BITSAT Physics Sample Paper-19

Duration: 40 Minutes **Maximum Marks: 90**

Instructions

- This paper contains **30** Multiple Choice Questions (Single Correct).
- Each correct answer carries **+3 marks**. Each incorrect answer carries: **−1** marks. Unattempted questions carry **0** marks.
- Only one option is correct for each question.
- Use of mobile phones, smartwatches, calculators, or any electronic gadgets is strictly prohibited.

Q1. A solid sphere of mass M and radius R is placed on a rough horizontal surface. A horizontal force F is applied at a height h above the center of the sphere. If the sphere rolls without slipping right from the start, and the direction of the frictional force acting on the sphere is in the backward direction (opposite to F), then which of the following is true?

(A) $h < \frac{2}{5}R$

(B) $h > \frac{2}{5}R$

(C) $h = \frac{2}{5}R$

(D) Friction is independent of h

Q2. An ideal gas undergoes a thermodynamic process represented by PV^3 = constant. If the initial temperature of the gas is T_0 and it is compressed to half of its initial volume, the final temperature of the gas will be:

(A) $2T_0$

(B) $4T_0$

(C) $\frac{T_0}{2}$

(D) $\frac{T_0}{4}$

Q3. A small block of mass m is released from rest from the top of a smooth sphere of radius R. The angular displacement θ from the vertical at which the block loses contact with the sphere satisfies:

(A) $\cos\theta = \frac{1}{3}$

(B) $\cos\theta = \frac{2}{3}$

(C) $\cos\theta = \frac{1}{2}$

(D) $\cos\theta = \frac{3}{5}$

Q4. In a photoelectric effect experiment, when light of wavelength λ is incident on a photosensitive surface, the stopping potential is V. When light of wavelength 2λ is incident on the same surface, the stopping potential becomes $V/3$. The threshold wavelength for this surface is:

(A) 3λ

(B) 4λ

(C) 5λ

(D) 1.5λ

Q5. A particle of charge q and mass m enters a region of uniform magnetic field $\vec{B} = B_0\hat{k}$ with an initial velocity $\vec{v} = v_x\hat{i} + v_z\hat{k}$ at the origin. The coordinates of the particle at time $t = \frac{\pi m}{qB_0}$ will be:

(A) $\left(0, \frac{2mv_x}{qB_0}, \frac{\pi m v_z}{qB_0}\right)$

(B) $\left(\frac{2mv_x}{qB_0}, 0, \frac{\pi m v_z}{qB_0}\right)$

(C) $\left(0, -\frac{2mv_x}{qB_0}, \frac{\pi m v_z}{qB_0}\right)$

(D) $\left(-\frac{2mv_x}{qB_0}, 0, 0\right)$

Q6. A parallel plate capacitor with plate area A and separation d is filled with two vertical slabs of dielectrics having dielectric constants K_1 and K_2, each of width $d/2$. The effective capacitance of the system is:

(A) $\frac{\epsilon_0 A}{d}\left(\frac{K_1+K_2}{2}\right)$

(B) $\frac{\epsilon_0 A}{d}\left(\frac{2K_1K_2}{K_1+K_2}\right)$

(C) $\frac{\epsilon_0 A}{d}(K_1 + K_2)$

(D) $\frac{\epsilon_0 A}{2d}\left(\frac{K_1 K_2}{K_1+K_2}\right)$

Q7. In a Young's double-slit experiment, the slits are illuminated by monochromatic light of wavelength 600 nm. A thin transparent sheet of refractive index 1.5 and thickness 6 μm is introduced in front of one of the slits. The number of fringes that shift past the central terminal point is:

(A) 5

(B) 10

(C) 15

(D) 20

Q8. A non-zero current I flows through a long straight wire along the z-axis. Another thin uniform wire loop of radius R carrying a current I_2 is placed in the xy-plane with its center at $(d, 0, 0)$ where $d > R$. The net magnetic force exerted by the straight wire on the loop is:

(A) Zero

(B) Proportional to $\frac{1}{d}$

(C) Proportional to $\frac{1}{d^2}$

(D) Proportional to $\ln(d)$

Q9. In an L-C-R series circuit connected to an AC source of voltage $V = V_0 \sin(\omega t)$, the voltage across the resistor, inductor, and capacitor are found to be 40 V, 100 V, and 70 V respectively. The peak value of the source voltage V_0 is:

(A) 50 V

(B) $50\sqrt{2}$ V

(C) 70 V

(D) $100\sqrt{2}$ V

Q10. The activity of a certain radioactive sample decreases to $1/8$ of its initial value in

9 days. The fraction of the initial nuclei remaining undecayed after an additional 6 days will be:

(A) $\frac{1}{16}$

(B) $\frac{1}{32}$

(C) $\frac{1}{64}$

(D) $\frac{1}{12}$

Q11. A constant force $F = 20$ N pushes a block of mass 2 kg up a rough inclined plane of inclination 30°. The coefficient of kinetic friction between the block and the plane is 0.2. The work done by the net force on the block as it slides a distance of 5 m up along the plane is (take $g = 10$ m/s^2, $\sqrt{3} \approx 1.732$):

(A) 100 J

(B) 50 J

(C) 32.7 J

(D) 17.3 J

Q12. A particle executes simple harmonic motion along the x-axis with an amplitude A. At what displacement from the mean position is its kinetic energy equal to three times its potential energy?

(A) $\pm\frac{A}{2}$

(B) $\pm\frac{A}{\sqrt{2}}$

(C) $\pm\frac{\sqrt{3}A}{2}$

(D) $\pm\frac{A}{4}$

Q13. Two bodies of masses m_1 and m_2 $(m_1 > m_2)$ are connected by a light inextensible string passing over a smooth, fixed pulley. The acceleration of the center of mass of the system is:

(A) $\left(\frac{m_1-m_2}{m_1+m_2}\right) g$

(B) $\left(\frac{m_1-m_2}{m_1+m_2}\right)^2 g$

(C) Zero

(D) $\frac{2m_1m_2}{(m_1+m_2)^2}g$

Q14. An electromagnetic wave is propagating in a medium with a velocity $\vec{v} = v\hat{i}$. The magnetic field oscillation at a certain point is given by $\vec{B} = B_0\cos(\omega t - kx)\hat{j}$. The corresponding electric field vector $\vec{E}$ is given by:

(A) $\vec{E} = vB_0\cos(\omega t - kx)\hat{k}$

(B) $\vec{E} = -vB_0\cos(\omega t - kx)\hat{k}$

(C) $\vec{E} = \frac{B_0}{v}\cos(\omega t - kx)\hat{k}$

(D) $\vec{E} = -\frac{B_0}{v}\cos(\omega t - kx)\hat{k}$

Q15. In a practical wheatstone bridge network, the balancing length from the left end is found to be l_1 when a standard resistance X is in the left gap and Y is in the right gap. If the wire has a non-uniform resistance per unit length such that $\rho(x) = \rho_0(1 + \alpha x)$, where x is measured from the left end, the true ratio X/Y is:

(A) $\frac{l_1}{100-l_1}$

(B) $\frac{l_1+\frac{\alpha}{2}l_1^2}{(100-l_1)+\frac{\alpha}{2}(100^2-l_1^2)}$

(C) $\frac{l_1+\alpha l_1^2}{(100-l_1)+\alpha(100-l_1)^2}$

(D) $\frac{2l_1+\alpha l_1^2}{2(100-l_1)+\alpha(100^2-l_1^2)}$

Q16. A particle of mass m moves in a circular orbit under the influence of a central attractive force given by $F = -\frac{k}{r^3}$. The total mechanical energy of the particle in an orbit of radius R (assuming potential energy at infinity to be zero) is:

