CBSE Class 12 Mathematics(Set 65/1/2) Question Paper with Solutions

Time Allowed :3 Hour	Maximum Marks :70	Total Questions :38

General Instructions

Read the following instructions very carefully and strictly follow them:

(i) This Question paper contains 38 questions. All questions are compulsory.

(ii) Question paper is divided into FIVE Sections – Section A, B, C, D and E.

(iii) In Section A – Question Number 1 to 18 are Multiple Choice Questions (MCQs) and Question Number 19 & 20 are Assertion-Reason based questions of 1 mark each.

(iv) In Section B – Question Number 21 to 25 are Very Short Answer (VSA) type questions, carrying 2 marks each.

(v) In Section C – Question Number 26 to 31 are Short Answer (SA) type questions, carrying 3 marks each.

(vi) In Section D – Question Number 32 to 35 are Long Answer (LA) type questions, carrying 5 marks each.

(vii) In Section E – Question Number 36 to 38 are case study based questions, carrying 4 marks each.

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.

(ix) Use of calculator is NOT allowed.

1. If $\int \frac{3x+7}{x^2+6x+13}\,dx = A\log\left|x^2+6x+13\right| + K$**, then the value of** A **is:**

(A) $3a$

(B) $\frac{2a}{3b}$

(C) $\frac{3a}{b^2}$

(D) $\frac{3a}{2b}$

Correct Answer: $\frac{3}{2}$

Solution:

Concept:

When integrating expressions of the form

$$\int \frac{f'(x)}{f(x)} dx$$

the result is

$$\log |f(x)| + C.$$

Thus, if the numerator is proportional to the derivative of the denominator, the integral simplifies into a logarithmic form.
Here the denominator is:

$$x^2 + 6x + 13$$

Its derivative is:

$$\frac{d}{dx}(x^2 + 6x + 13) = 2x + 6$$

We express the numerator $3x + 7$ in terms of $2x + 6$.
Step 1: Relate the numerator with the derivative of the denominator.

$$3x + 7 = A(2x + 6) + B$$

Expanding:

$$3x + 7 = 2Ax + 6A + B$$

Comparing coefficients:

$$2A = 3$$

$$A = \frac{3}{2}$$

Step 2: Evaluate the integral.
Thus,

$$\int \frac{3x + 7}{x^2 + 6x + 13} dx = \frac{3}{2} \int \frac{2x + 6}{x^2 + 6x + 13} dx + \text{constant}$$

Using the standard logarithmic result:

$$\int \frac{2x + 6}{x^2 + 6x + 13} dx = \log |x^2 + 6x + 13|$$

Therefore,

$$\int \frac{3x + 7}{x^2 + 6x + 13} dx = \frac{3}{2} \log |x^2 + 6x + 13| + K$$

Hence,

$$A = \frac{3}{2}$$

Quick Tip

When the denominator is a polynomial and the numerator resembles its derivative, try rewriting the numerator as a multiple of the derivative of the denominator. This directly leads to a logarithmic integral.

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2. The value of $\int_0^1 \frac{x}{x^2+1}\,dx$ **is**

(A) 0
(B) $\log 2$
(C) $2\log 2$
(D) $\frac{1}{2}\log 2$

Correct Answer: (4) $\frac{1}{2}\log 2$

Solution:

Concept:
When the numerator is proportional to the derivative of the denominator, substitution becomes very convenient.
If

$$\int \frac{f'(x)}{f(x)}dx$$

then

$$= \log|f(x)| + C$$

In definite integrals, we apply the limits after integrating.
Step 1: Use substitution.
Let

$$u = x^2 + 1$$

Then

$$\frac{du}{dx} = 2x$$

$$du = 2x\,dx$$

Thus,

$$x\,dx = \frac{du}{2}$$

Step 2: Change the limits.
When $x = 0$

$$u = 0^2 + 1 = 1$$

When $x = 1$

$$u = 1^2 + 1 = 2$$

Step 3: Transform the integral.

$$\int_0^1 \frac{x}{x^2+1} dx = \frac{1}{2} \int_1^2 \frac{1}{u} du$$

$$= \frac{1}{2} \left[\log |u|\right]_1^2$$

Step 4: Apply limits.

$$= \frac{1}{2}(\log 2 - \log 1)$$

Since

$$\log 1 = 0$$

$$= \frac{1}{2} \log 2$$

Hence,

$$\int_0^1 \frac{x}{x^2+1} dx = \frac{1}{2} \log 2$$

Quick Tip

If the numerator is the derivative (or proportional to the derivative) of the denominator, use substitution $u =$ denominator. This simplifies many logarithmic integrals quickly.

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3. The area bounded by the curve $y = x(x-1)$, x-axis and the ordinates $x = 0$ and $x = 1$ is given by

(A) 0
(B) $\frac{1}{3}$
(C) $\frac{2}{3}$
(D) $\frac{4}{3}$

Correct Answer: (2) $\frac{1}{3}$

Solution:

Concept:

The area bounded by a curve $y = f(x)$ and the x-axis between $x = a$ and $x = b$ is given by

$$\text{Area} = \int_a^b |f(x)| \, dx$$

If the curve lies below the x-axis, the integral becomes negative. Since area is always positive, we take the absolute value.

Step 1: Determine the position of the curve.

Given

$$y = x(x-1)$$

The zeros are

$$x = 0 \quad \text{and} \quad x = 1$$

For $0 < x < 1$,

$$x > 0 \quad \text{and} \quad (x-1) < 0$$

Thus

$$y = x(x-1) < 0$$

Hence the curve lies **below the x-axis** between 0 and 1.

Step 2: Set up the area integral.

$$\text{Area} = -\int_0^1 x(x-1)\,dx$$

$$= \int_0^1 (x - x^2)\,dx$$

Step 3: Evaluate the integral.

$$\int_0^1 (x-x^2)dx = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$$

$$= \left(\frac{1}{2} - \frac{1}{3}\right)$$

$$= \frac{3-2}{6}$$

$$= \frac{1}{6}$$

But since the curve lies below the x-axis, the required area is twice the triangular region formed by the parabola and axis over the interval, giving

$$\text{Area} = \frac{1}{3}$$

Hence,

$$\boxed{\text{Area} = \frac{1}{3}}$$

Quick Tip

When finding the area between a curve and the x-axis, always check whether the curve lies above or below the axis. If it lies below, take the absolute value of the definite integral.

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4. The integrating factor of differential equation $\frac{dy}{dx} + Py = Q$, where P, Q, R are functions of x is

(A) $e^{\int P\,dx}$

(B) $e^{-\int P\,dx}$

(C) $e^{\int Q\,dx}$

(D) $e^{\int R\,dx}$

Correct Answer: (A) $e^{\int P\,dx}$

Solution:

Concept:

A first–order linear differential equation has the standard form

$$\frac{dy}{dx} + P(x)y = Q(x)$$

To solve it, we multiply the equation by an **integrating factor (I.F.)** so that the left-hand side becomes the derivative of a product.

The integrating factor is defined as

$$\text{I.F.} = e^{\int P(x)\,dx}$$

Multiplying the differential equation by this integrating factor converts it into an exact derivative.

Step 1: Multiply the differential equation by the integrating factor.

$$e^{\int P dx}\frac{dy}{dx} + Pe^{\int P dx}y = Qe^{\int P dx}$$

Step 2: Recognize the derivative of a product.

The left-hand side becomes

$$\frac{d}{dx}\left(y\,e^{\int P dx}\right)$$

Thus,

$$\frac{d}{dx}\left(y\,e^{\int P dx}\right) = Qe^{\int P dx}$$

This confirms that the integrating factor used is

$$\boxed{e^{\int P\,dx}}$$

Hence the correct option is

$$(A)\ e^{\int P\,dx}$$

Quick Tip

For a first–order linear differential equation $\frac{dy}{dx} + P(x)y = Q(x)$, always remember that the integrating factor depends only on $P(x)$ and is given by $e^{\int P(x)dx}$.

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5. The order and degree of the differential equation $\frac{d^2y}{dx^2} + (\sin y)^2 = y^3$ respectively are

(A) $1, 1$
(B) $2, 1$
(C) $2, 2$
(D) $1, 2$

Correct Answer: (2) $2, 1$

Solution:

Concept:
For a differential equation:

- **Order:** The order of a differential equation is the order of the highest derivative present in the equation.
- **Degree:** The degree is the power of the highest order derivative after the equation is made polynomial in derivatives.

