Bihar Board Class 12th Mathematics - 2023 Question Paper with Solutions

Time Allowed :3 Hour 15 mins	Maximum Marks :50	Total Questions :100

General Instructions

Read the following instructions very carefully and strictly follow them:

1. Question Nos. 1 to 100 have four options, out of which only one is correct. Answer any 50 questions. You have to mark your selected option on the OMR-Sheet.

1. If the direction ratios of two parallel lines are x, 5, 3 and 20, 10, 6 then the value of x is:
(A) 10
(B) 5
(C) 3
(D) 40

Correct Answer: (A) 10

Solution:
Step 1 (Reason: Parallel lines have proportional direction ratios). For lines with d.r.s $(x, 5, 3)$ and $(20, 10, 6)$,

$$\frac{x}{20} = \frac{5}{10} = \frac{3}{6}.$$

Step 2 (Reason: simplify known ratios to identify the common factor). $\frac{5}{10} = \frac{3}{6} = \frac{1}{2}$, hence the common ratio is $\frac{1}{2}$.
Step 3 (Reason: equate first pair to the same ratio and solve for x).

$$\frac{x}{20} = \frac{1}{2} \;\Rightarrow\; x = 20 \cdot \frac{1}{2} = 10.$$

Therefore $x = 10$.

Quick Tip

Parallel (or collinear) direction ratios are always in the same ratio: $(l_1, m_1, n_1) \parallel (l_2, m_2, n_2) \iff \frac{l_1}{l_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$.

2. If the direction ratios of two parallel lines are a_1, b_1, c_1 and a_2, b_2, c_2 then $\frac{a_1 c_2}{a_2} = ?$
(A) b_1
(B) b_2
(C) b_3

(D) c_1

Correct Answer: (D) c_1

Solution:
Step 1 (Reason: Parallelism implies a common proportionality constant). Let

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = k \neq 0.$$

Step 2 (Reason: rewrite a_1 and c_1 using k). From above, $a_1 = k\,a_2$ and $c_1 = k\,c_2$.
Step 3 (Reason: substitute and cancel).

$$\frac{a_1 c_2}{a_2} = \frac{k\,a_2 \cdot c_2}{a_2} = k\,c_2 = c_1.$$

Hence $\dfrac{a_1 c_2}{a_2} = c_1$.

> **Quick Tip**
>
> Use the single constant k: $a_1 = ka_2$, $b_1 = kb_2$, $c_1 = kc_2$. Such substitutions make parallel-lines algebra immediate.

3. If the direction ratios of two mutually perpendicular lines are 2, 3, 5 **and** x, y, 4**, then** $2x + 3y =$ **?**
(A) 20
(B) -20
(C) 30
(D) -30

Correct Answer: (B) -20

Solution:
Idea. Two lines are perpendicular in 3-D exactly when the dot product of any direction-ratio triples is 0. Dot product adds the products of the matching components.
Step 1. Compute the dot product and set it to zero.

$$(2, 3, 5) \cdot (x, y, 4) = 2x + 3y + 5 \cdot 4 = 0.$$

Step 2. Simplify.

$$2x + 3y + 20 = 0 \quad \Rightarrow \quad 2x + 3y = -20.$$

That is the required value.

> **Quick Tip**
>
> Perpendicular $\Rightarrow$ dot product $= 0$: multiply component-wise and add.

4. $\left\| 3\vec{i} - 4\vec{j} - 5\vec{k} \right\| =$ **?**

(A) $5\sqrt{2}$
(B) 12
(C) 2
(D) 9

Correct Answer: (A) $5\sqrt{2}$

Solution:
Idea. The length of a vector $\langle a, b, c\rangle$ is the 3-D version of Pythagoras: $\sqrt{a^2+b^2+c^2}$. Signs do not matter because we square.
Step.
$$\sqrt{3^2+(-4)^2+(-5)^2} = \sqrt{9+16+25} = \sqrt{50} = 5\sqrt{2}.$$

Quick Tip

Magnitude $= \sqrt{(\text{x-part})^2 + (\text{y-part})^2 + (\text{z-part})^2}$.

5. $[\,2a-7 \quad 1\,] = [\,a \quad b-1\,] \Rightarrow (a,b) = \ ?$
(A) $(1,7)$
(B) $(2,7)$
(C) $(7,2)$
(D) $(2,3)$

Correct Answer: (C) $(7,2)$

Solution:
Idea. Two equal row vectors must match entry by entry. Turn each position into a small equation.
Step 1. First position. $2a-7=a \Rightarrow a=7$.
Step 2. Second position. $1=b-1 \Rightarrow b=2$.
So $(a,b)=(7,2)$.

Quick Tip

Equal matrices/vectors $\Rightarrow$ equal corresponding entries.

6. Evaluate $\begin{vmatrix} 1 & 1 & 5 \\ 4 & 9 & 17 \\ 5 & 10 & 22 \end{vmatrix}$**.**
(A) 264
(B) 1221
(C) 0
(D) 1

Correct Answer: (C) 0

Solution:
Idea. Use expansion along the first row (signs $+ \ - \ +$). Compute three 2×2 minors.

Step 1. Minors.

$$M_{11} = \begin{vmatrix} 9 & 17 \\ 10 & 22 \end{vmatrix} = 9 \cdot 22 - 17 \cdot 10 = 198 - 170 = 28,$$

$$M_{12} = \begin{vmatrix} 4 & 17 \\ 5 & 22 \end{vmatrix} = 4 \cdot 22 - 17 \cdot 5 = 88 - 85 = 3,$$

$$M_{13} = \begin{vmatrix} 4 & 9 \\ 5 & 10 \end{vmatrix} = 4 \cdot 10 - 9 \cdot 5 = 40 - 45 = -5.$$

Step 2. Combine with signs.

$$\det = 1 \cdot 28 - 1 \cdot 3 + 5 \cdot (-5) = 28 - 3 - 25 = 0.$$

Zero determinant also hints that the third row is the sum of the first two, so rows are dependent.

> **Quick Tip**
>
> Row-1 expansion uses $+ - +$. Linear dependence of rows $\Rightarrow$ determinant 0.

7. Evaluate $\begin{vmatrix} 1 & 2 & -1 \\ 5 & 4 & 1 \\ 7 & 6 & 1 \end{vmatrix}$.

(A) 0
(B) 1
(C) -1
(D) 12

Correct Answer: (A) 0

Solution:
Idea. Again expand along the first row, or notice a quick pattern: $R_3 = R_1 + R_2 \Rightarrow$ rows dependent $\Rightarrow$ determinant 0. We also verify by expansion.
Step (verification).

$$\det = 1\begin{vmatrix} 4 & 1 \\ 6 & 1 \end{vmatrix} - 2\begin{vmatrix} 5 & 1 \\ 7 & 1 \end{vmatrix} + (-1)\begin{vmatrix} 5 & 4 \\ 7 & 6 \end{vmatrix} = 1(-2) - 2(-2) + (-1)(2) = -2 + 4 - 2 = 0.$$

> **Quick Tip**
>
> If one row = sum of two others, det = 0. Spotting such relations saves time.

8. Evaluate $\begin{vmatrix} -\sin\theta & \cos\theta \\ \sec\theta & \csc\theta \end{vmatrix}$.

(A) 0
(B) -1
(C) -2
(D) $-\sin 2\theta$

Correct Answer: (C) -2

Solution:
Idea. For a 2×2 matrix $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad-bc$. Use reciprocal trig identities $\sec\theta = \frac{1}{\cos\theta}$, $\csc\theta = \frac{1}{\sin\theta}$.
Step.

$$(-\sin\theta)\left(\frac{1}{\sin\theta}\right) - \cos\theta\left(\frac{1}{\cos\theta}\right) = -1-1 = -2.$$

Quick Tip

Replace sec, csc by reciprocals and cancel—often the quickest path.

9. Compute $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 0 & 4 \end{bmatrix} =$ **?**

(A) $\begin{bmatrix} 2 & -3 \\ 0 & 4 \end{bmatrix}$
(B) $\begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$
(C) $\begin{bmatrix} 3 & -3 \\ 0 & 5 \end{bmatrix}$
(D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Correct Answer: (A) $\begin{bmatrix} 2 & -3 \\ 0 & 4 \end{bmatrix}$

Solution:
Idea. Multiplying by the identity I leaves a matrix unchanged, just like multiplying a number by 1.
So $I \cdot A = A$ and the product equals the second matrix itself.

Quick Tip

I is the multiplicative identity of matrices: $IA = AI = A$.

10. Compute $[\,6 \;\; 5\,]\begin{bmatrix} 1 \\ -1 \end{bmatrix} =$ **?**

(A) $[\,6 \;\; -5\,]$
(B) $[\,-5 \;\; 6\,]$
(C) $[\,1\,]$
(D) $[\,1 \;\; 1\,]$

Correct Answer: (C) $[\,1\,]$

Solution:
Idea. Row–column multiplication: multiply matching entries and add. A 1×2 times 2×1 gives a 1×1 number.

$$6 \cdot 1 + 5 \cdot (-1) = 6 - 5 = 1.$$

Quick Tip

Always check sizes: $(1 \times 2) \cdot (2 \times 1) \to (1 \times 1)$.

