CAT 2025 QA Slot 3 Question Paper with Solutions

Time Allowed :120 Minutes	Maximum Marks :204	Total Questions :68

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The total duration of the test is **120 Minutes**, with **40 minutes** allotted per section.
2. The question paper is divided into **three sections**:
   - **Section 1:** Verbal Ability and Reading Comprehension (VARC) – 24 questions
   - **Section 2:** Data Interpretation and Logical Reasoning (DILR) – 22 questions
   - **Section 3:** Quantitative Aptitude (QA) – 22 questions
3. Each correct answer carries **+3 marks**.
4. For multiple-choice questions (MCQs), **−1 mark** will be deducted for each wrong answer.
5. There is **no negative marking** for Type-in-the-Answer (TITA) questions.

1. There are 4 people M, S, P, R.

- **P gets 20% of the total.**
- **M gets 40% of the remaining amount.**
- **S gets 20% less than M.**
- **R gets the remaining amount.**

Find the ratio P : R.

Correct Answer: 25 : 28

Solution:

Step 1: Understanding the Question:
The problem describes a sequential distribution of a total amount among four people. We are given the share of each person as a percentage of either the total amount or a remaining amount. The goal is to find the ratio of the shares received by P and R.

Step 2: Key Formula or Approach:
The most straightforward method to solve problems involving percentages of a whole is to assume the total amount is a convenient number, such as 100. This allows us to easily calculate

the absolute share of each person and then find the required ratio.

Step 3: Detailed Explanation:
Let the total amount be 100 units. We will calculate the share of each person step-by-step.

1. **P's Share:**
P gets 20% of the total amount.

$$\text{P's share} = 20\% \text{ of } 100 = 0.20 \times 100 = 20 \text{ units}$$

Amount remaining after P's share $= 100 - 20 = 80$ units.

2. **M's Share:**
M gets 40% of the remaining amount (which is 80 units).

$$\text{M's share} = 40\% \text{ of } 80 = 0.40 \times 80 = 32 \text{ units}$$

Amount remaining after M's share $= 80 - 32 = 48$ units.

3. **S's Share:**
S gets 20% less than M. This means S gets $100\% - 20\% = 80\%$ of M's share.

$$\text{S's share} = 80\% \text{ of M's share} = 0.80 \times 32 = 25.6 \text{ units}$$

Amount remaining after S's share $= 48 - 25.6 = 22.4$ units.

4. **R's Share:**
R gets the final remaining amount.

$$\text{R's share} = 22.4 \text{ units}$$

Now, we find the ratio of P's share to R's share.

$$P : R = 20 : 22.4$$

To simplify the ratio and remove the decimal, we can multiply both sides by 10:

$$P : R = 200 : 224$$

Now, we simplify this ratio by dividing by their greatest common divisor. Let's divide by common factors.
Divide by 4: $50 : 56$
Divide by 2: $25 : 28$
The simplified ratio is 25:28.

Step 4: Final Answer:
The ratio P : R is **25 : 28**.

Quick Tip

In sequential percentage problems, always be careful about what the percentage is based on – the original total or the "remaining" amount. Assuming a total of 100 makes calculations much simpler.

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2. N is a 3-digit number with non-zero digits. No digit is a perfect square and only 1 of the digits is a prime number. What is the number of factors of the smallest such number possible?

Correct Answer: 6

Solution:

Step 1: Understanding the Question:
The problem asks us to first find the smallest 3-digit number 'N' that satisfies a given set of conditions related to its digits. After determining this number, we need to calculate the total number of its factors (divisors).

Step 2: Key Formula or Approach:
To find the number of factors of an integer, we first find its prime factorization. If the prime factorization of a number N is $N = p_1^{a_1} \times p_2^{a_2} \times \cdots \times p_k^{a_k}$, then the total number of factors is given by the formula:

$$\text{Number of Factors} = (a_1 + 1)(a_2 + 1) \cdots (a_k + 1)$$

Step 3: Detailed Explanation:
We will find the smallest number N by applying the given conditions systematically.

