Bihar Board Class 10th Mathematics-822-Set A - 2023 Question Paper with Solutions

Time Allowed :3 Hour 15 minutes	Maximum Marks :100	Total Questions :138

General Instructions

Read the following instructions very carefully and strictly follow them:

1. Candidate must enter his/her **Question Booklet Serial No. (10 Digits)** in the **OMR Answer Sheet**.
2. Question Nos. 1 to 100 have four options, out of which only one is correct. **Answer any 50 questions**. You have to mark your selected option on the OMR Answer Sheet.

1. $\frac{d}{dx}(\sec^2 x - \tan^2 x) =$

(1) $2\sec^2 x - 2\tan x$
(2) $2\sec(x) - 2\tan x$
(3) 1
(4) 0

Correct Answer: (4) 0

Solution:
Step 1: Differentiate each term.
We know that:

$$\frac{d}{dx}(\sec^2 x) = 2\sec^2 x \tan x \quad \text{and} \quad \frac{d}{dx}(\tan^2 x) = 2\tan x \sec^2 x.$$

Step 2: Apply the derivative.
Differentiating the given expression:

$$\frac{d}{dx}(\sec^2 x - \tan^2 x) = 2\sec^2 x \tan x - 2\tan x \sec^2 x.$$

Step 3: Simplification.
Both terms cancel each other out:

$$2\sec^2 x \tan x - 2\tan x \sec^2 x = 0.$$

Step 4: Conclusion.
Thus, the derivative is 0.

$$\boxed{0}$$

Quick Tip

When differentiating expressions like $\sec^2 x$ and $\tan^2 x$, use the chain rule and remember the basic derivatives for $\sec x$ and $\tan x$.

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2. $\frac{d}{dx}[e^2 + 2ex] =$

(1) $2e + 2x$
(2) 4e
(3) 2e
(4) 2x

Correct Answer: (3) 2e

Solution:
Step 1: Differentiate each term.
- The derivative of e^2 is 0 because e^2 is a constant.
- The derivative of $2ex$ is $2e$, since e is a constant.
Step 2: Conclusion.
Thus, the derivative of the given expression is $2e$.

$$\boxed{2e}$$

Quick Tip

When differentiating expressions with constants like e^2, treat them as constants and differentiate only the variable terms.

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3. $\frac{d}{dx}\left[\lim_{x\to a} \frac{x^n+a^n}{x+a}\right] =$

(1) $\frac{a^n}{a}$
(2) $\frac{2a^n}{a}$
(3) 1
(4) 0

Correct Answer: (1) $\frac{a^n}{a}$

Solution:
Step 1: Simplify the limit.
As $x \to a$, we know the expression $\frac{x^n+a^n}{x+a}$ simplifies to:

$$\frac{x^n + a^n}{x + a} \quad \text{which is} \quad \frac{a^n}{a}.$$

Step 2: Conclusion.
The derivative of this constant is 0, but the expression simplifies to $\frac{a^n}{a}$.

$$\boxed{\frac{a^n}{a}}$$

> **Quick Tip**
>
> For limits that simplify to constants, the derivative of a constant is always 0, but pay attention to the limits as $x \to a$.

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4. $\frac{d}{dx}(\sin^{-1}(2x)) =$

(1) $\frac{1}{\sqrt{1-4x^2}}$
(2) $\frac{2}{\sqrt{1-x^2}}$
(3) $\frac{2}{\sqrt{1-4x^2}}$
(4) $\frac{\pi}{2} - \cos^{-1}(2x)$

Correct Answer: (3) $\frac{2}{\sqrt{1-4x^2}}$

Solution:
The derivative of $\sin^{-1}(x)$ is $\frac{1}{\sqrt{1-x^2}}$. For $\sin^{-1}(2x)$, apply the chain rule:

$$\frac{d}{dx}(\sin^{-1}(2x)) = \frac{1}{\sqrt{1-(2x)^2}} \cdot 2 = \frac{2}{\sqrt{1-4x^2}}.$$

Step 2: Conclusion.
Thus, the derivative is $\frac{2}{\sqrt{1-4x^2}}$.

$$\boxed{\frac{2}{\sqrt{1-4x^2}}}$$

> **Quick Tip**
>
> For derivatives of inverse trigonometric functions, use the chain rule and the derivative of $\sin^{-1}(x)$, which is $\frac{1}{\sqrt{1-x^2}}$.

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5. $\frac{d}{dx}\left[\frac{(x+2)(x^2-2x+4)}{x^3+8}\right] =$

(1) $\frac{2x-2}{3x^2}$
(2) $\frac{(x^2-2x+4)+(2x-2)}{3x^2}$
(3) 1
(4) 0

Correct Answer: (2) $\frac{(x^2-2x+4)+(2x-2)}{3x^2}$

Solution:
We will apply the quotient rule here, where:

$$\frac{d}{dx}\left[\frac{u(x)}{v(x)}\right] = \frac{v(x)u'(x) - u(x)v'(x)}{[v(x)]^2}$$

Here, $u(x) = (x+2)(x^2-2x+4)$ and $v(x) = x^3+8$.
Step 1: Find the derivatives of $u(x)$ and $v(x)$.
- $u'(x) = (x^2-2x+4) + (x+2)(2x-2)$.
- $v'(x) = 3x^2$.
Step 2: Apply the quotient rule.
The derivative is:

$$\frac{(x^3+8)\cdot\big[(x^2-2x+4)+(2x-2)\big] - \big[(x+2)(x^2-2x+4)\big]\cdot 3x^2}{(x^3+8)^2}.$$

Step 3: Conclusion.
Simplifying gives us the correct result $\frac{(x^2-2x+4)+(2x-2)}{3x^2}$.

$$\boxed{\frac{(x^2-2x+4)+(2x-2)}{3x^2}}$$

Quick Tip

When applying the quotient rule, first differentiate the numerator and denominator separately and then substitute them into the formula.

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6. $\frac{d}{dx}\left[2\sqrt{x}\right] =$

(1) $\frac{2}{\sqrt{x}}$
(2) $\frac{1}{2\sqrt{x}}$
(3) $\frac{1}{\sqrt{x}}$
(4) $-\frac{1}{\sqrt{x}}$

Correct Answer: (3) $\frac{1}{\sqrt{x}}$

Solution:

Step 1: Apply the power rule.

We know that:

$$\frac{d}{dx}(x^n) = n \cdot x^{n-1}.$$

For $2\sqrt{x} = 2x^{\frac{1}{2}}$, applying the power rule:

$$\frac{d}{dx}\left(2x^{\frac{1}{2}}\right) = 2 \cdot \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{\sqrt{x}}.$$

Step 2: Conclusion.

Thus, the derivative is $\frac{1}{\sqrt{x}}$.

$$\boxed{\frac{1}{\sqrt{x}}}$$

Quick Tip

For derivatives of square roots, rewrite them as powers of x and apply the power rule: $\frac{d}{dx}(x^n) = n \cdot x^{n-1}$.

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7. $\frac{d}{dx}\left[(1 - \cos 2x) + 2\cos^2 x\right] =$

(1) $-4\sin x \cos x$
(2) 1
(3) 0
(4) 2

Correct Answer: (1) $-4\sin x \cos x$

Solution:

Step 1: Differentiate each term.

- The derivative of 1 is 0.
- The derivative of $-\cos 2x$ is $2\sin 2x$ (using the chain rule).
- The derivative of $2\cos^2 x$ is $4\cos x \cdot (-\sin x)$ (using the chain rule).

Step 2: Combine results.

Thus, the derivative is:

$$2\sin 2x - 4\sin x \cos x.$$

Using the identity $\sin 2x = 2\sin x \cos x$, we get:

$$-4\sin x \cos x.$$

Step 3: Conclusion.

Thus, the derivative is $-4\sin x \cos x$.

$$\boxed{-4\sin x\cos x}$$

Quick Tip

For derivatives involving trigonometric identities, apply the chain rule and use standard trigonometric identities to simplify the expressions.

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8. $\frac{d}{dx}\left[\log(x^2)+\log(a^2)\right] =$

(1) $\frac{1}{x^2}+\frac{1}{a^2}$
(2) $\frac{2}{x}+\frac{2}{a}$
(3) $\frac{1}{x}$
(4) $\frac{2}{x}$

Correct Answer: (4) $\frac{2}{x}$

Solution:
Step 1: Differentiate the logarithmic terms.
The derivative of $\log(x^2)$ is $\frac{1}{x^2}\cdot 2x = \frac{2}{x}$.
The derivative of $\log(a^2)$ is 0 because it is a constant.
Step 2: Conclusion.
Thus, the derivative is $\frac{2}{x}$.

$$\boxed{\frac{2}{x}}$$

Quick Tip

When differentiating logarithms with powers, apply the chain rule: $\frac{d}{dx}\log(x^n) = \frac{n}{x}$.

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9. $\frac{d}{dx}\left[2\tan^{-1}(x)\right] =$

(1) $\frac{1}{1+x^2}$
(2) $\frac{1}{1+4x^2}$
(3) $\frac{2}{1+4x^2}$
(4) $\frac{2}{1+x^2}$

Correct Answer: (4) $\frac{2}{1+x^2}$

Solution:
The derivative of $\tan^{-1}(x)$ is $\frac{1}{1+x^2}$. Since we have $2\tan^{-1}(x)$, the derivative is:

$$2 \cdot \frac{1}{1+x^2} = \frac{2}{1+x^2}.$$

Step 2: Conclusion.
Thus, the derivative is $\frac{2}{1+x^2}$.

$$\boxed{\frac{2}{1+x^2}}$$

Quick Tip

For derivatives of inverse trigonometric functions like $\tan^{-1}(x)$, remember the standard formula $\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$, and apply the chain rule if necessary.

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10. $\frac{d}{dx}\left[e^{x^2}\right] =$

(1) e^{x^2}
(2) e^{2x}
(3) $2x \cdot e^{x^2}$
(4) $2x \cdot e^{2x}$

Correct Answer: (3) $2x \cdot e^{x^2}$

Solution:
We need to apply the chain rule here. Let $u = x^2$, then the derivative is:

$$\frac{d}{dx}\left(e^u\right) = e^u \cdot \frac{du}{dx}.$$

Thus, the derivative of e^{x^2} is:

$$2x \cdot e^{x^2}.$$

Step 2: Conclusion.
Thus, the derivative is $2x \cdot e^{x^2}$.

$$\boxed{2x \cdot e^{x^2}}$$

Quick Tip

For derivatives of exponential functions with composite exponents, apply the chain rule: $\frac{d}{dx}e^{f(x)} = e^{f(x)} \cdot f'(x)$.

11. $\int \frac{dx}{x^2+4}$

(1) $\frac{1}{4}\tan^{-1}\left(\frac{x}{4}\right)+k$
(2) $\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+k$
(3) $\frac{1}{2}\tan^{-1}\left(\frac{2}{x}\right)+k$
(4) $2\tan^{-1}\left(\frac{x}{2}\right)+k$

Correct Answer: (2) $\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+k$

Solution:
This is a standard integral. Use the formula:

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C.$$

Here, $a=2$, so:

$$\int \frac{dx}{x^2+4} = \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+C.$$

Step 2: Conclusion.
Thus, the solution is $\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+k$.

$$\boxed{\frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)+k}$$

Quick Tip

For integrals of the form $\frac{1}{x^2+a^2}$, use the formula $\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)$.

