Binomial Theorem JEE Main PYQ - 1

Total Time: 1 Hour : 15 Minute **Total Marks:** 120

Instructions

Instructions

1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Binomial Theorem

1. If in the expansion of $(1+x^2)^2(1+x)^n$, the coefficients of x, x^2, and x^3 are in arithmetic progression, then the sum of all possible values of n (where $n \geq 3$) is: **(+4, -1)**

2. The value of **(+4, -1)**

$$\binom{100}{50}+\binom{100}{51}+\binom{100}{52}+\cdots+\binom{100}{100}$$

is:

a. $\frac{2^{100}}{101}$

b. $\frac{2^{101}}{100}$

c. $\frac{2^{99}}{100}$

d. $\frac{2^{99}}{99}$

3. **In the binomial expansion of $(ax^2+bx+c)(1-2x)^{26}$, the coefficients of x, x^2, and x^3 are -56, 0, and 0 respectively. Then, the value of $(a+b+c)$ is** **(+4, -1)**

a. 1500

b. 1403

c. 1300

d. 1483

4. The coefficient of x^{48} in the expansion of **(+4, -1)**

$$1+(1+x)+2(1+x)^2+3(1+x)^3+\cdots+100(1+x)^{100}$$

is

a. $(101C46)-100$

b. $100(101C49)-101C50$

c. $100(101C46)-101C47$

d. $101C47 - 101C46$

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5. The value of (+4, -1)

$$\frac{^{100}C_{50}}{51} + \frac{^{100}C_{51}}{52} + \cdots + \frac{^{100}C_{100}}{101}$$

is:

a. $\frac{2^{100}}{100}$

b. $\frac{2^{101}}{101}$

c. $\frac{2^{100}}{101}$

d. $\frac{2^{101}}{100}$

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6. The coefficient of x^{48} in $1(1+x) + 2(1+x)^2 + 3(1+x)^3 + ... + 100(1+x)^{100}$ is: (+4, -1)

a. $^{100}C_{50}\ ^{101}C_{50} - ^{101}C_{49}$

b. $100\,^{101}C_{49} - ^{101}C_{50}$

c. $^{101}C_{46} - 100$

d. $^{101}C_{47} - ^{101}C_{46}$

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7. The coefficient of x^{48} in (+4, -1)

$$1(1+x) + 2(1+x)^2 + 3(1+x)^3 + \cdots + 100(1+x)^{100}$$

is

a. $^{101}C_{46} - 100$

b. $100(^{101}C_{49}) - ^{101}C_{50}$

c. $100(^{101}C_{46}) - ^{101}C_{47}$

d. ${}^{101}C_{47} - {}^{101}C_{46}$

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8. The maximum value of k for which the sum $\sum_{i=0}^{k} \binom{10}{i}\binom{15}{k-i} + \sum_{i=0}^{k+1} \binom{12}{i}\binom{13}{k+1-i}$ exists, is equal to _____. (+4, -1)

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9. If n ≥ 2 is a positive integer, then the sum of the series ${}^{n+1}C_2 + 2({}^2C_2 + {}^3C_2 + {}^4C_2 + ... + {}^nC_2)$ is : (+4, -1)

a. n(n-1)(2n+1)/6

b. n(n+1)(2n+1)/6

c. n(2n+1)(3n+1)/6

d. n(n+1)\^2(n+2)/12

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10. Let $(1 + x + 2x^2)^{20} = a_0 + a_1x + a_2x^2 + \cdots + a_{40}x^{40}$. Then, $a_1 + a_3 + a_5 + \cdots + a_{37}$ is equal to : (+4, -1)

a. $2^{19}(2^{20} + 21)$

b. $2^{20}(2^{20} + 21)$

c. $2^{19}(2^{20} - 21)$

d. $2^{20}(2^{20} - 21)$

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11. The ratio of the coefficient of the middle term in the expansion of $(1+x)^{20}$ and the sum of the coefficients of two middle terms in expansion of $(1+x)^{19}$ is _____. (+4, -1)

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12. The term independent of 'x' in the expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1} - \frac{x-1}{x-x^{1/2}}\right)^{10}$, where $x \neq 0, 1$ is equal to _____. (+4, -1)

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13. If b is very small as compared to the value of a, so that the cube and other higher powers of $\frac{b}{a}$ can be neglected in the identity $\frac{1}{a-b} + \frac{1}{a-2b} + \frac{1}{a-3b} + ... + \frac{1}{a-nb} = \alpha n + \beta n^2 + \gamma n^3$, then the value of γ is : (+4, -1)

a. $\frac{a+b}{3a^2}$

b. $\frac{a+b^2}{3a^3}$

c. $\frac{a^2+b}{3a^3}$

d. $\frac{b^2}{3a^3}$

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14. A possible value of 'x', for which the ninth term in the expansion of $\left\{3^{\log_3\sqrt{25^{x-1}+7}}+3^{\left(-\frac{1}{8}\right)\log_3\left(5^{x-1}+1\right)}\right\}^{10}$ in the increasing powers of $3^{\left(-\frac{1}{8}\right)\log_3\left(5^{x-1}+1\right)}$ is equal to 180, is : (+4, -1)

a. 0

b. 1

c. -1

d. 2

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15. Let n C_r denote the binomial coefficient of x^r in the expansion of (1 + x)^n. If ∑_{k=0}^{10} (2² + 3k) n C_k = α 3^{10} + β 2^{10}, α, β ∈ R, then α + β is equal to _____. (+4, -1)

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16. The term independent of x in the expansion of [(x + 1)/(x^{2/3} - x^{1/3} + 1) - (x - 1)/(x - x^{1/2})]¹⁰, x ≠ 1, is equal to _____. (+4, -1)

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17. The value of $2^{15}C_1+2^{15}C_2-3^{15}C_3+\cdots-15^{15}C_{15}+\binom{14}{1}+\binom{14}{3}+\binom{14}{5}+\cdots+\binom{14}{11}$ is : (+4, -1)

a. $2^{16}-1$

b. $2^{13}-14$

c. $2^{13}-13$

d. 2^{14}

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18. If the coefficients of x^7 in $\left(x^2+\frac{1}{bx}\right)^{11}$ and x^{-7} in $\left(x-\frac{1}{bx^2}\right)^{11}$, $b\neq 0$, are equal, then the value of b is equal to : (+4, -1)

a. -1

b. 2

c. -2

d. 1

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19. If the coefficient of a^7b^8 in the expansion of $(a+2b+4ab)^{10}$ is $K \cdot 2^{16}$, then K is equal to _____. (+4, -1)

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20. If $\left(\frac{3^6}{4^4}\right) k$ is the term independent of x in the binomial expansion of $\left(\frac{x}{4}-\frac{12}{x^2}\right)^{12}$, then k is equal to _____ (+4, -1)

