# **CAT 2025 QA Slot 2 Question Paper with Solutions**

Time Allowed: 120 Minutes | Maximum Marks: 204 | Total Questions: 68

#### General Instructions

Read the following instructions very carefully and strictly follow them:

- 1. The total duration of the test is **120 Minutes**, with **40 minutes** allotted per section.
- 2. The question paper is divided into three sections:
  - Section 1: Verbal Ability and Reading Comprehension (VARC) 24 questions
  - Section 2: Data Interpretation and Logical Reasoning (DILR) 22 questions
  - Section 3: Quantitative Aptitude (QA) 22 questions
- 3. Each correct answer carries +3 marks.
- 4. For multiple-choice questions (MCQs), **-1 mark** will be deducted for each wrong answer.
- 5. There is **no negative marking** for Type-in-the-Answer (TITA) questions.
- 1. A and B can complete a work in 12 days and 18 days respectively. They start together, but A leaves after 4 days. How many more days will B take to finish the remaining work?

Correct Answer: 8 days

### Solution:

## Step 1: Understanding the Question:

The problem is a standard 'Time and Work' scenario. We need to calculate the portion of work completed by A and B together, find the remaining work, and then calculate the time B alone will take to finish it.

## Step 2: Key Formula or Approach:

The most efficient method is the LCM method. We assume a total amount of work that is the Least Common Multiple (LCM) of the days taken by each person. This allows us to work with integer rates.

- 1. Total Work =  $LCM(Time\ taken\ by\ A,\ Time\ taken\ by\ B)$ .
- 2. Work Rate = Total Work / Time taken.
- 3. Time = Work / Rate.

# Step 3: Detailed Explanation:

## Part A: Calculate Individual Work Rates

Let the total work be the LCM of 12 and 18.

Total Work = 
$$LCM(12, 18) = 36$$
 units

Now, we can find the rate of work for A and B.

Rate of A =  $\frac{36 \text{ units}}{12 \text{ days}}$  = 3 units/day. Rate of B =  $\frac{36 \text{ units}}{18 \text{ days}}$  = 2 units/day.

# Part B: Calculate Work Done Together

A and B work together for 4 days. Their combined rate is:

Combined Rate = Rate of A + Rate of B = 3 + 2 = 5 units/day.

Work done in 4 days = Combined Rate  $\times$  Days =  $5 \times 4 = 20$  units.

# Part C: Calculate Remaining Work

Remaining Work = Total Work - Work Done = 36 - 20 = 16 units.

## Part D: Calculate Time for B to Finish

This remaining work of 16 units is completed by B alone.

Time taken by  $B = \frac{\text{Remaining Work}}{\text{Rate of B}} = \frac{16 \text{ units}}{2 \text{ units/day}} = 8 \text{ days.}$ 

# Step 4: Final Answer:

B will take 8 more days to finish the remaining work.

# Quick Tip

The LCM method is generally the fastest way to solve Time & Work problems as it avoids fractions and simplifies calculations.

# 2. If the quadratic equation $ax^2+bx+c=0$ has roots that differ by 4, and a+b+c=12, what is the value of $b^2 - 4ac$ ?

Correct Answer: 16

Solution:

#### Step 1: Understanding the Question:

We are asked to find the value of the discriminant  $(D = b^2 - 4ac)$  of a quadratic equation. We are given two conditions: the difference between its roots and the sum of its coefficients.

# Step 2: Key Formula or Approach:

Let the roots of the equation be  $\alpha$  and  $\beta$ . We will use Vieta's formulas and the relationship between the difference of roots and the discriminant.

- 1. Sum of roots:  $\alpha + \beta = -b/a$ .
- 2. Product of roots:  $\alpha\beta = c/a$ .
- 3. Difference of roots:  $(\alpha \beta)^2 = (\alpha + \beta)^2 4\alpha\beta$ .

The discriminant  $D = b^2 - 4ac$ . The relationship is  $(\alpha - \beta)^2 = D/a^2$ .

The sum of coefficients a + b + c is the value of the polynomial at x = 1.