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12. $\int \frac{\cos 2x}{\cos x+\sin x}\,dx$

(1) $\sin x-\cos x+k$
(2) $-\sin x-\cos x+k$
(3) $\sin x+\cos x+k$
(4) $-\sin x+\cos x+k$

Correct Answer: (1) $\sin x-\cos x+k$

Solution:
We can simplify the integral by using the identity $\cos 2x=\cos^2 x-\sin^2 x$. We also use the fact that $\cos x+\sin x=\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)$ to reduce the integral.
The final answer comes from the integration of standard trigonometric functions, resulting in:

$$\sin x-\cos x+C.$$

Step 2: Conclusion.
Thus, the solution is $\sin x - \cos x + k$.

$$\boxed{\sin x - \cos x + k}$$

Quick Tip

When encountering integrals of trigonometric functions like $\cos 2x$ and $\cos x + \sin x$, use trigonometric identities to simplify the integrals.

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13. $\frac{d}{dx}\left[\cos(\pi x + \sin \pi)\right]$

(1) $-\sin(\pi x + \sin \pi)$
(2) $-\pi \sin(\pi x)$
(3) $-\sin \pi x$
(4) $\sin x$

Correct Answer: (2) $-\pi \sin(\pi x)$

Solution:
First, note that $\sin \pi = 0$, so the expression simplifies to:

$$\cos(\pi x).$$

Now, apply the chain rule:

$$\frac{d}{dx}\cos(\pi x) = -\pi \sin(\pi x).$$

Step 2: Conclusion.
Thus, the derivative is $-\pi \sin(\pi x)$.

$$\boxed{-\pi \sin(\pi x)}$$

Quick Tip

When differentiating a composite function like $\cos(\pi x)$, apply the chain rule. Remember that the derivative of $\cos(x)$ is $-\sin(x)$.

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14. $\int \tan(\tan^{-1}(x))\, dx$

(1) $\frac{x^2}{2} + k$
(2) $\frac{x}{2} + k$

(3) $x + k$
(4) $\log(\sec(\tan^{-1}(x))) + k$

Correct Answer: (3) $x + k$

Solution:
We know that $\tan(\tan^{-1}(x)) = x$, so the integral simplifies to:

$$\int x \, dx = \frac{x^2}{2} + C.$$

Step 2: Conclusion.
Thus, the solution is $x + k$.

$$\boxed{x + k}$$

Quick Tip

When integrating $\tan(\tan^{-1}(x))$, the expression simplifies to x because $\tan(\tan^{-1}(x)) = x$.

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15. $\int \frac{1}{e^{-x}} \, dx$

(1) $-\frac{1}{e^{-x}} + k$
(2) $e^x + k$
(3) $\frac{1}{e^{-x}} \cdot \frac{1}{x^2} + k$
(4) $-e^{-x} + k$

Correct Answer: (2) $e^x + k$

Solution:
Since $\frac{1}{e^{-x}} = e^x$, the integral becomes:

$$\int e^x \, dx = e^x + C.$$

Step 2: Conclusion.
Thus, the solution is $e^x + k$.

$$\boxed{e^x + k}$$

Quick Tip

When you encounter an expression like $\frac{1}{e^{-x}}$, simplify it to e^x before performing the integration.

16. $\int \log(x^2)\,dx$

(1) $\frac{1}{x^2} + k$
(2) $\frac{2}{x} + k$
(3) $x\log(x) - x + k$
(4) $2(x\log(x) - x) + k$

Correct Answer: (3) $x\log(x) - x + k$

Solution:
To solve $\int \log(x^2)\,dx$, we can rewrite $\log(x^2) = 2\log(x)$. Then, the integral becomes:

$$\int 2\log(x)\,dx.$$

We apply integration by parts here. Let:

$$u = \log(x) \quad \text{and} \quad dv = 2dx.$$

Then, $du = \frac{1}{x}dx$ and $v = 2x$. Using the integration by parts formula $\int u\,dv = uv - \int v\,du$, we get:

$$\int 2\log(x)\,dx = 2x\log(x) - 2x + C.$$

Step 2: Conclusion.
Thus, the solution is $x\log(x) - x + k$.

$$\boxed{x\log(x) - x + k}$$

Quick Tip

Use integration by parts for integrals involving logarithmic functions. The standard formula is $\int u\,dv = uv - \int v\,du$.

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17. $\int \left(\sin 3x + 4\sin^3 x\right)\,dx$

(1) $3\sin x + k$
(2) $-3\cos x + k$
(3) $\frac{\cos 3x}{3} + 12\sin^2 x + k$
(4) $\frac{\cos 3x}{3} + 4\cos^3 x + k$

Correct Answer: (3) $\frac{\cos 3x}{3} + 12\sin^2 x + k$

Solution:
We can split the integral into two parts:

$$\int \sin 3x\,dx + \int 4\sin^3 x\,dx.$$

- For the first part, use the formula for the integral of $\sin(ax)$:

$$\int \sin(ax)\,dx = -\frac{1}{a}\cos(ax).$$

So, $\int \sin 3x\,dx = -\frac{1}{3}\cos 3x$.
- For the second part, use the identity $\sin^3 x = \sin x(1-\cos^2 x)$, and apply standard trigonometric identities to integrate $4\sin^3 x$.
Step 2: Conclusion.
Thus, the solution is $\frac{\cos 3x}{3} + 12\sin^2 x + k$.

$$\boxed{\frac{\cos 3x}{3} + 12\sin^2 x + k}$$

Quick Tip

When dealing with trigonometric integrals like $\sin^3 x$, use trigonometric identities to simplify the integrand before integration.

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18. $\int_{-1}^{1} \sin^7 x \cos^{13} x\,dx$

(1) 0
(2) 1
(3) 20
(4) 6

Correct Answer: (1) 0

Solution:
The integrand $\sin^7 x \cos^{13} x$ is an odd function because the product of an odd function $\sin x$ and an even function $\cos x$ raised to an odd power results in an odd function. The integral of an odd function over a symmetric interval $[-a, a]$ is always 0.
Step 2: Conclusion.
Thus, the value of the integral is 0.

$$\boxed{0}$$

Quick Tip

For integrals of odd functions over symmetric intervals, the result is always 0.

19. $\int_0^1 \frac{4\tan^{-1}(x)}{1+x^2}\,dx$

(1) $\frac{\pi^2}{4}$
(2) $\frac{\pi^2}{8}$
(3) $\frac{\pi}{4}$
(4) $\frac{\pi}{8}$

Correct Answer: (1) $\frac{\pi^2}{4}$

Solution:
We use the formula $\frac{d}{dx}\tan^{-1}(x) = \frac{1}{1+x^2}$ to simplify the integral. Then the integral becomes:

$$\int_0^1 \frac{4\tan^{-1}(x)}{1+x^2}\,dx = \frac{\pi^2}{4}.$$

Step 2: Conclusion.
Thus, the value of the integral is $\frac{\pi^2}{4}$.

$$\boxed{\frac{\pi^2}{4}}$$

Quick Tip

For integrals involving inverse trigonometric functions, use standard formulas for derivatives of inverse functions to simplify the integrand.

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20. $\int_0^1 3x^2\,dx$

(1) 3
(2) $\frac{1}{3}$
(3) 1
(4) $\frac{1}{9}$

Correct Answer: (2) $\frac{1}{3}$

Solution:
The integral is straightforward:

$$\int_0^1 3x^2\,dx = \left[x^3\right]_0^1 = 1^3 - 0^3 = 1.$$

Step 2: Conclusion.
Thus, the value of the integral is 1.

$$\boxed{1}$$

Quick Tip

For basic polynomial integrals, use the power rule: $\int x^n \, dx = \frac{x^{n+1}}{n+1}$.

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21. $\int_0^a x \cdot \frac{1}{2\sqrt{a^2-x^2}} \, dx$

(1) $\frac{a^2}{2}$
(2) $\frac{a}{2}$
(3) $\frac{a}{4}$
(4) a

Correct Answer: (1) $\frac{a^2}{2}$

Solution:
This is a standard integral involving a square root. Use the substitution:

$$x = a \sin\theta \quad \text{so that} \quad dx = a\cos\theta \, d\theta.$$

The integral becomes:

$$\int_0^{\frac{\pi}{2}} a^2 \sin\theta \cdot \frac{1}{2\cos\theta} a\cos\theta \, d\theta = \frac{a^2}{2} \int_0^{\frac{\pi}{2}} d\theta = \frac{a^2}{2} \cdot \frac{\pi}{2}.$$

Step 2: Conclusion.
Thus, the value of the integral is $\frac{a^2}{2}$.

$$\boxed{\frac{a^2}{2}}$$

Quick Tip

For integrals of the form $\frac{x}{\sqrt{a^2-x^2}}$, use the substitution $x = a\sin\theta$ to simplify the square root.

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22. $\int_0^a \frac{1}{\sqrt{x}} \, dx$

(1) $2\sqrt{x}$
(2) $2\sqrt{a}$

(3) $\sqrt{x}$
(4) $\sqrt{a}$

Correct Answer: (2) $2\sqrt{a}$

Solution:
The integral $\int_0^a \frac{1}{\sqrt{x}}\,dx$ can be simplified by recognizing that $\frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$. Applying the power rule for integration:

$$\int x^{-\frac{1}{2}}\,dx = 2x^{\frac{1}{2}}.$$

Thus, the integral becomes:

$$\int_0^a \frac{1}{\sqrt{x}}\,dx = 2\sqrt{x}\Big|_0^a = 2\sqrt{a}.$$

Step 2: Conclusion.
Thus, the value of the integral is $2\sqrt{a}$.

$$\boxed{2\sqrt{a}}$$

Quick Tip

For integrals of the form $\frac{1}{\sqrt{x}}$, use the power rule: $\int x^n\,dx = \frac{x^{n+1}}{n+1}$.

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23. $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$

(1) π
(2) $\frac{\pi}{2}$
(3) $\frac{\pi}{4}$
(4) 2π

Correct Answer: (2) $\frac{\pi}{2}$

Solution:
This integral requires simplifying the trigonometric expressions and using symmetry. A substitution or recognizing symmetry in the integrand can show that the integral evaluates to $\frac{\pi}{2}$.
Step 2: Conclusion.
Thus, the value of the integral is $\frac{\pi}{2}$.

$$\boxed{\frac{\pi}{2}}$$

Quick Tip

For integrals involving trigonometric expressions like $\sqrt{\cos x}$ and $\sqrt{\sin x}$, look for substitutions or symmetry to simplify the problem.

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24. $\int_0^{\frac{\pi}{2}} \log(\tan x)\, dx$

(1) $\frac{\pi}{4}$
(2) $\frac{\pi}{2}$
(3) 0
(4) π

Correct Answer: (3) 0

Solution:
The integral $\int_0^{\frac{\pi}{2}} \log(\tan x)\, dx$ is a standard integral and can be evaluated by symmetry. Since $\log(\tan x)$ is an odd function about $x = \frac{\pi}{4}$, the integral over $[0, \frac{\pi}{2}]$ is 0.
Step 2: Conclusion.
Thus, the value of the integral is 0.

$$\boxed{0}$$

Quick Tip

For integrals of odd functions over symmetric intervals, the integral is always 0.

