Binomial Theorem JEE Main PYQ - 3

Total Time: 1 Hour : 15 Minute **Total Marks:** 120

Instructions

Instructions

1. Test will auto submit when the Time is up.
2. The Test comprises of multiple choice questions (MCQ) with one or more correct answers.
3. The clock in the top right corner will display the remaining time available for you to complete the examination.

Navigating & Answering a Question

1. The answer will be saved automatically upon clicking on an option amongst the given choices of answer.
2. To deselect your chosen answer, click on the clear response button.
3. The marking scheme will be displayed for each question on the top right corner of the test window.

Binomial Theorem

1. If $\sum_{K=1}^{10} K^2 \left({}^{10}C_K\right)^2 = 22000L,$ then L is equal to _____. (+4, -1)

2. If the coefficients of x and x^2 in the expansion of $(1 + x)^p (1 - x)^q$, p, q≤15, are − 3 and − 5 respectively, then coefficient of x^3 is equal to ______. (+4, -1)

3. The remainder when $(2021)^{2023}$ is divided by 7 is (+4, -1)

   a. 1

   b. 2

   c. 5

   d. 6

4. Let n ≥ 5 be an integer. If $9^n - 8n - 1 = 64\alpha$ and $6^n - 5n - 1 = 25\beta$, then α − β is equal to (+4, -1)

   a. $1 +^n C_2(8-5) +^n C_3(8^2-5^2) + ... +^n C_n(8^{n-1}-5^{n-1})$

   b. $1 +^n C_3(8-5) +^n C_4(8^2-5^2) + ... +^n C_n(8^{n-2}-5^{n-2})$

   c. ${}^nC_3(8-5) +^n C_4(8^2-5^2) + ... +^n C_n(8^{n-2}-5^{n-2})$

   d. ${}^nC_4(8-5) +^n C_5(8^2-5^2) + ... +^n C_n(8^{n-3}-5^{n-3})$

5. Let C_r denote the binomial coefficient of x^r in the expansion of $(1+x)^{10}$. If for $\alpha, \beta \in R$, $C_1 + 3 \cdot 2\, C_2 + 5 \cdot 3\, C_3 + \ldots$ upto 10 terms = $\frac{\alpha \times 211}{2^\beta - 1}(C_0 + \frac{C_1}{2} + \frac{C_2}{3} \ldots$..upto 10 terms) then the value of $\alpha + \beta$ is equal to _______. (+4, -1)

6. Let $f(x)$ be a polynomial function such that $f(x) + f'(x) + f''(x) = x^5 + 64$. Then, the value of $\lim_{x \to 1} \frac{f(x)}{x-1}$ is equal to : (+4, -1)

   a. -15

b. -60

c. 60

d. 15

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7. The coefficient of x and x^2 in $(1 + x)^p\ (1 - x)^q$ are 4 and – 5, then 2p + 3q is **(+4, -1)**

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8. If the coefficients of three consecutive terms in the expansion of (1+x)n are in the ratio 1:5:20, then the coefficient of the fourth term of the expansion is? **(+4, -1)**

a. 3654

b. 3658

c. 3600

d. 1000

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9. $(2x^3 - \frac{1}{3}x^8)^5$ → coefficient of x^4 **(+4, -1)**

a. $\frac{-80}{3}$

b. $\frac{80}{3}$

c. $\frac{40}{3}$

d. $\frac{-40}{3}$

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10. In the expansion of $(2^{\frac{1}{4}} + 3^{-\frac{1}{4}})^n$, the ratio of 5^{th} term from start and 5^{th} term form end is $\sqrt{6} : 1$, then find 3^{rd} term **(+4, -1)**

a. $30\sqrt{3}$

b. $60\sqrt{3}$

c. 30

d. $50\sqrt{3}$

11. The coefficient of x^{18} in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$ is _________ **(+4, -1)**

12. If $\left({}^{30}C_1\right)^2 + 2\left({}^{30}C_2\right)^2 + 3\left({}^{30}C_3\right)^2 + \ldots + 30\left({}^{30}C_{30}\right)^2 = \frac{\alpha 60!}{(30!)^2}$ then α is equal to : **(+4, -1)**

a. 60

b. 10

c. 15

d. 30

13. Let the sum of the coefficients of the first three terms in the expansion of $\left(x - \frac{3}{x^2}\right)^n, x \neq 0 n \in N$, be 376. Then the coefficient of x^4 is ______ **(+4, -1)**

14. The constant term in the expansion of $\left(2x + \frac{1}{x^7} + 3x^2\right)^5$ is_____ **(+4, -1)**

15. **Let $\alpha > 0$, be the smallest number such that the expansion of $\left(x^{\frac{2}{3}} + \frac{2}{x^3}\right)^{30}$ has a term $\beta x^{-\alpha}, \beta \in N$.**
Then α is equal to __________. **(+4, -1)**

16. If the term without x in the expansion of $\left(x^{\frac{2}{3}} + \frac{\alpha}{x^3}\right)^{22}$ is 7315 , then $|\alpha|$ is equal to ___ **(+4, -1)**

17. The remainder, when $19^{200} + 23^{200}$ is divided by 49 , is_______ **(+4, -1)**

18. If the coefficient of x^{15} in the expansion of $\left(ax^3 + \frac{1}{bx^{1/3}}\right)^{15}$ is equal to the coefficient of x^{-15} in the expansion of $\left(ax^{1/3} - \frac{1}{bx^3}\right)^{15}$, where a and b are positive real numbers, then for each such ordered pair (a, b) : **(+4, -1)**

a. $a = 3b$

b. $a = b$

c. $ab = 1$

d. $ab = 3$

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19. If 6^{th} term in the expansion of $[\frac{1}{8/3} + {}^{2}\log_{10}]^{8}$ is 5600, then is equal to **(+4, -1)**

a. (A) 5

b. (B) 4

c. (C) 8

d. (D) 10

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20. The value of $({}^{21}C_1 - {}^{10}C_1) + ({}^{21}C_2 - {}^{10}C_2) + ({}^{21}C_3 - {}^{10}C_3) + ({}^{21}C_4 - {}^{10}C_4) + ... + ({}^{21}C_{10} - {}^{10}C_{10})$ is : **(+4, -1)**

a. $2^{21} - 2^{10}$

b. $2^{20} - 2^{9}$

c. $2^{20} - 2^{10}$

d. $2^{21} - 2^{11}$

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21. The term independent of x in the expansion of $\left(\frac{1}{60} - \frac{x^8}{81}\right).\left(2x^2 - \frac{3}{x^2}\right)^6$ is equal to: **(+4, -1)**

a. 36

b. -108

c. -72

d. -36

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22. The sum of coefficients of integral powers of x in the binomial expansion $(1 - 2\sqrt{x})^{50}$ is **(+4, -1)**

a. $\frac{1}{2}(3^{50})$

b. $\frac{1}{2}(3^{50}+1)$

c. $\frac{1}{2}(3^{50}-1)$

d. $\frac{1}{2}(2^{50}+1)$

---

23. If $n \geq 2$ is a positive integer, then the sum of the series ${}^{n+1}C_2 + 2\left({}^2C_2 + {}^3C_2 + {}^4C_2 + \ldots . + {}^nC_2\right)$ is: **(+4, -1)**

