Bihar Board Class 10th Mathematics - 2023 Question Paper with Solutions

Time Allowed :3 Hour 15 mins	Maximum Marks :50	Total Questions :100

General Instructions

Read the following instructions very carefully and strictly follow them:

1. Question Nos. 1 to 100 have four options, out of which only one is correct. Answer any 50 questions. You have to mark your selected option on the OMR-Sheet.

1. What is the distance of the point $(15, 8)$ from the origin?

(1) 15
(2) 16
(3) 17
(4) 18

Correct Answer: (3) 17
Solution:
Step 1: Use the distance formula from the origin.
For a point (x, y), distance from origin $O(0, 0)$ is $\sqrt{x^2 + y^2}$.

Step 2: Substitute $x = 15$ and $y = 8$.

$$\sqrt{15^2 + 8^2} = \sqrt{225 + 64} = \sqrt{289} = 17.$$

Step 3: Conclude.
Hence, the required distance is 17.

Quick Tip

Distance from the origin is a special case of the distance formula: $d = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2}$.

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2. The graph of the straight line $y = 2x - 3$ passes through which of the following points?

(1) $(2, 2)$
(2) $(3, 4)$
(3) $(4, 1)$

(4) $(5, 7)$

Correct Answer: (4) $(5, 7)$
Solution:
Step 1: Use the line equation $y = 2x - 3$.
A point (x, y) lies on the line if its coordinates satisfy the equation $y = 2x - 3$.

Step 2: Check each option.
For $(2, 2)$: $2x - 3 = 2 \cdot 2 - 3 = 1 \neq 2$. Not on the line.
For $(3, 4)$: $2 \cdot 3 - 3 = 3 \neq 4$. Not on the line.
For $(4, 1)$: $2 \cdot 4 - 3 = 5 \neq 1$. Not on the line.
For $(5, 7)$: $2 \cdot 5 - 3 = 10 - 3 = 7$. Satisfied.

Step 3: Conclude.
Hence, the line passes through $(5, 7)$.

Quick Tip

To verify if a point lies on a line, substitute x and y into the line's equation and check if the equality holds.

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3. The point on the x-axis which is equidistant from the points $(-2, 0)$ and $(6, 0)$ is:

(1) $(0, 2)$
(2) $(2, 0)$
(3) $(3, 0)$
(4) $(0, 3)$

Correct Answer: (2) $(2, 0)$
Solution:
Step 1: Represent the required point on the x-axis.
Any point on the x-axis is of the form $(x, 0)$.

Step 2: Use the equidistant condition (distance formula).
Equidistant from $(-2, 0)$ and $(6, 0)$ means:

$$\sqrt{(x+2)^2 + (0-0)^2} = \sqrt{(x-6)^2 + (0-0)^2}.$$

Step 3: Square and solve for x.

$$(x+2)^2 = (x-6)^2 \Rightarrow x^2 + 4x + 4 = x^2 - 12x + 36 \Rightarrow 16x = 32 \Rightarrow x = 2.$$

Thus the point is $(2, 0)$.

Quick Tip

On a straight line, the point equidistant from two points lies at their midpoint. Here, the midpoint of $x = -2$ and $x = 6$ is $x = \frac{-2+6}{2} = 2$.

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4. The distance of the point $(13, 15)$ from the y-axis is:

(1) 13
(2) 15
(3) 2
(4) 28

Correct Answer: (1) 13
Solution:
Step 1: Recall the rule.
The perpendicular distance of a point (x, y) from the y-axis is $|x|$.

Step 2: Substitute the coordinates.
For $(13, 15)$, distance from the y-axis $= |13| = 13$.

Step 3: Conclude.
Hence, the required distance is 13.

Quick Tip

From the y-axis, use the absolute value of the x-coordinate; from the x-axis, use the absolute value of the y-coordinate.

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5. The coordinates of the ends of a diameter of a circle are $(-10, 6)$ and $(6, -10)$. Then the coordinates of its centre are:

(1) $(-8, -8)$
(2) $(-8, 4)$
(3) $(-2, -2)$
(4) $(2, 4)$

Correct Answer: (3) $(-2, -2)$
Solution:
Step 1: Use the midpoint formula for the centre.
For endpoints (x_1, y_1) and (x_2, y_2) of a diameter, the centre is

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right).$$

Step 2: Substitute the given points.

$$\left(\frac{-10+6}{2}, \frac{6+(-10)}{2}\right) = \left(\frac{-4}{2}, \frac{-4}{2}\right) = (-2,-2).$$

Step 3: Conclude.
Therefore, the centre of the circle is $(-2,-2)$.

Quick Tip

For any chord (including the diameter), the midpoint of its endpoints gives the point on the circle's perpendicular bisector; for a diameter, this midpoint is exactly the centre.

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6. The coordinates of the vertices of a triangle are $(2,4)$, $(0,6)$ and $(4,-1)$. Then the coordinates of the centroid of the triangle are:

(1) $(2,3)$
(2) $(3,2)$
(3) $(3,3)$
(4) $(2,2)$

Correct Answer: (1) $(2,3)$
Solution:
Step 1: Recall centroid formula.
For triangle with vertices $(x_1,y_1),(x_2,y_2),(x_3,y_3)$, centroid G is

$$G\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right).$$

Step 2: Substitute the given coordinates.

$$G\left(\frac{2+0+4}{3}, \frac{4+6+(-1)}{3}\right) = \left(\frac{6}{3}, \frac{9}{3}\right) = (2,3).$$

Step 3: Conclude.
Thus, the centroid of the triangle is $(2,3)$.

Quick Tip

Centroid is the average of the three vertices' coordinates: add the x's and y's separately and divide each by 3.

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7. The length of the perpendicular from the point $(13, 19)$ to the x-axis is:

(1) 13
(2) 19
(3) 32
(4) 6

Correct Answer: (2) 19
Solution:
Step 1: Recall the rule.
The perpendicular distance of a point (x, y) from the x-axis is $|y|$.

Step 2: Substitute the coordinates.
For $(13, 19)$, the distance from the x-axis is $|19| = 19$.

Step 3: Conclude.
Hence, the required length of the perpendicular is 19.

Quick Tip

Distance from the x-axis depends only on the y-coordinate: $d = |y|$. From the y-axis, use $d = |x|$.

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8. The mid-point of the line segment joining the points $(-2, 8)$ and $(-6, -4)$ lies in which quadrant?

(1) First
(2) Second
(3) Third
(4) Fourth

Correct Answer: (2) Second
Solution:
Step 1: Use the midpoint formula.
Midpoint M of (x_1, y_1) and (x_2, y_2) is $M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$.

Step 2: Substitute the given points.

$$M\left(\frac{-2+(-6)}{2}, \frac{8+(-4)}{2}\right) = \left(\frac{-8}{2}, \frac{4}{2}\right) = (-4, 2).$$

Step 3: Identify the quadrant.
Since $x < 0$ and $y > 0$, the point $(-4, 2)$ lies in the **Second** quadrant.

Quick Tip

Quadrant signs: I $(+,+)$, II $(-,+)$, III $(-,-)$, IV $(+,-)$. For midpoints, average the coordinates component-wise.

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9. If $P(0,0)$, $Q(8,0)$ and $R(0,12)$ are the vertices of $\triangle PQR$, then the area of $\triangle PQR$ is:

(1) 40
(2) 48
(3) 20
(4) 4

Correct Answer: (2) 48
Solution:
Step 1: Observe the triangle is right-angled on the axes.
Points $Q(8,0)$ and $R(0,12)$ lie on the coordinate axes, with $P(0,0)$ at the origin. Hence, PQ is along the x-axis with length 8 and PR is along the y-axis with length 12.

Step 2: Use area formula for a right triangle.

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 8 \times 12 = 48.$$

Step 3: Conclude.
Therefore, the area of $\triangle PQR$ is 48.

Quick Tip

When two vertices lie on the axes and the third is the origin, the triangle is right-angled at the origin; use $\frac{1}{2} \times$ (x-intercept) $\times$ (y-intercept).

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10. The coordinates of the vertices of a triangle are $(0,6)$, $(0,0)$ and $(8,0)$. Then the perimeter of the triangle is:

(1) 14
(2) 24
(3) 42
(4) none of these

Correct Answer: (2) 24
Solution:
Step 1: Find the side lengths using the distance formula.

$$AB : (0,6) \text{ to } (0,0) \Rightarrow \sqrt{(0-0)^2+(6-0)^2} = 6.$$
$$BC : (0,0) \text{ to } (8,0) \Rightarrow \sqrt{(8-0)^2+(0-0)^2} = 8.$$
$$CA : (8,0) \text{ to } (0,6) \Rightarrow \sqrt{(8-0)^2+(0-6)^2} = \sqrt{64+36} = 10.$$

Step 2: Add the side lengths to get the perimeter.

$$\text{Perimeter} = 6 + 8 + 10 = 24.$$

Step 3: Conclude.
Hence, the perimeter of the triangle is 24.

Quick Tip

When vertices lie on axes, two sides are axis-aligned (lengths are easy), and the third can often form a Pythagorean triple.

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11. If the graphs of two linear equations are coincident lines, then how many solutions do they have?

(1) One solution
(2) No solution
(3) Infinitely many solutions
(4) None of these

Correct Answer: (3) Infinitely many solutions
Solution:
Step 1: Understand "coincident lines."
Two lines are **coincident** if they lie exactly on top of each other; every point on one line is also on the other.

Step 2: Interpret in terms of solutions.
A solution to a pair of linear equations corresponds to a point common to both lines.

Since coincident lines share *all* points, there are **infinitely many** common points (solutions).

Step 3: Conclusion.
Therefore, a system represented by coincident lines is **consistent and dependent** with infinitely many solutions.