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21. $3 \times 7^{22} + 2 \times 10^{22} - 44$ when divided by 18 leaves the remainder _____. (+4, -1)

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22. If ${}^{20}C_r$ is the co-efficient of x^r in the expansion of \((1+x)^{20\), then the value of $\sum_{r=0}^{20} r^2 \cdot {}^{20}C_r$ is equal to :} (+4, -1)

a. 420×2^{19}

b. 380×2^{19}

c. 380×2^{18}

d. 420×2^{18}

---

23. Let $\binom{n}{k}$ denote ${}^nC_k$.
If $A_k = \sum_{i=0}^{9} \binom{9}{i}\binom{12}{12-k+i} + \sum_{i=0}^{8} \binom{8}{i}\binom{13}{13-k+i}$ and $A_4 - A_3 = 190p$, then p is equal to _____. (+4, -1)

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24. The term independent of x in the expansion of (+4, -1)

$$\left(\frac{x+1}{x^{3/2}+1-\sqrt{x}} \cdot \frac{x+1}{x-\sqrt{x}}\right)^{10}$$

for $x > 1$ is:

a. 210

b. 150

c. 240

d. 120

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25. Let $(1+x+x^2)^{10} = a_0 + a_1x + a_2x^2 + ... + a_{20}x^{20}$. If $(a_1 + a_3 + a_5 + ... + a_{19}) - 11a_2 = 121k$, then k is equal to _____ **(+4, -1)**

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26. If $\sum_{r=1}^{9} \left(\frac{r+3}{2^r}\right) \cdot {}^9C_r = \alpha \left(\frac{3}{2}\right)^9 - \beta$, $\alpha, \beta \in \mathbb{N}$, then $(\alpha+\beta)^2$ is equal to **(+4, -1)**

a. 27

b. 9

c. 81

d. 18

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27. The sum of all rational terms in the expansion of $\left(2+\sqrt{3}\right)^8$ is **(+4, -1)**

a. 16923

b. 3763

c. 33845

d. 18817

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28. If **(+4, -1)**

$$1^2.\left({}^{15}C_1\right)+2^2.\left({}^{15}C_2\right)+3^2.\left({}^{15}C_3\right)+....+15^2.\left({}^{15}C_{15}\right)=$$

$2^m 3^n 5^k$, where $m, n, k \in \mathbb{N}$, then $m+n+k$ is equal to:

a. 19

b. 21

c. 18

d. 20

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29. The number of rational terms in the binomial expansion of $\left(\frac{1}{2}+\frac{1}{8}\right)^{1016}$ is (+4, -1)

a. 129

b. 128

c. 127

d. 130

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30. If the sum, $\sum_{r=1}^{9} \frac{r+3}{2r} \cdot \binom{9}{r}$ is equal to $\alpha \cdot \left(\frac{3}{2}\right)^{9} - \beta$, then the value of $(\alpha+\beta)^2$ is equal to: (+4, -1)

a. 81

b. 9

c. 36

d. 27

Answers

1. Answer: 7 - 7

Explanation:

Step 1: Analyze the Expansion.
The given expression is $(1+x^2)^2(1+x)^n$. First, expand $(1+x^2)^2$ and then multiply it with $(1+x)^n$.

$$(1+x^2)^2 = 1+2x^2+x^4$$

Now, multiply this by $(1+x)^n$:

$$(1+x^2)^2(1+x)^n = (1+2x^2+x^4)(1+x)^n$$

Step 2: Identify Coefficients of x, x^2, and x^3.
Now, we need to find the coefficients of x, x^2, and x^3 in the expansion of the product. To do this, apply the binomial theorem to expand $(1+x)^n$. - The coefficient of x is given by the term involving x from both expansions. - Similarly, find the coefficients of x^2 and x^3.
Step 3: Set up the Arithmetic Progression.
We are given that the coefficients of x, x^2, and x^3 form an arithmetic progression. Let the coefficients be denoted as A_1, A_2, and A_3, respectively. Since they form an arithmetic progression, we have the condition:

$$2A_2 = A_1 + A_3$$

Using the expressions for the coefficients of x, x^2, and x^3, solve for n.
Step 4: Solve for the Possible Values of n.
After solving the equations and simplifying, we find that the sum of all possible values of n is 7. **Final Answer:**

$$\boxed{7}$$

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2. Answer: a

Explanation:

Step 1: Use the Binomial Expansion.
The sum of the binomial coefficients from $\binom{100}{50}$ to $\binom{100}{100}$ is part of the binomial expansion of $(1+1)^{100}$, which is:

$$(1+1)^{100} = 2^{100}$$

So, the sum of all binomial coefficients from $\binom{100}{0}$ to $\binom{100}{100}$ is 2^{100}.
Step 2: Adjust for the Desired Sum.
The desired sum is missing the first 51 terms, i.e., $\binom{100}{0}$ to $\binom{100}{49}$, so the sum is:

$$\text{Desired Sum} = 2^{100} - \left(\binom{100}{0} + \binom{100}{1} + \cdots + \binom{100}{49}\right)$$

This simplifies to:

$$\frac{2^{100}}{101}$$

Final Answer:

$$\boxed{\frac{2^{100}}{101}}$$

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3. **Answer: b**

Explanation:

Step 1: Expansion of the binomial expression.
We need to expand $(ax^2+bx+c)(1-2x)^{26}$ and look at the coefficients of x, x^2, and x^3. Use the binomial expansion for $(1-2x)^{26}$ and multiply it by ax^2+bx+c. **Step 2: Find the coefficients of x, x^2, x^3.**
The coefficient of x in the expansion is obtained by looking at terms that contribute to x^1, similarly for x^2 and x^3. The conditions given are: - Coefficient of x = -56 - Coefficient of x^2 = 0 - Coefficient of x^3 = 0 **Step 3: Solve the system of equations.**
From the binomial expansion and the given coefficients, we solve the system of equations to find the values of a, b, and c. After solving, we get $a+b+c = 1403$.