(A) $-\frac{k}{2R^2}$

(B) $\frac{k}{2R^2}$

(C) Zero

(D) $-\frac{k}{R^2}$

Q17. In the circuit diagram shown below, an ideal diode is connected in series with a 100 Ω resistor across a 5 V DC battery. If the diode is reverse-biased, the current through the circuit and the voltage drop across the diode are respectively:

(A) 50 mA, 0 V

(B) 0 mA, 5 V

(C) 50 mA, 5 V

(D) 0 mA, 0 V

Q18. Consider an electron in a hydrogen atom transitioning from an orbit with quantum number n to an orbit with quantum number $(n-1)$. For very large values of n, the frequency of the emitted radiation is approximately proportional to:

(A) $\frac{1}{n}$

(B) $\frac{1}{n^2}$

(C) $\frac{1}{n^3}$

(D) $\frac{1}{n^4}$

Q19. A solid cylinder of mass M and radius R is mounted on a frictionless horizontal axle. A string is wrapped around it, and a block of mass m hangs from the free end. The acceleration of the falling block is:

(A) $\frac{mg}{m+M}$

(B) $\frac{mg}{m+M/2}$

(C) $\frac{mg}{m+2M}$

(D) $\frac{2mg}{M}$

Q20. An ideal gas heat engine operates in a Carnot cycle between 227°C and 127°C. It absorbs 6×10^4 cal of heat at the higher temperature. The amount of heat converted into work is:

(A) 1.2×10^4 cal

(B) 4.8×10^4 cal

(C) 3.5×10^4 cal

(D) 2.4×10^4 cal

Q21. The period of oscillation of a simple pendulum is given by $T = 2\pi\sqrt{\frac{L}{g}}$. The measured value of L is 20.0 cm known to 1 mm accuracy and the time for 100 oscillations is measured to be 90 s using a wrist watch of 1 s resolution. The percentage error in the determination of g is closest to:

(A) 2.7%

(B) 3.8%

(C) 1.2%

(D) 5.4%

Q22. A small bar magnet of magnetic moment M is placed at the center of a thin flat circular coil of radius R containing N turns. The axis of the bar magnet is aligned perpendicular to the plane of the coil. If a current I is passed through the coil, the torque acting on the bar magnet is:

(A) $\frac{\mu_0 NIM}{2R}$

(B) $\frac{\mu_0 NIM}{R}$

(C) Zero

(D) $\frac{\mu_0 NIM}{4\pi R^2}$

Q23. A soap bubble of radius r is blown in an atmosphere of pressure P_0. The surface tension of the soap solution is T. The total absolute pressure inside the bubble is:

(A) $P_0 + \frac{2T}{r}$

(B) $P_0 + \frac{4T}{r}$

(C) $P_0 + \frac{T}{r}$

(D) $P_0 - \frac{4T}{r}$

Q24. A point charge $+q$ is placed at a distance d from an infinite, grounded, perfectly conducting plane surface. The electrostatic force of attraction between the charge and the plane is:

(A) $\frac{q^2}{4\pi\epsilon_0 d^2}$

(B) $\frac{q^2}{16\pi\epsilon_0 d^2}$

(C) $\frac{q^2}{8\pi\epsilon_0 d^2}$

(D) Zero

Q25. A vessel contains a mixture of 1 mole of oxygen gas (molar mass 32 g/mol) and 2 moles of argon gas (molar mass 40 g/mol) at a temperature T. Neglecting all vibrational modes, the total internal energy of the mixture is:

(A) $\frac{11}{2}RT$

(B) $\frac{9}{2}RT$

(C) $4RT$

(D) $5RT$

Q26. A ray of light is incident at an angle of 60° on one face of a prism of refracting angle 30°. The ray emerging from the prism is found to be deviated by an angle of 30° from its original path. The refractive index of the material of the prism is:

(A) $\sqrt{2}$

(B) $\sqrt{3}$

(C) 1.5

(D) 1.25

Q27. A wave pulse traveling along a stretched string is described by the equation $y(x,t) = \frac{A}{B+(x-vt)^2}$. At $t = 0$, a particle situated at $x = 0$ has a vertical velocity component. The maximum transverse velocity of any particle on the string is:

(A) $\frac{3\sqrt{3}Av}{8B^{3/2}}$

(B) $\frac{Av}{B}$

(C) $\frac{3Av}{4B}$

(D) $\frac{\sqrt{3}Av}{2B^{3/2}}$

Q28. Two batteries of EMFs $E_1 = 6$ V and $E_2 = 3$ V with internal resistances $r_1 = 1\ \Omega$ and $r_2 = 2\ \Omega$ respectively are connected in parallel across an external load resistance $R = 5\ \Omega$ such that their positive terminals are connected to the same node. The potential difference across the load resistor R is:

(A) 4.5 V

(B) 5.0 V

(C) 4.17 V

(D) 3.75 V

Q29. A block of mass M hanging from a vertical spring of spring constant k executes simple harmonic motion with a period T_1. If another block of mass m is dynamically placed on top of M at the instant it passes through its lowest point of oscillation, the new time period of the system becomes T_2. The ratio T_2/T_1 is:

(A) $\sqrt{\frac{M+m}{M}}$

(B) $\frac{M+m}{M}$

(C) $\sqrt{\frac{M}{M+m}}$

(D) Dependent on the initial amplitude of oscillation

Q30. A conducting rod of length L rotates with a uniform angular velocity ω about an axis passing through one of its ends and perpendicular to its length. A uniform magnetic field B exists parallel to the axis of rotation. The induced electromotive force between the two ends of the rod is:

(A) $BL^2\omega$

(B) $\frac{1}{2}BL^2\omega$

(C) $\frac{1}{4}BL^2\omega$

(D) Zero

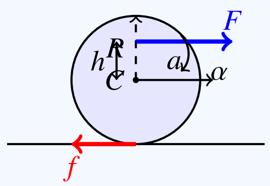
Detailed Solutions

Q1.

Solution

Concept:
Pure rolling without slipping implies that the acceleration of the center of mass (a) and the angular acceleration (α) satisfy the relation $a = R\alpha$[cite: 66]. By analyzing the translational and rotational dynamics around the center of mass, we can determine the exact direction and constraint on the frictional force[cite: 68].

Solution:
Step 1: Let the applied force F act horizontally toward the right at a height h above the center of mass[cite: 69]. The linear acceleration equation for the sphere is written as $F - f = Ma$ assuming the frictional force f acts in the backward direction[cite: 70].
Step 2: Considering the rotational dynamics about the center of mass, the torque equation is given by $Fh + fR = I\alpha$, where the moment of inertia of a solid sphere is $I = \frac{2}{5}MR^2$[cite: 71].
Step 3: Substituting the rolling condition $\alpha = \frac{a}{R}$ into the torque expression gives the relation $Fh + fR = (\frac{2}{5}MR^2)(\frac{a}{R}) = \frac{2}{5}MRa$[cite: 72].
Step 4: From the linear equation, we substitute $Ma = F - f$ into this relation, which yields $Fh + fR = \frac{2}{5}R(F - f)$[cite: 73]. Grouping the friction terms on one side results in $\frac{7}{5}fR = F(\frac{2}{5}R - h)$[cite: 74]. For the friction force f to be strictly positive (acting in the backward direction as defined), the condition $\frac{2}{5}R - h > 0$ must hold true, which simplifies directly to $h < \frac{2}{5}R$[cite: 74].



Final Answer:

$$\boxed{h < \frac{2}{5}R}$$

Answer: (A)

Go Back to Question 1

Q2.