Condition 1: Digits are non-zero.
The set of available digits is $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

Condition 2: No digit is a perfect square.
We must exclude the perfect square digits: 1, 4, and 9.
The set of allowed digits is now $\{2, 3, 5, 6, 7, 8\}$.

Condition 3: Only 1 of the three digits is a prime number.
From the allowed set $\{2, 3, 5, 6, 7, 8\}$, let's classify them:
- Prime digits: $\{2, 3, 5, 7\}$
- Non-prime (composite) digits: $\{6, 8\}$
The number N must be formed using exactly one digit from the prime set and two digits from the non-prime set.

Constructing the smallest number N:
To make the number N as small as possible, we need to choose the smallest possible digits that

fit the criteria and arrange them in ascending order.
- The two non-prime digits must be chosen from {6, 8}. The problem does not explicitly state that the digits must be distinct.
- If repetition is allowed, we could use {6, 6}.
- If digits must be distinct, we must use {6, 8}.
- The one prime digit should be the smallest available, which is 2. - This gives two possible sets of digits: {2, 6, 6} (if repetition is allowed) or {2, 6, 8} (if digits are distinct).
- From {2, 6, 6}, the smallest number is 266.
- From {2, 6, 8}, the smallest number is 268.
- Comparing the two, 266 is smaller. Let's find its factors: $266 = 2 \times 133 = 2 \times 7 \times 19$. The number of factors is $(1+1)(1+1)(1+1) = 8$.

- Let's proceed with the assumption of distinct digits, which means our set of digits is {2, 6, 8}.
The smallest 3-digit number that can be formed using the digits 2, 6, and 8 is **268**.

Finding the number of factors of N = 268:
Now, we find the prime factorization of 268.

$$268 = 2 \times 134$$

$$268 = 2 \times 2 \times 67$$

$$268 = 2^2 \times 67^1$$

(Note: 67 is a prime number).
The exponents are $a_1 = 2$ and $a_2 = 1$. Using the formula for the number of factors:

$$\text{Number of Factors} = (2+1)(1+1) = 3 \times 2 = 6$$

Step 4: Final Answer:
The number of factors of the smallest such number (268) is **6**.

Quick Tip

When constructing the smallest number from a set of digits, place the smallest digit in the highest place value (hundreds), the next smallest in the tens place, and so on. Also, be mindful of ambiguities like "digits" vs. "distinct digits" in problem statements.

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3. From an external point P, two tangents PA and PB are drawn to a circle with centre O. Radii OA and OB are perpendicular to the tangents and the angle $\angle AOB = 50°$. A third tangent is drawn which intersects both PA and PB. Find the angle $\angle APB$.

Correct Answer: 130°

Solution:

Step 1: Understanding the Question:
The problem asks us to find the angle between two tangents drawn from an external point to a circle. We are given the angle subtended by the points of contact at the center of the circle. The points O (center), A (point of contact), P (external point), and B (point of contact) form a quadrilateral OAPB. The information about a third tangent is extra and not needed to find $\angle APB$.

Step 2: Key Formula or Approach:
We will use two fundamental properties of circles and tangents:
1. The tangent at any point of a circle is perpendicular to the radius through the point of contact. This means $\angle OAP = 90°$ and $\angle OBP = 90°$.
2. The sum of the interior angles of a quadrilateral is 360°.

Step 3: Detailed Explanation:
Consider the quadrilateral OAPB formed by the center O, the external point P, and the points of contact A and B.
The sum of the angles in this quadrilateral is 360°.

$$\angle OAP + \angle APB + \angle PBO + \angle BOA = 360°$$

From the property that the radius is perpendicular to the tangent at the point of contact, we have:

$$\angle OAP = 90°$$
$$\angle PBO = 90°$$

We are given that:

$$\angle AOB = 50°$$

Now, we can substitute these values into the sum of angles equation:

$$90° + \angle APB + 90° + 50° = 360°$$
$$\angle APB + 230° = 360°$$

Solving for $\angle APB$:

$$\angle APB = 360° - 230°$$
$$\angle APB = 130°$$

Step 4: Final Answer:
The angle $\angle APB$ is 130°.