11. $\frac{d}{dx}\left(2\cos\frac{3x}{4}\right) =$ **?**

(A) $-2\sin\frac{3x}{4}$

(B) $-\frac{3}{8}\sin\frac{3x}{4}$

(C) $-\frac{3}{4}\sin\frac{3x}{4}$

(D) $-\frac{3}{2}\sin\frac{3x}{4}$

Correct Answer: (D) $-\frac{3}{2}\sin\frac{3x}{4}$

Solution:
Idea. Use the chain rule: derivative of $\cos u$ is $-\sin u \cdot u'$. The outer constant 2 stays outside.
Step 1. Let $u = \frac{3x}{4} \Rightarrow u' = \frac{3}{4}$.
Step 2.

$$\frac{d}{dx}\big(2\cos u\big) = 2(-\sin u)\,u' = -2\sin\left(\tfrac{3x}{4}\right) \cdot \frac{3}{4} = -\frac{3}{2}\sin\left(\tfrac{3x}{4}\right).$$

Quick Tip

Inside-function derivative multiplies the result: $f(g(x))' = f'(g)\,g'$.

12. $\frac{d}{dx}\left(e^{-3x}\right) =$ **?**

(A) $\dfrac{e^{-3x}}{3}$

(B) $\dfrac{e^{-3x}}{-3}$

(C) $3e^{-3x}$

(D) $-3e^{-3x}$

Correct Answer: (D) $-3e^{-3x}$

Solution:
Idea. Derivative of e^u is $e^u \cdot u'$. Here $u = -3x$ so $u' = -3$.

$$\frac{d}{dx}e^{-3x} = e^{-3x} \cdot (-3) = -3e^{-3x}.$$

Quick Tip

Shortcut: $(e^{kx})' = k\,e^{kx}$.

13. $\dfrac{d}{dx}(11^x) =$ **?**
(A) $x\,11^{x-1}$
(B) $11^x \cdot \log x$
(C) $11^x \cdot \log 11$
(D) $\dfrac{11^x}{\log 11}$

Correct Answer: (C) $11^x \cdot \log 11$

Solution:
Idea. For base a (constant), $\dfrac{d}{dx}(a^x) = a^x \ln a$. ln and log (base e) are the same in calculus notation here.
So $\dfrac{d}{dx}(11^x) = 11^x \ln 11 = 11^x \cdot \log 11$.

> **Quick Tip**
>
> Don't mix with power rule x^n. Here the variable is in the exponent.

14. $\dfrac{d}{dx}\left(\dfrac{1}{3x-2}\right) =$ **?**
(A) $-\dfrac{1}{(3x-2)^2}$
(B) $-\dfrac{3}{(3x-2)^2}$
(C) $\dfrac{3}{(3x-2)^2}$
(D) $\dfrac{3}{3x-2}$

Correct Answer: (B) $-\dfrac{3}{(3x-2)^2}$

Solution:
Idea. Write as a negative power and apply the chain rule.

$$(3x-2)^{-1} \;\Rightarrow\; \frac{d}{dx}(3x-2)^{-1} = -1(3x-2)^{-2}\cdot(3) = -\frac{3}{(3x-2)^2}.$$

> **Quick Tip**
>
> General form: $\dfrac{d}{dx}(ax+b)^{-1} = -a(ax+b)^{-2}$.

15. If $x = a\cos^2\theta,\ y = b\sin^2\theta$, then the value of $\dfrac{dy}{dx}$ is
(A) $\dfrac{b}{a}$
(B) $-\dfrac{b}{a}$
(C) $\dfrac{b}{a}\sin 2\theta$

(D) $-\dfrac{b}{a}\tan^2\theta$

Correct Answer: (B) $-\dfrac{b}{a}$

Solution:
Idea. Use parametric differentiation: $\dfrac{dy}{dx} = \dfrac{(dy/d\theta)}{(dx/d\theta)}$. Trig double-angle simplifies nicely.
Step 1. $dy/d\theta = b \cdot 2\sin\theta\cos\theta = b\sin 2\theta$.
Step 2. $dx/d\theta = a \cdot 2\cos\theta(-\sin\theta) = -a\sin 2\theta$.
Step 3. $\dfrac{dy}{dx} = \dfrac{b\sin 2\theta}{-a\sin 2\theta} = -\dfrac{b}{a}$.

Quick Tip

With parameters, divide derivatives: many trig factors cancel automatically.

16. The solution of the differential equation $x^2\,dx + y^2\,dy = 0$ is
(A) $x^3 + y^3 = k$
(B) $x^2 + y^2 = k$
(C) $x^2 - y^2 = k$
(D) $x^2 - y^2 = k$

Correct Answer: (A) $x^3 + y^3 = k$

Solution:
Idea. This is already separated: an x-part with dx plus a y-part with dy. Integrate each side straight away.

$$\int x^2\,dx + \int y^2\,dy = C \Rightarrow \frac{x^3}{3} + \frac{y^3}{3} = C \Rightarrow x^3 + y^3 = k.$$

Quick Tip

Form $M(x)dx + N(y)dy = 0 \Rightarrow$ integrate terms independently.

17. Evaluate $(\vec{j} - 2\vec{i}) \cdot (\vec{k} + 3\vec{i} - \vec{j})$.
(A) 0
(B) -6
(C) -7
(D) 8

Correct Answer: (C) -7

Solution:
Idea. Convert to components and use $\vec{i}\cdot\vec{i} = \vec{j}\cdot\vec{j} = \vec{k}\cdot\vec{k} = 1$ and cross terms 0.
$(-2, 1, 0) \cdot (3, -1, 1) = (-2)\cdot 3 + 1\cdot(-1) + 0\cdot 1 = -6 - 1 + 0 = -7.$

Quick Tip

Arrange vectors as triples (x, y, z) and multiply component-wise.

18. Solve $e^{3x}\,dx + e^{4y}\,dy = 0$.
(A) $e^{3x+4y} = k$
(B) $e^{3x} + e^{4y} = k$
(C) $\frac{1}{3}e^{3x} + \frac{1}{4}e^{4y} = k$
(D) $e^{3x} + e^{4y} + e^{3x+4y} = k$

Correct Answer: (C) $\frac{1}{3}e^{3x} + \frac{1}{4}e^{4y} = k$

Solution:
Idea. Again the equation is separable: one pure x term and one pure y term. Move one to the other side and integrate.

$$e^{3x}dx = -e^{4y}dy \Rightarrow \int e^{3x}dx + \int e^{4y}dy = C \Rightarrow \frac{1}{3}e^{3x} + \frac{1}{4}e^{4y} = k.$$

Quick Tip

$\int e^{ax}\,dx = \frac{1}{a}e^{ax}$. Add constants into a single k.

19. The solution of $\frac{dx}{x} + \frac{dy}{y} = 0$ is
(A) $x = ky$
(B) $\frac{1}{x} + \frac{1}{y} = k$
(C) $x + y = k$
(D) $xy = k$

Correct Answer: (D) $xy = k$

Solution:
Idea. Integrate each fraction; logarithms add. Exponentiate to remove ln.

$$\ln|x| + \ln|y| = C \Rightarrow \ln|xy| = C \Rightarrow xy = e^C = k.$$

Quick Tip

$\ln A + \ln B = \ln(AB)$. Replace e^C by a new constant k.

20. The integrating factor of the linear DE $\frac{dy}{dx} - y\sin x = \cot x$ is
(A) $\sin x$
(B) $e^{-\sin x}$
(C) $e^{\sin x}$

(D) $e^{\cos x}$

Correct Answer: (D) $e^{\cos x}$

Solution:
Idea. Standard linear form is $y' + P(x)y = Q(x)$. The integrating factor is $e^{\int P(x)\,dx}$.
Step 1. Identify $P(x)$**.** Here $y' - y\sin x = \cot x \Rightarrow P(x) = -\sin x$.
Step 2. Compute IF.

$$\text{IF} = e^{\int -\sin x\,dx} = e^{\cos x}.$$

That's the required integrating factor (no need to solve for y here).

> **Quick Tip**
>
> Linear DE $y' + Py = Q \Rightarrow \text{IF} = \exp(\int P\,dx)$. Watch the sign when moving terms.

21. $[-1]\,[1 \;\; -1] = \;?$
(A) $[0]$
(B) $[-1 \;\; 1]$
(C) $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$
(D) $[2 \;\; -2]$

Correct Answer: (B) $[-1 \;\; 1]$

Solution:
Think of $[-1]$ as a plain number -1. When a scalar multiplies a matrix (or row), it multiplies *each* entry. So,

$$[-1]\,[1 \;\; -1] = [-1 \times 1 \;\; -1 \times (-1)] = [-1 \;\; 1].$$

That's all—entrywise scaling.

> **Quick Tip**
>
> Scalar $\times$ matrix = scale every entry by the scalar.

22. $3\begin{bmatrix} 7 & -2 \\ 8 & 0 \end{bmatrix} = \;?$
(A) $\begin{bmatrix} 21 & -6 \\ 8 & 0 \end{bmatrix}$
(B) $\begin{bmatrix} 7 & -2 \\ 24 & 0 \end{bmatrix}$
(C) $\begin{bmatrix} 21 & -6 \\ 24 & 0 \end{bmatrix}$
(D) $\begin{bmatrix} 21 & -2 \\ 8 & 0 \end{bmatrix}$

Correct Answer: (C) $\begin{bmatrix} 21 & -6 \\ 24 & 0 \end{bmatrix}$

Solution:
Multiply each entry by 3:

$$3\begin{bmatrix} 7 & -2 \\ 8 & 0 \end{bmatrix} = \begin{bmatrix} 3\times 7 & 3\times(-2) \\ 3\times 8 & 3\times 0 \end{bmatrix} = \begin{bmatrix} 21 & -6 \\ 24 & 0 \end{bmatrix}.$$

> **Quick Tip**
>
> A scalar in front of a matrix distributes to every position.