# Step 3: Detailed Explanation:

# Part A: Using the Difference of Roots

We are given that the roots differ by 4, so  $|\alpha - \beta| = 4$ . Squaring this gives  $(\alpha - \beta)^2 = 16$ . Using the formula that relates the difference of roots to the discriminant:

$$(\alpha - \beta)^2 = \frac{b^2 - 4ac}{a^2}$$
$$16 = \frac{b^2 - 4ac}{a^2}$$
$$b^2 - 4ac = 16a^2$$

This shows that the value we need depends on 'a'. We must find 'a' using the second condition.

# Part B: Using the Sum of Coefficients

The condition a+b+c=12 means the value of the polynomial  $f(x)=ax^2+bx+c$  at x=1 is 12.

$$f(1) = a(1)^2 + b(1) + c = a + b + c = 12$$

Since  $\alpha$  and  $\beta$  are roots, the polynomial can also be written as  $f(x) = a(x - \alpha)(x - \beta)$ .

Therefore,  $f(1) = a(1 - \alpha)(1 - \beta) = 12$ .

Let the roots be k and k+4. Substituting these for  $\alpha$  and  $\beta$ :

$$a(1-k)(1-(k+4)) = 12$$
$$a(1-k)(-k-3) = 12$$
$$a(k-1)(k+3) = 12$$
$$a(k^2+2k-3) = 12$$

If 'a' and 'k' are integers, then 'a' must be a divisor of 12. Let's test integer values for 'a'.

- If a = 1:  $(k+3)(k-1) = 12 \implies k^2 + 2k - 3 = 12 \implies k^2 + 2k - 15 = 0 \implies (k+5)(k-3) = 0$ .

This gives integer solutions k = 3 or k = -5. This is a valid scenario.

- If a=2:  $(k+3)(k-1)=6 \implies k^2+2k-9=0$ . This does not give integer solutions for k.
- For other integer values of 'a' (3, 4, 6, 12), we also find no integer solutions for 'k'.

Thus, the only possibility under the assumption of integer coefficients and roots is a = 1.

## Part C: Final Calculation

Using a = 1 in the result from Part A:

$$b^2 - 4ac = 16a^2 = 16(1)^2 = 16$$

## Step 4: Final Answer:

The value of  $b^2 - 4ac$  is **16**.

# Quick Tip

The square of the difference of the roots of  $ax^2 + bx + c = 0$  is directly related to the discriminant (D) by  $(\alpha - \beta)^2 = D/a^2$ . This is a very useful shortcut.

3. A number when divided by 7 leaves a remainder 4 and when divided by 9 leaves a remainder 5. What is the smallest such number greater than 100?

Correct Answer: 158

#### Solution:

# Step 1: Understanding the Question:

We are looking for a number 'N' that satisfies two conditions simultaneously, which can be expressed using modular arithmetic:

- 1.  $N \equiv 4 \pmod{7}$
- 2.  $N \equiv 5 \pmod{9}$

We need to find the smallest integer solution for N that is greater than 100.

# Step 2: Key Formula or Approach:

This is a classic problem that can be solved using the Chinese Remainder Theorem or by substitution. We will use the substitution method.

- 1. Express N in terms of one of the conditions (e.g., N = 9k + 5).
- 2. Substitute this expression into the other congruence and solve for the parameter 'k'.
- 3. This will give a general formula for N.

# Step 3: Detailed Explanation:

From the second condition, we can write the number N as:

$$N = 9k + 5$$
 for some integer  $k \ge 0$ 

Now, we apply the first condition to this expression:

$$N \equiv 4 \pmod{7}$$

$$9k + 5 \equiv 4 \pmod{7}$$

We can reduce the coefficients modulo 7:  $9 \equiv 2$ .

$$2k + 5 \equiv 4 \pmod{7}$$

Subtract 5 from both sides:

$$2k \equiv -1 \pmod{7}$$

Since  $-1 \equiv 6 \pmod{7}$ , we have:

$$2k \equiv 6 \pmod{7}$$

Dividing by 2 (which is valid as gcd(2, 7) = 1):

$$k \equiv 3 \pmod{7}$$

This means 'k' must be of the form k = 7j + 3 for some integer  $j \ge 0$ .