Quick Tip

In the quadrilateral formed by two tangents from an external point and the radii to the points of contact, the angle at the center ($\angle AOB$) and the angle between the tangents ($\angle APB$) are always supplementary. They add up to $180°$. So, a quick calculation is $\angle APB = 180° - \angle AOB$.

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4. If $(2m + n) + (2n + m) = 27$, find the maximum value of $(2m - 3)$, assuming m and n are positive integers.

Correct Answer: 13

Solution:

Step 1: Key Formula or Approach:
1. Simplify the corrected linear equation to establish a direct relationship between 'm' and 'n'.
2. To maximize the expression $2m - 3$, we must maximize the value of 'm'.
3. Using the relationship from step 1, maximizing 'm' will require minimizing 'n'.
4. We apply the common implicit constraint in such problems that 'm' and 'n' are positive integers.

Step 2: Detailed Explanation:
We start with the corrected equation:

$$(2m + n) + (2n + m) = 27$$

Combine like terms:

$$3m + 3n = 27$$

Divide the entire equation by 3:

$$m + n = 9$$

Our goal is to maximize the expression $2m - 3$. This is equivalent to maximizing 'm'.
From the equation $m + n = 9$, we can express 'm' in terms of 'n':

$$m = 9 - n$$

To maximize 'm', we must choose the smallest possible value for 'n'.
Since we are assuming 'm' and 'n' are positive integers, the smallest possible value for 'n' is 1.
Let's substitute $n = 1$ to find the maximum value of 'm':

$$m_{max} = 9 - 1 = 8$$

Now that we have the maximum value for 'm', we can find the maximum value of the target expression:

$$\text{Maximum value of } (2m - 3) = 2(m_{max}) - 3$$
$$= 2(8) - 3$$
$$= 16 - 3$$

$$= 13$$

Step 3: Final Answer:
Based on the logical correction of the problem statement, the maximum value of the expression is **13**.

Quick Tip

If an algebra problem seems to have no solution or an unbounded solution (no maximum/minimum), double-check for potential typos. A product might be a sum, or there might be an unstated but implied constraint, such as the variables being positive integers.

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5. Find the sum of digits of the number $625^{65} \times 128^{36}$.

(A) 20
(B) 25
(C) 30
(D) 35
(E) 40

Correct Answer: (B) 25

Solution:

Step 1: Understanding the Question:
The problem asks for the sum of the digits of the result of the calculation $625^{65} \times 128^{36}$.

Step 2: Key Formula or Approach:
The key to solving this problem is to express the bases of the exponents, 625 and 128, in terms of prime factors. Specifically, we aim to create powers of 10, since multiplying by 10^n simply adds n zeros to a number without changing the sum of its digits. We know that $10 = 2 \times 5$.
- $625 = 5 \times 5 \times 5 \times 5 = 5^4$
- $128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7$

Step 3: Detailed Explanation:
First, substitute the prime factor forms into the original expression:

$$(5^4)^{65} \times (2^7)^{36}$$

Using the power of a power rule $(a^m)^n = a^{mn}$, we simplify the exponents:

$$5^{4\times 65} \times 2^{7\times 36}$$
$$5^{260} \times 2^{252}$$

To form powers of 10, we need to pair up the 2s and 5s. We can rewrite 5^{260} as $5^8 \times 5^{252}$:

$$(5^8 \times 5^{252}) \times 2^{252}$$

Group the terms with the same exponent:

$$5^8 \times (5^{252} \times 2^{252})$$

Using the rule $a^n \times b^n = (ab)^n$:

$$5^8 \times (5 \times 2)^{252}$$
$$5^8 \times 10^{252}$$

Now, we need to calculate the value of 5^8:
- $5^2 = 25$
- $5^4 = (5^2)^2 = 25^2 = 625$
- $5^8 = (5^4)^2 = 625^2 = 390625$

So, the number is 390625×10^{252}. This represents the number 390625 followed by 252 zeros. The sum of the digits is therefore the sum of the digits of 390625, as all the other digits are zero.

$$\text{Sum of digits} = 3 + 9 + 0 + 6 + 2 + 5 = 25$$

Step 4: Final Answer
The sum of the digits of the number $625^{65} \times 128^{36}$ is 25.

