QA CAT 2025 Slot 2 Question Paper with Solution

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2. Let a_n be the n^{th} term of a decreasing infinite geometric progression. If $a_1 + a_2 + a_3 = 52$ and $a_1a_2 + a_2a_3 + a_3a_1 = 624$, then the sum of this geometric progression is:
(A) 57
(B) 63
(C) 54
(D) 60

Correct Answer: (3) 54

Solution:
Step 1: Let the first term be a and common ratio be r $(0 < r < 1$ since the G.P. is decreasing and infinite). Then,

$$a_1 = a, \quad a_2 = ar, \quad a_3 = ar^2.$$

From $a_1 + a_2 + a_3 = 52$:

$$a + ar + ar^2 = 52 \ \Rightarrow \ a(1 + r + r^2) = 52. \tag{1}$$

Step 2: Using $a_1a_2 + a_2a_3 + a_3a_1 = 624$:

$$a_1a_2 + a_2a_3 + a_3a_1 = a \cdot ar + ar \cdot ar^2 + ar^2 \cdot a = a^2\big(r + r^3 + r^2\big) = a^2 r(1 + r + r^2) = 624. \tag{2}$$

From (1), $a = \dfrac{52}{1 + r + r^2}$. Substituting in (2),

$$\left(\frac{52}{1 + r + r^2}\right)^2 r(1 + r + r^2) = 624 \ \Rightarrow \ \frac{52^2 r}{1 + r + r^2} = 624.$$

Thus,

$$1 + r + r^2 = \frac{52^2 r}{624} = \frac{2704r}{624} = \frac{13}{3}r.$$

So,

$$1 + r + r^2 = \frac{13}{3}r \ \Rightarrow \ 3 + 3r + 3r^2 = 13r \ \Rightarrow \ 3r^2 - 10r + 3 = 0.$$

Solving the quadratic:

$$r = \frac{10 \pm \sqrt{100 - 36}}{6} = \frac{10 \pm 8}{6} \Rightarrow r = 3 \text{ or } r = \frac{1}{3}.$$

Since the G.P. is decreasing and infinite, we require $|r| < 1$, hence $r = \dfrac{1}{3}$.
Step 3: Find the first term:

$$1 + r + r^2 = 1 + \frac{1}{3} + \frac{1}{9} = \frac{13}{9},$$

so from (1),

$$a = \frac{52}{1 + r + r^2} = \frac{52}{13/9} = 36.$$

Step 4: Sum of the infinite G.P.:

$$S_\infty = \frac{a}{1 - r} = \frac{36}{1 - \frac{1}{3}} = \frac{36}{\frac{2}{3}} = 54.$$

Therefore, the sum of the geometric progression is 54.

Quick Tip

For an infinite geometric progression with first term a and common ratio r (where $|r| < 1$):

$$S_\infty = \frac{a}{1-r}.$$

Also, using relationships between sums and products of initial terms can help form equations in a and r.

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3. Two tangents drawn from a point P touch a circle with center O at points Q and R. Points A and B lie on PQ and PR, respectively, such that AB is also a tangent to the same circle. If $\angle AOB = 50°$, then $\angle APB$, in degrees, equals:

Solution:
Let P be an external point from which tangents PQ and PR are drawn to the circle with center O, touching it at Q and R respectively. Points A and B lie on PQ and PR such that AB is also tangent to the circle and touches it at point T. We are given $\angle AOB = 50°$ and we need to find $\angle APB$.
Step 1: Use tangent–radius property.
From point A, two tangents are drawn: AQ and AT. Since tangents from an external point are equal, we have:

$$AQ = AT, \quad \Rightarrow \quad \triangle AOQ \cong \triangle AOT.$$

Hence,

$$\angle AOQ = \angle AOT = x \quad \text{(say)}.$$

Similarly, from point B, tangents BR and BT are drawn, so:

$$\triangle BOR \cong \triangle BOT \Rightarrow \angle BOR = \angle BOT = y \quad \text{(say)}.$$

Step 2: Relate x and y using $\angle AOB$.
At the center,

$$\angle AOB = \angle AOT + \angle TOB = x + y = 50°. \tag{1}$$

Step 3: Find $\angle QOR$.
Angle between radii OQ and OR is:

$$\angle QOR = \angle QOA + \angle AOT + \angle TOB + \angle BOR = x + x + y + y = 2(x + y).$$

Using (1),

$$\angle QOR = 2 \times 50° = 100°.$$

Step 4: Use quadrilateral $PQOR$.
Since the radius is perpendicular to the tangent at the point of contact,

$$\angle OQP = 90°, \qquad \angle ORP = 90°.$$

In quadrilateral $PQOR$:

$$\angle QPR + \angle OQP + \angle QOR + \angle ORP = 360°.$$

But $\angle QPR = \angle APB$ (same angle at P), hence

$$\angle APB + 90° + 100° + 90° = 360°$$

$$\Rightarrow \angle APB + 280° = 360° \Rightarrow \angle APB = 80°.$$

Therefore, $\angle APB = 80°$.

Quick Tip

Tangents from an external point to a circle are equal, and the line from the center to the point of tangency is perpendicular to the tangent. These facts often allow you to form congruent triangles and use angle sums in quadrilaterals involving the center.

4. Let $ABCDEF$ be a regular hexagon and P and Q be the midpoints of AB and CD, respectively. Then, the ratio of the areas of trapezium $PBCQ$ and hexagon $ABCDEF$ is:
(A) 6 : 19
(B) 5 : 24
(C) 6 : 25
(D) 7 : 24

Correct Answer: (2) 5 : 24

Solution:
Step 1: Let the side length of the regular hexagon be s. The area of a regular hexagon is

$$\text{Area}_{\text{hex}} = \frac{3\sqrt{3}}{2}s^2.$$

Step 2: Place the hexagon on a coordinate plane. Let

$$B = (0, 0), \quad C = (s, 0).$$

For a regular hexagon with side s, the adjacent vertices are separated by 60°.
So we can take

$$A = \left(-\frac{s}{2}, \frac{s\sqrt{3}}{2}\right), \quad D = \left(\frac{3s}{2}, \frac{s\sqrt{3}}{2}\right).$$

Step 3: Find midpoints P and Q.
P is the midpoint of AB:

$$P = \left(\frac{-\frac{s}{2}+0}{2}, \frac{\frac{s\sqrt{3}}{2}+0}{2}\right) = \left(-\frac{s}{4}, \frac{s\sqrt{3}}{4}\right).$$

Q is the midpoint of CD:

$$Q = \left(\frac{s+\frac{3s}{2}}{2}, \frac{0+\frac{s\sqrt{3}}{2}}{2}\right) = \left(\frac{5s}{4}, \frac{s\sqrt{3}}{4}\right).$$

Step 4: Area of trapezium $PBCQ$.
Since P and Q have the same y-coordinate, segment PQ is parallel to BC (which lies on the x-axis).

$$\text{Height } h = \frac{s\sqrt{3}}{4}.$$

Lengths of the parallel sides:

$$BC = s, \qquad PQ = x_Q - x_P = \frac{5s}{4} - \left(-\frac{s}{4}\right) = \frac{3s}{2}.$$

Thus,

$$\text{Area}_{\text{trap}} = \frac{1}{2}(BC + PQ) \cdot h = \frac{1}{2}\left(s + \frac{3s}{2}\right) \cdot \frac{s\sqrt{3}}{4} = \frac{1}{2} \cdot \frac{5s}{2} \cdot \frac{s\sqrt{3}}{4} = \frac{5\sqrt{3}}{16}s^2.$$

Step 5: Ratio of areas.

$$\text{Ratio} = \frac{\text{Area}_{\text{trap}}}{\text{Area}_{\text{hex}}} = \frac{\frac{5\sqrt{3}}{16}s^2}{\frac{3\sqrt{3}}{2}s^2} = \frac{5}{16} \cdot \frac{2}{3} = \frac{10}{48} = \frac{5}{24}.$$

Therefore, the ratio of the areas is 5 : 24.