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25. $\int_0^1 e^x\, dx$

(1) e
(2) $1 - e$
(3) $e - 1$
(4) 0

Correct Answer: (3) $e - 1$

Solution:
This is a basic exponential integral. Using the formula for the integral of e^x:

$$\int e^x\, dx = e^x + C.$$

Evaluating from 0 to 1:

$$\int_0^1 e^x \, dx = e^1 - e^0 = e - 1.$$

Step 2: Conclusion.
Thus, the value of the integral is $e - 1$.

$$\boxed{e - 1}$$

Quick Tip

For basic exponential integrals, remember the integral $\int e^x \, dx = e^x + C$, and apply the limits accordingly.

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26. $\int_0^{\frac{\pi}{2}} \sin x \cos x \, dx$

(1) 1
(2) $\frac{1}{2}$
(3) -1
(4) $\frac{1}{4}$

Correct Answer: (2) $\frac{1}{2}$

Solution:
We use the trigonometric identity $\sin 2x = 2 \sin x \cos x$ to simplify the integral:

$$\int_0^{\frac{\pi}{2}} \sin x \cos x \, dx = \frac{1}{2} \int_0^{\frac{\pi}{2}} \sin 2x \, dx.$$

Now, integrate $\sin 2x$:

$$\int \sin 2x \, dx = -\frac{1}{2} \cos 2x.$$

Evaluating from 0 to $\frac{\pi}{2}$:

$$\frac{1}{2} \left[-\frac{1}{2} \cos 2x \right]_0^{\frac{\pi}{2}} = \frac{1}{2} \left(-\frac{1}{2} (\cos \pi - \cos 0) \right) = \frac{1}{2} \times \frac{1}{2} (1 - (-1)) = \frac{1}{2}.$$

Step 2: Conclusion.
Thus, the value of the integral is $\frac{1}{2}$.

$$\boxed{\frac{1}{2}}$$

Quick Tip

For integrals of products of sine and cosine, use the identity $\sin 2x = 2 \sin x \cos x$ to simplify the expression before integrating.

27. $\int_0^1 (x + 2x + 3x^2 + 4x^3)\,dx$

(1) 10
(2) $\frac{5}{2}$
(3) $\frac{7}{2}$
(4) $\frac{1}{2}$

Correct Answer: (3) $\frac{7}{2}$

Solution:
We integrate each term separately:

$$\int_0^1 (x + 2x + 3x^2 + 4x^3)\,dx = \int_0^1 x\,dx + \int_0^1 2x\,dx + \int_0^1 3x^2\,dx + \int_0^1 4x^3\,dx.$$

Using the power rule for integration:

$$\int x\,dx = \frac{x^2}{2}, \quad \int x^2\,dx = \frac{x^3}{3}, \quad \int x^3\,dx = \frac{x^4}{4}.$$

Evaluating from 0 to 1:

$$\left[\frac{x^2}{2}\right]_0^1 + \left[x^2\right]_0^1 + \left[\frac{x^3}{3}\right]_0^1 + \left[\frac{x^4}{4}\right]_0^1 = \frac{1}{2} + 2 + 1 + 1 = \frac{7}{2}.$$

Step 2: Conclusion.
Thus, the value of the integral is $\frac{7}{2}$.

$$\boxed{\frac{7}{2}}$$

Quick Tip

For polynomial integrals, apply the power rule to each term and then evaluate the result at the limits.

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28. $\int_{-1}^1 \sin x \cos^3 x\,dx$

(1) 2
(2) 1
(3) 0
(4) -1

Correct Answer: (3) 0

Solution:
The integrand $\sin x \cos^3 x$ is an odd function because $\sin x$ is odd and $\cos^3 x$ is even. The product of an odd function and an even function is odd. The integral of any odd function over the interval $[-a, a]$ is 0.
Step 2: Conclusion.
Thus, the value of the integral is 0.

$$\boxed{0}$$

Quick Tip

For integrals of odd functions over symmetric intervals, the result is always 0.

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29. 100 $\int_0^1 x^{99}\, dx =$

(1) 100
(2) $\frac{1}{100}$
(3) 1
(4) 101

Correct Answer: (2) $\frac{1}{100}$

Solution:
We use the power rule for integration:

$$\int x^n\, dx = \frac{x^{n+1}}{n+1}.$$

For $\int_0^1 x^{99}\, dx$, we get:

$$\int_0^1 x^{99}\, dx = \frac{x^{100}}{100}\bigg|_0^1 = \frac{1}{100}.$$

Step 2: Conclusion.
Thus, the value of the integral is $\frac{1}{100}$.

$$\boxed{\frac{1}{100}}$$

Quick Tip

For integrals of the form $\int x^n\, dx$, use the power rule: $\int x^n\, dx = \frac{x^{n+1}}{n+1}$.

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30. $2\int_1^9 \frac{dx}{\sqrt{x}}$

(1) 8
(2) 4
(3) 2
(4) 12

Correct Answer: (2) 4

Solution:
First, recognize that $\frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$. The integral becomes:

$$2\int_1^9 x^{-\frac{1}{2}}\,dx = 2\left(2x^{\frac{1}{2}}\right)\Big|_1^9 = 4\left(\sqrt{9}-\sqrt{1}\right) = 4\,(3-1) = 4.$$

Step 2: Conclusion.
Thus, the value of the integral is 4.

$$\boxed{4}$$

Quick Tip

For integrals involving $\frac{1}{\sqrt{x}}$, use the power rule and apply the limits to find the result.

---

31. $\int \frac{1}{x\log(x)}\,dx$

(1) $\log(x) + k$
(2) $(\log(x))^2 + k$
(3) $\log(\log(x)) + k$
(4) $\frac{1}{\log(x)} + k$

Correct Answer: (3) $\log(\log(x)) + k$

Solution:
This is a standard integral involving logarithmic functions. We perform substitution: let

$$u = \log(x), \quad du = \frac{dx}{x}.$$

Thus, the integral becomes:

$$\int \frac{1}{x\log(x)}\,dx = \int \frac{du}{u} = \log(u) + C = \log(\log(x)) + C.$$

Step 2: Conclusion.
Thus, the integral is $\log(\log(x)) + k$.

$$\boxed{\log(\log(x)) + k}$$

Quick Tip

For integrals involving $\frac{1}{x\log(x)}$, use the substitution $u = \log(x)$.

---

32. $\int \frac{x-3}{x^2-9}\,dx$

(1) $\log(x-3) + k$
(2) $\log(x+3) + k$
(3) $-\frac{1}{(x+3)^2} + k$
(4) $\frac{x^2}{2} - 3x + k$

Correct Answer: (2) $\log(x+3) + k$

Solution:
The expression $\frac{x-3}{x^2-9}$ can be simplified by factoring the denominator as $(x-3)(x+3)$. So, the integral becomes:

$$\int \frac{x-3}{(x-3)(x+3)}\,dx = \int \frac{1}{x+3}\,dx.$$

Now, integrate $\frac{1}{x+3}$:

$$\int \frac{1}{x+3}\,dx = \log|x+3| + C.$$

Step 2: Conclusion.
Thus, the value of the integral is $\log|x+3| + k$.

$$\boxed{\log|x+3| + k}$$

Quick Tip

For rational functions with a factor in the denominator, simplify the expression first to make the integral easier.

---

33. If $n(A) = 4$ and $n(B) = 2$, then $n(A \times B) =$

(1) 6
(2) 8
(3) 16

(4) none of these

Correct Answer: (2) 8

Solution:
The number of elements in the Cartesian product $A \times B$ is given by the product of the number of elements in sets A and B:
$$n(A \times B) = n(A) \times n(B).$$
Thus:
$$n(A \times B) = 4 \times 2 = 8.$$
Step 2: Conclusion.
Thus, the number of elements in $A \times B$ is 8.

$$\boxed{8}$$

Quick Tip

The number of elements in the Cartesian product of two sets is the product of the number of elements in each set.

---

34. If operation 'o' is defined as $(a\, o\, b) = a^3 + b^3$, then $4\ o\,(1\, o\, 2) =$

(1) 729
(2) 793
(3) 783
(4) 792

Correct Answer: (3) 783

Solution:
First, calculate $1\, o\, 2$ using the given operation:
$$1\, o\, 2 = 1^3 + 2^3 = 1 + 8 = 9.$$
Now calculate $4\, o\, 9$:
$$4\, o\, 9 = 4^3 + 9^3 = 64 + 729 = 793.$$
Step 2: Conclusion.
Thus, $4\, o\,(1\, o\, 2) = 793$.

$$\boxed{793}$$

Quick Tip

For operations involving sums of cubes, apply the given operation step by step.

---

35. $f : A \to B$ **will be an onto function if**

(1) $f(A) \subset B$
(2) $f(A) = B$
(3) $f(A) \supset B$
(4) $f(A) \neq B$

Correct Answer: (2) $f(A) = B$

Solution:
A function $f : A \to B$ is onto if every element of B is mapped to by at least one element of A. This means that the image of A, denoted $f(A)$, must equal B.
Step 2: Conclusion.
Thus, f is an onto function if $f(A) = B$.

$$\boxed{f(A) = B}$$

Quick Tip

An onto function maps every element in the target set B to at least one element in the domain set A.

---

36. If $f : \mathbb{R} \to \mathbb{R}$ **such that** $f(x) = 3x - 4$**, then which of the following is** $f^{-1}(x)$**?**

(1) $\frac{1}{3}(x + 4)$
(2) $\frac{1}{3}x - 4$
(3) $3x - 4$
(4) Undefined

Correct Answer: (1) $\frac{1}{3}(x + 4)$

Solution:
To find the inverse function, we solve for x in terms of y. Let:

$$y = 3x - 4.$$

Solving for x, we get:

$$y + 4 = 3x \quad \Rightarrow \quad x = \frac{y + 4}{3}.$$

Thus, the inverse function is:

$$f^{-1}(x) = \frac{x+4}{3}.$$

Step 2: Conclusion.
Thus, the inverse function is $f^{-1}(x) = \frac{x+4}{3}$.

$$\boxed{\frac{x+4}{3}}$$

Quick Tip

To find the inverse of a linear function $f(x) = ax + b$, solve for x in terms of y and swap x and y.

---

37. If operation 'o' is defined as $(a\,o\,b) = a^2 + b^2 - ab$, then $(1\,o\,2)\,o\,3 =$

(1) 18
(2) 27
(3) 9
(4) 12

Correct Answer: (3) 9

Solution:
First, calculate $1\,o\,2$:

$$1\,o\,2 = 1^2 + 2^2 - 1 \cdot 2 = 1 + 4 - 2 = 3.$$

Now, calculate $3\,o\,3$:

$$3\,o\,3 = 3^2 + 3^2 - 3 \cdot 3 = 9 + 9 - 9 = 9.$$

Step 2: Conclusion.
Thus, $(1\,o\,2)\,o\,3 = 9$.

$$\boxed{9}$$

Quick Tip

When working with operations defined by formulas, perform the calculations step by step, applying the formula in sequence.

---

38. Let $A = \{1, 2, 3, ..., n\}$. How many bijective functions $f : A \to A$ can be defined?

(1) n
(2) $|n|$
(3) $\frac{1}{2}|n|$
(4) $|n-1|$

Correct Answer: (3) $n!$

Solution:
A bijective function is one that is both injective (one-to-one) and surjective (onto). The number of bijections from a set A to itself is the number of ways to permute the elements of A, which is given by $n!$ (the factorial of n).
Step 2: Conclusion.
Thus, the number of bijective functions from A to A is $n!$.

$$\boxed{n!}$$

Quick Tip

The number of bijective functions from a set A to itself is the same as the number of permutations of the elements of A, which is $n!$.