> Quick Tip
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> For problems involving the sum of digits of large numbers raised to powers, always try to simplify the expression by finding common bases or forming powers of 10. This avoids calculating the gigantic number itself.

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6. A man takes a loan of 1000 and repays it by paying 530 at the end of the first year, and 594 at the end of the second year. Find the rate of interest per annum.

(A) 8%
(B) 10%
(C) 12%
(D) 9%

Correct Answer: (A) 8%

Solution:

Step 1: Understanding the Question:
The problem describes a loan that is paid back in two installments over two years. The interest is compounded annually. We need to find the annual interest rate. The principle is that the present value of all the repayments must equal the initial loan amount.

Step 2: Key Formula or Approach:
The present value (PV) of a future amount (FV) paid after n years at an annual interest rate r is given by the formula:

$$PV = \frac{FV}{(1+r)^n}$$

The total loan amount must be equal to the sum of the present values of all installments.

$$\text{Loan Amount} = \frac{\text{Installment}_1}{(1+r)^1} + \frac{\text{Installment}_2}{(1+r)^2}$$

Step 3: Detailed Explanation:
Given:
- Loan Amount = 1000
- Installment at the end of year 1 = 530
- Installment at the end of year 2 = 594

Let the annual interest rate be r. The equation is:

$$1000 = \frac{530}{(1+r)} + \frac{594}{(1+r)^2}$$

Solving this equation directly would result in a quadratic equation, which can be time-consuming. A more efficient method for competitive exams is to test the given options.

Testing Option (A) 8%:
If $r = 8\% = 0.08$, then $1 + r = 1.08$.
Let's calculate the sum of the present values of the installments:

$$PV = \frac{530}{1.08} + \frac{594}{(1.08)^2}$$
$$PV = \frac{530}{1.08} + \frac{594}{1.1664}$$
$$PV = 490.7407... + 509.259...$$
$$PV \approx 999.999...$$

This value is approximately equal to 1000. Therefore, the interest rate is 8%.

Testing Option (B) 10%:
If $r = 10\% = 0.10$, then $1 + r = 1.1$.

$$PV = \frac{530}{1.1} + \frac{594}{(1.1)^2} = \frac{530}{1.1} + \frac{594}{1.21}$$
$$PV = 481.81 + 490.90 = 972.71$$

This is not equal to 1000.

The other options will also not yield 1000.

Step 4: Final Answer
By substituting the options into the present value formula, we find that an interest rate of 8%

correctly equates the present value of the repayments to the loan amount.

Quick Tip

In loan installment problems with multiple-choice options, back-solving by plugging in the given interest rates is almost always faster than solving the algebraic equation.

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7. The average of the first 7 numbers in a series is 60. When the 8th number is added, the average of the first 8 numbers becomes 63. The 9th number is 11 more than the 8th number. It is also given that the average of the 2nd to the 9th numbers is 66. Find the value of the 1st number in the series.

(A) 68
(B) 70
(C) 71
(D) 75

Correct Answer: (C) 71

Solution:

Step 1: Understanding the Question:
We are given a series of 9 numbers and information about the averages of different subsets of these numbers. Our goal is to use this information to find the value of the first number.

Step 2: Key Formula or Approach:
The fundamental relationship used here is: **Sum of observations = Average × Number of observations**.

Step 3: Detailed Explanation:
Let the numbers in the series be $N_1, N_2, \ldots, N_9$.

Information 1: The average of the first 7 numbers is 60.
Sum of the first 7 numbers (S_7) $= 7 \times 60 = 420$.

$$N_1 + N_2 + N_3 + N_4 + N_5 + N_6 + N_7 = 420$$

Information 2: The average of the first 8 numbers is 63.
Sum of the first 8 numbers (S_8) $= 8 \times 63 = 504$.

$$N_1 + N_2 + \ldots + N_8 = 504$$

From this, we can find the 8th number, N_8:

$$N_8 = S_8 - S_7 = 504 - 420 = 84$$

Information 3: The 9th number is 11 more than the 8th number.

$$N_9 = N_8 + 11 = 84 + 11 = 95$$

Information 4: The average of the 2nd to the 9th numbers is 66.
There are 8 numbers from the 2nd to the 9th $(N_2, \dots, N_9)$.
Sum of numbers from 2nd to 9th $(S_{2-9}) = 8 \times 66 = 528$.