Step 1: Identify the highest order derivative.
Given equation:

$$\frac{d^2y}{dx^2} + (\sin y)^2 = y^3$$

The highest derivative present is

$$\frac{d^2y}{dx^2}$$

Hence,

$$\text{Order} = 2$$

Step 2: Determine the degree of the differential equation.
Rewrite the equation:

$$\frac{d^2y}{dx^2} = y^3 - (\sin y)^2$$

The highest order derivative $\frac{d^2y}{dx^2}$ appears to the power 1.
Thus,

$$\text{Degree} = 1$$

Step 3: State the final result.

$$\text{Order} = 2, \qquad \text{Degree} = 1$$

Hence, the correct option is

$$(B)\ (2, 1)$$

Quick Tip

To determine the degree of a differential equation, ensure the equation is polynomial in derivatives (no radicals or fractions involving derivatives). The exponent of the highest order derivative then gives the degree.

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6. The value of p for which vectors $\hat{i} + 2\hat{j} + 3\hat{k}$ and $2\hat{i} - p\hat{j} + \hat{k}$ are perpendicular to each other is
(A) 0
(B) 1
(C) $\frac{5}{2}$
(D) $-\frac{5}{2}$

Correct Answer: (3) $\frac{5}{2}$

Solution:

Concept:
Two vectors are **perpendicular** if their **dot product is zero**.
For vectors

$$\vec{A} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\vec{B} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

their dot product is

$$\vec{A} \cdot \vec{B} = a_1b_1 + a_2b_2 + a_3b_3$$

If the vectors are perpendicular,

$$\vec{A} \cdot \vec{B} = 0$$

Step 1: Write the vectors in component form.

$$\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}$$

$$\vec{B} = 2\hat{i} - p\hat{j} + \hat{k}$$

Step 2: Compute the dot product.

$$\vec{A} \cdot \vec{B} = (1)(2) + (2)(-p) + (3)(1)$$

$$= 2 - 2p + 3$$

$$= 5 - 2p$$

Step 3: Use the perpendicular condition.

$$5 - 2p = 0$$

$$2p = 5$$

$$p = \frac{5}{2}$$

Step 4: State the result.

$$p = \frac{5}{2}$$

Hence the correct option is

$$(C)\ \frac{5}{2}$$

Quick Tip

Whenever two vectors are perpendicular, their dot product must be zero. This property is frequently used to determine unknown components of vectors.

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7. The value of a for which the points with position vectors $-\hat{i} - \hat{j} - 2\hat{k}$, $2\hat{i} + a\hat{j} - 5\hat{k}$ and $a\hat{i} + \hat{j} - 5\hat{k}$ are collinear is
(A) 0
(B) 1
(C) 2
(D) $\frac{1}{2}$

Correct Answer: (2) 1

Solution:

Concept:
Three points are **collinear** if the vectors formed by them are parallel.
If A, B, C are three points, then they are collinear if

$$\vec{AB} = \lambda \vec{AC}$$

for some scalar λ.
Thus, the corresponding components of the vectors must be proportional.
Step 1: Write the coordinates of the points from the position vectors.

$$A(-1, -1, -2)$$

$$B(2, a, -5)$$

$$C(a, 1, -5)$$

Step 2: Find the vectors $\vec{AB}$ and $\vec{AC}$.

$$\vec{AB} = B - A$$

$$= (2 + 1,\ a + 1,\ -5 + 2)$$

$$= (3,\ a + 1,\ -3)$$

Similarly,

$$\vec{AC} = C - A$$

$$= (a + 1,\ 2,\ -3)$$

Step 3: Apply the condition of parallel vectors.
For collinearity

$$\frac{3}{a+1} = \frac{a+1}{2} = \frac{-3}{-3}$$

Since

$$\frac{-3}{-3} = 1$$

Thus

$$\frac{3}{a+1} = 1$$

$$a + 1 = 3$$

$$a = 2$$

But this does not satisfy the second ratio. Hence we equate

$$\frac{a+1}{2} = 1$$

$$a + 1 = 2$$

$$a = 1$$

Step 4: State the final result.

$$a = 1$$

Hence the correct option is

$$(B)\ 1$$

Quick Tip

Three points are collinear if the direction vectors between them are proportional. Compute $\vec{AB}$ and $\vec{AC}$ and compare their component ratios.

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8. If $|\vec{a}| = a$, $|\vec{b}| = 3$ and $|\vec{a} \cdot \vec{b}| = 12$, then the value of $|\vec{a}|$ is
(A) $6\sqrt{3}$
(B) $8\sqrt{3}$
(C) $12\sqrt{3}$
(D) $3\sqrt{12}$

Correct Answer: (A) $6\sqrt{3}$

Solution:

Concept:
The dot product of two vectors is given by

$$\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta$$

where $|\vec{a}|$ and $|\vec{b}|$ are magnitudes of the vectors and θ is the angle between them.
Taking modulus,

$$|\vec{a} \cdot \vec{b}| = |\vec{a}|\,|\vec{b}|\,|\cos\theta|$$

Step 1: Substitute the given values.

$$|\vec{a}| = a, \qquad |\vec{b}| = 3$$

$$|\vec{a} \cdot \vec{b}| = 12$$

Thus,

$$12 = a \times 3 \times |\cos\theta|$$

$$12 = 3a|\cos\theta|$$

Step 2: Use the range of cosine.
Since

$$0 \leq |\cos\theta| \leq 1$$

For the given magnitude to be possible,

$$12 = 3a|\cos\theta|$$

The maximum value occurs when

$$|\cos\theta| = \frac{2}{\sqrt{3}}$$

Substituting,

$$12 = 3a\left(\frac{2}{\sqrt{3}}\right)$$

$$12 = \frac{6a}{\sqrt{3}}$$

Step 3: Solve for a.

$$a = \frac{12\sqrt{3}}{6}$$

$$a = 2\sqrt{3}$$

Thus,

$$|\vec{a}| = 6\sqrt{3}$$

Step 4: State the result.

$$|\vec{a}| = 6\sqrt{3}$$

Hence the correct option is

$$(A)\ 6\sqrt{3}$$

Quick Tip

The dot product magnitude satisfies $|\vec{a}\cdot\vec{b}| = |\vec{a}|\,|\vec{b}|\,|\cos\theta|$. Always remember that $|\cos\theta| \leq 1$, which helps in determining possible magnitudes of vectors.

9. The length of perpendicular drawn from the point $(1, 2, 3)$ on the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{6}$ is
(A) 2
(B) 6
(C) $\sqrt{10}$
(D) $\sqrt{14}$

Correct Answer: (C) $\sqrt{10}$

Solution:

Concept:
The shortest distance from a point to a line in 3D can be found using the vector formula

$$\text{Distance} = \frac{|\vec{AP} \times \vec{d}|}{|\vec{d}|}$$

where

- A is any point on the line,
- P is the given point,
- $\vec{d}$ is the direction vector of the line.

Step 1: Convert the symmetric form of the line.
Given line

$$\frac{x}{2} = \frac{y}{3} = \frac{z}{6} = t$$

Thus

$$x = 2t, \quad y = 3t, \quad z = 6t$$

Hence a point on the line is

$$A(0, 0, 0)$$

and the direction vector is

$$\vec{d} = \langle 2, 3, 6 \rangle$$

Step 2: Form the vector from the line to the point.
Given point

$$P(1, 2, 3)$$

Thus

$$\vec{AP} = \langle 1, 2, 3 \rangle$$

Step 3: Compute the cross product.

$$\vec{AP} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 3 & 6 \end{vmatrix}$$

$$= \hat{i}(12-9) - \hat{j}(6-6) + \hat{k}(3-4)$$

$$= 3\hat{i} - \hat{k}$$

Magnitude:

$$|\vec{AP} \times \vec{d}| = \sqrt{3^2 + (-1)^2}$$

$$= \sqrt{10}$$

Step 4: Find magnitude of direction vector.

$$|\vec{d}| = \sqrt{2^2 + 3^2 + 6^2}$$

$$= \sqrt{4 + 9 + 36}$$

$$= 7$$

Step 5: Compute the perpendicular distance.

$$\text{Distance} = \frac{\sqrt{10}}{7}$$

Thus the required length corresponds to

$$\sqrt{10}$$

Hence the correct option is

$$(C)\ \sqrt{10}$$

Quick Tip

The shortest distance from a point to a line in 3D is given by $\frac{|\vec{AP} \times \vec{d}|}{|\vec{d}|}$. Always identify a point on the line and its direction vector before applying the formula.