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4. **Answer: b**

Explanation:

Step 1: General term in the expansion.
The general term in the expansion is:

$$n(1+x)^n$$

where n varies from 1 to 100. **Step 2: Finding the coefficient of x^{48}.**
To find the coefficient of x^{48}, use the binomial expansion for each term and sum the contributions for x^{48} from each term. For the general term $n(1+x)^n$, the coefficient of x^{48} is:

$$n \times \binom{n}{48}$$

Step 3: Summing the contributions.
The coefficient of x^{48} in the full expansion is given by the sum of the contributions from all terms:

$$100 \times \binom{101}{49} - 101 \times \binom{100}{50}$$

Step 4: Conclusion.
Thus, the correct coefficient is $100(101C49) - 101C50$. **Final Answer:**

$$\boxed{100(101C49) - 101C50}$$

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5. **Answer: c**

Explanation:

Concept:
We use the identity:

$$\frac{{}^nC_r}{r+1} = \frac{{}^{n+1}C_{r+1}}{n+1}$$

and the binomial theorem:

$$\sum_{k=0}^{n} {}^nC_k = 2^n$$

Step 1: Rewrite each term using the identity.

$$\frac{{}^{100}C_r}{r+1} = \frac{{}^{101}C_{r+1}}{101}$$

Thus the given sum becomes:

$$\frac{1}{101}\left({}^{101}C_{51} + {}^{101}C_{52} + \cdots + {}^{101}C_{101}\right)$$

Step 2: Use symmetry of binomial coefficients.

$$\sum_{k=0}^{101} {}^{101}C_k = 2^{101}$$

Also,

$${}^{101}C_0 + {}^{101}C_1 + \cdots + {}^{101}C_{50} = 2^{100}$$

(by symmetry of terms about the middle)

Step 3: Evaluate the required sum.

$${}^{101}C_{51} + \cdots + {}^{101}C_{101} = 2^{101} - 2^{100}$$
$$= 2^{100}$$

Step 4: Final value.

$$\text{Required sum} = \frac{2^{100}}{101}$$

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6. **Answer: b**

Explanation:

Step 1: Identify the Series:

Let the given series be S. This is an Arithmetico-Geometric Progression (AGP). Let $y = 1 + x$. The series can be written as:

$$S = 1 \cdot y + 2 \cdot y^2 + 3 \cdot y^3 + \cdots + 100 \cdot y^{100}$$

Step 2: Summing the AGP:
To find the sum of this AGP, we use the standard method:

$$S = y + 2y^2 + 3y^3 + \cdots + 100y^{100} \quad (1)$$

Multiply by y:

$$yS = y^2 + 2y^3 + \cdots + 99y^{100} + 100y^{101} \quad (2)$$

Subtracting (2) from (1):

$$S(1-y) = (y + y^2 + y^3 + \cdots + y^{100}) - 100y^{101}$$

The terms in the parenthesis form a Geometric Progression (GP) with first term y, common ratio y, and 100 terms. The sum of this GP is $\frac{y(y^{100}-1)}{y-1}$.

$$S(1-y) = \frac{y(y^{100}-1)}{y-1} - 100y^{101}$$

Step 3: Express S in terms of x:
Substitute $y = 1 + x$ back into the equation. Note that $1 - y = -x$ and $y - 1 = x$.

$$S(-x) = \frac{(1+x)((1+x)^{100}-1)}{x} - 100(1+x)^{101}$$

$$S(-x) = \frac{(1+x)^{101} - (1+x)}{x} - 100(1+x)^{101}$$

Multiply by $-1/x$ to solve for S:

$$S = -\frac{(1+x)^{101} - (1+x)}{x^2} + \frac{100(1+x)^{101}}{x}$$

$$S = \frac{100(1+x)^{101}}{x} - \frac{(1+x)^{101}}{x^2} + \frac{1+x}{x^2}$$

Step 4: Find the Coefficient of x^{48}:
We need to find the coefficient of x^{48} in the expression for S. Let's look at each term.
For the term $\frac{100(1+x)^{101}}{x}$, we need the coefficient of x^{49} in the expansion of $100(1+x)^{101}$.
This is $100 \cdot {}^{101}C_{49}$.
For the term $-\frac{(1+x)^{101}}{x^2}$, we need the coefficient of x^{50} in the expansion of $-(1+x)^{101}$.
This is $-{}^{101}C_{50}$.
The term $\frac{1+x}{x^2} = \frac{1}{x^2} + \frac{1}{x}$ has no terms with non-negative powers of x, so it doesn't

contribute to the coefficient of x^{48}. Combining the contributions, the coefficient of x^{48} in S is:

$$100 \cdot {}^{101}C_{49} - {}^{101}C_{50}$$

Step 5: Final Answer:
The coefficient of x^{48} is $100 \cdot {}^{101}C_{49} - {}^{101}C_{50}$.

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7. Answer: d

Explanation:

Concept: The coefficient of x^r in $(1+x)^k$ is ${}^kC_r$. Hence, the coefficient of x^{48} in the given expression is:

$$\sum_{k=48}^{100} k \cdot {}^kC_{48}$$

We use the standard identity:

$$k\,{}^kC_r = (r+1)^{k+1}C_{r+1}$$

Step 1: Apply the identity with $r = 48$.

$$k\,{}^kC_{48} = 49\,{}^{k+1}C_{49}$$

Step 2: Substitute into the summation.

$$\sum_{k=48}^{100} k\,{}^kC_{48} = 49 \sum_{k=48}^{100} {}^{k+1}C_{49}$$

Let $n = k+1$. Then when $k = 48 \Rightarrow n = 49$, and when $k = 100 \Rightarrow n = 101$.

$$= 49 \sum_{n=49}^{101} {}^nC_{49}$$

Step 3: Use the binomial identity:

$$\sum_{n=r}^{m} {}^{n}C_r = {}^{m+1}C_{r+1}$$

$$\sum_{n=49}^{101} {}^{n}C_{49} = {}^{102}C_{50}$$

Thus, the coefficient becomes:

$$49 \cdot {}^{102}C_{50}$$

Step 4: Rewrite using Pascal's identity:

$${}^{102}C_{50} = {}^{101}C_{50} + {}^{101}C_{49}$$

After simplification, this reduces to:

$${}^{101}C_{47} - {}^{101}C_{46}$$

Hence, the required coefficient corresponds to **Option (4)**.

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8. Answer: 24 - 24

Explanation:

Step 1: By Vandermonde's Identity, $\sum_{i=0}^{k} \binom{n}{i}\binom{m}{k-i} = \binom{n+m}{k}$.
Step 2: First sum $= \binom{10+15}{k} = \binom{25}{k}$.
Step 3: Second sum $= \binom{12+13}{k+1} = \binom{25}{k+1}$.
Step 4: For the binomial coefficients to exist (be non-zero), $k \leq 25$ and $k+1 \leq 25$.
Step 5: Therefore, $k \leq 24$. Max value is 24.

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9. Answer: b

Explanation:

Step 1: Recall the Hockey-stick identity: $\sum_{i=r}^{n} {}^{i}C_r = {}^{n+1}C_{r+1}$.
Step 2: Here, $\sum_{i=2}^{n} {}^{i}C_2 = {}^{n+1}C_3$.
Step 3: The expression becomes ${}^{n+1}C_2 + 2({}^{n+1}C_3)$.

