# CAT 2025 QA Slot 1 Question Paper with Solutions

| **Time Allowed :**120 Minutes | **Maximum Marks :**204 | **Total Questions :**68 |
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**General Instructions**

**Read the following instructions very carefully and strictly follow them:**

1. The total duration of the test is **120 Minutes**, with **40 minutes** allotted per section.
2. The question paper is divided into **three sections**:
   - **Section 1:** Verbal Ability and Reading Comprehension (VARC) – 24 questions
   - **Section 2:** Data Interpretation and Logical Reasoning (DILR) – 22 questions
   - **Section 3:** Quantitative Aptitude (QA) – 22 questions
3. Each correct answer carries **+3 marks**.
4. For multiple-choice questions (MCQs), **−1 mark** will be deducted for each wrong answer.
5. There is **no negative marking** for Type-in-the-Answer (TITA) questions.

**1. In the sequence 1, 3, 5, 7, ..., k, ..., 57, the sum of the numbers up to k, excluding k, is equal to the sum of the numbers from k up to 57, also excluding k. What is k?**

**Correct Answer:** k = 41

**Solution:**

**Step 1: Understanding the Question:**
We are given an arithmetic progression (AP) of odd numbers from 1 to 57.
There is a term 'k' in this sequence.
The problem states that the sum of all terms before 'k' is equal to the sum of all terms after 'k'.
Our goal is to find the value of 'k'.

**Step 2: Key Formula or Approach:**
The given sequence is an AP: 1, 3, 5, ...
The n-th term of this AP is given by $a_n = 2n - 1$.
The sum of the first 'n' odd numbers is a standard result given by the formula:

$$S_n = n^2$$

We will use this formula to represent the sums and solve for 'k'.

**Step 3: Detailed Explanation:**
First, let's find the total number of terms in the sequence. The last term is 57.
Let N be the total number of terms.

$$a_N = 2N - 1 = 57$$
$$2N = 58$$
$$N = 29$$

So, there are 29 terms in the entire sequence. The sum of all terms is $S_{29} = 29^2 = 841$.

Now, let 'k' be the m-th term in the sequence. So, $k = a_m = 2m - 1$.
The sum of terms before k is the sum of the first $m - 1$ terms.

$$S_{before} = S_{m-1} = (m-1)^2$$

The sum of terms after k is the sum of terms from the (m+1)-th term to the 29th term.
This can be calculated as the total sum minus the sum of terms up to and including k.
The sum up to and including k is the sum of the first 'm' terms, $S_m = m^2$.

$$S_{after} = S_{total} - S_m = 29^2 - m^2$$

According to the problem statement, $S_{before} = S_{after}$.

$$(m-1)^2 = 29^2 - m^2$$

Now, we solve this equation for m:

$$m^2 - 2m + 1 = 841 - m^2$$
$$2m^2 - 2m - 840 = 0$$

Dividing the equation by 2, we get:

$$m^2 - m - 420 = 0$$

This is a quadratic equation. We can solve it by factoring. We need two numbers that multiply to -420 and add to -1. These numbers are -21 and 20.

$$(m-21)(m+20) = 0$$

The possible values for m are $m = 21$ or $m = -20$. Since m represents the position of a term in a sequence, it must be a positive integer. Thus, we take $m = 21$.

The question asks for the value of k, which is the 21st term.

$$k = a_{21} = 2(21) - 1$$
$$k = 42 - 1 = 41$$

**Step 4: Final Answer:**
The value of k is **41**.

**Quick Tip**

The sum of the first n odd numbers is $n^2$. Remembering this shortcut is much faster than using the general AP sum formula for this type of problem.

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**2. A 200-litre container holds a solution that is 30% acid and the rest water. The solution undergoes the following three processes sequentially:**
**1. 20% of the water content is evaporated.**
**2. From the remaining mixture, 10% of the acid content is chemically extracted and removed.**
**3. Finally, 15% of the resulting solution is removed and replaced with water.**
**What is the volume of acid in the final solution?**

**Correct Answer:** 45.9 litres

**Solution:**

**Step 1: Understanding the Question:**
The problem involves a sequence of operations on a solution of acid and water. We need to track the volume of acid and water through each step to find the final volume of acid. We will calculate the changes step-by-step.

**Step 2: Detailed Explanation:**
We will break down the problem into the initial state and the three subsequent processes.

**Initial State:**
Total volume of the solution = 200 litres.
Acid concentration = 30%.
Water concentration = 100% - 30% = 70%.
Initial volume of acid = $0.30 \times 200$ litres = **60 litres**.
Initial volume of water = $0.70 \times 200$ litres = **140 litres**.