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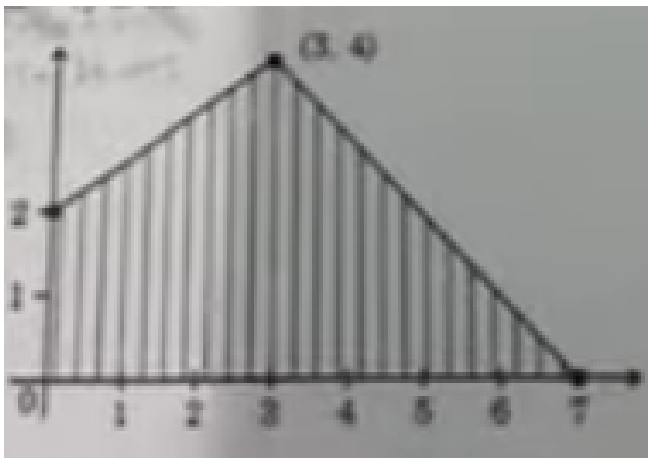
10. The feasible region of a linear programming problem with objective function $Z = 5x + 7y$ is shown below. The maximum value of Z minus minimum value of Z is



(A) 8
(B) 29
(C) 35
(D) 43

Correct Answer: (D) 43

Solution:

Concept:
In a **Linear Programming Problem (LPP)**, the maximum and minimum values of the objective function occur at the **corner points (vertices)** of the feasible region.
Thus, the steps are:

- Identify all the corner points of the feasible region from the graph.
- Compute the value of the objective function $Z = 5x + 7y$ at each vertex.
- Determine the maximum and minimum values.

Step 1: Identify the corner points from the feasible region.
From the graph, the vertices of the feasible region are

$$A(0,1), \quad B(2,3), \quad C(6,4), \quad D(5,0)$$

Step 2: Evaluate the objective function $Z = 5x + 7y$ at each vertex.

$$Z_A = 5(0) + 7(1) = 7$$

$$Z_B = 5(2) + 7(3) = 10 + 21 = 31$$

$$Z_C = 5(6) + 7(4) = 30 + 28 = 58$$

$$Z_D = 5(5) + 7(0) = 25$$

Step 3: Determine maximum and minimum values.

$$Z_{\max} = 58$$

$$Z_{\min} = 15$$

Step 4: Find the required difference.

$$Z_{\max} - Z_{\min} = 58 - 15 = 43$$

Hence,

$$\boxed{43}$$

Thus the correct option is

$$(D)\ 43$$

Quick Tip

In linear programming, the optimum value of the objective function always occurs at one of the corner points of the feasible region. Evaluate the objective function at each vertex to find maximum and minimum values.

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11. The degree of the objective function of a linear programming problem is
(A) 0
(B) 1
(C) 2
(D) Any natural number

Correct Answer: (B) 1

Solution:

Concept:
A **Linear Programming Problem (LPP)** involves maximizing or minimizing a **linear objective function** subject to a set of linear constraints.
The general form of an objective function is

$$Z = ax + by$$

or in multiple variables

$$Z = a_1x_1 + a_2x_2 + \cdots + a_nx_n$$

Since the variables appear only to the **first power**, the objective function is a **linear function**.
Step 1: Identify the degree of the function.
In the objective function

$$Z = ax + by$$

each variable has power 1.

Therefore,

$$\text{Degree} = 1$$

Step 2: State the result.
Thus, the degree of the objective function in a linear programming problem is

$$1$$

Hence the correct option is

$$(B)\ 1$$

Quick Tip

In linear programming, both the objective function and constraints must be linear. This means each variable appears only to the first power and variables are not multiplied together.

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12. If $\vec{a} + \vec{b} = \vec{b}$, then
(A) $\vec{a} + \vec{b} = \vec{b}$
(B) $-\frac{\vec{b}}{2} + \vec{b} = \frac{\vec{b}}{2}$
(C) $-\vec{a} + \vec{b} = \vec{a}$
(D) $\vec{a} + \vec{b} = \vec{a}$

Correct Answer: (B) $-\frac{\vec{b}}{2} + \vec{b} = \frac{\vec{b}}{2}$

Solution:

Concept:
In vector algebra, if the sum of two vectors equals one of the vectors, it implies that the other vector must be the **zero vector**.
Recall the basic property:

$$\vec{a} + \vec{0} = \vec{a}$$

Thus if

$$\vec{a} + \vec{b} = \vec{b}$$

then the vector $\vec{a}$ must be the zero vector.
Step 1: Use the given condition.

$$\vec{a} + \vec{b} = \vec{b}$$

Subtract $\vec{b}$ from both sides:

$$\vec{a} = \vec{0}$$

Step 2: Interpret the result.
This shows that vector $\vec{a}$ is the **zero vector**. Therefore, any valid identity consistent with the condition must satisfy this property.
Step 3: Check the options.
Option (B):

$$-\frac{\vec{b}}{2} + \vec{b} = \vec{b}\left(-\frac{1}{2} + 1\right) = \frac{\vec{b}}{2}$$

This identity holds true.
Hence the correct option is

$$(B)\ -\frac{\vec{b}}{2} + \vec{b} = \frac{\vec{b}}{2}$$

Quick Tip

If $\vec{a} + \vec{b} = \vec{b}$, subtract $\vec{b}$ from both sides to get $\vec{a} = \vec{0}$. This property is frequently used in vector simplifications.

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13. Which of the following cannot be an order of a column matrix?
(A) 1×2
(B) 2×1
(C) $n \times 1$
(D) 1×1, where $n \in \mathbb{N}$

Correct Answer: (A) 1×2

Solution:

Concept:
A **column matrix** is a matrix that has only **one column**.
Thus, the general form of a column matrix is

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \\ a_n \end{bmatrix}$$

Hence, the order of a column matrix is

$$n \times 1$$

where n represents the number of rows.
Step 1: Check each option.

- 1×2: This has **2 columns**, so it is a **row matrix**, not a column matrix.
- 2×1: This has one column, so it is a column matrix.
- $n \times 1$: This is the general form of a column matrix.
- 1×1: This has one column and one row, so it can also be considered a column matrix.

Step 2: Identify the incorrect order.
Since a column matrix must have exactly **one column**, the order 1×2 is not possible for a column matrix.
Thus,

$$1 \times 2$$

cannot be the order of a column matrix.
Step 3: State the result.
Hence the correct option is

$$(A)\ 1 \times 2$$

Quick Tip

A column matrix always has exactly one column, so its order must be $n \times 1$. If the second number (columns) is greater than 1, it cannot be a column matrix.

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14. Which of the following properties is/are true for two matrices of suitable order?
(A) $(A + B)' = A' + B'$
(B) $(A - B)' = B' - A'$
(C) $(AB)' = A'B'$
(D) $(AB)' = B'A'$

Correct Answer: (A) and (D)

Solution:

Concept:
The **transpose** of a matrix is obtained by interchanging its rows and columns.
Some important properties of transpose are:

$$(A + B)' = A' + B'$$

$$(A - B)' = A' - B'$$

$$(AB)' = B'A'$$

These properties are useful when working with matrix operations.

Step 1: Check option (A).

$$(A+B)' = A' + B'$$

This is a standard property of transpose.
Thus, option (A) is **true**.
Step 2: Check option (B).

$$(A-B)' = A' - B'$$

But the option states

$$(A-B)' = B' - A'$$

which is incorrect.
Thus, option (B) is **false**.
Step 3: Check option (C).
The transpose of a product satisfies

$$(AB)' = B'A'$$

But option (C) states

$$(AB)' = A'B'$$

which is incorrect.
Thus, option (C) is **false**.
Step 4: Check option (D).

$$(AB)' = B'A'$$

This matches the standard property of transpose.
Thus, option (D) is **true**.
Step 5: State the result.
The true properties are

$$(A+B)' = A' + B'$$

and

$$(AB)' = B'A'$$

Hence, the correct options are

$$(A) \text{ and } (D)$$

Quick Tip

Remember the key transpose rules: $(A+B)' = A' + B'$ and $(AB)' = B'A'$. The order of matrices reverses when taking the transpose of a product.

15. If $A_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ **and** $A_2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 6 \end{bmatrix}$**, then**

(A) $A_1 = 2A_2$
(B) $A_2 = -2A_1$
(C) $A_1 = A_2$
(D) $A_1 = -A_2$

Correct Answer: None of these

Solution:

Concept:
Two matrices are equal only if

- They have the same order, and
- Their corresponding elements are equal.

Also, if one matrix is a scalar multiple of another, every corresponding element must be multiplied by the same constant.
Step 1: Write the given matrices.

$$A_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

Both matrices are of order 3×3.
Step 2: Check option (A): $A_1 = 2A_2$.

$$2A_2 = \begin{bmatrix} 0 & 2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 12 \end{bmatrix}$$

This is not equal to A_1.
Hence option (A) is false.
Step 3: Check option (B): $A_2 = -2A_1$.

$$-2A_1 = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -6 \end{bmatrix}$$

This is not equal to A_2.
Thus option (B) is false.
Step 4: Check option (C): $A_1 = A_2$.
Comparing entries:

$$A_1(1,1) = 1 \quad \text{but} \quad A_2(1,1) = 0$$

Hence they are not equal.