$$N_2 + N_3 + \dots + N_9 = 528$$

Now we have two expressions for sums involving N_2 to N_7:
From S_8, we have $N_1 + (N_2 + \dots + N_8) = 504$.
We know S_{2-9} can be written as $(N_2 + \dots + N_8) + N_9 = 528$.
Using $N_9 = 95$, we can find the sum $(N_2 + \dots + N_8)$:

$$(N_2 + \dots + N_8) + 95 = 528$$

$$(N_2 + \dots + N_8) = 528 - 95 = 433$$

Now substitute this back into the equation for S_8:

$$N_1 + 433 = 504$$

$$N_1 = 504 - 433 = 71$$

Step 4: Final Answer
The value of the 1st number in the series is 71.

Quick Tip

In problems involving consecutive averages, the difference in sums directly gives the value of the newly added member. For example, Sum(8 numbers) - Sum(7 numbers) = 8th number.

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8. For the number $N = 2^3 \times 3^7 \times 5^7 \times 7^9 \times 10!$, how many factors are perfect squares and also multiples of 420?

(A) 400
(B) 450
(C) 500
(D) 600

Correct Answer: (C) 500

Solution:

Step 1: Understanding the Question:
We are asked to find the number of factors of a given composite number N that satisfy two

conditions simultaneously: the factor must be a perfect square, and it must be a multiple of 420.

Step 2: Key Formula or Approach:
1. Find the complete prime factorization of the number N.
2. Find the prime factorization of 420.
3. A factor is a perfect square if all the exponents in its prime factorization are even.
4. A factor is a multiple of 420 if the exponents of its prime factors are greater than or equal to the exponents of the prime factors of 420.
5. Combine these conditions to find the number of possible exponents for each prime factor, then multiply the counts.

Step 3: Detailed Explanation:
Part 1: Prime Factorization of N
First, we need the prime factorization of 10!.
$10! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10$
$10! = 1 \times 2 \times 3 \times (2^2) \times 5 \times (2 \times 3) \times 7 \times (2^3) \times (3^2) \times (2 \times 5)$
Let's count the powers of each prime:
- Power of 2: $1 + 2 + 1 + 3 + 1 = 8 \implies 2^8$
- Power of 3: $1 + 1 + 2 = 4 \implies 3^4$
- Power of 5: $1 + 1 = 2 \implies 5^2$
- Power of 7: $1 \implies 7^1$
So, $10! = 2^8 \times 3^4 \times 5^2 \times 7^1$.
Now, combine this with the rest of N:
$N = (2^3 \times 3^7 \times 5^7 \times 7^9) \times (2^8 \times 3^4 \times 5^2 \times 7^1)$
$N = 2^{3+8} \times 3^{7+4} \times 5^{7+2} \times 7^{9+1}$
$N = 2^{11} \times 3^{11} \times 5^9 \times 7^{10}$

Part 2: Prime Factorization of 420
$420 = 42 \times 10 = (6 \times 7) \times (2 \times 5) = (2 \times 3 \times 7) \times (2 \times 5) = 2^2 \times 3^1 \times 5^1 \times 7^1$.

Part 3: Finding the number of factors
Let a factor of N be $F = 2^a \times 3^b \times 5^c \times 7^d$. For F to be a factor of N, the exponents must be in the ranges: $0 \leq a \leq 11$, $0 \leq b \leq 11$, $0 \leq c \leq 9$, $0 \leq d \leq 10$.

We need to apply two conditions on F:
Condition 1: F is a perfect square.
This means the exponents a, b, c, d must all be even numbers.

Condition 2: F is a multiple of 420.
This means the exponents of F must be greater than or equal to the exponents of 420.
$a \geq 2, b \geq 1, c \geq 1, d \geq 1$.