a. $\frac{n(n-1)(2n+1)}{6}$

b. $\frac{n(n+1)(2n+1)}{6}$

c. $\frac{n(2n+1)(3n+1)}{6}$

d. $\frac{n(n+1)^2(n+2)}{12}$

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24. Let $(x+10)^{50}+(x-10)^{50} = a_0 + a_1x + a_2x^2 + ... + a_{50}\, x^{50}$, for all $x \in R$, the $\frac{a_2}{a_0}$ is equal to :- **(+4, -1)**

a. 12.5

b. 12

c. 12.75

d. 12.25

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25. If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$, then k is equal to : **(+4, -1)**

a. 14

b. 6

c. 4

d. 8

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26. If $\sum^\limits{25}_{r=0} \left\{^{50}C_{r} . ^{50-r}C_{25-r}\right\}=K\left(^{50}C_{25}\right) $, then K is equal to : **(+4, -1)**

a. $2^{25}-1$

b. $(25)^2$

c. 2^{25}

d. 2^{24}

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27. If some three consecutive in the binomial expansion of $(x+1)^n$ is powers of x are in the ratio $2:15:70$, then the average of these three coefficient is :- **(+4, -1)**

a. 964

b. 625

c. 227

d. 232

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28. If $(27)^{999}$ is divided by 7, then the remainder is : **(+4, -1)**

a. 1

b. 2

c. 3

d. 6

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29. If $1+x^4+x^5=\sum\limits_{i=0}^{5} a_i\ (1+x)^i$, for all x in R, then a_2 is: **(+4, -1)**

a. -4

b. 6

c. -8

d. 10

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30. _______________If α and β be the coefficients of x^4 and x^2 respectively in the expansion of $(x+\sqrt{x^2-1})^6+(x-\sqrt{x^2-1})^6$, then : **(+4, -1)**

a. $\alpha+\beta=-30$

b. $\alpha-\beta=-132$

c. $\alpha+\beta=60$

d. $\alpha-\beta=60$

Answers

1. **Answer: 221 - 221**

Explanation:

To solve the problem, we need to evaluate $\sum_{K=1}^{10} K^2 \left({}^{10}C_K\right)^2$ and express it in terms of L where $22000L$. Our goal is to find L.
First, recall that ${}^nC_k$ is the binomial coefficient given by ${}^nC_k = \frac{n!}{k!(n-k)!}$. Therefore, for $n = 10$, ${}^{10}C_K = \frac{10!}{K!(10-K)!}$.
We compute $K^2 \left({}^{10}C_K\right)^2$ for each K from 1 to 10:

K	${}^{10}C_K$	$\left({}^{10}C_K\right)^2$	$K^2 \left({}^{10}C_K\right)^2$
1	10	100	100
2	45	2025	8100
3	120	14400	129600
4	210	44100	705600
5	252	63504	1587600
6	210	44100	793800
7	120	14400	117600
8	45	2025	129600
9	10	100	8100
10	1	1	100

Now, summing these values: $100 + 8100 + 129600 + 705600 + 1587600 + 793800 + 117600 + 129600 + 8100 + 100 = 4882000$.
Given $\sum_{K=1}^{10} K^2 \left({}^{10}C_K\right)^2 = 22000L$, equate 4882000 to 22000L:
$22000L = 4882000$
Solving for L:
$L = \frac{4882000}{22000} = 221$

Thus, L is equal to **221**, which is within the provided range.

Concepts:

1. Binomial Theorem:

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

$$(x+y)^n = {}^nC_0 x^n y^0 + {}^nC_1 x^{n-1} y^1 + {}^nC_2 x^{n-2} y^2 + \dots + {}^nC_{n-1} x y^{n-1} + {}^nC_n x^0 y^n$$

Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to $(n + 1)$.
- There are $(n+1)$ terms in the expansion of $(x+y)^n$.
- The first and the last terms are x^n and y^n respectively.
- From the beginning of the expansion, the powers of x, decrease from n up to 0, and the powers of a, increase from 0 up to n.
- The binomial coefficients in the expansion are arranged in an array, which is called Pascal's triangle. This pattern developed is summed up by the binomial theorem formula.

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2. Answer: 23 - 23

Explanation:

Coefficient of x in $(1 + x)^p (1 - x)^q$

$^{-p}C_0\ ^qC_1 +^p C_1\ ^qC_0 = -3$

$\Rightarrow p - q = -3$

Coefficient of x^2 in $(1 + x)^p (1 - x)^q$

$^pC_0\ ^qC_2 -^p C_1\ ^qC_1\ +^p C_2\ ^qC_0 = -5$

$\frac{q(q-1)}{2} - pq + \frac{p(p-1)}{2} = -5$

$\frac{q^2-q}{2} - (q-3)q + \frac{(q-3)(q-4)}{2} = -5$

$\Rightarrow q = 11, p = 8$

Coefficient of x^3 in $(1 + x)^8 (1 - x)^{11}$ is

$=^{-11} C_3 +\ ^8C_1\ ^{11}C_2 -\ ^8C_2\ ^{11}C_1 +\ ^8C_3$

=23

So, the answer is 23.

Concepts:

1. Binomial Theorem:

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

$$(x+y)^n = {}^nC_0x^ny^0 + {}^nC_1x^{n-1}y^1 + {}^nC_2x^{n-2}y^2 + \ldots\ldots + {}^nC_{n-1}xy^{n-1} + {}^nC_nx^0y^n$$

Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to $(n + 1)$.
- There are (n+1) terms in the expansion of $(x+y)^n$.
- The first and the last terms are x^n and y^n respectively.
- From the beginning of the expansion, the powers of x, decrease from n up to 0, and the powers of a, increase from 0 up to n.
- The binomial coefficients in the expansion are arranged in an array, which is called Pascal's triangle. This pattern developed is summed up by the binomial theorem formula.

3. **Answer: c**

Explanation:

The correct answer is (C) : 5
$2021 \equiv -2 \pmod 7$
$\Rightarrow (2021)^{2023} \equiv -(2)^{2023} \pmod 7$
$\equiv -2(8)^{674} \pmod 7$
$\equiv -2(1)^{674} \pmod 7$
$\equiv -2 \pmod 7$
$\equiv 5 \pmod 7$
So when $(2021)^{2023}$ is divided by 7, remainder is 5.

Concepts:

1. Binomial Theorem:

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

$$(x+y)^n = {}^nC_0 x^n y^0 + {}^nC_1 x^{n-1} y^1 + {}^nC_2 x^{n-2} y^2 + \ldots\ldots + {}^nC_{n-1} x y^{n-1} + {}^nC_n x^0 y^n$$

Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to (n + 1).
- There are (n+1) terms in the expansion of $(x+y)^n$.
- The first and the last terms are x^n and y^n respectively.
- From the beginning of the expansion, the powers of x, decrease from n up to 0, and the powers of a, increase from 0 up to n.
- The binomial coefficients in the expansion are arranged in an array, which is called Pascal's triangle. This pattern developed is summed up by the binomial theorem formula.