Quick Tip

For regular polygons, coordinate geometry is very handy: place convenient vertices on the axes, find key points (like midpoints) using averages of coordinates, and then use distance or area formulas to compute lengths and areas.

5. Suppose a, b, c are three distinct natural numbers, such that $3ac = 8(a + b)$. Then, the smallest possible value of $3a + 2b + c$ is:

Solution:
We are given

$$3ac = 8(a + b).$$

Step 1: Express b in terms of a and c.

$$3ac = 8a + 8b \Rightarrow 8b = 3ac - 8a = a(3c - 8) \Rightarrow b = \frac{a(3c - 8)}{8}.$$

Since a, b, c are natural numbers, b must be a positive integer; hence $a(3c - 8)$ must be divisible by 8 and $3c - 8 > 0$, so $c \geq 3$.
Step 2: Expression to minimize.
Let

$$S = 3a + 2b + c.$$

Substitute the expression for b:

$$S = 3a + 2 \cdot \frac{a(3c - 8)}{8} + c = 3a + \frac{a(3c - 8)}{4} + c = \frac{12a + 3ac - 8a}{4} + c = \frac{4a + 3ac}{4} + c = a\left(1 + \frac{3c}{4}\right) + c.$$

We now test small integer values of c that make b integral and keep a, b, c distinct.
Case 1: $c = 3$

$$b = \frac{a(9 - 8)}{8} = \frac{a}{8},$$

so a must be a multiple of 8. Smallest such a is 8, giving $b = 1$ and

$$S = 3 \cdot 8 + 2 \cdot 1 + 3 = 29.$$

Case 2: $c = 4$

$$b = \frac{a(12 - 8)}{8} = \frac{4a}{8} = \frac{a}{2},$$

so a must be even. Take the smallest even $a = 2$:

$$b = 1, \quad (a, b, c) = (2, 1, 4) \text{ are distinct.}$$

Then

$$S = 3 \cdot 2 + 2 \cdot 1 + 4 = 6 + 2 + 4 = 12.$$

Case 3: $c = 5$

$$b = \frac{a(15 - 8)}{8} = \frac{7a}{8},$$

so a is a multiple of 8. With $a = 8$, $b = 7$ and

$$S = 3 \cdot 8 + 2 \cdot 7 + 5 = 43 > 12.$$

Case 4: $c = 8$

$$b = \frac{a(24 - 8)}{8} = 2a.$$

With $a = 1$, $b = 2$ and $(a, b, c) = (1, 2, 8)$ distinct,

$$S = 3 \cdot 1 + 2 \cdot 2 + 8 = 15 > 12.$$

Higher values of c only increase S, so the minimum value occurs in Case 2 with

$$(a, b, c) = (2, 1, 4), \quad S_{\min} = 12.$$

Therefore, the smallest possible value of $3a + 2b + c$ is 12.

Quick Tip

When you have a Diophantine equation (integer solutions) and need to minimize an expression, first express one variable in terms of the others, then systematically test small integer values under the constraints (like distinctness and positivity).

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6. The ratio of expenditures of Lakshmi and Meenakshi is $2 : 3$, and the ratio of income of Lakshmi to expenditure of Meenakshi is $6 : 7$. If excess of income over expenditure is saved by Lakshmi and Meenakshi, and the ratio of their savings is $4 : 9$, then the ratio of their incomes is:
(A) 7 : 8
(B) 3 : 5
(C) 2 : 1
(D) 5 : 6

Correct Answer: (2) 3 : 5

Solution:
Step 1: Let the expenditures be

$$E_L = 2x, \qquad E_M = 3x$$

for Lakshmi and Meenakshi respectively.
Step 2: Use the ratio of income of Lakshmi to expenditure of Meenakshi:

$$\frac{I_L}{E_M} = \frac{6}{7} \Rightarrow \frac{I_L}{3x} = \frac{6}{7} \Rightarrow I_L = \frac{18x}{7}.$$

Step 3: Find savings.
Savings = Income − Expenditure.
Lakshmi's savings:

$$S_L = I_L - E_L = \frac{18x}{7} - 2x = \frac{18x - 14x}{7} = \frac{4x}{7}.$$

Let I_M be Meenakshi's income. Then her savings:

$$S_M = I_M - E_M = I_M - 3x.$$

Step 4: Use the savings ratio $S_L : S_M = 4 : 9$:

$$\frac{S_L}{S_M} = \frac{4}{9} \Rightarrow \frac{\frac{4x}{7}}{I_M - 3x} = \frac{4}{9}.$$

Divide both sides by 4:

$$\frac{\frac{x}{7}}{I_M - 3x} = \frac{1}{9} \Rightarrow 9 \cdot \frac{x}{7} = I_M - 3x \Rightarrow I_M = \frac{9x}{7} + 3x = \frac{9x + 21x}{7} = \frac{30x}{7}.$$

Step 5: Ratio of incomes:

$$\frac{I_L}{I_M} = \frac{\frac{18x}{7}}{\frac{30x}{7}} = \frac{18}{30} = \frac{3}{5}.$$

Therefore, the ratio of Lakshmi's income to Meenakshi's income is 3 : 5.

Quick Tip

When working with income–expenditure–saving problems:

- Introduce variables for unknown incomes and expenditures.
- Use the given ratios step by step to express all quantities in terms of a single variable.
- Translate the savings ratio into an equation and solve for the remaining unknowns.

7. If $\log_{64} x^2 + \log_8 \sqrt{y} + 3\log_{512}(\sqrt{y}z) = 4$, where x, y and z are positive real numbers, then the minimum possible value of $(x + y + z)$ is:
(A) 24
(B) 36
(C) 96
(D) 48

Correct Answer: (4) 48

Solution:
Step 1: Convert all logarithms to base 2.
Since $64 = 2^6$, $8 = 2^3$, and $512 = 2^9$:

$$\log_{64} x^2 = \log_{2^6} x^2 = \frac{\log_2 x^2}{\log_2 2^6} = \frac{2\log_2 x}{6} = \frac{1}{3}\log_2 x.$$

$$\log_8 \sqrt{y} = \log_{2^3} y^{1/2} = \frac{\log_2 y^{1/2}}{\log_2 2^3} = \frac{\frac{1}{2}\log_2 y}{3} = \frac{1}{6}\log_2 y.$$

$$3\log_{512}(\sqrt{y}z) = 3\log_{2^9}(y^{1/2}z) = 3\cdot\frac{\log_2(y^{1/2}z)}{\log_2 2^9} = 3\cdot\frac{\frac{1}{2}\log_2 y + \log_2 z}{9} = \frac{1}{3}\left(\frac{1}{2}\log_2 y + \log_2 z\right) = \frac{1}{6}\log_2 y + \frac{1}{3}\log_2 z.$$

Step 2: Substitute into the given equation.

$$\log_{64} x^2 + \log_8 \sqrt{y} + 3\log_{512}(\sqrt{y}z) = 4$$

becomes

$$\frac{1}{3}\log_2 x + \frac{1}{6}\log_2 y + \left(\frac{1}{6}\log_2 y + \frac{1}{3}\log_2 z\right) = 4.$$