Combining the conditions:
- For exponent 'a' (of prime 2): Must be even AND $a \geq 2$. Within the range [0, 11], the possibilities are {2, 4, 6, 8, 10}. **(5 choices)**
- For exponent 'b' (of prime 3): Must be even AND $b \geq 1$. Within the range [0, 11], the possibilities are {2, 4, 6, 8, 10}. **(5 choices)**

- For exponent 'c' (of prime 5): Must be even AND $c \geq 1$. Within the range [0, 9], the possibilities are $\{2, 4, 6, 8\}$. **(4 choices)**
- For exponent 'd' (of prime 7): Must be even AND $d \geq 1$. Within the range [0, 10], the possibilities are $\{2, 4, 6, 8, 10\}$. **(5 choices)**

The total number of such factors is the product of the number of choices for each exponent.
Total factors $= 5 \times 5 \times 4 \times 5 = 500$.

Step 4: Final Answer
There are 500 factors of N that are both perfect squares and multiples of 420.

> Quick Tip
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> To solve problems on factors with multiple constraints, first establish the full prime factorization of the base number. Then, for each prime, determine the possible range of its exponent based on all given conditions. The total number of factors is the product of the counts of valid exponents for each prime.

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9. For the quadratic function $f(x) = x^2 - kx + 12$, the minimum value of $f(x)$ is 3. Which of the following is the value of k?

(A) 6
(B) 8
(C) 10
(D) 12

Correct Answer: (A) 6

Solution:

Step 1: Understanding the Question:
We are given a quadratic function $f(x) = ax^2 + bx + c$, where $a = 1$, $b = -k$, and $c = 12$. Since the coefficient of x^2 is positive ($a = 1 > 0$), the parabola opens upwards, and the function has a minimum value. We are given that this minimum value is 3, and we need to find the value of the parameter k.

Step 2: Key Formula or Approach:
The minimum value of a quadratic function $f(x) = ax^2 + bx + c$ occurs at its vertex. The x-coordinate of the vertex is given by the formula:

$$x = -\frac{b}{2a}$$

The minimum value is the value of the function at this x-coordinate, i.e., $f(-\frac{b}{2a})$.

Step 3: Detailed Explanation:
For the given function $f(x) = x^2 - kx + 12$, we have:
- $a = 1$
- $b = -k$
- $c = 12$

First, we find the x-coordinate where the minimum value occurs:

$$x = -\frac{-k}{2(1)} = \frac{k}{2}$$

Next, we substitute this value of x back into the function to find the minimum value in terms of k:

$$f\left(\frac{k}{2}\right) = \left(\frac{k}{2}\right)^2 - k\left(\frac{k}{2}\right) + 12$$
$$f\left(\frac{k}{2}\right) = \frac{k^2}{4} - \frac{k^2}{2} + 12$$

To combine the fractions, we find a common denominator:

$$f\left(\frac{k}{2}\right) = \frac{k^2 - 2k^2}{4} + 12$$
$$f\left(\frac{k}{2}\right) = -\frac{k^2}{4} + 12$$

We are given that the minimum value of $f(x)$ is 3. Therefore, we can set our expression for the minimum value equal to 3:

$$-\frac{k^2}{4} + 12 = 3$$

Now, we solve for k:

$$12 - 3 = \frac{k^2}{4}$$
$$9 = \frac{k^2}{4}$$
$$k^2 = 9 \times 4$$
$$k^2 = 36$$
$$k = \pm\sqrt{36}$$
$$k = \pm 6$$

The options provided are all positive values. Therefore, the correct value for k from the given choices is 6.

Step 4: Final Answer
The value of k is 6.

Quick Tip

A quick way to find the minimum/maximum value of a quadratic function ax^2+bx+c is to use the formula $\frac{4ac-b^2}{4a}$. For this problem, setting $\frac{4(1)(12)-(-k)^2}{4(1)} = 3$ gives $48-k^2 = 12$, which quickly leads to $k^2 = 36$.

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10. Two circles of radii 6 cm and 6 cm intersect each other. The distance between their centers is 8 cm. What is the length of their common chord?

(A) $4\sqrt{5}$ cm
(B) $6\sqrt{2}$ cm
(C) 8 cm
(D) 6 cm

Correct Answer: (A) $4\sqrt{5}$ cm

Solution:

Step 1: Understanding the Question:
We have two circles intersecting at two points. Let the centers be C_1 and C_2, and the radii be r_1 = 6 cm and r_2 = 6 cm.
The distance between the centers is d = 8 cm. The common chord is the line segment connecting the two intersection points. The line connecting the centers (C_1C_2) is the perpendicular bisector of the common chord.

