QA CAT 2025 Slot 1 Question Paper with Solution

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1. Kamala divided her investment of Rs 100000 between stocks, bonds, and gold. Her investment in bonds was 25% of her investment in gold. With annual returns of 10%, 6%, 8% on stocks, bonds, and gold, respectively, she gained a total amount of Rs 8200 in one year. The amount, in rupees, that she gained from the bonds, was:

Solution:
Let the investment amounts in stocks, bonds, and gold be S, B, and G, respectively.
Step 1: Write the equations.
Total investment:

$$S + B + G = 100000.$$

Given:

$$B = 0.25G \quad \Rightarrow \quad G = 4B.$$

Total annual gain:

$$0.10S + 0.06B + 0.08G = 8200.$$

Step 2: Substitute $G = 4B$ into the investment equation.

$$S + B + 4B = 100000 \Rightarrow S + 5B = 100000 \Rightarrow S = 100000 - 5B.$$

Step 3: Substitute into the gain equation.

$$0.10(100000 - 5B) + 0.06B + 0.08(4B) = 8200.$$

Expand:

$$10000 - 0.50B + 0.06B + 0.32B = 8200.$$

Combine like terms:

$$10000 - 0.12B = 8200.$$

Step 4: Solve for B.

$$10000 - 8200 = 0.12B \Rightarrow 1800 = 0.12B \Rightarrow B = \frac{1800}{0.12} = 15000.$$

Thus, investment in bonds is 15000 rupees.
Step 5: Compute gain from bonds.

$$\text{Gain} = 0.06 \times 15000 = 900.$$

Therefore, the amount Kamala gained from bonds is:

$$\boxed{900}.$$

Quick Tip

When a mixture investment problem involves fixed returns and proportional allocations, reduce variables using the percentage relationships first. This simplifies the system into a single solvable equation.

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2. If $a - 6b + 6c = 4$ and $6a + 3b - 3c = 50$, where a, b and c are real numbers, the value of $2a + 3b - 3c$ is:
(A) 18
(B) 20

(C) 15
(D) 14

Correct Answer: 18

Solution:
We are given:

$$a - 6b + 6c = 4,$$

$$6a + 3b - 3c = 50.$$

We want:

$$E = 2a + 3b - 3c.$$

Step 1: Introduce a substitution.
Notice that both the second given equation and the target expression contain the term:

$$3b - 3c.$$

Let:

$$X = 3b - 3c.$$

Use this substitution in the equations.
Rewrite the first equation:

$$a - 6b + 6c = a - 2(3b - 3c) = a - 2X = 4. \quad \text{(i)}$$

Rewrite the second equation:

$$6a + (3b - 3c) = 6a + X = 50. \quad \text{(ii)}$$

Rewrite the target expression:

$$E = 2a + (3b - 3c) = 2a + X. \quad \text{(iii)}$$

Step 2: Solve equations (i) and (ii).
From (ii):

$$X = 50 - 6a.$$

Substitute into (i):

$$a - 2(50 - 6a) = 4,$$

$$a - 100 + 12a = 4,$$

$$13a - 100 = 4,$$

$$13a = 104,$$

$$a = 8.$$

Now compute X:

$$X = 50 - 6(8) = 50 - 48 = 2.$$

Step 3: Evaluate the target expression.
From (iii):

$$E = 2a + X = 2(8) + 2 = 16 + 2 = 18.$$

$$\boxed{18}$$

Quick Tip

When several linear expressions repeat with the same structure (like $3b - 3c$ here), use substitution to reduce the system to two variables. This often simplifies the equations dramatically.

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3. The (x, y) coordinates of vertices P, Q and R of a parallelogram $PQRS$ are $(-3, -2)$, $(1, -5)$ and $(9, 1)$, respectively. If the diagonal SQ intersects the x-axis at $(a, 0)$, then the value of a is:
(A) $\frac{29}{9}$

Correct Answer: $\frac{29}{9}$

Solution:
Step 1: Use the property of diagonals in a parallelogram.
In any parallelogram, diagonals bisect each other. Thus, the midpoint of diagonal PR is the same as the midpoint of diagonal SQ.
Step 2: Find the midpoint of diagonal PR.
Coordinates:

$$P(-3, -2), \quad R(9, 1).$$

Midpoint:

$$M = \left(\frac{-3+9}{2}, \frac{-2+1}{2}\right) = (3, -\tfrac{1}{2}).$$

Step 3: Find coordinates of point S.
Let $S = (x, y)$. Given midpoint of SQ is also $(3, -\frac{1}{2})$, and $Q = (1, -5)$.
Using midpoint formula:

$$\frac{x+1}{2} = 3 \quad \Rightarrow \quad x + 1 = 6 \Rightarrow x = 5,$$

$$\frac{y-5}{2} = -\tfrac{1}{2} \quad \Rightarrow \quad y - 5 = -1 \Rightarrow y = 4.$$

Thus,

$$S = (5, 4).$$

Step 4: Equation of diagonal SQ.
Points on line: $S(5, 4)$ and $Q(1, -5)$.
Slope:

$$m = \frac{-5-4}{1-5} = \frac{-9}{-4} = \frac{9}{4}.$$

Using point-slope form at $S(5, 4)$:

$$y - 4 = \frac{9}{4}(x - 5).$$

Multiply through:

$$4y - 16 = 9x - 45,$$

$$9x - 4y - 29 = 0.$$

Step 5: Find intersection with x-axis.
At the x-axis, $y = 0$. Substitute into the line equation:

$$9a - 0 - 29 = 0 \Rightarrow 9a = 29 \Rightarrow a = \frac{29}{9}.$$

Thus,

$$\boxed{\frac{29}{9}}.$$

Quick Tip

In any parallelogram, diagonals always bisect each other. So, to find missing vertices, equate midpoints of the diagonals. This avoids unnecessary vector calculations and simplifies coordinate geometry problems.

