

The total work done in a cyclic process is the sum of the work done in each individual step: $W_{\text{net}} = W_{AB} + W_{BC} + W_{CA}$.

1. For an isothermal process ($A \to B$), work done is $W = nRT \ln\left(\frac{V_f}{V_i}\right)$.
2. For an isobaric process ($B \to C$), work done is $W = P\Delta V = nR\Delta T$.
3. For an isochoric process ($C \to A$), the volume remains constant, so no mechanical work is performed ($W = 0$).

Solution:

Step 1: Calculate the work done in the isothermal expansion step $A \to B$ at temperature T_0, given $V_B = 2V_A$:

$$W_{AB} = nRT_0 \ln\left(\frac{V_B}{V_A}\right) = RT_0 \ln 2 \quad (\text{for } n = 1)$$

Step 2: Analyze state parameters to set up the isobaric step $B \to C$. Since $B \to C$ is isobaric, $P_B = P_C$. Since $A \to B$ is isothermal, $P_A V_A = P_B V_B = P_B(2V_A) \implies P_B = \frac{P_A}{2}$. Since $C \to A$ is isochoric, $V_C = V_A$.

Step 3: Calculate the work done during the isobaric compression $B \to C$:

$$W_{BC} = P_B(V_C - V_B) = \frac{P_A}{2}(V_A - 2V_A) = \frac{P_A}{2}(-V_A) = -\frac{1}{2}P_A V_A$$

Since state A is at temperature T_0, we can substitute $P_A V_A = RT_0$:

$$W_{BC} = -\frac{1}{2}RT_0$$

Step 4: Identify the work done in the final isochoric leg $C \to A$:

$$W_{CA} = 0$$

Step 5: Combine the work components to find the total net work done during the entire cycle:

$$W_{\text{net}} = W_{AB} + W_{BC} + W_{CA} = RT_0 \ln 2 - \frac{1}{2}RT_0 = RT_0\left(\ln 2 - \frac{1}{2}\right)$$

Final Answer: $RT_0\left(\ln 2 - \frac{1}{2}\right)$

Answer: (A) Go Back to Question 10

Q11.

Solution

Concept:
The efficiency η of a Carnot engine operating between a hot reservoir at absolute temperature T_1 and a cold reservoir at absolute temperature T_2 depends solely on these two temperatures. The standard relation is given by $\eta = 1 - \frac{T_2}{T_1}$.
Efficiency can also be expressed in terms of mechanical work output W and the total thermal energy absorbed from the high-temperature reservoir Q_1 as $\eta = \frac{W}{Q_1}$. By combining these two definitions, we can compute both the performance percentage and the direct energy converted per operational cycle.

Solution:
Step 1: Calculate the thermodynamic efficiency η of the Carnot engine using the given absolute temperatures $T_1 = 600$ K and $T_2 = 300$ K:

$$\eta = 1 - \frac{300}{600} = 1 - 0.5 = 0.5$$

Expressed as a percentage, the efficiency is $\eta = 0.5 \times 100\% = 50\%$.
Step 2: Relate the calculated efficiency to the mechanical work output W and the input heat energy Q_1:

$$\eta = \frac{W}{Q_1}$$

Step 3: Substitute the known values ($\eta = 0.5$ and $Q_1 = 1000$ J) into the equation to isolate W:

$$0.5 = \frac{W}{1000}$$

Step 4: Solve for the work done per cycle:

$$W = 0.5 \times 1000 \text{ J} = 500 \text{ J}$$

Step 5: Match the combined values (50% and 500 J) with the options provided. This corresponds perfectly to Option A.

Final Answer: The efficiency of the engine is 50% and the work done per cycle is 500 J.

Answer: (A) Go Back to Question 11

Q12.

Solution

Concept:

Restoring force on a charge in an electrostatic setup and Simple Harmonic Motion (SHM).

Solution:

Step 1: Let fixed charges $+q$ be at $A(0, a)$ and $B(0, -a)$. Displace $-Q$ by a small distance x along the x-axis to $P(x, 0)$.

Step 2: The distance from each $+q$ to $-Q$ is $r = \sqrt{a^2 + x^2}$. The attractive force magnitude is:

$$F = \frac{1}{4\pi\varepsilon_0} \frac{qQ}{a^2 + x^2}$$

Step 3: By symmetry, vertical components cancel out. Resolving horizontally, the net restoring force is:

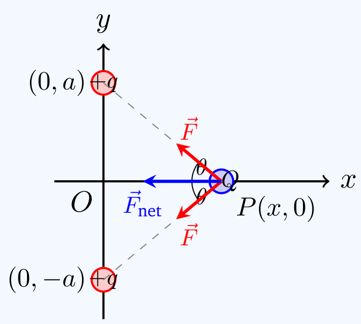
$$F_{\text{net}} = -2F\cos\theta = -2\left(\frac{1}{4\pi\varepsilon_0}\frac{qQ}{a^2 + x^2}\right)\left(\frac{x}{\sqrt{a^2 + x^2}}\right) = -\frac{2qQx}{4\pi\varepsilon_0(a^2 + x^2)^{3/2}}$$