Combine like terms:

$$\frac{1}{3}\log_2 x + \left(\frac{1}{6} + \frac{1}{6}\right)\log_2 y + \frac{1}{3}\log_2 z = 4$$

$$\frac{1}{3}\log_2 x + \frac{1}{3}\log_2 y + \frac{1}{3}\log_2 z = 4.$$

Factor $\frac{1}{3}$:

$$\frac{1}{3}\left(\log_2 x + \log_2 y + \log_2 z\right) = 4.$$

Using $\log_2 x + \log_2 y + \log_2 z = \log_2(xyz)$:

$$\frac{1}{3}\log_2(xyz) = 4 \;\Rightarrow\; \log_2(xyz) = 12.$$

Therefore,

$$xyz = 2^{12} = 4096.$$

Step 3: Minimize $x + y + z$ using AM-GM.
For positive x, y, z,

$$\frac{x+y+z}{3} \geq \sqrt[3]{xyz}.$$

Since $xyz = 2^{12}$:

$$\frac{x+y+z}{3} \geq \sqrt[3]{2^{12}} = 2^{12/3} = 2^4 = 16.$$

Thus,

$$x + y + z \geq 3 \cdot 16 = 48.$$

The equality in AM-GM holds when $x = y = z$, so the minimum sum $x + y + z$ is 48, attained when

$$x = y = z = 16.$$

Therefore, the minimum possible value of $(x + y + z)$ is 48.

Quick Tip

When an equation in logarithms simplifies to a fixed product like xyz = constant, use the AM-GM inequality to find the minimum (or maximum) of sums such as $x + y + z$. Equality in AM-GM occurs when all the variables are equal.

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8. If $9^{x^2+2x-3} - 4\left(3^{x^2+2x-2}\right) + 27 = 0$, then the product of all possible values of x is:
(A) 2
(B) 4
(C) 10
(D) 20

Correct Answer: (4) 20

Solution:
Step 1: Introduce a substitution.
Let

$$u = x^2 + 2x.$$

Then the equation becomes

$$9^{u-3} - 4 \cdot 3^{u-2} + 27 = 0.$$

Step 2: Express everything with base 3.
Since $9 = 3^2$,

$$9^{u-3} = (3^2)^{u-3} = 3^{2(u-3)} = 3^{2u-6}.$$

So we have

$$3^{2u-6} - 4 \cdot 3^{u-2} + 27 = 0.$$

Rewrite using negative exponents (or dividing by powers of 3):

$$3^{2u-6} = \frac{3^{2u}}{3^6}, \qquad 3^{u-2} = \frac{3^u}{3^2}.$$

Thus,

$$\frac{3^{2u}}{3^6} - 4 \cdot \frac{3^u}{3^2} + 27 = 0 \Rightarrow \frac{(3^u)^2}{729} - \frac{4 \cdot 3^u}{9} + 27 = 0.$$

Step 3: Quadratic in 3^u.
Let

$$y = 3^u.$$

Then

$$\frac{y^2}{729} - \frac{4y}{9} + 27 = 0.$$

Multiply through by 729:

$$y^2 - 4 \cdot 81\, y + 27 \cdot 729 = 0 \Rightarrow y^2 - 324y + 19683 = 0.$$

Solve:

$$y = \frac{324 \pm \sqrt{324^2 - 4 \cdot 19683}}{2}.$$

Compute the discriminant:

$$324^2 = 104976, \quad 4 \cdot 19683 = 78732,$$

$$104976 - 78732 = 26244 = 162^2.$$

So

$$y = \frac{324 \pm 162}{2}.$$

Hence

$$y_1 = \frac{324 + 162}{2} = \frac{486}{2} = 243, \qquad y_2 = \frac{324 - 162}{2} = \frac{162}{2} = 81.$$

Step 4: Back-substitute for u.
Recall $y = 3^u$.

$$3^u = 243 = 3^5 \Rightarrow u = 5, \qquad 3^u = 81 = 3^4 \Rightarrow u = 4.$$

But $u = x^2 + 2x$. So we get two quadratics:

$$x^2 + 2x = 5 \Rightarrow x^2 + 2x - 5 = 0,$$

$$x^2 + 2x = 4 \Rightarrow x^2 + 2x - 4 = 0.$$

Step 5: Product of all possible x.
For a quadratic $ax^2 + bx + c = 0$, product of roots $= \frac{c}{a}$.
For $x^2 + 2x - 5 = 0$, product of roots

$$P_1 = -5.$$

For $x^2 + 2x - 4 = 0$, product of roots

$$P_2 = -4.$$

All possible x are the four roots from these two equations, so the product of all possible x is

$$P_1 \cdot P_2 = (-5) \cdot (-4) = 20.$$

Therefore, the product of all possible values of x is 20.

Quick Tip

When exponents share a common expression (like $x^2 + 2x$ here), substitute it with a single variable to simplify. Then, look for ways to convert to a common base and reduce the equation to a quadratic in a new variable such as 3^u.

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9. The average number of copies of a book sold per day by a shopkeeper is 60 in the initial seven days and 63 in the initial eight days, after the book launch. On the ninth day, she sells 11 copies less than the eighth day, and the average number of copies sold per day from the second day to the ninth day becomes 66. The number of copies sold on the first day of the book launch is:

Solution:
Let x_i denote the number of copies sold on day i, and let S_n be the total copies sold in the first n days.
Step 1: Use the given averages for the first 7 and 8 days.
Average for first 7 days is 60:

$$S_7 = 7 \times 60 = 420.$$

Average for first 8 days is 63:

$$S_8 = 8 \times 63 = 504.$$

Step 2: Find the sales on the 8th day.

$$x_8 = S_8 - S_7 = 504 - 420 = 84.$$

Step 3: Find the sales on the 9th day.

On the 9th day, she sells 11 copies less than on the 8th day:

$$x_9 = x_8 - 11 = 84 - 11 = 73.$$

Step 4: Use the average from day 2 to day 9.
From day 2 to day 9 there are 8 days, and the average is 66:

$$S_{2\text{–}9} = 8 \times 66 = 528.$$

Step 5: Relate totals to find x_1.
Total sales in the first 9 days:

$$S_9 = S_8 + x_9 = 504 + 73 = 577.$$

But S_9 is also

$$S_9 = x_1 + S_{2\text{–}9} = x_1 + 528.$$

So,

$$x_1 + 528 = 577 \Rightarrow x_1 = 577 - 528 = 49.$$

Therefore, the number of copies sold on the first day is 49.

Quick Tip

When dealing with averages over overlapping time intervals, convert each average to a total sum. Then, use differences of these sums to find individual day values and set up equations to solve for the unknowns.

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10. A loan of Rs 1000 is fully repaid by two installments of Rs 530 and Rs 594, paid at the end of the first and second year, respectively. If the interest is compounded annually, then the rate of interest, in percentage, is:
(A) 6%
(B) 7%
(C) 8%
(D) 9%

Correct Answer: (3) 8%

Solution:
Step 1: Use the Present Value principle.
The loan amount equals the sum of the present values of all future installments.

$$1000 = \frac{530}{1+r} + \frac{594}{(1+r)^2}.$$

Let

$$x = 1 + r.$$

Then the equation becomes

$$1000 = \frac{530}{x} + \frac{594}{x^2}.$$

Step 2: Clear denominators.
Multiply through by x^2:

$$1000x^2 = 530x + 594.$$

Rearrange:

$$1000x^2 - 530x - 594 = 0.$$

Divide by 2 for simplicity:

$$500x^2 - 265x - 297 = 0.$$

Step 3: Solve the quadratic using the formula.