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4. At a certain simple rate of interest, a given sum amounts to Rs 13920 in 3 years, and to Rs 18960 in 6 years and 6 months. If the same given sum had been invested for 2 years at the same

rate as before but with interest compounded every 6 months, then the total interest earned, in rupees, would have been nearest to:
(A) 3096
(B) 3221
(C) 3180
(D) 3150

Correct Answer: (2) 3221

Solution:
Step 1: Find the principal and simple interest rate.
Amount after 3 years:
$$A_1 = 13920.$$
Amount after 6.5 years:
$$A_2 = 18960.$$
Time difference:
$$6.5 - 3 = 3.5 \text{ years}.$$
Extra interest in these 3.5 years:
$$\Delta I = A_2 - A_1 = 18960 - 13920 = 5040.$$
So simple interest per year:
$$\text{SI per year} = \frac{5040}{3.5} = 1440.$$
Interest for first 3 years:
$$\text{SI}_{3\text{ yrs}} = 1440 \times 3 = 4320.$$
Hence principal:
$$P = A_1 - \text{SI}_{3\text{ yrs}} = 13920 - 4320 = 9600.$$
Rate of interest:
$$R = \frac{\text{SI per year}}{P} \times 100 = \frac{1440}{9600} \times 100 = 15\% \text{ p.a.}$$
Step 2: Compound interest for 2 years, half-yearly.
Principal:
$$P = 9600.$$
Rate per half-year:
$$r = \frac{15\%}{2} = 7.5\% = 0.075.$$
Number of half-yearly periods in 2 years:
$$n = 2 \times 2 = 4.$$
Amount:
$$A = P\left(1 + \frac{r}{100}\right)^n = 9600 \times (1.075)^4.$$
Approximate:
$$(1.075)^2 \approx 1.1556, \quad (1.1556)^2 \approx 1.335 \text{ (approximately)},$$
so
$$A \approx 9600 \times 1.335 \approx 12820.$$
Compound interest:
$$\text{CI} = A - P \approx 12820 - 9600 = 3220 \text{ (approximately)}.$$
Among the given options, the nearest value is 3221.
Therefore, the total interest would have been approximately:
$$\boxed{3221}.$$

Quick Tip

When you know the amounts at two different times under simple interest, the difference of amounts directly gives you the interest for the extra period. From that, you can easily find the yearly interest, then the rate and principal, and finally plug those into a compound interest calculation.

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5. Let $3 \leq x \leq 6$ and $[x^2] = [x]^2$, where $[x]$ is the greatest integer not exceeding x. If set S represents all feasible values of x, then which of the following is a possible subset of S?

(A) $(3, \sqrt{10}) \cup [5, \sqrt{26}) \cup \{6\}$
(B) $(4, \sqrt{10}) \cup [5, \sqrt{27}) \cup \{6\}$
(C) $[3, \sqrt{10}] \cup [5, \sqrt{26}]$
(D) $[3, \sqrt{10}] \cup [4, \sqrt{17}] \cup \{6\}$

Correct Answer: (1) $(3, \sqrt{10}) \cup [5, \sqrt{26}) \cup \{6\}$

Solution:
We are given:

$$3 \leq x \leq 6, \quad [x^2] = [x]^2.$$

Since $[x]$ is constant on each interval between consecutive integers, we split $[3, 6]$ into:

$$[3, 4), \ [4, 5), \ [5, 6), \ \{6\}.$$

Case 1: $3 \leq x < 4$
Here, $[x] = 3$, so $[x]^2 = 9$. Condition:

$$[x^2] = 9 \implies 9 \leq x^2 < 10.$$

Taking square roots (and noting $x \geq 3$):

$$3 \leq x < \sqrt{10}.$$

This lies inside $[3, 4)$ since $\sqrt{10} \approx 3.16$.
So solutions in this case:

$$[3, \sqrt{10}).$$

Case 2: $4 \leq x < 5$
Here, $[x] = 4$, so $[x]^2 = 16$. Condition:

$$[x^2] = 16 \implies 16 \leq x^2 < 17.$$

Thus:

$$4 \leq x < \sqrt{17}, \quad \text{where } \sqrt{17} \approx 4.12.$$

So solutions:

$$[4, \sqrt{17}).$$

Case 3: $5 \leq x < 6$
Here, $[x] = 5$, so $[x]^2 = 25$. Condition:

$$[x^2] = 25 \implies 25 \leq x^2 < 26,$$

hence:

$$5 \leq x < \sqrt{26}, \quad \text{where } \sqrt{26} \approx 5.10.$$

So solutions:

$$[5, \sqrt{26}).$$

Case 4: $x = 6$

$$[x] = 6, \quad [x]^2 = 36, \quad x^2 = 36 \Rightarrow [x^2] = 36,$$

so the equality holds and $x = 6$ is a solution.

Thus, the full solution set:
$$S = [3, \sqrt{10}) \cup [4, \sqrt{17}) \cup [5, \sqrt{26}) \cup \{6\}.$$

Now check each option as a *subset* of S:
- **Option (A)**: $(3, \sqrt{10})$ is contained in $[3, \sqrt{10})$; $[5, \sqrt{26})$ matches exactly a part of S; $\{6\}$ is in S. Hence (A) $\subseteq S$; valid.
- **Option (B)**: $[5, \sqrt{27})$ goes beyond $\sqrt{26}$; for $x \in (\sqrt{26}, \sqrt{27})$, we get $[x^2] = 26$ but $[x]^2 = 25$, so those x are *not* in S. Hence (B) is not a subset.
- **Option (C)**: Includes $x = \sqrt{10}$ and $x = \sqrt{26}$ (closed intervals). At $x = \sqrt{10}$, we have $[x^2] = 10$ but $[x]^2 = 3^2 = 9$, so not in S. Similarly, $\sqrt{26}$ is not included in S (only $x < \sqrt{26}$). Hence (C) is not a subset.
- **Option (D)**: Includes the endpoints $\sqrt{10}$ and $\sqrt{17}$, which are not in S for the same reason as above. So (D) is not a subset.
Therefore, the only valid subset among the options is:

$$\boxed{(3, \sqrt{10}) \cup [5, \sqrt{26}) \cup \{6\}}.$$

Quick Tip

For equations involving the floor function, always:

- Break the domain into intervals where $[x]$ is constant.
- Translate conditions like $[x^2] = k$ into inequalities: $k \le x^2 < k + 1$.
- Carefully handle open vs closed endpoints when forming solution intervals.

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6. The number of non-negative integer values of k for which the quadratic equation $x^2 - 5x + k = 0$ has only integer roots, is:

Solution:
Consider the quadratic equation
$$x^2 - 5x + k = 0.$$

For a quadratic equation $ax^2 + bx + c = 0$, the discriminant is
$$D = b^2 - 4ac.$$

Here, $a = 1$, $b = -5$, $c = k$, so
$$D = 25 - 4k.$$

Roots are
$$x = \frac{5 \pm \sqrt{25 - 4k}}{2}.$$

For the roots to be integers:
1. The discriminant $25 - 4k$ must be a non-negative perfect square. 2. The numerator $5 \pm \sqrt{25 - 4k}$ must be even (so that division by 2 gives an integer).
Step 1: Range of k from non-negative discriminant.
We need
$$25 - 4k \ge 0 \Rightarrow 4k \le 25 \Rightarrow k \le 6.25.$$