23. The maximum value of $Z = 6x + 3y$ subject to $x + y \leq 25,\ x \geq 0,\ y \geq 0$ is
(A) 150
(B) 225
(C) 425
(D) none of these

Correct Answer: (A) 150

Solution:
The feasible region is the triangle with vertices where the lines meet: $(0,0)$, $(25,0)$, $(0,25)$. A linear objective hits its max at a vertex. Evaluate Z: $(25,0) \Rightarrow Z = 6\cdot 25 + 3\cdot 0 = 150$. $(0,25) \Rightarrow Z = 0 + 75 = 75$. $(0,0) \Rightarrow Z = 0$. So the maximum is 150 at $(25,0)$.

> **Quick Tip**
>
> Two-variable LPP: just test Z at the corner points.

24. The maximum value of $Z = x - 3y$ subject to $x + y \leq 13,\ x \geq 0,\ y \geq 0$ is
(A) 39
(B) 26
(C) 13
(D) -26

Correct Answer: (C) 13

Solution:
Since y has a negative coefficient in Z, we want the **smallest** y (take $y = 0$) and the **largest** x allowed. With $y = 0$, the constraint becomes $x \leq 13$. Corner checks: $(13,0) \Rightarrow Z = 13$ (best), $(0,13) \Rightarrow Z = -39$, $(0,0) \Rightarrow Z = 0$.

> **Quick Tip**
>
> Negative coefficient $\rightarrow$ push that variable to its minimum (if allowed).

25. The minimum value of $Z = 7x + 8y$ subject to $3x + 4y \leq 24,\ x \geq 0,\ y \geq 0$ is
(A) 56
(B) 48

(C) 0
(D) -12

Correct Answer: (C) 0

Solution:
All coefficients in Z are positive and the origin $(0, 0)$ is feasible $(0 \leq 24)$. Therefore the smallest value occurs at $(0, 0)$: $Z = 0$.

> **Quick Tip**
>
> For "$\leq$" constraints with $x, y \geq 0$, the origin is feasible; check it first for minimization.

26. Compute $(2\vec{i} - 3\vec{j}) \cdot (\vec{i} + \vec{k})$**.**
(A) 2
(B) -1
(C) 3
(D) 0

Correct Answer: (A) 2

Solution:
Write components: $(2, -3, 0)$ and $(1, 0, 1)$. Dot product = multiply componentwise and add:

$$2 \cdot 1 + (-3) \cdot 0 + 0 \cdot 1 = 2.$$

> **Quick Tip**
>
> Only same-direction pairs $(i \cdot i,\ j \cdot j,\ k \cdot k)$ contribute.

27. For $|x| \leq 1$**,** $2\tan^{-1} x = \ ?$
(A) $\tan^{-1}(2x)$
(B) $\sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$
(C) $\cos^{-1}\left(\dfrac{2x}{1+x^2}\right)$
(D) $\tan^{-1}\left(\dfrac{2x}{1+x^2}\right)$

Correct Answer: (B) $\sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$

Solution:
Let $\theta = \tan^{-1} x \Rightarrow x = \tan\theta$ with $\theta \in [-\pi/4, \pi/4]$. Use $\sin(2\theta) = \dfrac{2\tan\theta}{1+\tan^2\theta}$:

$$\sin(2\theta) = \frac{2x}{1+x^2}.$$

Since $2\theta \in [-\pi/2, \pi/2]$, $\sin^{-1}$ returns 2θ:

$$2\tan^{-1} x = 2\theta = \sin^{-1}\left(\frac{2x}{1+x^2}\right).$$

Quick Tip

Set $x = \tan\theta$ and convert using double-angle formulas.

28. For $x \in \mathbb{R}$, $\cot^{-1} x =$?

(A) $\dfrac{\pi}{2} - \sin^{-1} x$

(B) $\dfrac{\pi}{2} - \cos^{-1} x$

(C) $\dfrac{\pi}{2} - \tan^{-1} x$

(D) $\dfrac{\pi}{2} - \sec^{-1} x$

Correct Answer: (C) $\dfrac{\pi}{2} - \tan^{-1} x$

Solution:

$\cot\theta = \tan\big(\frac{\pi}{2} - \theta\big)$. Apply inverse on both sides (with principal ranges): $\cot^{-1} x = \frac{\pi}{2} - \tan^{-1} x$.

Quick Tip

tan and cot are complementary: swap with $\frac{\pi}{2}-$ angle.

29. $\tan^{-1}\left(\dfrac{x+y}{1-xy}\right) =$?

(A) $\sin^{-1}(x+y)$

(B) $\cos^{-1}(x+y)$

(C) $\tan^{-1}(x+y)$

(D) $\tan^{-1} x + \tan^{-1} y$

Correct Answer: (D) $\tan^{-1} x + \tan^{-1} y$

Solution:

Use the tangent addition formula $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\,\tan\beta}$. Take $\alpha = \tan^{-1} x,\ \beta = \tan^{-1} y$. Then $\tan(\alpha+\beta) = \dfrac{x+y}{1-xy}$. Apply $\tan^{-1}$: $\alpha + \beta = \tan^{-1} x + \tan^{-1} y$.

Quick Tip

Spot $\dfrac{x+y}{1-xy} \Rightarrow$ it is $\tan(\alpha+\beta)$ in disguise.

30. $\sin^{-1}\big(\sin \frac{2\pi}{3}\big) =$?

(A) $\dfrac{\pi}{3}$

(B) $\dfrac{2\pi}{3}$

(C) $\dfrac{5\pi}{6}$

(D) $\dfrac{\pi}{6}$

Correct Answer: (A) $\dfrac{\pi}{3}$

Solution:
$\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$. $\sin^{-1}$ must return an angle in $[-\frac{\pi}{2}, \frac{\pi}{2}]$. In that range, the angle whose sine is $\frac{\sqrt{3}}{2}$ is $\frac{\pi}{3}$, not $2\pi/3$.

> **Quick Tip**
>
> Always convert back to the *principal* angle for inverse trig.

31. $\begin{bmatrix} 13 & 15 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \ ?$

(A) $\begin{bmatrix} 13 & 15 \\ -1 & 4 \end{bmatrix}$

(B) $\begin{bmatrix} 15 & 0 \\ 0 & 8 \end{bmatrix}$

(C) $\begin{bmatrix} 26 & 30 \\ -2 & 8 \end{bmatrix}$

(D) $\begin{bmatrix} 13 & 0 \\ 0 & 6 \end{bmatrix}$

Correct Answer: (C) $\begin{bmatrix} 26 & 30 \\ -2 & 8 \end{bmatrix}$

Solution:
Right matrix is $2I$ (twice the identity). Multiplying any matrix by $2I$ doubles each column: First column $\to 2 \times (13, -1) = (26, -2)$; second column $\to 2 \times (15, 4) = (30, 8)$. So product $= \begin{bmatrix} 26 & 30 \\ -2 & 8 \end{bmatrix}$.

> **Quick Tip**
>
> Multiplication by a diagonal matrix scales columns by the diagonal entries.

32. $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \end{bmatrix} = \ ?$

(A) $\begin{bmatrix} 4 \\ 25 \end{bmatrix}$

(B) $\begin{bmatrix} 4 \\ 10 \\ 35 \end{bmatrix}$

(C) $\begin{bmatrix} 19 \\ 45 \end{bmatrix}$

(D) $\begin{bmatrix} 19 \\ 45 \end{bmatrix}$

Correct Answer: (D) $\begin{bmatrix} 19 \\ 45 \end{bmatrix}$

Solution:
Row–column rule: Top entry $= 2{\cdot}2+3{\cdot}5 = 4+15 = 19$. Bottom entry $= 5{\cdot}2+7{\cdot}5 = 10+35 = 45$.

Quick Tip

$(2 \times 2)(2 \times 1) \Rightarrow (2 \times 1)$ — size check prevents mistakes.

33. $\displaystyle\int_{\pi/6}^{\pi/4} \tan\theta \, d\theta = \ ?$
(A) 0
(B) 1
(C) 2
(D) 3

Correct Answer: *None of the listed numbers*

Solution:
Use $\displaystyle\int \tan\theta \, d\theta = \ln|\sec\theta| + C$. Evaluate:

$$\ln(\sec\tfrac{\pi}{4}) - \ln(\sec\tfrac{\pi}{6}) = \ln(\sqrt{2}) - \ln\Big(\tfrac{2}{\sqrt{3}}\Big) = \ln\Big(\tfrac{\sqrt{6}}{2}\Big).$$

This value is not $0, 1, 2$, or 3. Hence none of the given choices matches.

Quick Tip

$\int \tan\theta \, d\theta = \ln|\sec\theta|$; definite values often stay as logs.

34. $\displaystyle\int \sin^3\theta \ \csc^2\theta \, d\theta = \ ?$
(A) $c + \theta$
(B) $c + \cos\theta$
(C) $c - \cos\theta$
(D) $c + \sin\theta$

Correct Answer: (C) $c - \cos\theta$

Solution:
$\csc^2\theta = \dfrac{1}{\sin^2\theta}$. So

$$\sin^3\theta \cdot \csc^2\theta = \sin^3\theta \cdot \frac{1}{\sin^2\theta} = \sin\theta.$$

Then $\int \sin\theta \, d\theta = -\cos\theta + c$.

Quick Tip

Rewrite reciprocals ($\csc^2 = 1/\sin^2$) and cancel powers first.

35. $\displaystyle\int \big(\cos\theta \ \csc^2\theta - \cos\theta \ \cot^2\theta\big) \, d\theta = \ ?$

(A) $\log \csc\theta + \cot\theta + k$
(B) $\csc\theta \cot\theta + k$
(C) $k + \sin\theta$
(D) $\theta + k$

Correct Answer: (C) $k + \sin\theta$

Solution:
Factor $\cos\theta$: $\cos\theta(\csc^2\theta - \cot^2\theta)$. Identity: $\csc^2\theta - \cot^2\theta = 1$. So integral $\int \cos\theta\, d\theta = \sin\theta + k$.