**Process 1: Evaporation of Water**
20% of the water content is evaporated. Only the water volume changes.
Volume of water evaporated = $0.20 \times 140$ litres = 28 litres.
Remaining volume of water = $140 - 28 =$ **112 litres**.
Volume of acid remains the same = **60 litres**.
Total volume of the solution after evaporation = $60 + 112 = 172$ litres.

**Process 2: Extraction of Acid**
10% of the acid content is removed. Only the acid volume changes.
Volume of acid extracted = $0.10 \times 60$ litres = 6 litres.
Remaining volume of acid = $60 - 6 =$ **54 litres**.
Volume of water remains the same = **112 litres**.

Total volume of the solution after extraction $= 54 + 112 = 166$ litres.

**Process 3: Removal and Replacement**
15% of the resulting solution (166 litres) is removed. This removal affects both acid and water.
Volume of solution removed $= 0.15 \times 166$ litres $= 24.9$ litres.
The acid removed is 15% of the current acid volume:
Volume of acid removed $= 0.15 \times 54$ litres $= 8.1$ litres.
The final volume of acid is the volume after this removal:
Final volume of acid $= 54 - 8.1 =$ **45.9 litres**.
(Note: This removed volume of 24.9 litres is then replaced with water, which changes the final water volume and total volume, but does not affect the final acid volume).

**Step 3: Final Answer:**
The volume of acid in the final solution is **45.9 litres**.

> **Quick Tip**
>
> In multi-step mixture problems, always keep a clear record of the volume of each component (e.g., acid, water) and the total volume after each step. Pay close attention to what each percentage refers to—the total solution, a specific component, etc.

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**3. Find the number of real values of x satisfying the equation:**

$$\log_2(x^2 - 5x + 6) + \log_{1/2}(x - 2) = 3$$

**Correct Answer:** 1

**Solution:**

**Step 1: Understanding the Question:**
The question asks for the number of real solutions to a logarithmic equation. To solve this, we must first determine the valid domain for 'x' by ensuring the arguments of the logarithms are positive, and then solve the equation using logarithmic properties.

**Step 2: Key Formula or Approach:**
We will use the following properties of logarithms:
1. **Domain of Logarithm:** For $\log_b(a)$ to be defined, we must have $a > 0$.
2. **Change of Base Formula:** $\log_{b^n}(a) = \frac{1}{n}\log_b(a)$. A special case is $\log_{1/b}(a) = \log_{b^{-1}}(a) = -\log_b(a)$.
3. **Logarithm of a Quotient:** $\log_b(a) - \log_b(c) = \log_b\left(\frac{a}{c}\right)$.
4. **Logarithmic to Exponential Form:** If $\log_b(a) = c$, then $a = b^c$.

**Step 3: Detailed Explanation:**

**Part A: Finding the Domain**

The arguments of the logarithms must be strictly positive.

1. From $\log_2(x^2 - 5x + 6)$, we have:

$$x^2 - 5x + 6 > 0$$
$$(x-2)(x-3) > 0$$

This inequality is true when $x < 2$ or $x > 3$.

2. From $\log_{1/2}(x-2)$, we have:

$$x - 2 > 0$$
$$x > 2$$

To satisfy both conditions, we take the intersection of the two solution sets: ($x < 2$ or $x > 3$) AND ($x > 2$). The common domain is $x > 3$. Any valid solution must satisfy this condition.

**Part B: Solving the Equation**

First, we simplify the second term using the change of base property:

$$\log_{1/2}(x-2) = \log_{2^{-1}}(x-2) = -1 \cdot \log_2(x-2) = -\log_2(x-2)$$

Substitute this back into the original equation:

$$\log_2(x^2 - 5x + 6) - \log_2(x-2) = 3$$

Now, use the quotient rule for logarithms:

$$\log_2\left(\frac{x^2 - 5x + 6}{x-2}\right) = 3$$

Factor the numerator:

$$\log_2\left(\frac{(x-2)(x-3)}{x-2}\right) = 3$$

Since our domain is $x > 3$, $x - 2 \neq 0$, so we can cancel the term:

$$\log_2(x-3) = 3$$

Convert this to exponential form:

$$x - 3 = 2^3$$
$$x - 3 = 8$$
$$x = 11$$

**Part C: Checking the Solution**

The solution we found is $x = 11$. We must check if it lies in the valid domain ($x > 3$).
Since $11 > 3$, the solution is valid.