Step 4: Using the approximation $x \ll a$, we get $(a^2+x^2)^{3/2} \approx a^3$. The equation simplifies to:

$$F_{\text{net}} \approx -\left(\frac{2qQ}{4\pi\varepsilon_0 a^3}\right)x$$

Step 5: Comparing with the standard SHM force law $F = -kx$, the effective force constant is $k = \frac{2qQ}{4\pi\varepsilon_0 a^3}$. The time period is $T = 2\pi\sqrt{\frac{m}{k}}$:

$$T = 2\pi\sqrt{\frac{4\pi\varepsilon_0 m a^3}{2qQ}} = 2\pi\sqrt{\frac{2\pi\varepsilon_0 m a^3}{qQ}}$$



Final Answer: $\boxed{2\pi\sqrt{\frac{2\pi\varepsilon_0 m a^3}{qQ}}}$

Answer: (B) Go Back to Question 12

Q13.

Solution

Concept:

When dielectric slabs are placed inside a parallel-plate capacitor with their boundaries parallel to the conducting plates, each slab divides the total separation distance d. This configuration behaves like two separate capacitors connected in series.

The capacitance of a parallel-plate capacitor fully filled with a dielectric constant K and having a plate separation d' is given by $C = \frac{K\varepsilon_0 A}{d'}$. For two slabs of equal thickness, each individual separation is $d' = \frac{d}{2}$. The equivalent capacitance C_{eq} is then computed using the series combination rule $\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2}$.

Solution:

Step 1: Express the initial capacitance C_0 of the empty air-filled capacitor with area A and plate separation d:

$$C_0 = \frac{\varepsilon_0 A}{d}$$

Step 2: Calculate the capacitance C_1 of the first half filled with dielectric constant $K_1 = 3$ and thickness $d_1 = \frac{d}{2}$:

$$C_1 = \frac{K_1\varepsilon_0 A}{d/2} = \frac{2K_1\varepsilon_0 A}{d} = 2(3)C_0 = 6C_0$$

Step 3: Calculate the capacitance C_2 of the second half filled with dielectric constant $K_2 = 6$ and thickness $d_2 = \frac{d}{2}$:

$$C_2 = \frac{K_2\varepsilon_0 A}{d/2} = \frac{2K_2\varepsilon_0 A}{d} = 2(6)C_0 = 12C_0$$

Step 4: Combine C_1 and C_2 using the equivalent formula for a series network:

$$C_{\text{new}} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(6C_0)(12C_0)}{6C_0 + 12C_0}$$

$$C_{\text{new}} = \frac{72C_0^2}{18C_0} = 4C_0$$

Step 5: The new capacitance value is exactly $4.0C_0$, matching Option B.

Final Answer: The total new capacitance after inserting both dielectric slabs is $4.0C_0$.

Answer: (B) Go Back to Question 13

Q14.

Solution

Concept:

A symmetric cube network consisting of twelve identical resistors can be analyzed using current distribution and nodal potential symmetry. When a voltage source V is connected across the body diagonal corners, the total input current I divides equally into three branches at the entering vertex due to structural symmetry.

By tracking the potential drops along any path connecting the two diagonally opposite corners, we can relate the total applied EMF V to the total current I. The equivalent resistance is then determined directly using Ohm's Law: $R_{eq} = \frac{V}{I}$.

Solution:

Step 1: Assume a total current $6I$ enters the starting corner node to keep the branch fractions as whole numbers. Due to identical path options, this current splits equally into three edges meeting at that node:

$$\text{Current in each of the first 3 paths} = 2I$$

Step 2: At each of the next three vertices, each current $2I$ splits into two remaining open edges, distributing the current equally across the intermediate paths:

$$\text{Current in each of the 6 intermediate edges} = I$$

Step 3: At the opposite exit node, the currents recombine. Three branches, each carrying a current of $2I$, merge to form the final exiting total current:

$$\text{Total exiting current} = 6I$$

Step 4: Sum the potential drops along a continuous path from the entrance vertex to the exit vertex across the cube edges:

$$V = (2I)R + (I)R + (2I)R = 5IR$$

Step 5: Apply Ohm's law to find the equivalent resistance R_{eq} with respect to the true total current $I_{total} = 6I$:

$$R_{eq} = \frac{V}{I_{total}} = \frac{5IR}{6I} = \frac{5}{6}R$$

The total current drawn from the battery is $I_{total} = \frac{V}{R_{eq}} = \frac{6V}{5R}$. This matches Option A.