Quick Tip

Parallel distinct lines ⇒ no solution; intersecting lines ⇒ one solution; coincident lines ⇒ infinitely many solutions.

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12. The graphs of the equations $2x - y - 3 = 0$ and $12x + 7y - 5 = 0$ are which type of straight lines?

(1) Coincident straight lines
(2) Parallel straight lines
(3) Intersecting straight lines
(4) None of these

Correct Answer: (3) Intersecting straight lines
Solution:
Step 1: Convert each equation to slope–intercept form $y = mx + c$.
For $2x - y - 3 = 0 \Rightarrow y = 2x - 3$, slope $m_1 = 2$.
For $12x + 7y - 5 = 0 \Rightarrow 7y = -12x + 5 \Rightarrow y = -\frac{12}{7}x + \frac{5}{7}$, slope $m_2 = -\frac{12}{7}$.

Step 2: Compare slopes.
Since $m_1 \neq m_2$ $(2 \neq -\frac{12}{7})$, the lines are neither parallel nor coincident; hence they **intersect**.

Step 3: Conclude.
Therefore, the two given lines are **intersecting straight lines**.

Quick Tip

In 2D, two non-vertical lines: equal slopes ⇒ parallel; equal slopes and equal intercepts ⇒ coincident; unequal slopes ⇒ intersecting.

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13. Which type of straight line will be the graph of $x - y = 0$?

(1) Parallel to x-axis
(2) Parallel to y-axis
(3) Passing through origin
(4) None of these

Correct Answer: (3) Passing through origin
Solution:
Step 1: Rearrange the equation.
$x - y = 0 \Rightarrow y = x$. This is a line with slope 1.

Step 2: Identify intercept.
When $x = 0$, $y = 0$. Hence the line passes through $(0, 0)$, i.e., the origin.

Step 3: Conclude.
Therefore, the graph is a straight line through the origin (making a 45° angle with the positive x-axis).

Quick Tip

If a line can be written as $y = mx$ (no constant term), it always passes through the origin. A constant c in $y = mx + c$ shifts it off the origin.

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14. Which of the following is a quadratic equation?

(1) $(x + 2)(x - 2) = x^2 - 4x^3$
(2) $(x + 2)^2 = 3(x + 4)$
(3) $(2x^2 + 3) = (5 + x)(2x^2 - 3)$
(4) $2x + \dfrac{1}{2x} = 4x^2$

Correct Answer: (2) $(x + 2)^2 = 3(x + 4)$
Solution:
Step 1: Test each option by bringing all terms to one side and checking the highest power of x.

(1) $(x + 2)(x - 2) = x^2 - 4x^3 \Rightarrow x^2 - 4 = x^2 - 4x^3 \Rightarrow 4x^3 - 4 = 0$.
Highest degree $= 3$ (cubic) $\Rightarrow$ Not quadratic.

(2) $(x + 2)^2 = 3(x + 4) \Rightarrow x^2 + 4x + 4 = 3x + 12 \Rightarrow x^2 + x - 8 = 0$.
Highest degree $= 2$ (quadratic) $\Rightarrow$ **Quadratic**.

(3) $(2x^2 + 3) = (5 + x)(2x^2 - 3)$. RHS contains $x \cdot 2x^2 = 2x^3 \Rightarrow$ degree 3. After simplifying, equation is cubic $\Rightarrow$ Not quadratic.

(4) $2x + \dfrac{1}{2x} = 4x^2$. Multiply by $2x$: $4x^2 + 1 = 8x^3 \Rightarrow 8x^3 - 4x^2 - 1 = 0$.
Highest degree $= 3$ (cubic) $\Rightarrow$ Not quadratic.

Step 2: Conclude.
Only option (2) reduces to a degree-2 equation, hence it is quadratic.

Quick Tip

To identify a quadratic, first clear denominators if any, expand, and collect like terms. If the highest power of x is exactly 2, it's quadratic.

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15. If one root of the quadratic equation $2x^2 + px - 3 = 0$ is -3, then the value of p will be:

(1) 3
(2) 5
(3) 4
(4) 6

Correct Answer: (2) 5
Solution:
Step 1: Use the fact that a root satisfies the equation.
If $x = -3$ is a root, substitute $x = -3$ into $2x^2 + px - 3 = 0$:

$$2(-3)^2 + p(-3) - 3 = 0 \Rightarrow 18 - 3p - 3 = 0.$$

Step 2: Solve for p.

$$15 - 3p = 0 \Rightarrow 3p = 15 \Rightarrow p = 5.$$

Step 3: Conclude.
Therefore, $p = 5$.

Quick Tip

When a value r is a root of $ax^2 + bx + c = 0$, substituting $x = r$ gives an equation in the unknown parameter(s). Alternatively, you can use the factor theorem $a(r)^2 + b(r) + c = 0$.

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16. For what values of k are the roots of the quadratic equation $9x^2 + 3kx + 4 = 0$ real and equal?

(1) ± 4
(2) ± 7
(3) ± 9

(4) ± 6

Correct Answer: (1) ± 4
Solution:
Step 1: Use the discriminant condition for equal real roots.
For $ax^2 + bx + c = 0$, roots are real and equal when $D = b^2 - 4ac = 0$.
Here $a = 9$, $b = 3k$, $c = 4$.

Step 2: Set the discriminant to zero and solve for k.

$$D = (3k)^2 - 4 \cdot 9 \cdot 4 = 9k^2 - 144 = 0 \;\Rightarrow\; 9k^2 = 144 \;\Rightarrow\; k^2 = 16 \;\Rightarrow\; k = \pm 4.$$

Step 3: Conclude.
Hence, the required values of k are $\boxed{\pm 4}$.

Quick Tip

"Real and equal roots" $\Rightarrow$ discriminant $D = 0$; "real and distinct" $\Rightarrow D > 0$; "no real roots" $\Rightarrow D < 0$.

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17. If α, β are the roots of the equation $x^2 + 3px + 2p^2 = 0$ and $\alpha^2 + \beta^2 = 5$, then the value of p is:

(1) ± 3
(2) ± 2
(3) ± 1
(4) ± 5

Correct Answer: (3) ± 1
Solution:
Step 1: Use Vieta's formulas for the quadratic $x^2 + 3px + 2p^2 = 0$.
Sum of roots: $\alpha + \beta = -3p$.
Product of roots: $\alpha\beta = 2p^2$.

Step 2: Express $\alpha^2 + \beta^2$ in terms of p.

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (-3p)^2 - 2(2p^2) = 9p^2 - 4p^2 = 5p^2.$$

Step 3: Use the given condition $\alpha^2 + \beta^2 = 5$.

$$5p^2 = 5 \;\Rightarrow\; p^2 = 1 \;\Rightarrow\; p = \pm 1.$$

Step 4: Conclude.
Hence, the required values of p are $\boxed{\pm 1}$.

Quick Tip

For a monic quadratic $x^2 + bx + c = 0$, use Vieta: $\alpha + \beta = -b$, $\alpha\beta = c$. Also, $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$.

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18. The roots of the quadratic equation $a^2p^2x^2 - q^2 = 0$ are:

(1) $\dfrac{a^2p^2}{q^2}$
(2) $\dfrac{ap}{q}$
(3) $\dfrac{q^2}{ap}$
(4) $\pm\dfrac{q}{ap}$

Correct Answer: (4) $\pm\dfrac{q}{ap}$
Solution:
Step 1: Recognize the equation as a difference of squares.
$a^2p^2x^2 - q^2 = (apx)^2 - q^2 = (apx - q)(apx + q)$.

Step 2: Set each factor equal to zero and solve for x.
From $apx - q = 0 \Rightarrow x = \dfrac{q}{ap}$.
From $apx + q = 0 \Rightarrow x = -\dfrac{q}{ap}$.

Step 3: Conclude.
Hence, the roots are $x = \pm\dfrac{q}{ap}$.

Quick Tip

Whenever you see $A^2 - B^2 = 0$, factor it as $(A - B)(A + B) = 0$ to get roots quickly.

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19. The ratio of the sum of the roots and the product of the roots of the quadratic equation $x^2 - 15x + 50 = 0$ is:

(1) 3 : 10
(2) 3 : 25

(3) 3 : 50
(4) 5 : 3

Correct Answer: (1) 3 : 10
Solution:
Step 1: Use Vieta's formulas for $ax^2 + bx + c = 0$.
Sum of roots $= -\dfrac{b}{a}$, product of roots $= \dfrac{c}{a}$. For $x^2-15x+50 = 0$, we have $a = 1, b = -15, c = 50$.

Step 2: Compute sum and product.
$$\text{Sum} = -\frac{-15}{1} = 15, \quad \text{Product} = \frac{50}{1} = 50.$$

Step 3: Form and simplify the ratio.
$$\text{Sum : Product} = 15 : 50 = \frac{15}{5} : \frac{50}{5} = 3 : 10.$$

Quick Tip

For a monic quadratic $x^2 + bx + c = 0$, sum of roots $= -b$ and product $= c$ — no solving required.

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20. If -5 and -1 are the roots of a quadratic equation, then the equation will be:

(1) $x^2 + 6x + 5 = 0$
(2) $x^2 - 6x + 5 = 0$
(3) $x^2 - 6x - 5 = 0$
(4) $x^2 + 6x - 5 = 0$

Correct Answer: (1) $x^2 + 6x + 5 = 0$
Solution:
Step 1: Use sum and product of roots for a monic quadratic.
If roots are $\alpha = -5$ and $\beta = -1$, then

$$\alpha + \beta = -6, \quad \alpha\beta = 5.$$

Step 2: Form the quadratic using $x^2 - (\alpha + \beta)x + \alpha\beta = 0$.

$$x^2 - (-6)x + 5 = 0 \;\Rightarrow\; x^2 + 6x + 5 = 0.$$

Step 3: Conclude.
Hence, the required equation is $x^2 + 6x + 5 = 0$.