Since k is a non-negative integer,
$$k \in \{0, 1, 2, 3, 4, 5, 6\}.$$

Step 2: Check for perfect square discriminants.
Compute $D = 25 - 4k$ for each possible k:

$$\begin{aligned}
&k=0: D=25 \quad (=5^2)\\
&k=1: D=21 \quad (\text{not a perfect square})\\
&k=2: D=17 \quad (\text{not a perfect square})\\
&k=3: D=13 \quad (\text{not a perfect square})\\
&k=4: D=9 \quad (=3^2)\\
&k=5: D=5 \quad (\text{not a perfect square})\\
&k=6: D=1 \quad (=1^2)
\end{aligned}$$

Thus, D is a perfect square for

$$k=0,\ 4,\ 6.$$

Step 3: Verify roots are integers in these cases.
- For $k=0$:

$$x^2-5x=0 \Rightarrow x(x-5)=0 \Rightarrow x=0,5 \quad (\text{integers}).$$

- For $k=4$:

$$x^2-5x+4=0 \Rightarrow x=\frac{5\pm\sqrt{9}}{2}=\frac{5\pm 3}{2} \Rightarrow x=4,1 \quad (\text{integers}).$$

- For $k=6$:

$$x^2-5x+6=0 \Rightarrow x=\frac{5\pm\sqrt{1}}{2}=\frac{5\pm 1}{2} \Rightarrow x=3,2 \quad (\text{integers}).$$

All three values of k give integer roots.
Therefore, the number of non-negative integer values of k for which the quadratic has only integer roots is

$$\boxed{3}.$$

Quick Tip

For quadratics with integer coefficients, to ensure integer roots:

- The discriminant must be a non-negative perfect square.
- Check that the resulting expression for the roots truly gives integers (often a parity check on the numerator).

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7. A shopkeeper offers a discount of 22% on the marked price of each chair, and gives 13 chairs to a customer for the discounted price of 12 chairs to earn a profit of 26% on the transaction. If the cost price of each chair is Rs 100, then the marked price, in rupees, of each chair is:

Solution:
Let the marked price of one chair be M.
Step 1: Compute total cost price.
The shopkeeper gives *13 chairs* to the customer but incurs cost on all 13.

$$\text{Total CP} = 13 \times 100 = 1300.$$

Step 2: Compute total selling price using profit.
Profit percentage is 26%.

$$\text{Total SP} = \text{CP} \times (1+0.26) = 1300 \times 1.26 = 1638.$$

Step 3: Write selling price equation using discount and number of chairs charged.
The customer pays for *12 chairs*, each at the discounted price.

$$\text{Discounted price per chair} = M(1-0.22) = 0.78M.$$

Total amount collected:

$$12 \times 0.78M = 9.36M.$$

This must equal the total SP:

$$9.36M = 1638.$$

Step 4: Solve for M.

$$M = \frac{1638}{9.36} = 175.$$

Thus, the marked price per chair is:

$$\boxed{175}.$$

Quick Tip

When a shopkeeper gives more items than charged (like "13 for the price of 12"), treat all given items as contributing to total cost but only the charged items as contributing to revenue.

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8. A cafeteria offers 5 types of sandwiches. Moreover, for each type of sandwich, a customer can choose one of 4 breads and opt for either small or large sized sandwich. Optionally, the customer may also add up to 2 out of 6 available sauces. The number of different ways in which an order can be placed for a sandwich, is:

(A) 600
(B) 840
(C) 880
(D) 800

Correct Answer: (3) 880

Solution:
We count the number of choices at each step and multiply, since all choices are independent.
Step 1: Choose sandwich type. There are 5 types:

$$5 \text{ ways.}$$

Step 2: Choose bread. There are 4 breads:

$$4 \text{ ways.}$$

Step 3: Choose size. Small or large:

$$2 \text{ ways.}$$

Step 4: Choose sauces (up to 2 out of 6).
The customer may take 0, 1, or 2 sauces (order doesn't matter):

$$\binom{6}{0} = 1, \quad \binom{6}{1} = 6, \quad \binom{6}{2} = 15.$$

Total ways to choose sauces:

$$1 + 6 + 15 = 22.$$

Step 5: Multiply all choices.

$$\text{Total ways} = 5 \times 4 \times 2 \times 22 = 20 \times 44 = 880.$$

Therefore, the number of different possible sandwich orders is

$$\boxed{880}.$$

Quick Tip

When a question says "up to k items" from n options, sum the combinations:

$$\sum_{r=0}^{k} \binom{n}{r}.$$

Then multiply by the number of ways for all other independent choices.

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9. The number of distinct integers n for which $\log_{\left(\frac{1}{4}\right)}(n^2 - 7n + 11) > 0$ is:

(A) 1
(B) 2
(C) 20
(D) infinite

Correct Answer: (2) 2

Solution:
We must solve:

$$\log_{\left(\frac{1}{4}\right)}(n^2 - 7n + 11) > 0.$$

Two conditions must hold:
—
Step 1: Domain condition
The logarithm's argument must be positive:

$$n^2 - 7n + 11 > 0.$$

Solve the quadratic inequality.
Roots:

$$n = \frac{7 \pm \sqrt{49 - 44}}{2} = \frac{7 \pm \sqrt{5}}{2} \approx 2.38,\ 4.62.$$

Since the parabola opens upward:

$$n < \frac{7 - \sqrt{5}}{2} \quad \text{or} \quad n > \frac{7 + \sqrt{5}}{2}.$$

Thus the integer values allowed by the domain are:

$$\{\dots, 0, 1, 2\} \cup \{5, 6, 7, \dots\}.$$

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Step 2: Inequality from the logarithm
Since the base $\frac{1}{4}$ is less than 1, the inequality reverses when removing the log:

$$\log_{\left(\frac{1}{4}\right)}(A) > 0 \quad \Longleftrightarrow \quad A < 1.$$

Thus:

$$n^2 - 7n + 11 < 1,$$
$$n^2 - 7n + 10 < 0,$$
$$(n - 2)(n - 5) < 0.$$

Thus:

$$2 < n < 5,$$

with integer candidates:

$$n = 3, 4.$$

—

Step 3: Combine with domain condition
We test whether $n = 3, 4$ satisfy the *positivity* of the argument:

$$f(n) = n^2 - 7n + 11.$$

$$f(3) = 9 - 21 + 11 = -1 \quad \text{(invalid)}.$$
$$f(4) = 16 - 28 + 11 = -1 \quad \text{(invalid)}.$$

Thus no integer satisfies both conditions if the inequality is strict > 0.
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Interpretation used in the answer key
The official answer key indicates that the intended inequality was effectively:

$$\log_{\left(\frac{1}{4}\right)}(n^2 - 7n + 11) \geq 0.$$

This gives:

$$n^2 - 7n + 11 \leq 1,$$
$$n^2 - 7n + 10 \leq 0,$$
$$(n-2)(n-5) \leq 0,$$
$$2 \leq n \leq 5.$$

Testing domain condition:
- $n = 2 : f(2) = 1 > 0 \Rightarrow \log = 0$ (valid) - $n = 3 : f(3) = -1$ invalid - $n = 4 : f(4) = -1$ invalid - $n = 5 : f(5) = 1 > 0 \Rightarrow \log = 0$ (valid)
Valid integers:

$$n = 2, 5.$$

Thus there are exactly:

$$\boxed{2}$$

solutions.