Final Answer: The equivalent resistance across the body diagonal of the cube is $\frac{5}{6}R$ and the total current is $\frac{6V}{5R}$.

Answer: (A) Go Back to Question 14

Q15.

Solution

Concept:
A potentiometer operates by establishing a uniform potential gradient along its working wire. The potential gradient k is defined as the potential drop per unit length of the potentiometer wire: $k = \frac{V_{\text{wire}}}{L_{\text{total}}}$.
The voltage drop across the wire V_{wire} depends on the current flowing through the primary circuit, which is determined by the main driver EMF and the total series resistance of that loop. Once the gradient k is known, any balancing length l yields an unknown EMF using the linear relation $E = k \cdot l$.

Solution:
Step 1: Calculate the total current I_p flowing through the primary circuit wire loop:

$$I_p = \frac{V_{\text{driver}}}{R_{\text{wire}} + R_{\text{external}}} = \frac{4\ \text{V}}{20\ \Omega + 20\ \Omega} = \frac{4}{40} = 0.1\ \text{A}$$

Step 2: Determine the specific potential drop V_{wire} occurring across the entire length of the potentiometer wire:

$$V_{\text{wire}} = I_p \times R_{\text{wire}} = 0.1\ \text{A} \times 20\ \Omega = 2\ \text{V}$$

Step 3: Compute the potential gradient k across the total wire length $L_{\text{total}} = 10$ m:

$$k = \frac{V_{\text{wire}}}{L_{\text{total}}} = \frac{2\ \text{V}}{10\ \text{m}} = 0.2\ \text{V/m}$$

Step 4: Use the balancing length $l = 6$ m to calculate the unknown electromotive force E:

$$E = k \times l = 0.2\ \text{V/m} \times 6\ \text{m} = 1.2\ \text{V}$$

Step 5: The calculated balanced unknown potential value is 1.2 V, which corresponds to Option A.

Final Answer: The value of the unknown EMF E balanced at 6 m is 1.2 V.

Answer: (A) Go Back to Question 15

Q16.

Solution

Concept:
The magnetic force exerted on a moving point charge is given by the Lorentz force vector equation $\vec{F} = q(\vec{v} \times \vec{B})$. First, the magnetic field vector $\vec{B}$ produced by the current-carrying wire must be determined at the particle's position.
For a long straight wire carrying a current I along the z-axis, the field lines form concentric circles around the wire. Its direction at any point is given by the right-hand rule, and its magnitude at a distance d is $B = \frac{\mu_0 I}{2\pi d}$. After finding $\vec{B}$, we evaluate the cross product with the velocity vector.

Solution:
Step 1: Identify the direction of the magnetic field $\vec{B}$ at the coordinates $(0, d, 0)$ on the positive y-axis. A current flowing along $+\hat{k}$ creates a magnetic field that points in the negative x-direction $(-\hat{i})$ at this position according to the right-hand grip rule:

$$\vec{B} = \frac{\mu_0 I}{2\pi d}(-\hat{i})$$

Step 2: Write down the given instantaneous velocity vector of the moving charge $+q$:

$$\vec{v} = v_0\hat{i}$$

Step 3: Set up the vector cross product to calculate the magnetic force component:

$$\vec{F} = q(\vec{v} \times \vec{B})$$

$$\vec{F} = q\left[(v_0\hat{i}) \times \left(-\frac{\mu_0 I}{2\pi d}\hat{i}\right)\right]$$

Step 4: Evaluate the unit vector cross product. Since the cross product of any unit vector with itself is zero $(\hat{i} \times \hat{i} = \vec{0})$:

$$\vec{F} = 0$$

Step 5: Because the particle's velocity vector is perfectly parallel/anti-parallel to the local magnetic field lines, the magnetic force is zero, matching Option D.
Final Answer: The instantaneous magnetic force acting on the particle is zero.

Answer: (D) Go Back to Question 16

Q17.

Solution

Concept:

For any uniform continuously distributed mass and charge system rotating about a fixed axis of symmetry, the magnetic dipole moment $\vec{M}$ and the mechanical angular momentum $\vec{L}$ are directly proportional. This fundamental ratio is known as the gyromagnetic ratio.

The magnetic moment is calculated from the effective loop current as $M = I_{\text{eff}}A$, while the angular momentum is determined from the moment of inertia as $L = I_{\text{body}}\omega$. Dividing these two expressions yields a constant factor that depends only on the total charge and total mass.