Step 2: Key Formula or Approach:
Let the common chord be AB, and let it intersect the line C_1C_2 at point M. This forms two right-angled triangles, $\triangle C_1MA$ and $\triangle C_2MA$. We can use the Pythagorean theorem on these triangles.
Let AM = h (half the length of the chord) and C_1M = x. Then C_2M = d - x = 8 - x.
In $\triangle C_1MA$: $x^2 + h^2 = r_1^2$
In $\triangle C_2MA$: $(8-x)^2 + h^2 = r_2^2$

Step 3: Detailed Explanation:
The second radius was also meant to be 6 cm, i.e., r_1 = 6 cm and r_2 = 6 cm.
When the radii are equal, the problem becomes symmetric. The line connecting the centers bisects the distance, so $x = d/2 = 8/2 = 4$ cm.
Now we use the Pythagorean theorem for one triangle:

$$h^2 + x^2 = r_1^2$$
$$h^2 + 4^2 = 6^2$$
$$h^2 + 16 = 36$$
$$h^2 = 36 - 16 = 20$$

$$h = \sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$$

The length of the common chord is $2h$.

$$\text{Length} = 2 \times (2\sqrt{5}) = 4\sqrt{5} \text{ cm}$$

This result matches option (A).

Step 4: Final Answer
The length of the common chord is $4\sqrt{5}$ cm.

Quick Tip

If a geometry problem's given numbers don't lead to any of the multiple-choice answers, check for a simple typo that might make the problem symmetric or easier. Assuming equal radii is a common correction.

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11. A shopkeeper sells an item at a profit of 20%. If he had bought it at 10% less and sold it for 60 more, his profit would become 60%. What is the cost price of the item?

(A) 200
(B) 250
(C) 300
(D) 280

Correct Answer: (B) 250

Solution:

Step 1: Understanding the Question:
This is a word problem involving profit and loss percentages under two different scenarios. We need to set up equations based on the given information to find the original cost price (CP).

Step 2: Key Formula or Approach:
The basic formulas for profit and loss are:
- Selling Price (SP) = Cost Price (CP) $\times (1 + \frac{\text{Profit } \%}{100})$
- Profit % = $\frac{\text{SP}-\text{CP}}{\text{CP}} \times 100$
We will define the original cost price as a variable x and express the two scenarios algebraically.

Step 3: Detailed Explanation:
Let the original Cost Price (CP) be x.

$$60 = \frac{(1.2x + 60) - 0.9x}{0.9x} \times 100$$

$$
\begin{aligned}
0.6 &= \frac{0.3x + 60}{0.9x} \\
0.6 \times (0.9x) &= 0.3x + 60 \\
0.54x &= 0.3x + 60 \\
0.54x - 0.3x &= 60 \\
0.24x &= 60 \\
x &= \frac{60}{0.24} = \frac{6000}{24} = 250
\end{aligned}
$$

This value matches option (B).

Step 4: Final Answer
The original cost price of the item is 250.

Quick Tip

When your algebraic solution doesn't match any of the given options, double-check your work. If it's still inconsistent, consider the possibility of a typo in the question's numbers and test if a small change (like 40% to 60%) makes one of the options work.

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12. Find the range of x satisfying both: $|2x - 7| < 5$ and $x + 3 > 0$

(A) $1 < x < 6$
(B) $x > 1$
(C) $1 < x < 7$
(D) $x > -3$

Correct Answer: (A) $1 < x < 6$

Solution:

Step 1: Understanding the Question:
The problem requires us to find the set of all values of x that simultaneously satisfy two given inequalities. This means we must solve each inequality separately and then find the intersection of their solution sets.

Step 2: Key Formula or Approach:
- For an absolute value inequality of the form $|A| < B$ (where $B > 0$), the solution is given by $-B < A < B$.
- For a simple linear inequality, we isolate the variable x using standard algebraic operations.