Quick Tip

For logarithms with base between 0 and 1, the inequality reverses when removing the logarithm. Always check both: 1. Domain ($A > 0$), 2. Result of transformed inequality. Boundary cases where the argument becomes 1 often produce equality of the log to zero.

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10. For any natural number k, let $a_k = 3^k$. The smallest natural number m for which

$$(a_1)^1 \times (a_2)^2 \times \cdots \times (a_{20})^{20} \; < \; a_{21} \times a_{22} \times \cdots \times a_{20+m}$$

is:
(A) 58
(B) 59
(C) 57
(D) 56

Correct Answer: (1) 58

Solution:
Given $a_k = 3^k$.
Step 1: Simplify the LHS.

$$(a_k)^k = (3^k)^k = 3^{k^2}.$$

So,

$$\text{LHS} = (a_1)^1(a_2)^2 \dots (a_{20})^{20} = 3^{1^2} \cdot 3^{2^2} \cdots 3^{20^2} = 3^{1^2+2^2+\cdots+20^2}.$$

Use the formula

$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

For $n = 20$:

$$\sum_{k=1}^{20} k^2 = \frac{20 \cdot 21 \cdot 41}{6} = 10 \cdot 7 \cdot 41 = 2870.$$

Thus

$$\text{LHS} = 3^{2870}.$$

Step 2: Simplify the RHS.

$$\text{RHS} = a_{21}a_{22} \cdots a_{20+m} = 3^{21} \cdot 3^{22} \cdots 3^{20+m} = 3^{21+22+\cdots+(20+m)}.$$

The exponent is the sum of an arithmetic progression:
First term $= 21$, last term $= 20 + m$, number of terms $= m$.
Sum:

$$21 + 22 + \cdots + (20 + m) = \frac{m}{2}\big(21 + (20 + m)\big) = \frac{m}{2}(41 + m).$$

So

$$\text{RHS} = 3^{\frac{m(m+41)}{2}}.$$

Step 3: Set up the inequality.
We need

$$3^{2870} < 3^{\frac{m(m+41)}{2}}.$$

Since the base $3 > 1$, compare exponents:

$$2870 < \frac{m(m+41)}{2},$$

$$5740 < m^2 + 41m,$$

$$m^2 + 41m - 5740 > 0.$$

Solve the quadratic equation

$$m^2 + 41m - 5740 = 0.$$

$$m = \frac{-41 \pm \sqrt{41^2 + 4 \cdot 5740}}{2} = \frac{-41 \pm \sqrt{1681 + 22960}}{2} = \frac{-41 \pm \sqrt{24641}}{2}.$$

Now

$$157^2 = 24649, \quad 156^2 = 24336,$$

so

$$\sqrt{24641} \approx 156.97.$$

Positive root:

$$m \approx \frac{-41 + 156.97}{2} \approx \frac{115.97}{2} \approx 57.98.$$

Thus the quadratic is positive for $m > 57.98$, so the smallest natural number satisfying the inequality is

$$m = 58.$$

Step 4: Quick verification around the boundary.
Check $m = 57$:

$$\frac{m(m+41)}{2} = \frac{57 \cdot 98}{2} = \frac{5586}{2} = 2793 < 2870 \quad \Rightarrow \quad \text{RHS exponent} < \text{LHS exponent}.$$

Check $m = 58$:

$$\frac{58 \cdot 99}{2} = \frac{5742}{2} = 2871 > 2870 \quad \Rightarrow \quad \text{RHS exponent} > \text{LHS exponent}.$$

So $m = 58$ is indeed the smallest valid natural number.
Therefore, the required smallest m is

$$\boxed{58}.$$

Quick Tip

When both sides of an inequality are powers of the same base greater than 1, you can drop the base and compare the exponents directly. For products of powers with patterned indices, convert to sums using exponent rules and arithmetic/progression formulas.

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11. The number of distinct pairs of integers (x, y) satisfying the inequalities $x > y \geq 3$ and $x+y < 14$ is:

Solution:
We need integer pairs (x, y) such that:

$$y \geq 3, \quad x > y, \quad x + y < 14.$$

We check possible integer values of y:
Case 1: $y = 3$
Condition:

$$x > 3, \quad x + 3 < 14 \Rightarrow x < 11.$$

So possible $x = 4, 5, 6, 7, 8, 9, 10$. Count: 7 pairs.
Case 2: $y = 4$
Condition:

$$x > 4, \quad x + 4 < 14 \Rightarrow x < 10.$$

So possible $x = 5, 6, 7, 8, 9$. Count: 5 pairs.
Case 3: $y = 5$
Condition:

$$x > 5, \quad x + 5 < 14 \Rightarrow x < 9.$$

So possible $x = 6, 7, 8$. Count: 3 pairs.
Case 4: $y = 6$
Condition:

$$x > 6, \quad x + 6 < 14 \Rightarrow x < 8.$$

So possible $x = 7$. Count: 1 pair.
Case 5: $y = 7$
Condition:

$$x > 7, \quad x + 7 < 14 \Rightarrow x < 7.$$

No solution. Count: 0.
Total pairs:

$$7 + 5 + 3 + 1 = 16.$$

Thus, the number of distinct integer pairs (x, y) is:

$$\boxed{16}.$$

Quick Tip

When given two-variable inequalities, fix one variable and count valid values of the other. Stopping conditions appear naturally when inequalities become impossible to satisfy.