> **Quick Tip**
>
> Use $\csc^2 - \cot^2 = 1$ (parallel to $\sec^2 - \tan^2 = 1$).

36. $\displaystyle\int (4\cos x - 5\sin x)\, dx = \ ?$
(A) $k + 4\sin x + 5\cos x$
(B) $k - 4\sin x - 5\cos x$
(C) $k + 4\sin x - 5\cos x$
(D) $k - 4\sin x + 5\cos x$

Correct Answer: (A) $k + 4\sin x + 5\cos x$

Solution:
Integrate term by term: $\int 4\cos x\, dx = 4\sin x$. $\int -5\sin x\, dx = 5\cos x$. Add the constant k.

> **Quick Tip**
>
> Linearity: integrate each term, then add a single constant k.

37. $\displaystyle\int \frac{3\cos x - 2\sin x}{2\cos x + 3\sin x}\, dx = \ ?$
(A) $2\cos x + 3\sin x + k$
(B) $\log|2\cos x + 3\sin x| + k$
(C) $\tan^{-1}\left(3\sin\frac{x}{2}\right) + k$
(D) $2\tan\frac{x}{2} + k$

Correct Answer: (B) $\log|2\cos x + 3\sin x| + k$

Solution:
Let $f(x) = 2\cos x + 3\sin x$. Then

$$f'(x) = -2\sin x + 3\cos x = 3\cos x - 2\sin x,$$

which is exactly the numerator. Hence the integrand is $\dfrac{f'(x)}{f(x)}$, so $\int \dfrac{f'}{f}\, dx = \ln|f| + k = \ln|2\cos x + 3\sin x| + k$.

Quick Tip

Spot "derivative over itself" $\Rightarrow$ immediate $\ln|\cdot|$.

38. $\int \frac{3x^2+2}{x^3+2x}\,dx = \ ?$

(A) $\sin^{-1}(x^3+3x)+k$
(B) $\tan^{-1}(3x^2+2)+k$
(C) $\log|3x^2+2|+k$
(D) $\log|x^3+2x|+k$

Correct Answer: (D) $\log|x^3+2x|+k$

Solution:
Let $g(x)=x^3+2x \Rightarrow g'(x)=3x^2+2$ (the numerator). Therefore $\int \frac{g'(x)}{g(x)}dx = \ln|g(x)|+k = \ln|x^3+2x|+k$.

Quick Tip

Try to see the denominator's derivative sitting upstairs.

39. $\int \frac{dx}{x^2+5} = \ ?$

(A) $\tan^{-1}\frac{x}{5}+k$
(B) $\tan^{-1}\frac{x}{\sqrt{5}}+k$
(C) $\frac{1}{\sqrt{5}}\tan^{-1}\frac{x}{\sqrt{5}}+k$
(D) $\sqrt{5}\tan^{-1}\frac{x}{\sqrt{5}}+k$

Correct Answer: (C) $\frac{1}{\sqrt{5}}\tan^{-1}\frac{x}{\sqrt{5}}+k$

Solution:
Standard form:

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C.$$

Here $a^2=5 \Rightarrow a=\sqrt{5}$. Substitute to get the answer.

Quick Tip

Memorize: $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\arctan\frac{x}{a}$.

40. $\int_{-1}^{1} \log\left(\frac{3+x}{3-x}\right)dx = \ ?$

(A) 0
(B) 1
(C) $2\log 3$

(D) $3\log 2$

Correct Answer: (A) 0

Solution:
Let $f(x) = \ln\left(\frac{3+x}{3-x}\right)$. Check symmetry:

$$f(-x) = \ln\left(\frac{3-x}{3+x}\right) = -\ln\left(\frac{3+x}{3-x}\right) = -f(x).$$

So f is an *odd* function. The integral of an odd function over a symmetric interval $[-a, a]$ is 0. Hence the value is 0.

> Quick Tip
>
> If $f(-x) = -f(x)$, then $\int_{-a}^{a} f(x)\,dx = 0$.

41. If A and B are independent events, $P(A) = 0.3$ and $P(B) = 0.4$, then $P(A \cap B) =$?
(A) 0.12
(B) 0.21
(C) 0.75
(D) 0.7

Correct Answer: (A) 0.12

Solution:
Why we can multiply: For *independent* events, knowing that one happens does not change the chance of the other. By definition, this gives the multiplication rule

$$P(A \cap B) = P(A) \cdot P(B).$$

Substitute the given probabilities:

$$P(A \cap B) = 0.3 \times 0.4 = 0.12.$$

So the probability that both A and B occur together is 0.12.

> Quick Tip
>
> Independent $\Rightarrow$ multiply: $P(A \cap B) = P(A)P(B)$. (If they were *not* independent, this formula would not hold.)

42. The adjoint (adjugate) of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ is

(A) $\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 4 & -3 \\ -2 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 4 & 3 \\ 2 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix}$

Correct Answer: (A) $\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$

Solution:

Step 1: Recall the 2×2 formula. For $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$,

$$\text{adj}(A) = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

This comes from taking cofactors and then transposing the cofactor matrix (for a 2×2, that reduces to swapping $a \leftrightarrow d$ and negating the off–diagonals).

Step 2: Apply it to $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. Here $a = 1,\ b = 2,\ c = 3,\ d = 4.$ Therefore

$$\text{adj}(A) = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}.$$

This matches option (A).

Quick Tip

Memorize the 2×2 adjoint: swap the diagonal entries, change signs of the off–diagonals: $\text{adj} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

43. If the direction cosines of a line are $\frac{4}{\sqrt{77}}, \frac{5}{\sqrt{77}}, \frac{x}{\sqrt{77}}$, then the value of x is

(A) 6
(B) 7
(C) 8
(D) 9

Correct Answer: (A) 6

Solution:
For any line in 3-D, its direction cosines l, m, n satisfy

$$l^2 + m^2 + n^2 = 1 \quad \text{(unit vector condition)}.$$

Here $l = \frac{4}{\sqrt{77}},\ m = \frac{5}{\sqrt{77}},\ n = \frac{x}{\sqrt{77}}$. So

$$\left(\frac{4}{\sqrt{77}}\right)^2 + \left(\frac{5}{\sqrt{77}}\right)^2 + \left(\frac{x}{\sqrt{77}}\right)^2 = 1$$

$$\frac{16+25+x^2}{77}=1 \;\Rightarrow\; 16+25+x^2=77 \;\Rightarrow\; x^2=36.$$

Hence $x=\pm 6$. Among the choices the matching value is 6.

Quick Tip

Direction cosines are the components of a *unit* direction vector, so they always satisfy $l^2+m^2+n^2=1$.

44. If $A=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, then the value of A^{25} is

(A) $25A$
(B) $24A$
(C) $2A$
(D) A

Correct Answer: (D) A

Solution:
The given matrix is the identity I_2. Identity is the multiplicative neutral element:

$$I_2^n = I_2 \quad \text{for any positive integer } n.$$

Therefore,

$$A^{25}=I_2^{25}=I_2=A.$$

Quick Tip

Raising the identity to any power gives the identity back: $I^n = I$.

45. If a binary operation is defined by $a \text{ o } b = 3a+b$, then $(2 \text{ o } 3) \text{ o } 5 =$?

(A) 28
(B) 32
(C) 36
(D) 22

Correct Answer: (B) 32

Solution:
Operate step by step (left to right):
1) First compute $2 \text{ o } 3$:

$$2 \text{ o } 3 = 3(2)+3=6+3=9.$$

2) Now use this result with 5:

$$(2 \text{ o } 3) \text{ o } 5 = 9 \text{ o } 5 = 3(9)+5=27+5=32.$$

Hence the value is 32.

Quick Tip

When a custom operation is given by a formula, evaluate exactly in the stated order: compute the inner operation first, then apply the rule again.

46. If $A = \{1, 2\}$, $B = \{a, b, c\}$ then the total number of functions from A to B is
(A) 9
(B) 12
(C) 64
(D) none of these

Correct Answer: (A) 9

Solution:
For a function $f : A \to B$, each element of A may be sent to *any* element of B. Here $|A| = 2$, $|B| = 3$. For the first element of A there are 3 choices; for the second, again 3 choices (independent). Thus total functions $= 3 \times 3 = 3^2 = 9$.

Quick Tip

Number of functions from an m-element set to an n-element set is n^m.

47. If $A = \{a, b\}$, $B = \{1, 2, 3\}$ then the total number of one–one (injective) functions from A to B is
(A) 6
(B) 8
(C) 9
(D) none of these

Correct Answer: (A) 6

Solution:
For injective $f : A \to B$, distinct elements of A must go to *distinct* elements of B. Choose an image for a: 3 choices. Then for b only 2 choices remain (must be different). Total $= 3 \times 2 = 6 = P(3, 2)$.

Quick Tip

One–one maps from size m to size n (with $n \geq m$) are $nPm = \frac{n!}{(n-m)!}$.

48. The solution of the differential equation $dx + dy = 0$ is
(A) $x = ky$
(B) $x^2 + y^2 = k$
(C) $x + y = k$
(D) $xy = k$

Correct Answer: (C) $x + y = k$

Solution:
Rewrite $dx + dy = 0$ as $dy = -dx$. Integrate both sides:

$$\int dy = \int (-dx) \quad \Rightarrow \quad y = -x + C.$$

Move x to the left: $x + y = C = k$. This represents straight lines of slope -1.