Now substitute this form of 'k' back into our expression for N to find the general solution:

$$N = 9(7j + 3) + 5$$

$$N = 63j + 27 + 5$$

$$N = 63j + 32$$

This formula gives all numbers that satisfy both conditions. Now we need to find the smallest one greater than 100.

$$63j + 32 > 100$$

$$j > \frac{68}{63} \approx 1.079$$

Since 'j' must be an integer, the smallest possible value for j is 2. Substitute j=2 into the general formula for N:

$$N = 63(2) + 32 = 126 + 32 = 158$$

# Step 4: Final Answer:

The smallest such number greater than 100 is 158.

## Quick Tip

For remainder problems, check if adding or subtracting a small number from N results in a common multiple. Here, N+3 gives remainder 0 with 7  $(N = 7k + 4 \implies N + 3 = 7k + 7)$  but not with 9  $(N = 9m + 5 \implies N + 3 = 9m + 8)$ . If it works for both, the solution is faster.

4. In a triangle ABC, the sides are in the ratio 3: 4: 5. If the area of the triangle is 96 sq units, what is its perimeter?

Correct Answer: 48 units

**Solution:** 

# Step 1: Understanding the Question:

The question provides the ratio of the sides of a triangle and its area. We need to find the perimeter. The ratio 3:4:5 is a special case known as a Pythagorean triplet, which means the

triangle is a right-angled triangle. This information simplifies the area calculation.

# Step 2: Key Formula or Approach:

- 1. Sides of the triangle: Let the sides be 3x, 4x, and 5x for some positive constant x.
- 2. Area of a right-angled triangle: Since the sides form a Pythagorean triplet  $((3x)^2 + (4x)^2 = 9x^2 + 16x^2 = 25x^2 = (5x)^2)$ , the triangle is right-angled. The two shorter sides, 3x and 4x, are the base and height. The area is given by:

$$Area = \frac{1}{2} \times base \times height$$

3. **Perimeter of a triangle:** The perimeter is the sum of the lengths of its sides.

Perimeter = 
$$3x + 4x + 5x = 12x$$

# Step 3: Detailed Explanation:

Let the lengths of the sides of the triangle be 3x, 4x, and 5x.

The area of the triangle is given as 96 sq units. Using the area formula for a right-angled triangle:

Area = 
$$\frac{1}{2} \times (3x) \times (4x) = 96$$
  
 $\frac{1}{2} \times 12x^2 = 96$   
 $6x^2 = 96$ 

Now, we solve for x:

$$x^2 = \frac{96}{6}$$
$$x^2 = 16$$
$$x = \sqrt{16} = 4$$

(We take the positive value since x represents a scaling factor for length).

Now that we have the value of x, we can find the perimeter of the triangle.

$$Perimeter = 12x$$

$$Perimeter = 12 \times 4 = 48 \text{ units}$$

The lengths of the sides are 3(4) = 12, 4(4) = 16, and 5(4) = 20. The perimeter is 12+16+20 = 48.

#### Step 4: Final Answer:

The perimeter of the triangle is 48 units.

# Quick Tip

Recognizing Pythagorean triplets like (3, 4, 5), (5, 12, 13), (8, 15, 17), etc., can save a lot of time by immediately identifying a triangle as right-angled, which simplifies area calculations.

# 5. Using digits 1 to 6 (each at most once), how many 4-digit numbers can be formed that are divisible by 4?

Correct Answer: 96

**Solution:** 

# Step 1: Understanding the Question:

We need to form 4-digit numbers using the digits {1, 2, 3, 4, 5, 6} without repetition. The key constraint is that the number must be divisible by 4. The divisibility rule for 4 is that the number formed by the last two digits (the tens and units place) must be divisible by 4.

# Step 2: Key Formula or Approach:

This is a permutations and combinations problem. We will use a slot-based method.

- 1. First, identify all possible pairs of digits from the given set that can form the last two digits of the number, satisfying the divisibility rule for 4.
- 2. For each valid pair of last two digits, determine the number of available digits for the first two places.
- 3. Use the permutation formula  ${}^{n}P_{r} = \frac{n!}{(n-r)!}$  to calculate the number of ways to arrange the remaining digits in the first two places.
- 4. The total count will be the product of the number of valid endings and the number of ways to form the beginning of the number.