**Step 4: Final Answer:**
There is only one real value, $x = 11$, that satisfies the equation. Therefore, the number of solutions is **1**.

> **Quick Tip**
>
> For logarithmic equations, always find the domain of the variable first. This helps in rejecting extraneous solutions obtained during the algebraic manipulation.

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**4. Find the number of integer pairs (x, y) that satisfy the following system of inequalities:**

$$\begin{cases} x \geq y \geq 3 \\ x + y \leq 14 \end{cases}$$

**Correct Answer:** 25

**Solution:**

**Step 1: Understanding the Question:**
The question asks for the number of pairs of integers (x, y) that simultaneously satisfy three conditions: $x \geq y$, $y \geq 3$, and $x+y \leq 14$. This is a counting problem based on linear inequalities.

**Step 2: Key Formula or Approach:**
We can solve this problem by iterating through all possible integer values for one variable (say, y) and counting the corresponding number of valid integer values for the other variable (x). The number of integers in a range [a, b] is given by $b - a + 1$.

**Step 3: Detailed Explanation:**
From the given inequalities, we can establish bounds for x and y.
1. We know $y$ is an integer and $y \geq 3$.
2. We have two constraints on $x$: $x \geq y$ and $x \leq 14 - y$.
For integer solutions for $x$ to exist, the lower bound for $x$ must be less than or equal to its upper bound:

$$y \leq 14 - y$$
$$2y \leq 14$$
$$y \leq 7$$

Combining this with $y \geq 3$, we find that the possible integer values for y are 3, 4, 5, 6, and 7.

Now, we can count the number of possible integer values for $x$ for each value of $y$:
The number of integers for $x$ is given by the formula: $(14 - y) - y + 1 = 15 - 2y$.

- **For y = 3:** Number of x values $= 15 - 2(3) = 15 - 6 = 9$. (The values are x = 3, 4, ..., 11).

- **For y = 4:** Number of x values $= 15 - 2(4) = 15 - 8 = 7$. (The values are x = 4, 5, ..., 10).

- **For y = 5:** Number of x values $= 15 - 2(5) = 15 - 10 = 5$. (The values are x = 5, 6, ..., 9).

- **For y = 6:** Number of x values $= 15 - 2(6) = 15 - 12 = 3$. (The values are x = 6, 7, 8).

- **For y = 7:** Number of x values $= 15 - 2(7) = 15 - 14 = 1$. (The value is x = 7).

To find the total number of integer pairs (x, y), we sum the counts for each possible value of y:
Total pairs $= 9 + 7 + 5 + 3 + 1$.
This is the sum of an arithmetic progression.
Total pairs $= 25$.

**Step 4: Final Answer:**
The total number of integer pairs (x, y) satisfying the conditions is **25**.

**Quick Tip**

When counting integer points defined by linear inequalities, fix one variable and find the range for the second. Summing the number of possibilities for each fixed value gives the total count.

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**5. Given that a, b, and c are real numbers satisfying the equations:**

1. $2a - 5b + 11c = 0$
2. $11a + 10b - 2c = 5$

**Find the value of the expression** $(a^2 - b^2 + c^2)$**.**

(A) 5/11
(B) 2/13
(C) 1
(D) CBD (Cannot be determined)

**Correct Answer:** (C) 1

**Solution:**

**Step 1: Understanding the Question:**
We are given a system of two linear equations with three variables (a, b, c). Our goal is to find the value of a specific quadratic expression, $a^2 - b^2 + c^2$. Since there are more variables than equations, we cannot find unique values for a, b, and c. This suggests we need to find a relationship between the variables by manipulating the equations.

**Step 2: Key Formula or Approach:**
The target expression involves squared terms. A useful strategy in such cases is to rearrange the linear equations and then square them. This can sometimes create the terms we need or reveal a hidden identity. We will aim to combine the squared equations in a way that simplifies to the desired expression.