Quick Tip

Given roots α, β, the monic quadratic is $x^2 - (\alpha+\beta)x + \alpha\beta = 0$. This avoids expanding factors each time.

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21. Evaluate $\dfrac{\cos 60^\circ + 1}{\cos 60^\circ - 1}$.

(1) 2
(2) -2
(3) 3
(4) -3

Correct Answer: (4) -3
Solution:
Step 1: Use the exact value of $\cos 60^\circ$.
$\cos 60^\circ = \dfrac{1}{2}$.

Step 2: Substitute and simplify.

$$\frac{\cos 60^\circ + 1}{\cos 60^\circ - 1} = \frac{\frac{1}{2} + 1}{\frac{1}{2} - 1} = \frac{\frac{3}{2}}{-\frac{1}{2}} = -3.$$

Step 3: Conclude.
Hence, the required value is -3.

Quick Tip

Memorize $\cos 60^\circ = \frac{1}{2}$ and $\sin 30^\circ = \frac{1}{2}$; they simplify many trig ratios instantly.

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22. If $\dfrac{A}{5} = 12^\circ$**, then the value of** $3\csc^2 A$ **will be:**

(1) $2\sqrt{3}$
(2) 3
(3) 4
(4) $4\sqrt{3}$

Correct Answer: (3) 4
Solution:
Step 1: Find angle A.
$\dfrac{A}{5} = 12^\circ \Rightarrow A = 60^\circ$.

Step 2: Evaluate $\csc^2 A$.

$\sin 60^\circ = \dfrac{\sqrt{3}}{2} \Rightarrow \csc 60^\circ = \dfrac{1}{\sin 60^\circ} = \dfrac{2}{\sqrt{3}}$.

Hence, $\csc^2 60^\circ = \left(\dfrac{2}{\sqrt{3}}\right)^2 = \dfrac{4}{3}$.

Step 3: Multiply by 3.

$3 \csc^2 60^\circ = 3 \cdot \dfrac{4}{3} = 4$.

Quick Tip

Convert the given angle first; with special angles like $30^\circ, 45^\circ, 60^\circ$, use exact trig values to simplify quickly.

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23. Evaluate $\sin 54^\circ - \cos 36^\circ$.

(1) 0
(2) 1
(3) 2
(4) -1

Correct Answer: (1) 0
Solution:
Step 1: Use the co-function identity.
$\sin(90^\circ - \theta) = \cos\theta$. With $\theta = 36^\circ$, we get $\sin 54^\circ = \cos 36^\circ$.

Step 2: Substitute and simplify.

$$\sin 54^\circ - \cos 36^\circ = \cos 36^\circ - \cos 36^\circ = 0.$$

Quick Tip

Remember co-function pairs: $\sin(90^\circ - \theta) = \cos\theta$ and $\cos(90^\circ - \theta) = \sin\theta$.

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24. Evaluate $\cos 1^\circ \cdot \cos 2^\circ \cdot \cos 3^\circ \cdots \cos 90^\circ$.

(1) 0
(2) 1
(3) -1
(4) $\sqrt{2}$

Correct Answer: (1) 0
Solution:
Step 1: Note the factor $\cos 90^\circ$.
We know $\cos 90^\circ = 0$.

Step 2: Use zero-product property.
Since the product includes the factor $\cos 90^\circ = 0$, the entire product equals 0.

Quick Tip

When a product includes any factor equal to 0, the whole product is 0 — no further computation needed.

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25. The value of $\dfrac{\cos 37^\circ}{\sin 53^\circ} + \dfrac{\cot 34^\circ}{\tan 56^\circ}$ is:

(1) 0
(2) 1
(3) 2
(4) -1

Correct Answer: (3) 2
Solution:
Step 1: Use co-function identities.
Since $53^\circ = 90^\circ - 37^\circ$, we have $\sin 53^\circ = \cos 37^\circ$.
Thus, $\dfrac{\cos 37^\circ}{\sin 53^\circ} = \dfrac{\cos 37^\circ}{\cos 37^\circ} = 1$.

Step 2: Simplify the second term.
Because $56^\circ = 90^\circ - 34^\circ$, $\tan 56^\circ = \cot 34^\circ$.
Hence, $\dfrac{\cot 34^\circ}{\tan 56^\circ} = \dfrac{\cot 34^\circ}{\cot 34^\circ} = 1$.

Step 3: Add the results.
Total $= 1 + 1 = 2$.

Quick Tip

Co-function pairs: $\sin(90^\circ - \theta) = \cos\theta$, $\tan(90^\circ - \theta) = \cot\theta$, $\sec(90^\circ - \theta) = \csc\theta$. They make many evaluations trivial.

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26. Evaluate $2(\sin 45^\circ - \cos 45^\circ)$.

(1) 0

(2) 1
(3) 2
(4) -2

Correct Answer: (1) 0
Solution:
Step 1: Recall exact values of $\sin 45^\circ$ and $\cos 45^\circ$.
We know $\sin 45^\circ = \dfrac{1}{\sqrt{2}}$, $\cos 45^\circ = \dfrac{1}{\sqrt{2}}$.

Step 2: Substitute into the expression.

$$2(\sin 45^\circ - \cos 45^\circ) = 2\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) = 2(0) = 0.$$

Step 3: Conclude.
Therefore, the value is 0.

Quick Tip

For 45°, sine and cosine are equal, so any difference like $\sin 45^\circ - \cos 45^\circ$ vanishes to zero.

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27. The value of $\csc^2\theta - \cot^2\theta$ is:

(1) 1
(2) 7
(3) 49
(4) 0

Correct Answer: (1) 1
Solution:
Step 1: Recall the Pythagorean identity in trigonometry.
For any angle θ (where defined), $\csc^2\theta = 1 + \cot^2\theta$.

Step 2: Substitute into the expression.

$$\csc^2\theta - \cot^2\theta = (1 + \cot^2\theta) - \cot^2\theta = 1.$$

Step 3: Conclude.
Thus, the required value is 1.

Quick Tip

Memorize $\csc^2\theta = 1 + \cot^2\theta$ and $\sec^2\theta = 1 + \tan^2\theta$. They help reduce many expressions instantly.

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28. If $\sin 48^\circ = p$, then the value of $\tan 48^\circ$ is:

(1) $\dfrac{p}{\sqrt{1-p^2}}$

(2) $\dfrac{\sqrt{1-p^2}}{p}$

(3) $\dfrac{p}{\sqrt{1+p^2}}$

(4) $\dfrac{\sqrt{1+p^2}}{p}$

Correct Answer: (1) $\dfrac{p}{\sqrt{1-p^2}}$

Solution:

Step 1: Express $\cos 48^\circ$ in terms of p.

Given $\sin 48^\circ = p$ and 48° is acute, so

$$\cos 48^\circ = \sqrt{1 - \sin^2 48^\circ} = \sqrt{1-p^2}.$$

Step 2: Use $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$.

$$\tan 48^\circ = \frac{\sin 48^\circ}{\cos 48^\circ} = \frac{p}{\sqrt{1-p^2}}.$$

Step 3: Conclude.

Hence, $\tan 48^\circ = \dfrac{p}{\sqrt{1-p^2}}$.

Quick Tip

For an acute angle, $\cos\theta = \sqrt{1-\sin^2\theta}$ and $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$. Always choose the positive square root.

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29. If $\tan 27^\circ \cdot \tan 63^\circ = \sin A$, then the value of A is:

(1) 27°

(2) 63°

(3) 90°
(4) 36°

Correct Answer: (3) 90°
Solution:
Step 1: Use co-function identity for tangent.
Since $63^\circ = 90^\circ - 27^\circ$, we have $\tan 63^\circ = \cot 27^\circ$.

Step 2: Evaluate the product.

$$\tan 27^\circ \cdot \tan 63^\circ = \tan 27^\circ \cdot \cot 27^\circ = 1.$$

Step 3: Equate to $\sin A$ and solve.
Given $\tan 27^\circ \cdot \tan 63^\circ = \sin A \Rightarrow \sin A = 1$.
Therefore $A = 90^\circ$ (in the principal range).

Quick Tip

$\tan(90^\circ - \theta) = \cot\theta$. Hence $\tan\theta \cdot \tan(90^\circ - \theta) = 1$, a handy shortcut for many angle products.

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30. Evaluate $\dfrac{\sin^2(90^\circ - \theta) + \sin^2\theta}{\csc^2(90^\circ - \theta) - \tan^2\theta}$.

(1) 1
(2) 0
(3) 2
(4) -1

Correct Answer: (1) 1
Solution:
Step 1: Use co-function identities.
$\sin(90^\circ - \theta) = \cos\theta$ and $\csc(90^\circ - \theta) = \sec\theta$.

Step 2: Simplify numerator and denominator.
Numerator: $\sin^2(90^\circ - \theta) + \sin^2\theta = \cos^2\theta + \sin^2\theta = 1$.
Denominator: $\csc^2(90^\circ - \theta) - \tan^2\theta = \sec^2\theta - \tan^2\theta = 1$ (Pythagorean identity).

Step 3: Conclude.

$$\frac{1}{1} = 1.$$

Quick Tip

Remember $\sin(90^\circ - \theta) = \cos\theta$ and the identity $\sec^2\theta - \tan^2\theta = 1$; such pairs often collapse expressions to a constant.

---

31. If the radius of a wheel is $\frac{35}{44}$ metre, then the distance covered in 2 revolutions is:

(1) 10 m
(2) 35 m
(3) 22 m
(4) 40 m

Correct Answer: (1) 10 m
Solution:
Step 1: Use the circumference formula.
Distance in one revolution $= 2\pi r$. For 2 revolutions, distance $= 4\pi r$.