Solution

Concept: The question relates a thermodynamic process governed by a polytropic equation. By converting the state variables from pressure and volume to temperature and volume using the ideal gas law, the final temperature can be explicitly calculated for a given volume change.

Solution:

(a) The given polytropic process follows the equation $PV^3 = C$, where C is a constant. The ideal gas equation states that $PV = nRT$, which allows us to substitute the pressure variable as $P = \frac{nRT}{V}$.

(b) Substituting this expression into the process equation gives $\left(\frac{nRT}{V}\right) V^3 = C$, which simplifies to $TV^2 = \frac{C}{nR}$. Since n and R are constants, the relation can be written as $TV^2 = \text{constant}$.

(c) For the initial and final states of the compression process, we set up the ratio equation $T_0 V_0^2 = T_f V_f^2$. This allows us to isolate the final temperature as $T_f = T_0 \left(\frac{V_0}{V_f}\right)^2$.

(d) The problem states that the ideal gas is compressed to half of its initial volume, which gives the volume ratio $\frac{V_0}{V_f} = 2$.

(e) Substituting this value into our equation yields $T_f = T_0(2)^2 = 4T_0$. This calculation shows that compressing the volume by half under this process quadruples the absolute temperature.

Final Answer: Option (B) is correct.

Answer: (B)

Go Back to Question 2

Q3.

Solution

Concept: A particle sliding down a smooth curved surface undergoes circular motion. It maintains contact as long as the normal force is greater than zero. The point of separation is determined by setting the normal reaction force to zero while satisfying conservation of mechanical energy.

Solution:

(a) Let the block start from rest at the top of the smooth sphere. When it reaches an angular displacement θ relative to the vertical axis, its vertical drop height is given by $h = R(1 - \cos\theta)$.

(b) Applying the law of conservation of mechanical energy between the initial top position and the angular position θ, the potential energy lost equals the kinetic energy gained: $mgR(1 - \cos\theta) = \frac{1}{2}mv^2$, which simplifies to $v^2 = 2gR(1 - \cos\theta)$.

(c) The centripetal force required for circular motion is provided by the radial components of the forces acting on the block, yielding the dynamic equation $mg\cos\theta - N = \frac{mv^2}{R}$.

(d) The block loses contact with the smooth sphere when the normal reaction force becomes zero ($N = 0$). Substituting this boundary condition simplifies the force equation to $mg\cos\theta = \frac{mv^2}{R}$, which gives $v^2 = gR\cos\theta$.

(e) Equating the two independent expressions for v^2 yields $2gR(1 - \cos\theta) = gR\cos\theta$. Canceling common terms results in $2 - 2\cos\theta = \cos\theta$, which directly simplifies to $3\cos\theta = 2$, or $\cos\theta = \frac{2}{3}$.

Final Answer: Option (B) is correct.

Answer: (B)

Go Back to Question 3

Q4.

Solution

Concept: The photoelectric effect is described by Einstein photoelectric equation, which relates the kinetic energy of emitted photoelectrons to the incident photon energy and the work function of the metal surface. The stopping potential is a direct measure of this maximum kinetic energy.

Solution:

(a) Einstein equation is written as $eV = \frac{hc}{\lambda} - \phi$, where V is the stopping potential, λ is the incident wavelength, and ϕ is the work function. The work function can be expressed in terms of the threshold wavelength as $\phi = \frac{hc}{\lambda_0}$.

(b) For the first case, the equation is $eV = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$. For the second case with incident wavelength 2λ, the stopping potential becomes $V/3$, yielding the second equation $e\left(\frac{V}{3}\right) = \frac{hc}{2\lambda} - \frac{hc}{\lambda_0}$.

(c) Multiplying the second equation by 3 to align the terms with the first equation gives $eV = \frac{3hc}{2\lambda} - \frac{3hc}{\lambda_0}$.

(d) Equating both expressions for eV results in $\frac{hc}{\lambda} - \frac{hc}{\lambda_0} = \frac{3hc}{2\lambda} - \frac{3hc}{\lambda_0}$. We can cancel the common factor hc from all terms in the equation.

(e) Rearranging the remaining terms to separate the variables yields $\frac{3}{\lambda_0} - \frac{1}{\lambda_0} = \frac{3}{2\lambda} - \frac{1}{\lambda}$, which simplifies to $\frac{2}{\lambda_0} = \frac{1}{2\lambda}$. Solving for the threshold wavelength gives $\lambda_0 = 4\lambda$.

Final Answer: Option (B) is correct.

Answer: (B)

Go Back to Question 4

Q5.

Solution

Concept: A charged particle moving in a uniform magnetic field experiences a Lorentz force perpendicular to its velocity and the magnetic field. This force affects only the component of velocity perpendicular to the field, causing circular motion in that plane, while the parallel velocity component remains unaffected.

Solution:

(a) The magnetic field is given as $\vec{B} = B_0\hat{k}$. The initial velocity vector has two components: $v_x\hat{i}$ in the xy-plane and $v_z\hat{k}$ along the z-axis. The parallel component v_z remains constant throughout the motion because the magnetic force along $\hat{k}$ is zero.

(b) Thus, the z-coordinate at any time t increases linearly as $z(t) = v_z t$. Substituting the given target time $t = \frac{\pi m}{qB_0}$ gives the final z-coordinate as $z = \frac{\pi m v_z}{qB_0}$.

(c) In the xy-plane, the velocity component v_x undergoes uniform circular motion. The cyclotron angular frequency is $\omega = \frac{qB_0}{m}$. The given time $t = \frac{\pi m}{qB_0}$ corresponds exactly to half a period of revolution ($\omega t = \pi$).

(d) At $t = 0$, the particle moves along $+\hat{i}$ from the origin. The magnetic force at the origin is $\vec{F} = q(\Delta v_x\hat{i} \times B_0\hat{k}) = -qv_xB_0\hat{j}$, which forces the particle into a circular arc in the fourth quadrant ($-\hat{j}$ direction).

(e) After completing half a circle (π radians), the particle reaches its maximum displacement along the y-axis, which corresponds to a distance of two radii ($2R$). Since the orbit curves into the negative y-region, the coordinates become $x = 0$ and $y = -2R = -\frac{2mv_x}{qB_0}$.

Final Answer: Option (C) is correct.

Answer: (C)

Go Back to Question 5

Q6.

Solution

Concept: When dielectric slabs are placed inside a parallel plate capacitor, they modify the capacitance. Slabs stacked side-by-side perpendicular to the plates divide the total separation distance, effectively creating individual capacitors connected in a series configuration.

Solution:

(a) The capacitor is filled with two vertical slabs, each of width $\frac{d}{2}$, spanning across the total plate area A. This geometry means the arrangement forms two distinct capacitors connected in series, where each individual capacitor has a plate separation of $\frac{d}{2}$.

(b) The capacitance of the first section containing the dielectric constant K_1 is given by the formula $C_1 = \frac{K_1 \epsilon_0 A}{d/2} = \frac{2K_1 \epsilon_0 A}{d}$.

(c) Similarly, the capacitance of the second section containing the dielectric constant K_2 is expressed as $C_2 = \frac{K_2 \epsilon_0 A}{d/2} = \frac{2K_2 \epsilon_0 A}{d}$.

(d) For a series combination of two capacitors, the equivalent capacitance C_{eq} is calculated using the reciprocal formula: $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$, which simplifies to $C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$.

(e) Substituting the values of C_1 and C_2 into the expression gives $C_{eq} = \frac{\left(\frac{2K_1 \epsilon_0 A}{d}\right)\left(\frac{2K_2 \epsilon_0 A}{d}\right)}{\frac{2K_1 \epsilon_0 A}{d} + \frac{2K_2 \epsilon_0 A}{d}} = \frac{\epsilon_0 A}{d}\left(\frac{2K_1 K_2}{K_1 + K_2}\right)$.