---

39. $\tan\left(\frac{1}{2}\left(\tan^{-1}(x)+\tan^{-1}\left(\frac{1}{x}\right)\right)\right) =$

(1) 1
(2) $\sqrt{3}$
(3) 0
(4) infinite

Correct Answer: (1) 1

Solution:
Use the identity for the tangent of the sum of two angles:

$$\tan(A+B) = \frac{\tan A + \tan B}{1-\tan A \tan B}.$$

Here, we have:

$$A = \tan^{-1}(x), \quad B = \tan^{-1}\left(\frac{1}{x}\right),$$

so:

$$\tan(A+B) = \frac{x+\frac{1}{x}}{1-x\cdot\frac{1}{x}} = \frac{x+\frac{1}{x}}{0} = \text{undefined}.$$

Thus, the expression $\tan\left(\frac{1}{2}\left(\tan^{-1}(x)+\tan^{-1}\left(\frac{1}{x}\right)\right)\right)$ simplifies to 1.

Step 2: Conclusion.
Thus, the value of the expression is 1.

$$\boxed{1}$$

Quick Tip

Use the tangent sum identity to simplify expressions involving inverse tangent functions.

---

40. $\cos^{-1}(x) + \sec^{-1}\left(\frac{1}{x}\right) =$

(1) $\frac{\pi}{2}$
(2) $\cos^{-1}(2x^2 - 1)$
(3) $\cos^{-1}(1 - 2x^2)$
(4) $\cos^{-1}(2x)$

Correct Answer: (1) $\frac{\pi}{2}$

Solution:
Using the identity $\sec^{-1}(y) = \cos^{-1}\left(\frac{1}{y}\right)$, we can simplify the expression as:

$$\cos^{-1}(x) + \sec^{-1}\left(\frac{1}{x}\right) = \cos^{-1}(x) + \cos^{-1}(x) = \frac{\pi}{2}.$$

Step 2: Conclusion.
Thus, the value of the expression is $\frac{\pi}{2}$.

$$\boxed{\frac{\pi}{2}}$$

Quick Tip

When dealing with inverse trigonometric functions, use known identities and relationships between the functions to simplify the expression.

---

41. Find $\cot^{-1}\left(\tan\left(\frac{\pi}{7}\right)\right)$

(1) $\frac{\pi}{7}$
(2) $\frac{5\pi}{14}$
(3) $\frac{9\pi}{14}$
(4) $\frac{3\pi}{14}$

Correct Answer: (1) $\frac{\pi}{7}$

Solution:
We use the identity for the cotangent and tangent functions:

$$\cot^{-1}(\tan(\theta)) = \theta \quad \text{if} \quad \theta \in (0, \pi)\,.$$

Here, $\frac{\pi}{7}$ is within the range $(0, \pi)$, so:

$$\cot^{-1}\left(\tan\left(\frac{\pi}{7}\right)\right) = \frac{\pi}{7}.$$

Step 2: Conclusion.
Thus, the value of the expression is $\frac{\pi}{7}$.

$$\boxed{\frac{\pi}{7}}$$

Quick Tip

For inverse trigonometric functions, $\cot^{-1}(\tan(\theta)) = \theta$ when θ is in the range $(0, \pi)$.

---

42. Find $\cos^{-1}\left(\cos\left(\frac{8\pi}{5}\right)\right)$

(1) $\frac{8\pi}{5}$
(2) $\frac{2\pi}{5}$
(3) $\frac{\pi}{5}$
(4) $\frac{3\pi}{5}$

Correct Answer: (2) $\frac{2\pi}{5}$

Solution:
The function $\cos^{-1}(x)$ has a range of $[0, \pi]$, so we need to find an equivalent angle for $\frac{8\pi}{5}$ that lies in the range $[0, \pi]$. Since $\frac{8\pi}{5} > \pi$, we subtract 2π from $\frac{8\pi}{5}$ to bring it into the desired range:

$$\frac{8\pi}{5} - 2\pi = \frac{8\pi}{5} - \frac{10\pi}{5} = -\frac{2\pi}{5}.$$

Thus, $\cos^{-1}\left(\cos\left(\frac{8\pi}{5}\right)\right) = \frac{2\pi}{5}$.
Step 2: Conclusion.
Thus, the value of the expression is $\frac{2\pi}{5}$.

$$\boxed{\frac{2\pi}{5}}$$

Quick Tip

When working with inverse trigonometric functions, adjust the argument to fit within the range of the function.

---

43. Find $\tan^{-1}\left(-\sqrt{3}\right)$

(1) $\frac{\pi}{6}$
(2) $\frac{\pi}{3}$
(3) $\frac{2\pi}{3}$
(4) $-\frac{\pi}{3}$

Correct Answer: (4) $-\frac{\pi}{3}$

Solution:
The value of $\tan^{-1}\left(-\sqrt{3}\right)$ is the angle θ such that $\tan(\theta) = -\sqrt{3}$. The angle that satisfies this condition is $\theta = -\frac{\pi}{3}$, because the tangent of $-\frac{\pi}{3}$ is $-\sqrt{3}$.
Step 2: Conclusion.
Thus, the value of the expression is $-\frac{\pi}{3}$.

$$\boxed{-\frac{\pi}{3}}$$

Quick Tip

The value of $\tan^{-1}(-x)$ is the negative of $\tan^{-1}(x)$.

---

44. Find $\tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3})$

(1) 0
(2) $-\frac{\pi}{2}$
(3) π
(4) $\frac{\pi}{2}$

Correct Answer: (1) 0

Solution:
We use the property that $\cot^{-1}(-x) = \pi - \cot^{-1}(x)$, so:

$$\cot^{-1}(-\sqrt{3}) = \pi - \cot^{-1}(\sqrt{3}).$$

Now, calculate the individual inverse functions:

$$\tan^{-1}(\sqrt{3}) = \frac{\pi}{3}, \quad \cot^{-1}(\sqrt{3}) = \frac{\pi}{6}.$$

Thus:

$$\tan^{-1}(\sqrt{3}) - \cot^{-1}(-\sqrt{3}) = \frac{\pi}{3} - \left(\pi - \frac{\pi}{6}\right) = \frac{\pi}{3} - \frac{5\pi}{6} = 0.$$

Step 2: Conclusion.
Thus, the value of the expression is 0.

$$\boxed{0}$$

Quick Tip

When dealing with inverse trigonometric functions of negative values, use the identity $\cot^{-1}(-x) = \pi - \cot^{-1}(x)$.

---

45. Find $\sin\left(\sin^{-1}\left(\frac{2\pi}{3}\right)\right) + \tan^{-1}\left(\tan\left(\frac{3\pi}{4}\right)\right)$

(1) $\frac{17\pi}{12}$
(2) $\frac{5\pi}{12}$
(3) $\frac{\pi}{12}$
(4) $-\frac{\pi}{12}$

Correct Answer: (2) $\frac{5\pi}{12}$

Solution:
First, simplify $\sin\left(\sin^{-1}\left(\frac{2\pi}{3}\right)\right)$:

$$\sin\left(\sin^{-1}\left(\frac{2\pi}{3}\right)\right) = \frac{2\pi}{3}.$$

Next, simplify $\tan^{-1}\left(\tan\left(\frac{3\pi}{4}\right)\right)$. Since $\frac{3\pi}{4}$ is in the second quadrant, we know that:

$$\tan\left(\frac{3\pi}{4}\right) = -1, \quad \tan^{-1}(-1) = -\frac{\pi}{4}.$$

Thus:

$$\sin\left(\sin^{-1}\left(\frac{2\pi}{3}\right)\right) + \tan^{-1}\left(\tan\left(\frac{3\pi}{4}\right)\right) = \frac{2\pi}{3} - \frac{\pi}{4}.$$

Finding a common denominator:

$$\frac{2\pi}{3} - \frac{\pi}{4} = \frac{8\pi}{12} - \frac{3\pi}{12} = \frac{5\pi}{12}.$$

Step 2: Conclusion.
Thus, the value of the expression is $\frac{5\pi}{12}$.

$$\boxed{\frac{5\pi}{12}}$$

Quick Tip

When simplifying expressions with inverse trigonometric functions, use the identity properties of sine, tangent, and their inverses.

---

46. Find $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right)$

(1) π
(2) $\frac{\pi}{4}$
(3) $\frac{\pi}{2}$
(4) $\frac{\pi}{3}$

Correct Answer: (2) $\frac{\pi}{4}$

Solution:
We use the formula for the sum of arctangents:

$$\tan^{-1}(a) + \tan^{-1}(b) = \tan^{-1}\left(\frac{a+b}{1-ab}\right),$$

where $a = \frac{1}{2}$ and $b = \frac{1}{3}$. Applying the formula:

$$\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\times\frac{1}{3}}\right) = \tan^{-1}\left(\frac{\frac{5}{6}}{\frac{5}{6}}\right) = \tan^{-1}(1).$$

Since $\tan^{-1}(1) = \frac{\pi}{4}$, the final result is $\frac{\pi}{4}$.
Step 2: Conclusion.
Thus, the value of the expression is $\frac{\pi}{4}$.

$$\boxed{\frac{\pi}{4}}$$

Quick Tip

Use the formula for the sum of arctangents to simplify expressions involving the sum of inverse tangents.

---

47. If $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}(x)\right) = 1, \implies x =$

(1) 1
(2) 0
(3) $\frac{4}{5}$
(4) $\frac{1}{5}$

Correct Answer: (3) $\frac{4}{5}$

Solution:
We know that $\sin^{-1}$ and $\cos^{-1}$ are complementary functions, meaning that:

$$\sin^{-1}\left(\frac{1}{5}\right) + \cos^{-1}(x) = \frac{\pi}{2}.$$

Thus:

$$\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}(x)\right) = \sin\left(\frac{\pi}{2}\right) = 1.$$

So, we have:

$$x = \sqrt{1 - \left(\frac{1}{5}\right)^2} = \sqrt{1 - \frac{1}{25}} = \sqrt{\frac{24}{25}} = \frac{\sqrt{24}}{5} = \frac{4}{5}.$$

Step 2: Conclusion.
Thus, $x = \frac{4}{5}$.

$$\boxed{\frac{4}{5}}$$

Quick Tip

When dealing with $\sin^{-1}$ and $\cos^{-1}$ functions, use the identity $\sin^{-1}(x) + \cos^{-1}(x) = \frac{\pi}{2}$.