Step 2: Substitute $r = \frac{35}{44}$ m and $\pi = \frac{22}{7}$.

$$\text{Distance} = 4\pi r = 4 \times \frac{22}{7} \times \frac{35}{44} = \frac{140}{44} \times \frac{22}{7} = \frac{35}{11} \times \frac{22}{7} = \frac{770}{77} = 10 \text{ m}.$$

Step 3: Conclude.
Hence, the distance covered in 2 revolutions is 10 metres.

Quick Tip

Distance in n revolutions $= n \times 2\pi r$. For clean numbers, use $\pi = \frac{22}{7}$ when r has factors 7 or 11.

---

32. If O is the centre and r is the radius of a circle and $\angle AOB = \theta$ (in degrees), then the length of arc AB is equal to:

(1) $\frac{\pi r^2\theta}{180}$
(2) $\frac{\pi r\theta}{360}$
(3) $\frac{\pi r\theta}{180}$
(4) $\frac{\pi r^2\theta}{360}$

Correct Answer: (3) $\frac{\pi r\theta}{180}$

Solution:
Step 1: Use the arc length formula for central angle in degrees.
Length of arc s corresponding to central angle θ° is

$$s = \frac{\theta}{360^\circ} \times 2\pi r.$$

Step 2: Simplify the expression.

$$s = \frac{\theta}{360} \cdot 2\pi r = \frac{\pi r \theta}{180}.$$

Step 3: Conclude.
Hence, the length of arc AB is $\frac{\pi r \theta}{180}$.

Quick Tip

For angles in degrees, arc length $s = \frac{\theta}{360} \times 2\pi r$. For angles in radians, $s = r\theta$.

---

33. How many solid spheres of radius 1 cm can be made from a solid sphere of radius 8 cm (assuming no loss of material)?

(1) 256
(2) 512
(3) 1024
(4) 576

Correct Answer: (2) 512
Solution:
Step 1: Use volume conservation.
Number of small spheres $= \dfrac{\text{Volume of big sphere}}{\text{Volume of one small sphere}}$.
Step 2: Write volumes using $V = \frac{4}{3}\pi r^3$.

$$N = \frac{\frac{4}{3}\pi(8)^3}{\frac{4}{3}\pi(1)^3} = \frac{512}{1} = 512.$$

Step 3: Conclude.
Thus, 512 small spheres can be made.

Quick Tip

When reshaping solids without loss, the count scales with the ratio of volumes. For similar shapes, that's the cube of the linear scale: $N = (R/r)^3$.

---

34. The base diameter of a cone is 10 cm and its height is 12 cm. Then the volume of the cone is:

(1) 400π cm^3
(2) 300π cm^3
(3) 100π cm^3
(4) 200π cm^3

Correct Answer: (3) 100π cm^3
Solution:
Step 1: Write the volume formula of a cone.
$V = \frac{1}{3}\pi r^2 h.$

Step 2: Convert diameter to radius.
Diameter $= 10$ cm $\Rightarrow$ radius $r = \frac{10}{2} = 5$ cm; height $h = 12$ cm.

Step 3: Substitute and compute.

$$V = \frac{1}{3}\pi(5)^2(12) = \frac{1}{3}\pi \cdot 25 \cdot 12 = \frac{300}{3}\pi = 100\pi \text{ cm}^3.$$

Step 4: Conclude.
Therefore, the volume is 100π cm^3.

Quick Tip

Always halve the diameter to get the radius before using $V = \frac{1}{3}\pi r^2 h$.

---

35. The external radius of a metallic pipe is 4 cm and its internal radius is 3 cm. If its length is 10 cm, then the volume of metal is:

(1) 120 cm^3
(2) 220 cm^3
(3) 440 cm^3
(4) 1540 cm^3

Correct Answer: (2) 220 cm^3

Solution:
Step 1: Use the formula for volume of a hollow cylinder.
Volume $V = \pi h(R^2 - r^2)$, where R is external radius, r is internal radius, and h is length.

Step 2: Substitute the given values.
$R = 4$ cm, $r = 3$ cm, $h = 10$ cm:

$$V = \pi \times 10 \times (4^2 - 3^2) = 10\pi \times (16 - 9) = 10\pi \times 7 = 70\pi \text{ cm}^3.$$

Step 3: Evaluate.
Using $\pi = \dfrac{22}{7}$: $V = 70 \times \dfrac{22}{7} = 220$ cm^3.

Quick Tip

For a pipe (hollow cylinder), subtract areas first: $\pi(R^2 - r^2)$, then multiply by length h.

---

36. The areas of the bases of a cone and a cylinder are equal and their curved surface areas are also equal. If the height of the cylinder is 2 metre, then the slant height of the cone is:

(1) 2 metre
(2) 3 metre
(3) 4 metre
(4) 5 metre

Correct Answer: (3) 4 metre
Solution:
Step 1: Use "equal base areas."
If base areas are equal, their radii are equal. Let the common radius be r.

Step 2: Use "equal curved surface areas."
Cylinder CSA $= 2\pi rh$.
Cone CSA $= \pi r\ell$ (where ℓ is the slant height).
Given they are equal: $2\pi rh = \pi r\ell \Rightarrow \ell = 2h$.

Step 3: Substitute the cylinder's height.
$h = 2$ m $\Rightarrow \ell = 2 \times 2 = 4$ m.

Quick Tip

Equal base areas $\Rightarrow$ equal radii. If $\text{CSA}_{\text{cyl}} = 2\pi rh$ equals $\text{CSA}_{\text{cone}} = \pi r\ell$, then $\ell = 2h$ immediately.

37. If the volume of a cube is 125 cm^3 then the ratio of the side of the cube and the space diagonal of the cube is:

(1) $1 : \sqrt{3}$
(2) $5 : \sqrt{3}$
(3) $25 : \sqrt{3}$
(4) $15 : \sqrt{3}$

Correct Answer: (1) $1 : \sqrt{3}$
Solution:
Step 1: Find the side length from volume.
For a cube with side a, volume $V = a^3$. Given $a^3 = 125 \Rightarrow a = \sqrt[3]{125} = 5$ cm.

Step 2: Write the space diagonal of a cube.
Space diagonal $d = a\sqrt{3} = 5\sqrt{3}$ cm.

Step 3: Form the required ratio (side : diagonal).

$$a : d = 5 : 5\sqrt{3} = 1 : \sqrt{3}.$$

Quick Tip

Cube facts: $V = a^3$, face diagonal $= a\sqrt{2}$, space diagonal $= a\sqrt{3}$. Ratios often simplify by canceling a.

---

38. If the total surface area of a hemisphere is 462 cm^2, then its diameter is:

(1) 7 cm
(2) 14 cm
(3) 21 cm
(4) 22 cm

Correct Answer: (2) 14 cm
Solution:
Step 1: Use the total surface area (TSA) of a hemisphere.
TSA $= 3\pi r^2$. Given $3\pi r^2 = 462$.

Step 2: Solve for r.
Taking $\pi = \dfrac{22}{7}$:

$$3 \cdot \frac{22}{7} r^2 = 462 \;\Rightarrow\; r^2 = \frac{462 \cdot 7}{66} = 49 \;\Rightarrow\; r = 7 \text{ cm}.$$

Step 3: Find the diameter.
Diameter $= 2r = 2 \times 7 = 14$ cm.

> **Quick Tip**
>
> For a hemisphere, TSA $= 2\pi r^2 + \pi r^2 = 3\pi r^2$. Once r is known, the diameter is simply $2r$.

---

39. The radii of two cylinders are in the ratio $2:3$ and their heights are in the ratio $5:3$. Then the ratio of their volumes is:

(1) $27:20$
(2) $20:27$
(3) $4:9$
(4) $9:4$

Correct Answer: (2) $20:27$
Solution:
Step 1: Recall the volume formula of a cylinder.
$V = \pi r^2 h$. For two cylinders, $V_1 : V_2 = (r_1^2 h_1) : (r_2^2 h_2)$.

Step 2: Substitute the given ratios.
$r_1 : r_2 = 2:3 \Rightarrow r_1^2 : r_2^2 = 4:9$.
$h_1 : h_2 = 5:3$.

Step 3: Multiply component-wise.

$$V_1 : V_2 = (4 \times 5) : (9 \times 3) = 20:27.$$

> **Quick Tip**
>
> For similar solids where $V \propto r^2 h$ (cylinders), square the radius ratio and multiply by the height ratio to get the volume ratio.

---

40. If the radius of a sphere becomes 3 times, then its volume will become:

(1) 3 times
(2) 6 times
(3) 9 times

(4) 27 times

Correct Answer: (4) 27 times
Solution:
Step 1: Recall the volume formula of a sphere.
$V = \frac{4}{3}\pi r^3$. Thus, volume is proportional to r^3.

Step 2: Apply the scaling.
If r becomes $3r$, then the new volume $V' \propto (3r)^3 = 27r^3$.

Step 3: Conclude.
Hence, the volume becomes 27 times the original.

Quick Tip

For similar 3D shapes, volumes scale with the cube of the linear scale factor: if $r \to kr$, then $V \to k^3 V$.

---

41. $\frac{\sqrt{81}}{2}$ **is:**

(1) a rational number
(2) an irrational number
(3) an integer
(4) none of these

Correct Answer: (1) a rational number
Solution:
Step 1: Simplify the given expression.
$\sqrt{81} = 9$. Hence,

$$\frac{\sqrt{81}}{2} = \frac{9}{2}.$$

Step 2: Check the nature of the number.
$\frac{9}{2} = 4.5$. It is not an integer, but it can be expressed as a ratio of two integers, i.e., $\frac{9}{2}$.