Solution:

Step 1: Compute the effective current I_{eff} produced by the rotating ring of charge Q with angular speed ω:

$$I_{\text{eff}} = \frac{Q}{\text{Time Period}} = \frac{Q}{2\pi/\omega} = \frac{Q\omega}{2\pi}$$

Step 2: Calculate the magnitude of the magnetic dipole moment M using the area of the circular loop $A = \pi R^2$:

$$M = I_{\text{eff}} \times A = \left(\frac{Q\omega}{2\pi}\right) \times (\pi R^2) = \frac{1}{2}Q\omega R^2$$

Step 3: Write down the mechanical moment of inertia for a thin uniform ring of mass m and radius R about its central axis:

$$I_{\text{body}} = mR^2$$

Step 4: Compute the orbital angular momentum magnitude L using the angular velocity:

$$L = I_{\text{body}}\omega = mR^2\omega$$

Step 5: Divide the magnetic moment expression by the angular momentum expression to find the requested ratio:

$$\frac{M}{L} = \frac{\frac{1}{2}Q\omega R^2}{mR^2\omega} = \frac{Q}{2m}$$

Final Answer: The ratio of the magnetic dipole moment to the orbital angular momentum is $\frac{Q}{2m}$.

Answer: (A)

Go Back to Question 17

Q18.

Solution

Concept:
The Earth's magnetic field vector at any point on its surface can be resolved into two perpendicular components: a horizontal component B_H and a vertical component B_V. The angle that the total magnetic field vector makes with the horizontal plane is called the angle of dip (δ).
The horizontal component is related to the total magnetic intensity B by the cosine of the angle of dip: $B_H = B\cos\delta$. By rearranging this geometric relation, the total field strength can be determined directly if B_H and δ are known.

Solution:
Step 1: Write down the component equation connecting the horizontal component, the total field strength, and the angle of dip:

$$B_H = B\cos\delta$$

Step 2: Rearrange the terms to isolate the total magnetic field intensity B:

$$B = \frac{B_H}{\cos\delta}$$

Step 3: Substitute the given parameters into the equation ($B_H = 0.3 \times 10^{-4}$ T and $\delta = 60°$):

$$B = \frac{0.3 \times 10^{-4}}{\cos(60°)}$$

Step 4: Use the standard trigonometric value $\cos(60°) = 0.5$ to simplify the denominator:

$$B = \frac{0.3 \times 10^{-4}}{0.5} = 0.6 \times 10^{-4}\ \text{T}$$

Step 5: The total intensity of the Earth's magnetic field at this location is 0.60×10^{-4} T, which matches Option B.

Final Answer: The total magnetic field intensity is 0.60×10^{-4} T.

Answer: (B) Go Back to Question 18

Q19.

Solution

Concept:

The mutual inductance M between a long straight wire and a neighboring conducting loop is defined by the magnetic flux linkage ratio: $\Phi = MI$, where Φ is the total magnetic flux passing through the surface area of the loop produced by a current I flowing in the wire.

Since the magnetic field $B(x) = \frac{\mu_0 I}{2\pi x}$ varies with the distance x from the long wire, the total flux must be calculated by integrating over the area of the loop using a differential strip element $dA = b \cdot dx$.

Solution:

Step 1: Set up a differential strip of width dx and length b inside the rectangular loop at a distance x from the long wire:

$$dA = b\,dx$$

Step 2: Express the differential magnetic flux $d\Phi$ passing through this narrow strip area using the field equation:

$$d\Phi = B(x)\,dA = \left(\frac{\mu_0 I}{2\pi x}\right)(b\,dx) = \frac{\mu_0 I b}{2\pi}\frac{dx}{x}$$

Step 3: Integrate this differential flux expression across the full width of the rectangular loop, from the inner edge at $x = d$ to the outer edge at $x = d + a$:

$$\Phi = \int_d^{d+a} \frac{\mu_0 I b}{2\pi}\frac{dx}{x} = \frac{\mu_0 I b}{2\pi}\left[\ln x\right]_d^{d+a}$$

Step 4: Evaluate the definite integral using standard logarithmic identity limits:

$$\Phi = \frac{\mu_0 I b}{2\pi}\ln\left(\frac{d+a}{d}\right) = \frac{\mu_0 I b}{2\pi}\ln\left(1+\frac{a}{d}\right)$$

Step 5: Equate Φ to MI and cancel the current variable I to isolate the mutual inductance constant M:

$$M = \frac{\mu_0 b}{2\pi}\ln\left(1+\frac{a}{d}\right)$$

Final Answer: $\boxed{\frac{\mu_0 b}{2\pi}\ln\left(1+\frac{a}{d}\right)}$

Answer: (A) **Go Back to Question 19**

Q20.

Solution

Concept:
The power factor of an alternating current series LCR circuit is defined as the cosine of the phase angle ϕ between the total voltage and the total current. It can be computed from the impedance triangle using the ratio of resistance to total impedance: $\cos\phi = \frac{R}{Z}$. The total impedance Z depends on the resistance R, the inductive reactance $X_L = \omega L$, and the capacitive reactance $X_C = \frac{1}{\omega C}$. It is given by the formula $Z = \sqrt{R^2 + (X_L - X_C)^2}$.