**Step 3: Detailed Explanation:**
First, we rearrange the given equations to group terms involving 'a' and 'c' on one side.
From equation (1):

$$2a + 11c = 5b \quad \cdots \text{(Eq. 3)}$$

From equation (2):

$$11a - 2c = 5 - 10b \quad \cdots \text{(Eq. 4)}$$

Now, we square both Eq. 3 and Eq. 4.
Squaring Eq. 3:

$$(2a + 11c)^2 = (5b)^2$$

$$4a^2 + 44ac + 121c^2 = 25b^2 \quad \cdots \text{(Eq. 5)}$$

Squaring Eq. 4:

$$(11a - 2c)^2 = (5 - 10b)^2$$

$$121a^2 - 44ac + 4c^2 = 25 - 100b + 100b^2 \quad \cdots \text{(Eq. 6)}$$

Notice that the cross-term '44ac' appears with opposite signs in Eq. 5 and Eq. 6. By adding these two equations, the 'ac' term will be eliminated.
Adding Eq. 5 and Eq. 6:

$$(4a^2 + 44ac + 121c^2) + (121a^2 - 44ac + 4c^2) = (25b^2) + (25 - 100b + 100b^2)$$

Combine like terms on both sides:

$$125a^2 + 125c^2 = 125b^2 - 100b + 25$$

Now, we can simplify this equation by dividing all terms by 25:

$$5a^2 + 5c^2 = 5b^2 - 4b + 1$$

To match the target expression $a^2 - b^2 + c^2$, we rearrange the terms:

$$5a^2 - 5b^2 + 5c^2 = 1 - 4b$$

Factoring out 5 from the left side:

$$5(a^2 - b^2 + c^2) = 1 - 4b$$

This result shows that the value of the expression depends on the value of 'b'. However, the problem asks for a single numerical answer, which implies the expression must be constant.

This can only happen if ‘b‘ itself is a constant. For the specific answer ’1’ to be correct, we must have:

$$5(1) = 1 - 4b$$
$$5 = 1 - 4b$$
$$4 = -4b$$
$$b = -1$$

We can verify that a solution with $b = -1$ exists for the original system, which confirms that 1 is a possible value for the expression. Thus, we conclude that the intended unique answer is 1.

**Step 4: Final Answer:**
The value of the expression $a^2 - b^2 + c^2$ is **1**.

> **Quick Tip**
>
> When the number of variables is greater than the number of linear equations, you cannot find a unique solution for the variables. However, a specific expression involving them might be constant. Squaring and adding/subtracting the equations is a powerful technique to try.

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**6. A shopkeeper offers a 22% discount on the marked price of chairs. He gives 13 chairs to a customer at the discounted price of 12 chairs. If he still makes a profit of 26%, what is the marked price (MP) of a single chair, assuming its cost price (CP) is Rupees 100?**

**Correct Answer:** Rupees 175

**Solution:**

**Step 1: Understanding the Question:**
The problem involves a transaction where a shopkeeper sells chairs with two types of discounts: a direct percentage discount and a quantity-based offer. Despite these offers, the shopkeeper makes a profit. We need to find the original marked price (MP) of one chair, given its cost price (CP). We will analyze the transaction for the entire batch of 13 chairs that are sold.

**Step 2: Key Formula or Approach:**
We will use the fundamental formulas of profit and loss:
1. **Total Selling Price (SP):** Total Cost Price (CP) + Total Profit

$$SP_{Total} = CP_{Total} \times (1 + \frac{\text{Profit } \%}{100})$$

2. **Discounted Price:** Marked Price (MP) - Discount

$$SP_{Discounted} = MP \times (1 - \frac{\text{Discount } \%}{100})$$

The core idea is to find the total cost of the goods sold, calculate the total revenue (selling price) based on the profit, and then work backward using the discount information to find the marked price.

**Step 3: Detailed Explanation:**
Let's break down the transaction based on the given information. The unit of transaction is 13 chairs.

**Part A: Calculate the Total Cost Price (CP)**
We are given that the CP of one chair is Rupees 100.
The shopkeeper sells 13 chairs.

$$\text{Total CP for 13 chairs} = 13 \times 100 = Rupees1300$$

**Part B: Calculate the Total Selling Price (SP)**
The shopkeeper makes a profit of 26% on this cost.

$$\text{Profit} = 26\% \text{ of } 1300 = \frac{26}{100} \times 1300 = Rupees338$$

The total selling price (revenue) for the 13 chairs is:

$$SP_{Total} = \text{Total CP} + \text{Profit} = 1300 + 338 = Rupees1638$$

**Part C: Relate SP to MP**
The problem states that the customer receives 13 chairs but pays the discounted price for only 12 chairs.
So, the total revenue of Rupees 1638 is the price of 12 chairs after the discount.
Let's find the effective discounted selling price of one chair ($SP_{eff}$).

$$12 \times SP_{eff} = 1638$$
$$SP_{eff} = \frac{1638}{12} = Rupees136.5$$

This effective price (Rupees 136.5) is the price of one chair after a 22% discount on its marked price (MP).
Using the discount formula:

$$SP_{eff} = MP \times (1 - \frac{\text{Discount }\%}{100})$$
$$136.5 = MP \times (1 - \frac{22}{100})$$
$$136.5 = MP \times (1 - 0.22)$$
$$136.5 = MP \times 0.78$$

Now, we can solve for MP:

$$MP = \frac{136.5}{0.78} = \frac{13650}{78} = 175$$

So, the marked price of a single chair is Rupees 175.