Step 3: Conclusion.
Hence, $\frac{\sqrt{81}}{2}$ is a rational number.

Quick Tip

If a number can be expressed in the form $\frac{p}{q}$ (where p, q are integers and $q \neq 0$), it is a rational number even if it's not an integer.

---

42. What is the exponent of 2 in the prime factorization of 192?

(1) 2
(2) 3
(3) 6
(4) 5

Correct Answer: (3) 6
Solution:
Step 1: Factorize 192.
$192 = 64 \times 3 = 2^6 \times 3$.

Step 2: Read off the exponent of 2.
In $192 = 2^6 \times 3$, the exponent of 2 is 6.

Step 3: Conclude.
Therefore, the required exponent is 6.

Quick Tip

Break numbers into convenient powers: $192 = 64 \times 3 = 2^6 \times 3$. Spotting perfect powers speeds up prime factorization.

---

43. If in the division algorithm $a = bq + r$, $a = 72$, $q = 8$ and $r = 0$, then what is the value of b?

(1) 9
(2) 8
(3) 7
(4) 4

Correct Answer: (1) 9
Solution:
Step 1: Recall the division algorithm.
It states $a = bq + r$ with $0 \leq r < b$. Here $r = 0$, so $a = bq$.

Step 2: Substitute the given values and solve for b.

$72 = b \cdot 8 \Rightarrow b = \dfrac{72}{8} = 9.$

Step 3: Conclude.
Therefore, $b = 9$.

> **Quick Tip**
>
> When the remainder $r = 0$, the dividend is exactly divisible: $a = bq$, so $b = \dfrac{a}{q}$.

---

44. Which of the following has a terminating decimal expansion?

(1) $\dfrac{2}{15}$
(2) $\dfrac{11}{160}$
(3) $\dfrac{17}{60}$
(4) $\dfrac{6}{35}$

Correct Answer: (2) $\dfrac{11}{160}$
Solution:
Step 1: Use the terminating-decimal criterion.
A rational number $\dfrac{p}{q}$ in lowest terms has a terminating decimal expansion iff the prime factorization of q contains only 2's and 5's, i.e., $q = 2^m 5^n$.

Step 2: Check each denominator (in lowest terms).
$\dfrac{2}{15}$: $15 = 3 \cdot 5$ (contains 3) $\Rightarrow$ non-terminating.
$\dfrac{11}{160}$: $160 = 2^5 \cdot 5$ (only 2 and 5) $\Rightarrow$ **terminating**.
$\dfrac{17}{60}$: $60 = 2^2 \cdot 3 \cdot 5$ (contains 3) $\Rightarrow$ non-terminating.
$\dfrac{6}{35}$: $35 = 5 \cdot 7$ (contains 7) $\Rightarrow$ non-terminating.

Step 3: Conclude.
Only $\dfrac{11}{160}$ meets the criterion, so it has a terminating decimal expansion.

> **Quick Tip**
>
> Reduce the fraction first. If the denominator has any prime other than 2 or 5, the decimal is non-terminating (recurring).

---

45. If q is a positive integer, which of the following is *not* an odd positive integer?

(1) $8q + 1$
(2) $8q + 4$
(3) $8q + 3$
(4) $8q + 7$

Correct Answer: (2) $8q + 4$
Solution:
Step 1: Use parity of multiples of 8.
Since $q \in \mathbb{Z}^+$, $8q$ is divisible by 8 and hence is **even**.

Step 2: Check each expression's parity.
$8q + 1 = \text{even} + 1 = \text{odd}$.
$8q + 4 = \text{even} + 4 = \textbf{even}$.
$8q + 3 = \text{even} + 3 = \text{odd}$.
$8q + 7 = \text{even} + 7 = \text{odd}$.

Step 3: Conclude.
Only $8q + 4$ is not odd; it is even.

Quick Tip

Even + even = even, even + odd = odd. Any multiple of 2 (like $8q$) is even.

---

46. The HCF of two consecutive even numbers is:

(1) 0
(2) 1
(3) 2
(4) 4

Correct Answer: (3) 2
Solution:
Step 1: Represent consecutive even numbers.
Let the two consecutive even numbers be $2n$ and $2n + 2 = 2(n + 1)$.

Step 2: Take out the common factor.
$\gcd(2n,\, 2n + 2) = \gcd\big(2n,\, 2(n + 1)\big) = 2 \cdot \gcd\big(n,\, n + 1\big)$.

Step 3: Use property of consecutive integers.
$\gcd(n,\, n + 1) = 1$ (consecutive integers are co-prime). Hence, the HCF is $2 \times 1 = 2$.

Quick Tip

Consecutive integers are co-prime. For consecutive even numbers $2n$ and $2n+2$, a factor 2 is always common, so HCF $= 2$.

---

47. The sum of a rational number and an irrational number is which type of number?

(1) An integer
(2) Irrational number
(3) Natural number
(4) None of these

Correct Answer: (2) Irrational number
Solution:
Step 1: Let r be rational and i be irrational.
Assume for contradiction that $r + i$ is rational.

Step 2: Use closure of rationals under subtraction.
If $r + i$ were rational, then $i = (r + i) - r$ would be the difference of two rationals, hence rational—contradiction.

Step 3: Conclude.
Therefore, $r + i$ must be **irrational**.

Quick Tip

Rationals are closed under addition/subtraction; irrationals are not. Adding any nonzero rational to an irrational keeps it irrational.

---

48. The product of two numbers is 8670 and their HCF is 17. What is their LCM?

(1) 102
(2) 85
(3) 107
(4) 510

Correct Answer: (4) 510
Solution:
Step 1: Use the relation between HCF, LCM and product.
For two positive integers a and b:

$$a \times b = \text{HCF}(a, b) \times \text{LCM}(a, b).$$

Step 2: Substitute the given values and solve for LCM.

$$8670 = 17 \times \text{LCM} \;\Rightarrow\; \text{LCM} = \frac{8670}{17} = 510.$$

Step 3: Conclude.
Hence, the LCM is 510.

Quick Tip

Always remember: HCF $\times$ LCM = product of the two numbers. It saves time when either HCF or LCM is known.

---

49. Which of the following is a rational number?

(1) $\sqrt{64+36}$
(2) $\sqrt{25+25}$
(3) $\sqrt{49+49}$
(4) $\sqrt{36+36}$

Correct Answer: (1) $\sqrt{64+36}$
Solution:
Step 1: Evaluate each expression under the square root.
$\sqrt{64+36} = \sqrt{100} = 10$ (rational).
$\sqrt{25+25} = \sqrt{50} = 5\sqrt{2}$ (irrational).
$\sqrt{49+49} = \sqrt{98} = 7\sqrt{2}$ (irrational).
$\sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$ (irrational).

Step 2: Conclude.
Only option (1) simplifies to an integer, hence it is rational.

Quick Tip

A square root is rational only if the radicand is a perfect square (e.g., $100, 144$). Otherwise, it's irrational.

---

50. If $130 = 15 \times 8 + 10$ and $15 = 5 \times 3 + 0$, then HCF(130, 15) will be:

(1) 8
(2) 5

(3) 130
(4) 15

Correct Answer: (2) 5
Solution:
Step 1: Apply Euclid's division algorithm.
From $130 = 15 \cdot 8 + 10$, the remainder is 10.
Next, divide 15 by 10: $15 = 10 \cdot 1 + 5$.
Finally, $10 = 5 \cdot 2 + 0$. The last non-zero remainder is 5.

Step 2: Conclude.
Hence, $\gcd(130, 15) = 5$.

Quick Tip

In Euclid's algorithm, the HCF is the last non-zero remainder in the sequence of divisions.

---

51. Simplify $1 - \sin^4\theta$:

(1) $\cos^2\theta\,(1 + \sin^2\theta)$
(2) $\cos^2\theta\,(1 - \sin^2\theta)$
(3) $\cos^2\theta\,(1 - \cos^2\theta)$
(4) $\sin^2\theta\,(1 + \sin^2\theta)$

Correct Answer: (1) $\cos^2\theta\,(1 + \sin^2\theta)$
Solution:

$$1 - \sin^4\theta = (1 - \sin^2\theta)(1 + \sin^2\theta) = \cos^2\theta(1 + \sin^2\theta).$$

Quick Tip

Use difference of squares and $1 - \sin^2\theta = \cos^2\theta$.

---

52. In $\triangle ABC$, points X on AB and Y on AC satisfy $XY \parallel BC$. If $AX : XB = 2 : 3$, then $AY : YC =$?

(1) 3 : 2
(2) 2 : 3
(3) 1 : 3
(4) 3 : 1

Correct Answer: (2) 2 : 3
Solution: By Basic Proportionality Theorem, $\dfrac{AX}{XB} = \dfrac{AY}{YC}$. Hence $AY : YC = 2 : 3$.

Quick Tip

When a segment is parallel to a triangle's side, it divides the other two sides proportionally.

---

53. The ratio of the areas of two similar triangles is 121 : 64. The ratio of their corresponding medians is:

(1) 11 : 8
(2) 8 : 11
(3) 121 : 64
(4) 12 : 91

Correct Answer: (1) 11 : 8
Solution: In similar figures, any corresponding lengths are in the ratio of the square root of the area ratio: $\sqrt{121/64} = 11/8$.

Quick Tip

Area ratio = (linear scale factor)^2. Take square roots to switch to lengths.

---

54. Two similar triangles have perimeters 30 cm and 20 cm. If a side of the first triangle is 18 cm, the corresponding side of the second is:

(1) 10 cm
(2) 8 cm
(3) 9 cm
(4) 12 cm

Correct Answer: (4) 12 cm
Solution: Side ratio = perimeter ratio = 30 : 20 = 3 : 2. Thus required side = $18 \times \frac{2}{3} = 12$ cm.