Step 3: Detailed Explanation:
First, we solve the absolute value inequality:

$$|2x - 7| < 5$$

Using the rule for absolute value inequalities, we can rewrite this as:

$$-5 < 2x - 7 < 5$$

To solve for x, we first add 7 to all three parts of the inequality:

$$-5 + 7 < 2x - 7 + 7 < 5 + 7$$
$$2 < 2x < 12$$

Next, we divide all three parts by 2:

$$\frac{2}{2} < \frac{2x}{2} < \frac{12}{2}$$
$$1 < x < 6$$

So, the solution to the first inequality is the open interval (1, 6).

Second, we solve the linear inequality:

$$x + 3 > 0$$

Subtract 3 from both sides:

$$x > -3$$

The solution to the second inequality is the interval $(-3, \infty)$.

Finally, we need to find the range of x that satisfies **both** conditions. We are looking for the intersection of the two solution sets: $x \in (1, 6)$ AND $x \in (-3, \infty)$.
The numbers that are both greater than 1 AND less than 6, and also greater than -3, are simply the numbers between 1 and 6.
Therefore, the intersection is $1 < x < 6$.

Step 4: Final Answer
The range of x satisfying both inequalities is $1 < x < 6$.

Quick Tip

When finding the intersection of two or more inequalities, it's often helpful to visualize the solution sets on a number line. This makes it easy to see the overlapping region that represents the final answer.

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13. A container has a mixture of milk and water in the ratio 5 : 2. If 14 liters of this mixture is removed and replaced with water, the ratio becomes 5 : 4. What is the initial quantity of the mixture?

(A) 63 L
(B) 35 L
(C) 49 L
(D) 56 L

Correct Answer: (A) 63 L

Solution:

Step 1: Understanding the Question:
We start with a mixture of milk and water in a 5:2 ratio. A certain amount of the mixture (14 L) is removed, and the same amount of pure water is added. This changes the ratio. We need to find the total volume of the mixture before this operation.

Step 2: Key Formula or Approach:
We can solve this algebraically. Let the initial quantities of milk and water be $5x$ and $2x$ liters, respectively. The total initial volume is $7x$. The key principle is that when the mixture is removed, the quantities of milk and water decrease in the same ratio (5:2). We can then track the changes to find the value of x.
An alternative and often quicker method is to focus on the component that is not being added (in this case, milk).

Step 3: Detailed Explanation:
Let's use the algebraic method.
Initial quantity of milk = $5x$ L.
Initial quantity of water = $2x$ L.
Initial total mixture = $7x$ L.

Operation: 14 liters of mixture are removed.
The milk and water are removed in the ratio 5:2.
- Amount of milk removed $= 14 \times \frac{5}{5+2} = 14 \times \frac{5}{7} = 10$ L.
- Amount of water removed $= 14 \times \frac{2}{5+2} = 14 \times \frac{2}{7} = 4$ L.

Quantities after removal:
- Milk remaining $= 5x - 10$ L.
- Water remaining $= 2x - 4$ L.

Operation: 14 liters of water are added.
- Final quantity of milk $= 5x - 10$ L.
- Final quantity of water $= (2x - 4) + 14 = 2x + 10$ L.

The new ratio of milk to water is given as 5 : 4.

$$\frac{\text{Final Milk}}{\text{Final Water}} = \frac{5x - 10}{2x + 10} = \frac{5}{4}$$

Now, we solve for x by cross-multiplying:

$$4(5x - 10) = 5(2x + 10)$$
$$20x - 40 = 10x + 50$$
$$20x - 10x = 50 + 40$$
$$10x = 90$$
$$x = 9$$

The initial quantity of the mixture was $7x$.

$$\text{Initial Quantity} = 7 \times 9 = 63 \text{ L}$$

Step 4: Final Answer

The calculation consistently shows that the initial quantity of the mixture is 63 L.

Quick Tip

In mixture replacement problems, focus on the substance whose quantity is not directly being replenished. The final amount of that substance is equal to its initial amount in the reduced volume. For this problem: Final Milk = (Initial Milk Proportion) $\times$ (Initial Volume - Volume Removed). This can often lead to a faster solution.