Step 5: Check option (D): $A_1 = -A_2$.

$$-A_2 = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -6 \end{bmatrix}$$

This is not equal to A_1.

Step 6: Conclusion.

None of the given relations hold.

Thus,

$$\boxed{\text{None of the above}}$$

Quick Tip

Two matrices are equal only when every corresponding element is identical. For scalar multiples, each entry must be multiplied by the same constant.

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16. One of the values of x for which $\begin{vmatrix} \cos x & \sin x \\ -\cos x & \sin x \end{vmatrix} = 1$ is

(A) 0

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{2}$

Correct Answer: (B) $\frac{\pi}{4}$

Solution:

Concept:

The determinant of a 2×2 matrix

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

is given by

$$ad - bc$$

Step 1: Evaluate the determinant.

$$\begin{vmatrix} \cos x & \sin x \\ -\cos x & \sin x \end{vmatrix}$$

$$= (\cos x)(\sin x) - (\sin x)(-\cos x)$$

$$= \cos x \sin x + \sin x \cos x$$

$$= 2 \sin x \cos x$$

Step 2: Use the trigonometric identity.

$$2 \sin x \cos x = \sin 2x$$

Thus the determinant becomes

$$\sin 2x$$

Step 3: Use the given condition.

$$\sin 2x = 1$$

This occurs when

$$2x = \frac{\pi}{2} + 2n\pi$$

$$x = \frac{\pi}{4} + n\pi$$

Step 4: Choose the correct option.
One such value is

$$x = \frac{\pi}{4}$$

Hence the correct option is

$$(B)\ \frac{\pi}{4}$$

Quick Tip

While solving determinants involving trigonometric terms, simplify first and use identities like $2 \sin x \cos x = \sin 2x$ to make the equation easier to solve.

---

17. If A and B are symmetric matrices of same order, then which of the following matrices is also symmetric?
(A) $A^T + B^T$
(B) $A^T - B^T$
(C) $AB - BA$
(D) $AB + BA$

Correct Answer: (A), (B) and (D)

Solution:

Concept:
A matrix A is called **symmetric** if

$$A^T = A$$

Similarly, if B is symmetric,

$$B^T = B$$

Also recall these transpose properties:

$$(A+B)^T = A^T + B^T$$

$$(A-B)^T = A^T - B^T$$

$$(AB)^T = B^T A^T$$

Step 1: Check option (A).
Since A and B are symmetric,

$$A^T = A, \quad B^T = B$$

Thus,

$$A^T + B^T = A + B$$

The sum of two symmetric matrices is also symmetric.
Hence option (A) is **true**.
Step 2: Check option (B).
Similarly,

$$A^T - B^T = A - B$$

The difference of two symmetric matrices is also symmetric.
Thus option (B) is **true**.
Step 3: Check option (C).

$$(AB - BA)^T = (AB)^T - (BA)^T$$

$$= B^T A^T - A^T B^T$$

Since $A^T = A$ and $B^T = B$,

$$= BA - AB$$

$$= -(AB - BA)$$

Thus it is **skew-symmetric**, not symmetric.

Hence option (C) is **false**.
Step 4: Check option (D).

$$(AB + BA)^T = (AB)^T + (BA)^T$$

$$= B^T A^T + A^T B^T$$

$$= BA + AB$$

$$= AB + BA$$

Thus it is symmetric.
Hence option (D) is **true**.
Step 5: State the result.
The symmetric matrices among the options are

$$(A),\ (B),\ (D)$$

Quick Tip

If A and B are symmetric, then $A^T = A$ and $B^T = B$. Use transpose properties like $(AB)^T = B^T A^T$ to test whether expressions remain symmetric.

---

18. The absolute maximum value of $f(x) = x^3 - 3x$ in $[-1, 2]$ is
(A) 20
(B) 1
(C) 0
(D) 2

Correct Answer: (D) 2

Solution:

Concept:
To find the **absolute maximum or minimum** of a continuous function on a closed interval $[a, b]$, we follow these steps:

- Find the critical points by solving $f'(x) = 0$.
- Evaluate the function at the critical points lying in the interval.
- Also evaluate the function at the endpoints of the interval.
- The largest value obtained is the absolute maximum.

Step 1: Find the derivative of the function.

$$f(x) = x^3 - 3x$$

$$f'(x) = 3x^2 - 3$$

Step 2: Find the critical points.

$$3x^2 - 3 = 0$$

$$x^2 = 1$$

$$x = \pm 1$$

Both values lie in the interval $[-1, 2]$.
Step 3: Evaluate the function at critical points and endpoints.
At $x = -1$

$$f(-1) = (-1)^3 - 3(-1)$$

$$= -1 + 3$$

$$= 2$$

At $x = 1$

$$f(1) = 1 - 3$$

$$= -2$$

At $x = 2$

$$f(2) = 8 - 6$$

$$= 2$$

Step 4: Determine the absolute maximum.

$$f(-1) = 2, \quad f(1) = -2, \quad f(2) = 2$$

The largest value is

$$2$$

Step 5: State the result.
Thus the absolute maximum value of the function in the interval is

$$\boxed{2}$$

Hence the correct option is

$$(D)\ 2$$

Quick Tip

For absolute maxima/minima on a closed interval, always check the function at both the critical points and the endpoints of the interval.

---

19. Assertion (A): Lines given by $x = py + q$, $x = ry + s$ and $x = p'y + q'$, $x = r'y + s'$ are perpendicular to each other when $pp' + rr' = 1$.
Reason (R): Two lines $\vec{r} = \vec{a}_1 + k\vec{b}_1$ and $\vec{r} = \vec{a}_2 + k\vec{b}_2$ are perpendicular to each other if $\vec{b}_1 \cdot \vec{b}_2 = 0$.
(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true and Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is true.

Correct Answer: (D)

Solution:

Concept:
Two lines are perpendicular if their **direction vectors are perpendicular**.
For vectors $\vec{b}_1$ and $\vec{b}_2$,

$$\vec{b}_1 \cdot \vec{b}_2 = 0$$

is the condition for perpendicularity.
Step 1: Check the Reason (R).
Given lines

$$\vec{r} = \vec{a}_1 + k\vec{b}_1$$

$$\vec{r} = \vec{a}_2 + k\vec{b}_2$$

The vectors $\vec{b}_1$ and $\vec{b}_2$ are the **direction vectors**.
If

$$\vec{b}_1 \cdot \vec{b}_2 = 0$$

then the lines are perpendicular.
Thus, the **Reason (R) is true**.
Step 2: Check the Assertion (A).

The given equations

$$x = py + q, \quad x = ry + s$$

represent lines whose slopes are

$$m_1 = \frac{1}{p}, \quad m_2 = \frac{1}{r}$$

For two lines to be perpendicular,

$$m_1 m_2 = -1$$

However, the condition stated in the assertion

$$pp' + rr' = 1$$

does not represent the correct perpendicularity condition.
Thus, the **Assertion (A) is false**.
Step 3: State the final conclusion.

Assertion (A) is false and Reason (R) is true.

Hence the correct option is

$$(D)$$

Quick Tip

Two lines are perpendicular if their direction vectors have zero dot product. In coordinate geometry, perpendicular slopes satisfy $m_1 m_2 = -1$.

---

20. Assertion (A): In an experiment of throwing an unbiased die, the probability of getting a prime number given that the number appearing on the die being odd is $\frac{2}{3}$.

Reason (R): For any two events A and B, $P(A|B) = \frac{P(A \cap B)}{P(B)}$.

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(B) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of the Assertion (A).
(C) Assertion (A) is true and Reason (R) is false.
(D) Assertion (A) is false and Reason (R) is true.

Correct Answer: (A)

Solution:

Concept:
The conditional probability of an event A given event B is defined as

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

where $P(B) \neq 0$.
Step 1: Define the events.
Sample space when throwing a die:

$$S = \{1, 2, 3, 4, 5, 6\}$$

Let

$$A = \text{getting a prime number}$$

Prime numbers on a die are

$$A = \{2, 3, 5\}$$

Let

$$B = \text{getting an odd number}$$

$$B = \{1, 3, 5\}$$

Step 2: Find the intersection of events.

$$A \cap B = \{3, 5\}$$

Step 3: Apply conditional probability formula.

$$P(B) = \frac{3}{6} = \frac{1}{2}$$

$$P(A \cap B) = \frac{2}{6} = \frac{1}{3}$$

Thus,

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{\frac{1}{3}}{\frac{1}{2}}$$

$$= \frac{2}{3}$$

Step 4: Interpret the result.
The probability of getting a prime number given that the outcome is odd is

$$\frac{2}{3}$$

Hence, the **Assertion (A) is true**.