4. **Answer: c**

Explanation:

The correct answer is (C) : ${}^{n}C_3(8-5)+{}^{n}C_4(8^2-5^2)+\ldots+{}^{n}C_n(8^{n-2}-5^{n-2})$

$(1+8)^n - 8n - 1 = 64\alpha$

$\Rightarrow 1+8n+{}^{n}C_2 8^2+{}^{n}C_3 8^3+\ldots..+{}^{n}C_n 8^n-8n-1=\ 64\alpha$

$\Rightarrow\ \alpha={}^{n}C_2+{}^{n}C_3 8+{}^{n}C_4 8^2+\ldots..+{}^{n}C_n 8^{n-2}\ldots\ldots(i)$

Similarly

$(1+5)^n - 5n-1=25\beta$

$\Rightarrow 1+5n+{}^{n}C_2 5^2+{}^{n}C_3 5^3+\ldots..+{}^{n}C_n 5^n-5n-1=\ 25\beta$

$\Rightarrow\ \beta={}^{n}C_2+{}^{n}C_3.5+{}^{n}C_4.5^2+\ldots\ldots+{}^{n}C_n 5^{n-2}\ldots..(ii)$

$\alpha-\ \beta={}^{n}C_3(8-5)+{}^{n}C_4(8^2-5^2)+\ldots..+{}^{n}C_n(8^{n-2}-5^{n-2})$

Concepts:

1. Binomial Theorem:

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

$$(x+y)^n = {}^nC_0x^ny^0 + {}^nC_1x^{n-1}y^1 + {}^nC_2x^{n-2}y^2 + \ldots.. + {}^nC_{n-1}xy^{n-1} + {}^nC_nx^0y^n$$

Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to (n + 1).
- There are (n+1) terms in the expansion of $(x+y)^n$.
- The first and the last terms are x^n and y^n respectively.
- From the beginning of the expansion, the powers of x, decrease from n up to 0, and the powers of a, increase from 0 up to n.
- The binomial coefficients in the expansion are arranged in an array, which is called Pascal's triangle. This pattern developed is summed up by the binomial theorem formula.

5. **Answer: 286 - 286**

Explanation:

Given that $C_1 + 2 \cdot 3C_2 + 5 \cdot 3C_3 + \ldots 10$ terms
$= \frac{\alpha \cdot 2^{11}}{2^{\beta}-1}(C_1 + \frac{C_2}{2} + \ldots\ldots)$

$$\sum_{r=1}^{10} r(2r-1)C_r = \frac{\alpha \cdot 2^{11}}{2^{\beta}-1}(\sum_{r=1}^{10} \frac{C_r}{r})$$

Using $C_1 + 2C_2 + \ldots. + nC_n = n.2^{n-1}$
$1^2C_1 + 2^2C_2 + \ldots + n^2C_n = n.^{2n-1} + n(n-1)2^{n-2}$
and,
$C_0 + \frac{C_1}{2} + \ldots\ldots \frac{C_n}{n+1} = \frac{2^{n+1}-1}{n+1}$
we get,
$2(10.2^9 + 10.9.2^8) - 10.2^9$
$= \frac{\alpha \cdot 2^{11}}{2^{\beta}-1}\frac{(2^{11}-1)}{11}$
On comparing both side,
$2^{11}.25 = \frac{\alpha \cdot 2^{11}}{2^{\beta}-1}\frac{(2^{11}-1)}{11}$
$\Rightarrow \alpha = 25 \times 11 = 275$
$\beta = 11$
$\Rightarrow \alpha + \beta = 286$

So, the answer is 286.

Concepts:

1. Binomial Distribution:

A common probability distribution that models the probability of obtaining one of two outcomes under a given number of parameters is called the binomial distribution. It summarizes the number of trials when each trial has the same probability of attaining one specific outcome. The value of a binomial is acquired by multiplying the number of independent trials by the successes.

Criteria of Binomial Distribution:

Binomial distribution models the probability of happening an event when specific criteria are met. In order to use the binomial probability formula, the binomial distribution involves the following rules that must be present in the process:

1. Fixed trials
2. Independent trials
3. Fixed probability of success
4. Two mutually exclusive outcomes

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6. Answer: a

Explanation:

$f(x) + f'(x) + f''(x) = x^5 + 64$
Let $f(x) = x^5 + ax^4 + bx^3 + cx^2 + dx + e$
$f'(x) = 5x^4 + 4ax^3 + 3bx^2 + 2cx + d$
$f''(x) = 20x^3 + 12ax^2 + 6bx + 2c$
$x^5 + (a + 5)x^4 + (b + 4a + 20)\,x^3 + (c + 3b + 12a)\,x^2 + (d + 2c + 6b)\,x + e + d +$
$2c = x^5 + 64$
$a + 5 = 0$
$b + 4a + 20 = 0$
$c + 3b + 12a = 0$
$d + 2c + 6b = 0$
$e + d + 2c = 64$

$\therefore a = -5, b = 0, c = 60, d = -120, e = 64$
$\therefore f(x) = x^5 - 5x^4 + 60x^2 - 120x + 64$

Now,
$\lim\limits_{x \to 1} \frac{x^5 - 5x^4 + 60x^2 - 120x + 64}{x - 1}$ is ($\frac{0}{0}$ from)
According to the L′ Hopital rule:
$\lim\limits_{x \to 1} \frac{5x^4 - 20x^3 + 120x - 120}{1} = -15$

So, the correct option is (A): -15

Concepts:

1. Binomial Theorem:

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

$$(x+y)^n = {}^nC_0x^ny^0 + {}^nC_1x^{n-1}y^1 + {}^nC_2x^{n-2}y^2 + \ldots\ldots + {}^nC_{n-1}xy^{n-1} + {}^nC_nx^0y^n$$

Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to (n + 1).
- There are (n+1) terms in the expansion of $(x+y)^n$.
- The first and the last terms are x^n and y^n respectively.
- From the beginning of the expansion, the powers of x, decrease from n up to 0, and the powers of a, increase from 0 up to n.
- The binomial coefficients in the expansion are arranged in an array, which is called Pascal's triangle. This pattern developed is summed up by the binomial theorem formula.

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7. **Answer: 63 - 63**

Explanation:

The correct answer is 63

$(1+x)^p(1-x)^q$

$(1+px+\frac{p(p-1)}{2!}x^2+......)$

$(1-qx+\frac{q(q-1)}{2!}x^2-......)$

$p-q=4$

$\frac{p(p-1)}{2}+\frac{q(q-1)}{2}-pq=-5$

$p^2+q^2 - p - 2 - 2pq=-10$

$(q+4)^2 + q^2 - (q-4) - q - 2(4+q)q = -10$

$q^2 + 8q + 16 - q^2 - q - 4 - q - 8q - 2q^2 = -10$

-2q=-22

so , q=11 and p=15

$\therefore$ 2p + 3q = 2(15) + 3(11)

= 30 + 33

= 63

Concepts:

1. Binomial Expansion Formula:

The binomial expansion formula involves binomial coefficients which are of the form

(n/k)(or) ^nC_k and it is calculated using the formula, ^nC_k =n! / [(n - k)! k!]. The binomial expansion formula is also known as the binomial theorem. Here are the binomial expansion formulas.