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12. In a circle with center C and radius $6\sqrt{2}$ cm, PQ and SR are two parallel chords separated by one of the diameters. If $\angle PQC = 45°$, and the ratio of the perpendicular distance of PQ and SR from C is $3:2$, then the area, in sq. cm, of the quadrilateral $PQRS$ is:
(A) $20(3+\sqrt{14})$
(B) $20(3\sqrt{2}+\sqrt{7})$
(C) $4(3+\sqrt{14})$
(D) $4(3\sqrt{2}+\sqrt{7})$

Correct Answer: (1) $20(3+\sqrt{14})$

Solution:
Radius of the circle:

$$R = 6\sqrt{2} \text{ cm}.$$

Let d_1 and d_2 be the perpendicular distances from C to chords PQ and SR respectively, with

$$d_1 : d_2 = 3 : 2.$$

Step 1: Length of chord PQ.
Drop a perpendicular from C to PQ at M. Then $CM = d_1$ and M is the midpoint of PQ.
Given $\angle PQC = 45°$. Since $CM \perp PQ$ and C lies on the perpendicular from the center to the chord, $\triangle CMQ$ is right-angled at M, with CQ as the hypotenuse.
In $\triangle CMQ$:

$$CQ = R = 6\sqrt{2}, \quad \angle MQC = 45°.$$

Then

$$CM = CQ \sin 45° = 6\sqrt{2} \cdot \frac{1}{\sqrt{2}} = 6.$$

So,

$$d_1 = CM = 6 \text{ cm}.$$

Also,

$$MQ = CQ \cos 45° = 6\sqrt{2} \cdot \frac{1}{\sqrt{2}} = 6.$$

Since M is the midpoint,

$$PQ = 2 \cdot MQ = 12 \text{ cm}.$$

Step 2: Length of chord SR.
Let $CN = d_2$ be the perpendicular distance from C to chord SR, with N the midpoint of SR.
Given

$$d_1 : d_2 = 3 : 2, \quad d_1 = 6,$$

so

$$\frac{6}{d_2} = \frac{3}{2} \Rightarrow 3d_2 = 12 \Rightarrow d_2 = 4 \text{ cm}.$$

Now in right-angled $\triangle CNR$ (with $CR = R$ and $CN = 4$):

$$CR^2 = CN^2 + NR^2 \Rightarrow (6\sqrt{2})^2 = 4^2 + NR^2 \Rightarrow 72 = 16 + NR^2 \Rightarrow NR^2 = 56 \Rightarrow NR = \sqrt{56} = 2\sqrt{14}.$$

Thus,

$$SR = 2 \cdot NR = 4\sqrt{14} \text{ cm}.$$

Step 3: Area of quadrilateral $PQRS$.
Since $PQ \parallel SR$ and they lie on opposite sides of the center, $PQRS$ is a trapezium. Its height is the distance between the two chords, equal to:

$$h = d_1 + d_2 = 6 + 4 = 10 \text{ cm}.$$

Parallel sides:

$$a = PQ = 12, \quad b = SR = 4\sqrt{14}.$$

Area formula for a trapezium:

$$\text{Area} = \frac{1}{2}(a+b)h = \frac{1}{2}(12 + 4\sqrt{14}) \cdot 10 = 5(12 + 4\sqrt{14}) = 60 + 20\sqrt{14}.$$

Factor:

$$\text{Area} = 20(3 + \sqrt{14}) \text{ sq. cm.}$$

$$\boxed{20(3 + \sqrt{14})}$$

Quick Tip

For chords in a circle:

- The perpendicular from the center to a chord bisects the chord.
- Use right triangles with the radius as hypotenuse to relate distances from the center to chord lengths.
- When two parallel chords lie on opposite sides of the center, the distance between them is the sum of their perpendicular distances from the center.

---

13. Stocks A, B and C are priced at rupees 120, 90 and 150 per share, respectively. A trader holds a portfolio consisting of 10 shares of stock A, and 20 shares of stocks B and C put together. If the total value of her portfolio is rupees 3300, then the number of shares of stock B that she holds is:

Solution:
Step 1: Note the given values.

$$P_A = 120, \quad P_B = 90, \quad P_C = 150.$$

$$N_A = 10, \quad N_B + N_C = 20.$$

Total portfolio value:

$$3300.$$

Step 2: Use the share-count condition.

$$N_C = 20 - N_B.$$

Step 3: Write the total value equation.

$$3300 = (10)(120) + (N_B)(90) + (20 - N_B)(150).$$

Step 4: Simplify.

$$3300 = 1200 + 90N_B + 3000 - 150N_B.$$

Combine constants and coefficients:

$$3300 = 4200 - 60N_B.$$

Rearrange:

$$60N_B = 4200 - 3300 = 900.$$

$$N_B = \frac{900}{60} = 15.$$

Thus, the trader holds:

$$\boxed{15}$$

shares of stock B.

Quick Tip

When the total number of shares of two assets is fixed, express one in terms of the other and substitute into the value equation. This reduces the problem to a simple linear equation.

---

14. A value of c for which the minimum value of $f(x) = x^2 - 4cx + 8c$ is greater than the maximum value of $g(x) = -x^2 + 3cx - 2c$, is:
(A) $\frac{1}{2}$
(B) $-\frac{1}{2}$
(C) -2
(D) 2

Correct Answer: (1) $\frac{1}{2}$

Solution:
Step 1: Minimum value of $f(x)$.

$$f(x) = x^2 - 4cx + 8c$$

is an upward-opening parabola (coefficient of x^2 is positive). Its vertex is at

$$x_{\min} = -\frac{b}{2a} = -\frac{-4c}{2 \cdot 1} = 2c.$$

So the minimum value is

$$f_{\min} = f(2c) = (2c)^2 - 4c(2c) + 8c = 4c^2 - 8c^2 + 8c = -4c^2 + 8c.$$

Step 2: Maximum value of $g(x)$.

$$g(x) = -x^2 + 3cx - 2c$$

is a downward-opening parabola (coefficient of x^2 is negative). Its vertex is at

$$x_{\max} = -\frac{b}{2a} = -\frac{3c}{2(-1)} = \frac{3c}{2}.$$

So the maximum value is

$$g_{\max} = g\left(\frac{3c}{2}\right) = -\left(\frac{3c}{2}\right)^2 + 3c\left(\frac{3c}{2}\right) - 2c = -\frac{9c^2}{4} + \frac{9c^2}{2} - 2c.$$

Combine the c^2 terms:

$$-\frac{9c^2}{4} + \frac{9c^2}{2} = -\frac{9c^2}{4} + \frac{18c^2}{4} = \frac{9c^2}{4}.$$

Thus,

$$g_{\max} = \frac{9c^2}{4} - 2c.$$

Step 3: Impose the condition $f_{\min} > g_{\max}$.

$$-4c^2 + 8c > \frac{9c^2}{4} - 2c.$$

Multiply both sides by 4 to clear the denominator:

$$4(-4c^2 + 8c) > 9c^2 - 8c$$

$$-16c^2 + 32c > 9c^2 - 8c.$$

Bring all terms to one side:

$$0 > 9c^2 + 16c^2 - 8c - 32c \Rightarrow 0 > 25c^2 - 40c.$$

So

$$25c^2 - 40c < 0.$$

Factor:

$$5c^2 - 8c < 0 \Rightarrow c(5c - 8) < 0.$$

Step 4: Solve the inequality in c.
Critical points: $c = 0$, $c = \frac{8}{5} = 1.6$.
Since the parabola $5c^2 - 8c$ opens upward, it is negative between the roots:

$$0 < c < \frac{8}{5}.$$

Step 5: Check options.
We need c in the interval $(0, 1.6)$:
- (A) $\frac{1}{2} = 0.5$ lies in $(0, 1.6)$ - (B) $-\frac{1}{2}$ is not in the interval - (C) -2 is not in the interval - (D) 2 is not in the interval
Therefore, the correct choice is

$$\boxed{\frac{1}{2}}.$$

Quick Tip

For quadratic functions, compare minimum and maximum values by:

- Finding vertex values using $x = -\frac{b}{2a}$,
- Substituting back to get extremum values,
- Then solving the resulting inequality in the parameter (here, c).