Final Answer: Option (B) is correct.

Answer: (B)

Go Back to Question 6

Q7.

Solution

Concept:

Introducing a transparent sheet in front of one slit in a Young's double-slit experiment changes the optical path length of that light beam[cite: 132]. This extra optical path causes the entire interference fringe pattern to shift across the screen[cite: 133].

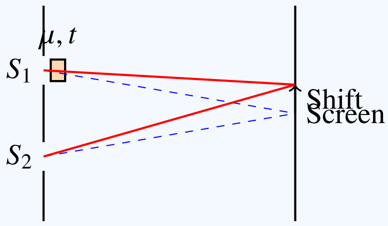
Solution:

Step 1: When a transparent sheet of thickness t and refractive index μ is placed in the path of one of the interfering beams, it introduces an additional optical path difference given by $\Delta x = (\mu - 1)t$[cite: 134].

Step 2: This additional path difference shifts the central maximum and the entire fringe system[cite: 135]. The linear shift on the screen translates into a shift of a certain number of fringes, denoted by N[cite: 136].

Step 3: The relationship between the path difference and the number of shifted fringes is given by the equation $\Delta x = N\lambda$, where λ is the wavelength of the monochromatic light source[cite: 137].

Step 4: Equating both expressions gives $(\mu - 1)t = N\lambda$[cite: 138]. We can isolate the number of fringes shifted past the central point as $N = \frac{(\mu-1)t}{\lambda}$[cite: 139]. Substituting the given values ($\mu = 1.5$, $t = 6 \times 10^{-6}$ m, and $\lambda = 600 \times 10^{-9}$ m) yields $N = \frac{(1.5-1)\times 6\times 10^{-6}}{600\times 10^{-9}} = \frac{3\times 10^{-6}}{6\times 10^{-7}} = 5$[cite: 140].



Final Answer:

$$\boxed{5}$$

Answer: (A)

Go Back to Question 7

Q8.

Solution

Concept: The magnetic force exerted on a current-carrying loop by an external magnetic field is determined by integrating the differential force $d\vec{F} = I(d\vec{l} \times \vec{B})$ along the closed path of the loop. Symmetry properties of the field determine the net vector sum.

Solution:

(a) The long straight wire lies along the z-axis and carries a steady current I. The magnetic field $\vec{B}$ produced by this wire at any point in space is directed azimuthally around the z-axis according to the right-hand rule.

(b) In the xy-plane ($z = 0$), the magnetic field lines form concentric circles centered at the origin, meaning the magnetic field vector $\vec{B}$ at any point in this plane lies entirely within the xy-plane itself.

(c) The circular loop carrying current I_2 is also placed entirely within the xy-plane, centered at the coordinate $(d, 0, 0)$. Therefore, the line element vector $d\vec{l}$ of this loop also lies entirely within the xy-plane.

(d) The differential magnetic force on an element of the loop is given by $d\vec{F} = I_2(d\vec{l} \times \vec{B})$. Since both vectors $d\vec{l}$ and $\vec{B}$ lie in the xy-plane, their cross product must point along the z-axis ($\pm\hat{k}$).

(e) By analyzing symmetric pairs of elements on the circular loop, the field magnitude is symmetric relative to the x-axis, causing equal and opposite forces along the z-direction that cancel out completely when integrated over the entire closed loop, resulting in a net force of zero.

Final Answer: Option (A) is correct.

Answer: (A)

Go Back to Question 8

Q9.

Solution

Concept: In an alternating current series L-C-R circuit, the voltages across the resistor, inductor, and capacitor are not in phase with each other. They must be combined as vector-like quantities using a phasor diagram to find the total effective source voltage.

Solution:

(a) In a series L-C-R circuit, the current is identical through all components. The voltage across the resistor (V_R) is in phase with the current, while the voltage across the inductor (V_L) leads the current by 90° and the voltage across the capacitor (V_C) lags by 90°.

(b) This phase relationship implies that V_L and V_C are exactly 180° out of phase with each other. The net reactive voltage is therefore given by the difference component, $(V_L - V_C)$.

(c) The total effective root-mean-square (rms) voltage V_{rms} of the AC source is found using the phasor addition formula: $V_{rms} = \sqrt{V_R^2 + (V_L - V_C)^2}$.

(d) Substituting the given values ($V_R = 40$ V, $V_L = 100$ V, and $V_C = 70$ V) into the formula gives $V_{rms} = \sqrt{40^2 + (100 - 70)^2} = \sqrt{40^2 + 30^2} = \sqrt{1600 + 900} = \sqrt{2500} = 50$ V.

(e) The question asks for the peak value of the source voltage V_0. For a sinusoidal AC source, the peak voltage is related to the rms voltage by $V_0 = V_{rms}\sqrt{2}$. Substituting our value gives $V_0 = 50\sqrt{2}$ V.

Final Answer: Option (B) is correct.

Answer: (B)

Go Back to Question 9

Q10.

Solution

Concept: Radioactive decay follows first-order kinetics, described by the exponential law $A(t) = A_0 e^{-\lambda t}$ or expressed in terms of the half-life as $N(t) = N_0 \left(\frac{1}{2}\right)^{t/T_{1/2}}$. The remaining fraction depends on the total elapsed time measured in units of half-lives.

Solution:

(a) The activity of a sample is directly proportional to the number of active undecayed nuclei ($A \propto N$). The problem states that the activity decreases to $\frac{1}{8}$ of its initial value in 9 days.

(b) Using the half-life relation, we can write $\frac{N}{N_0} = \frac{1}{8} = \left(\frac{1}{2}\right)^3$. This shows that an interval of 9 days corresponds exactly to three half-lives ($3T_{1/2} = 9$ days), which gives a single half-life of $T_{1/2} = 3$ days.

(c) The sample then experiences an additional decay period of 6 days. The total elapsed time from the very beginning of the experiment is $t_{total} = 9 + 6 = 15$ days.

(d) We can calculate the total number of half-lives that have passed during this combined time interval as $n = \frac{t_{total}}{T_{1/2}} = \frac{15}{3} = 5$ half-lives.

(e) The fraction of the initial nuclei remaining undecayed after 5 half-lives is given by the formula $\frac{N_{final}}{N_0} = \left(\frac{1}{2}\right)^5 = \frac{1}{32}$.

Final Answer: Option (B) is correct.

Answer: (B)

Go Back to Question 10

Q11.

Solution

Concept: The work-energy theorem states that the work done by the net force acting on a particle is equal to the change in its kinetic energy. Alternatively, the work done by the net force can be calculated by finding the vector dot product of the net force acting on the object and its linear displacement vector along the plane.

Solution:

(a) First, we resolve the forces acting on the mass parallel to the inclined plane. The component of gravity acting down the incline is given by the expression $mg \sin\theta = 2 \times 10 \times \sin(30°) = 20 \times 0.5 = 10$ N.

(b) Next, we determine the normal force acting perpendicular to the inclined surface, which balances the perpendicular component of gravity: $N = mg \cos\theta = 2 \times 10 \times \cos(30°) = 20 \times \frac{\sqrt{3}}{2} = 10\sqrt{3}$ N.

(c) Using the calculated normal force and the given coefficient of kinetic friction, the frictional force opposing the motion up the incline is $f_k = \mu_k N = 0.2 \times 10\sqrt{3} = 2\sqrt{3}$ N. Substituting $\sqrt{3} \approx 1.732$ yields $f_k \approx 3.464$ N.