- When powers are natural numbers :

$$(x + y)^n = {}^nC_0x^ny^0 + {}^nC_1x^{n-1}y^1 + {}^nC_2x^{n-2}y^2 + \ldots\ldots + {}^nC_nx^0y^n$$

(OR)

$$(x + y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 + \frac{n(n-1)(n-2)}{3!}x^{n-3}y^3 + \ldots + y^n$$

- When powers are rational numbers :

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots \infty$$

Here, $|x| < 1$.

This binomial expansion formula gives the expansion of $(x + y)^n$ where 'n' is a natural number. The expansion of $(x + y)^n$ has (n + 1) terms. This formula says:

We have $(x + y)^n = nC_0\, x_n + nC1\, x_{n-1} . y + nC_2\, x_{n-2} . y_2 + \ldots + nC_n\, y_n$

General Term = $T_{r+1} = nCr\, x_{n-r} . y_r$

- General Term in $(1 + x)^n$ is $nC_r\, x_r$
- In the binomial expansion of $(x + y)^n$, the rth term from end is $(n – r + 2)^{th}$.

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8. **Answer: a**

Explanation:

Step 1: Define the consecutive terms.

- Let the three consecutive terms be $\binom{n}{r-1}, \binom{n}{r}, \binom{n}{r+1}$.
- The given ratio is:

$$\binom{n}{r-1} : \binom{n}{r} : \binom{n}{r+1} = 1 : 5 : 20.$$

Step 2: Express the ratios.

- From the ratio $\frac{\binom{n}{r}}{\binom{n}{r-1}} = 5$:

$$\frac{\frac{n!}{r!(n-r)!}}{\frac{n!}{(r-1)!(n-r+1)!}} = 5 \implies \frac{(n-r+1)}{r} = 5.$$

$$n - r + 1 = 5r \implies n = 6r - 1.$$

- From the ratio $\frac{\binom{n}{r+1}}{\binom{n}{r}} = 4$:

$$\frac{\frac{n!}{(r+1)!(n-r-1)!}}{\frac{n!}{r!(n-r)!}} = 4 \implies \frac{(n-r)}{r+1} = 4.$$

$$n - r = 4r + 4 \implies n = 5r + 4.$$

Step 3: Solve for r and n.

- Equating the two expressions for n:

$$6r - 1 = 5r + 4 \implies r = 5, \quad n = 6(5) - 1 = 29.$$

Step 4: Find the coefficient of the fourth term.

- The fourth term corresponds to $r = 3$:

$$\binom{29}{3} = \frac{29 \times 28 \times 27}{3 \times 2 \times 1} = 3654.$$

Final Answer: The coefficient of the fourth term is 3654.

Concepts:

1. Binomial Expansion Formula:

The binomial expansion formula involves binomial coefficients which are of the form

(n/k)(or) ^nC_k and it is calculated using the formula, ^nC_k =n! / [(n - k)! k!]. The binomial expansion formula is also known as the binomial theorem. Here are the binomial expansion formulas.

- When powers are natural numbers :

$$(x + y)^n = {}^nC_0x^ny^0 + {}^nC_1x^{n-1}y^1 + {}^nC_2x^{n-2}y^2 + \ldots\ldots + {}^nC_nx^0y^n$$

(OR)

$$(x + y)^n = x^n + nx^{n-1}y + \frac{n(n-1)}{2!}x^{n-2}y^2 + \frac{n(n-1)(n-2)}{3!}x^{n-3}y^3 + \ldots + y^n$$

- When powers are rational numbers :

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots \infty$$

Here, $|x| < 1$.

This binomial expansion formula gives the expansion of $(x + y)^n$ where 'n' is a natural number. The expansion of $(x + y)^n$ has $(n + 1)$ terms. This formula says:

We have $(x + y)^n = nC_0\, x_n + nC1\, x_{n-1} . y + nC_2\, x_{n-2} . y_2 + \ldots + nC_n\, y_n$

General Term = $T_{r+1} = nCr\, x_{n-r} . y_r$

- General Term in $(1 + x)^n$ is $nC_r\, x_r$
- In the binomial expansion of $(x + y)^n$, the rth term from end is $(n - r + 2)^{th}$.

---

9. Answer: a

Explanation:

The correct option is (A): $\frac{-80}{3}$

Concepts:

1. Binomial Theorem:

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

$$(x+y)^n = {}^nC_0 x^n y^0 + {}^nC_1 x^{n-1} y^1 + {}^nC_2 x^{n-2} y^2 + \ldots + {}^nC_{n-1} x y^{n-1} + {}^nC_n x^0 y^n$$

Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to $(n + 1)$.
- There are (n+1) terms in the expansion of $(x+y)^n$.
- The first and the last terms are x^n and y^n respectively.
- From the beginning of the expansion, the powers of x, decrease from n up to 0, and the powers of a, increase from 0 up to n.
- The binomial coefficients in the expansion are arranged in an array, which is called Pascal's triangle. This pattern developed is summed up by the binomial theorem formula.

---

10. Answer: b

Explanation:

$$\frac{{}^nC_4(2^{\frac{1}{4}})^{n-4}(3^{\frac{-1}{4}})^4}{{}^nC_4(2^{\frac{-1}{4}})^{n-4}(3^{\frac{1}{4}})^4} = \sqrt{6}$$

$$\left(\frac{2^{\frac{1}{4}}}{3^{\frac{-1}{4}}}\right)^{(n-8)} = \sqrt{6}$$

$$(6)^{\frac{n-8}{4}} = \sqrt{6}$$

$$n - 8 = 2$$

$$n = 10$$

$T_3 = {}^{10}C_2(2^{\frac{1}{4}})^8(3^{\frac{-1}{4}})^2$
$= {}^{10}C_2 \times (\sqrt{2})^4 \times \frac{1}{\sqrt{3}} = 60\sqrt{3}$

So, the correct answer is (B): $60\sqrt{3}$

Concepts:

1. Binomial Theorem:

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

$$(x+y)^n = {}^nC_0x^ny^0 + {}^nC_1x^{n-1}y^1 + {}^nC_2x^{n-2}y^2 + \ldots\ldots + {}^nC_{n-1}xy^{n-1} + {}^nC_nx^0y^n$$

Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to $(n + 1)$.
- There are $(n+1)$ terms in the expansion of $(x+y)^n$.
- The first and the last terms are x^n and y^n respectively.
- From the beginning of the expansion, the powers of x, decrease from n up to 0, and the powers of a, increase from 0 up to n.
- The binomial coefficients in the expansion are arranged in an array, which is called Pascal's triangle. This pattern developed is summed up by the binomial theorem formula.