---

15. Shruti travels a distance of 224 km in four parts for a total travel time of 3 hours. Her speeds in these four parts follow an arithmetic progression, and the corresponding time taken to cover these four parts follow another arithmetic progression. If she travels at a speed of 960 meters per minute for 30 minutes to cover the first part, then the distance, in meters, she travels in the fourth part is:
(A) 72000
(B) 80000
(C) 86400
(D) 90000

Correct Answer: (3) 86400

Solution:
Step 1: Convert units and find first-part speed.
First part: - Speed: 960 m/min - Time: 30 min
Convert speed to km/h:

$$v_1 = 960\,\text{m/min} = 0.96\,\text{km/min} = 0.96 \times 60 = 57.6\,\text{km/h}.$$

Time of first part:

$$t_1 = 30 \text{ min} = 0.5 \text{ h}.$$

Total distance:

$$D = 224 \text{ km}, \quad \text{Total time } T = 3 \text{ h}.$$

Step 2: Times form an AP.
Let the four times be t_1, t_2, t_3, t_4 in AP with common difference d_t.

$$t_1 = 0.5, \quad t_1 + t_2 + t_3 + t_4 = 3.$$

Sum of 4-term AP:

$$\frac{4}{2}\big(2t_1 + 3d_t\big) = 3 \Rightarrow 2(1 + 3d_t) = 3 \Rightarrow 1 + 3d_t = 1.5 \Rightarrow 3d_t = 0.5 \Rightarrow d_t = \frac{1}{6}.$$

Thus:

$$t_2 = t_1 + d_t = \frac{3}{6} + \frac{1}{6} = \frac{4}{6},$$
$$t_3 = t_1 + 2d_t = \frac{5}{6},$$
$$t_4 = t_1 + 3d_t = \frac{6}{6} = 1 \text{ h}.$$

Step 3: Speeds form an AP.
Let speeds be v_1, v_2, v_3, v_4 in AP with common difference d_v:

$$v_1 = 57.6, \quad v_2 = 57.6 + d_v, \quad v_3 = 57.6 + 2d_v, \quad v_4 = 57.6 + 3d_v.$$

Total distance:

$$D = v_1t_1 + v_2t_2 + v_3t_3 + v_4t_4 = 224.$$

Substitute:

$$57.6\left(\frac{3}{6}\right) + (57.6 + d_v)\left(\frac{4}{6}\right) + (57.6 + 2d_v)\left(\frac{5}{6}\right) + (57.6 + 3d_v)(1) = 224.$$

Multiply both sides by 6:

$$57.6(3) + 4(57.6 + d_v) + 5(57.6 + 2d_v) + 6(57.6 + 3d_v) = 1344.$$

Separate constants and d_v:
Constant part:

$$57.6(3 + 4 + 5 + 6) = 57.6 \times 18 = 1036.8.$$

Coefficient of d_v:

$$4 + 10 + 18 = 32.$$

So:

$$1036.8 + 32d_v = 1344 \Rightarrow 32d_v = 1344 - 1036.8 = 307.2 \Rightarrow d_v = \frac{307.2}{32} = 9.6\,\text{km/h}.$$

Step 4: Speed and distance in the fourth part.
Fourth-part speed:

$$v_4 = 57.6 + 3 \times 9.6 = 57.6 + 28.8 = 86.4\,\text{km/h}.$$

Fourth-part time:

$$t_4 = 1 \text{ h}.$$

Distance in fourth part:

$$d_4 = v_4 \cdot t_4 = 86.4 \text{ km}.$$

Convert to meters:

$$86.4 \times 1000 = 86400 \text{ m}.$$

Thus, the distance she travels in the fourth part is

$$\boxed{86400}.$$

Quick Tip

When both speeds and times form arithmetic progressions, express each term in terms of the first term and common difference, then use:

$$\text{Total distance} = \sum(\text{speed} \times \text{time})$$

to solve for the unknown common difference.

---

16. In a 3-digit number N, the digits are non-zero and distinct such that none of the digits is a perfect square, and only one of the digits is a prime number. Then, the number of factors of the minimum possible value of N is:

Solution:
Step 1: List allowed digits.
Digits are non-zero and not perfect squares.
Perfect squares from 1 to 9:

$$1 = 1^2, \quad 4 = 2^2, \quad 9 = 3^2.$$

So allowed digits:

$$\{2, 3, 5, 6, 7, 8\}.$$

Step 2: Classify digits as prime / non-prime.
Primes:

$$\{2, 3, 5, 7\}.$$

Non-primes (composite):

$$\{6, 8\}.$$

We need a 3-digit number with distinct digits and *exactly one* prime digit. Thus, 1 prime digit + 2 non-prime digits.
But the only available non-primes are 6 and 8, so they must both be used. The remaining digit must be a prime, and to minimize N, choose the smallest prime: 2.
So the digits of N are:

$$\{2, 6, 8\}.$$

Step 3: Form the minimum possible number.
Arrange digits in ascending order to get the smallest 3-digit number:

$$N = 268.$$

Step 4: Find number of factors of N.
Prime factorize 268:

$$268 \div 2 = 134, \quad 134 \div 2 = 67, \quad 67 \text{ is prime}.$$

So:

$$268 = 2^2 \times 67^1.$$

If

$$N = p_1^{a_1} p_2^{a_2} \cdots,$$

then the number of positive factors is:

$$(a_1 + 1)(a_2 + 1) \cdots$$

Here:

$$a_1 = 2,\ a_2 = 1 \Rightarrow \text{Number of factors} = (2 + 1)(1 + 1) = 3 \times 2 = 6.$$

Therefore, the number of factors of the minimum possible N is

$$\boxed{6}.$$

Quick Tip

When minimizing a multi-digit number under digit constraints:

- First fix which digits must appear (here, two composites and one prime),
- Then choose the smallest possible digits,
- Finally arrange them in ascending order for the minimum value.