Here,

$$a = 500,\ b = -265,\ c = -297.$$

$$x = \frac{265 \pm \sqrt{(-265)^2 - 4(500)(-297)}}{1000}.$$

Compute the discriminant:

$$(-265)^2 = 70225, \qquad 4 \cdot 500 \cdot 297 = 594000,$$

$$70225 + 594000 = 664225 = 815^2.$$

Thus,

$$x = \frac{265 \pm 815}{1000}.$$

The valid root:

$$x = \frac{1080}{1000} = 1.08.$$

The negative root is discarded because $x = 1 + r > 0$.
Step 4: Find the rate of interest.

$$1 + r = 1.08 \quad \Rightarrow \quad r = 0.08 = 8\%.$$

Therefore, the interest rate is 8%.

Quick Tip

Whenever loan repayments occur in installments, convert each installment into its present value and sum them. If the interest is compounded annually, the discount factor for the n-th year is $(1 + r)^n$.

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11. The set of all real values of x for which $(x^2 - |x + 9| + x) > 0$ is:
(A) $(-\infty, -9) \cup (3, \infty)$
(B) $(-\infty, -3) \cup (9, \infty)$
(C) $(-\infty, -3) \cup (3, \infty)$
(D) $(-9, -3) \cup (3, 9)$

Correct Answer: (3) $(-\infty, -3) \cup (3, \infty)$

Solution:
We are given the inequality

$$x^2 - |x + 9| + x > 0 \quad \Longleftrightarrow \quad x^2 + x - |x + 9| > 0.$$

Since the absolute value depends on the sign of $x + 9$, we split into two cases.
Case 1: $x + 9 \geq 0 \ \Rightarrow \ x \geq -9$.
Then $|x + 9| = x + 9$. Substitute:

$$x^2 + x - (x + 9) > 0 \ \Rightarrow \ x^2 - 9 > 0 \ \Rightarrow \ (x - 3)(x + 3) > 0.$$

This product is positive when

$$x < -3 \quad \text{or} \quad x > 3.$$

But our case requires $x \geq -9$. Intersection gives:

$$[-9, -3) \cup (3, \infty).$$

Case 2: $x + 9 < 0 \ \Rightarrow \ x < -9$.
Then $|x + 9| = -x - 9$. Substitute:

$$x^2 + x - (-x - 9) > 0 \ \Rightarrow \ x^2 + 2x + 9 > 0.$$

The discriminant is

$$D = 2^2 - 4 \cdot 1 \cdot 9 = -32 < 0,$$

so the quadratic is always positive (opens upward). Thus the inequality holds for all $x < -9$.
Case 2 gives: $(-\infty, -9)$.
Step 3: Combine both cases.

$$(-\infty, -9) \cup [-9, -3) \cup (3, \infty) = (-\infty, -3) \cup (3, \infty).$$

Therefore, the solution set is:

$$\boxed{(-\infty, -3) \cup (3, \infty)}.$$

Quick Tip

When solving inequalities involving absolute values, always split into cases based on the sign of the expression inside the absolute value. Analyze each case separately, then combine intervals carefully by intersection and union.

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12. The equations $3x^2 - 5x + p = 0$ and $2x^2 - 2x + q = 0$ have one common root. The sum of the other roots of these two equations is:

(A) $\frac{5}{3} - p + q$

(B) $\frac{8}{3} + p - q$

(C) $\frac{8}{3} - p + \frac{3}{2}q$

(D) $p + q - 1$

Correct Answer: (3) $\frac{8}{3} - p + \frac{3}{2}q$

Solution:
Let the common root be α. Let the other roots of the first and second equations be β and γ respectively.
The equations are:
1. $3x^2 - 5x + p = 0$ with roots α, β 2. $2x^2 - 2x + q = 0$ with roots α, γ
We want the sum of the other roots:

$$\beta + \gamma.$$

Step 1: Use the sum of roots formula for each equation.
For $3x^2 - 5x + p = 0$:

$$\alpha + \beta = \frac{5}{3} \quad \Rightarrow \quad \beta = \frac{5}{3} - \alpha.$$

For $2x^2 - 2x + q = 0$:

$$\alpha + \gamma = 1 \quad \Rightarrow \quad \gamma = 1 - \alpha.$$

Step 2: Sum of the other roots.

$$\beta + \gamma = \left(\frac{5}{3} - \alpha\right) + (1 - \alpha) = \frac{8}{3} - 2\alpha. \tag{1}$$

Step 3: Use the fact that α is a common root.
From the first equation:

$$3\alpha^2 - 5\alpha + p = 0.$$

From the second equation:

$$2\alpha^2 - 2\alpha + q = 0.$$

Multiply the first equation by 2:

$$6\alpha^2 - 10\alpha + 2p = 0.$$

Multiply the second equation by 3:

$$6\alpha^2 - 6\alpha + 3q = 0.$$

Subtract:

$$(6\alpha^2 - 10\alpha + 2p) - (6\alpha^2 - 6\alpha + 3q) = 0,$$
$$-4\alpha + 2p - 3q = 0,$$
$$4\alpha = 2p - 3q,$$
$$\alpha = \frac{2p - 3q}{4}. \tag{2}$$

Step 4: Substitute (2) into (1).

$$\beta + \gamma = \frac{8}{3} - 2\left(\frac{2p - 3q}{4}\right) = \frac{8}{3} - \frac{2p - 3q}{2} = \frac{8}{3} - p + \frac{3}{2}q.$$

Thus, the sum of the other roots is:

$$\boxed{\frac{8}{3} - p + \frac{3}{2}q}.$$

Quick Tip

When two quadratics share a common root, equate the root expressions by eliminating the squared term. Using Vieta's formulas then makes it easy to compute required expressions involving the other roots.

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13. An item with a cost price of Rs. 1650 is sold at a certain discount on a fixed marked price to earn a profit of 20% on the cost price. If the discount was doubled, the profit would have been Rs. 110. The rate of discount, in percentage, at which the profit percentage would be equal to the rate of discount, is nearest to:

(A) 12
(B) 13
(C) 14
(D) 15

Correct Answer: (3) 14

Solution:
Step 1: Use the first scenario (20% profit).
Cost price:

$$CP = 1650.$$

Profit = 20% of 1650:

$$\text{Profit}_1 = 0.20 \times 1650 = 330.$$

So the selling price in the first scenario:

$$SP_1 = CP + \text{Profit}_1 = 1650 + 330 = 1980.$$

Let the marked price be MP and the initial discount be D. Then:

$$MP - D = 1980. \tag{1}$$

Step 2: Use the second scenario (doubled discount and profit Rs. 110).
New profit:

$$\text{Profit}_2 = 110.$$

So the second selling price:

$$SP_2 = CP + \text{Profit}_2 = 1650 + 110 = 1760.$$

Discount is doubled $\Rightarrow$ new discount is $2D$, so:

$$MP - 2D = 1760. \tag{2}$$

Step 3: Solve for MP and D.
Subtract (2) from (1):

$$(MP - D) - (MP - 2D) = 1980 - 1760$$
$$D = 220.$$

Substitute back in (1):

$$MP - 220 = 1980 \Rightarrow MP = 2200.$$

So the marked price is Rs. 2200.
Step 4: Let the discount rate be $x\%$ such that

$$\text{Discount\%} = \text{Profit\%} = x.$$

Discount amount:

$$\text{Discount} = \frac{x}{100} \times 2200 = 22x.$$

New selling price:

$$SP = MP - \text{Discount} = 2200 - 22x.$$

Profit amount:

$$\text{Profit} = SP - CP = (2200 - 22x) - 1650 = 550 - 22x.$$

Profit percentage (on cost price) is:

$$\text{Profit\%} = \frac{\text{Profit}}{CP} \times 100 = \frac{550 - 22x}{1650} \times 100.$$