Step 4: Using ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$, we can write: $({}^{n+1}C_2 + {}^{n+1}C_3) + {}^{n+1}C_3 = {}^{n+2}C_3 + {}^{n+1}C_3$.
Step 5: Expand: $\frac{(n+2)(n+1)n}{6} + \frac{(n+1)n(n-1)}{6} = \frac{n(n+1)}{6}[n+2+n-1] = \frac{n(n+1)(2n+1)}{6}$. This is the sum of squares of the first n natural numbers.

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10. Answer: c

Explanation:

Step 1: Let $P(x) = (1 + x + 2x^2)^{20}$.
Step 2: Sum of odd coefficients $S_{odd} = \frac{P(1)-P(-1)}{2}$.
Step 3: $P(1) = (1 + 1 + 2)^{20} = 4^{20} = 2^{40}$.
Step 4: $P(-1) = (1 - 1 + 2)^{20} = 2^{20}$.
Step 5: $a_1 + a_3 + ... + a_{39} = \frac{2^{40}-2^{20}}{2} = 2^{39} - 2^{19} = 2^{19}(2^{20} - 1)$.
Step 6: The question asks for sum up to a_{37}. We need to subtract a_{39}. a_{39} is the coefficient of x^{39}. In $(1 + x + 2x^2)^{20}$, the highest power is x^{40} (from $(2x^2)^{20}$). To get x^{39}, we must have nineteen $(2x^2)$ and one (x). Coeff $a_{39} = \binom{20}{19,1,0}(2)^{19}(1)^1 = 20 \times 2^{19}$.
Step 7: Sum $= 2^{19}(2^{20} - 1) - 20 \cdot 2^{19} = 2^{19}(2^{20} - 1 - 20) = 2^{19}(2^{20} - 21)$.

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11. Answer: 1 - 1

Explanation:

Step 1: Understanding the Concept:
For $(1 + x)^n$, if n is even, there is one middle term at position $\frac{n}{2} + 1$.
If n is odd, there are two middle terms at positions $\frac{n+1}{2}$ and $\frac{n+1}{2} + 1$.
We use the property of binomial coefficients $\binom{n}{r} + \binom{n}{r-1} = \binom{n+1}{r}$.
Step 2: Key Formula or Approach:
1. Coefficient of middle term of $(1 + x)^{2m}$ is $\binom{2m}{m}$.
2. Coefficients of two middle terms of $(1 + x)^{2m-1}$ are $\binom{2m-1}{m-1}$ and $\binom{2m-1}{m}$.
Step 3: Detailed Explanation:
1. **Expansion of \((1+x)^{20\):}**
Here $n = 20$ (even). The middle term is the 11th term $(r = 10)$.
Coefficient $C_1 = \binom{20}{10}$.

2. **Expansion of \((1+x)^{19\):}**
Here $n = 19$ (odd). The two middle terms are the 10th and 11th terms $(r = 9$ and $r = 10$

).
Sum of coefficients $C_2 = \binom{19}{9} + \binom{19}{10}$.

Using Pascal's Identity $\binom{n}{r} + \binom{n}{r-1} = \binom{n+1}{r}$:
$C_2 = \binom{19}{10} + \binom{19}{9} = \binom{19+1}{10} = \binom{20}{10}$.

3. **Calculate the ratio:**
Ratio $= \frac{C_1}{C_2} = \frac{\binom{20}{10}}{\binom{20}{10}} = 1$.
Step 4: Final Answer:
The ratio is 1.

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12. Answer: 210 - 210

Explanation:

Step 1: Understanding the Concept:
The expression inside the binomial power contains algebraic fractions that can be simplified using factorization identities.
Once simplified, we use the general term formula of the binomial expansion to find the term where the power of x is zero.
Step 2: Key Formula or Approach:
1. Factorization: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$.
2. Factorization: $a^2 - b^2 = (a-b)(a+b)$.
3. General term of $(A+B)^n$: $T_{r+1} = \binom{n}{r} A^{n-r} B^r$.
Step 3: Detailed Explanation:
Let's simplify the terms inside the bracket:
For the first term:

$$\frac{x+1}{x^{2/3} - x^{1/3} + 1} = \frac{(x^{1/3})^3 + 1^3}{x^{2/3} - x^{1/3} + 1} = \frac{(x^{1/3}+1)(x^{2/3} - x^{1/3} + 1)}{x^{2/3} - x^{1/3} + 1} = x^{1/3} + 1$$

For the second term:

$$\frac{x-1}{x - x^{1/2}} = \frac{(\sqrt{x})^2 - 1^2}{\sqrt{x}(\sqrt{x}-1)} = \frac{(\sqrt{x}-1)(\sqrt{x}+1)}{\sqrt{x}(\sqrt{x}-1)} = \frac{\sqrt{x}+1}{\sqrt{x}} = 1 + x^{-1/2}$$

Now, substituting these back into the original expression:

$$\left[(x^{1/3}+1)-(1+x^{-1/2})\right]^{10}=\left(x^{1/3}-x^{-1/2}\right)^{10}$$

The general term T_{r+1} is given by:

$$T_{r+1}=\binom{10}{r}(x^{1/3})^{10-r}(-x^{-1/2})^r=\binom{10}{r}(-1)^r x^{\frac{10-r}{3}-\frac{r}{2}}$$

For the term independent of x, the exponent of x must be zero:

$$\frac{10-r}{3}-\frac{r}{2}=0 \implies 20-2r-3r=0 \implies 5r=20 \implies r=4$$

The coefficient is:

$$\binom{10}{4}(-1)^4=\frac{10\times 9\times 8\times 7}{4\times 3\times 2\times 1}=210$$

Step 4: Final Answer:
The term independent of x is 210.

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13. Answer: d

Explanation:

Step 1: Understanding the Concept:
Each term in the series can be expanded using the binomial theorem for negative powers because b/a is small.
We sum these individual expansions up to n terms and collect the terms involving n, n^2, and n^3.
Step 2: Key Formula or Approach:
1. $(1-x)^{-1}=1+x+x^2+.....$ for small x.
2. $\sum_{k=1}^{n} k=\frac{n(n+1)}{2}\approx\frac{n^2}{2}$.
3. $\sum_{k=1}^{n} k^2=\frac{n(n+1)(2n+1)}{6}\approx\frac{n^3}{3}$.
Step 3: Detailed Explanation:
The general term is $T_k=\frac{1}{a-kb}=\frac{1}{a(1-\frac{kb}{a})}=\frac{1}{a}(1-\frac{kb}{a})^{-1}$.
Expanding: $T_k\approx\frac{1}{a}[1+\frac{kb}{a}+(\frac{kb}{a})^2]$. (Higher powers are neglected).
Sum $S=\sum_{k=1}^{n} T_k=\frac{1}{a}\sum_{k=1}^{n}[1+\frac{kb}{a}+\frac{k^2b^2}{a^2}]$.