Quick Tip

For similar polygons, perimeters scale exactly as any side.

---

55. The area of an equilateral triangle is $36\sqrt{3}$ cm^2. Its side is:

(1) 6 cm
(2) $3\sqrt{3}$ cm
(3) 12 cm

(4) $\sqrt{3}$ cm

Correct Answer: (3) 12 cm

Solution: $\frac{\sqrt{3}}{4}a^2 = 36\sqrt{3} \Rightarrow a^2 = 144 \Rightarrow a = 12$ cm.

Quick Tip

Memorize $A_{\text{equi}} = \frac{\sqrt{3}}{4}a^2$.

---

56. The number of common tangents of two intersecting circles is:

(1) 4
(2) 2
(3) 1
(4) 3

Correct Answer: (2) 2
Solution: Intersecting circles have exactly two common tangents (both direct).

Quick Tip

Disjoint: 4, externally tangent: 3, intersecting: 2, internally tangent: 1, one inside without touch: 0.

---

57. A circle has radius 7 cm. From external point P, tangent $PT = 24$ cm. If O is centre, find OP.

(1) 30 cm
(2) 28 cm
(3) 25 cm
(4) 18 cm

Correct Answer: (3) 25 cm
Solution: $OT \perp PT$. In right $\triangle OPT$:

$$OP^2 = OT^2 + PT^2 = 7^2 + 24^2 = 49 + 576 = 625 \Rightarrow OP = 25 \text{ cm}.$$

Quick Tip

Radius to the point of tangency is perpendicular to the tangent—use Pythagoras.

---

58. From an external point P, the length of one tangent to a circle is 17 cm. The other tangent from P to the same circle has length:

(1) 34 cm
(2) 51 cm
(3) 17 cm
(4) 17/2 cm

Correct Answer: (3) 17 cm
Solution: Tangents from the same external point are equal: $PT_1 = PT_2 = 17$ cm.

Quick Tip

Equal tangents from an external point is a standard theorem—instant result.

---

59. A circle has radius 7 cm. If the area of a minor sector is 14 cm^2, find the area of the major sector.

(1) 140 cm^2
(2) 150 cm^2
(3) 125 cm^2
(4) 200 cm^2

Correct Answer: (1) 140 cm^2
Solution: Total area $= \pi r^2 = \frac{22}{7} \cdot 7^2 = 154$ cm^2.
Major area $= 154 - 14 = 140$ cm^2.

Quick Tip

Major sector area = total circle area − minor sector area.

---

60. The area of a circle is 154 cm^2. Its diameter is:

(1) 14 cm
(2) 28 cm
(3) 7 cm
(4) 21 cm

Correct Answer: (1) 14 cm
Solution: $\pi r^2 = 154 \Rightarrow r^2 = 154 \cdot \frac{7}{22} = 49 \Rightarrow r = 7$ cm.
Diameter $= 2r = 14$ cm.

Quick Tip

When areas involve 154, try $\pi = \frac{22}{7}$ to spot perfect squares quickly.

---

61. The discriminant of the quadratic equation $x^2 + 4x + b = 0$ is

(1) $16 - 4b$
(2) $4b - 16$
(3) $b^2 - 16$
(4) $16 - b^2$

Correct Answer: (1) $16 - 4b$
Solution:
Step 1: Recall the discriminant formula.
For $ax^2 + bx + c = 0$, the discriminant is $D = b^2 - 4ac$.

Step 2: Identify coefficients.
Here $a = 1,\ b = 4,\ c = b$.

Step 3: Substitute values.

$$D = 4^2 - 4(1)(b) = 16 - 4b$$

Quick Tip

For quadratic equations in the form $x^2 + px + q = 0$, the discriminant is $p^2 - 4q$.

---

62. Which of the following are in A.P.?

(1) 0.4, 0.44, 0.444, ...
(2) 1, 11, 111, ...
(3) 2, 4, 8, 16, ...
(4) 0, -4, -8, -12, ...

Correct Answer: (4) 0, -4, -8, -12, ...
Solution:
Step 1: Check sequence (1).
$0.44 - 0.4 = 0.04,\ 0.444 - 0.44 = 0.004$, not constant. Not an A.P.

Step 2: Check sequence (2).
$11 - 1 = 10,\ 111 - 11 = 100$, not constant. Not an A.P.

Step 3: Check sequence (3).
Differences $4 - 2 = 2,\ 8 - 4 = 4,\ 16 - 8 = 8$, not constant. This is a G.P.

Step 4: Check sequence (4).
Differences $-4 - 0 = -4,\ -8 - (-4) = -4,\ -12 - (-8) = -4$, constant. Hence this is an A.P.

Quick Tip

A sequence is an A.P. if the difference between consecutive terms is always the same.

---

63. The common difference of an A.P. $\sqrt{18}, \sqrt{50}, \sqrt{98}, \sqrt{162}, \ldots$ **is**

(1) 2
(2) $2\sqrt{2}$
(3) 3
(4) $2\sqrt{3}$

Correct Answer: (2) $2\sqrt{2}$
Solution:
Step 1: Simplify radicals.
$\sqrt{18} = 3\sqrt{2},\ \sqrt{50} = 5\sqrt{2},\ \sqrt{98} = 7\sqrt{2},\ \sqrt{162} = 9\sqrt{2}.$

Step 2: Check differences.
$5\sqrt{2} - 3\sqrt{2} = 2\sqrt{2},\ 7\sqrt{2} - 5\sqrt{2} = 2\sqrt{2}.$ Constant difference.

Thus, $d = 2\sqrt{2}$.

Quick Tip

Always try to simplify square roots to find patterns in A.P. sequences.

---

64. What is the 11th term of the A.P. 2, 5, 8, 11, ...**?**

(1) 24
(2) 30
(3) 32
(4) 14

Correct Answer: (3) 32
Solution:
Step 1: Identify terms. First term $a = 2$, common difference $d = 3$.

Step 2: Use formula.

$$a_n = a + (n-1)d$$

Step 3: Substitute.

$$a_{11} = 2 + (11 - 1) \cdot 3 = 2 + 30 = 32$$

Quick Tip

The nth term of an A.P. is given by $a + (n - 1)d$.

---

65. If the nth term of an A.P. is $8n - 2$, then the 7th term is

(1) 54
(2) 50
(3) 30
(4) 40

Correct Answer: (1) 54
Solution:
Step 1: Substitute $n = 7$.

$$a_7 = 8(7) - 2 = 56 - 2 = 54$$

Quick Tip

When the general term is given, just substitute n to find the required term.

---

66. If in an A.P. $a_{30} - a_{20} = 50$, then the common difference is

(1) 4
(2) 15
(3) 5
(4) 10

Correct Answer: (3) 5
Solution:
Step 1: Write formulas.
$a_{30} = a + 29d, \quad a_{20} = a + 19d.$

Step 2: Subtract.

$$a_{30} - a_{20} = (a + 29d) - (a + 19d) = 10d$$

Step 3: Use condition.

$$10d = 50 \quad \Rightarrow \quad d = 5$$

Quick Tip

In A.P., $a_m - a_n = (m - n)d$. This is a shortcut for differences.

---

67. If $(2x - 1)$, $(3x + 2)$ and $(6x - 1)$ are three consecutive terms of an A.P., then the value of x is

(1) 3
(2) 2
(3) 4
(4) 1

Correct Answer: (1) 3
Solution:
Step 1: Use A.P. property.
In an A.P., the middle term equals the average of its neighbors:

$$3x + 2 = \frac{(2x - 1) + (6x - 1)}{2} = \frac{8x - 2}{2} = 4x - 1.$$

Step 2: Solve for x.

$$3x + 2 = 4x - 1 \ \Rightarrow \ x = 3.$$

Quick Tip

For three consecutive A.P. terms A, B, C, the relation $2B = A + C$ (or $B = \frac{A+C}{2}$) is the quickest route.

---

68. The number of terms in the A.P. $2, 6, 10, 14, \ldots, 82$ **is**

(1) 15
(2) 21
(3) 20
(4) 22

Correct Answer: (2) 21
Solution:
Step 1: Identify a, d, l**.**
$a = 2,\ d = 4,\ l = 82.$

Step 2: Use last-term formula.
$l = a + (n-1)d \Rightarrow 82 = 2 + (n-1) \cdot 4.$

Step 3: Solve for n**.**
$80 = 4(n-1) \Rightarrow n - 1 = 20 \Rightarrow n = 21.$

Quick Tip

When first term a, last term l and d are known: $n = \dfrac{l-a}{d} + 1.$

---

69. Which term of the A.P. $72, 63, 54, \ldots$ **is zero?**

(1) 8th
(2) 9th
(3) 10th
(4) 11th

Correct Answer: (2) 9th
Solution:
Step 1: Identify a, d**.**
$a = 72,\ d = 63 - 72 = -9.$

Step 2: Set $a_n = 0$**.**
$a_n = a + (n-1)d = 72 + (n-1)(-9) = 0.$

Step 3: Solve.
$72 - 9n + 9 = 0 \Rightarrow 81 - 9n = 0 \Rightarrow n = 9.$

Quick Tip

To find when an A.P. hits a target value T, solve $a + (n-1)d = T$.

---

70. Distance between the points $(a\cos\theta, 0)$ and $(0, a\sin\theta)$ is

(1) a
(2) $2a$
(3) $3a$
(4) $4a$

Correct Answer: (1) a
Solution:
Step 1: Use the distance formula.

$$d = \sqrt{(a\cos\theta - 0)^2 + (0 - a\sin\theta)^2} = \sqrt{a^2\cos^2\theta + a^2\sin^2\theta}.$$

Step 2: Apply identity.
$\cos^2\theta + \sin^2\theta = 1 \Rightarrow d = \sqrt{a^2} = a$ (assuming $a \geq 0$).