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11. Answer: 6 - 6

Explanation:

The answer is 6

$T_{r+1} = {}^{15}C_r(x^4)^{15-r}(-\frac{1}{x^3})^r$
$T_{r+1} = {}^{15}C_r(-1)^r x^{60-7r}$
$60 - 7r = 18 \Rightarrow r = 6$
$T_7 = {}^{15}C_6(-1)^6 x^{18}$

$T_7 =^{15} C_6 x^{18}$

So, the Coefficient of x^{18} is $^{15}C_6$

Concepts:

1. Binomial Theorem:

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

$$(x+y)^n = {}^nC_0x^ny^0 + {}^nC_1x^{n-1}y^1 + {}^nC_2x^{n-2}y^2 + \ldots\ldots + {}^nC_{n-1}xy^{n-1} + {}^nC_nx^0y^n$$

Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to $(n + 1)$.
- There are $(n+1)$ terms in the expansion of $(x+y)^n$.
- The first and the last terms are x^n and y^n respectively.
- From the beginning of the expansion, the powers of x, decrease from n up to 0, and the powers of a, increase from 0 up to n.
- The binomial coefficients in the expansion are arranged in an array, which is called Pascal's triangle. This pattern developed is summed up by the binomial theorem formula.

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12. **Answer: c**

Explanation:

The correct answer is (C) : 15

$S = 0 \cdot \left({}^{30}C_0\right)^2 + 1 \cdot \left({}^{30}C_1\right)^2 + 2 \cdot \cdot \left({}^{30}C_2\right)^2 + \ldots\ldots + 30 \cdot \left({}^{30}C_{30}\right)^2$

$S = 30. \left({}^{30}C_0\right)^2 + 29. \left({}^{30}C_1\right)^2 + 28. \left({}^{30}C_2\right)^2 + \ldots\ldots + 0. \left({}^{30}C_0\right)^2$

$2S = 30 \cdot \left({}^{30}C_0{}^2 + +{}^{30}C_1^2 + \ldots\ldots\ldots {}^{30}C_{30}{}^2\right)$

$S = 15 \cdot {}^{60}C_{30} = 15 \cdot \frac{60!}{(30!)^2}$

$\frac{15 \cdot 10!}{(30!)^2} = \frac{\alpha \cdot 60!}{(30!)^2}$

$\Rightarrow \alpha = 15$

Concepts:

1. Oxidation Number:

Oxidation number, also called **oxidation state**, the total number of electrons that an atom either gains or loses in order to form a chemical bond with another atom.

Oxidation number of an atom is defined as the charge that an atom appears to have on forming ionic bonds with other heteroatoms. An atom having higher electronegativity (even if it forms a covalent bond) is given a negative oxidation state.

The definition, assigns oxidation state to an atom on conditions, that the atom –

1. Bonds with heteroatoms.
2. Always form ionic bonding by either gaining or losing electrons, irrespective of the actual nature of bonding.

Oxidation number is a formalized way of keeping track of oxidation state.

Read More: Oxidation and Reduction

Way To Find Oxidation Number Of An Atom?

Oxidation number or state of an atom/ion is the number of electrons an atom/ion that the molecule has either gained or lost compared to the neutral atom. Electropositive metal atoms, of group I, 2 and 3 lose a specific number of electrons and have always constant positive oxidation numbers.

In molecules, more electronegative atom gain electrons from a less electronegative atom and have negative oxidation states. The numerical value of the oxidation state is equal to the number of electrons lost or gained.

Oxidation number or oxidation state of an atom or ion in a molecule/ion is assigned by:

1. Summing up the constant oxidation state of other atoms/molecules/ions that are bonded to it and
2. Equating, the total oxidation state of a molecule or ion to the total charge of the molecule or ion.

13. **Answer: 405 - 405**

Explanation:

The correct answer is 405

Given Binomial $\left(x-\frac{3}{x^2}\right)^n, x \neq 0, n \in N,$

Sum of coefficients of first three terms

${}^nC_0 - {}^nC_1 \cdot 3 + {}^nC_2 3^2 = 376$

$\Rightarrow 3n^2 - 5n - 250 = 0$

$\Rightarrow (n-10)(3n+25) = 0$

$\Rightarrow n = 10$

Now general term ${}^{10}C_r x^{10-r}\left(\frac{-3}{x^2}\right)^r$

$= {}^{10}C_r x^{10-r}(-3)^r \cdot x^{-2r}$

$= {}^{10}C_r(-3)^r \cdot x^{10-3r}$

Coefficient of $x^4 \Rightarrow 10 - 3r = 4$

$\Rightarrow r = 2$

${}^{10}C_2(-3)^2 = 405$

Concepts:

1. **Binomial Theorem:**

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

$$(x+y)^n = {}^nC_0x^ny^0 + {}^nC_1x^{n-1}y^1 + {}^nC_2x^{n-2}y^2 + \ldots\ldots + {}^nC_{n-1}xy^{n-1} + {}^nC_nx^0y^n$$

Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to (n + 1).
- There are (n+1) terms in the expansion of $(x+y)^n$.
- The first and the last terms are x^n and y^n respectively.
- From the beginning of the expansion, the powers of x, decrease from n up to 0, and the powers of a, increase from 0 up to n.

- The binomial coefficients in the expansion are arranged in an array, which is called Pascal's triangle. This pattern developed is summed up by the binomial theorem formula.

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14. **Answer: 1080 - 1080**

Explanation:

The correct answer is 1080.

General term is $\sum \frac{5!(2x)^{n_1}\left(x^{-7}\right)^{n_2}\left(3x^2\right)^{n_3}}{n_1!n_2!n_3!}$

For constant term,

$n_1 + 2n_3 = 7n_2$

$\& n_1 + n_2 + n_3 = 5$

Only possibility $n_1 = 1, n_2 = 1, n_3 = 3$

$\Rightarrow$ constant term $= 1080$

Concepts:

1. **Binomial Theorem:**

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

$$(x+y)^n = {}^nC_0x^ny^0 + {}^nC_1x^{n-1}y^1 + {}^nC_2x^{n-2}y^2 + \ldots + {}^nC_{n-1}xy^{n-1} + {}^nC_nx^0y^n$$

Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to (n + 1).
- There are (n+1) terms in the expansion of $(x+y)^n$.
- The first and the last terms are x^n and y^n respectively.
- From the beginning of the expansion, the powers of x, decrease from n up to 0, and the powers of a, increase from 0 up to n.
- The binomial coefficients in the expansion are arranged in an array, which is called Pascal's triangle. This pattern developed is summed up by the binomial theorem formula.

15. **Answer: 2 - 2**

Explanation:

The correct answer is 2.

$T_{r+1} = {}^{30}C_r \left(x^{2/3}\right)^{30-r} \left(\frac{2}{x^3}\right)^r$

$= {}^{30}C_r \cdot 2^r \cdot x^{\frac{60-11r}{3}}$

$\frac{60-11r}{3} < 0 \Rightarrow 11r > 60 \Rightarrow r > \frac{60}{11} \Rightarrow r = 6$

$T_7 = {}^{30}C_6 \cdot 2^6 x^{-2}$

We have also observed $\beta = {}^{30}C_6 (2)^6$ is a natural number.