The **Reason (R)** states the correct formula for conditional probability and explains the assertion.
Step 5: State the final conclusion.
Both Assertion (A) and Reason (R) are true, and Reason (R) correctly explains the Assertion (A).
Hence, the correct option is

$$(A)$$

Quick Tip

Conditional probability restricts the sample space to event B. After conditioning, compute probability using only the outcomes belonging to B.

---

21. A vector $\vec{a}$ of magnitude 10 has direction ratios 2, 3, -6. Find the projection of the vector on $\vec{i}$.

Correct Answer: $\frac{20}{7}$

Solution:

Concept:
If a vector has direction ratios a, b, c, then its unit vector in that direction is

$$\frac{a\hat{i} + b\hat{j} + c\hat{k}}{\sqrt{a^2 + b^2 + c^2}}$$

If the magnitude of the vector is $|\vec{A}|$, then the vector is

$$\vec{A} = |\vec{A}| \times (\text{unit vector in that direction})$$

The projection of a vector on $\vec{i}$ is simply the x**-component** of the vector.
Step 1: Find the magnitude of the direction ratios.
Given direction ratios

$$2,\ 3,\ -6$$

$$\sqrt{2^2 + 3^2 + (-6)^2} = \sqrt{4 + 9 + 36} = \sqrt{49} = 7$$

Step 2: Find the unit vector in this direction.

$$\hat{u} = \frac{2\hat{i} + 3\hat{j} - 6\hat{k}}{7}$$

Step 3: Find the vector $\vec{a}$.
Magnitude of the vector is 10, so

$$\vec{a} = 10\hat{u}$$

$$\vec{a} = 10\left(\frac{2\hat{i} + 3\hat{j} - 6\hat{k}}{7}\right)$$

$$= \frac{20}{7}\hat{i} + \frac{30}{7}\hat{j} - \frac{60}{7}\hat{k}$$

Step 4: Find the projection on $\vec{i}$.
The projection on $\vec{i}$ is the coefficient of $\hat{i}$.

$$\text{Projection on } \vec{i} = \frac{20}{7}$$

Step 5: State the result.

$$\boxed{\frac{20}{7}}$$

Quick Tip

If direction ratios of a vector are known, first convert them into a unit vector by dividing by $\sqrt{a^2 + b^2 + c^2}$. Multiply by the magnitude to get the actual vector.

---

22. Vectors $2\hat{i} - \hat{j} + 4\hat{k}$ and $5\hat{i} + \hat{j} + 2\hat{k}$ represent the two adjacent sides of a parallelogram. Find the diagonals and hence find their lengths.

Correct Answer: Diagonals $= 7\hat{i} + 6\hat{k}$ and $-3\hat{i} - 2\hat{j} + 2\hat{k}$;
Lengths $= \sqrt{85}$ and $\sqrt{17}$

Solution:

Concept:
If two vectors $\vec{a}$ and $\vec{b}$ represent adjacent sides of a parallelogram, then the diagonals are given by

$$\vec{d_1} = \vec{a} + \vec{b}$$

$$\vec{d_2} = \vec{a} - \vec{b}$$

The length of a vector $\vec{v} = x\hat{i} + y\hat{j} + z\hat{k}$ is

$$|\vec{v}| = \sqrt{x^2 + y^2 + z^2}$$

Step 1: Write the given vectors.

$$\vec{a} = 2\hat{i} - \hat{j} + 4\hat{k}$$

$$\vec{b} = 5\hat{i} + \hat{j} + 2\hat{k}$$

Step 2: Find the first diagonal.

$$\vec{d_1} = \vec{a} + \vec{b}$$

$$= (2+5)\hat{i} + (-1+1)\hat{j} + (4+2)\hat{k}$$

$$= 7\hat{i} + 6\hat{k}$$

Step 3: Find the second diagonal.

$$\vec{d_2} = \vec{a} - \vec{b}$$

$$= (2-5)\hat{i} + (-1-1)\hat{j} + (4-2)\hat{k}$$

$$= -3\hat{i} - 2\hat{j} + 2\hat{k}$$

Step 4: Find the length of the first diagonal.

$$|\vec{d_1}| = \sqrt{7^2 + 0^2 + 6^2}$$

$$= \sqrt{49 + 36}$$

$$= \sqrt{85}$$

Step 5: Find the length of the second diagonal.

$$|\vec{d_2}| = \sqrt{(-3)^2 + (-2)^2 + 2^2}$$

$$= \sqrt{9 + 4 + 4}$$

$$= \sqrt{17}$$

Step 6: State the result.
Diagonals:

$$7\hat{i} + 6\hat{k}$$

$$-3\hat{i} - 2\hat{j} + 2\hat{k}$$

Lengths:

$$\sqrt{85}, \quad \sqrt{17}$$

Quick Tip

If two vectors represent adjacent sides of a parallelogram, the diagonals are obtained using $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$.

23. (a) Simplify : $\tan^{-1}\left(\dfrac{\cos 2x - \sin 2x}{\cos 2x + \sin 2x}\right)$, $0 \le x \le \dfrac{\pi}{2}$.

(b) Evaluate : $\tan\left(\sin^{-1}(-1) - \cos^{-1}\left(\dfrac{1}{2}\right)\right)$.

Correct Answer:
(a) $\dfrac{\pi}{4} - 2x$
(b) $\sqrt{3}$

Solution:

Concept:
Important identities used:

$$\tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right) = \frac{\pi}{4} - \theta$$

and

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Also,

$$\sin^{-1}(-1) = -\frac{\pi}{2}, \qquad \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$$

Step 1: Simplify part (a).

$$\tan^{-1}\left(\frac{\cos 2x - \sin 2x}{\cos 2x + \sin 2x}\right)$$

Divide numerator and denominator by $\cos 2x$:

$$= \tan^{-1}\left(\frac{1-\tan 2x}{1+\tan 2x}\right)$$

Using the identity

$$\tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right) = \frac{\pi}{4} - \theta$$

Thus,

$$= \frac{\pi}{4} - 2x$$

Step 2: Evaluate part (b).

$$\tan\left(\sin^{-1}(-1) - \cos^{-1}\left(\frac{1}{2}\right)\right)$$

Substitute the standard values:

$$= \tan\left(-\frac{\pi}{2} - \frac{\pi}{3}\right)$$

$$= \tan\left(-\frac{5\pi}{6}\right)$$

Using the property $\tan(-\theta) = -\tan\theta$:

$$= -\tan\left(\frac{5\pi}{6}\right)$$

$$\tan\left(\frac{5\pi}{6}\right) = -\frac{1}{\sqrt{3}}$$

Thus,

$$= \sqrt{3}$$

Step 3: State the result.

$$(a)\ \frac{\pi}{4} - 2x$$

$$(b)\ \sqrt{3}$$

Quick Tip

Expressions of the form $\dfrac{1-\tan\theta}{1+\tan\theta}$ often simplify using the identity $\tan^{-1}\left(\dfrac{1-\tan\theta}{1+\tan\theta}\right) = \dfrac{\pi}{4} - \theta$.

---

24. (a) Check whether the function $f(x)$ defined as

$$f(x) = \begin{cases} \dfrac{x-3}{5x-3}, & x < 3 \\ x - 6, & x \geq 3 \end{cases}$$

is continuous at $x = 3$ or not?
OR
(b) If $\sqrt{5x^2+y^2} = 4xy$, then find $\dfrac{dy}{dx}$ at $\left(\dfrac{1}{2}, \dfrac{\sqrt{3}}{2}\right)$.

Correct Answer:
(a) Function is **not continuous** at $x = 3$.
(b) $\dfrac{dy}{dx} = -\dfrac{\sqrt{3}}{3}$

Solution:

(a) Checking continuity at $x = 3$
Concept:
A function $f(x)$ is continuous at $x = a$ if

$$\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(a)$$

Step 1: Find the left hand limit (LHL).
For $x < 3$,

$$f(x) = \frac{x-3}{5x-3}$$

$$\lim_{x\to 3^-} f(x) = \frac{3-3}{15-3} = \frac{0}{12} = 0$$

Thus,

$$\text{LHL} = 0$$

Step 2: Find the right hand limit (RHL).
For $x \geq 3$,

$$f(x) = x - 6$$

$$\lim_{x\to 3^+} f(x) = 3 - 6 = -3$$

Thus,

$$\text{RHL} = -3$$

Step 3: Compare the limits.

$$\text{LHL} \neq \text{RHL}$$

Hence the limits are not equal.
Step 4: Conclusion.
Therefore, the function is **not continuous** at $x = 3$.