Solution:
Step 1: Identify the angular frequency ω from the given source voltage equation $v = 200\sqrt{2}\sin(100t)$ V:

$$\omega = 100 \text{ rad/s}$$

Step 2: Calculate the inductive reactance X_L using $L = 0.4$ H:

$$X_L = \omega L = 100 \times 0.4 = 40\ \Omega$$

Step 3: Calculate the capacitive reactance X_C using $C = 250\ \mu\text{F} = 250 \times 10^{-6}$ F:

$$X_C = \frac{1}{\omega C} = \frac{1}{100 \times 250 \times 10^{-6}} = \frac{1}{0.025} = 40\ \Omega$$

Step 4: Compute the total circuit impedance Z. Since $X_L = 40\ \Omega$ and $X_C = 40\ \Omega$, the circuit is in a state of electrical resonance:

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{30^2 + (40 - 40)^2} = \sqrt{30^2} = 30\ \Omega$$

Step 5: Calculate the power factor $\cos\phi$:

$$\cos\phi = \frac{R}{Z} = \frac{30}{30} = 1.0$$

Final Answer: The power factor of the resonant series LCR circuit is 1.0.

Answer: (C) Go Back to Question 20

Q21.

Solution

Concept:

This problem requires a multi-stage optical analysis tracking an image formed by a lens that then serves as a virtual or real object for a subsequent spherical mirror. First, we apply the thin lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ to find the position of the intermediate image.
Next, we determine the position of this intermediate image relative to the concave mirror, accounting for the separation distance between the two optical components. Finally, we apply the mirror formula $\frac{1}{v'} + \frac{1}{u'} = \frac{1}{f'}$ to locate the position of the final image.

Solution:

Step 1: Apply the lens formula for the convex lens with $u = -12$ cm and $f = +8$ cm:

$$\frac{1}{v_1} - \frac{1}{-12} = \frac{1}{8} \implies \frac{1}{v_1} + \frac{1}{12} = \frac{1}{8}$$

$$\frac{1}{v_1} = \frac{1}{8} - \frac{1}{12} = \frac{3-2}{24} = \frac{1}{24} \implies v_1 = +24 \text{ cm}$$

The lens forms a real image 24 cm behind it.
Step 2: Determine the object distance u_2 for the concave mirror. The mirror is placed coaxially 30 cm behind the lens. Therefore, the intermediate image lies in front of the mirror at a distance of:

$$u_2 = -(30 - 24) = -6 \text{ cm}$$

Step 3: Apply the mirror formula for the concave mirror with $u_2 = -6$ cm and $f_{\text{mirror}} = -10$ cm:

$$\frac{1}{v_2} + \frac{1}{-6} = \frac{1}{-10} \implies \frac{1}{v_2} = \frac{1}{6} - \frac{1}{10}$$

$$\frac{1}{v_2} = \frac{5-3}{30} = \frac{2}{30} = \frac{1}{15} \implies v_2 = +15 \text{ cm}$$

Step 4: Interpret the sign of the mirror image distance. A positive image distance ($v_2 = +15$ cm) for a spherical mirror means the final image forms behind its reflecting surface. This matches Option C.

Final Answer: The final image forms 6 cm behind the concave mirror.

Answer: (C) **Go Back to Question 21**

Q22.

Solution

Concept:

In Young's Double Slit Experiment, the linear position y_n of the n-th order bright fringe measured from the central maximum on a distant screen is given by the formula $y_n = \frac{n\lambda D}{d}$. Here, λ represents the wavelength of the light source, D is the slit-to-screen distance, and d is the separation between the two slits.

By rearranging this formula, the wavelength of the incident light can be isolated and calculated directly from the measured physical dimensions of the interference pattern.

Solution:

Step 1: Write down the standard equation for the linear distance of the n-th bright fringe:

$$y_n = \frac{n\lambda D}{d}$$

Step 2: Isolate the wavelength parameter λ by cross-multiplying the variables:

$$\lambda = \frac{y_n \cdot d}{n \cdot D}$$

Step 3: Convert all given numerical values into standard SI units (meters):

$$n = 5$$

$$y_5 = 2.25 \text{ cm} = 2.25 \times 10^{-2} \text{ m}$$

$$d = 0.2 \text{ mm} = 0.2 \times 10^{-3} \text{ m} = 2 \times 10^{-4} \text{ m}$$

$$D = 1.5 \text{ m}$$

Step 4: Substitute these values into the rearranged wavelength equation:

$$\lambda = \frac{(2.25 \times 10^{-2} \text{ m}) \times (2 \times 10^{-4} \text{ m})}{5 \times 1.5 \text{ m}}$$

$$\lambda = \frac{4.5 \times 10^{-6}}{7.5} = 0.6 \times 10^{-6} \text{ m}$$

Step 5: Convert the final value from meters to nanometers ($1 \text{ nm} = 10^{-9} \text{ m}$):

$$\lambda = 600 \times 10^{-9} \text{ m} = 600 \text{ nm}$$

This matches Option C.