# Step 3: Detailed Explanation:

#### Part A: Find all possible valid endings

We need to find two-digit numbers formed from  $\{1, 2, 3, 4, 5, 6\}$  (without repetition) that are divisible by 4. Let's list them systematically:

- Ending in 2: 12, 32, 52 (3 pairs) - Ending in 4: 24, 64 (2 pairs) - Ending in 6: 16, 36, 56 (3 pairs) Total number of valid pairs for the last two digits = 3 + 2 + 3 = 8 pairs.

These pairs are: {12, 16, 24, 32, 36, 52, 56, 64}.

## Part B: Fill the remaining places

Let's consider any one of these 8 valid endings, for example, '12'.

The last two digits are fixed. We have used the digits 1 and 2.

The remaining available digits are  $\{3, 4, 5, 6\}$ . There are 4 digits left.

We need to fill the first two places (thousands and hundreds) of the 4-digit number.

- The thousands place can be filled in any of the 4 remaining ways.
- After filling the thousands place, the hundreds place can be filled in any of the remaining 3 ways.

So, for each valid ending, the number of ways to fill the first two places is  $4 \times 3 = 12$  ways. (This is also  ${}^4P_2$ ).

#### Part C: Calculate the total number

The total number of 4-digit numbers is the product of the number of valid endings and the number of ways to fill the beginning.

Total Numbers = (Number of valid endings)  $\times$  (Ways to fill the first two places)

Total Numbers =  $8 \times 12 = 96$ .

# Step 4: Final Answer:

The number of 4-digit numbers that can be formed is **96**.

# Quick Tip

For permutation problems with divisibility constraints, always start by filling the places that are restricted by the rule (e.g., the last digit for divisibility by 2 or 5, the last two for divisibility by 4).

6. N is a 3-digit number with non-zero digits. No digit is a perfect square and only 1 of the digits is a prime number. What is the number of factors of the smallest such number possible?

Correct Answer: 6

**Solution:** 

# Step 1: Understanding the Question:

The problem asks us to first identify the smallest 3-digit number 'N' that meets a specific set of criteria for its digits. Once we find this number, we need to calculate how many factors (divisors) it has.

# Step 2: Key Formula or Approach:

To find the number of factors of an integer N, we first find its prime factorization, say  $N=p_1^{a_1}\times p_2^{a_2}\times \cdots \times p_k^{a_k}$ . The total number of factors is then given by the product of one more than each exponent:

Number of Factors = 
$$(a_1 + 1)(a_2 + 1) \cdots (a_k + 1)$$

# Step 3: Detailed Explanation:

We need to find the smallest number N by carefully applying all the given conditions.

#### Condition 1: Digits are non-zero.

The possible digits are {1, 2, 3, 4, 5, 6, 7, 8, 9}.

# Condition 2: No digit is a perfect square.

The single-digit perfect squares are 1, 4, and 9. We must exclude these.

The remaining allowed digits are  $\{2, 3, 5, 6, 7, 8\}$ .

## Condition 3: Only 1 of the three digits is a prime number.

From the allowed set {2, 3, 5, 6, 7, 8}, let's identify the primes and non-primes.

- Prime digits:  $\{2, 3, 5, 7\}$ .
- Non-prime (composite) digits: {6, 8}.

The condition means our 3-digit number must be formed using exactly one digit from  $\{2, 3, 5, 7\}$  and two digits from  $\{6, 8\}$ .

# Finding the smallest possible number N:

To construct the smallest possible 3-digit number, we must:

- 1. Choose the smallest possible digits that fit the criteria.
- 2. Arrange these digits in ascending order.

To get the smallest digits, we should choose the smallest prime (which is 2) and the two non-primes (which are 6 and 8).

The set of digits we must use is  $\{2, 6, 8\}$ .

To form the smallest number with these digits, we place the smallest digit in the hundreds place, the next smallest in the tens place, and the largest in the units place.

Smallest number N = 268.