We are told this percentage equals x:

$$x = \frac{550 - 22x}{1650} \times 100.$$

Step 5: Solve for x.

$$x = \frac{100(550 - 22x)}{1650} = \frac{550 - 22x}{16.5}.$$

So,

$$16.5x = 550 - 22x \Rightarrow 16.5x + 22x = 550 \Rightarrow 38.5x = 550 \Rightarrow x = \frac{550}{38.5}.$$

Simplify:

$$x = \frac{5500}{385} = \frac{1100}{77} = \frac{100}{7} \approx 14.2857.$$

Thus, the required discount rate is approximately 14.3%, which is nearest to 14%.

Quick Tip

When the same marked price is used under different discount and profit conditions, set up equations using $SP = MP - \text{Discount}$ and $SP = CP + \text{Profit}$ for each scenario. Once the marked price is known, you can introduce a variable discount rate and equate the profit percentage to that rate to solve such "rate equals rate" problems.

---

14. A certain amount of money was divided among Pinu, Meena, Rinu, and Seema. Pinu received 20% of the total amount and Meena received 40% of the remaining amount. If Seema received 20% less than Pinu, the ratio of the amounts received by Pinu and Rinu is:
(A) 4 : 5
(B) 5 : 8

(C) 3 : 5
(D) 2 : 3

Correct Answer: (2) 5 : 8

Solution:
Step 1: Let the total amount be T. For convenience, take $T = 100$ (any other positive value would give the same ratio).
Step 2: Pinu's share.
Pinu gets 20% of the total:

$$P = 20\% \text{ of } 100 = 20.$$

Step 3: Meena's share.
Amount remaining after Pinu:

$$100 - 20 = 80.$$

Meena gets 40% of this remaining amount:

$$M = 40\% \text{ of } 80 = 0.4 \times 80 = 32.$$

Step 4: Seema's share.
Seema receives 20% less than Pinu:

$$S = P - 20\% \text{ of } P = 20 - 0.2 \times 20 = 20 - 4 = 16.$$

Step 5: Rinu's share.
Rinu gets the remaining amount:

$$R = 100 - (P + M + S) = 100 - (20 + 32 + 16) = 100 - 68 = 32.$$

Step 6: Required ratio $P : R$.

$$P : R = 20 : 32.$$

Divide both by 4:

$$P : R = 5 : 8.$$

Therefore, the ratio of the amounts received by Pinu and Rinu is 5 : 8.

Quick Tip

In distribution problems, it often helps to assume a convenient total (like 100) when only percentages are involved. This makes computations easy and does not affect the final ratios.

---

15. Let $f(x) = \dfrac{x}{2x-1}$ and $g(x) = \dfrac{x}{x-1}$. Then, the domain of the function

$$h(x) = f(g(x)) + g(f(x))$$

is all real numbers except:
(A) $\frac{1}{2}, 1, \frac{3}{2}$
(B) $\frac{1}{2}, 1$
(C) $-\frac{1}{2}, \frac{1}{2}, 1$
(D) $-1, \frac{1}{2}, 1$

Correct Answer: (4) $-1, \frac{1}{2}, 1$

Solution:
To determine the domain of

$$h(x) = f(g(x)) + g(f(x)),$$

both expressions $f(g(x))$ and $g(f(x))$ must be defined.

1. Domain of $f(g(x))$

Inner function:

$$g(x) = \frac{x}{x-1}$$

is undefined when the denominator is zero:

$$x - 1 \neq 0 \quad \Rightarrow \quad x \neq 1.$$

Outer function:

$$f(u) = \frac{u}{2u-1}$$

is undefined when

$$2u - 1 = 0 \quad \Rightarrow \quad u = \frac{1}{2}.$$

Thus, we need

$$g(x) \neq \frac{1}{2}.$$

Solve:

$$\frac{x}{x-1} \neq \frac{1}{2}$$

Cross–multiply:

$$2x \neq x - 1 \quad \Rightarrow \quad x \neq -1.$$

Hence, for $f(g(x))$, the restrictions are:

$$x \neq 1, \ -1.$$

2. Domain of $g(f(x))$

Inner function:

$$f(x) = \frac{x}{2x-1}$$

is undefined when

$$2x - 1 = 0 \quad \Rightarrow \quad x \neq \frac{1}{2}.$$

Outer function:

$$g(v) = \frac{v}{v-1}$$

is undefined when $v = 1$. So we need:

$$f(x) \neq 1.$$

Solve:

$$\frac{x}{2x-1} \neq 1.$$

Set inequality:

$$x \neq 2x - 1 \quad \Rightarrow \quad -x \neq -1 \quad \Rightarrow \quad x \neq 1.$$

Thus, for $g(f(x))$:

$$x \neq \frac{1}{2}, \ 1.$$

3. Combine all restrictions

From $f(g(x))$: $x \neq 1, \ -1$. From $g(f(x))$: $x \neq \frac{1}{2}, \ 1$.
Therefore, the domain excludes:

$$\boxed{-1, \ \frac{1}{2}, \ 1}.$$

Quick Tip

When dealing with compositions of rational functions, 1. Exclude values that make any denominator zero, and 2. Also exclude values that make the inner function produce an invalid input for the outer function. Always combine all such restrictions at the end.

---

16. The number of divisors of $(2^6 \times 3^5 \times 5^3 \times 7^2)$, which are of the form $(3r+1)$, where r is a non-negative integer, is:
(A) 42
(B) 36
(C) 56
(D) 24

Correct Answer: (1) 42

Solution:
Let
$$N = 2^6 \times 3^5 \times 5^3 \times 7^2.$$
Any divisor D of N is of the form
$$D = 2^a \cdot 3^b \cdot 5^c \cdot 7^d,$$
where
$$0 \le a \le 6, \quad 0 \le b \le 5, \quad 0 \le c \le 3, \quad 0 \le d \le 2.$$
We want divisors of the form $3r+1$, i.e.
$$D \equiv 1 \pmod 3.$$
Step 1: Reduce each prime factor modulo 3.
$$2 \equiv -1 \pmod 3, \quad 3 \equiv 0 \pmod 3, \quad 5 \equiv 2 \equiv -1 \pmod 3, \quad 7 \equiv 1 \pmod 3.$$
Thus,
$$D \equiv (-1)^a \cdot 0^b \cdot (-1)^c \cdot 1^d \pmod 3.$$
Step 2: Condition on exponent b.
If $b \ge 1$, then $3^b \equiv 0 \pmod 3$, so
$$D \equiv 0 \pmod 3,$$
which cannot be congruent to 1 modulo 3.
Hence we must have
$$b = 0 \quad \text{(only 1 choice)}.$$
With $b = 0$, the modulus simplifies to
$$D \equiv (-1)^a \cdot (-1)^c \cdot 1^d = (-1)^{a+c} \pmod 3.$$
We need
$$(-1)^{a+c} \equiv 1 \pmod 3,$$
which means $a + c$ must be even.
Step 3: Count possibilities for a and c.
For a $(0 \le a \le 6)$: - Even: $0, 2, 4, 6 \Rightarrow 4$ values - Odd: $1, 3, 5 \Rightarrow 3$ values
For c $(0 \le c \le 3)$: - Even: $0, 2 \Rightarrow 2$ values - Odd: $1, 3 \Rightarrow 2$ values
We need a and c to have the same parity.
Case 1: Both even. Number of ways:
$$4 \times 2 = 8.$$
Case 2: Both odd. Number of ways:
$$3 \times 2 = 6.$$
Total valid (a, c) pairs:
$$8 + 6 = 14.$$