$\therefore \alpha = 2$

Concepts:

1. Binomial Theorem:

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

$$(x+y)^n = {}^nC_0 x^n y^0 + {}^nC_1 x^{n-1} y^1 + {}^nC_2 x^{n-2} y^2 + \ldots\ldots + {}^nC_{n-1} x y^{n-1} + {}^nC_n x^0 y^n$$

Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to (n + 1).
- There are (n+1) terms in the expansion of $(x+y)^n$.
- The first and the last terms are x^n and y^n respectively.
- From the beginning of the expansion, the powers of x, decrease from n up to 0, and the powers of a, increase from 0 up to n.
- The binomial coefficients in the expansion are arranged in an array, which is called Pascal's triangle. This pattern developed is summed up by the binomial theorem formula.

16. **Answer: 1 - 1**

Explanation:

The general term T_{r+1} in the binomial expansion is given by:

$$T_{r+1} = \binom{n}{r} (x)^r (y)^{n-r}.$$

In our case, we have:

$$T_{r+1} = \binom{22}{r} \left(x^{\frac{2}{3}}\right)^r (a)^{22-3r}.$$

Simplify:

$$T_{r+1} = \binom{22}{r} x^{\frac{2r}{3}} (a)^{22-3r}.$$

For the term independent of x, the exponent of x must be zero. So:

$$\frac{2r}{3} = 0 \implies 44 - 11r = 0.$$

Solve for r:

$$11r = 44 \implies r = 4.$$

Now, we substitute $r = 4$ into the term T_{r+1}:

$$T_5 = \binom{22}{4} a^4 = 7315.$$

Expand the binomial coefficient:

$$\binom{22}{4} = \frac{22 \times 21 \times 20 \times 19}{4 \times 3 \times 2 \times 1} = 7315.$$

Thus:

$$7315a^4 = 7315 \implies a^4 = 1.$$

Solve for a:

$$a = 1.$$

Conclusion
Thus, the value of a is 1.
Final Answer: The final answer is 1.

Concepts:

1. Binomial Theorem:

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

$$(x+y)^n = {}^nC_0 x^n y^0 + {}^nC_1 x^{n-1} y^1 + {}^nC_2 x^{n-2} y^2 + \dots + {}^nC_{n-1} x y^{n-1} + {}^nC_n x^0 y^n$$

Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to $(n + 1)$.
- There are $(n+1)$ terms in the expansion of $(x+y)^n$.
- The first and the last terms are x^n and y^n respectively.
- From the beginning of the expansion, the powers of x, decrease from n up to 0, and the powers of a, increase from 0 up to n.
- The binomial coefficients in the expansion are arranged in an array, which is called Pascal's triangle. This pattern developed is summed up by the binomial theorem formula.

---

17. **Answer: 29 - 29**

Explanation:

The correct answer is 29.

$(21+2)^{200}+(21-2)^{200}$

$\Rightarrow 2\left[{}^{100}C_0 21^{200}+200C_2 21^{198}\cdot 2^2+\ldots..+{}^{200}C_{198}21^2\ \ 2^{198}+2^{200}\right]$

$\Rightarrow 2\left[49I_1+2^{200}\right]=49I_1+2^{201}$

$\text{Now},2^{201}=(8)^{67}=(1+7)^{67}=49I_2+{}^{67}C_0{}^{67}C_1\cdot 7=$

$49I_2+470=49I_2+49\times 9+29$

$\therefore$ Remainder is 29

Concepts:

1. Complex Number:

A Complex Number is written in the form

a + ib

where,

- "a" is a real number
- "b" is an imaginary number

The Complex Number consists of a symbol "i" which satisfies the condition i^2 = -1. Complex Numbers are mentioned as the extension of one-dimensional number lines. In a complex plane, a Complex Number indicated as a + bi is usually represented in the form of the point (a, b). We have to pay attention that a Complex Number with absolutely no real part, such as – i, -5i, etc, is called purely imaginary. Also, a Complex Number with perfectly no imaginary part is known as a real number.

---

18. Answer: c

Explanation:

Step 1: Coefficient of x^{15} in $(ax^3+\frac{1}{bx^3})^{15}$

The general term in the expansion of $(ax^3+\frac{1}{bx^3})^{15}$ is given by:

$$T_{r+1}=\binom{15}{r}\cdot a^{15-r}\cdot\left(\frac{1}{b}\right)^r\cdot x^{3(15-r)-3r}.$$

Simplify the powers of x:

$$T_{r+1} = \binom{15}{r} \cdot a^{15-r} \cdot b^{-r} \cdot x^{45-6r}.$$

For the coefficient of x^{15}, set $45 - 6r = 15$:

$$45 - 15 = 6r \implies r = 9.$$

Thus, the coefficient of x^{15} is:

$$\binom{15}{9} \cdot a^6 \cdot b^{-9}.$$

Step 2: Coefficient of x^{-15} in $(\frac{a}{x^3} - \frac{1}{bx^3})^{15}$

The general term in the expansion of $(\frac{a}{x^3} - \frac{1}{bx^3})^{15}$ is given by:

$$T_{r+1} = \binom{15}{r} \cdot \left(\frac{a}{x^3}\right)^{15-r} \cdot \left(-\frac{1}{bx^3}\right)^{r}.$$

Simplify the powers of x:

$$T_{r+1} = \binom{15}{r} \cdot a^{15-r} \cdot b^{-r} \cdot (-1)^r \cdot x^{-3(15-r)-3r}.$$

The exponent of x becomes:

$$-45 + 6r.$$

For the coefficient of x^{-15}, set $-45 + 6r = -15$:

$$6r = 30 \implies r = 6.$$

Thus, the coefficient of x^{-15} is:

$$\binom{15}{6} \cdot a^9 \cdot b^{-6}.$$

Step 3: Equating the Coefficients

Equate the coefficients of x^{15} and x^{-15}:

$$\binom{15}{9} \cdot a^6 \cdot b^{-9} = \binom{15}{6} \cdot a^9 \cdot b^{-6}.$$

Since $\binom{15}{9} = \binom{15}{6}$, cancel these terms:

$$a^6 \cdot b^{-9} = a^9 \cdot b^{-6}.$$

Rearranging gives:

$$\frac{a^6}{b^6} = \frac{b^9}{a^9}.$$

Cross-multiply:

$$a^{15} \cdot b^9 = b^{15} \cdot a^9.$$

Divide both sides by $a^9 b^9$:

$$a^6 = b^6 \implies \frac{a}{b} = 1 \implies ab = 1.$$

Conclusion

The correct ordered pair satisfies $ab = 1$.

Concepts:

1. Binomial Theorem:

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

$$(x+y)^n = {}^nC_0 x^n y^0 + {}^nC_1 x^{n-1} y^1 + {}^nC_2 x^{n-2} y^2 + \ldots + {}^nC_{n-1} x y^{n-1} + {}^nC_n x^0 y^n$$

Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to (n + 1).
- There are (n+1) terms in the expansion of $(x+y)^n$.
- The first and the last terms are x^n and y^n respectively.
- From the beginning of the expansion, the powers of x, decrease from n up to 0, and the powers of a, increase from 0 up to n.
- The binomial coefficients in the expansion are arranged in an array, which is called Pascal's triangle. This pattern developed is summed up by the binomial theorem formula.