(d) The net force acting on the block along the direction of motion up the incline is equal to the applied pushing force minus both the opposing gravitational component and the kinetic friction: $F_{net} = F - mg \sin\theta - f_k = 20 - 10 - 3.464 = 6.536$ N.

(e) The work done by this net force as the block shifts a displacement distance of 5 m up the plane is calculated as $W_{net} = F_{net} \times d = 6.536 \times 5 = 32.68$ J. Rounding this value to the nearest tenth gives approximately 32.7 J.

Final Answer: Option (C) is correct.

Answer: (C)

Go Back to Question 11

Q12.

Solution

Concept: In simple harmonic motion, the total mechanical energy is conserved and equals the sum of the instantaneous kinetic energy and potential energy at any displacement x. These energies are functions of displacement, where potential energy varies as x^2 and kinetic energy varies as $(A^2 - x^2)$.

Solution:

(a) The potential energy of a particle executing simple harmonic motion at a displacement x from its mean equilibrium position is given by the formula $U = \frac{1}{2}kx^2$, where k represents the force constant of the oscillating system.

(b) The kinetic energy of the particle at that same displacement position is given by the formula $K = \frac{1}{2}k(A^2 - x^2)$, where A denotes the maximum displacement amplitude of the harmonic motion.

(c) The problem specifies a condition where the kinetic energy is exactly equal to three times the potential energy, which translates to the algebraic equation $K = 3U$.

(d) Substituting the explicit displacement expressions for both forms of energy into this condition yields the equation $\frac{1}{2}k(A^2 - x^2) = 3\left(\frac{1}{2}kx^2\right)$.

(e) We can simplify this equation by canceling the common factor $\frac{1}{2}k$ from both sides, leaving $A^2 - x^2 = 3x^2$. Combining the displacement terms on one side gives $A^2 = 4x^2$, which simplifies to $x^2 = \frac{A^2}{4}$. Taking the square root gives the displacement $x = \pm\frac{A}{2}$.

Final Answer: Option (A) is correct.

Answer: (A)

Go Back to Question 12

Q13.

Solution

Concept:

For two body systems connected by a string passing over a fixed pulley, the acceleration of each block is found by standard force equations. The acceleration of the center of mass depends on individual accelerations weighted by their respective masses.

Solution:

Step 1: Formulate the equations of motion for individual hanging masses. Let $m_1 > m_2$, meaning mass m_1 accelerates downwards with magnitude a and mass m_2 moves upwards with acceleration magnitude a.

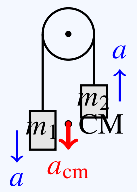
Step 2: Write the force relationships: $m_1 g - T = m_1 a$ and $T - m_2 g = m_2 a$. Solving these standard linear equations simultaneously yields the common magnitude of acceleration:

$$a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) g$$

Step 3: Define vector components along the vertical direction. Taking downward direction as positive, the vector acceleration of the first block is $\vec{a}_1 = a\hat{j}$ and for the second block is $\vec{a}_2 = -a\hat{j}$.

Step 4: Substitute these components into the definition of the center of mass acceleration vector $\vec{a}_{\text{cm}} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2}$. This gives:

$$a_{\text{cm}} = \frac{m_1(a) + m_2(-a)}{m_1 + m_2} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) a = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 g$$



Final Answer:

$$\boxed{\left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 g}$$

Answer: (B)

Go Back to Question 13

Q14.

Solution

Concept: Electromagnetic waves consist of oscillating electric and magnetic field vectors that are perpendicular to each other and perpendicular to the direction of wave propagation. The directional relationship between these three vectors is rigidly governed by the cross-product vector pointing relation.

Solution:

(a) The direction of propagation of an electromagnetic wave is given by the unit vector of its velocity, which is specified as $\hat{v} = \hat{i}$. The cross product of the electric field unit vector $\hat{E}$ and the magnetic field unit vector $\hat{B}$ must point along this propagation direction, satisfying $\hat{E} \times \hat{B} = \hat{i}$.

(b) The given magnetic field vector is oriented along the positive y-axis, meaning its unit direction is $\hat{B} = \hat{j}$. Substituting this into our cross-product relation gives the directional constraint $\hat{E} \times \hat{j} = \hat{i}$.

(c) According to standard unit vector cross-product rules, we know that $(-\hat{k}) \times \hat{j} = \hat{i}$. This mathematically implies that the oscillating electric field vector must be oriented along the negative z-axis $(-\hat{k})$.

(d) The amplitudes of the fields in a dielectric medium are linked by the velocity of the wave, satisfying the magnitude relation $E_0 = vB_0$.

(e) Combining the magnitude relation with the spatial and temporal phase factor $\cos(\omega t - kx)$ and the negative z-axis direction vector yields the complete vector expression for the electric field: $\vec{E} = -vB_0 \cos(\omega t - kx)\hat{k}$.

Final Answer: Option (B) is correct.

Answer: (B)

Go Back to Question 14

Q15.

Solution

Concept: A Wheatstone bridge balances when the ratio of the resistances in the two main gaps equals the ratio of the resistances of the corresponding segments of the bridge wire. When the wire is non-uniform, the resistance of each segment must be found by integrating the non-uniform resistivity function over the segment length.

Solution:

(a) The balancing condition for a Wheatstone bridge with resistance X in the left gap and Y in the right gap is given by the ratio $\frac{X}{Y} = \frac{R_{left}}{R_{right}}$, where R_{left} is the resistance of the wire from 0 to l_1 and R_{right} is the resistance from l_1 to 100.

(b) Because the wire has a non-uniform resistance per unit length given by $\rho(x) = \rho_0(1 + \alpha x)$, we cannot simply use the lengths. Instead, we must find the resistance of the left segment by integrating this function from $x = 0$ to $x = l_1$.

(c) Performing the integration for the left segment gives $R_{left} = \int_0^{l_1} \rho_0(1 + \alpha x)dx = \rho_0 \left[x + \frac{\alpha x^2}{2}\right]_0^{l_1} = \rho_0 \left(l_1 + \frac{\alpha}{2} l_1^2\right)$.

(d) Similarly, the resistance of the remaining right segment of the wire is calculated by integrating the same resistivity function over the boundaries from $x = l_1$ to $x = 100$: $R_{right} = \int_{l_1}^{100} \rho_0(1 + \alpha x)dx = \rho_0 \left[(100 - l_1) + \frac{\alpha}{2}(100^2 - l_1^2)\right]$.

(e) Substituting these integrated expressions into the balancing ratio eliminates the constant scale factor ρ_0, resulting in the true ratio $\frac{X}{Y} = \frac{2l_1 + \alpha l_1^2}{2(100 - l_1) + \alpha(100^2 - l_1^2)}$ after multiplying the numerator and denominator by 2.

Final Answer: Option (D) is correct.

Answer: (D)

Go Back to Question 15

Q16.

Solution

Concept: The mechanical energy of a particle in a central force orbit is the sum of its kinetic energy and its potential energy. The kinetic energy is determined from the centripetal dynamic equation of motion, while the potential energy is found by integrating the conservative force function from infinity.

Solution:

(a) The central attractive force acting on the particle in its circular orbit of radius R is given by $F = -\frac{k}{R^3}$. This attractive force provides the necessary centripetal acceleration for circular motion: $\frac{mv^2}{R} = \frac{k}{R^3}$.

(b) Multiplying both sides by $\frac{R}{2}$ allows us to isolate the expression for the kinetic energy of the orbiting particle: $K = \frac{1}{2}mv^2 = \frac{k}{2R^2}$.

(c) The potential energy U of the particle is found by integrating the force function relative to position from the reference point at infinity to the orbital radius R: $U = -\int_{\infty}^{R} F dr = -\int_{\infty}^{R} \left(-\frac{k}{r^3}\right) dr$.