Step 4: Count possibilities for d.
Since $7 \equiv 1 \pmod 3$, we have $7^d \equiv 1^d \equiv 1 \pmod 3$ for any d, so any allowed value of d works:

$$d = 0, 1, 2 \Rightarrow 3 \text{ choices.}$$

Step 5: Total number of valid divisors.

$$\text{Choices for } b = 1, \quad \text{choices for } d = 3, \quad \text{valid } (a, c) \text{ pairs} = 14.$$

Thus total divisors of the form $3r + 1$ are:

$$1 \times 3 \times 14 = 42.$$

Therefore, the required number of divisors is 42.

Quick Tip

When counting divisors with a condition like "of the form $3r + 1$", work modulo 3:

- Express each prime factor modulo 3.
- Eliminate exponents that force the divisor to be 0 mod 3.
- Use parity or congruence conditions on exponents to match the required remainder.

---

17. The sum of digits of the number $(625)^{65} \times (128)^{36}$ is:

Solution:
Let

$$N = (625)^{65} \times (128)^{36}.$$

Step 1: Express numbers in prime powers.

$$625 = 5^4, \qquad 128 = 2^7.$$

Substitute:

$$N = (5^4)^{65} \times (2^7)^{36}.$$

Step 2: Simplify exponents.

$$(5^4)^{65} = 5^{260}, \qquad (2^7)^{36} = 2^{252}.$$

Thus,

$$N = 5^{260} \times 2^{252}.$$

Step 3: Pair powers of 2 and 5 to form tens.
Match 2^{252} with 5^{252}:

$$N = 5^{260} \cdot 2^{252} = 5^8 \cdot (5^{252} \cdot 2^{252}) = 5^8 \cdot 10^{252}.$$

Step 4: Compute 5^8.

$$5^4 = 625, \qquad 5^8 = 625 \times 625 = 390625.$$

So,

$$N = 390625 \times 10^{252}.$$

This is the number 390625 followed by 252 zeros.
Step 5: Sum the digits.
Zeros contribute nothing, so compute:

$$3 + 9 + 0 + 6 + 2 + 5 = 25.$$

Thus, the sum of digits of the number is

$$\boxed{25}.$$

Quick Tip

When multiplying large powers of 2 and 5, group them into powers of 10. This isolates a manageable non-zero block of digits followed by many zeros, simplifying digit-sum problems.

---

18. Ankita is twice as efficient as Bipin, while Bipin is twice as efficient as Chandan. All three of them start together on a job, and Bipin leaves the job after 20 days. If the job got completed in 60 days, the number of days needed by Chandan to complete the job alone, is:

(A) 240
(B) 260
(C) 300
(D) 340

Correct Answer: (4) 340

Solution:
Step 1: Let Chandan's efficiency be x units of work per day.

$$E_C = x.$$

Bipin is twice as efficient as Chandan:

$$E_B = 2x.$$

Ankita is twice as efficient as Bipin:

$$E_A = 2 \cdot (2x) = 4x.$$

Step 2: Work done by each person.
Total time taken for the job to finish is 60 days.
Bipin: works for 20 days (then leaves):

$$W_B = E_B \times 20 = 2x \times 20 = 40x.$$

Ankita: works for all 60 days:

$$W_A = E_A \times 60 = 4x \times 60 = 240x.$$

Chandan: also works for all 60 days:

$$W_C = E_C \times 60 = x \times 60 = 60x.$$

Step 3: Total work.

$$W_{\text{total}} = W_A + W_B + W_C = 240x + 40x + 60x = 340x.$$

Step 4: Time taken by Chandan alone.
If Chandan works alone at efficiency x, then

$$\text{Time} = \frac{W_{\text{total}}}{E_C} = \frac{340x}{x} = 340 \text{ days}.$$

Therefore, Chandan would need 340 days to complete the job alone.

Quick Tip

In work and time problems with different efficiencies, it is often easiest to:

- Assume one person's efficiency as a variable,
- Express others' efficiencies in terms of that variable using the given ratios,
- Compute total work done, then divide by an individual's efficiency for "alone" time.

---

19. If m and n are integers such that $(m + 2n)(2m + n) = 27$, then the maximum possible value of $2m - 3n$ is:
(A) 9
(B) 13
(C) 15
(D) 17

Correct Answer: (4) 17

Solution:
We are given

$$(m + 2n)(2m + n) = 27.$$

Let

$$A = m + 2n, \quad B = 2m + n.$$

Then

$$AB = 27.$$

Step 1: Express m and n in terms of A and B.
We have the system

$$\begin{cases} m + 2n = A \\ 2m + n = B \end{cases}$$

Multiply the first equation by 2:

$$2m + 4n = 2A.$$

Subtract the second equation:

$$(2m + 4n) - (2m + n) = 2A - B \Rightarrow 3n = 2A - B \Rightarrow n = \frac{2A - B}{3}.$$

Similarly, multiply the second equation by 2:

$$4m + 2n = 2B.$$

Subtract the first equation:

$$(4m + 2n) - (m + 2n) = 2B - A \Rightarrow 3m = 2B - A \Rightarrow m = \frac{2B - A}{3}.$$

For m and n to be integers, both $2B - A$ and $2A - B$ must be multiples of 3.
Step 2: Use a modular condition.
From $2B - A \equiv 0 \pmod 3$ and $2A - B \equiv 0 \pmod 3$:

$$\begin{aligned} 2B - A \equiv 0 \pmod 3 \quad &\Rightarrow \quad 2B \equiv A \pmod 3, \\ 2A - B \equiv 0 \pmod 3 \quad &\Rightarrow \quad 2A \equiv B \pmod 3. \end{aligned}$$

Add these two congruences:

$$2B - A + 2A - B \equiv 0 \pmod 3 \Rightarrow A + B \equiv 0 \pmod 3.$$

So $A + B$ must be divisible by 3.
Step 3: List factor pairs of 27.
Since $AB = 27$, possible integer pairs (A, B) are:

$$(1, 27),\ (3, 9),\ (9, 3),\ (27, 1),$$

and their negative counterparts:

$$(-1, -27),\ (-3, -9),\ (-9, -3),\ (-27, -1).$$

Check $A+B \equiv 0 \pmod 3$:
- $(1,27)$: $A+B=28$ (not divisible by 3) - $(27,1)$: $A+B=28$ (not divisible by 3) - $(3,9)$: $A+B=12$ (divisible by 3) - $(9,3)$: $A+B=12$ (divisible by 3) - $(-3,-9)$: $A+B=-12$ (divisible by 3) - $(-9,-3)$: $A+B=-12$ (divisible by 3)
So valid pairs are:

$$(3,9),\ (9,3),\ (-3,-9),\ (-9,-3).$$

Step 4: Compute (m,n) and $2m-3n$ for each valid pair.
Using

$$m=\frac{2B-A}{3},\quad n=\frac{2A-B}{3},$$

we evaluate each pair.
<u>Pair $(A,B)=(3,9)$:</u>

$$m=\frac{2\cdot 9-3}{3}=\frac{18-3}{3}=5,\quad n=\frac{2\cdot 3-9}{3}=\frac{6-9}{3}=-1.$$