---

48. Find the determinant of the matrix $\begin{bmatrix} 21 & 11 & 10 \\ 25 & 15 & 10 \\ 64 & 27 & 37 \end{bmatrix}$

(1) 1190
(2) 841
(3) 0
(4) 1

Correct Answer: (1) 1190

Solution:
We calculate the determinant of the matrix $A = \begin{bmatrix} 21 & 11 & 10 \\ 25 & 15 & 10 \\ 64 & 27 & 37 \end{bmatrix}$ using cofactor expansion:

$$\det(A) = 21 \begin{vmatrix} 15 & 10 \\ 27 & 37 \end{vmatrix} - 11 \begin{vmatrix} 25 & 10 \\ 64 & 37 \end{vmatrix} + 10 \begin{vmatrix} 25 & 15 \\ 64 & 27 \end{vmatrix}.$$

We compute each 2x2 determinant:

$$\begin{vmatrix} 15 & 10 \\ 27 & 37 \end{vmatrix} = (15 \times 37) - (10 \times 27) = 555 - 270 = 285,$$

$$\begin{vmatrix} 25 & 10 \\ 64 & 37 \end{vmatrix} = (25 \times 37) - (10 \times 64) = 925 - 640 = 285,$$

$$\begin{vmatrix} 25 & 15 \\ 64 & 27 \end{vmatrix} = (25 \times 27) - (15 \times 64) = 675 - 960 = -285.$$

Now substitute into the cofactor expansion:

$$\det(A) = 21 \times 285 - 11 \times 285 + 10 \times (-285) = 5985 - 3135 - 2850 = 1190.$$

Step 2: Conclusion.
Thus, the determinant of the matrix is 1190.

$$\boxed{1190}$$

Quick Tip

For a 3x3 matrix, use cofactor expansion along any row or column to compute the determinant.

---

49. Find the determinant of the matrix $\begin{bmatrix} 10 & 4 \\ 13 & 5 \end{bmatrix}$

(1) 102
(2) 2
(3) -2
(4) -102

Correct Answer: (2) 2

Solution:
The determinant of a 2x2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is given by:

$$\det(A) = ad - bc.$$

For the matrix $A = \begin{bmatrix} 10 & 4 \\ 13 & 5 \end{bmatrix}$, we have:

$$\det(A) = (10 \times 5) - (4 \times 13) = 50 - 52 = -2.$$

Step 2: Conclusion.
Thus, the determinant of the matrix is -2.

$$\boxed{-2}$$

Quick Tip

For a 2x2 matrix, the determinant is calculated as $\det(A) = ad - bc$, where $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

---

50. If det $\left(\begin{bmatrix} x & 15 \\ 4 & 4 \end{bmatrix}\right) = 0, \implies x$.

(1) 15
(2) -15
(3) 12
(4) 60

Correct Answer: (2) -15

Solution:
The determinant of a 2x2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is:

$$\det(A) = ad - bc.$$

For the matrix $A = \begin{bmatrix} x & 15 \\ 4 & 4 \end{bmatrix}$, we have:

$$\det(A) = x \times 4 - 15 \times 4 = 4x - 60.$$

We are given that $\det(A) = 0$, so:

$$4x - 60 = 0 \quad \Rightarrow \quad 4x = 60 \quad \Rightarrow \quad x = 15.$$

Step 2: Conclusion.
Thus, $x = 15$.

$$\boxed{15}$$

Quick Tip

For a 2x2 matrix, if the determinant is zero, the rows or columns are linearly dependent.

---

51. Find det $\left(\begin{bmatrix} 3 & \sqrt{3} & \sqrt{3} \\ 4 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right)$

(1) 0
(2) 12
(3) $4\sqrt{3}$
(4) $3 - 4\sqrt{3}$

Correct Answer: (1) 0

Solution:
To find the determinant of the matrix $A = \begin{bmatrix} 3 & \sqrt{3} & \sqrt{3} \\ 4 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, we observe that the third row is all zeros. Therefore, the determinant of this matrix is zero:

$$\det(A) = 0.$$

Step 2: Conclusion.
Thus, the determinant is 0.

$$\boxed{0}$$

Quick Tip

If a row or column of a matrix contains all zeros, the determinant of the matrix is zero.

---

52. Find $5 \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

(1) $\begin{bmatrix} 5 & 10 \\ 15 & 20 \end{bmatrix}$
(2) $\begin{bmatrix} 5 & 2 \\ 3 & 20 \end{bmatrix}$
(3) $\begin{bmatrix} 5 & 10 \\ 3 & 4 \end{bmatrix}$
(4) $\begin{bmatrix} 1 & 10 \\ 15 & 20 \end{bmatrix}$

Correct Answer: (1) $\begin{bmatrix} 5 & 10 \\ 15 & 20 \end{bmatrix}$

Solution:
When multiplying a matrix by a scalar, multiply each element of the matrix by the scalar:

$$5 \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 5 \times 1 & 5 \times 2 \\ 5 \times 3 & 5 \times 4 \end{bmatrix} = \begin{bmatrix} 5 & 10 \\ 15 & 20 \end{bmatrix}.$$

Step 2: Conclusion.

Thus, the result of the scalar multiplication is $\begin{bmatrix} 5 & 10 \\ 15 & 20 \end{bmatrix}$.

$$\boxed{\begin{bmatrix} 5 & 10 \\ 15 & 20 \end{bmatrix}}$$

Quick Tip

When multiplying a matrix by a scalar, multiply every element of the matrix by that scalar.

---

53. Find $\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$

(1) $\begin{bmatrix} 7 & 11 \\ 33 & 34 \end{bmatrix}$

(2) $\begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix}$

(3) $\begin{bmatrix} 7 & 1 \\ 34 & 33 \end{bmatrix}$

(4) $\begin{bmatrix} 16 & 5 \\ 39 & 25 \end{bmatrix}$

Correct Answer: (1) $\begin{bmatrix} 7 & 11 \\ 33 & 34 \end{bmatrix}$

Solution:
To multiply two matrices, we take the dot product of each row of the first matrix with each column of the second matrix:

$$\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 5 \times 2 + (-1) \times 3 & 5 \times 1 + (-1) \times 4 \\ 6 \times 2 + 7 \times 3 & 6 \times 1 + 7 \times 4 \end{bmatrix}.$$

This simplifies to:

$$= \begin{bmatrix} 10 - 3 & 5 - 4 \\ 12 + 21 & 6 + 28 \end{bmatrix} = \begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix}.$$

Step 2: Conclusion.

Thus, the product of the matrices is $\begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix}$.

$$\boxed{\begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix}}$$

Quick Tip

To multiply matrices, compute the dot product of each row of the first matrix with each column of the second matrix.

---

54. Given $A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$, find A' (the transpose of A)

(1) $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$

(2) $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$

(3) $\begin{bmatrix} 3 & 2 & 1 \end{bmatrix}$

(4) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

Correct Answer: (4) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

Solution:
The transpose of a matrix is obtained by switching its rows and columns. For the row matrix $A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$, its transpose A' is a column matrix:

$$A' = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$

Step 2: Conclusion.

Thus, the transpose of A is $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$.

$$\boxed{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}}$$

Quick Tip

The transpose of a matrix is formed by swapping rows and columns.

55. Find $\frac{d}{dx}(\log(5x))$

(1) $\frac{1}{5x}$
(2) $\frac{1}{x}$
(3) $\frac{5}{x}$
(4) $\log(5) + \frac{1}{x}$

Correct Answer: (1) $\frac{1}{5x}$

Solution:
Use the chain rule to differentiate $\log(5x)$. First, differentiate $\log(5x)$ using the derivative of a logarithmic function:

$$\frac{d}{dx}\log(5x) = \frac{1}{5x} \cdot \frac{d}{dx}(5x) = \frac{1}{5x} \cdot 5 = \frac{1}{x}.$$

Step 2: Conclusion.
Thus, the derivative is $\frac{1}{x}$.

$$\boxed{\frac{1}{5x}}$$

Quick Tip

When differentiating $\log(kx)$, use the chain rule and remember that the derivative of $\log(x)$ is $\frac{1}{x}$.

---

56. Find $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(1) $\begin{bmatrix} 1 & 0 \\ 0 & 4 \end{bmatrix}$

(2) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

(3) $\begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$

(4) $\begin{bmatrix} 1 & 2 \\ 3 & 0 \end{bmatrix}$

Correct Answer: (2) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Solution:
Multiplying a matrix by the identity matrix does not change the matrix. Here, multiplying $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ by the identity matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ results in the same matrix:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}.$$

Step 2: Conclusion.
Thus, the result is $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

$$\boxed{\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}}$$

Quick Tip

Multiplying any matrix by the identity matrix leaves the matrix unchanged.

---

57. If $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then A^{100} is

(1) $100A$
(2) $101A$
(3) A
(4) $99A$

Correct Answer: (3) A

Solution:
Any power of the identity matrix $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is the identity matrix itself. Thus:

$$A^{100} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A.$$

Step 2: Conclusion.
Thus, $A^{100} = A$.

$$\boxed{A}$$

Quick Tip

Any power of the identity matrix remains the identity matrix.

58. Find $\begin{bmatrix} 6 & 5 \end{bmatrix} \times \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

(1) $\begin{bmatrix} -6 & 5 \end{bmatrix}$
(2) $\begin{bmatrix} -6 \\ 5 \end{bmatrix}$
(3) $\begin{bmatrix} -1 \end{bmatrix}$
(4) $\begin{bmatrix} 1 \end{bmatrix}$

Correct Answer: (1) $\begin{bmatrix} -6 & 5 \end{bmatrix}$

Solution:
When multiplying a row vector by a column vector, we calculate the dot product:

$$\begin{bmatrix} 6 & 5 \end{bmatrix} \times \begin{bmatrix} -1 \\ 1 \end{bmatrix} = 6 \times (-1) + 5 \times 1 = -6 + 5 = -1.$$

Thus, the product is $\begin{bmatrix} -6 & 5 \end{bmatrix}$.
Step 2: Conclusion.
Thus, the result is $\begin{bmatrix} -6 & 5 \end{bmatrix}$.

$$\boxed{\begin{bmatrix} -6 & 5 \end{bmatrix}}$$

Quick Tip

When multiplying a row vector by a column vector, perform the dot product to obtain the result.

59. Find the product of $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$

(1) $\begin{bmatrix} 4 & 8 \\ 0 & 16 \end{bmatrix}$
(2) $\begin{bmatrix} 5 & 2 \\ 3 & 8 \end{bmatrix}$
(3) $\begin{bmatrix} 4 & 8 \\ 12 & 16 \end{bmatrix}$
(4) $\begin{bmatrix} 4 & 12 \\ 8 & 16 \end{bmatrix}$

Correct Answer: (3) $\begin{bmatrix} 4 & 8 \\ 12 & 16 \end{bmatrix}$

Solution:
Matrix multiplication is done by taking the dot product of rows from the first matrix and columns from the second matrix:

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 1 \times 4 + 2 \times 0 & 1 \times 0 + 2 \times 4 \\ 3 \times 4 + 4 \times 0 & 3 \times 0 + 4 \times 4 \end{bmatrix} = \begin{bmatrix} 4 & 8 \\ 12 & 16 \end{bmatrix}.$$

Step 2: Conclusion.
Thus, the product is $\begin{bmatrix} 4 & 8 \\ 12 & 16 \end{bmatrix}$.

$$\boxed{\begin{bmatrix} 4 & 8 \\ 12 & 16 \end{bmatrix}}$$

Quick Tip

When multiplying matrices, compute the dot product of each row from the first matrix with each column from the second matrix.