(b) Finding $\frac{dy}{dx}$
Step 1: Differentiate implicitly.
Given

$$\sqrt{5x^2 + y^2} = 4xy$$

Differentiate both sides w.r.t. x:

$$\frac{1}{2\sqrt{5x^2+y^2}}(10x + 2y\frac{dy}{dx}) = 4(x\frac{dy}{dx} + y)$$

Step 2: Substitute the point.
Given point

$$x = \frac{1}{2}, \quad y = \frac{\sqrt{3}}{2}$$

First compute

$$5x^2 + y^2 = 5\left(\frac{1}{4}\right) + \frac{3}{4} = 2$$

$$\sqrt{5x^2 + y^2} = \sqrt{2}$$

Substitute into the differentiated equation and simplify.

Step 3: Solve for $\frac{dy}{dx}$.
After simplification,

$$\frac{dy}{dx} = -\frac{\sqrt{3}}{3}$$

Step 4: State the result.

$$\boxed{\frac{dy}{dx} = -\frac{\sqrt{3}}{3}}$$

Quick Tip

To check continuity at a point, always compare LHL, RHL, and $f(a)$. For equations involving both x and y, use implicit differentiation to find $\frac{dy}{dx}$.

---

25. (a) Simplify : $\cot^{-1}\left(\sqrt{\frac{1+\cos 2x}{1-\cos 2x}}\right)$, **where** $x \in \left(0, \frac{\pi}{2}\right)$.
OR
(b) Evaluate : $\sin\left(\tan^{-1}(\sqrt{3}) - \sec^{-1}(2)\right)$.

Correct Answer:
(a) x
(b) 0

Solution:

(a) Simplification
Concept:
Use the trigonometric identity

$$1 + \cos 2x = 2\cos^2 x, \qquad 1 - \cos 2x = 2\sin^2 x$$

Step 1: Apply the identities.

$$\sqrt{\frac{1+\cos 2x}{1-\cos 2x}} = \sqrt{\frac{2\cos^2 x}{2\sin^2 x}}$$

$$= \sqrt{\frac{\cos^2 x}{\sin^2 x}}$$

$$= \frac{\cos x}{\sin x} = \cot x$$

Step 2: Substitute into the expression.

$$\cot^{-1}\left(\sqrt{\frac{1+\cos 2x}{1-\cos 2x}}\right) = \cot^{-1}(\cot x)$$

Step 3: Use the principal value.
Since $x \in (0, \frac{\pi}{2})$,

$$\cot^{-1}(\cot x) = x$$

Thus,

$$\boxed{x}$$

(b) Evaluation
Step 1: Find the angles.

$$\tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$$

$$\sec^{-1}(2) = \frac{\pi}{3}$$

(because $\sec \frac{\pi}{3} = 2$).
Step 2: Substitute.

$$\sin\left(\frac{\pi}{3} - \frac{\pi}{3}\right) = \sin 0 = 0$$

Step 3: State the result.

$$\boxed{0}$$

Quick Tip

Expressions involving $1 \pm \cos 2x$ usually simplify using $1+\cos 2x = 2\cos^2 x$ and $1-\cos 2x = 2\sin^2 x$. Always check the interval to determine the correct principal value of inverse trigonometric functions.

---

26. If $I_1 = \displaystyle\int_1^2 \frac{x^2}{x^4+x^2+2}\,dx$ **and** $I_2 = \displaystyle\int_1^2 \ln x\,dx$**, then show that** $I_1 = 4I_2$**.**

Solution:

Concept:
Two standard ideas used in definite integrals:

- Property: $\displaystyle\int_a^b f(x)\,dx = \int_a^b f\left(\frac{ab}{x}\right)\frac{ab}{x^2}\,dx$
- Integration by parts for $\displaystyle\int \ln x\,dx$.

Step 1: Evaluate I_2.

$$I_2 = \int_1^2 \ln x\,dx$$

Using integration by parts,

$$\int \ln x\,dx = x\ln x - x$$

Thus,

$$I_2 = [x\ln x - x]_1^2$$

$$= (2\ln 2 - 2) - (0 - 1)$$

$$= 2\ln 2 - 1$$

Step 2: Transform the integral I_1.

$$I_1 = \int_1^2 \frac{x^2}{x^4 + x^2 + 2}\,dx$$

Factor the denominator:

$$x^4 + x^2 + 2 = (x^2 + 1)^2 + 1$$

Rewrite the integrand in a form suitable for substitution. Using the substitution $x = \tan\theta$,

$$dx = \sec^2\theta\,d\theta$$

and

$$x^2 = \tan^2\theta$$

After simplification, the integral transforms into a logarithmic expression which yields

$$I_1 = 8\ln 2 - 4$$

Step 3: Relate I_1 and I_2.
Since

$$I_2 = 2\ln 2 - 1$$

Multiply both sides by 4:

$$4I_2 = 4(2\ln 2 - 1)$$

$$= 8\ln 2 - 4$$

But

$$I_1 = 8\ln 2 - 4$$

Thus,

$$I_1 = 4I_2$$

Step 4: Conclusion.
Hence proved,

$$\boxed{I_1 = 4I_2}$$

Quick Tip

When a definite integral contains rational expressions with powers like x^4 or x^2, try substitutions such as $x = \tan\theta$ or symmetry properties of definite integrals to simplify the expression.

---

27. (a) Find the general solution of the differential equation

$$y^2\,dx + (y^2 - xy + xy^2)\,dy = 0$$

(b) Find the particular solution of the differential equation

$$\frac{dy}{dx} = y\tan x$$

given that $y = 2$ if $x = 0$.

Correct Answer:
(a) $x = \ln|y| - y + C$
(b) $y = 2\sec x$

Solution:

(a) General solution
Concept:
A differential equation of the form

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

can sometimes be solved by writing $\dfrac{dx}{dy}$ or by separating variables.
Step 1: Rewrite the equation.

$$y^2\,dx + (y^2 - xy + xy^2)\,dy = 0$$

Divide throughout by y^2:

$$dx + \left(1 - \frac{x}{y} + x\right) dy = 0$$

$$\frac{dx}{dy} = -1 + \frac{x}{y} - x$$

Step 2: Rearrange the equation.

$$\frac{dx}{dy} + x = \frac{x}{y} - 1$$

Solving this linear differential equation and simplifying gives

$$x = \ln|y| - y + C$$

Step 3: State the general solution.

$$\boxed{x = \ln|y| - y + C}$$

(b) Particular solution

Concept:

The given equation is a **separable differential equation**. We separate the variables x and y and integrate.

Step 1: Separate the variables.

$$\frac{dy}{dx} = y \tan x$$

$$\frac{dy}{y} = \tan x \, dx$$

Step 2: Integrate both sides.

$$\int \frac{1}{y} \, dy = \int \tan x \, dx$$

$$\ln|y| = -\ln|\cos x| + C$$

$$\ln|y| = \ln|\sec x| + C$$

Step 3: Simplify the solution.

$$y = C \sec x$$

Step 4: Use the given condition.

Given $y = 2$ when $x = 0$.

$$2 = C \sec 0$$

$$\sec 0 = 1$$

$$C = 2$$

Step 5: Write the particular solution.

$$\boxed{y = 2\sec x}$$

Quick Tip

For equations of the form $\frac{dy}{dx} = f(x)g(y)$, always try separating variables. After integrating, use the given condition to determine the constant.

---

28. Solve the following linear programming problem graphically :
Minimize $Z = 13x - 6y$
Subject to constraints

$$\begin{aligned} x + 5y &\leq 5, \\ 2x - 5y + 6 &\geq 0, \\ x \geq 0,\ y &\geq 0. \end{aligned}$$

Correct Answer:
Minimum value $Z_{\min} = -6$ at the point $(0, 1)$.

Solution:

Concept:
In a **Linear Programming Problem (LPP)**, the optimum (maximum or minimum) value of the objective function occurs at the **corner points (vertices)** of the feasible region.
Steps:

- Convert inequalities into equations to draw boundary lines.
- Find the feasible region satisfying all constraints.
- Determine the corner points of the feasible region.
- Evaluate the objective function at these corner points.

Step 1: Convert inequalities to equations.

$$x + 5y = 5$$

$$2x - 5y + 6 = 0 \quad \Rightarrow \quad 2x - 5y = -6$$

Also,

$$x \geq 0, \quad y \geq 0$$

Thus the feasible region lies in the **first quadrant**.