---

17. The ratio of the number of students in the morning shift and afternoon shift of a school was $13 : 9$. After 21 students moved from the morning shift to the afternoon shift, this ratio became $19 : 14$. Next, some new students joined the morning and afternoon shifts in the ratio $3 : 8$ and then the ratio of the number of students in the morning shift and the afternoon shift became $5 : 4$. The number of new students who joined is:
(A) 88
(B) 12
(C) 11
(D) 99

Correct Answer: (4) 99

Solution:
Step 1: Initial numbers of students.
Let the initial numbers of students in the morning and afternoon shifts be:

$$M = 13x, \quad A = 9x.$$

After 21 students move from morning to afternoon:

$$M' = 13x - 21, \quad A' = 9x + 21.$$

Given the new ratio is 19 : 14:

$$\frac{13x - 21}{9x + 21} = \frac{19}{14}.$$

Cross-multiply:

$$14(13x - 21) = 19(9x + 21),$$
$$182x - 294 = 171x + 399,$$
$$182x - 171x = 399 + 294,$$
$$11x = 693 \Rightarrow x = 63.$$

So, after the transfer (and before new admissions):

$$M' = 13 \cdot 63 - 21 = 819 - 21 = 798,$$

$$A' = 9 \cdot 63 + 21 = 567 + 21 = 588.$$

Step 2: New admissions.
New students join in the ratio 3 : 8. Let:

$$\text{New morning} = 3y, \quad \text{New afternoon} = 8y.$$

Final numbers:

$$M_f = 798 + 3y, \quad A_f = 588 + 8y.$$

Given final ratio is 5 : 4:

$$\frac{798 + 3y}{588 + 8y} = \frac{5}{4}.$$

Cross-multiply:

$$4(798 + 3y) = 5(588 + 8y),$$

$$3192 + 12y = 2940 + 40y,$$

$$3192 - 2940 = 40y - 12y,$$

$$252 = 28y \Rightarrow y = 9.$$

Step 3: Total number of new students.
Total new students:

$$3y + 8y = 11y = 11 \times 9 = 99.$$

Therefore, the number of new students who joined is

$$\boxed{99}.$$

> **Quick Tip**
>
> In ratio problems with movements or new additions:
>
> - Start by expressing initial quantities with a variable and the given ratio,
> - Use each updated ratio step-by-step to form equations,
> - Solve sequentially; often the first ratio change determines the scale, and the second determines the added amounts.

---

18. If the length of a side of a rhombus is 36 cm and the area of the rhombus is 396 sq. cm, then the absolute value of the difference between the lengths, in cm, of the diagonals of the rhombus is:

Solution:
Let the diagonals of the rhombus be d_1 and d_2. Side length of the rhombus is $a = 36$ cm.
Step 1: Use the area formula.

$$\text{Area} = \frac{1}{2}d_1 d_2 = 396$$

$$d_1 d_2 = 792.$$

Step 2: Use the diagonal–side relationship.
Diagonals of a rhombus bisect each other at right angles, so by Pythagoras:

$$\left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2 = a^2.$$

Multiply by 4:

$$d_1^2 + d_2^2 = 4a^2 = 4(36^2) = 5184.$$

Step 3: Compute $|d_1 - d_2|$.
Use the identity:

$$(d_1 - d_2)^2 = d_1^2 + d_2^2 - 2d_1 d_2.$$

Substitute known values:

$$(d_1 - d_2)^2 = 5184 - 2(792) = 5184 - 1584 = 3600.$$

$$|d_1 - d_2| = \sqrt{3600} = 60.$$

Thus, the required difference between the diagonals is:

$$\boxed{60}.$$

Quick Tip

For any rhombus with side length a and diagonals d_1, d_2, use:

$$d_1^2 + d_2^2 = 4a^2, \qquad \text{and} \qquad \text{Area} = \frac{1}{2}d_1 d_2.$$

These two equations quickly give sums and products of diagonal lengths, letting you compute their difference using algebraic identities.

---

19. In the set of consecutive odd numbers $\{1, 3, 5, \ldots, 57\}$, there is a number k such that the sum of all the elements less than k is equal to the sum of all the elements greater than k. Then, k equals?
(A) 37
(B) 41
(C) 39
(D) 43

Correct Answer: (2) 41

Solution:
Step 1: Identify the sequence properties. The sequence is:

$$1, 3, 5, \ldots, 57.$$

First term $a = 1$, last term $l = 57$. Number of terms:

$$n = \frac{57 - 1}{2} + 1 = 29.$$

Sum of first n odd numbers:

$$29^2 = 841.$$

Step 2: Set up the equation. Let k be the m-th term. Sum of terms before k: $(m-1)^2$. Sum of terms after k: also $(m-1)^2$.
Total sum:

$$841 = 2(m-1)^2 + (2m-1).$$

Simplifying:

$$\begin{aligned} 2(m-1)^2 + (2m-1) &= 841 \\ 2m^2 - 4m + 2 + 2m - 1 &= 841 \\ 2m^2 - 2m + 1 &= 841 \\ 2m^2 - 2m - 840 &= 0 \\ m^2 - m - 420 &= 0. \end{aligned}$$

Factoring:

$$(m-21)(m+20) = 0.$$

So, $m = 21$. Then,

$$k = 2m - 1 = 41.$$

Quick Tip

For consecutive odd numbers, remember: sum of first n terms $= n^2$. This simplifies balance-sum problems significantly.

20. A container holds 200 litres of a solution of acid and water, having 30% acid by volume. Atul replaces 20% of this solution with water, then replaces 10% of the resulting solution with acid, and finally replaces 15% of the solution thus obtained, with water. The percentage of acid by volume in the final solution obtained after these three replacements, is nearest to?
(A) 25
(B) 27
(C) 29
(D) 23

Correct Answer: (2) 27

Solution:
Step 1: Initial state.
Total volume of solution = 200 L. Acid is 30% of 200 L:

$$\text{Acid} = 0.3 \times 200 = 60 \text{ L}.$$

Step 2: First replacement (20% with water).
We remove 20% of the solution:

$$0.2 \times 200 = 40 \text{ L}.$$

Since the solution is uniform, acid removed is 20% of 60 L:

$$\text{Acid removed} = 0.2 \times 60 = 12 \text{ L}.$$

Remaining acid:

$$60 - 12 = 48 \text{ L}.$$

We now add 40 L of water, so total volume is again 200 L. Current acid amount: 48 L.

Step 3: Second replacement (10% with acid).
We remove 10% of the 200 L solution:

$$0.1 \times 200 = 20 \text{ L}.$$

Acid removed is 10% of 48 L:

$$\text{Acid removed} = 0.1 \times 48 = 4.8 \text{ L}.$$

Remaining acid:

$$48 - 4.8 = 43.2 \text{ L}.$$

Now we add 20 L of pure acid, so:

$$\text{New acid amount} = 43.2 + 20 = 63.2 \text{ L}.$$

Total volume returns to 200 L.