Quick Tip

Combine the Pythagorean identity $\sin^2\theta + \cos^2\theta = 1$ with the distance formula for quick simplification.

---

71. If $p(x) = 3x^3 + x^2 + 2x + 5$ is divided by $q(x) = x^2 + 2x + 1$, then the degree of the quotient will be

(1) 1
(2) 2
(3) 3
(4) 4

Correct Answer: (1) 1
Solution:
Step 1: Use degree rule for polynomial division.
For nonzero polynomials, deg(quotient) = deg(dividend) − deg(divisor) when deg(dividend) ≥ deg(divisor).
Step 2: Apply to given polynomials.
$\deg p = 3,\ \deg q = 2 \Rightarrow \deg(\text{quotient}) = 3 - 2 = 1.$

Quick Tip

While dividing polynomials, the quotient's degree is the difference of degrees (provided leading terms don't cancel).

---

72. If polynomial $g(x) = 3x^2 - 7x + 2$, then the value of $g(-1)$ is

(1) -8
(2) 12
(3) -12
(4) 0

Correct Answer: (2) 12
Solution:
Step 1: Substitute $x = -1$.
$g(-1) = 3(-1)^2 - 7(-1) + 2 = 3 + 7 + 2 = 12.$

Quick Tip

Evaluate a polynomial at $x = a$ by direct substitution; watch signs carefully.

---

73. The zeroes of the polynomial $x^2 - 16$ are

(1) $+4, -4$
(2) $+4, +4$
(3) $-4, -4$
(4) none of these

Correct Answer: (1) $+4, -4$
Solution:
Step 1: Factorize as difference of squares.
$x^2 - 16 = (x - 4)(x + 4).$
Step 2: Set each factor to zero.
$x = 4$ or $x = -4.$

Quick Tip

Remember $a^2 - b^2 = (a - b)(a + b)$ to factor quickly.

---

74. If α and β are zeroes of the polynomial $p(x) = x^2 - 3x + 5$, then the value of $4(\alpha + \beta)$ is

(1) 12
(2) -12
(3) 20
(4) -20

Correct Answer: (1) 12
Solution:
Step 1: Use sum of roots formula.
For $x^2 + px + q$, $\alpha + \beta = -p$. Here $p = -3 \Rightarrow \alpha + \beta = 3$.
Step 2: Multiply by 4.
$4(\alpha + \beta) = 4 \times 3 = 12$.

Quick Tip

For $ax^2 + bx + c$: $\alpha + \beta = -\frac{b}{a}$, $\alpha\beta = \frac{c}{a}$.

---

75. If the product of zeroes of a polynomial $x^2 - 9x + 2a$ is 8, then the value of a will be

(1) 4
(2) -4
(3) 9
(4) -9

Correct Answer: (1) 4
Solution:
Step 1: Use product of roots.
For $x^2 + px + q$, $\alpha\beta = q$. Here $q = 2a$.
Step 2: Set equal to 8.
$2a = 8 \Rightarrow a = 4$.

Quick Tip

In a monic quadratic, the constant term equals the product of the roots.

---

76. If one zero of a polynomial $p(x)$ is -1, then a factor of $p(x)$ will be

(1) $x - 1$
(2) $x + 1$
(3) $\dfrac{1}{x + 1}$
(4) $\dfrac{1}{x - 1}$

Correct Answer: (2) $x + 1$
Solution:
Step 1: Factor theorem.
If $x = a$ is a zero, then $(x - a)$ is a factor.
Step 2: Apply to $a = -1$.
Factor is $(x - (-1)) = (x + 1)$.

Quick Tip

"Zero $a \Rightarrow$ factor $(x - a)$" is the essence of the Factor Theorem.

---

77. Which of the following quadratic polynomials has sum of zeroes -3 and product of zeroes 2?

(1) $x^2 + 3x + 2$
(2) $x^2 + 2x - 3$
(3) $x^2 - 3x - 2$
(4) $x^2 - 3x + 2$

Correct Answer: (1) $x^2 + 3x + 2$
Solution:
Step 1: Build quadratic from sum S and product P.
A monic quadratic with sum S and product P is $x^2 - Sx + P$.
Step 2: Substitute $S = -3,\ P = 2$.
$x^2 - (-3)x + 2 = x^2 + 3x + 2$.

Quick Tip

Given sum S and product P, write $x^2 - Sx + P$ directly.

---

78. If α, β, γ are zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d = 0$, then the value of $\alpha\beta\gamma$ is

(1) $\frac{b}{a}$
(2) $-\frac{c}{a}$
(3) $-\frac{d}{a}$
(4) $\frac{c}{a}$

Correct Answer: (3) $-\frac{d}{a}$

Solution:
Step 1: Use Vieta's relation for cubics.
For $ax^3 + bx^2 + cx + d$: $\alpha\beta\gamma = -\dfrac{d}{a}$.
Step 2: State result.
Hence $\alpha\beta\gamma = -\dfrac{d}{a}$.

Quick Tip

Signs in Vieta alternate: for cubic, product of roots is $-d/a$.

---

79. The degree of the polynomial $(y^2 - 3y + 1)\,(y^5 - 4y^3 + y^2 + 3y)$ is

(1) 6
(2) 7
(3) 3
(4) 4

Correct Answer: (2) 7
Solution:
Step 1: Degrees of factors.
First factor has degree 2; second has degree 5.
Step 2: Degree of product.
$\deg(\text{product}) = 2 + 5 = 7$.

Quick Tip

For nonzero polynomials, $\deg(PQ) = \deg P + \deg Q$.

---

80. If $3x - 2y = 12$ and $4x - 5y = 16$, then

(1) $x = 4,\ y = 0$
(2) $x = 0,\ y = 4$
(3) $x = 4,\ y = 2$
(4) $x = 1,\ y = 10$

Correct Answer: (1) $x = 4,\ y = 0$
Solution:
Step 1: Eliminate y.
Multiply $3x - 2y = 12$ by 5: $15x - 10y = 60$.
Multiply $4x - 5y = 16$ by 2: $8x - 10y = 32$.
Step 2: Subtract equations.
$(15x - 10y) - (8x - 10y) = 60 - 32 \Rightarrow 7x = 28 \Rightarrow x = 4$.

Step 3: Back-substitute.
$3(4) - 2y = 12 \Rightarrow 12 - 2y = 12 \Rightarrow y = 0$.

Quick Tip

To solve two linear equations, align coefficients (by multiplication) and subtract to eliminate one variable.

---

81. The perimeter of a semicircle whose radius is $3r$ is

(1) $3\pi r + 3r$
(2) $3\pi r + 6r$
(3) $3\pi r + 9r$
(4) $3\pi r$

Correct Answer: (1) $3\pi r + 3r$
Solution:
Step 1: Formula for semicircle perimeter.
Perimeter $= \pi r + 2r$ (half the circumference plus the diameter).
Step 2: Substitute radius $3r$.

$$P = \pi(3r) + 2(3r) = 3\pi r + 6r$$

Step 3: Match with options.
Oops—actual correct choice is (2) $3\pi r + 6r$.

Quick Tip

For semicircle perimeter: add half circumference (πr) and diameter ($2r$).

---

82. Which of the following numbers is not the probability of any event?

(1) 0.7
(2) 2.5
(3) 75%
(4) $\frac{4}{5}$

Correct Answer: (2) 2.5
Solution:
Step 1: Recall probability range.
Probability always lies between 0 and 1.
Step 2: Check each option.

0.7 is valid, 75But 2.5 $>$ 1, invalid.

Quick Tip

Probability values must satisfy $0 \leq P(E) \leq 1$.

---

83. If E' is the complementary event of an event E, then which of the following is true?

(1) $P(E) = P(E')$
(2) $P(E) + P(E') = 0$
(3) $P(E) + P(E') = 1$
(4) None of these

Correct Answer: (3) $P(E) + P(E') = 1$
Solution:
Step 1: Complement rule.
By definition, $E + E' = \Omega$ (the sample space).
Step 2: Apply probability axiom.

$$P(E) + P(E') = P(\Omega) = 1$$

Quick Tip

Complementary events always add up to 1.

---

84. In throwing a die one time, what is the probability of not appearing 2?

(1) 0
(2) 16
(3) $\frac{5}{6}$
(4) $\frac{1}{2}$

Correct Answer: (3) $\frac{5}{6}$
Solution:
Step 1: Total outcomes of a die.
There are 6 equally likely outcomes.
Step 2: Favorable for not getting 2.
5 outcomes: 1,3,4,5,6.
Step 3: Probability.

$$P(\text{not } 2) = \frac{5}{6}$$

Quick Tip

Probability $= \frac{\text{Favorable}}{\text{Total}}$. Exclude unwanted outcome(s).

---

85. A card is drawn at random from a well-shuffled deck of 52 cards. What is the probability of getting a black king?

(1) $\frac{1}{13}$
(2) $\frac{1}{26}$
(3) $\frac{2}{39}$
(4) None of these

Correct Answer: (2) $\frac{1}{26}$
Solution:
Step 1: Total cards.
52 cards in a deck.
Step 2: Black kings.
There are 2 black kings (spades, clubs).
Step 3: Probability.

$$P = \frac{2}{52} = \frac{1}{26}$$

Quick Tip

Always count favorable cases carefully; a standard deck has 2 black suits.

---

86. If the median of $24, 27, 28, 31, 34, x, 37, 40, 42, 45, 50$ is 35, then the value of x is

(1) 35
(2) 36
(3) 34.5
(4) 35.5

Correct Answer: (2) 36
Solution:
Step 1: Count terms.
There are 11 numbers (odd). Median = 6th term when ordered.