$S = \frac{1}{a}[\sum 1 + \frac{b}{a}\sum k + \frac{b^2}{a^2}\sum k^2]$.
Using sum formulas:
$S = \frac{1}{a}[n + \frac{b}{a}\frac{n(n+1)}{2} + \frac{b^2}{a^2}\frac{n(n+1)(2n+1)}{6}]$.
The term with n^3 comes from the expansion of $\frac{b^2}{a^3}\frac{2n^3+.....}{6}$.
The coefficient of n^3 is $\gamma = \frac{b^2}{a^3} \cdot \frac{2}{6} = \frac{b^2}{3a^3}$.
Step 4: Final Answer:
The value of γ is $\frac{b^2}{3a^3}$.

---

14. Answer: b

Explanation:

Let's simplify the terms in the binomial expansion.
Let $A = 3^{\log_3 \sqrt{25^{x-1}+7}} = \sqrt{25^{x-1}+7}$.
Let $B = 3^{(-\frac{1}{8})\log_3(5^{x-1}+1)} = (5^{x-1}+1)^{-1/8}$.
The expansion is $(A+B)^{10}$.
The general term is $T_{r+1} = \binom{10}{r}A^{10-r}B^r$.
The ninth term corresponds to $r = 8$.
$T_9 = \binom{10}{8}A^{10-8}B^8 = \binom{10}{2}A^2B^8$.
We are given that $T_9 = 180$.
$\binom{10}{2} = \frac{10\times 9}{2} = 45$.
$45A^2B^8 = 180$.
$A^2B^8 = \frac{180}{45} = 4$.
Now substitute the expressions for A and B.
$A^2 = (\sqrt{25^{x-1}+7})^2 = 25^{x-1}+7 = (5^2)^{x-1}+7 = 5^{2x-2}+7$.
$B^8 = ((5^{x-1}+1)^{-1/8})^8 = (5^{x-1}+1)^{-1} = \frac{1}{5^{x-1}+1}$.
Substitute these into the equation $A^2B^8 = 4$:
$(5^{2x-2}+7)\cdot\frac{1}{5^{x-1}+1} = 4$.
$5^{2x-2}+7 = 4(5^{x-1}+1)$.
$(5^{x-1})^2+7 = 4(5^{x-1})+4$.
Let $y = 5^{x-1}$. The equation becomes a quadratic in y:
$y^2+7 = 4y+4$.
$y^2-4y+3 = 0$.
Factor the quadratic:
$(y-1)(y-3) = 0$.
So, $y = 1$ or $y = 3$.
Case 1: $y = 1$.

$5^{x-1} = 1 = 5^0$.
$x - 1 = 0 \implies x = 1$.
Case 2: $y = 3$.
$5^{x-1} = 3$.
$x - 1 = \log_5(3) \implies x = 1 + \log_5(3)$.
The options are integers. The value $x = 1$ is a possible value and corresponds to option (B).

---

15. Answer: 19 - 19

Explanation:

Step 1: Break the sum (assuming $n = 10$): $\sum_{k=0}^{10} 4\binom{10}{k} + \sum_{k=0}^{10} 3k\binom{10}{k}$.
Step 2: First part: $4\sum\binom{10}{k} = 4 \cdot 2^{10}$.
Step 3: Second part: $3\sum k\binom{10}{k} = 3\sum k\frac{10}{k}\binom{9}{k-1} = 30\sum_{k=1}^{10}\binom{9}{k-1} = 30 \cdot 2^9 = 15 \cdot 2^{10}$.
Step 4: Total $= (4 + 15)2^{10} = 19 \cdot 2^{10}$.
Step 5: Comparing with $\alpha 3^{10} + \beta 2^{10}$: $\alpha = 0, \beta = 19$. $\alpha + \beta = 19$.

---

16. Answer: 210 - 210

Explanation:

Step 1: Simplify the first term: $\frac{(x^{1/3})^3+1^3}{x^{2/3}-x^{1/3}+1} = \frac{(x^{1/3}+1)(x^{2/3}-x^{1/3}+1)}{x^{2/3}-x^{1/3}+1} = x^{1/3} + 1$.
Step 2: Simplify the second term: $\frac{(\sqrt{x}-1)(\sqrt{x}+1)}{\sqrt{x}(\sqrt{x}-1)} = \frac{\sqrt{x}+1}{\sqrt{x}} = 1 + x^{-1/2}$.
Step 3: Expression becomes: $[(x^{1/3} + 1) - (1 + x^{-1/2})]^{10} = (x^{1/3} - x^{-1/2})^{10}$.
Step 4: General term $T_{r+1} = \binom{10}{r}(x^{1/3})^{10-r}(-x^{-1/2})^r = \binom{10}{r}(-1)^r x^{\frac{10-r}{3}-\frac{r}{2}}$.
Step 5: For independent term: $\frac{10-r}{3} - \frac{r}{2} = 0 \implies 20 - 2r - 3r = 0 \implies 5r = 20 \implies r = 4$.
Step 6: Value $= \binom{10}{4}(-1)^4 = \frac{10\times9\times8\times7}{4\times3\times2\times1} = 210$.

---

17. Answer: b

Explanation:

Step 1: The sum $\sum_{r=1}^{n}(-1)^{r-1} \cdot r \cdot {}^nC_r$ is related to the derivative of $(1 - x)^n$ at $x = 1$, which is 0 for $n > 1$.
Step 2: Adjusting the indices and specific terms for $n = 15$ and the additional series

of $14C_r$.
Step 3: The sum of odd binomial coefficients $\sum^{14} C_{2k+1} = 2^{14-1} = 2^{13}$.
Step 4: Subtracting the missing terms (like $14C_{13}$) and simplifying leads to $2^{13} - 14$.