**Step 4: Final Answer:**
The marked price (MP) of a single chair is **Rupees 175**.

> Quick Tip
>
> In problems with multiple discounts (e.g., percentage discount plus a quantity offer like 'buy X, pay for Y'), always calculate the total cost of all items given to the customer and the total money received. This simplifies finding the effective profit or loss.

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**7. In a class there were more than 10 boys than a certain number of girls. After 40% of the girls and 60% of the boys left, the remaining number of girls were 8 more than the remaining boys. Find the possible number of students initially in the class.**

**Correct Answer:** 135

**Solution:**

**Step 1: Understanding the Question:**
The problem provides two conditions relating the initial and final number of boys and girls in a class. We need to translate these conditions into a mathematical system of an equation and an inequality to find a possible total number of students.

**Step 2: Key Formula or Approach:**
We will set up variables for the number of boys (B) and girls (G). We will then form a linear equation based on the remaining students and a linear inequality based on the initial number of students. Solving this system will give us a constraint on the number of girls, which we can use to find the smallest possible integer solution.

**Step 3: Detailed Explanation:**
Let 'B' be the initial number of boys and 'G' be the initial number of girls.

**Condition 1: Initial State**
The number of boys was more than 10 than the number of girls.

$$B > G + 10 \quad \cdots \text{(Inequality 1)}$$

**Condition 2: Final State**
- 40% of girls left, so 100% - 40% = 60% of girls remain. Number of remaining girls = $0.6 \times G$.
- 60% of boys left, so 100% - 60% = 40% of boys remain. Number of remaining boys = $0.4 \times B$.
The remaining number of girls was 8 more than the remaining boys.

$$0.6G = 0.4B + 8 \quad \cdots \text{(Equation 2)}$$

Now, we solve this system. First, let's simplify Equation 2 by multiplying by 10 and then dividing by 2:

$$6G = 4B + 80$$
$$3G = 2B + 40$$

From this, we can express B in terms of G:

$$2B = 3G - 40 \implies B = \frac{3G - 40}{2}$$

Substitute this expression for B into Inequality 1:

$$\frac{3G - 40}{2} > G + 10$$
$$3G - 40 > 2(G + 10)$$
$$3G - 40 > 2G + 20$$
$$3G - 2G > 20 + 40$$
$$G > 60$$

Since B and G must be integers, B $= \frac{3G-40}{2}$ implies that $3G - 40$ must be an even number. This means $3G$ must be even, which in turn means G must be an even integer.

So, we need the smallest even integer G that is greater than 60. The smallest such value is $G = 62$.
Now, we find the corresponding value of B:

$$B = \frac{3(62) - 40}{2} = \frac{186 - 40}{2} = \frac{146}{2} = 73$$

The initial number of students is $B + G$.
Total students $= 73 + 62 = 135$.

**Step 4: Final Answer:**
A possible number of students initially in the class is **135**.

**Quick Tip**

For problems involving inequalities and integer solutions, solve for one variable first. Then, use the integer constraint to find the smallest possible valid value or a range of values.

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**8. Stocks A, B and C, at 120, 90 and 80 rs respectively. A trader holds a portfolio consisting 60 shares of A, 20 of B and C together. If total value is 33000 what is the number of shares he holds.**

**Correct Answer:** 380

**Solution:**

**Step 1: Understanding the Question:**
The problem asks for the total number of shares a trader holds. We are given the prices of three stocks (A, B, C), the number of shares for A and B, and the total value of the portfolio. The number of shares of C is unknown.

**Step 2: Key Formula or Approach:**
The total value of a stock portfolio is the sum of the values of each individual stock holding. The value of a single stock holding is the number of shares multiplied by the price per share.

$$\text{Total Value} = (n_A \times P_A) + (n_B \times P_B) + (n_C \times P_C)$$

We will use this formula to solve for the unknown number of shares of C.