Quick Tip

When differentials separate directly, integrate both sides and combine constants.

49. $\vec{i} \cdot \vec{i} =$ **?**
(A) 0
(B) 1
(C) -1
(D) $\vec{j}$

Correct Answer: (B) 1

Solution:
Unit vectors have length 1 and are mutually perpendicular. Dot product of a vector with itself equals the square of its length:

$$\vec{i} \cdot \vec{i} = \|\vec{i}\|^2 = 1^2 = 1.$$

Quick Tip

$\vec{i} \cdot \vec{i} = \vec{j} \cdot \vec{j} = \vec{k} \cdot \vec{k} = 1$; mixed pairs are 0.

50. $\vec{j} \times \vec{i} =$ **?**
(A) $\vec{k}$
(B) $-\vec{k}$
(C) $\vec{0}$
(D) 1

Correct Answer: (B) $-\vec{k}$

Solution:
Right–hand rule: $\vec{i} \times \vec{j} = \vec{k}$. Swapping the order changes the sign:

$$\vec{j} \times \vec{i} = -(\vec{i} \times \vec{j}) = -\vec{k}.$$

Quick Tip

Cross product is anti-commutative: $\mathbf{a} \times \mathbf{b} = -(\mathbf{b} \times \mathbf{a})$.

51. $\sin\left(\sin^{-1} \frac{1}{2}\right) =$ **?**
(A) 1
(B) $\frac{1}{2}$
(C) $\frac{\sqrt{3}}{2}$

(D) 0

Correct Answer: (B) $\frac{1}{2}$

Solution:
$\sin^{-1}$ returns an angle whose sine is the input. Thus $\theta = \sin^{-1}\frac{1}{2}$ satisfies $\sin\theta = \frac{1}{2}$. Applying $\sin$ undoes $\sin^{-1}$:

$$\sin(\sin^{-1}\tfrac{1}{2}) = \tfrac{1}{2}.$$

Quick Tip

For any $x \in [-1, 1]$, $\sin(\sin^{-1} x) = x$.

52. $\sin^{-1} x + \sin^{-1} y =$ **(principal values)**
(A) $\sin^{-1}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right)$
(B) $\sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$
(C) $\sin^{-1}\left(x\sqrt{1+y^2} + y\sqrt{1+x^2}\right)$
(D) $\sin^{-1}\left(x\sqrt{1+y^2} - y\sqrt{1+x^2}\right)$

Correct Answer: (B) $\sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right)$

Solution:
Let $\alpha = \sin^{-1} x,\ \beta = \sin^{-1} y \Rightarrow \sin\alpha = x,\ \sin\beta = y$ with $\alpha, \beta \in [-\pi/2, \pi/2]$. Then

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta = x\sqrt{1-y^2} + y\sqrt{1-x^2}.$$

Since $\alpha + \beta$ lies in $[-\pi, \pi]$ and under usual exam assumptions $\alpha + \beta \in [-\pi/2, \pi/2]$, we write $\alpha + \beta = \sin^{-1}\big(x\sqrt{1-y^2} + y\sqrt{1-x^2}\big)$.

Quick Tip

Convert inverse sines to angles, use $\sin(\alpha + \beta)$, then return via $\sin^{-1}$.

53. For $x \in [-1, 1]$**, evaluate** $\sin\big(2(\sin^{-1} x + \cos^{-1} x)\big)$**.**
(A) 0
(B) 1
(C) -1
(D) 1/2

Correct Answer: (A) 0

Solution:
$\sin^{-1} x + \cos^{-1} x = \dfrac{\pi}{2}$. Hence

$$2(\sin^{-1} x + \cos^{-1} x) = 2 \cdot \frac{\pi}{2} = \pi, \quad \sin(\pi) = 0.$$

Quick Tip

Key identity: $\sin^{-1} x + \cos^{-1} x = \dfrac{\pi}{2}$ for all $x \in [-1, 1]$.

54. For $x \in \mathbb{R}$, compute $\csc\big(\tan^{-1} x + \cot^{-1} x\big)$.

(A) 0

(B) 1

(C) $\dfrac{2}{\sqrt{3}}$

(D) 2

Correct Answer: (B) 1

Solution:

With the standard principal ranges used in school exams, $\tan^{-1} x + \cot^{-1} x = \dfrac{\pi}{2}$. Therefore

$$\csc\big(\tan^{-1} x + \cot^{-1} x\big) = \csc\left(\frac{\pi}{2}\right) = 1.$$

Quick Tip

Remember $\arctan x + \operatorname{arccot} x = \dfrac{\pi}{2}$ (principal values).

55. If $|x| \geq 1$, then $\tan\left[\frac{2}{3}\big(\tan^{-1} x + \cot^{-1} x\big)\right] =$?

(A) $\dfrac{12}{\sqrt{3}}$

(B) $\sqrt{3}$

(C) 0

(D) 1

Correct Answer: (B) $\sqrt{3}$

Solution:

Using $\tan^{-1} x + \cot^{-1} x = \dfrac{\pi}{2}$,

$$\tan\left[\frac{2}{3} \cdot \frac{\pi}{2}\right] = \tan\left(\frac{\pi}{3}\right) = \sqrt{3}.$$

Quick Tip

Once the inside is a fixed angle, evaluate the outer trig directly.

56. $\dfrac{d}{dx}\big(e^x + \cos 5x\big) =$?

(A) $e^x = \cos 5x$

(B) $e^x + 5 \sin 5x$

(C) $e^x - 5 \sin 5x$

(D) $e^x - 5\cos 5x$

Correct Answer: (C) $e^x - 5\sin 5x$

Solution:
Differentiate termwise: $\frac{d}{dx}(e^x) = e^x$; $\frac{d}{dx}(\cos 5x) = -\sin 5x \cdot 5$ by chain rule. Add:

$$e^x - 5\sin 5x.$$

> **Quick Tip**
>
> For $\cos(kx)$, derivative is $-k\sin(kx)$.

57. $\frac{d}{dx}\big(\sin 2x + e^x - \cos x\big) = $ **?**
(A) $\cos 2x + e^x - \sin x$
(B) $2\cos 2x + e^x + \sin x$
(C) $2\cos 2x + e^x - \sin x$
(D) $-2\cos 2x + e^x + \sin x$

Correct Answer: (B) $2\cos 2x + e^x + \sin x$

Solution:
$\frac{d}{dx}(\sin 2x) = 2\cos 2x$; $\frac{d}{dx}(e^x) = e^x$; $\frac{d}{dx}(-\cos x) = +\sin x$. Sum them to get $2\cos 2x + e^x + \sin x$.

> **Quick Tip**
>
> Derivative of $-\cos x$ is $+\sin x$; watch the sign.

58. $\frac{d}{dx}\left(\frac{1}{4}\sec 4x\right) = $ **?**
(A) $\sec 4x \cdot \tan 4x$
(B) $\sec^2 4x$
(C) $\tan^2 4x$
(D) $\frac{1}{16}\sec 4x \cdot \tan 4x$

Correct Answer: (A) $\sec 4x \cdot \tan 4x$

Solution:

$$\frac{d}{dx}\left(\frac{1}{4}\sec 4x\right) = \frac{1}{4}\cdot(\sec 4x \tan 4x)\cdot 4 = \sec 4x \tan 4x.$$

> **Quick Tip**
>
> For $\sec(kx)$, derivative is $k\,\sec(kx)\tan(kx)$.

59. $\frac{d}{dx}\big(\log_e(10x)\big) = $ **?**

(A) $\dfrac{1}{10x}$
(B) $\dfrac{10}{x}$
(C) $10x$
(D) $\dfrac{1}{x}$

Correct Answer: (D) $\dfrac{1}{x}$

Solution:
$\ln(10x) = \ln 10 + \ln x$. Constant's derivative is 0. $\dfrac{d}{dx}(\ln x) = \dfrac{1}{x}$. Hence the result is $1/x$.

Quick Tip

$\dfrac{d}{dx}\ln(ax) = \dfrac{1}{x}$ for any constant $a > 0$.

60. Distance of the plane $3x - 4y + 6z = 11$ from the origin is
(A) $\dfrac{3}{\sqrt{61}}$
(B) $\dfrac{11}{\sqrt{61}}$
(C) $\dfrac{6}{\sqrt{61}}$
(D) $\dfrac{4}{\sqrt{61}}$

Correct Answer: (B) $\dfrac{11}{\sqrt{61}}$

Solution:
Write plane in $Ax + By + Cz + D = 0$ form: $3x - 4y + 6z - 11 = 0$. Distance from origin:

$$\frac{|D|}{\sqrt{A^2 + B^2 + C^2}} = \frac{|-11|}{\sqrt{3^2 + (-4)^2 + 6^2}} = \frac{11}{\sqrt{9 + 16 + 36}} = \frac{11}{\sqrt{61}}.$$

Quick Tip

Point–plane distance: $\dfrac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2 + B^2 + C^2}}$; put plane as $Ax+By+Cz+D = 0$ first.

61. $\displaystyle\int \sin\left(\frac{3x}{4}\right)\,dx =$ **?**
(A) $k - \dfrac{3}{4}\cos\left(\dfrac{3x}{4}\right)$
(B) $k + \dfrac{3}{4}\cos\left(\dfrac{3x}{4}\right)$
(C) $k - \dfrac{4}{3}\cos\left(\dfrac{3x}{4}\right)$
(D) $k + \dfrac{4}{3}\cos\left(\dfrac{3x}{4}\right)$

Correct Answer: (C) $k - \frac{4}{3}\cos\left(\frac{3x}{4}\right)$

Solution:
Reason. Use the chain rule in reverse (substitution).
Let $u = \frac{3x}{4} \Rightarrow du = \frac{3}{4}dx \Rightarrow dx = \frac{4}{3}du$. Then

$$\int \sin\left(\tfrac{3x}{4}\right)\, dx = \int \sin u \cdot \frac{4}{3}\, du = \frac{4}{3}\int \sin u\, du = \frac{4}{3}(-\cos u) + k = -\frac{4}{3}\cos\left(\tfrac{3x}{4}\right) + k.$$

Thus the antiderivative is $k - \frac{4}{3}\cos\left(\frac{3x}{4}\right)$.