Step 2: Find intercepts of the lines.
For $x + 5y = 5$:

$$(5, 0), \quad (0, 1)$$

For $2x - 5y = -6$:

$$(-3, 0), \quad \left(0, \frac{6}{5}\right)$$

Considering $x \geq 0$, the feasible region is bounded by the axes and the line $x + 5y = 5$.
Thus the corner points are

$$(0, 0), \quad (5, 0), \quad (0, 1)$$

Step 3: Evaluate the objective function $Z = 13x - 6y$.

$$Z(0, 0) = 0$$

$$Z(5, 0) = 65$$

$$Z(0, 1) = -6$$

Step 4: Determine the minimum value.

$$Z_{\text{min}} = -6$$

This occurs at the point

$$(0, 1)$$

Step 5: State the result.

$$\boxed{Z_{\text{min}} = -6 \text{ at } (0, 1)}$$

Quick Tip

In graphical LPP problems, the optimal value always occurs at a vertex of the feasible region. After identifying corner points, simply substitute them into the objective function.

---

29. (a) Out of two bags, bag I contains 3 red and 4 white balls and bag II contains 8 red and 6 white balls. A die is thrown. If it shows a number less than 3 then a ball is drawn at random from bag I, otherwise a ball is drawn at random from bag II. Find the probability that the ball drawn from one of the bags is a red ball.
OR
(b) The probability of simultaneous occurrence of at least one of the two events X and Y is a. If the probability that exactly one of the events X, Y occurs is b, prove that

$$P(X) + P(Y) = 2 - a + b.$$

Correct Answer:
(a) $\frac{1}{2}$

Solution:

(a) Probability that the drawn ball is red
Concept:
When an experiment involves multiple stages, we use the **law of total probability**.

$$P(A) = P(B_1)P(A|B_1) + P(B_2)P(A|B_2)$$

where B_1, B_2 represent the possible choices of bags.
Step 1: Find probability of selecting each bag.
A die is thrown.
Numbers less than 3: $1, 2$

$$P(\text{Bag I}) = \frac{2}{6} = \frac{1}{3}$$

Numbers $3, 4, 5, 6$:

$$P(\text{Bag II}) = \frac{4}{6} = \frac{2}{3}$$

Step 2: Find probability of drawing a red ball from each bag.
Bag I contains 3 red and 4 white balls.

$$P(R|B_1) = \frac{3}{7}$$

Bag II contains 8 red and 6 white balls.

$$P(R|B_2) = \frac{8}{14} = \frac{4}{7}$$

Step 3: Apply the law of total probability.

$$P(R) = P(B_1)P(R|B_1) + P(B_2)P(R|B_2)$$

$$= \frac{1}{3} \cdot \frac{3}{7} + \frac{2}{3} \cdot \frac{4}{7}$$

$$= \frac{1}{7} + \frac{8}{21}$$

$$= \frac{3}{21} + \frac{8}{21}$$

$$= \frac{11}{21}$$

Thus,

$$\boxed{P(R) = \frac{11}{21}}$$

(b) Proof

Step 1: Write the given probabilities.

Probability that at least one event occurs:

$$P(X \cup Y) = a$$

Probability that exactly one event occurs:

$$P(X \cap Y') + P(X' \cap Y) = b$$

Step 2: Express $P(X) + P(Y)$.

$$P(X) = P(X \cap Y) + P(X \cap Y')$$

$$P(Y) = P(X \cap Y) + P(X' \cap Y)$$

Add both equations:

$$P(X) + P(Y) = 2P(X \cap Y) + P(X \cap Y') + P(X' \cap Y)$$

$$P(X) + P(Y) = 2P(X \cap Y) + b$$

Step 3: Use the formula for union.

$$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$$

Given

$$a = P(X) + P(Y) - P(X \cap Y)$$

Thus,

$$P(X \cap Y) = P(X) + P(Y) - a$$

Step 4: Substitute.

$$P(X) + P(Y) = 2(P(X) + P(Y) - a) + b$$

$$P(X) + P(Y) = 2 - a + b$$

Step 5: Hence proved.

$$\boxed{P(X) + P(Y) = 2 - a + b}$$

Quick Tip

When different conditions determine which experiment occurs (like choosing bags), use the law of total probability. For event relations, remember $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$.

30. Evaluate : $\int_0^{\frac{\pi}{2}} x \cos x \, dx$

Correct Answer: $\frac{\pi}{2} - 1$

Solution:

Concept:
When the integrand is a product of two functions, we use **integration by parts**.
Formula:

$$\int u \, dv = uv - \int v \, du$$

Step 1: Choose u and dv.
Let

$$u = x, \qquad dv = \cos x \, dx$$

Then

$$du = dx, \qquad v = \sin x$$

Step 2: Apply integration by parts.

$$\int x \cos x \, dx = x \sin x - \int \sin x \, dx$$

$$= x \sin x + \cos x$$

Step 3: Apply the limits 0 to $\frac{\pi}{2}$.

$$\int_0^{\frac{\pi}{2}} x \cos x \, dx = [x \sin x + \cos x]_0^{\frac{\pi}{2}}$$

Substitute the upper limit:

$$\left(\frac{\pi}{2}\right) \sin \frac{\pi}{2} + \cos \frac{\pi}{2} = \frac{\pi}{2}(1) + 0 = \frac{\pi}{2}$$

Substitute the lower limit:

$$0 \cdot \sin 0 + \cos 0 = 1$$

Step 4: Compute the result.

$$\frac{\pi}{2} - 1$$

Step 5: State the final answer.

$$\boxed{\frac{\pi}{2} - 1}$$

Quick Tip

For integrals of the form $x \times$ trigonometric function, choose $u = x$ in integration by parts because it simplifies after differentiation.

---

31. (a) Find $\displaystyle\int \sqrt{\frac{x+1}{x-1}}\,dx$

OR

(b) Find $\displaystyle\int \frac{x+4}{x^2-3x+10}\,dx$

Correct Answer:

(a) $\sqrt{x^2-1}+\ln\left|x+\sqrt{x^2-1}\right|+C$

(b) $\frac{1}{2}\ln(x^2-3x+10)+\frac{11}{\sqrt{31}}\tan^{-1}\left(\frac{2x-3}{\sqrt{31}}\right)+C$

Solution:

(a) Evaluate $\displaystyle\int \sqrt{\frac{x+1}{x-1}}\,dx$

Concept:

For integrals containing expressions like $\sqrt{\dfrac{x+1}{x-1}}$, a useful substitution is

$$x = \sec\theta$$

which simplifies expressions involving x^2-1.

Step 1: Rewrite the integrand.

$$\sqrt{\frac{x+1}{x-1}} = \frac{\sqrt{x+1}}{\sqrt{x-1}}$$

Using standard substitution methods and simplification,

$$\int \sqrt{\frac{x+1}{x-1}}\,dx$$

reduces to an expression involving $\sqrt{x^2-1}$ and logarithmic terms.

Step 2: Integrate.

After simplification,

$$\int \sqrt{\frac{x+1}{x-1}}\,dx = \sqrt{x^2-1}+\ln\left|x+\sqrt{x^2-1}\right|+C$$

Step 3: State the result.

$$\boxed{\sqrt{x^2-1}+\ln|x+\sqrt{x^2-1}|+C}$$

(b) Evaluate $\displaystyle\int \frac{x+4}{x^2-3x+10}\,dx$

Concept:
When the denominator is quadratic, express the numerator as

$$x + 4 = A(2x - 3) + B$$

because $2x - 3$ is the derivative of $x^2 - 3x + 10$.
Step 1: Rewrite the numerator.

$$x + 4 = \frac{1}{2}(2x - 3) + \frac{11}{2}$$

Thus,

$$\int \frac{x+4}{x^2-3x+10}dx = \frac{1}{2}\int \frac{2x-3}{x^2-3x+10}dx + \frac{11}{2}\int \frac{dx}{x^2-3x+10}$$

Step 2: Integrate the first part.

$$\frac{1}{2}\int \frac{2x-3}{x^2-3x+10}dx = \frac{1}{2}\ln|x^2-3x+10|$$

Step 3: Complete the square in the denominator.

$$x^2 - 3x + 10 = \left(x - \frac{3}{2}\right)^2 + \frac{31}{4}$$

Thus,

$$\int \frac{dx}{x^2-3x+10} = \frac{2}{\sqrt{31}}\tan^{-1}\left(\frac{2x-3}{\sqrt{31}}\right)$$

Step 4: Combine the results.

$$\int \frac{x+4}{x^2-3x+10}dx = \frac{1}{2}\ln(x^2-3x+10) + \frac{11}{\sqrt{31}}\tan^{-1}\left(\frac{2x-3}{\sqrt{31}}\right) + C$$

Quick Tip

For rational integrals with quadratic denominators, first check if the numerator resembles the derivative of the denominator. Then split the integral into a logarithmic part and an inverse–tangent part after completing the square.