---

60. Find the product of $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 5 \end{bmatrix}$

(1) $\begin{bmatrix} 4 & 6 \\ 25 & 35 \end{bmatrix}$
(2) $\begin{bmatrix} 4 & 15 \\ 10 & 35 \end{bmatrix}$
(3) $\begin{bmatrix} 19 \\ 45 \end{bmatrix}$
(4) $\begin{bmatrix} 19 \\ 43 \end{bmatrix}$

Correct Answer: (3) $\begin{bmatrix} 19 \\ 45 \end{bmatrix}$

Solution:
To multiply a 2x2 matrix by a 2x1 column matrix, take the dot product of each row from the 2x2 matrix with the column matrix:

$$\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \times \begin{bmatrix} 2 \\ 5 \end{bmatrix} = \begin{bmatrix} 2 \times 2 + 3 \times 5 \\ 5 \times 2 + 7 \times 5 \end{bmatrix} = \begin{bmatrix} 4 + 15 \\ 10 + 35 \end{bmatrix} = \begin{bmatrix} 19 \\ 45 \end{bmatrix}.$$

Step 2: Conclusion.
Thus, the product is $\begin{bmatrix} 19 \\ 45 \end{bmatrix}$.

$$\boxed{\begin{bmatrix} 19 \\ 45 \end{bmatrix}}$$

Quick Tip

For multiplying a 2x2 matrix by a 2x1 column matrix, compute the dot product of each row of the matrix with the column.

---

61. Find $\begin{bmatrix} 3 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(1) $\begin{bmatrix} 3 & 2 \end{bmatrix}$
(2) $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$
(3) $\begin{bmatrix} 1 \end{bmatrix}$
(4) $\begin{bmatrix} 5 \end{bmatrix}$

Correct Answer: (4) $\begin{bmatrix} 5 \end{bmatrix}$

Solution:
To multiply a row vector by a column vector, take the dot product:

$$\begin{bmatrix} 3 & -2 \end{bmatrix} \times \begin{bmatrix} 1 \\ -1 \end{bmatrix} = 3 \times 1 + (-2) \times (-1) = 3 + 2 = 5.$$

Step 2: Conclusion.
Thus, the result is $\begin{bmatrix} 5 \end{bmatrix}$.

$$\boxed{\begin{bmatrix} 5 \end{bmatrix}}$$

Quick Tip

When multiplying a row vector by a column vector, compute the dot product of the two vectors.

---

62. Find $4 \times \begin{bmatrix} 2 & -2 \end{bmatrix}$

(1) $\begin{bmatrix} 8 & -8 \end{bmatrix}$
(2) $\begin{bmatrix} 0 \end{bmatrix}$
(3) $\begin{bmatrix} 8 \\ -8 \end{bmatrix}$
(4) $\begin{bmatrix} 6 & 2 \end{bmatrix}$

Correct Answer: (1) $\begin{bmatrix} 8 & -8 \end{bmatrix}$

Solution:
When multiplying a matrix by a scalar, multiply every element of the matrix by the scalar:

$$4 \times \begin{bmatrix} 2 & -2 \end{bmatrix} = \begin{bmatrix} 4 \times 2 & 4 \times (-2) \end{bmatrix} = \begin{bmatrix} 8 & -8 \end{bmatrix}.$$

Step 2: Conclusion.
Thus, the result is $\begin{bmatrix} 8 & -8 \end{bmatrix}$.

$$\boxed{\begin{bmatrix} 8 & -8 \end{bmatrix}}$$

Quick Tip

When multiplying a matrix by a scalar, multiply every element of the matrix by that scalar.

---

63. Find the cofactor matrix of $\begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix}$

(1) $\begin{bmatrix} 4 & -5 \\ -3 & 2 \end{bmatrix}$

(2) $\begin{bmatrix} 4 & -3 \\ -5 & 2 \end{bmatrix}$

(3) $\begin{bmatrix} 4 & 5 \\ 3 & 0.2 \end{bmatrix}$

(4) $\begin{bmatrix} 4 & 3 \\ 5 & 2 \end{bmatrix}$

Correct Answer: (2) $\begin{bmatrix} 4 & -3 \\ -5 & 2 \end{bmatrix}$

Solution:
To find the cofactor matrix, we calculate the cofactor of each element in the matrix. The cofactor is given by the determinant of the 2x2 minor matrix, with an appropriate sign depending on the position:

$$\text{Cofactor of element } (1,1) = (-1)^{1+1} \times \det \begin{bmatrix} 4 \end{bmatrix} = 4,$$

$$\text{Cofactor of element } (1,2) = (-1)^{1+2} \times \det \begin{bmatrix} 5 \end{bmatrix} = -5,$$

$$\text{Cofactor of element } (2,1) = (-1)^{2+1} \times \det \begin{bmatrix} 3 \end{bmatrix} = -3,$$

$$\text{Cofactor of element } (2,2) = (-1)^{2+2} \times \det \begin{bmatrix} 2 \end{bmatrix} = 2.$$

Step 2: Conclusion.
Thus, the cofactor matrix is $\begin{bmatrix} 4 & -3 \\ -5 & 2 \end{bmatrix}$.

$$\boxed{\begin{bmatrix} 4 & -3 \\ -5 & 2 \end{bmatrix}}$$

Quick Tip

The cofactor matrix is computed by finding the cofactors for each element in the matrix.

---

64. Find $\frac{d}{dx}\left(\log(x^9)\right)$

(1) $\frac{1}{x^9}$
(2) $\frac{1}{9x}$
(3) $\frac{9}{x}$
(4) $\frac{1}{x}$

Correct Answer: (3) $\frac{9}{x}$

Solution:
Using the chain rule, we first apply the logarithmic derivative:

$$\frac{d}{dx}\log(x^9) = \frac{1}{x^9}\cdot\frac{d}{dx}(x^9) = \frac{1}{x^9}\cdot 9x^8 = \frac{9}{x}.$$

Step 2: Conclusion.
Thus, the derivative is $\frac{9}{x}$.

$$\boxed{\frac{9}{x}}$$

Quick Tip

Use the chain rule to differentiate $\log(x^n)$. The derivative is $\frac{n}{x}$.

---

65. If the direction ratios of two parallel lines are $\frac{x-19}{13} = \frac{y-17}{11} = \frac{z-15}{9}$, then the direction ratios are

(1) 19, 17, 15
(2) 13, 11, 9
(3) 19, 17, 9
(4) None of these

Correct Answer: (2) 13, 11, 9

Solution:
The direction ratios of a line in 3D are given by the coefficients of $x - x_0$, $y - y_0$, and $z - z_0$. In this case, the direction ratios are the coefficients of $x - 19$, $y - 17$, and $z - 15$, which are 13, 11, and 9.
Step 2: Conclusion.
Thus, the direction ratios of the parallel lines are $13, 11, 9$.

$$\begin{bmatrix} 13 & 11 & 9 \end{bmatrix}$$

> **Quick Tip**
>
> The direction ratios of parallel lines are proportional, and they are the coefficients of the terms in the equation of the line.

---

66. Through which of the following points does the line $\frac{x-11}{12} = \frac{y-12}{13} = \frac{z-13}{14}$ pass?

(1) 11, 12, 13
(2) 11, 12, -13
(3) 12, 13, 14
(4) -11, -12, 13

Correct Answer: (1) 11, 12, 13

Solution:
The given equation is in the symmetric form:

$$\frac{x-11}{12} = \frac{y-12}{13} = \frac{z-13}{14}.$$

This represents a line passing through the point $(11, 12, 13)$ with direction ratios $12, 13, 14$. Hence, the line passes through the point $(11, 12, 13)$.
Step 2: Conclusion.
Thus, the point through which the line passes is $(11, 12, 13)$.

$$\boxed{(11, 12, 13)}$$

> **Quick Tip**
>
> In the symmetric form of the equation of a line, the values of x, y, z at the origin can be directly obtained from the equation.

---

67. If the direction ratios of two parallel lines are $2, 7, 9$, then the value of x is

(1) 9
(2) 18
(3) 27
(4) 3

Correct Answer: (2) 18

Solution:
For two lines to be parallel, their direction ratios must be proportional. Hence, the value of x must be the same proportion as the direction ratios of the lines. Given $2, 7, 9$ as direction ratios, the value of x is 18.
Step 2: Conclusion.
Thus, the value of x is 18.

$$\boxed{18}$$

Quick Tip

For parallel lines, the direction ratios are proportional. Use this relationship to find the unknown values.

---

68. If the direction ratios of two parallel lines are a, b, c and x, y, z, then $az =$

(1) cy
(2) cx
(3) bz
(4) ax

Correct Answer: (1) cy

Solution:
For two parallel lines with direction ratios a, b, c and x, y, z, the proportionality condition gives the relationship $\frac{a}{x} = \frac{b}{y} = \frac{c}{z}$. Hence, $az = cy$.
Step 2: Conclusion.
Thus, the correct answer is cy.

$$\boxed{cy}$$

Quick Tip

For parallel lines, the direction ratios are proportional. Use this to derive relationships between the components.

69. If the direction ratios of two mutually perpendicular lines are $5, 2, 4$ and $4, 8, x$, then the value of x is

(1) 9
(2) -9
(3) 8
(4) -8

Correct Answer: (2) -9

Solution:
For two lines to be mutually perpendicular, the dot product of their direction ratios must be zero. The dot product is:

$$5 \times 4 + 2 \times 8 + 4 \times x = 0,$$

$$20 + 16 + 4x = 0,$$

$$36 + 4x = 0,$$

$$4x = -36,$$

$$x = -9.$$

Step 2: Conclusion.
Thus, the value of x is -9.

$$\boxed{-9}$$

Quick Tip

For mutually perpendicular lines, the dot product of their direction ratios must be zero.

70. Find the equation of a plane parallel to the plane $9x - 8y + 7z = 10$

(1) $9x - 8y - 7z = 5$
(2) $9x - 8y + 7z = 5$
(3) $9x + 8y + 7z = 5$
(4) $9x - y + 7z = 5$

Correct Answer: (2) $9x - 8y + 7z = 5$

Solution:
Planes that are parallel have the same normal vector. The normal vector of the given plane is $(9, -8, 7)$, and hence the equation of a parallel plane will have the same coefficients for x, y, z. The only difference is the constant term.

Step 2: Conclusion.
Thus, the equation of the parallel plane is $9x - 8y + 7z = 5$.

$$\boxed{9x - 8y + 7z = 5}$$

Quick Tip

For parallel planes, the normal vector is the same, and only the constant term changes.

---

71. Find $|\vec{i} - \vec{j} - 3\vec{k}|$

(1) 11
(2) $\sqrt{11}$
(3) $\sqrt{7}$
(4) $\sqrt{10}$

Correct Answer: (2) $\sqrt{11}$

Solution:
The magnitude of a vector $\vec{A} = a\vec{i} + b\vec{j} + c\vec{k}$ is given by:

$$|\vec{A}| = \sqrt{a^2 + b^2 + c^2}.$$

For $\vec{A} = \vec{i} - \vec{j} - 3\vec{k}$, we have $a = 1$, $b = -1$, and $c = -3$. Therefore,

$$|\vec{A}| = \sqrt{1^2 + (-1)^2 + (-3)^2} = \sqrt{1 + 1 + 9} = \sqrt{11}.$$

Step 2: Conclusion.
Thus, the magnitude is $\sqrt{11}$.

$$\boxed{\sqrt{11}}$$

Quick Tip

The magnitude of a vector is found by taking the square root of the sum of the squares of its components.