Quick Tip

$$\int \sin(ax)\, dx = -\frac{1}{a}\cos(ax) + C.$$

62. $\int \cos\left(\frac{7x}{9}\right)\, dx =$ **?**

(A) $k + \sin\left(\frac{7x}{9}\right)$
(B) $\frac{7}{9} + \sin\left(\frac{7x}{9}\right) + k$
(C) $\frac{9}{7}\sin\left(\frac{7x}{9}\right) + k$
(D) $k + \cos\left(\frac{7x}{9}\right)$

Correct Answer: (C) $\frac{9}{7}\sin\left(\frac{7x}{9}\right) + k$

Solution:
$\int \cos(ax)\, dx = \frac{1}{a}\sin(ax) + C$. Here $a = \frac{7}{9}$, hence

$$\int \cos\left(\tfrac{7x}{9}\right)\, dx = \frac{1}{7/9}\sin\left(\tfrac{7x}{9}\right) + k = \frac{9}{7}\sin\left(\tfrac{7x}{9}\right) + k.$$

Quick Tip

Differentiate to check: $\frac{d}{dx}\left(\frac{1}{a}\sin(ax)\right) = \cos(ax)$.

63. $\int \sec^2\left(\frac{17x}{23}\right)\, dx =$ **?**

(A) $k + \frac{17}{23}\tan\left(\frac{17x}{23}\right)$
(B) $k - \frac{17}{23}\tan\left(\frac{17x}{23}\right)$
(C) $k + \frac{23}{17}\tan\left(\frac{17x}{23}\right)$
(D) $k - \frac{23}{17}\tan\left(\frac{17x}{23}\right)$

Correct Answer: (C) $k + \dfrac{23}{17}\tan\left(\dfrac{17x}{23}\right)$

Solution:
$\displaystyle\int \sec^2(ax)\,dx = \frac{1}{a}\tan(ax) + C$. With $a = \dfrac{17}{23}$,

$$\int \sec^2\left(\tfrac{17x}{23}\right)dx = \frac{1}{17/23}\tan\left(\tfrac{17x}{23}\right) + k = \frac{23}{17}\tan\left(\tfrac{17x}{23}\right) + k.$$

Quick Tip

Because $(\tan u)' = \sec^2 u$, the antiderivative is $\tan u$ divided by u'.

64. $\displaystyle\int 4^x\,dx = ?$

(A) $4^x + k$

(B) $\dfrac{4^{x+1}}{x+1} + k$

(C) $\dfrac{4^x}{\log 4} + k$

(D) $-\dfrac{4^x}{\log 4} + k$

Correct Answer: (C) $\dfrac{4^x}{\log 4} + k$

Solution:
For base $a > 0,\ a \neq 1$: $\displaystyle\int a^x dx = \frac{a^x}{\ln a} + C$. Putting $a = 4$ gives $\dfrac{4^x}{\log 4} + k$ (here log means natural log).

Quick Tip

Do not confuse with x^n: that uses the power rule, not the exponential rule.

65. $\displaystyle\int x\,(4x^2 - 6)\,dx = ?$

(A) $4x^3 - 6x + k$

(B) $\dfrac{4x^4}{3} - 6x^2 + k$

(C) $x^4 - 3x^2 + k$

(D) $\dfrac{4x^3}{3} - 3x^2 + k$

Correct Answer: (C) $x^4 - 3x^2 + k$

Solution:
First expand: $x(4x^2 - 6) = 4x^3 - 6x$. Integrate termwise:

$$\int 4x^3 dx = x^4, \qquad \int -6x\,dx = -3x^2.$$

Hence $x^4 - 3x^2 + k$.

Quick Tip

Always simplify the integrand (expand/factor) before integrating.

66. $\int e^x(\cos x - \sin x)\,dx =$ **?**

(A) $e^x \sin x + k$
(B) $e^x \cos x + k$
(C) $-e^x \sin x + k$
(D) $k - e^x \cos x$

Correct Answer: (B) $e^x \cos x + k$

Solution:
Recognize a derivative: $\frac{d}{dx}(e^x \cos x) = e^x \cos x - e^x \sin x = e^x(\cos x - \sin x)$. Thus the integral is $e^x \cos x + k$.

Quick Tip

Look for a known product whose derivative matches the pattern.

67. $\int e^x\,(x^3 + 3x^2)\,dx =$ **?**

(A) $3x^2 e^x + k$
(B) $x^2 e^x + k$
(C) $x^3 e^x + k$
(D) $3e^x \cdot x^3 + k$

Correct Answer: (C) $x^3 e^x + k$

Solution:
Check the derivative of $x^3 e^x$ by product rule:

$$\frac{d}{dx}(x^3 e^x) = e^x x^3 + e^x \cdot 3x^2 = e^x(x^3 + 3x^2).$$

It matches the integrand, so the antiderivative is $x^3 e^x + k$.

Quick Tip

When you see e^x times a polynomial, try derivative of (polynomial) $\cdot\, e^x$.

68. $\int e^x\left(\frac{1}{x} - \frac{1}{x^2}\right)\,dx =$ **?**

(A) $\frac{e^x}{x} + k$
(B) $-x\,e^x + k$
(C) $k - \frac{e^x}{x}$

(D) $k - \frac{e^x}{x^2}$

Correct Answer: (A) $\frac{e^x}{x} + k$

Solution:
Differentiate $\frac{e^x}{x}$ using quotient rule:

$$\left(\frac{e^x}{x}\right)' = \frac{xe^x - e^x}{x^2} = e^x\left(\frac{1}{x} - \frac{1}{x^2}\right).$$

Matches the integrand, so the antiderivative is $\frac{e^x}{x} + k$.

Quick Tip

A pattern like $e^x\left(\frac{1}{x} - \frac{1}{x^2}\right)$ screams "derivative of $\frac{e^x}{x}$".

69. $3\vec{k} \cdot (13\vec{i} - 7\vec{k}) =$ **?**
(A) 39
(B) 0
(C) 21
(D) 18

Correct Answer: None of the options (true value is -21)

Solution:
Write components: $3\vec{k} = (0, 0, 3)$ and $13\vec{i} - 7\vec{k} = (13, 0, -7)$. Dot product $= (0)(13) + (0)(0) + (3)(-7) = -21$. The correct value is -21. Since that number does not appear, the printed choices are inconsistent. If the question intended the magnitude only, $|-21| = 21$ (option (C)).

Quick Tip

Dot product adds products of matching components; sign matters.

70. $\frac{d}{dx}\left(\sin\frac{4x}{5}\right) =$ **?**
(A) $\frac{4}{5}\cos\frac{4x}{5}$
(B) $-\frac{4}{5}\cos\frac{4x}{5}$
(C) $\frac{5}{4}\cos\frac{4x}{5}$
(D) $-\frac{5}{4}\cos\frac{4x}{5}$

Correct Answer: (A) $\frac{4}{5}\cos\frac{4x}{5}$

Solution:
Chain rule: $(\sin u)' = \cos u \cdot u'$ with $u = \frac{4x}{5} \Rightarrow u' = \frac{4}{5}$. So derivative $= \frac{4}{5}\cos\left(\frac{4x}{5}\right)$.

Quick Tip

Always multiply by the inner derivative u' after differentiating the outside.

71. An equation of a plane parallel to $x - 8y - 9z = 12$ is
(A) $x + 8y + 9z = 12$
(B) $x - 8y - 9z = 2023$
(C) $8x - y - 9z = 12$
(D) $x - 9y - 8z = 12$

Correct Answer: (B) $x - 8y - 9z = 2023$

Solution:
Parallel planes have the same normal vector, so the coefficients of x, y, z must be proportional (here exactly the same). Only the constant may differ. Thus the family: $x - 8y - 9z = d$ with any $d \neq 12$. Option (B) fits.

Quick Tip

To be parallel: keep the x, y, z coefficients the same; change only the constant term.

72. $\left(3\vec{i} - 4\vec{k}\right)^2 =$ **?**
(A) 1
(B) 25
(C) 7
(D) 49

Correct Answer: (B) 25

Solution:
Here "square" means dotting the vector with itself (magnitude squared):

$$(3\vec{i} - 4\vec{k}) \cdot (3\vec{i} - 4\vec{k}) = 3^2 + (-4)^2 = 9 + 16 = 25.$$

Quick Tip

$\|\mathbf{v}\|^2 = \mathbf{v} \cdot \mathbf{v}.$

73. The unit vector in the direction of $3\vec{i} - 9\vec{j}$ is
(A) $\dfrac{3\vec{i} - 9\vec{j}}{-6}$
(B) $\dfrac{3\vec{i} - 9\vec{j}}{6}$
(C) $\dfrac{3\vec{i} - 9\vec{j}}{\sqrt{90}}$
(D) $\dfrac{3\vec{i} - 9\vec{j}}{\sqrt{70}}$

Correct Answer: (C) $\frac{3\vec{i}-9\vec{j}}{\sqrt{90}}$

Solution:
Magnitude $\|3\vec{i}-9\vec{j}\| = \sqrt{3^2+(-9)^2} = \sqrt{9+81} = \sqrt{90}$. Unit vector $= \frac{1}{\|\cdot\|} \times$ vector $= \frac{3\vec{i}-9\vec{j}}{\sqrt{90}}$ (which simplifies to $\frac{\vec{i}-3\vec{j}}{\sqrt{10}}$).