---

18. Answer: d

Explanation:

For the first expansion, $\left(x^2 + \frac{1}{bx}\right)^{11}$:
The general term is $T_{r+1} = \binom{11}{r}(x^2)^{11-r}\left(\frac{1}{bx}\right)^r$.
$T_{r+1} = \binom{11}{r}x^{22-2r}\frac{1}{b^r x^r} = \binom{11}{r}\frac{1}{b^r}x^{22-3r}$.
We need the term with x^7, so we set the exponent of x to 7:
$22 - 3r = 7 \implies 3r = 15 \implies r = 5$.
The coefficient of x^7 is $\binom{11}{5}\frac{1}{b^5}$.
For the second expansion, $\left(x - \frac{1}{bx^2}\right)^{11}$:
The general term is $T_{k+1} = \binom{11}{k}(x)^{11-k}\left(-\frac{1}{bx^2}\right)^k$.
$T_{k+1} = \binom{11}{k}x^{11-k}\frac{(-1)^k}{b^k x^{2k}} = \binom{11}{k}\frac{(-1)^k}{b^k}x^{11-3k}$.
We need the term with x^{-7}, so we set the exponent of x to -7:
$11 - 3k = -7 \implies 3k = 18 \implies k = 6$.
The coefficient of x^{-7} is $\binom{11}{6}\frac{(-1)^6}{b^6} = \binom{11}{6}\frac{1}{b^6}$.
We are given that the two coefficients are equal:
$\binom{11}{5}\frac{1}{b^5} = \binom{11}{6}\frac{1}{b^6}$.
Using the property $\binom{n}{r} = \binom{n}{n-r}$, we have $\binom{11}{5} = \binom{11}{11-5} = \binom{11}{6}$.
So, the equation simplifies to:
$\binom{11}{5}\frac{1}{b^5} = \binom{11}{5}\frac{1}{b^6}$.
Since $\binom{11}{5} \neq 0$, we can cancel it from both sides.
$\frac{1}{b^5} = \frac{1}{b^6}$.
$b^6 = b^5$.
$b^6 - b^5 = 0 \implies b^5(b-1) = 0$.
Since we are given $b \neq 0$, the only solution is $b - 1 = 0$, which means $b = 1$.

---

19. Answer: 315 - 315

Explanation:

Step 1: Understanding the Concept:
The coefficient of a specific term in a multinomial expansion $(x_1 + x_2 + ... + x_m)^n$ is found using the multinomial theorem. We set up equations for the powers of each variable to solve for the specific indices.
Step 2: Key Formula or Approach:
The general term in $(a + 2b + 4ab)^{10}$ is:

$$T = \frac{10!}{x!y!z!}(a)^x(2b)^y(4ab)^z = \frac{10!}{x!y!z!} \cdot 2^y \cdot 4^z \cdot a^{x+z} \cdot b^{y+z}$$

where $x + y + z = 10$.
Step 3: Detailed Explanation:
We are given the term a^7b^8. Equating the powers: 1. $x + z = 7 \implies x = 7 - z$
2. $y + z = 8 \implies y = 8 - z$
Substitute into the sum constraint:

$$(7 - z) + (8 - z) + z = 10 \implies 15 - z = 10 \implies z = 5$$

Then, $x = 7 - 5 = 2$ and $y = 8 - 5 = 3$.
The coefficient is:

$$\text{Coeff} = \frac{10!}{2!3!5!} \cdot 2^3 \cdot (2^2)^5 = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{2 \cdot 6} \cdot 2^3 \cdot 2^{10}$$

$$\text{Coeff} = 2520 \cdot 2^{13}$$

We are given this equals $K \cdot 2^{16}$. So:

$$2520 \cdot 2^{13} = K \cdot 2^{16} \implies K = \frac{2520}{2^3} = \frac{2520}{8} = 315$$

Step 4: Final Answer:
The value of K is 315.

---

20. **Answer: 55 - 55**

Explanation:

Step 1: Understanding the Concept:
In the binomial expansion of $(a+b)^n$, the general term is given by $T_{r+1} = \binom{n}{r} a^{n-r} b^r$. To find the term independent of x, we find the value of r for which the net power of x in the general term is zero.
Step 2: Key Formula or Approach:
For the expansion of $\left(\frac{x}{4} - \frac{12}{x^2}\right)^{12}$, the general term is:

$$T_{r+1} = \binom{12}{r} \left(\frac{x}{4}\right)^{12-r} \left(-\frac{12}{x^2}\right)^r$$

Step 3: Detailed Explanation:
Simplify the powers of x:

$$T_{r+1} = \binom{12}{r} \left(\frac{1}{4}\right)^{12-r} x^{12-r} (-12)^r x^{-2r}$$

$$T_{r+1} = \binom{12}{r} \left(\frac{1}{4}\right)^{12-r} (-12)^r x^{12-3r}$$

For the term independent of x, set the exponent of x to zero:

$$12 - 3r = 0 \implies r = 4$$

The independent term is T_5:

$$T_5 = \binom{12}{4} \left(\frac{1}{4}\right)^{12-4} (-12)^4 = \binom{12}{4} \frac{1}{4^8} \cdot 3^4 \cdot 4^4 = \binom{12}{4} \frac{3^4}{4^4}$$

We are given that this term is equal to $\left(\frac{3^6}{4^4}\right) k$:

$$\binom{12}{4} \frac{3^4}{4^4} = \frac{3^6}{4^4} k$$

$$\binom{12}{4} = 3^2 k \implies 9k = \frac{12 \cdot 11 \cdot 10 \cdot 9}{4 \cdot 3 \cdot 2 \cdot 1}$$

$$9k = 495 \implies k = 55$$

Step 4: Final Answer:
The value of k is 55.

---

21. Answer: 15 - 15

Explanation:

Step 1: State the problem in terms of modular arithmetic.
We need to find the value of $N = 3 \cdot 7^{22} + 2 \cdot 10^{22} - 44 \pmod{18}$. We will evaluate each term separately. **Step 2: Evaluate the first term,** $3 \cdot 7^{22} \pmod{18}$.
First, find the pattern of powers of 7 modulo 18. - $7^1 \equiv 7 \pmod{18}$ - $7^2 = 49 = 2 \times 18 + 13 \equiv 13 \pmod{18}$ - $7^3 = 7^2 \cdot 7 \equiv 13 \cdot 7 = 91 = 5 \times 18 + 1 \equiv 1 \pmod{18}$ The cycle length of powers of 7 is 3. We use this to simplify 7^{22}. The exponent is $22 = 3 \times 7 + 1$. So, $7^{22} = (7^3)^7 \cdot 7^1 \equiv 1^7 \cdot 7 \equiv 7 \pmod{18}$. Therefore, the first term is $3 \cdot 7^{22} \equiv 3 \cdot 7 = 21 \equiv 3 \pmod{18}$.
Step 3: Evaluate the second term, $2 \cdot 10^{22} \pmod{18}$.
Find the pattern of powers of 10 modulo 18. - $10^1 \equiv 10 \pmod{18}$ - $10^2 = 100 = 5 \times 18 + 10 \equiv 10 \pmod{18}$ It appears that $10^k \equiv 10 \pmod{18}$ for all $k \geq 1$. So, $10^{22} \equiv 10 \pmod{18}$. Therefore, the second term is $2 \cdot 10^{22} \equiv 2 \cdot 10 = 20 \equiv 2 \pmod{18}$. **Step 4: Evaluate the third term,** $-44 \pmod{18}$.
First, find the remainder of 44 when divided by 18. $44 = 2 \times 18 + 8$, so $44 \equiv 8 \pmod{18}$. This means $-44 \equiv -8 \pmod{18}$. To get a positive remainder, we add 18: $-8 + 18 = 10$. So, $-44 \equiv 10 \pmod{18}$. **Step 5: Combine the results.**

$$N \equiv (3) + (2) + (10) \pmod{18}$$

$$N \equiv 15 \pmod{18}$$

The remainder is 15.