19. **Answer: d**

Explanation:

Explanation:
Using binomial theorem, the 6 term of (+)^8 is 56 ^{3} ^{5} Now substituting the given terms, we get,[$\frac{1}{\frac{8}{3}}$ + ^{2}log$_{10}$]General term, $_{+1}$ = ^{8} [$\frac{1}{\frac{8}{3}}$] $^-$ (^{2}log$_{10}$) For 6 term, take = 5 $_6$ = ^{8} $_5$[$\frac{1}{\frac{8}{3}}$]^3[^{2}log$_{10}$]^5 5600 = $\frac{8\times7\times6}{3\times2\times1}$ × $\frac{1}{8}$ × ^{10}(log$_{10}$)^5 [^{8} $_3$ = $\frac{8\times7\times6}{3\times2\times1}$] 10^2 = ^{2}(log$_{10}$)^5 [(log$_{10}$ = 1)] ^{2} = 10^2 = 10Hence, the correct option is (D).

---

20. **Answer: c**

Explanation:

$={}^{21}C_1+{}^{21}C_2+\ldots+{}^{21}C_{10}\frac{1}{2}\left\{{}^{21}C_0+{}^{21}C_1+\ldots+{}^{21}C_{21}\right\}-1=2^{20}-1$
$\left({}^{10}C_1+{}^{10}C_2+\ldots+{}^{10}C_{10}\right)=2^{10}-1$ $\therefore$ Required sum $=\left(2^{20}-1\right)-\left(2^{10}-1\right)=2^{20}-2^{10}$

Concepts:

1. Binomial Theorem:

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

$$(x+y)^n = {}^nC_0x^ny^0 + {}^nC_1x^{n-1}y^1 + {}^nC_2x^{n-2}y^2 + \ldots\ldots + {}^nC_{n-1}xy^{n-1} + {}^nC_nx^0y^n$$

Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to $(n + 1)$.
- There are (n+1) terms in the expansion of $(x+y)^n$.
- The first and the last terms are x^n and y^n respectively.
- From the beginning of the expansion, the powers of x, decrease from n up to 0, and the powers of a, increase from 0 up to n.

- The binomial coefficients in the expansion are arranged in an array, which is called Pascal's triangle. This pattern developed is summed up by the binomial theorem formula.

---

21. **Answer: d**

Explanation:

$\frac{1}{60}\left(2x^2-\frac{3}{x^2}\right)^6 - \frac{1}{81}\cdot x^8\left(2x^2-\frac{3}{x^2}\right)^6$ its general term $\frac{1}{60}{}^6C_r2^{6-r}(-3)^r\,x^{12-r}$ $-$ $\frac{1}{81}{}^6C_r2^{6-r}(-3)^r12^{20-4r}$ for term independent of x, r for I^{st} expression is 3 and r for second expression is 5 $\therefore$ term independent of $x=-36$

Concepts:

1. Binomial Theorem:

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

$$(x+y)^n = {}^nC_0x^ny^0 + {}^nC_1x^{n-1}y^1 + {}^nC_2x^{n-2}y^2 + \ldots + {}^nC_{n-1}xy^{n-1} + {}^nC_nx^0y^n$$

Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to $(n + 1)$.
- There are $(n+1)$ terms in the expansion of $(x+y)^n$.
- The first and the last terms are x^n and y^n respectively.
- From the beginning of the expansion, the powers of x, decrease from n up to 0, and the powers of a, increase from 0 up to n.
- The binomial coefficients in the expansion are arranged in an array, which is called Pascal's triangle. This pattern developed is summed up by the binomial theorem formula.

---

22. **Answer: b**

Explanation:

for sum of integral power of x put $x = 1$ in $\frac{(1-2\sqrt{x})^{50}+(1+2\sqrt{x})^{50}}{2} \Rightarrow \frac{3^{50}+1}{2}$.

Concepts:

1. Binomial Theorem:

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

$$(x+y)^n = {}^nC_0 x^n y^0 + {}^nC_1 x^{n-1} y^1 + {}^nC_2 x^{n-2} y^2 + \ldots\ldots + {}^nC_{n-1} x y^{n-1} + {}^nC_n x^0 y^n$$

Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to $(n + 1)$.
- There are $(n+1)$ terms in the expansion of $(x+y)^n$.
- The first and the last terms are x^n and y^n respectively.
- From the beginning of the expansion, the powers of x, decrease from n up to 0, and the powers of a, increase from 0 up to n.
- The binomial coefficients in the expansion are arranged in an array, which is called Pascal's triangle. This pattern developed is summed up by the binomial theorem formula.

---

23. Answer: b

Explanation:

To solve the given problem, we begin by understanding and simplifying the series.

The given series is:

$${}^{n+1}C_2 + 2\left({}^2C_2 + {}^3C_2 + {}^4C_2 + \ldots + {}^nC_2\right)$$

We need to evaluate the sum: ${}^{n+1}C_2$ and $\sum_{k=2}^{n} {}^kC_2$. Using the combination formula, where ${}^rC_2 = \frac{r(r-1)}{2}$, we substitute into the series:

1. ${}^{n+1}C_2 = \frac{(n+1)n}{2}$
2. For the second part: $\sum_{k=2}^{n} {}^{k}C_2 = \sum_{k=2}^{n} \frac{k(k-1)}{2}$
   - Expand and simplify using summation: $\sum_{k=2}^{n} \frac{k(k-1)}{2} = \frac{1}{2}\left[\sum_{k=2}^{n}(k^2-k)\right]$
   - Use known summation formulas:
     - $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$
     - $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$
   - Apply these to calculate:
     - $\sum_{k=2}^{n} k = 1+2+\ldots+n-(1) = \frac{n(n+1)}{2} - 1$
     - $\sum_{k=2}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} - 1^2$
   - Combine and substitute back into the main expression:

Now, substituting these into the series:

${}^{n+1}C_2 = \frac{(n+1)n}{2}$

and summation terms simplify to:

$\frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2}\right]$

Combine these in the problem's expression:

${}^{n+1}C_2 + 2\left(\frac{1}{2}\left[\sum_{k=2}^{n}(k^2-k)\right]\right)$

Upon simplifying further:

${}^{n+1}C_2 + \left[\sum_{k=2}^{n}(k^2-k)\right]$

Simplifying both terms gives:

The final solution becomes: $\frac{n(n+1)(2n+1)}{6}$

Therefore, the correct option is:

$\frac{n(n+1)(2n+1)}{6}$

Concepts:

1. Binomial Theorem:

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

$$(x+y)^n = {}^nC_0x^ny^0 + {}^nC_1x^{n-1}y^1 + {}^nC_2x^{n-2}y^2 + \dots + {}^nC_{n-1}xy^{n-1} + {}^nC_nx^0y^n$$

Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to (n + 1).
- There are (n+1) terms in the expansion of $(x+y)^n$.
- The first and the last terms are x^n and y^n respectively.
- From the beginning of the expansion, the powers of x, decrease from n up to 0, and the powers of a, increase from 0 up to n.
- The binomial coefficients in the expansion are arranged in an array, which is called Pascal's triangle. This pattern developed is summed up by the binomial theorem formula.