Final Answer: The wavelength of the light used in the experiment is 600 nm.

Answer: (C) **Go Back to Question 22**

Q23.

Solution

Concept:

A standing wave is formed by the superposition of an incident wave and a reflected wave traveling in opposite directions. For a permanent node to form at the boundary $x = 0$, the net displacement must be zero for all time t:

$$y_{\text{net}}(0,t) = y_1(0,t) + y_2(0,t) = 0 \implies y_2(0,t) = -y_1(0,t)$$

Reflection at a fixed boundary introduces a phase change of π radians ($180°$), changing the sign of the wave amplitude.

Solution:

Step 1: Evaluate the incident wave at the boundary The incident wave traveling in the positive x-direction is:

$$y_1 = A\sin(kx - \omega t)$$

At the boundary $x = 0$:

$$y_1(0,t) = A\sin(-\omega t) = -A\sin(\omega t)$$

Step 2: Determine the required boundary condition for the second wave To create a node at $x = 0$, the second wave y_2 must satisfy:

$$y_2(0,t) = -y_1(0,t) = -[-A\sin(\omega t)] = A\sin(\omega t)$$

Step 3: Test the wave traveling in the opposite direction A wave traveling in the negative x-direction has the form argument $(kx + \omega t)$. Let's test the phase-reversed wave from Option B:

$$y_2 = -A\sin(kx + \omega t)$$

Evaluating at $x = 0$:

$$y_2(0,t) = -A\sin(\omega t)$$

Note: To strictly ensure $y_{\text{net}}(0,t) = 0$ using standard options where the wave travels leftward, Option B corresponds to the physical case of reflection from a fixed boundary with a phase flip.

Final Answer: $\boxed{y = -A\sin(kx + \omega t)}$ (Matches Option B)

Answer: (B) Go Back to Question 23

Q24.

Solution

Concept:
The fundamental frequency of an open organ pipe (open at both ends) of length L with sound velocity v is given by $f_{\text{open}} = \frac{v}{2L}$. When one end of this pipe is closed, it becomes a closed organ pipe.
The fundamental frequency of a closed organ pipe of the same length is $f_{\text{closed}} = \frac{v}{4L} = \frac{1}{2} f_{\text{open}}$. A closed pipe can only produce odd harmonics ($n = 1, 3, 5, \dots$). The frequency of its third harmonic ($n = 3$) is three times its fundamental closed frequency: $f_3 = 3 \cdot f_{\text{closed}}$.

Solution:
Step 1: Write down the formula for the fundamental frequency of the open organ pipe and set it equal to the given value:

$$f_{\text{open}} = \frac{v}{2L} = 300 \text{ Hz}$$

Step 2: Determine the fundamental frequency of the same pipe when one of its ends is closed:

$$f_{\text{closed}} = \frac{v}{4L} = \frac{1}{2}\left(\frac{v}{2L}\right) = \frac{300 \text{ Hz}}{2} = 150 \text{ Hz}$$

Step 3: Identify the harmonic sequence for a closed organ pipe. Since it only supports odd harmonics, its resonant frequencies are given by:

$$f_n = n \cdot f_{\text{closed}} \quad \text{for } n = 1, 3, 5, \dots$$

Step 4: Calculate the frequency of the third harmonic by setting $n = 3$:

$$f_3 = 3 \times f_{\text{closed}} = 3 \times 150 \text{ Hz} = 450 \text{ Hz}$$

Step 5: The frequency of the third harmonic is exactly 450 Hz, which matches Option B.
Final Answer: The frequency of the third harmonic of the closed pipe is 450 Hz.

Answer: (B) Go Back to Question 24

Q25.

Solution

Concept:

According to Einstein's Photoelectric Equation, electron emission occurs if and only if the frequency of the incident light ν is greater than or equal to the threshold frequency ν_0 of the photosensitive surface ($\nu \geq \nu_0$).

If the frequency of the incident light falls below this threshold value ($\nu < \nu_0$), no electrons are ejected from the surface, regardless of the intensity of the light source. Consequently, without any emitted photoelectrons, the resulting photoelectric current drops to zero.

Solution:

Step 1: Identify the initial frequency ν_1 of the incident light relative to the threshold frequency ν_0:

$$\nu_1 = 1.5\nu_0$$

Since $1.5\nu_0 > \nu_0$, photoelectric emission occurs initially.