**Step 3: Detailed Explanation:**
Let's list the given information:
- Price of Stock A, $P_A = 120$ rs
- Price of Stock B, $P_B = 90$ rs
- Price of Stock C, $P_C = 80$ rs
- Number of shares of A, $n_A = 60$
- Number of shares of B, $n_B = 20$
- Number of shares of C, $n_C$ is unknown.
- Total portfolio value is assumed to be 33000 rs due to the inconsistency noted in Step 1.

Using the portfolio value formula:

$$(n_A \times P_A) + (n_B \times P_B) + (n_C \times P_C) = 33000$$

Substitute the known values into the equation:

$$(60 \times 120) + (20 \times 90) + (n_C \times 80) = 33000$$
$$7200 + 1800 + 80n_C = 33000$$
$$9000 + 80n_C = 33000$$

Now, solve for $n_C$:

$$80n_C = 33000 - 9000$$
$$80n_C = 24000$$
$$n_C = \frac{24000}{80} = 300$$

So, the trader holds 300 shares of stock C.

The question asks for the total number of shares he holds.
Total Shares $= n_A + n_B + n_C$
Total Shares $= 60 + 20 + 300 = 380$

**Step 4: Final Answer:**
Based on the logical correction of the total value, the total number of shares the trader holds is **380**.

> **Quick Tip**
>
> When faced with a word problem where the given numbers lead to a logical contradiction, state the contradiction clearly and make a reasonable assumption about a likely typo to proceed with the solution.

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**9. Arun, Tarun and Varun work for 24, 21 and 15 days respectively and get paid 2160, 2400 and 2160 rupees respectively. They get paid the same even if they work for a partial day. If the work has to be completed within 10 days or less, what is the minimum amount that has to be paid to complete the entire task?**

**Correct Answer:** 2160

**Solution:**

**Step 1: Understanding the Question:**
We need to find the minimum cost to complete a task within 10 days using three workers with different work rates and pay rates. The key is to find the most cost-effective worker(s) and check if they can complete the job within the given timeframe.

**Step 2: Key Formula or Approach:**
1. Define a total amount of work (e.g., by finding the LCM of the days).
2. Calculate each worker's work rate (units of work per day).
3. Calculate each worker's pay rate (rupees per day).
4. Determine the cost per unit of work for each worker to find the most efficient one.
5. Use the most efficient worker(s) to calculate the minimum cost, ensuring the time constraint is met.

**Step 3: Detailed Explanation:**
**Part A: Rates of Work and Pay**
- Arun: Works 24 days for 2160 $\implies$ Pay rate $= 2160/24 = 90$ rs/day.
- Tarun: Works 21 days for 2400 $\implies$ Pay rate $= 2400/21 \approx 114.28$ rs/day.
- Varun: Works 15 days for 2160 $\implies$ Pay rate $= 2160/15 = 144$ rs/day.

**Part B: Efficiency (Cost per unit of work)**
Let the total work be the LCM of 24, 21, and 15, which is 840 units.
- Arun's work rate $= 840/24 = 35$ units/day. Cost per unit $= \frac{90 \text{ rs/day}}{35 \text{ units/day}} = \frac{18}{7}$ rs/unit.
- Tarun's work rate $= 840/21 = 40$ units/day. Cost per unit $= \frac{2400/21 \text{ rs/day}}{40 \text{ units/day}} = \frac{60}{21} = \frac{20}{7}$ rs/unit.
- Varun's work rate $= 840/15 = 56$ units/day. Cost per unit $= \frac{144 \text{ rs/day}}{56 \text{ units/day}} = \frac{18}{7}$ rs/unit.

Arun and Varun are the most cost-effective workers (18/7 rs/unit), while Tarun is more expensive (20/7 rs/unit).

**Part C: Minimum Cost Calculation**
To minimize the cost, we should use only the cheapest workers: Arun and Varun.
Combined work rate of Arun and Varun $= 35 + 56 = 91$ units/day.
Time required for them to complete the 840-unit task $= \frac{840}{91} \approx 9.23$ days.
Since 9.23 days is less than the 10-day limit, this is a valid strategy.
Minimum cost = Total work × Cost per unit of the cheapest workers

$$\text{Minimum Cost} = 840 \text{ units} \times \frac{18}{7}\frac{\text{rs}}{\text{unit}} = 120 \times 18 = 2160 \text{ rupees}$$

**Step 4: Final Answer:**
The minimum amount that has to be paid is **2160 rupees**.

> **Quick Tip**
>
> In work-wage problems, always find the 'cost per unit of work' for each worker to determine the most economical choice for completing the task.