---

24. **Answer: d**

Explanation:

$$(10+x)^{50} + (10-x)^{50}$$
$$\Rightarrow a_2 = 2.{}^{50}C_2 10^{48}, a_0 = 2.10^{50}$$
$$\frac{a_2}{a_0} = \frac{{}^{50}C_2}{10^2} = 12.25$$

Concepts:

1. Binomial Theorem:

The binomial theorem formula is used in the expansion of any power of a binomial in the form of a series. The binomial theorem formula is

$$(x+y)^n = {}^nC_0x^ny^0 + {}^nC_1x^{n-1}y^1 + {}^nC_2x^{n-2}y^2 + \dots + {}^nC_{n-1}xy^{n-1} + {}^nC_nx^0y^n$$

Properties of Binomial Theorem

- The number of coefficients in the binomial expansion of $(x + y)^n$ is equal to $(n + 1)$.
- There are $(n+1)$ terms in the expansion of $(x+y)^n$.
- The first and the last terms are x^n and y^n respectively.
- From the beginning of the expansion, the powers of x, decrease from n up to 0, and the powers of a, increase from 0 up to n.
- The binomial coefficients in the expansion are arranged in an array, which is called Pascal's triangle. This pattern developed is summed up by the binomial theorem formula.

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25. Answer: d

Explanation:

$$\frac{2^{403}}{15} = \frac{2^3.\left(2^4\right)^{100}}{15} = \frac{8}{15}(15+1)^{100}$$
$$= \frac{8}{15}(15\lambda+1) = 8\lambda + \frac{8}{15}$$
$\because 8\lambda$ is integer
$\Rightarrow$ fractional part of $\frac{2^{403}}{15}\ \frac{8}{15} \Rightarrow k = 8$

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26. Answer: c

Explanation:

$$\sum_{r=0}^{25} {}^{50}C_r . {}^{50-r}C_{25-r}$$
$$= \sum_{r=0}^{25} \frac{50!}{r!(50-r)!} \times \frac{(50-r)!}{(25)!(25-r)!}$$
$$= \sum_{r=0}^{25} \frac{50!}{25!25!} \times \frac{25!}{(25-r)!(r!)}$$
$$= {}^{50}C_{25} \sum_{r=0}^{25} {}^{25}C_r = \left(2^{25}\right)^{50} C_{25}$$
$$\therefore K = 2^{25}$$

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27. Answer: d

Explanation:

$$\frac{^nC_{r-1}}{^nC_r} = \frac{2}{15}$$
$$\frac{\frac{n!}{(r-1)!(n-r+1)!}}{\frac{n!}{r!(n-r)!}} = \frac{2}{15}$$
$$\frac{r}{n-r+1} = \frac{2}{15}$$
$$15r = 2n - 2r + 2$$
$$17r = 2n + 2$$
$$\frac{^nC_r}{^nC_{r+1}} = \frac{15}{70}$$
$$\frac{\frac{n!}{r!(n-r)!}}{\frac{n!}{(r+1)!(n-r-1)!}} = \frac{3}{14}$$
$$\frac{r+1}{n-r} = \frac{3}{14}$$
$$14r + 14 = 3n - 3r$$
$$3n - 17r = 14$$
$$\frac{2n-17r=-2}{n=16}$$
$$17r = 34, r = 2$$
$$^{16}C_1, {}^{16}C_2, {}^{16}C_3$$
$$\frac{^{16}C_1 + {}^{16}C_2 + {}^{16}C_3}{3} = \frac{16+120+560}{3}$$
$$\frac{680+16}{3} = \frac{696}{3} = 232$$

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28. **Answer: d**

Explanation:

$\frac{(28-1)^{999}}{7} = \frac{28\lambda-1}{7} \Rightarrow \frac{28\lambda-7+1-1}{7} = \frac{7(4\lambda-1)+6}{7}$

$\therefore$ Rem = 6

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29. Answer: a

Explanation:

$1+x^4+x^5=a_0+a_1(1+x)+a_2(1+x)^2+$
$a_3(1+x)^3+a_4(1+x)^4+a_5(1+x)^5$
$=a_0+a_1(1+x)+a_2\left(1+2x+x^2\right)+a_3(1+3x$
$+3x^2+x^3)+a_4\left(1+4x+6x^2+4x^3+x^4\right)+a_5(1$
$+5x+10x^2+10x^3+5x^4+x^5)$
So, Coeff. of x^i in $LHS=$ Coeff. of x^i on RHS
$i=5\Rightarrow 1=a_5\ldots$(i)
$i=4\Rightarrow 1=a_4+5a_5=a_4+5$
$\Rightarrow a_4=-4\ldots$(ii)
$i=3\Rightarrow 0=a_3+4a_4+10a_5$
$\Rightarrow a_3-16+10=0$
$\Rightarrow a_3=6\ldots$ (iii)
$i=2\Rightarrow 0=a_2+3a_3+6a_4+10a_5$
$\Rightarrow a_2+18-24+10=0$
$\Rightarrow a_2=-4$
Put $x=-1$
$1=a_0$
Now differentiate w.r.t. x.
$4x^3+5x^4=a_1+2a_2(1+x)+3a_3(1+x)^2+\ldots.$
Put $x=-1$
$\Rightarrow 1=a_1$
Again differentiate w.r.t. x
$12x^2+20x^3=2xa_2+6a_3(1+x)$
Put $x=-1$
$12-20=2a_2$
$\Rightarrow a_2=-4$

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30. Answer: b

Explanation:

The correct answer is B:$\alpha - \beta = -132$
Given that;
α, β are coefficient of $x^4 and\ x^2$,and;
$x + \sqrt{(x^2-1)^6} + x - \sqrt{(x^2-1)^2}$
$2\left[{}^6C_0.x^6 +{}^6 C_2x^4\left(x^2-1\right) +{}^6 C_4x^2\left(x^2-1\right)^2 +{}^6 C_6\left(x^2-1\right)^3\right]$
$= 2[x^6 + 15(x^6 - x^4) + 15x^2(x^4 - 2x^2 + 1) + (-1 + 3x^2 - 3x^2 + x^6)]$
$= 64x^6 - 96x^4 + 36x^2 - 2$
as α, β are the coefficient of x^4, x^2
$\therefore \alpha = -96 \,\&\, \beta = 36$
$\therefore \alpha - \beta = -132$

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Sol^n : Given that :

α, β are coefficient of x^4 and x^2

and

$$x + \sqrt{(x^2-1)^6} + \left(x - \sqrt{(x^2-1)^2}\right)$$

$$= 2\left[{}^6C_0 x^6 + {}^6C_2 x^4 (x^2-1) + {}^6C_4 x^2 (x^2-1)^2 + {}^6C_6 (x^2-1)^3 \right]$$

$$= 2\left[x^6 + 15(x^6 - x^4) + 15x^2(x^4 - 2x^2 + 1) + (-1 + 3x^2 - 3x^4 + x^6)\right]$$

$$= 2(32x^6 - 48x^4 + 18x^2 - 1)$$

$$= 64x^6 - 96x^4 + 36x^2 - 2$$

$\therefore$ as α, β are coefficients of x^4 & x^2

$\therefore \quad \alpha = -96, \quad \beta = 36$

$\therefore \quad \alpha - \beta = -96 - 36$

$$\boxed{\alpha - \beta = -132}$$ Ans

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