Evaluating this definite integral gives $U = k\left[\frac{r^{-2}}{-2}\right]_{\infty}^{R} = -\frac{k}{2R^2}$.

(a) The total mechanical energy E of the particle in its orbit is defined as the sum of its kinetic and potential energy contributions: $E = K + U$.

(b) Substituting our calculated expressions for K and U into this summation yields $E = \frac{k}{2R^2} + \left(-\frac{k}{2R^2}\right) = 0$. For an inverse-cubic attractive force field, the kinetic and potential components cancel, making the total mechanical energy zero.

Final Answer: Option (C) is correct.

Answer: (C)

Go Back to Question 16

Q4.

Solution

Concept:

An ideal diode conducts electric current with zero resistance under forward-biased configurations. Conversely, when subjected to a reverse-biased condition, it acts as an ideal open switch, exhibiting infinite structural resistance and completely stopping any charge flow through its branch.

Solution:

Step 1: Identify the operating state of the ideal diode in the given arrangement. The problem explicitly states that the semiconductor diode is connected across a 5 V DC power source in a reverse-biased configuration[cite: 507, 508].

Step 2: Apply the circuit parameters of an ideal reverse-biased diode. Since its effective electrical resistance becomes infinitely large, it blocks all conventional current. Thus, the total circuit current is:

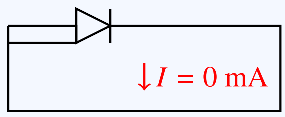
$$I = 0 \text{ mA}$$

Step 3: Analyze the distribution of potential drops across the loop elements using Kirchhoff's voltage law. The voltage drop across the series resistor is determined by Ohm's law: $V_R = I \times R = 0 \text{ mA} \times 100\ \Omega = 0$ V.

Step 4: Since the resistor accounts for zero potential drop, the entire electromotive force supplied by the external battery must drop across the open terminals of the reverse-biased diode:

$$V_{\text{diode}} = V_{\text{battery}} - V_R = 5 \text{ V} - 0 \text{ V} = 5 \text{ V}$$

Diode (Reverse-Biased)



Open Circuit Configuration

Final Answer:

$$\boxed{0 \text{ mA},\ 5 \text{ V}}$$

Answer: (B)

Go Back to Question 17

Q5.

Solution

Concept: The frequency of radiation emitted during an atomic transition between energy levels is determined by the energy difference between those states according to the Bohr model. For transitions between highly excited states (large principal quantum numbers), the frequency can be approximated using binomial expansions.

Solution:

(a) According to the Bohr model of the hydrogen atom, the energy of an electron in an orbit with principal quantum number n is given by $E_n = -\frac{R_H ch}{n^2}$, where R_H is the Rydberg constant.

(b) The frequency ν of the photon emitted during a transition from state n to state $(n-1)$ is given by the energy difference equation: $\nu = \frac{\Delta E}{h} = R_H c\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]$.

(c) Finding a common denominator and simplifying the terms inside the brackets yields the expression: $\nu = R_H c\left[\frac{n^2-(n-1)^2}{n^2(n-1)^2}\right] = R_H c\left[\frac{2n-1}{n^2(n-1)^2}\right]$.

(d) The question specifies that we are analyzing this behavior for very large values of n ($n \gg 1$). In this asymptotic limit, we can approximate the numerator terms as $2n - 1 \approx 2n$, and the denominator terms as $n - 1 \approx n$.

(e) Substituting these large-n approximations back into the frequency expression gives $\nu \approx R_H c\left[\frac{2n}{n^2 \times n^2}\right] = R_H c\left[\frac{2n}{n^4}\right] = \frac{2R_H c}{n^3}$. This demonstrates that the emission frequency is inversely proportional to n^3.

Final Answer: Option (C) is correct.

Answer: (C)

Go Back to Question 18

Q6.

Solution

Concept:

The acceleration of a falling body in a coupled rotational-translational setup depends on Newton's second law combined with torque definitions. The wrapped string creates a constraint relating linear acceleration to angular acceleration.

Solution:

Step 1: Set up the linear translation equation of motion for the hanging block of mass m dropping vertically under gravity. Taking downward direction as positive, the net force equation yields $mg - T = ma$, where T is the string tension.

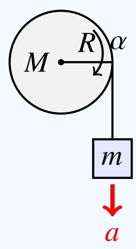
Step 2: Set up the rotational dynamics equation for the solid cylinder of mass M and radius R rotating about its frictionless horizontal axle[cite: 522]. The torque provided by tension is $\tau = TR = I\alpha$, where the moment of inertia is $I = \frac{1}{2}MR^2$.

Step 3: Relate linear acceleration to angular acceleration using the non-slip constraint equation $a = R\alpha$. Substitute this into the torque relation:

$$TR = \left(\frac{1}{2}MR^2\right)\left(\frac{a}{R}\right) \implies T = \frac{1}{2}Ma$$

Step 4: Substitute the tension expression into the linear dynamic equation from Step 1 to solve for acceleration:

$$mg - \frac{1}{2}Ma = ma \implies mg = a\left(m + \frac{M}{2}\right) \implies a = \frac{mg}{m + M/2}$$



Final Answer:

$$\boxed{\frac{mg}{m + M/2}}$$

Answer: (B)

Go Back to Question 19

Q7.

Solution

Concept: The thermodynamic efficiency of a Carnot heat engine depends exclusively on the absolute thermodynamic temperatures of its hot heat source and cold heat sink. The definition of efficiency connects the work output to the total heat input absorbed at the higher operating temperature.

Solution:

(a) First, we must convert the given operating temperatures from the Celsius scale to the absolute Kelvin scale. The temperature of the hot source is $T_H = 227°C + 273 = 500$ K, and the temperature of the cold sink is $T_C = 127°C + 273 = 400$ K.

(b) The thermodynamic efficiency η of a reversible Carnot engine is calculated from these absolute temperatures using the standard formula: $\eta = 1 - \frac{T_C}{T_H}$.

(c) Substituting our converted absolute temperatures into this efficiency equation yields $\eta = 1 - \frac{400}{500} = 1 - 0.8 = 0.2$. This means the engine converts 20% of its absorbed heat into useful mechanical work.

(d) Efficiency is also defined as the ratio of the useful work output performed by the engine to the total heat energy absorbed at the hot source: $\eta = \frac{W}{Q_H}$.

(e) Rearranging this definition to solve for the work component gives $W = \eta \times Q_H$. Substituting the calculated efficiency value and the given heat input ($Q_H = 6 \times 10^4$ cal) results in $W = 0.2 \times 6 \times 10^4$ cal $= 1.2 \times 10^4$ cal.

Final Answer: Option (A) is correct.

Answer: (A)

Go Back to Question 20

Q8.

Solution

Concept: The calculation of fractional or percentage error in a derived physical quantity is determined using the method of partial differentiation. For a product or quotient of measured variables, the maximum relative error is found by summing the individual relative errors multiplied by their respective power exponents.

Solution:

(a) The formula for the time period of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Squaring both sides of this equation allows us to isolate the acceleration due to gravity as $g = \frac{4\pi^2 L}{T^2}$.

(b) The total time for n oscillations is measured as $t = nT$, which implies the period is $T = \frac{t}{n}$. Substituting this into our expression for g yields $g = \frac{4\pi^2 L n^2}{t^2}$. Since $4\pi^2$ and n are exact numbers, they do not contribute to the measurement uncertainty.

(c) Taking the natural logarithm on both sides and differentiating gives the maximum relative error equation for g: $\frac{\Delta g}{g} = \frac{\Delta L}{L} + 2\frac{\Delta t}{t}$.