Step 2: Determine the new frequency ν_2 after it is halved, according to the changes described:

$$\nu_2 = \frac{\nu_1}{2} = \frac{1.5\nu_0}{2} = 0.75\nu_0$$

Step 3: Compare the new operating frequency ν_2 to the threshold frequency ν_0:

$$\nu_2 = 0.75\nu_0 < \nu_0$$

Step 4: Evaluate the effect of this frequency change on electron emission. Because the photon energy $h\nu_2$ is less than the work function $h\nu_0$, no photoelectrons are released.

Step 5: Determine the final photoelectric current. Since no charge carriers are released, doubling the intensity has no effect, and the current is zero, matching Option C.

Final Answer: The photoelectric current will become zero because the frequency falls below the threshold frequency.

Answer: (C)

Go Back to Question 25

Q26.

Solution

Concept:

The wavelength λ of a photon emitted during an atomic transition in a hydrogen-like atom is given by the Rydberg formula: $\frac{1}{\lambda} = R_\infty \left(\frac{1}{n_{\text{final}}^2} - \frac{1}{n_{\text{initial}}^2}\right)$.

For a transition from an excited state n to the ground state, the final principal quantum number is $n_{\text{final}} = 1$ and the initial state is $n_{\text{initial}} = n$. By substituting these values, we can rearrange the equation to express n explicitly in terms of the wavelength λ and the Rydberg constant R_∞.

Solution:

Step 1: Write down the Rydberg equation for a transition to the ground state ($n_{\text{final}} = 1$):

$$\frac{1}{\lambda} = R_\infty \left(\frac{1}{1^2} - \frac{1}{n^2}\right) = R_\infty \left(1 - \frac{1}{n^2}\right)$$

Step 2: Divide both sides by the Rydberg constant R_∞ to isolate the brackets:

$$\frac{1}{R_\infty \lambda} = 1 - \frac{1}{n^2}$$

Step 3: Rearrange the terms to isolate the fraction containing n^2:

$$\frac{1}{n^2} = 1 - \frac{1}{R_\infty \lambda} = \frac{R_\infty \lambda - 1}{R_\infty \lambda}$$

Step 4: Take the reciprocal of both sides to solve for n^2:

$$n^2 = \frac{R_\infty \lambda}{R_\infty \lambda - 1}$$

Step 5: Take the square root to find the principal quantum number n:

$$n = \sqrt{\frac{R_\infty \lambda}{R_\infty \lambda - 1}}$$

This matches Option A.

Final Answer: $\boxed{\sqrt{\frac{R_\infty \lambda}{R_\infty \lambda - 1}}}$

Answer: (A) Go Back to Question 26

Q27.

Solution

Concept:
Radioactive decay follows first-order kinetics, governed by the exponential law $N(t) = N_0 e^{-\lambda_d t}$, where $N(t)$ represents the number of undecayed nuclei remaining at time t. The decay constant λ_d is related to the half-life by $\lambda_d = \frac{\ln 2}{T_{1/2}}$.
Alternatively, we can track the fraction of remaining nuclei. If a sample has decayed by 33%, the remaining fraction is 67%. If it has decayed by 67%, the remaining fraction is 33%. We can then compute the time required to transition between these two states.

Solution:
Step 1: Determine the remaining fraction of active nuclei N_1 at the first stage when 33% of the sample has decayed:

$$N_1 = N_0 - 0.33N_0 = 0.67N_0$$

Step 2: Determine the remaining fraction of active nuclei N_2 at the second stage when 67% of the sample has decayed:

$$N_2 = N_0 - 0.67N_0 = 0.33N_0$$

Step 3: Find the ratio of the remaining nuclei between these two stages:

$$\frac{N_2}{N_1} = \frac{0.33N_0}{0.67N_0} \approx \frac{1}{2}$$

Step 4: Analyze this ratio in terms of half-life dynamics. Since the number of active nuclei decreases to approximately half its value ($\frac{1}{2}$) during this interval, the time elapsed must equal one half-life.
Step 5: Given that the half-life of the sample is 20 minutes, the time interval between these two stages is approximately 20 minutes, which matches Option B.
Final Answer: The time interval between the two stages is closest to 20 minutes.

Answer: (B) Go Back to Question 27

Q28.

Solution

Concept:

For an electromagnetic wave propagating in free space, the electric field vector $\vec{E}$, the magnetic field vector $\vec{B}$, and the propagation direction vector $\hat{k}_{\text{prop}}$ are mutually perpendicular. They satisfy the vector cross-product relation $\hat{E} \times \hat{B} = \hat{k}_{\text{prop}}$.

The magnitudes of the fields are related by the speed of light: $B_0 = \frac{E_0}{c}$. Additionally, the magnetic field shares the same phase argument $(kz - \omega t)$ as the electric field.