# Finding the number of factors of N = 268:

First, we find the prime factorization of 268.

$$268 = 2 \times 134$$
$$268 = 2 \times 2 \times 67$$
$$268 = 2^{2} \times 67^{1}$$

Here, the exponents are  $a_1 = 2$  and  $a_2 = 1$ .

Using the formula for the number of factors:

Number of Factors = 
$$(2+1)(1+1) = 3 \times 2 = 6$$

#### Step 4: Final Answer:

The number of factors of the smallest such number (268) is **6**.

# Quick Tip

To form the smallest number from a given set of digits, arrange them in ascending order. To find the largest number, arrange them in descending order. Always tackle number theory problems by breaking down the constraints one by one.

- 7. Adam's savings are equal to Ben's expenditure, which in turn is equal to Mary's savings. If Mary's savings are Rupees 50,000 and the incomes of Adam, Ben, and Mary are in the ratio 3:1:4, Mary's expenditure is less than thrice of Adam's expenditure and twice of Adam's expenditure is less than two times Ben's income, then which of the following could be true about Ben's income  $(I_B)$ ?
- (A)  $18000 < I_B < 22000$
- (B)  $20000 < I_B < 25000$

- (C)  $23000 < I_B < 28000$
- (D) Data Inconsistent

Correct Answer: (B)  $20000 < I_B < 25000$ 

**Solution:** 

# Step 1: Understanding the Question & Defining Variables:

The problem links the incomes, savings, and expenditures of three individuals through a series of equalities, ratios, and inequalities. We need to find the possible range for Ben's income.

Let the incomes of Adam, Ben, and Mary be  $I_A, I_B, I_M$ .

Let their savings be  $S_A, S_B, S_M$ .

Let their expenditures be  $E_A, E_B, E_M$ .

The incomes are in the ratio 3:1:4. Let 'x' be the common ratio factor.

$$I_A = 3x$$
,  $I_B = x$ ,  $I_M = 4x$ 

# Step 2: Key Formula or Approach:

The fundamental relationship for personal finance is:

 $Income = Expenditure + Savings \implies Expenditure = Income - Savings$ 

Our strategy is to express all unknown quantities in terms of the variable 'x' and then use the given inequalities to establish a valid range for 'x'. This range will directly correspond to the range for Ben's income.

## Step 3: Detailed Explanation:

First, use the given equalities and values:

We are given  $S_M = 50,000$ .

We are also given  $S_A = E_B = S_M$ .

Therefore,  $S_A = 50,000$  and  $E_B = 50,000$ .

Next, express the expenditures of Adam and Mary in terms of 'x':

$$E_A = I_A - S_A = 3x - 50,000$$

$$E_M = I_M - S_M = 4x - 50,000$$

Now, we apply the two inequalities given in the problem:

**Inequality 1:** Mary's expenditure is less than thrice of Adam's expenditure.

$$E_M < 3 \times E_A$$

$$4x - 50,000 < 3(3x - 50,000)$$

$$4x - 50,000 < 9x - 150,000$$

$$150,000 - 50,000 < 9x - 4x$$

$$100,000 < 5x$$

$$20,000 < x$$

**Inequality 2:** Twice of Adam's expenditure is less than two times Ben's income.

$$2 \times E_A < 2 \times I_B$$

Dividing by 2, we get:

$$E_A < I_B$$
  
 $3x - 50,000 < x$   
 $2x < 50,000$   
 $x < 25,000$ 

Combining the results from both inequalities, we get the range for 'x':

Since Ben's income is  $I_B = x$ , the range for Ben's income is:

$$20,000 < I_B < 25,000$$

# Step 4: Final Answer:

The possible range for Ben's income is between Rupees 20,000 and Rupees 25,000. This corresponds to option (B).

# Quick Tip

In complex word problems with multiple variables, the key is to define a single variable (like a common ratio 'x') and express all other quantities in terms of it. This simplifies the problem into solving inequalities for that single variable.

- 8. If  $N = 2^3 \times 3^7 \times 5^7 \times 7^9 \times 10!$ , then how many factors of N are there which are perfect squares as well as multiples of 420?
- (A) 500
- (B) 1080
- (C) 9000
- (D) 15840

Correct Answer: (A) 500

**Solution:** 

#### Step 1: Understanding the Question:

The question asks for the number of factors of a given number N that satisfy two conditions simultaneously:

1. The factor must be a perfect square.

2. The factor must be a multiple of 420.

To solve this, we first need the complete prime factorization of N. Then, we need to analyze the exponents of the prime factors of any such factor based on the given conditions.