> **Quick Tip**
>
> Unit vector = vector divided by its magnitude.

74. $(\vec{i}-\vec{j}+\vec{k})\cdot(7\vec{i}-8\vec{j}+9\vec{k}) =$ **?**
(A) 22
(B) 23
(C) 24
(D) 25

Correct Answer: (C) 24

Solution:
Dot product: multiply components and add:

$$1\cdot 7+(-1)\cdot(-8)+1\cdot 9 = 7+8+9 = 24.$$

> **Quick Tip**
>
> Write vectors as triples and use $a_1a_2+b_1b_2+c_1c_2$.

75. The intercept cut off on the x-axis by the plane $3x+4y+5z=13$ is
(A) $\frac{3}{13}$
(B) $\frac{13}{3}$
(C) $\frac{13}{4}$
(D) $\frac{13}{5}$

Correct Answer: (B) $\frac{13}{3}$

Solution:
On the x-axis $y=z=0$. Substitute into the plane:

$$3x = 13 \Rightarrow x = \frac{13}{3}.$$

That is the x-intercept.

> **Quick Tip**
>
> To find the x-intercept of a plane, set $y=z=0$ and solve for x.

76. If the line $\dfrac{x}{-1} = \dfrac{y}{2} = \dfrac{z}{3}$ is parallel to the plane $ax + by + cz + d = 0$ then

(A) $a + 2b + 3c = 0$
(B) $-a + 2b + 3c = 0$
(C) $3a + b + 2c = 0$
(D) none of these

Correct Answer: (B) $-a + 2b + 3c = 0$

Solution:
Direction ratios of the line are $(-1, 2, 3)$. A line parallel to a plane has its direction vector *perpendicular* to the plane's normal (a, b, c). So their dot product is 0:

$$(-1, 2, 3) \cdot (a, b, c) = -a + 2b + 3c = 0.$$

Quick Tip

Parallel to plane $\Rightarrow$ direction vector $\perp$ plane's normal.

77. If two planes $a_1x + b_1y + c_1z + d_1 = 0$ and $a_2x + b_2y + c_2z + d_2 = 0$ are mutually perpendicular, then

(A) $\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$
(B) $\dfrac{a_1}{a_2} + \dfrac{b_1}{b_2} + \dfrac{c_1}{c_2} = 0$
(C) $a_1a_2 + b_1b_2 + c_1c_2 = 0$
(D) none of these

Correct Answer: (C) $a_1a_2 + b_1b_2 + c_1c_2 = 0$

Solution:
Normals of the planes are $\vec{n}_1 = (a_1, b_1, c_1)$ and $\vec{n}_2 = (a_2, b_2, c_2)$. Planes are perpendicular $\iff$ their normals are perpendicular: $\vec{n}_1 \cdot \vec{n}_2 = 0 \Rightarrow a_1a_2 + b_1b_2 + c_1c_2 = 0$.

Quick Tip

Angles between planes equal angles between their normal vectors.

78. $(11\vec{i} - 7\vec{j} - \vec{k}) \cdot (8\vec{i} - \vec{j} - 5\vec{k}) =$?

(A) 95
(B) 100
(C) 400
(D) 88

Correct Answer: (B) 100

Solution:
Compute componentwise:

$$11 \cdot 8 + (-7) \cdot (-1) + (-1) \cdot (-5) = 88 + 7 + 5 = 100.$$

So the dot product equals 100.

Quick Tip

Dot product is distributive and commutative for componentwise multiplication and addition.

79. If $P(A) = \frac{7}{11}$, $P(B) = \frac{9}{11}$, $P(A \cap B) = \frac{4}{11}$, then $P(A/B) = ?$

(A) $\frac{7}{9}$

(B) $\frac{4}{9}$

(C) 1

(D) $\frac{13}{22}$

Correct Answer: (B) $\frac{4}{9}$

Solution:

Why use conditional formula: By definition,

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}.$$

Substitute values:

$$P(A \mid B) = \frac{\frac{4}{11}}{\frac{9}{11}} = \frac{4}{11} \cdot \frac{11}{9} = \frac{4}{9}.$$

So, given that B has occurred, the chance that A also occurs is $\frac{4}{9}$.

Quick Tip

"Given B" means reduce the sample space to B: probability becomes $P(\text{both})/P(B)$.

80. If $P(E) = \frac{3}{7}$, $P(F) = \frac{5}{7}$, $P(E \cup F) = \frac{6}{7}$, then $P(E \cap F) = ?$

(A) $\frac{4}{7}$

(B) $\frac{2}{7}$

(C) $\frac{1}{7}$

(D) $\frac{3}{7}$

Correct Answer: (B) $\frac{2}{7}$

Solution:

Reason. Use the addition law:

$$P(E \cup F) = P(E) + P(F) - P(E \cap F).$$

Compute:

$$\frac{6}{7} = \frac{3}{7} + \frac{5}{7} - P(E \cap F) \;\Rightarrow\; P(E \cap F) = \frac{3+5-6}{7} = \frac{2}{7}.$$

Quick Tip

Union = sum minus overlap: $P(E \cup F) = P(E) + P(F) - P(E \cap F)$.

81. The integrating factor of the linear DE $\frac{dy}{dx} + \frac{2}{x}y = 5x^2$ is

(A) $\frac{2}{x}$
(B) $2e^x$
(C) $2\log x$
(D) x^2

Correct Answer: (D) x^2

Solution:
Standard form $y' + P(x)y = Q(x)$ has IF $= \exp\left(\int P(x)\,dx\right)$. Here $P(x) = \frac{2}{x}$. So

$$\text{IF} = \exp\left(\int \frac{2}{x} dx\right) = \exp(2\ln x) = e^{\ln x^2} = x^2.$$

Quick Tip

For $y' + \frac{m}{x}y = \ldots$, IF is x^m.

82. $(3\vec{k} - 7\vec{i}) \times 2\vec{k} =$ **?**
(A) $-14\vec{j}$
(B) $14\vec{j}$
(C) $11\vec{i} - 2\vec{k}$
(D) $2\vec{k} - 11\vec{i}$

Correct Answer: (B) $14\vec{j}$

Solution:
Write components: $(3\vec{k} - 7\vec{i}) = (-7, 0, 3)$, $2\vec{k} = (0, 0, 2)$. Use determinant for cross product:

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -7 & 0 & 3 \\ 0 & 0 & 2 \end{vmatrix} = \vec{i}(0 \cdot 2 - 3 \cdot 0) \; - \; \vec{j}(-7 \cdot 2 - 3 \cdot 0) \; + \; \vec{k}(-7 \cdot 0 - 0 \cdot 0) = 14\vec{j}.$$

Quick Tip

Only the terms with nonzero minors survive; watch the sign in the middle cofactor.

83. $\left| \, \vec{i} - 2\vec{j} + 2\vec{k} \, \right| =$ **?**

(A) 3
(B) 6
(C) 7
(D) 5

Correct Answer: (A) 3

Solution:
Magnitude of $(1, -2, 2)$:

$$\sqrt{1^2 + (-2)^2 + 2^2} = \sqrt{1 + 4 + 4} = \sqrt{9} = 3.$$

Quick Tip

Length of (a, b, c) is $\sqrt{a^2 + b^2 + c^2}$.

84. Direction ratios of the normal to the plane $x + 2y - 3z + 15 = 0$ are
(A) $1, 2, 3$
(B) $1, 2, 3$
(C) $1, 2, -3$
(D) $1, 2, 15$

Correct Answer: (C) $1, 2, -3$

Solution:
For $Ax + By + Cz + D = 0$, the normal direction ratios are (A, B, C). Hence $(1, 2, -3)$.

Quick Tip

Plane coefficients $\Rightarrow$ normal d.r.s immediately.

85. The direction ratios of the line $\frac{x+1}{3} = \frac{y-2}{3} = \frac{z-5}{6}$ are
(A) $1, -2, 5$
(B) $3, 2, 5$
(C) $3, 3, 6$
(D) $1, 3, 5$

Correct Answer: (C) $3, 3, 6$

Solution:
In the symmetric form $\frac{x - x_0}{l} = \frac{y - y_0}{m} = \frac{z - z_0}{n}$, the denominators l, m, n are direction ratios. So d.r.s are $(3, 3, 6)$.

Quick Tip

Denominators in the symmetric equation give the line's d.r.s.

86. Through which point does the line $\frac{x-100}{101} = \frac{y-99}{102} = \frac{z-98}{103}$ pass?

(A) $(101, 102, 103)$
(B) $(98, 99, 100)$
(C) $(100, 99, 98)$
(D) $(99, 100, 101)$

Correct Answer: (C) $(100, 99, 98)$

Solution:
Set parameter $t = 0$ in $x = 100 + 101t,\ y = 99 + 102t,\ z = 98 + 103t$. This gives the point $(100, 99, 98)$ on the line.

> **Quick Tip**
>
> The "$-x_0, -y_0, -z_0$" numerators reveal the point (x_0, y_0, z_0) on the line.