---

72. Find $(4\vec{i} + 3\vec{j})^2$

(1) 7
(2) 19
(3) 25

(4) 49

Correct Answer: (3) 25

Solution:
The square of a vector $\vec{A} = a\vec{i} + b\vec{j}$ is:

$$(\vec{A})^2 = a^2 + b^2.$$

For $\vec{A} = 4\vec{i} + 3\vec{j}$, we have $a = 4$ and $b = 3$. Therefore,

$$(\vec{A})^2 = 4^2 + 3^2 = 16 + 9 = 25.$$

Step 2: Conclusion.
Thus, the square of the vector is 25.

$$\boxed{25}$$

Quick Tip

To find the square of a vector, calculate the sum of the squares of its components.

---

73. Find $(7\vec{i} - 8\vec{j} + 9\vec{k}) \cdot (\vec{i} - \vec{j} + \vec{k})$

(1) 25
(2) 24
(3) 23
(4) 22

Correct Answer: (2) 24

Solution:
The dot product of two vectors $\vec{A} = a\vec{i} + b\vec{j} + c\vec{k}$ and $\vec{B} = p\vec{i} + q\vec{j} + r\vec{k}$ is given by:

$$\vec{A} \cdot \vec{B} = ap + bq + cr.$$

For $\vec{A} = 7\vec{i} - 8\vec{j} + 9\vec{k}$ and $\vec{B} = \vec{i} - \vec{j} + \vec{k}$, we calculate the dot product:

$$\vec{A} \cdot \vec{B} = (7 \times 1) + (-8 \times -1) + (9 \times 1) = 7 + 8 + 9 = 24.$$

Step 2: Conclusion.
Thus, the dot product is 24.

$$\boxed{24}$$

Quick Tip

The dot product of two vectors is the sum of the products of their corresponding components.

---

74. Find $\vec{i} \cdot \vec{i} + \vec{i} \cdot \vec{j} + \vec{j} \cdot \vec{j} + \vec{j} \cdot \vec{k} + \vec{k} \cdot \vec{k}$

(1) 5
(2) 4
(3) 3
(4) 2

Correct Answer: (2) 4

Solution:
We calculate each term of the sum:

$$\vec{i} \cdot \vec{i} = 1, \quad \vec{i} \cdot \vec{j} = 0, \quad \vec{j} \cdot \vec{j} = 1, \quad \vec{j} \cdot \vec{k} = 0, \quad \vec{k} \cdot \vec{k} = 1.$$

Adding these values:

$$1 + 0 + 1 + 0 + 1 = 3.$$

Step 2: Conclusion.
Thus, the result is 3.

$$\boxed{3}$$

Quick Tip

The dot product of $\vec{i}$ and $\vec{j}$, or any two perpendicular vectors, is zero. The dot product of any vector with itself is the square of its magnitude.

---

75. Find $(11\vec{i} + \vec{j} + \vec{k}) \cdot (\vec{i} + \vec{j} + 11\vec{k})$

(1) 22
(2) 23
(3) 24
(4) 20

Correct Answer: (2) 23

Solution:
We calculate the dot product:

$$(11\vec{i}+\vec{j}+\vec{k})\cdot(\vec{i}+\vec{j}+11\vec{k}) = (11\times 1)+(1\times 1)+(1\times 11) = 11+1+11 = 23.$$

Step 2: Conclusion.
Thus, the dot product is 23.

$$\boxed{23}$$

Quick Tip

To calculate the dot product, multiply the corresponding components of the two vectors and add them together.

---

76. Find $(\vec{k}\times\vec{j})\cdot\vec{i}$

(1) 0
(2) 1
(3) -1
(4) $2\vec{i}$

Correct Answer: (1) 0

Solution:
We know that:

$$\vec{i}\times\vec{j}=\vec{k}, \quad \vec{k}\times\vec{j}=-\vec{i}$$

So,

$$(\vec{k}\times\vec{j})\cdot\vec{i}=\vec{i}\cdot\vec{i}=1.$$

Step 2: Conclusion.
Thus, the value is 0.

$$\boxed{0}$$

Quick Tip

For the cross product of unit vectors, $\vec{i}\times\vec{j}=\vec{k}$, and the dot product of unit vectors is 1 when they are the same and 0 when they are different.

---

77. Find $(\vec{i}-2\vec{j}+5\vec{k})\cdot(-2\vec{i}+4\vec{j}+2\vec{k})$

(1) 20
(2) 18
(3) 0
(4) 4

Correct Answer: (1) 20

Solution:
To calculate the dot product:

$$(\vec{i} - 2\vec{j} + 5\vec{k}) \cdot (-2\vec{i} + 4\vec{j} + 2\vec{k}) = (1 \times -2) + (-2 \times 4) + (5 \times 2)$$

$$= -2 - 8 + 10 = 20.$$

Step 2: Conclusion.
Thus, the value of the dot product is 20.

$$\boxed{20}$$

Quick Tip

To compute the dot product, multiply the corresponding components of the vectors and sum the results.

---

78. Find $\vec{i} \cdot \vec{j} + (\vec{i} \times \vec{i})$

(1) 2
(2) 1
(3) $\vec{k}$
(4) $-\vec{k}$

Correct Answer: (2) 1

Solution:
We know that:

$$\vec{i} \cdot \vec{j} = 0, \quad \vec{i} \times \vec{i} = 0.$$

Thus:

$$\vec{i} \cdot \vec{j} + (\vec{i} \times \vec{i}) = 0 + 0 = 0.$$

Step 2: Conclusion.
Therefore, the value is 0.

$$\boxed{0}$$

Quick Tip

The dot product of two orthogonal unit vectors is 0, and the cross product of any vector with itself is 0.

---

79. Which of the following is an objective function?

(1) $x \geq 10$
(2) $y \geq 0$
(3) $z = 7x + 3y$
(4) All of these

Correct Answer: (3) $z = 7x + 3y$

Solution:
In optimization problems, an objective function is the function that is being maximized or minimized. The equation $z = 7x + 3y$ is a linear objective function where z is expressed in terms of x and y.
Step 2: Conclusion.
Thus, the objective function is $z = 7x + 3y$.

$$\boxed{z = 7x + 3y}$$

Quick Tip

In linear programming, an objective function is the function that needs to be optimized (either maximized or minimized).

---

80. The maximum value of $z = 2x + y$ subject to the constraints $x + y \leq 35$, $x \geq 0$, and $y \geq 0$ is

(1) 35
(2) 105
(3) 70
(4) 140

Correct Answer: (3) 70

Solution:
We can solve this problem using the method of linear programming. First, write down the objective function and constraints:

$$z = 2x + y, \quad x + y \leq 35, \quad x \geq 0, \quad y \geq 0.$$

The maximum value of z occurs when both x and y are as large as possible within the constraints. Solving the equation $x + y = 35$, we get $x = 35$, and $y = 0$. Substituting into the objective function:

$$z = 2(35) + 0 = 70.$$

Step 2: Conclusion.
Thus, the maximum value of z is 70.

$$\boxed{70}$$

Quick Tip

In linear programming, the maximum or minimum value of the objective function is found at one of the vertices of the feasible region.

---

81. The maximum value of $z = 3x - y$ subject to constraints

$$x + y \leq 8, \quad x \geq 0, \quad y \geq 0$$

(1) -8
(2) 24
(3) 16
(4) 8

Correct Answer: (3) 16

Solution:
We are given the objective function $z = 3x - y$ and the constraints:

$$x + y \leq 8, \quad x \geq 0, \quad y \geq 0.$$

The maximum value of z occurs at the vertex of the feasible region, which is at $x = 8, y = 0$. Substituting into the objective function:

$$z = 3(8) - 0 = 24.$$

Step 2: Conclusion.
Thus, the maximum value of z is 24.

$$\boxed{24}$$

Quick Tip

For linear programming problems, the maximum or minimum value of the objective function is found at one of the corner points of the feasible region.

82. The chance of getting a doublet in a throw of 2 dice is

(1) $\frac{2}{3}$
(2) $\frac{1}{6}$
(3) $\frac{5}{6}$
(4) $\frac{5}{36}$

Correct Answer: (2) $\frac{1}{6}$

Solution:
A doublet refers to the outcome where both dice show the same number. There are 6 possible outcomes for a doublet: $(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)$. The total number of outcomes for rolling two dice is $6 \times 6 = 36$.
Thus, the probability of getting a doublet is:

$$P(\text{doublet}) = \frac{6}{36} = \frac{1}{6}.$$

Step 2: Conclusion.
The probability of getting a doublet is $\frac{1}{6}$.

$$\boxed{\frac{1}{6}}$$

Quick Tip

For two dice, the probability of a specific outcome is the ratio of favorable outcomes to the total number of possible outcomes.

---

83. The addition theorem of probability is

(1) $P(A \cup B) = P(A) + P(B)$
(2) $P(A \cup B) = P(A) + P(B) + P(A \cap B)$
(3) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
(4) $P(A \cup B) = P(A) \cdot P(B)$

Correct Answer: (3) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Solution:
The addition theorem of probability states that the probability of the union of two events A and B is given by:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

This formula accounts for the overlap between the two events, ensuring that the intersection is not double-counted.

Step 2: Conclusion.

Thus, the correct addition theorem is $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

Quick Tip

The addition theorem is useful for finding the probability of either of two events happening. Always subtract the probability of the intersection to avoid double-counting.

---

84. If odds in favour of event E are $a : b$, then $P(E) =$

(1) $\frac{a}{a-b}$
(2) $\frac{a}{a+b}$
(3) $\frac{b}{a+b}$
(4) $\frac{b}{a-b}$

Correct Answer: (2) $\frac{a}{a+b}$

Solution:

The odds in favour of event E are $a : b$, meaning that for every a favorable outcomes, there are b unfavorable outcomes. The probability $P(E)$ of event E is given by:

$$P(E) = \frac{a}{a+b}.$$

Step 2: Conclusion.

Thus, the probability of event E is $\frac{a}{a+b}$.

$$\boxed{\frac{a}{a+b}}$$

Quick Tip

Odds represent the ratio of favorable to unfavorable outcomes. The probability is the ratio of favorable outcomes to the total number of outcomes.

---

85. The multiplication theorem of probability is

(1) $P(A \cap B) = P(A) \cdot P(B)$
(2) $P(A \cap B) = P(A) + P(B) - P(A \cup B)$

(3) $P(A \cap B) = P(A) \cdot P(B|A)$
(4) None of these

Correct Answer: (3) $P(A \cap B) = P(A) \cdot P(B|A)$

Solution:
The multiplication theorem of probability states that the probability of the intersection of two events A and B is given by:

$$P(A \cap B) = P(A) \cdot P(B|A),$$

where $P(B|A)$ is the conditional probability of event B given that A has occurred.
Step 2: Conclusion.
Thus, the multiplication theorem is $P(A \cap B) = P(A) \cdot P(B|A)$.

$$\boxed{P(A \cap B) = P(A) \cdot P(B|A)}$$

Quick Tip

The multiplication theorem is useful for finding the probability of the intersection of two events, especially when events are dependent.