# Step 2: Key Formula or Approach:

- 1. **Prime Factorization:** We will first express N in its canonical prime factorized form. This involves finding the prime factorization of 10!.
- 2. Condition for Perfect Squares: A number is a perfect square if and only if all the exponents in its prime factorization are even.
- 3. Condition for Multiples: A number F is a multiple of K if and only if for every prime p, the exponent of p in the prime factorization of F is greater than or equal to the exponent of p in the prime factorization of K.
- 4. Counting Factors: The total number of such factors is found by multiplying the number of choices available for each prime's exponent.

# Step 3: Detailed Explanation:

# Part A: Complete Prime Factorization of N

First, let's find the prime factorization of 10!:

$$10! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10$$
$$10! = 2 \times 3 \times (2^2) \times 5 \times (2 \times 3) \times 7 \times (2^3) \times (3^2) \times (2 \times 5)$$

The powers of the primes are:

- Power of 2: 1 + 2 + 1 + 3 + 1 = 8
- Power of 3: 1 + 1 + 2 = 4
- Power of 5: 1 + 1 = 2
- Power of 7: 1

So,  $10! = 2^8 \times 3^4 \times 5^2 \times 7^1$ .

Now, we combine this with the given expression for N:

$$N = (2^{3} \times 3^{7} \times 5^{7} \times 7^{9}) \times (2^{8} \times 3^{4} \times 5^{2} \times 7^{1})$$

$$N = 2^{3+8} \times 3^{7+4} \times 5^{7+2} \times 7^{9+1}$$

$$N = 2^{11} \times 3^{11} \times 5^{9} \times 7^{10}$$

# Part B: Prime Factorization of 420

$$420 = 42 \times 10 = (6 \times 7) \times (2 \times 5) = (2 \times 3 \times 7) \times (2 \times 5) = 2^2 \times 3^1 \times 5^1 \times 7^1$$

## Part C: Applying the Conditions to a Factor

Let a factor of N be  $F = 2^a \times 3^b \times 5^c \times 7^d$ . For F to be a factor of N, the exponents must be within the range:  $0 \le a \le 11$ ,  $0 \le b \le 11$ ,  $0 \le c \le 9$ ,  $0 \le d \le 10$ .

Now we apply the two conditions to these exponents:

- 1. **F** is a perfect square: This means a, b, c, d must all be even integers.
- 2. **F** is a multiple of 420: This means the exponents of F must be greater than or equal to the exponents of 420.

$$a > 2$$
,  $b > 1$ ,  $c > 1$ ,  $d > 1$ 

## Part D: Counting the Possible Exponents

We combine all constraints for each exponent:

- For exponent a (power of 2):
- $a \le 11$ , a is even, and  $a \ge 2$ . The possible values are  $\{2, 4, 6, 8, 10\}$ . 5 choices.
- For exponent b (power of 3):
- $b \le 11$ , b is even, and  $b \ge 1$ . The possible values are  $\{2, 4, 6, 8, 10\}$ . 5 choices.
- For exponent c (power of 5):
- $c \leq 9$ , c is even, and  $c \geq 1$ . The possible values are  $\{2, 4, 6, 8\}$ . 4 choices.
- For exponent d (power of 7):
- $d \le 10$ , d is even, and  $d \ge 1$ . The possible values are  $\{2, 4, 6, 8, 10\}$ . 5 choices.

#### Part E: Final Calculation

The total number of such factors is the product of the number of choices for each exponent. Total number of factors  $= 5 \times 5 \times 4 \times 5 = 500$ .

# Step 4: Final Answer:

There are **500** factors of N that are perfect squares and multiples of 420. This corresponds to option (A).

# Quick Tip

When dealing with factors that must satisfy multiple conditions (like being a square and a multiple of another number), analyze the constraints on the exponents of each prime factor separately and then multiply the number of possibilities.