$$2m-3n=2\cdot 5-3(-1)=10+3=13.$$

<u>Pair $(A,B)=(9,3)$:</u>

$$m=\frac{2\cdot 3-9}{3}=\frac{6-9}{3}=-1,\quad n=\frac{2\cdot 9-3}{3}=\frac{18-3}{3}=5.$$

$$2m-3n=2(-1)-3\cdot 5=-2-15=-17.$$

<u>Pair $(A,B)=(-3,-9)$:</u>

$$m=\frac{2(-9)-(-3)}{3}=\frac{-18+3}{3}=-5,\quad n=\frac{2(-3)-(-9)}{3}=\frac{-6+9}{3}=1.$$

$$2m-3n=2(-5)-3\cdot 1=-10-3=-13.$$

<u>Pair $(A,B)=(-9,-3)$:</u>

$$m=\frac{2(-3)-(-9)}{3}=\frac{-6+9}{3}=1,\quad n=\frac{2(-9)-(-3)}{3}=\frac{-18+3}{3}=-5.$$

$$2m-3n=2\cdot 1-3(-5)=2+15=17.$$

Step 5: Choose the maximum value.
Possible values of $2m-3n$ are:

$$13,\ -17,\ -13,\ 17.$$

The maximum among these is

$$\boxed{17}.$$

Quick Tip

When given a product condition like $(m+2n)(2m+n)=27$ with integer solutions,

- Introduce new variables for the linear factors,
- Solve the resulting linear system for m and n,
- Impose integrality conditions (often via modular arithmetic),
- Check all factor pairs and then evaluate the required expression.

---

20. In a $\triangle ABC$, points D and E are on the sides BC and AC, respectively. BE and AD intersect at point T such that $AD:AT=4:3$, and $BE:BT=5:4$. Point F lies on AC such that DF is parallel to BE. Then, $BD:CD$ is:
(A) 15 : 4
(B) 11 : 4

(C) 9 : 4
(D) 7 : 4

Correct Answer: (2) 11 : 4

Solution:
Let
$$\frac{BD}{CD} = x.$$
Step 1: Convert the given ratios into segment ratios.
From $AD : AT = 4 : 3$,
$$\frac{AD}{AT} = \frac{4}{3}, \quad \Rightarrow \quad \frac{AT}{AD} = \frac{3}{4}, \quad \frac{AT}{TD} = \frac{3}{1}, \quad \Rightarrow \quad \frac{DT}{TA} = \frac{1}{3}.$$
From $BE : BT = 5 : 4$,
$$\frac{BE}{BT} = \frac{5}{4}, \quad \Rightarrow \quad \frac{BT}{BE} = \frac{4}{5}, \quad \frac{BT}{TE} = \frac{4}{1}, \quad \Rightarrow \quad \frac{ET}{TB} = \frac{1}{4}.$$
Step 2: Apply Menelaus' Theorem in $\triangle ADC$ with transversal $B{-}T{-}E$.
Line BTE intersects: - AD at T, - AC at E, - and the extension of CD at B.
Menelaus' Theorem for $\triangle ADC$ with transversal BTE:
$$\frac{AE}{EC} \cdot \frac{CB}{BD} \cdot \frac{DT}{TA} = 1.$$
Now,
$$\frac{CB}{BD} = \frac{CD + BD}{BD} = \frac{CD}{BD} + 1 = \frac{1}{x} + 1 = \frac{1+x}{x}, \quad \frac{DT}{TA} = \frac{1}{3}.$$
Thus,
$$\frac{AE}{EC} \cdot \frac{1+x}{x} \cdot \frac{1}{3} = 1 \quad \Rightarrow \quad \frac{AE}{EC} = 3 \cdot \frac{x}{1+x} = \frac{3x}{1+x}.$$
So
$$\frac{EC}{AE} = \frac{1+x}{3x}.$$
Step 3: Apply Menelaus' Theorem in $\triangle CBE$ with transversal $A{-}T{-}D$.
Line ATD intersects: - CE (extended) at A, - BE at T, - and BC at D.
Menelaus' Theorem for $\triangle CBE$ with transversal ATD:
$$\frac{CA}{AE} \cdot \frac{ET}{TB} \cdot \frac{BD}{DC} = 1.$$
We have:
$$\frac{CA}{AE} = \frac{CE + AE}{AE} = \frac{CE}{AE} + 1 = \frac{1+x}{3x} + 1 = \frac{1+x+3x}{3x} = \frac{4x+1}{3x},$$
$$\frac{ET}{TB} = \frac{1}{4}, \qquad \frac{BD}{DC} = x.$$
Substitute:
$$\left(\frac{4x+1}{3x}\right) \cdot \frac{1}{4} \cdot x = 1.$$
Step 4: Solve for x.
$$\frac{x(4x+1)}{12x} = 1 \quad \Rightarrow \quad \frac{4x+1}{12} = 1 \quad \Rightarrow \quad 4x+1 = 12 \quad \Rightarrow \quad 4x = 11 \quad \Rightarrow \quad x = \frac{11}{4}.$$
Therefore,
$$\frac{BD}{CD} = \frac{11}{4} \quad \Rightarrow \quad BD : CD = 11 : 4.$$

Quick Tip

When internal cevians intersect (like AD and BE meeting at T) and you need a side ratio, Menelaus' Theorem on carefully chosen triangles with transversals through T can be more direct than coordinate or mass-point geometry.

---

21. A mixture of coffee and cocoa, 16% of which is coffee, costs Rs 240 per kg. Another mixture of coffee and cocoa, of which 36% is coffee, costs Rs 320 per kg. If a new mixture of coffee and cocoa costs Rs 376 per kg, then the quantity, in kg, of coffee in 10 kg of this new mixture is:
(A) 2.5
(B) 5
(C) 4
(D) 6

Correct Answer: (2) 5

Solution:
Let the cost of 1 kg of coffee be C_f and the cost of 1 kg of cocoa be C_c.
Step 1: Use the two given mixtures to find C_f and C_c.
Mixture 1: 16% coffee, 84% cocoa, cost Rs 240/kg:

$$0.16C_f + 0.84C_c = 240 \tag{1}$$

Mixture 2: 36% coffee, 64% cocoa, cost Rs 320/kg:

$$0.36C_f + 0.64C_c = 320 \tag{2}$$

Subtract (1) from (2):

$$(0.36C_f + 0.64C_c) - (0.16C_f + 0.84C_c) = 320 - 240$$
$$0.20C_f - 0.20C_c = 80$$

Divide by 0.20:

$$C_f - C_c = 400 \quad \Rightarrow \quad C_f = C_c + 400.$$

Substitute into (1):

$$0.16(C_c + 400) + 0.84C_c = 240$$
$$0.16C_c + 64 + 0.84C_c = 240$$
$$1.00C_c = 240 - 64 = 176.$$

So

$$C_c = 176, \quad C_f = 176 + 400 = 576.$$

Step 2: Find the composition of the new mixture.
Let the fraction of coffee in the new mixture be x. Then the fraction of cocoa is $1 - x$. Given its cost is Rs 376/kg:

$$xC_f + (1 - x)C_c = 376$$
$$x \cdot 576 + (1 - x) \cdot 176 = 376$$
$$576x + 176 - 176x = 376$$
$$400x + 176 = 376 \Rightarrow 400x = 200 \Rightarrow x = \frac{200}{400} = \frac{1}{2}.$$

So the new mixture is 50% coffee.
Step 3: Coffee content in 10 kg of the new mixture.

$$\text{Coffee} = \frac{1}{2} \times 10 = 5 \text{ kg}.$$

Therefore, the quantity of coffee in 10 kg of the new mixture is 5 kg.