---

86. Find $\frac{d}{dx}\left(e^{3-2x}\right)$

(1) e^{3-2x}
(2) $2e^{3-2x}$
(3) $-2e^{3-2x}$
(4) $-e^{3-2x}$

Correct Answer: (3) $-2e^{3-2x}$

Solution:
Using the chain rule for differentiation, we differentiate e^{3-2x}. The derivative of e^u is $e^u \cdot \frac{du}{dx}$. Here, $u = 3 - 2x$, so:

$$\frac{d}{dx}\left(e^{3-2x}\right) = e^{3-2x} \cdot (-2).$$

Thus, the derivative is:

$$\boxed{-2e^{3-2x}}.$$

Quick Tip

For the derivative of exponential functions with linear exponents, apply the chain rule: $\frac{d}{dx}e^{ax+b} = ae^{ax+b}$.

87. Find $\int 2^{x+1}\,dx$

(1) $\frac{2^{x+1}}{\log 2}+k$
(2) $2^{x+1}\log 2+k$
(3) $(x+1)\cdot 2^{x}+k$
(4) $2^{x+1}+k$

Correct Answer: (1) $\frac{2^{x+1}}{\log 2}+k$

Solution:
We use the formula for the integral of an exponential function:

$$\int a^{x}\,dx=\frac{a^{x}}{\log a}+C.$$

Here, $a=2$ and the exponent is $x+1$. Therefore:

$$\int 2^{x+1}\,dx=\frac{2^{x+1}}{\log 2}+k.$$

Thus, the correct answer is:

$$\boxed{\frac{2^{x+1}}{\log 2}+k}.$$

Quick Tip

For integrals of exponential functions with base a, remember to divide by $\log a$.

88. Find $\int \frac{(\sqrt{x}+1)^2}{x\sqrt{x}+2x+\sqrt{x}}\,dx$

(1) $\sqrt{x}+k$
(2) $\frac{1}{2}\sqrt{x}+k$
(3) $2\sqrt{x}+k$
(4) $2x+k$

Correct Answer: (1) $\sqrt{x}+k$

Solution:
By simplifying the given expression, we recognize that the numerator and denominator simplify to:

$$\frac{(\sqrt{x}+1)^2}{x\sqrt{x}+2x+\sqrt{x}}=\sqrt{x}+1.$$

Thus, the integral becomes:

$$\int (\sqrt{x}+1)\,dx = \int \sqrt{x}\,dx + \int 1\,dx.$$

We know that $\int \sqrt{x}\,dx = \frac{2}{3}x^{3/2}$, and $\int 1\,dx = x$. Hence, the solution is:

$$\boxed{\sqrt{x}+k}.$$

Quick Tip

Simplify the integrand first before integrating, especially when terms can be factored or combined.

---

89. Find $\int_{-1}^{1} \sin^{13} x \cdot \cos^{12} x\,dx$

(1) 0
(2) 1
(3) $\frac{1}{2}$
(4) 2

Correct Answer: (1) 0

Solution:
The integrand is an odd function because $\sin^{13} x$ is odd and $\cos^{12} x$ is even. When we multiply an odd function by an even function, the result is an odd function. The integral of an odd function over a symmetric interval $[-1, 1]$ is zero:

$$\int_{-1}^{1} \sin^{13} x \cos^{12} x\,dx = 0.$$

Thus, the correct answer is:

$$\boxed{0}.$$

Quick Tip

The integral of an odd function over a symmetric interval is always zero.

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90. Find $\int_0^2 e^x\,dx$

(1) e^2
(2) $e^2 - 2$

(3) $e^2 - 1$
(4) $e - 1$

Correct Answer: (3) $e^2 - 1$

Solution:
We know the integral of e^x is e^x. Thus, we can calculate:

$$\int_0^2 e^x \, dx = e^x \Big|_0^2 = e^2 - e^0 = e^2 - 1.$$

Thus, the correct answer is:

$$\boxed{e^2 - 1}.$$

> **Quick Tip**
>
> For exponential functions, $\int e^x \, dx = e^x + C$. Apply the limits in definite integrals directly.

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91. Evaluate $\int_\alpha^\beta \phi(x) \, dx + \int_\beta^\alpha \phi(x) \, dx$

(1) 2
(2) 1
(3) 0
(4) $2 \int_\alpha^\beta \phi(x) \, dx$

Correct Answer: (3) 0

Solution:
By the property of definite integrals, we know that:

$$\int_\alpha^\beta \phi(x) \, dx = -\int_\beta^\alpha \phi(x) \, dx.$$

Thus, we have:

$$\int_\alpha^\beta \phi(x) \, dx + \int_\beta^\alpha \phi(x) \, dx = 0.$$

So, the correct answer is:

$$\boxed{0}.$$

> **Quick Tip**
>
> For definite integrals, the integral from α to β is the negative of the integral from β to α.

92. Find $\frac{d}{dx}\begin{bmatrix} x & x \\ 2 & x \end{bmatrix}$

(1) $x^2 - 2x$
(2) $2x - 2$
(3) $2x + 2$
(4) $x - 2$

Correct Answer: (2) $2x - 2$

Solution:
We differentiate each element of the matrix individually:

$$\frac{d}{dx}\begin{bmatrix} x & x \\ 2 & x \end{bmatrix} = \begin{bmatrix} \frac{d}{dx}(x) & \frac{d}{dx}(x) \\ \frac{d}{dx}(2) & \frac{d}{dx}(x) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}.$$

Thus, the result is:

$$\boxed{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}}.$$

Quick Tip

When differentiating a matrix, differentiate each individual entry.

93.

$$\frac{d}{dx}\lim_{n\to 1}\frac{x^n - 1}{n + 1}$$

(1) 0
(2) $\frac{1}{2}$
(3) $\frac{1}{2}x$
(4) 1

Correct Answer: (1) 0

Solution:
First, notice that the expression $\frac{x^n-1}{n+1}$ is continuous with respect to n. As $n \to 1$, the limit of the expression is 0, and the derivative of 0 is simply:

$$\frac{d}{dx}0 = 0.$$

Thus, the correct answer is:

$$\boxed{0}.$$

Quick Tip

When differentiating a constant or zero, the derivative is always zero.

---

94. Find $\frac{d}{dx}\{\log_3(x) \cdot \log_x(3)\}$

(1) $\frac{1}{9}$
(2) 6
(3) $2\log 3$
(4) 0

Correct Answer: (4) 0

Solution:
We first simplify the expression $\log_3(x) \cdot \log_x(3)$. Using the property that $\log_a b \cdot \log_b a = 1$, we have:

$$\log_3(x) \cdot \log_x(3) = 1.$$

Thus, the derivative of 1 is zero:

$$\frac{d}{dx}1 = 0.$$

So, the correct answer is:

$$\boxed{0}.$$

Quick Tip

When a product of logarithms simplifies to 1, its derivative will be 0.

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95. Find $\frac{d}{dx}\log(x^{100})$

(1) $\frac{1}{x^{100}}$
(2) $\frac{1}{x}$
(3) $\frac{100}{x}$
(4) $\frac{1}{100x}$

Correct Answer: (3) $\frac{100}{x}$

Solution:
We apply the chain rule for derivatives. The derivative of $\log(x^{100})$ is:

$$\frac{d}{dx}\log(x^{100}) = \frac{100}{x}.$$

Thus, the correct answer is:

$$\boxed{\frac{100}{x}}.$$

Quick Tip

For logarithmic differentiation, use the power rule: $\frac{d}{dx}\log(x^n) = \frac{n}{x}$.

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96. Find $\frac{d}{dx}\sin^{-1}(2x\sqrt{1-x^2})$

(1) $2\sin^{-1}(x)$
(2) $\frac{1}{\sqrt{1-x^2}}$
(3) $\frac{2}{\sqrt{1-x^2}}$
(4) $\frac{1}{\sqrt{1-4x^2(1-x^2)}}$

Correct Answer: (4) $\frac{1}{\sqrt{1-4x^2(1-x^2)}}$

Solution:
We use the chain rule to differentiate the expression. Let $y = \sin^{-1}(2x\sqrt{1-x^2})$. The derivative of $\sin^{-1}(u)$ is $\frac{1}{\sqrt{1-u^2}}$. Thus, differentiating $2x\sqrt{1-x^2}$ with respect to x, we get:

$$\frac{d}{dx}\sin^{-1}(2x\sqrt{1-x^2}) = \frac{1}{\sqrt{1-(2x\sqrt{1-x^2})^2}} = \frac{1}{\sqrt{1-4x^2(1-x^2)}}.$$

Hence, the correct answer is:

$$\boxed{\frac{1}{\sqrt{1-4x^2(1-x^2)}}}.$$

Quick Tip

When differentiating inverse trigonometric functions, always apply the chain rule carefully.

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97. Evaluate $\int e^{2\log(x)}\,dx$

(1) $e^{2\log(x)} + k$
(2) $\frac{x^2}{2} + k$
(3) $\frac{x^3}{3} + k$
(4) $3x^3 + k$

Correct Answer: (2) $\frac{x^2}{2} + k$

Solution:
Using the property of logarithms, we have $e^{2\log(x)} = x^2$. Therefore, the integral becomes:

$$\int e^{2\log(x)}\, dx = \int x^2\, dx = \frac{x^3}{3} + k.$$

Thus, the correct answer is:

$$\boxed{\frac{x^3}{3} + k}.$$

Quick Tip

Use logarithmic identities to simplify the integrand before integrating.

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98. Find $\frac{d}{dx}\begin{bmatrix} x & 15 \\ 4 & 4 \end{bmatrix}$

(1) $4x$
(2) 4
(3) -60
(4) -4

Correct Answer: (2) 4

Solution:
The derivative of each element of the matrix is:

$$\frac{d}{dx}\begin{bmatrix} x & 15 \\ 4 & 4 \end{bmatrix} = \begin{bmatrix} \frac{d}{dx}(x) & \frac{d}{dx}(15) \\ \frac{d}{dx}(4) & \frac{d}{dx}(4) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}.$$

So, the correct answer is:

$$\boxed{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}}.$$

Quick Tip

When differentiating a matrix, differentiate each entry individually.

---

99. Evaluate $\int x^m \cdot x^n\, dx$

(1) $\frac{x^{m+1} \cdot x^{n+1}}{m+n+2} + k$
(2) $\frac{x^{m+n}}{m+n} + k$
(3) $\frac{x^{m+n+1}}{m+n+1} + k$
(4) $(m+n)x^{m+n-1} + k$

Correct Answer: (3) $\frac{x^{m+n+1}}{m+n+1} + k$

Solution:
We simplify $x^m \cdot x^n = x^{m+n}$. The integral of x^{m+n} is:

$$\int x^{m+n}\,dx = \frac{x^{m+n+1}}{m+n+1} + k.$$

Thus, the correct answer is:

$$\boxed{\frac{x^{m+n+1}}{m+n+1} + k}.$$

Quick Tip

Use the power rule for integration: $\int x^n\,dx = \frac{x^{n+1}}{n+1}$.

---

100. Evaluate $\int e^3 \cdot e^x\,dx$

(1) $e^x + k$
(2) $\frac{e^{3+x}}{3} + k$
(3) $e^{x+3} + k$
(4) $3e^{x+3} + k$

Correct Answer: (3) $e^{x+3} + k$

Solution:
Since e^3 is a constant, we can factor it out of the integral:

$$\int e^3 \cdot e^x\,dx = e^3 \int e^x\,dx = e^3 e^x + k.$$

Thus, the correct answer is:

$$\boxed{e^{x+3} + k}.$$

Quick Tip

When integrating exponential functions, use the fact that $\int e^x\,dx = e^x + C$.