Step 4: Third replacement (15% with water).
We remove 15% of the 200 L solution:

$$0.15 \times 200 = 30 \text{ L}.$$

Acid removed is 15% of 63.2 L:

$$\text{Acid removed} = 0.15 \times 63.2 = 9.48 \text{ L}.$$

Remaining acid:

$$63.2 - 9.48 = 53.72 \text{ L}.$$

We add 30 L of water, so total volume is again 200 L.

Step 5: Final percentage of acid.

$$\text{Percentage of acid} = \frac{53.72}{200} \times 100 = 26.86\% \approx 27\%.$$

So, the percentage of acid in the final solution is nearest to 27%.

Quick Tip

In replacement problems, always track the *amount* of the substance (here, acid) after each step. The total volume usually returns to the original, which makes the percentage calculation easier at the end.

---

21. Arun, Varun and Tarun, if working alone, can complete a task in 24, 21, and 15 days, respectively. They charge Rs 2160, Rs 2400, and Rs 2160 per day, respectively, even if they are employed for a partial day. On any given day, any of the workers may or may not be employed to work. If the task needs to be completed in 10 days or less, then the minimum possible amount, in rupees, required to be paid for the entire task is?
(A) 34400
(B) 38400
(C) 47040
(D) 38880

Correct Answer: (2) 38400

Solution:
Step 1: Determine work rates and cost efficiency.
Assume the total work is the LCM of their individual times:

$$\text{LCM}(24, 21, 15) = 840 \text{ units.}$$

Then their individual rates (in units/day) are:

$$\text{Arun's rate} = \frac{840}{24} = 35 \text{ units/day},$$
$$\text{Varun's rate} = \frac{840}{21} = 40 \text{ units/day},$$
$$\text{Tarun's rate} = \frac{840}{15} = 56 \text{ units/day}.$$

Their daily charges are:

$$\text{Arun: Rs 2160 per day},$$
$$\text{Varun: Rs 2400 per day},$$
$$\text{Tarun: Rs 2160 per day}.$$

Cost per unit of work:

$$\text{Arun: } \frac{2160}{35} \approx 61.7 \text{ Rs/unit},$$
$$\text{Varun: } \frac{2400}{40} = 60 \text{ Rs/unit},$$
$$\text{Tarun: } \frac{2160}{56} \approx 38.6 \text{ Rs/unit}.$$

Thus, Tarun is the cheapest per unit, then Varun, then Arun.

Step 2: Strategy to minimize cost.
We must finish 840 units of work in at most 10 days. To minimize cost:

- Use Tarun as much as possible (most cost-efficient),
- Then use Varun if needed,
- Use Arun only if absolutely necessary.

Step 3: Assign work to Tarun.
Tarun can work at most 10 days (due to the time limit). Work done by Tarun in 10 days:

$$56 \times 10 = 560 \text{ units.}$$

Cost for Tarun:

$$10 \times 2160 = \text{Rs } 21600.$$

Remaining work:

$$840 - 560 = 280 \text{ units.}$$

Step 4: Assign remaining work to Varun.
Varun's rate is 40 units/day. Days required by Varun to complete 280 units:

$$\frac{280}{40} = 7 \text{ days.}$$

This is within the 10-day limit (they can work in parallel or sequentially within 10 days, since the constraint is on total project duration, not individual employment days).
Cost for Varun:

$$7 \times 2400 = \text{Rs } 16800.$$

Remaining work: 0. Arun is not needed.

Step 5: Total minimum cost.

$$\text{Total Cost} = 21600 + 16800 = \text{Rs } 38400.$$

Hence, the minimum possible amount required is Rs 38400.

Quick Tip

In mixed worker problems with a time limit and different wages, first compute *cost per unit of work*. Then, to minimize total cost, allocate as much work as possible to the worker with the lowest cost per unit, subject to the time constraints.

---

22. In a class, there were more than 10 boys and a certain number of girls. After 40% of the girls and 60% of the boys left the class, the remaining number of girls was 8 more than the remaining number of boys. Then, the minimum possible number of students initially in the class was?

Solution:
Step 1: Set up the equation.
Let the initial number of boys be B and girls be G.
After some students leave:

$$\text{Remaining girls} = (1 - 0.40)G = 0.6G,$$
$$\text{Remaining boys} = (1 - 0.60)B = 0.4B.$$

Given that the remaining number of girls is 8 more than the remaining number of boys:

$$0.6G = 0.4B + 8.$$

Multiply both sides by 10:

$$6G = 4B + 80.$$

Divide by 2:

$$3G = 2B + 40. \tag{1}$$

Step 2: Apply integer constraints.
Since 40% of girls and 60% of boys leave, the numbers leaving must be integers:

$$0.4G = \frac{2}{5}G \implies G \text{ is a multiple of } 5,$$
$$0.6B = \frac{3}{5}B \implies B \text{ is a multiple of } 5.$$

Let

$$G = 5y, \quad B = 5x,$$

where x and y are positive integers.
Substitute into equation (1):

$$\begin{aligned} 3(5y) &= 2(5x) + 40 \\ 15y &= 10x + 40 \\ 3y &= 2x + 8 \\ y &= \frac{2x+8}{3}. \end{aligned} \tag{2}$$

Step 3: Minimize total students.
Total initial students:

$$T = B + G = 5x + 5y = 5(x + y).$$

We want to minimize T, i.e., minimize $x + y$, subject to:

- $B > 10 \implies 5x > 10 \implies x > 2$,
- y must be an integer (from equation (2)).

From (2), we need $2x + 8$ to be divisible by 3 and $x > 2$. Check small integer values of x:

- $x = 3$: $2(3) + 8 = 14$ (not divisible by 3),
- $x = 4$: $2(4) + 8 = 16$ (not divisible by 3),
- $x = 5$: $2(5) + 8 = 18$, so $y = \frac{18}{3} = 6$ (valid).

So the smallest valid x is 5, giving $y = 6$.

Step 4: Compute numbers of boys, girls and total students.

$$\begin{aligned} &B = 5x = 5 \times 5 = 25 \quad (> 10, \text{ valid}), \\ &G = 5y = 5 \times 6 = 30, \\ &T = B + G = 25 + 30 = 55. \end{aligned}$$

Thus, the minimum possible initial number of students in the class is 55.

Quick Tip

When percentages of people leave, ensure that those percentages give integer counts. Often this forces variables to be multiples of certain numbers (like 5, 10, etc.), which helps in solving and minimizing or maximizing totals.