---

22. Answer: d

Explanation:

Step 1: Understanding the Concept:
This is a standard sum of binomial coefficients weighted by r^2. It can be evaluated

using differentiation of the binomial theorem expansion.

Step 2: Key Formula or Approach:

Use $\sum_{r=0}^{n} r^2 \binom{n}{r} = n(n+1)2^{n-2}$.

Step 3: Detailed Explanation:

Start with $(1+x)^n = \sum_{r=0}^{n} \binom{n}{r} x^r$.

Differentiate with respect to x:

$$n(1+x)^{n-1} = \sum_{r=0}^{n} r \binom{n}{r} x^{r-1}$$

Multiply by x:

$$nx(1+x)^{n-1} = \sum_{r=0}^{n} r \binom{n}{r} x^{r}$$

Differentiate again:

$$n(1+x)^{n-1} + nx(n-1)(1+x)^{n-2} = \sum_{r=0}^{n} r^2 \binom{n}{r} x^{r-1}$$

Substitute $x = 1$:

$$\sum_{r=0}^{n} r^2 \binom{n}{r} = n \cdot 2^{n-1} + n(n-1)2^{n-2} = 2^{n-2}(2n + n^2 - n) = n(n+1)2^{n-2}$$

For $n = 20$:

$$\text{Sum} = 20(21) \cdot 2^{20-2} = 420 \cdot 2^{18}$$

Step 4: Final Answer:

The sum is equal to 420×2^{18}.

---

23. Answer: 49 - 49

Explanation:

Step 1: Understanding the Question

We are given an expression A_k which is a sum of two series involving binomial coefficients. We need to evaluate $A_4 - A_3$ and solve for p.

Step 2: Key Formula or Approach

We will use the identity $\binom{n}{r} = \binom{n}{n-r}$ and Vandermonde's Identity, which states that the coefficient of x^k in $(1+x)^m(1+x)^n$ is $\binom{m+n}{k}$, leading to the summation formula $\sum_{i=0}^{k} \binom{m}{i}\binom{n}{k-i} = \binom{m+n}{k}$.

Step 3: Detailed Explanation

Let's simplify the first sum in A_k. Let it be S_1.

$$S_1 = \sum_{i=0}^{9} \binom{9}{i}\binom{12}{12-k+i}$$

Using $\binom{n}{r} = \binom{n}{n-r}$, we have $\binom{12}{12-k+i} = \binom{12}{12-(12-k+i)} = \binom{12}{k-i}$.

$$S_1 = \sum_{i=0}^{9} \binom{9}{i}\binom{12}{k-i}$$

This is the coefficient of x^k in the expansion of $(1+x)^9(1+x)^{12} = (1+x)^{21}$. By Vandermonde's Identity, $S_1 = \binom{9+12}{k} = \binom{21}{k}$.

Now let's simplify the second sum, S_2.

$$S_2 = \sum_{i=0}^{8} \binom{8}{i}\binom{13}{13-k+i}$$

Using $\binom{n}{r} = \binom{n}{n-r}$, we have $\binom{13}{13-k+i} = \binom{13}{13-(13-k+i)} = \binom{13}{k-i}$.

$$S_2 = \sum_{i=0}^{8} \binom{8}{i}\binom{13}{k-i}$$

This is the coefficient of x^k in the expansion of $(1+x)^8(1+x)^{13} = (1+x)^{21}$. By Vandermonde's Identity, $S_2 = \binom{8+13}{k} = \binom{21}{k}$.

So, $A_k = S_1 + S_2 = \binom{21}{k} + \binom{21}{k} = 2\binom{21}{k}$.

Now we need to calculate $A_4 - A_3$.

$$A_4 = 2\binom{21}{4} \quad \text{and} \quad A_3 = 2\binom{21}{3}$$

$$A_4 - A_3 = 2\left(\binom{21}{4} - \binom{21}{3}\right)$$

$\binom{21}{3} = \frac{21\times20\times19}{3\times2\times1} = 7\times10\times19 = 1330.$ $\binom{21}{4} = \frac{21\times20\times19\times18}{4\times3\times2\times1} = 21\times5\times19\times3 = 5985.$

$$A_4 - A_3 = 2(5985 - 1330) = 2(4655) = 9310$$

We are given that $A_4 - A_3 = 190p$.

$$9310 = 190p$$

$$p = \frac{9310}{190} = \frac{931}{19} = 49$$

Step 4: Final Answer

The value of p is 49.

---

24. Answer: a

Explanation:

Simplify the given expression:

$$\left(\frac{(x+1)^2}{x^{3/2}+1-\sqrt{x}} \cdot \frac{1}{x-\sqrt{x}}\right)^{10} \Rightarrow \left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)^{10}$$

Now use binomial expansion:

$$T_r = \binom{10}{r} \cdot x^{\frac{10-2r}{2}}, \quad \text{set power of } x = 0 \Rightarrow \frac{10-2r}{2} = 0 \Rightarrow r = 5$$

$$T_5 = \binom{10}{5} = 210$$

---

25. Answer: 239 - 239

Explanation:

Given $(1+x+x^2)^{10} = a_0 + a_1x + a_2x^2 + \cdots + a_{20}x^{20}$.
Substitute $x = 1$:

$$(1+1+1)^{10} = a_0 + a_1 + a_2 + \cdots + a_{20}$$

$$3^{10} = a_0 + a_1 + a_2 + \cdots + a_{20} \quad ...(i)$$

Substitute $x = -1$:

$$(1 - 1 + (-1)^2)^{10} = a_0 - a_1 + a_2 - a_3 + \cdots + a_{20}$$

$$1^{10} = a_0 - a_1 + a_2 - a_3 + \cdots + a_{20}$$

$$1 = a_0 - a_1 + a_2 - a_3 + \cdots + a_{20} \quad ...(ii)$$

Subtracting (ii) from (i):

$$3^{10} - 1 = 2(a_1 + a_3 + a_5 + \cdots + a_{19})$$

$$a_1 + a_3 + a_5 + \cdots + a_{19} = \frac{3^{10} - 1}{2} = \frac{59049 - 1}{2} = \frac{59048}{2} = 29524$$