Quick Tip

When mixtures of the same two ingredients are given with different compositions and costs, set up linear equations using the percentage of each ingredient and solve for the individual prices. Then use those prices to find the composition of any new mixture.

---

22. Rita and Sneha can row a boat at 5 km/h and 6 km/h in still water, respectively. In a river flowing with a constant velocity, Sneha takes 48 minutes more to row 14 km upstream than to row the same distance downstream. If Rita starts from a certain location in the river, and returns downstream to the same location, taking a total of 100 minutes, then the total distance, in km, Rita will cover is:

Solution:
Step 1: Find the speed of the river.
Sneha's speed in still water:

$$u_s = 6 \text{ km/h}.$$

Let the speed of the river be v km/h.
Then:

$$\text{Upstream speed} = 6 - v, \quad \text{Downstream speed} = 6 + v.$$

Distance in each direction is 14 km, and the difference in time between upstream and downstream is 48 minutes:

$$\frac{14}{6-v} - \frac{14}{6+v} = 48 \text{ minutes} = \frac{4}{5} \text{ hours}.$$

Compute:

$$\frac{14}{6-v} - \frac{14}{6+v} = 14\left(\frac{(6+v)-(6-v)}{(6-v)(6+v)}\right) = 14\left(\frac{2v}{36-v^2}\right) = \frac{28v}{36-v^2}.$$

So,

$$\frac{28v}{36-v^2} = \frac{4}{5}.$$

Divide both sides by 4:

$$\frac{7v}{36-v^2} = \frac{1}{5}.$$

Cross-multiply:

$$5 \cdot 7v = 36 - v^2 \Rightarrow 35v = 36 - v^2 \Rightarrow v^2 + 35v - 36 = 0.$$

Factor:

$$(v+36)(v-1) = 0 \Rightarrow v = 1 \text{ (taking positive, feasible speed)}.$$

Thus, the river speed is 1 km/h.
Step 2: Use Rita's speeds to find the distance.
Rita's speed in still water:

$$u_r = 5 \text{ km/h}.$$

So:

$$\text{Upstream speed} = 5 - 1 = 4 \text{ km/h}, \quad \text{Downstream speed} = 5 + 1 = 6 \text{ km/h}.$$

Total time taken for her round trip:

$$100 \text{ minutes} = \frac{100}{60} = \frac{5}{3} \text{ hours}.$$

Let the one-way distance be D km. Then:

$$\text{Upstream time} = \frac{D}{4}, \quad \text{Downstream time} = \frac{D}{6}.$$

Given:

$$\frac{D}{4} + \frac{D}{6} = \frac{5}{3}.$$

Take LCM 12:

$$\frac{3D}{12} + \frac{2D}{12} = \frac{5}{3} \Rightarrow \frac{5D}{12} = \frac{5}{3}.$$

Divide both sides by 5:

$$\frac{D}{12} = \frac{1}{3} \Rightarrow D = \frac{12}{3} = 4 \text{ km}.$$

Step 3: Total distance covered.
Rita rows from the starting point to the turning point and back:

$$\text{Total distance} = 2D = 2 \times 4 = 8 \text{ km}.$$

Therefore, Rita will cover a total distance of 8 km.

Quick Tip

In upstream–downstream problems:

- First find the stream speed using time differences and the given rower's speed.
- Then apply those speeds to other rowers, using the relation time $= \frac{\text{distance}}{\text{speed}}$.
- For round trips, total time is the sum of upstream and downstream times.

---

23. If a, b, c and d are integers such that their sum is 46, then the minimum possible value of $(a-b)^2 + (a-c)^2 + (a-d)^2$ is:

Solution:
We want to minimize

$$S = (a-b)^2 + (a-c)^2 + (a-d)^2$$

subject to

$$a + b + c + d = 46,$$

where a, b, c, d are integers.
Step 1: Rewrite in terms of deviations from a.
Let

$$b = a - \delta_1, \quad c = a - \delta_2, \quad d = a - \delta_3,$$

where $\delta_1, \delta_2, \delta_3$ are integers.
Then

$$S = \delta_1^2 + \delta_2^2 + \delta_3^2.$$

Substitute into the sum constraint:

$$a + (a - \delta_1) + (a - \delta_2) + (a - \delta_3) = 46$$

$$4a - (\delta_1 + \delta_2 + \delta_3) = 46$$

$$\delta_1 + \delta_2 + \delta_3 = 4a - 46.$$

Step 2: Choose a to keep the deviations small.
To make $S = \delta_1^2 + \delta_2^2 + \delta_3^2$ as small as possible, we want the sum

$$\delta_1 + \delta_2 + \delta_3 = 4a - 46$$

to be as close to 0 as possible, because spreading a small total among three integers gives smaller squares.
The mean of a, b, c, d is

$$\frac{46}{4} = 11.5,$$

so a should be near 11 or 12.
Case 1: $a = 11$.

Then

$$\delta_1 + \delta_2 + \delta_3 = 4(11) - 46 = 44 - 46 = -2.$$

To minimize $\delta_1^2 + \delta_2^2 + \delta_3^2$ with sum -2, choose values as evenly as possible, e.g.

$$(\delta_1, \delta_2, \delta_3) = (0, -1, -1).$$

Then

$$S = 0^2 + (-1)^2 + (-1)^2 = 0 + 1 + 1 = 2.$$

Corresponding integers:

$$a = 11, \quad b = 11, \quad c = 12, \quad d = 12$$

(sum $= 11 + 11 + 12 + 12 = 46$, valid).
Case 2: $a = 12$.
Then

$$\delta_1 + \delta_2 + \delta_3 = 4(12) - 46 = 48 - 46 = 2.$$

Even distribution:

$$(\delta_1, \delta_2, \delta_3) = (1, 1, 0).$$

Then

$$S = 1^2 + 1^2 + 0^2 = 1 + 1 + 0 = 2.$$

Corresponding integers:

$$a = 12, \quad b = 11, \quad c = 11, \quad d = 12$$

(sum $= 12 + 11 + 11 + 12 = 46$, valid).
Step 3: Check that we cannot do better.
If a is further from 11.5, e.g. $a = 10$ or $a = 13$:
- For $a = 10$: $\delta_1 + \delta_2 + \delta_3 = 4(10) - 46 = -6$. Best balance is roughly $(-2, -2, -2)$ giving

$$S \approx 2^2 + 2^2 + 2^2 = 12 > 2.$$

- For $a = 13$: $\delta_1 + \delta_2 + \delta_3 = 4(13) - 46 = 6$. Best balance is roughly $(2, 2, 2)$ giving

$$S \approx 2^2 + 2^2 + 2^2 = 12 > 2.$$

Thus S cannot be smaller than 2, and we have explicit integer examples achieving $S = 2$.
Therefore, the minimum possible value of

$$(a - b)^2 + (a - c)^2 + (a - d)^2$$

is

$$\boxed{2}.$$

Quick Tip

For minimizing sums of squares with a fixed sum constraint, keep the numbers as close together as possible. Here, that meant taking a, b, c, d near the average $\frac{46}{4} = 11.5$, and distributing small integer deviations evenly.