Step 2: Arrange.
Sequence is already ordered: 24,27,28,31,34,x,37,40,42,45,50.
Median = 6th term = x.
Step 3: Use condition.
Median = 35 $\Rightarrow x = 35$.
Wait, option check: 35 corresponds to (1), not (2). Correct is (1) 35.

Quick Tip

For odd number of terms, median is the middle term directly.

---

87. The cumulative frequency table is useful in the determination of the following?

(1) Mean
(2) Median
(3) Mode
(4) None of these

Correct Answer: (2) Median
Solution:
Step 1: Recall usage.
Cumulative frequency helps locate median class by finding the middle observation.

Quick Tip

Cumulative frequency distribution is primarily for median calculation.

---

88. Which relationship is true for Mean, Median and Mode?

(1) Mean - Mode = 3 (Mean - Median)
(2) Mean - Mode = 3 (Mean + Median)
(3) Mean - Mode = 3 (Mode - Median)
(4) None of these

Correct Answer: (1) Mean - Mode = 3 (Mean - Median)
Solution:
Step 1: Recall empirical relation.
Karl Pearson's empirical formula:

$$\text{Mode} = 3 \times \text{Median} - 2 \times \text{Mean}$$

Step 2: Rearrange.

Mean − Mode = 3(Mean − Median).

Quick Tip

The relation Mean - Mode = 3(Mean - Median) is an empirical formula, not always exact.

---

89. In the following distribution, what is the modal class?

Class	Frequency
$0-10$	15
$10-20$	20
$20-30$	45
$30-40$	15
$40-50$	25

(1) 10-20
(2) 20-30
(3) 40-50
(4) None of these

Correct Answer: (2) 20-30
Solution:
Step 1: Recall definition.
Modal class = class with highest frequency.
Step 2: Identify.
Frequencies: max is 45 for class 20-30.

Quick Tip

The modal class is the class interval with the maximum frequency.

---

90. If the mean of $x+2, 2x+3, 4x+5, 5x+2$ is 18, then the value of x is

(1) 5
(2) 6
(3) 3
(4) 15

Correct Answer: (2) 6
Solution:
Step 1: Write mean formula.

$$\frac{(x+2)+(2x+3)+(4x+5)+(5x+2)}{4} = 18$$

Step 2: Simplify numerator.
$x + 2 + 2x + 3 + 4x + 5 + 5x + 2 = 12x + 12$.
Step 3: Equation.
$\frac{12x+12}{4} = 18 \Rightarrow 12x + 12 = 72 \Rightarrow x = 5$.
Wait—actually value is 5, so correct answer is (1).

Quick Tip

Double-check simplification; group like terms before solving mean equations.

---

91. If point $R(a, b)$ lies on the line segment joining $P(0, 0)$ and $Q(0, 2)$, which of the following is true?

(1) $a = 0$
(2) $a = 2$
(3) $b = 0$
(4) None of these

Correct Answer: (1) $a = 0$
Solution:
Step 1: Observe the segment PQ.
Both P and Q have x-coordinate 0, so the segment PQ lies on the vertical line $x = 0$.
Step 2: Conclude for any point on the segment.
Any point $R(a, b)$ on this segment must satisfy $a = 0$ and $0 \leq b \leq 2$.

Quick Tip

If two end points share the same x (or y) coordinate, the whole segment is a vertical (or horizontal) line with that coordinate fixed.

---

92. The point of intersection of the straight lines $x = 2$ and $y = -3$ is

(1) $(2, -3)$
(2) $(-2, -3)$
(3) $(2, 3)$
(4) $(3, -2)$

Correct Answer: (1) $(2, -3)$

Solution:
Step 1: Intersection of two lines.
Solve simultaneously: $x = 2$ and $y = -3$.
Step 2: Read coordinates.
Point is $(2, -3)$.

Quick Tip

Lines of the form $x = a$ and $y = b$ intersect at (a, b) immediately—no algebra needed.

---

93. If $\tan\theta = \frac{15}{8}$, then the value of $\sin\theta$ will be

(1) $\frac{8}{17}$
(2) $\frac{8}{15}$
(3) $\frac{15}{17}$
(4) $\frac{17}{8}$

Correct Answer: (3) $\frac{15}{17}$
Solution:
Step 1: Form a right triangle.
Let opposite $= 15$, adjacent $= 8$. Then hypotenuse $= \sqrt{15^2 + 8^2} = \sqrt{289} = 17$.
Step 2: Compute sine.
$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{15}{17}$.

Quick Tip

From $\tan\theta = \frac{p}{b}$, set a triangle with legs p, b to get $\sin\theta = \frac{p}{\sqrt{p^2+b^2}}$.

---

94. If $(\tan\theta + \cot\theta) = 6$, then the value of $\tan^2\theta + \cot^2\theta$ is

(1) 25
(2) 27
(3) 24
(4) 34

Correct Answer: (4) 34
Solution:
Step 1: Use identity.

Let $x = \tan\theta$. Then $x + \dfrac{1}{x} = 6$.
Step 2: Square both sides.
$\left(x + \dfrac{1}{x}\right)^2 = x^2 + 2 + \dfrac{1}{x^2} = 36 \Rightarrow x^2 + \dfrac{1}{x^2} = 34.$
Thus $\tan^2\theta + \cot^2\theta = 34$.

Quick Tip

Whenever $x + \frac{1}{x}$ is known, squaring gives $x^2 + \frac{1}{x^2} = (x + \frac{1}{x})^2 - 2$.

---

95. If $\sec\theta + \tan\theta + 1 = 0$, then the value of $\sec\theta - \tan\theta$ is

(1) 1
(2) -1
(3) 0
(4) 2

Correct Answer: (2) -1
Solution:
Step 1: Use $\sec^2\theta - \tan^2\theta = 1$.
$(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$.
Step 2: Substitute the given value.
$\sec\theta + \tan\theta = -1 \Rightarrow \sec\theta - \tan\theta = \dfrac{1}{-1} = -1.$

Quick Tip

Products $(\sec \pm \tan)(\sec \mp \tan) = 1$ are handy consequences of $\sec^2 - \tan^2 = 1$.

---

96. If $\sin\theta = \dfrac{\sqrt{3}}{2}$, then the value of $\csc\theta + \cot\theta$ is

(1) $2 + \sqrt{3}$
(2) $2\sqrt{3}$
(3) $\sqrt{2}$
(4) $\sqrt{3}$

Correct Answer: (4) $\sqrt{3}$
Solution:
Step 1: Compute co-functions.
$\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{2}{\sqrt{3}} = \dfrac{2\sqrt{3}}{3}.$

$\cos\theta = \sqrt{1-\sin^2\theta} = \frac{1}{2} \Rightarrow \cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$.

Step 2: Add.

$\csc\theta + \cot\theta = \frac{2\sqrt{3}}{3} + \frac{\sqrt{3}}{3} = \sqrt{3}$.

Quick Tip

When $\sin\theta$ or $\cos\theta$ is standard, compute others using $\sin^2\theta + \cos^2\theta = 1$.

---

98. If $\sin\theta = \cos\theta,\ 0^\circ \le \theta \le 90^\circ$, then the value of θ is

(1) 30°
(2) 45°
(3) 60°
(4) 90°

Correct Answer: (2) 45°

Solution:

Step 1: Use equality.

$\sin\theta = \cos\theta \Rightarrow \tan\theta = 1$.

Step 2: Principal value in first quadrant.

$\theta = 45^\circ$.

Quick Tip

In 0°–90°, $\sin\theta = \cos\theta$ only at $\theta = 45^\circ$.

---

99. $\frac{\sin 30^\circ + \cos 45^\circ - \tan 60^\circ}{\cot 30^\circ - \sin 45^\circ - \cos 60^\circ} =$

(1) -1
(2) 0
(3) 1
(4) 2

Correct Answer: (1) -1

Solution:

Step 1: Substitute standard values.

$\sin 30^\circ = \frac{1}{2},\ \cos 45^\circ = \frac{\sqrt{2}}{2},\ \tan 60^\circ = \sqrt{3},\ \cot 30^\circ = \sqrt{3},\ \cos 60^\circ = \frac{1}{2}$.

Numerator $= \frac{1}{2} + \frac{\sqrt{2}}{2} - \sqrt{3}$.

Denominator $= \sqrt{3} - \frac{\sqrt{2}}{2} - \frac{1}{2}$.

Step 2: Multiply by 2.

$\frac{1+\sqrt{2}-2\sqrt{3}}{2\sqrt{3}-\sqrt{2}-1} = -1$ since the numerator is the negative of the denominator.

Quick Tip

When many surds appear, clear small fractions (like halves) first—often reveals cancellations.

---

100. $\tan 30^\circ \cdot \sin 30^\circ \cdot \cot 60^\circ \cdot \csc 30^\circ =$

(1) $1/\sqrt{3}$
(2) $\sqrt{3}$
(3) $1/(2\sqrt{3})$
(4) $1/3$

Correct Answer: (4) $1/3$
Solution:
Step 1: Insert standard values.
$\tan 30^\circ = \frac{1}{\sqrt{3}}$, $\sin 30^\circ = \frac{1}{2}$, $\cot 60^\circ = \frac{1}{\sqrt{3}}$, $\csc 30^\circ = 2$.
Step 2: Multiply.
$\frac{1}{\sqrt{3}} \cdot \frac{1}{2} \cdot \frac{1}{\sqrt{3}} \cdot 2 = \frac{1}{3}$.

Quick Tip

Know the six trigonometric values at $30^\circ, 45^\circ, 60^\circ$; they simplify most exam problems instantly.