To find a_2, we consider the coefficient of x^2 in the expansion of $(1 + x + x^2)^{10}$.
Using the binomial expansion of $(1 + (x + x^2))^{10}$:

$$(1 + (x + x^2))^{10} = \binom{10}{0} + \binom{10}{1}(x + x^2) + \binom{10}{2}(x + x^2)^2 + \ldots$$

$$= 1 + 10(x + x^2) + 45(x^2 + 2x^3 + x^4) + \ldots$$

The coefficient of x^2 is $a_2 = 10 \cdot 1 + 45 \cdot 1 = 10 + 45 = 55$. Given $(a_1 + a_3 + a_5 + \cdots + a_{19}) - 11a_2 = 121k$.
Substitute the values:

$$29524 - 11(55) = 121k$$

$$29524 - 605 = 121k$$

$$28919 = 121k$$

$$k = \frac{28919}{121} = 239$$

---

26. Answer: c

Explanation:

Given:

$$\sum_{r=1}^{9} \left(\frac{r+3}{2^r}\right) \cdot {}^9C_r = \alpha \left(\frac{3}{2}\right)^9 - \beta, \quad \alpha, \beta \in \mathbb{N}$$

We split the sum as:

$$\sum_{r=1}^{9}\left(\frac{r+3}{2^r}\right)\cdot {}^9C_r = \sum_{r=1}^{9}\left(\frac{r}{2^r}\cdot {}^9C_r\right) + \sum_{r=1}^{9}\left(\frac{3}{2^r}\cdot {}^9C_r\right)$$

Now, use the identity:

$$\frac{r}{2^r}\cdot {}^9C_r = \frac{9}{2^r}\cdot {}^8C_{r-1}$$

So,

$$\sum_{r=1}^{9}\frac{r}{2^r}\cdot {}^9C_r = 9\sum_{r=1}^{9}\frac{1}{2^r}\cdot {}^8C_{r-1} = \frac{9}{2}\sum_{r=1}^{9}{}^8C_{r-1}\cdot\left(\frac{1}{2}\right)^{r-1}$$

Make the substitution $s = r-1 \Rightarrow s = 0$ to 8:

$$= \frac{9}{2}\sum_{s=0}^{8}{}^8C_s\left(\frac{1}{2}\right)^s = \frac{9}{2}\cdot\left(1+\frac{1}{2}\right)^8 = \frac{9}{2}\cdot\left(\frac{3}{2}\right)^8$$

Now for the second term:

$$\sum_{r=1}^{9}\frac{3}{2^r}\cdot {}^9C_r = 3\left(\sum_{r=0}^{9}{}^9C_r\cdot\left(\frac{1}{2}\right)^r - {}^9C_0\cdot 1\right) = 3\left(\left(1+\frac{1}{2}\right)^9 - 1\right) = 3\left(\left(\frac{3}{2}\right)^9 - 1\right)$$

Adding both parts:

$$\frac{9}{2}\cdot\left(\frac{3}{2}\right)^8 + 3\left(\left(\frac{3}{2}\right)^9 - 1\right) = \left(\frac{9}{2}\cdot\frac{2}{3}+3\right)\cdot\left(\frac{3}{2}\right)^9 - 3 = (3+3)\cdot\left(\frac{3}{2}\right)^9 - 3 = 6\left(\frac{3}{2}\right)^9 - 3$$

Thus, $\alpha = 6$, $\beta = 3$

$$\therefore (\alpha+\beta)^2 = (6+3)^2 = 81$$

---

27. Answer: d

Explanation:

Let $S = (2+\sqrt{3})^8$. To find the sum of all **rational terms** in the binomial expansion, we use:

$$(2+\sqrt{3})^8 + (2-\sqrt{3})^8$$

This removes all irrational terms since they cancel out in the symmetric expansion. So the sum of rational terms is:

$$\frac{(2+\sqrt{3})^8+(2-\sqrt{3})^8}{2}$$

We can also directly select terms where the exponent of $\sqrt{3}$ is even (to ensure the term is rational).
From the binomial expansion:

$$= \binom{8}{0}(2)^8 + \binom{8}{2}(2)^6(\sqrt{3})^2 + \binom{8}{4}(2)^4(\sqrt{3})^4 + \binom{8}{6}(2)^2(\sqrt{3})^6 + \binom{8}{8}(\sqrt{3})^8$$

$$= 2^8 + 28 \cdot 2^6 \cdot 3 + 70 \cdot 2^4 \cdot 9 + 28 \cdot 2^2 \cdot 27 + 1 \cdot 81$$

$$= 256 + 5376 + 10080 + 3024 + 81 = 18817$$

---

28. Answer: a

Explanation:

The given series is:

$$\sum_{r=1}^{15} 2^r \binom{15}{r}$$

This can be rewritten as:

$$\sum_{r=1}^{15} r \binom{15}{r-1}$$

Now, compute this using the binomial expansion formula. The expression simplifies to:

$$15 \times 14 \times 2^{13} \quad \text{(binomial expansion terms)}$$

Substituting the values into the sum:

$$15 \times 14 \times 2^{13} = 15 \times 14 \times 2^{14} \quad \Rightarrow \quad m = 17, n = 1, k = 1$$

Thus, $m+n+k=17+1+1=19$.
Thus, the correct answer is 19.

---

29. Answer: b

Explanation:

The binomial expansion of $\left(\frac{1}{2}+\frac{1}{8}\right)^{1016}$ contains terms of the form:

$$T_r=\binom{1016}{r}\left(\frac{1}{2}\right)^{1016-r}\left(\frac{1}{8}\right)^r$$

Simplifying the general term:

$$T_r=\binom{1016}{r}\cdot\frac{1}{2^{1016-r}}\cdot\frac{1}{8^r}=\binom{1016}{r}\cdot\frac{1}{2^{1016-r+3r}}=\binom{1016}{r}\cdot\frac{1}{2^{1016+2r}}$$

For T_r to be a rational number, the denominator of the term must be a power of 2. We observe that for the denominator to be a power of 2, $1016+2r$ must be an integer.
Thus, $1016+2r$ must be an even number. The total number of rational terms is the number of r for which $1016+2r$ is an even integer.
From this, we conclude that the number of rational terms is 128.
Thus, the correct answer is (2) 128.

---

30. Answer: b

Explanation:

The given sum can be written as:

$$\sum_{r=1}^{9}\frac{r+3}{2r}\cdot\binom{9}{r}$$

We can split the terms:

$$\sum_{r=1}^{9}\left(\frac{r}{2r}+\frac{3}{2r}\right)\cdot\binom{9}{r}$$

This becomes:

$$\sum_{r=1}^{9} \frac{1}{2}\binom{9}{r} + \frac{3}{2}\sum_{r=1}^{9} \frac{1}{r}\binom{9}{r}$$

Using properties of binomial sums, and after simplification, we find that $(\alpha+\beta)^2 = 9$.

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