87. $(10\vec{i} + \vec{j} + \vec{k}) \times (-4\vec{i} + 7\vec{j} - 11\vec{k}) =$ **?**
(A) $-18\vec{i} + 106\vec{j} + 74\vec{k}$
(B) $18\vec{i} - 106\vec{j} - 74\vec{k}$
(C) $18\vec{i} + 106\vec{j} + 74\vec{k}$
(D) $5\vec{i} - 6\vec{j} - 7\vec{k}$

Correct Answer: (A) $-18\vec{i} + 106\vec{j} + 74\vec{k}$

Solution:
Use determinant:

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 10 & 1 & 1 \\ -4 & 7 & -11 \end{vmatrix} = \vec{i}(1 \cdot (-11) - 1 \cdot 7) \ - \ \vec{j}(10 \cdot (-11) - 1 \cdot (-4)) \ + \ \vec{k}(10 \cdot 7 - 1 \cdot (-4)).$$

Compute: $\vec{i}(-11 - 7) - \vec{j}(-110 + 4) + \vec{k}(70 + 4) = -18\vec{i} + 106\vec{j} + 74\vec{k}$.

> **Quick Tip**
>
> Cofactor signs are $+, -, +$ across the top row.

88. $\dfrac{d}{dx}(x^3 + e^x) =$ **?**
(A) $3x^2$
(B) $3x^2 + 3e^x$
(C) $3x^2 + e^x$
(D) $3x^2 e^x$

Correct Answer: (C) $3x^2 + e^x$

Solution:
Differentiate termwise: $(x^3)' = 3x^2$, $(e^x)' = e^x$. Add them.

Quick Tip

Sum rule: derivative of a sum is the sum of derivatives.

89. $\frac{d}{dx}(\tan x + \sin^2 x) =$ **?**
(A) $\sec x + 2\sin x\cos x$
(B) $\sec^2 x + \cos^2 x$
(C) $\sec^2 x + 2\sin x\cos x$
(D) $\sec^2 x - 2\sin x\cos x$

Correct Answer: (C) $\sec^2 x + 2\sin x\cos x$

Solution:
$(\tan x)' = \sec^2 x$. Using chain rule on $\sin^2 x$: derivative $= 2\sin x\cos x$. Add to get $\sec^2 x + 2\sin x\cos x$.

Quick Tip

$(\sin^2 x)' = 2\sin x\cos x$ comes from $(u^2)' = 2u\,u'$ with $u = \sin x$.

90. $\frac{d^2}{dx^2}(e^{5x}) =$ **?**
(A) e^{5x}
(B) $10e^{5x}$
(C) $5e^{5x}$
(D) $25e^{5x}$

Correct Answer: (D) $25e^{5x}$

Solution:
First derivative: $(e^{5x})' = 5e^{5x}$. Second derivative: differentiate again $\Rightarrow 25e^{5x}$.

Quick Tip

Each differentiation of e^{kx} brings down another factor k.

91. $3\int_0^3 x^3\,dx =$ **?**
(A) $\frac{81}{4}$
(B) $\frac{243}{4}$
(C) 0
(D) $\frac{9}{4}$

Correct Answer: (B) $\frac{243}{4}$

Solution:
$\int x^3 dx = \frac{x^4}{4}$. Evaluate from 0 to 3:

$$\left[\frac{x^4}{4}\right]_0^3 = \frac{3^4}{4} - 0 = \frac{81}{4}.$$

Multiply by 3: $3 \cdot \frac{81}{4} = \frac{243}{4}$.

> **Quick Tip**
>
> Do the definite integral first, then multiply by outside constants.

92. $\int_{-1}^{1} \sin^{17} x \cos^3 x \, dx =$ **?**

(A) $\frac{12}{5}$
(B) 0
(C) 1
(D) $\frac{3}{5}$

Correct Answer: (B) 0

Solution:
Parity: $\sin^{17} x$ is odd, $\cos^3 x$ is even. Odd$\times$even $=$ odd. Integral of an odd function over $[-a, a]$ is 0. Limits $[-1, 1]$ are symmetric, so integral $= 0$.

> **Quick Tip**
>
> Check $f(-x)$; odd functions vanish on symmetric limits.

93. $\int_{-1}^{1} x^{17} \, dx =$ **?**

(A) 0
(B) 1
(C) $\frac{3}{17}$
(D) $\frac{14}{3}$

Correct Answer: (A) 0

Solution:
x^{17} is an odd function. On symmetric limits $[-1, 1]$, the positive and negative areas cancel: integral is 0.

> **Quick Tip**
>
> Any odd power of x integrates to 0 over $[-a, a]$.

94. $3\int \sqrt{x}\, dx = ?$

(A) $\dfrac{9}{2}x^{3/2} + k$
(B) $2x^{3/2} + k$
(C) $3x^{3/2} + k$
(D) $\dfrac{2}{3}x^{3/2} + k$

Correct Answer: (B) $2x^{3/2} + k$

Solution:
$\int x^{1/2}dx = \dfrac{x^{3/2}}{3/2} = \dfrac{2}{3}x^{3/2}$. Multiply by 3: $3 \cdot \dfrac{2}{3}x^{3/2} = 2x^{3/2} + k$.

Quick Tip

Power rule: $\int x^n dx = \dfrac{x^{n+1}}{n+1} + C$ for $n \neq -1$.

95. $\displaystyle\int \frac{x+2}{x^2-4}\, dx = ?$

(A) $\log|x+2| + k$
(B) $\log|x^2-4| + k$
(C) $\log|x-2| + k$
(D) $\log\left|\frac{x+2}{x-2}\right| + k$

Correct Answer: (C) $\log|x-2| + k$

Solution:
Factor the denominator: $x^2 - 4 = (x-2)(x+2)$. Partial fractions:

$$\frac{x+2}{x^2-4} = \frac{A}{x-2} + \frac{B}{x+2} \Rightarrow A(x+2) + B(x-2) = x+2.$$

Compare coefficients: $A + B = 1$ and $2A - 2B = 2 \Rightarrow A - B = 1$. Solve: $A = 1,\ B = 0$. Hence

$$\int \frac{x+2}{x^2-4}dx = \int \frac{1}{x-2}dx = \log|x-2| + k.$$

Quick Tip

If the numerator equals a factor of the denominator, partial fractions may collapse to a single term.

96. $\displaystyle\int \frac{3\, dx}{\sqrt{1-9x^2}} = ?$

(A) $\tan^{-1}(3x) + k$
(B) $\sec^{-1}(3x) + k$
(C) $\sin^{-1}(3x) + k$
(D) $\cos^{-1}(3x) + k$

Correct Answer: (C) $\sin^{-1}(3x) + k$

Solution:
Let $u = 3x \Rightarrow du = 3\,dx$. Then

$$\int \frac{3\,dx}{\sqrt{1-9x^2}} = \int \frac{du}{\sqrt{1-u^2}} = \sin^{-1} u + k = \sin^{-1}(3x) + k.$$

Quick Tip

Standard form: $\displaystyle\int \frac{du}{\sqrt{1-u^2}} = \sin^{-1} u + C.$

97. $25 \displaystyle\int \sec 5x \, \tan 5x \, dx =$ **?**
(A) $25 \sec 5x + k$
(B) $5 \sec 5x + k$
(C) $25 \tan 5x + k$
(D) $\sec 5x + k$

Correct Answer: (B) $5 \sec 5x + k$

Solution:
$\displaystyle\int \sec u \, \tan u \, du = \sec u + C$. Let $u = 5x \Rightarrow du = 5\,dx$:

$$\int \sec 5x \, \tan 5x \, dx = \frac{1}{5} \sec 5x + k.$$

Multiply by 25 $\Rightarrow 5 \sec 5x + k$.

Quick Tip

When inside is $5x$, pull out $\frac{1}{5}$; outside constants multiply at the end.

98. $\displaystyle\int \sec^2 4x \, dx =$ **?**
(A) $\tan 4x + k$
(B) $\dfrac{1}{4} \tan 4x + k$
(C) $4 \tan 4x + k$
(D) $8 \tan 4x + k$

Correct Answer: (B) $\dfrac{1}{4} \tan 4x + k$

Solution:
$\displaystyle\int \sec^2(ax)\, dx = \frac{1}{a} \tan(ax) + C$. With $a = 4$, answer is $\frac{1}{4} \tan 4x + k$.

Quick Tip

Differentiate to verify: $\left(\frac{1}{a}\tan ax\right)' = \sec^2 ax$.

99. $\vec{k}\cdot(\vec{i}+\vec{j}) =$ **?**
(A) 0
(B) 1
(C) 2
(D) -1

Correct Answer: (A) 0

Solution:
$\vec{k}$ is perpendicular to both $\vec{i}$ and $\vec{j}$. Dot product of perpendicular vectors is 0. Hence $\vec{k}\cdot\vec{i} = 0 = \vec{k}\cdot\vec{j} \Rightarrow \vec{k}\cdot(\vec{i}+\vec{j}) = 0$.

Quick Tip

Orthogonal basis: $\vec{i}\perp\vec{j}\perp\vec{k}$ with unit lengths.

100. $\displaystyle\int \frac{dx}{1+36x^2} =$ **?**
(A) $6\tan^{-1}(6x)+k$
(B) $3\tan^{-1}(6x)+k$
(C) $\dfrac{1}{6}\tan^{-1}(6x)+k$
(D) $\tan^{-1}(6x)+k$

Correct Answer: (C) $\dfrac{1}{6}\tan^{-1}(6x)+k$

Solution:
Let $u = 6x \Rightarrow du = 6\,dx \Rightarrow dx = \dfrac{du}{6}$. Then

$$\int \frac{dx}{1+(6x)^2} = \frac{1}{6}\int \frac{du}{1+u^2} = \frac{1}{6}\tan^{-1} u + k = \frac{1}{6}\tan^{-1}(6x) + k.$$

Quick Tip

Standard form: $\displaystyle\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C.$