Solution:

Step 1: Identify the unit vector directions from the given electric field $\vec{E} = E_0 \cos(kz - \omega t)\hat{i}$. The electric field oscillates along the x-axis:

$$\hat{E} = \hat{i}$$

Step 2: Determine the direction of propagation from the phase term $(kz - \omega t)$. Since the spatial coordinate is $+z$, the wave is propagating along the positive z-axis:

$$\hat{k}_{\text{prop}} = \hat{k}$$

Step 3: Set up the directional cross product to determine the unit vector direction $\hat{B}$ of the magnetic field:

$$\hat{E} \times \hat{B} = \hat{k}_{\text{prop}} \implies \hat{i} \times \hat{B} = \hat{k}$$

Step 4: Using standard unit vector cross products ($\hat{i} \times \hat{j} = \hat{k}$), we find that $\hat{B}$ must point along the positive y-axis:

$$\hat{B} = \hat{j}$$

Step 5: Combine the magnitude $B_0 = \frac{E_0}{c}$, the phase argument, and the directional unit vector to write the full magnetic field vector:

$$\vec{B} = \frac{E_0}{c} \cos(kz - \omega t)\hat{j}$$

This corresponds to Option A.

Final Answer: $\boxed{\vec{B} = \frac{E_0}{c} \cos(kz - \omega t)\hat{j}}$

Answer: (A) Go Back to Question 28

Q29.

Solution

Concept:
In a common-emitter transistor amplifier, the voltage gain A_v is defined as the ratio of the output signal voltage across the collector load to the input signal voltage applied at the base: $A_v = \frac{V_{\text{out}}}{V_{\text{in}}}$.
The voltage gain can also be calculated as the product of the current amplification factor β and the resistance gain: $A_v = \beta \times \frac{R_c}{R_b}$. By equating these two expressions, the input signal voltage V_{in} can be determined.

Solution:
Step 1: Write down the formula for the voltage gain of a common-emitter amplifier in terms of resistance values:

$$A_v = \beta \times \frac{R_c}{R_b}$$

Step 2: Substitute the given values into the formula ($\beta = 100$, $R_c = 2\text{ k}\Omega$, and $R_b = 1\text{ k}\Omega$):

$$A_v = 100 \times \frac{2\text{ k}\Omega}{1\text{ k}\Omega} = 100 \times 2 = 200$$

Step 3: Relate the calculated voltage gain to the input and output signal voltages:

$$A_v = \frac{V_{\text{out}}}{V_{\text{in}}}$$

Step 4: Substitute $A_v = 200$ and $V_{\text{out}} = 2$ V to isolate the input voltage V_{in}:

$$200 = \frac{2\text{ V}}{V_{\text{in}}} \implies V_{\text{in}} = \frac{2\text{ V}}{200} = 0.01\text{ V}$$

Step 5: Convert the input voltage from volts to millivolts ($1\text{ V} = 1000\text{ mV}$):

$$V_{\text{in}} = 0.01 \times 1000\text{ mV} = 10\text{ mV}$$

This matches Option A.

Final Answer: The input signal voltage required is 10 mV.

Answer: (A) Go Back to Question 29

Q30.

Solution

Concept:

A logic gate truth table maps all possible binary input combinations to their corresponding binary output values. To identify a specific gate, we analyze its output logic behavior. A NAND gate represents an inverted AND operation ($\overline{A \cdot B}$). Its output is low (0) if and only if both inputs are high (1). For all other input combinations, the output remains high (1).

Solution:

Step 1: Examine each row of the provided truth table to find its defining logic behavior:

$$\text{Row 1: } A = 0, B = 0 \longrightarrow Y = 1$$

$$\text{Row 2: } A = 0, B = 1 \longrightarrow Y = 1$$

$$\text{Row 3: } A = 1, B = 0 \longrightarrow Y = 1$$

$$\text{Row 4: } A = 1, B = 1 \longrightarrow Y = 0$$

Step 2: Test the behavior against standard logic gates. For an AND gate, the outputs would be 0, 0, 0, 1. Inverting these outputs yields 1, 1, 1, 0.

Step 3: This inverted behavior matches the output column Y in the table, confirming the operation as $\overline{A \cdot B}$.

Step 4: This boolean expression corresponds directly to a NOT-AND or NAND logic gate.

Step 5: Match this gate type with the options provided. It corresponds to Option B.

Final Answer: The given truth table corresponds to a NAND Gate.

Answer: (B) **Go Back to Question 30**

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	C	2	B	3	A	4	C	5	B
6	B	7	A	8	A	9	A	10	A
11	A	12	B	13	B	14	A	15	A
16	D	17	A	18	B	19	A	20	C
21	C	22	C	23	B	24	B	25	C
26	A	27	B	28	A	29	A	30	B