(d) Next, we identify the absolute errors from the instrument resolutions. The absolute error in length is the accuracy of the scale, $\Delta L = 1$ mm $= 0.1$ cm. The absolute error in total time is the stopwatch resolution, $\Delta t = 1$ s.

(e) Substituting the measured values ($L = 20.0$ cm, $t = 90$ s) into the percentage error equation yields $\frac{\Delta g}{g} \times 100 = \left(\frac{0.1}{20.0} + 2 \times \frac{1}{90}\right) \times 100 = (0.005 + 0.0222) \times 100 = 0.0272 \times 100 = 2.72\%$. This value is closest to 2.7%.

Final Answer: Option (A) is correct.

Answer: (A)

Go Back to Question 21

Q9.

Solution

Concept: The magnetic torque experienced by a magnetic dipole placed inside an external magnetic field is mathematically defined by the vector cross-product operation $\vec{\tau} = \vec{M} \times \vec{B}$. The magnitude of this torque depends directly on the angle between the dipole moment vector and the magnetic field vector.

Solution:

(a) A circular coil of radius R containing N turns carries a steady electrical current I. According to the Biot-Savart law and the right-hand rule, the magnetic field vector $\vec{B}$ generated at the exact geometric center of this loop points along the central axis of the coil.

(b) Therefore, the direction of the magnetic field vector at the center is mathematically parallel to the unit vector defining the axis of the circular coil, which can be expressed as $\vec{B} = B\hat{n}_{axis}$.

(c) The small bar magnet acts as a localized magnetic dipole with a magnetic moment vector $\vec{M}$. The problem states that the physical axis of this bar magnet is aligned perpendicular to the flat plane of the circular coil.

(d) Since the normal axis of a flat loop is also perpendicular to its plane, the magnetic moment vector $\vec{M}$ of the bar magnet is oriented perfectly parallel to the central axis of the coil. This means $\vec{M}$ is parallel to the magnetic field vector $\vec{B}$.

(e) The mathematical expression for the torque magnitude is $\tau = MB\sin\theta$, where θ is the spatial angle between vectors $\vec{M}$ and $\vec{B}$. Because the vectors are parallel, the angle is $\theta = 0°$. Since $\sin(0°) = 0$, the net torque acting on the bar magnet is exactly zero.

Final Answer: Option (C) is correct.

Answer: (C)

Go Back to Question 22

Q10.

Solution

Concept:
The total mechanical pressure inside a spherical fluid interface bounding an inner volume relies on balancing surface tension forces against spatial area constraints. A soap bubble possesses two concentric surface interfaces exposed directly to surrounding air.

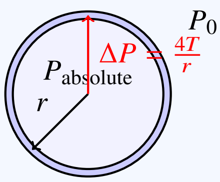
Solution:
Step 1: Consider a spherical soap bubble of radius r blown in an environment with ambient atmospheric pressure P_0[cite: 555]. Because a soap bubble is a hollow thin shell, it contains both an internal and an external boundary layer in contact with air.
Step 2: Recall the formula for excess pressure inside a fluid sphere with two active surface boundaries. The internal pressure must counteract the contracting force of both boundaries, which creates an excess pressure relation:

$$\Delta P = P_{\text{absolute}} - P_0 = \frac{4T}{r}$$

Step 3: Isolate the total inside pressure variable by shifting the external atmospheric baseline value to the right-hand side of the expression. This linear addition yields the final absolute structural pressure configuration:

$$P_{\text{absolute}} = P_0 + \frac{4T}{r}$$

Step 4: Note that for a single-surfaced liquid drop or a submerged air bubble, the excess pressure is only $\frac{2T}{r}$ due to having only one boundary layer. The presence of two layers doubles the required pressure.



Two active surface interfaces

Final Answer:

$$\boxed{P_0 + \frac{4T}{r}}$$

Answer: (B)

Go Back to Question 23

Q11.

Solution

Concept: The electrostatic field configuration created by a point charge positioned near an infinite, grounded, conducting plane can be solved using the method of images. This technique replaces the complex induced surface charge distribution with a single imaginary point charge behind the boundary plane.

Solution:

(a) A real point charge $+q$ is positioned at a perpendicular distance d in front of an infinite, grounded, perfectly conducting plane surface. The presence of $+q$ causes a redistribution of free electrons, inducing a localized negative surface charge density on the conductor.

(b) According to the uniqueness theorem of electrostatics, the boundary conditions can be duplicated by removing the conducting plane and introducing an imaginary image charge $-q$ located symmetrically at a distance d behind the original position of the boundary plane.

(c) The total separation distance between the real positive point charge $+q$ and its negative image charge $-q$ is equal to the sum of their individual boundary distances: $r = d + d = 2d$.

(d) The electrostatic force of attraction between the real charge and the conducting plane is equivalent to the direct Coulomb force acting between the real charge and this image charge: $F = \frac{q \cdot |-q|}{4\pi \epsilon_0 r^2}$.

(e) Substituting the total separation distance $r = 2d$ into the Coulomb equation yields $F = \frac{q^2}{4\pi \epsilon_0 (2d)^2} = \frac{q^2}{16\pi \epsilon_0 d^2}$. This attractive force acts perpendicular to the plane, pulling the real charge toward the conducting surface.

Final Answer: Option (B) is correct.

Answer: (B)

Go Back to Question 24

Q12.

Solution

Concept: The total internal energy of an ideal gas mixture is equal to the sum of the individual internal energies of its constituent gases. The internal energy of an ideal gas depends on its temperature, number of moles, and its specific molecular degrees of freedom.

Solution:

(a) The total internal energy of a non-interacting ideal gas mixture is given by the additive relation $U_{total} = U_1 + U_2$, where U_1 is the internal energy of the oxygen gas and U_2 is the internal energy of the argon gas.

(b) The general formula for the internal energy of n moles of an ideal gas at a thermodynamic temperature T is $U = \frac{f}{2}nRT$, where f represents the active molecular degrees of freedom.

(c) Oxygen (O_2) is a diatomic molecule. Neglecting high-temperature vibrational modes as specified, it has exactly 5 active degrees of freedom (3 translational and 2 rotational). For $n_1 = 1$ mole, its energy is $U_1 = \frac{5}{2}(1)RT = \frac{5}{2}RT$.

(d) Argon (Ar) is a noble gas and exists as a monostatic, monatomic gas. It possesses only 3 translational degrees of freedom ($f = 3$). For $n_2 = 2$ moles, its internal energy is calculated as $U_2 = \frac{3}{2}(2)RT = 3RT$.

(e) Summing the two individual energy contributions to find the total internal energy of the mixture yields $U_{total} = \frac{5}{2}RT + 3RT = \frac{5+6}{2}RT = \frac{11}{2}RT$.

Final Answer: Option (A) is correct.

Answer: (A)

Go Back to Question 25

Q13.

Solution

Concept: The geometric optics of a prism link the angle of incidence, angle of emergence, refracting angle of the prism, and the total angle of deviation through exact angle tracking equations. Refractive index can then be calculated using Snell law at the interfaces.

Solution:

(a) The fundamental structural equation for a light ray passing through a triangular optical prism relates the angles as follows: $i + e = A + \delta$, where i is the incidence angle, e is the emergence angle, A is the prism angle, and δ is the deviation angle.

(b) Substituting the values given in the problem statement ($i = 60°$, $A = 30°$, and $\delta = 30°$) into this fundamental equation allows us to solve for the emergence angle: $60° + e = 30° + 30° = 60°$. This yields $e = 0°$.