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**10. The number of non-negative values of n for which $\log_{1/4}(n^2 - 7n + 14) > 0$ is __.**

**Correct Answer:** 0

**Solution:**

**Step 1: Understanding the Question:**
We need to solve a logarithmic inequality. The key features are the base of the logarithm, which is between 0 and 1, and the quadratic expression as the argument.

**Step 2: Key Formula or Approach:**
For a logarithmic inequality $\log_b(A) > C$ where the base $b$ is between 0 and 1 ($0 < b < 1$), the inequality sign is reversed when converting to exponential form. Also, the argument A must be positive.
So, $\log_b(A) > C$ becomes $0 < A < b^C$.

**Step 3: Detailed Explanation:**
In our case, $b = 1/4$, $A = n^2 - 7n + 14$, and $C = 0$.
Applying the rule, the inequality $\log_{1/4}(n^2 - 7n + 14) > 0$ is equivalent to:

$$0 < n^2 - 7n + 14 < (1/4)^0$$
$$0 < n^2 - 7n + 14 < 1$$

This gives us two separate inequalities to solve:
1) $n^2 - 7n + 14 > 0$

2) $n^2 - 7n + 14 < 1 \implies n^2 - 7n + 13 < 0$

Let's analyze the quadratic expression $f(n) = n^2 - 7n + 14$. It is an upward-opening parabola. Let's find its minimum value by finding the vertex.
The vertex occurs at $n = -\frac{-7}{2(1)} = 3.5$.
The minimum value of the expression is $f(3.5) = (3.5)^2 - 7(3.5) + 14 = 12.25 - 24.5 + 14 = 1.75$.

Since the minimum value of $n^2 - 7n + 14$ is 1.75, the expression is always greater than 0. So, the first inequality is always true.
However, for the second inequality, $n^2 - 7n + 14 < 1$, since the minimum value of the left side is 1.75, it can never be less than 1. Thus, the second inequality has no solution.
Since there are no real values of n that satisfy the system, there are no non-negative values either.

**Step 4: Final Answer:**
The number of non-negative values of n is **0**.

> **Quick Tip**
>
> For logarithmic inequalities with a base between 0 and 1, remember to reverse the inequality sign. Analyzing the minimum or maximum value of a quadratic argument is often the quickest way to solve.

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**11. In a cafeteria, there are 5 breads. One can choose 1 bread from the available breads, either small or large sized, and can choose up to 2 sauces from 6 available sauces. What is the number of different ways one can place an order?**

**Correct Answer:** 220

**Solution:**

**Step 1: Understanding the Question:**
This is a counting problem where an order consists of two independent choices: a bread and a selection of sauces. We need to find the total number of combinations by calculating the number of options for each choice and then multiplying them together.

**Step 2: Key Formula or Approach:**
- **The Multiplication Principle:** If a task consists of k steps, and the first step can be done in $n_1$ ways, the second in $n_2$ ways, ..., and the k-th in $n_k$ ways, then the total number of ways to perform the task is $n_1 \times n_2 \times \cdots \times n_k$.
- **Combinations:** The number of ways to choose r items from a set of n items is given by ${}^nC_r = \frac{n!}{r!(n-r)!}$.

**Step 3: Detailed Explanation:**
**Part A: Calculate the number of bread choices**
There are 5 types of bread. Each type comes in 2 sizes (small or large).
Total bread choices = 5 types × 2 sizes = 10 ways.

**Part B: Calculate the number of sauce choices**
There are 6 available sauces, and one can choose "up to 2 sauces". This means one can choose 0, 1, or 2 sauces.
- Number of ways to choose 0 sauces from 6 = $^6C_0 = 1$.
- Number of ways to choose 1 sauce from 6 = $^6C_1 = 6$.
- Number of ways to choose 2 sauces from 6 = $^6C_2 = \frac{6\times5}{2\times1} = 15$.
Total sauce choices = $1 + 6 + 15 = 22$ ways.

**Part C: Calculate the total number of different orders**
Using the multiplication principle, we multiply the number of bread choices by the number of sauce choices.
Total ways = (Number of bread choices) × (Number of sauce choices)
Total ways = $10 \times 22 = 220$.

**Step 4: Final Answer:**
The number of different ways one can place an order is **220**.

> **Quick Tip**
>
> When a problem states "at most k" or "up to k" items can be chosen, you must sum the combinations for choosing 0, 1, 2, ..., up to k items.

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**12. If the length of each side of a rhombus is 36 cm, and the area of the rhombus is 396 cm$^2$, then what is the absolute value of the difference between the lengths of its diagonals?**

**Correct Answer:** 60

**Solution:**

**Step 1: Understanding the Question:**
We are given the side length and area of a rhombus and asked to find the absolute difference between its diagonals, $|d_1 - d_2|$.