(c) An angle of emergence of $e = 0°$ means the light ray exits the second face of the prism perfectly perpendicular to the surface. According to geometry, the internal refraction angle at this second face must also be $r_2 = 0°$.

(d) The refracting angle of a prism is related to the internal angles of refraction by the geometric equation $A = r_1 + r_2$. Substituting our calculated values ($A = 30°$ and $r_2 = 0°$) reveals that the first internal refraction angle is $r_1 = 30°$.

(e) Finally, we apply Snell law at the first refracting interface of the prism: $\mu = \frac{\sin i}{\sin r_1}$. Substituting the angles gives $\mu = \frac{\sin(60°)}{\sin(30°)} = \frac{\sqrt{3}/2}{1/2} = \sqrt{3}$.

Final Answer: Option (B) is correct.

Answer: (B)

Go Back to Question 26

Q14.

Solution

Concept: The mathematical function $y(x,t)$ describes the transverse displacement profile of a wave propagating along a medium. The transverse particle velocity is found by taking the partial derivative of this displacement function with respect to time, and its maximum value is located using calculus.

Solution:

(a) The given mathematical expression for the wave pulse displacement profile is $y(x,t) = A[B + (x - vt)^2]^{-1}$. The transverse velocity v_p of any individual particle along the string is defined by the partial derivative $v_p = \frac{\partial y}{\partial t}$.

(b) Applying the chain rule for differentiation with respect to time t yields the particle velocity function: $v_p = -A[B + (x - vt)^2]^{-2} \cdot 2(x - vt)(-v) = \frac{2Av(x-vt)}{[B+(x-vt)^2]^2}$.

(c) To simplify tracking the maximum value of this function, we can define a single localized position-time variable $z = x - vt$. This substitution transforms the particle velocity expression into the single-variable function $v_p(z) = \frac{2Avz}{(B+z^2)^2}$.

(d) To find the point where $v_p(z)$ reaches its maximum value, we differentiate this expression with respect to z and set the derivative to zero ($\frac{dv_p}{dz} = 0$). Applying the quotient rule gives the structural condition $B - 3z^2 = 0$, which yields $z = \sqrt{\frac{B}{3}}$.

(e) Substituting this critical value $z = \sqrt{\frac{B}{3}}$ back into the velocity equation gives $v_{p,max} = \frac{2Av\sqrt{B/3}}{(B+B/3)^2} = \frac{2Av\sqrt{B}/\sqrt{3}}{(4B/3)^2} = \frac{2Av\sqrt{B}/\sqrt{3}}{16B^2/9} = \frac{18Av\sqrt{B}}{16\sqrt{3}B^2} = \frac{3\sqrt{3}Av}{8B^{1/2}B} = \frac{3\sqrt{3}Av}{8B^{3/2}}$.

Final Answer: Option (A) is correct.

Answer: (A)

Go Back to Question 27

Q15.

Solution

Concept:

The electrical distribution across a parallel DC battery network containing internal resistances can be analyzed using Millman's theorem or Kirchhoff's laws. The equivalent electromotive force and parallel resistance determine the load voltage.

Solution:

Step 1: Identify the circuit values given: $E_1 = 6$ V, $r_1 = 1\ \Omega$, $E_2 = 3$ V, $r_2 = 2\ \Omega$, and the external load resistance $R = 5\ \Omega$[cite: 602]. The positive terminals link to the same terminal node.

Step 2: Use Millman's theorem to find the parallel equivalent electromotive force (E_{eq}) and internal resistance (r_{eq}) across the combinations feeding the node:

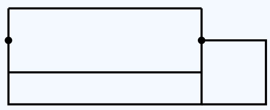
$$E_{eq} = \frac{\frac{E_1}{r_1} + \frac{E_2}{r_2}}{\frac{1}{r_1} + \frac{1}{r_2}} = \frac{\frac{6}{1} + \frac{3}{2}}{\frac{1}{1} + \frac{1}{2}} = \frac{7.5}{1.5} = 5\text{ V}$$

$$\frac{1}{r_{eq}} = \frac{1}{1} + \frac{1}{2} = 1.5 \implies r_{eq} = \frac{2}{3}\ \Omega \approx 0.67\ \Omega$$

Step 3: Frame the simplified circuit as an equivalent single source ($E_{eq} = 5$ V) linked in series with $r_{eq} = \frac{2}{3}\ \Omega$ across the main external load resistance $R = 5\ \Omega$.

Step 4: Calculate the terminal potential difference across the load using the voltage divider relationship:

$$V = E_{eq} \times \frac{R}{R + r_{eq}} = 5 \times \frac{5}{5 + \frac{2}{3}} = 5 \times \frac{15}{17} = \frac{75}{17} \approx 4.17\text{ V}$$



Final Answer:

$$\boxed{4.17\text{ V}}$$

Answer: (C)

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Q16.

Solution

Concept: The time period of a mass-spring system executing simple harmonic motion depends purely on the total mass suspended from the spring and the mechanical spring constant. It is fundamentally independent of the displacement amplitude or the specific kinetic velocity of the system.

Solution:

(a) The initial time period of oscillation for a single block of mass M suspended from a vertical spring of force constant k is given by the standard formula $T_1 = 2\pi\sqrt{\frac{M}{k}}$.

(b) The problem states that a second block of mass m is placed on top of the moving mass M at the precise instant the system passes through its lowest point of oscillation.

(c) At the lowest point of simple harmonic motion, the instantaneous velocity of mass M is exactly zero ($v = 0$). Therefore, adding mass m at this specific moment introduces no momentum mismatch or loss of kinetic energy.

(d) Once the block of mass m rests securely on top of mass M, they move together as a single coupled inertial mass system. The new total oscillating mass of this spring system becomes $M_{new} = M + m$.

(e) The new time period of the coupled system is calculated by substituting this total mass into the period formula: $T_2 = 2\pi\sqrt{\frac{M+m}{k}}$. Taking the ratio of the new period to the initial period eliminates the common factor $2\pi/\sqrt{k}$, yielding $\frac{T_2}{T_1} = \sqrt{\frac{M+m}{M}}$.

Final Answer: Option (A) is correct.

Answer: (A)

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Q17.

Solution

Concept: A straight conductor moving through a magnetic field experiences a motional electromotive force due to the magnetic Lorentz force acting on its free conduction electrons. For a rotating rod, the linear speed varies along its length, requiring integration to find the total potential difference.

Solution:

(a) Consider a small differential element of the conducting rod of length dr located at a distance r from the fixed axis of rotation. As the rod rotates with a uniform angular velocity ω, the linear velocity of this element is given by $v = r\omega$.

(b) This element moves perpendicular to the uniform external magnetic field B. The differential motional electromotive force $d\epsilon$ induced across this small element is given by the formula $d\epsilon = Bvdr$.

(c) Substituting the expression for linear velocity into this differential relation gives the expression $d\epsilon = B(r\omega)dr = B\omega rdr$.

(d) To find the total induced electromotive force ϵ between the two ends of the rod, we integrate this differential expression over the entire length of the rod, from the pivot end $r = 0$ to the outer tip $r = L$.

(e) Performing the definite integration yields the final equation: $\epsilon = \int_0^L B\omega rdr = B\omega \int_0^L rdr = B\omega \left[\frac{r^2}{2}\right]_0^L = \frac{1}{2}BL^2\omega$. This confirms that the total potential difference generated depends on the square of the length.

Final Answer: Option (B) is correct.

Answer: (B)

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Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	B	3	B	4	B	5	C
6	B	7	A	8	A	9	B	10	B
11	C	12	A	13	B	14	B	15	D
16	C	17	B	18	C	19	B	20	A
21	A	22	C	23	B	24	B	25	A
26	B	27	A	28	C	29	A	30	B