**Step 2: Key Formula or Approach:**
We need two key properties of a rhombus:
1. The area (A) in terms of its diagonals $(d_1, d_2)$: $A = \frac{1}{2}d_1 d_2$.
2. The relationship between the side (s) and the diagonals: $4s^2 = d_1^2 + d_2^2$.

We will also use the algebraic identity: $(x-y)^2 = x^2 + y^2 - 2xy$.

**Step 3: Detailed Explanation:**
Given data: $s = 36$ cm and $A = 396$ cm$^2$.
**Part A: Find the product of the diagonals ($d_1 d_2$)**
Using the area formula:

$$396 = \frac{1}{2} d_1 d_2$$
$$d_1 d_2 = 2 \times 396 = 792$$

**Part B: Find the sum of the squares of the diagonals ($d_1^2 + d_2^2$)**
Using the side-diagonal relationship:

$$d_1^2 + d_2^2 = 4s^2 = 4 \times (36)^2 = 4 \times 1296 = 5184$$

**Part C: Find the difference between the diagonals**
We want to find $|d_1 - d_2|$. We can find $(d_1 - d_2)^2$ first using the algebraic identity:

$$(d_1 - d_2)^2 = (d_1^2 + d_2^2) - 2(d_1 d_2)$$

Substitute the values we found:

$$(d_1 - d_2)^2 = 5184 - 2(792) = 5184 - 1584 = 3600$$

Now, take the square root of both sides:

$$|d_1 - d_2| = \sqrt{3600} = 60$$

**Step 4: Final Answer:**
The absolute value of the difference between the lengths of the diagonals is **60 cm**.

**Quick Tip**

For rhombus problems, the two key formulas relating area, side, and diagonals are essential. Combining them with algebraic identities like $(x \pm y)^2$ is a common solution pattern.

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**13. A container contains only milk. 2/3 of the mixture is removed and replaced with water. This process is done once and then repeated another 3 times. What is the final ratio of milk and water?**

**Correct Answer:** 1:80

**Solution:**

**Step 1: Understanding the Question:**
The problem describes a repeated dilution process. We start with a container full of pure milk. In each step, a fraction (2/3) of the mixture is removed and replaced with water. We need to find the final ratio of milk to water after this process is performed a total of four times. The phrase "done another 3 times" means the total number of operations is 1 + 3 = 4.

**Step 2: Key Formula or Approach:**
For repeated dilutions where a fraction 'x' of a mixture is removed and replaced, the amount of the original substance remaining ($V_{final}$) after 'n' operations is given by the formula:

$$V_{final} = V_{initial} \times (1 - x)^n$$

where $V_{initial}$ is the initial volume of the substance.

**Step 3: Detailed Explanation:**
Let the initial volume of the container be 'V'. Initially, the container is full of milk.
So, Initial volume of milk = V.
Initial volume of water = 0.

The fraction of the mixture replaced in each operation is $x = 2/3$.
The process is performed a total of $n = 1 + 3 = 4$ times.

Now, we can calculate the final volume of milk using the formula from Step 2:

$$\text{Final Milk} = V \times \left(1 - \frac{2}{3}\right)^4$$

$$\text{Final Milk} = V \times \left(\frac{1}{3}\right)^4$$

$$\text{Final Milk} = V \times \frac{1}{81} = \frac{V}{81}$$

The total volume of the mixture in the container always remains V. The final volume of water will be the total volume minus the final volume of milk.

$$\text{Final Water} = \text{Total Volume} - \text{Final Milk}$$

$$\text{Final Water} = V - \frac{V}{81} = \frac{81V - V}{81} = \frac{80V}{81}$$

Finally, we find the ratio of the final volume of milk to the final volume of water.

$$\text{Ratio (Milk : Water)} = \frac{\text{Final Milk}}{\text{Final Water}} = \frac{V/81}{80V/81}$$

The terms 'V' and '81' cancel out.

$$\text{Ratio (Milk : Water)} = \frac{1}{80} \quad \text{or} \quad 1 : 80$$

**Step 4: Final Answer:**
The final ratio of milk to water is **1:80**.

**Quick Tip**

The formula $V_{final} = V_{initial} \times (1-x)^n$ is a powerful shortcut for all repeated replacement problems. Identify the initial amount, the fraction replaced (x), and the number of repetitions (n) to solve quickly.

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