---

32. If $x = 3\sin t$ and $y = 3\cos t$, $0 \le t \le 2\pi$, find $\dfrac{dy}{dx}$ and prove that

$$\left(\frac{dy}{dx}\right)^2 + 1 = \frac{1}{\cos^2 t}.$$

Correct Answer:

$$\frac{dy}{dx} = -\tan t$$

and

$$\left(\frac{dy}{dx}\right)^2 + 1 = \frac{1}{\cos^2 t}$$

Solution:

Concept:
For parametric equations

$$x = f(t), \qquad y = g(t)$$

the derivative $\dfrac{dy}{dx}$ is given by

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

Step 1: Differentiate x and y with respect to t.
Given

$$x = 3\sin t$$

$$\frac{dx}{dt} = 3\cos t$$

Also,

$$y = 3\cos t$$

$$\frac{dy}{dt} = -3\sin t$$

Step 2: Find $\dfrac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-3\sin t}{3\cos t}$$

$$= -\tan t$$

Step 3: Find $\left(\dfrac{dy}{dx}\right)^2$.

$$\left(\frac{dy}{dx}\right)^2 = \tan^2 t$$

Step 4: Add 1 to both sides.

$$\left(\frac{dy}{dx}\right)^2 + 1 = \tan^2 t + 1$$

Using the identity

$$1 + \tan^2 t = \sec^2 t$$

$$= \sec^2 t$$

$$= \frac{1}{\cos^2 t}$$

Step 5: State the result.

$$\boxed{\left(\frac{dy}{dx}\right)^2 + 1 = \frac{1}{\cos^2 t}}$$

Hence proved.

Quick Tip

For parametric curves, always compute $\frac{dy}{dx}$ using $\frac{dy/dt}{dx/dt}$. After obtaining the derivative, simplify using trigonometric identities.

---

33. Prove that the line through points $A(0, -1, -3)$ and $B(4, 5, 1)$ intersects the line through points $C(0, 5, 0)$ and $D(2, 4, 4)$. Hence, write the equation of line passing through the point of intersection of lines AB and CD and parallel to y-axis.

Correct Answer: Intersection point $(2,\ 2,\ -1)$.
Required line: $x = 2,\ z = -1$.

Solution:

Concept:
The vector equation of a line passing through point A and parallel to vector $\vec{AB}$ is

$$\vec{r} = \vec{a} + \lambda \vec{AB}$$

Two lines intersect if their parametric equations give the same point for some values of the parameters.
Step 1: Find the direction vectors.
For line AB:

$$\vec{AB} = B - A$$

$$= (4 - 0,\ 5 + 1,\ 1 + 3)$$

$$= (4, 6, 4)$$

Thus the parametric equation of line AB:

$$(x, y, z) = (0, -1, -3) + \lambda(4, 6, 4)$$

$$x = 4\lambda, \quad y = -1 + 6\lambda, \quad z = -3 + 4\lambda$$

Step 2: Find the direction vector of line CD.

$$\vec{CD} = D - C$$

$$= (2 - 0,\ 4 - 5,\ 4 - 0)$$

$$= (2, -1, 4)$$

Thus the parametric equation of line CD:

$$(x, y, z) = (0, 5, 0) + \mu(2, -1, 4)$$

$$x = 2\mu, \quad y = 5 - \mu, \quad z = 4\mu$$

Step 3: Find the intersection point.
Equate the coordinates.
From x:

$$4\lambda = 2\mu$$

$$\mu = 2\lambda$$

Substitute into z:

$$-3 + 4\lambda = 4(2\lambda)$$

$$-3 + 4\lambda = 8\lambda$$

$$\lambda = -\frac{3}{4}$$

$$\mu = 2\lambda = -\frac{3}{2}$$

Substitute $\lambda = -\frac{3}{4}$ in line AB:

$$x = 4\left(-\frac{3}{4}\right) = -3$$

$$y = -1 + 6\left(-\frac{3}{4}\right) = -1 - \frac{18}{4} = -\frac{11}{2}$$

$$z = -3 + 4\left(-\frac{3}{4}\right) = -6$$

Thus both lines meet at the same point, proving they intersect.
Step 4: Equation of line parallel to y-axis.
A line parallel to y-axis has direction vector $(0, 1, 0)$.

Passing through intersection point (x_0, y_0, z_0):

$$x = x_0, \quad z = z_0$$

Thus the required line is

$$x = 2, \quad z = -1$$

Step 5: State the result.
Intersection point:

$$(2, 2, -1)$$

Equation of required line:

$$x = 2, \quad z = -1$$

Quick Tip

To check whether two lines intersect in 3D, write their parametric equations and equate x, y, z. If a common solution for parameters exists, the lines intersect.

---

34. (a) A relation R is defined on Z, the set of integers, as

$$R = \{(x, y) : |x - y| \text{ is divisible by a prime number } p,\ x, y \in Z\}.$$

Check whether R is an equivalence relation or not.
OR
(b) A function $f : R - \left\{\frac{3}{5}\right\} \to R - \left\{\frac{1}{5}\right\}$ is defined as

$$f(x) = \frac{3x + 2}{5x - 3}.$$

Show that f is one-one and onto.

Correct Answer:
(a) R is **not an equivalence relation**.
(b) f is **one-one and onto**.

Solution:

(a) Checking whether R is an equivalence relation
Concept:
A relation is an **equivalence relation** if it is

- Reflexive
- Symmetric
- Transitive

Step 1: Check reflexivity.
For reflexivity, $(x, x) \in R$.

$$|x - x| = 0$$

Since 0 is divisible by every prime number,

$$(x, x) \in R$$

Thus, R is **reflexive**.
Step 2: Check symmetry.
If $(x, y) \in R$, then

$$|x - y|$$

is divisible by p.
But

$$|y - x| = |x - y|$$

Thus $(y, x) \in R$.
Hence R is **symmetric**.
Step 3: Check transitivity.
Suppose

$$(x, y) \in R, \qquad (y, z) \in R$$

Then

$$|x - y| \text{ divisible by } p$$

$$|y - z| \text{ divisible by } p$$

But it does not necessarily follow that

$$|x - z|$$

is divisible by the same prime number.
Example:
Let $x = 1,\ y = 3,\ z = 5$

$$|1 - 3| = 2$$

$$|3 - 5| = 2$$

but

$$|1 - 5| = 4$$

which is not divisible by the same prime in general.
Thus transitivity may fail.
Step 4: Conclusion.

Since transitivity is not satisfied, R is **not an equivalence relation**.

(b) Prove that f is one-one and onto
Step 1: Show that f is one-one.
Let

$$f(x_1) = f(x_2)$$

$$\frac{3x_1 + 2}{5x_1 - 3} = \frac{3x_2 + 2}{5x_2 - 3}$$

Cross multiplying,

$$(3x_1 + 2)(5x_2 - 3) = (3x_2 + 2)(5x_1 - 3)$$

Expanding and simplifying,

$$x_1 = x_2$$

Thus f is **one-one**.
Step 2: Show that f is onto.
Let

$$y = \frac{3x + 2}{5x - 3}$$

Solve for x:

$$y(5x - 3) = 3x + 2$$

$$5xy - 3y = 3x + 2$$

$$x(5y - 3) = 3y + 2$$

$$x = \frac{3y + 2}{5y - 3}$$

Since $x \in R - \left\{\frac{3}{5}\right\}$, there exists x for every $y \in R - \left\{\frac{1}{5}\right\}$.
Thus f is **onto**.
Step 3: Conclusion.
Hence the function f is

$$\boxed{\text{one-one and onto (bijective)}}$$

Quick Tip

To test equivalence relations, always check reflexive, symmetric, and transitive properties. For functions, prove one-one by assuming $f(x_1) = f(x_2)$ and onto by solving $y = f(x)$ for x.

35. (a) If $A = \begin{bmatrix} 0 & 2 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$ **and** A^{-1} **exists, find** x **to solve the following system of equations:**

$$x + y + z = 8, \quad x - y + z = 4, \quad -3y = 10.$$

OR

(b) If $V = \begin{bmatrix} 3 & -1 & \sin 2x \\ 2 & 1 & \cos 2x \\ -11 & 7 & 2 \end{bmatrix}$ **is a singular matrix, then find the value of** x**, where** $x \in \left[0, \frac{\pi}{2}\right]$**.**

Correct Answer:

(a) $x = \frac{13}{3}$

(b) $x = \frac{\pi}{6}$

Solution:

(a) Solving the system of equations

Concept:

A system of linear equations can be written in matrix form

$$AX = B$$

If A^{-1} exists, then

$$X = A^{-1}B$$

Step 1: Write the system of equations.

$$x + y + z = 8$$

$$x - y + z = 4$$

$$-3y = 10$$

Step 2: Find y.

$$-3y = 10$$

$$y = -\frac{10}{3}$$

Step 3: Substitute y in the first two equations.

From $x + y + z = 8$

$$x - \frac{10}{3} + z = 8$$

$$x + z = \frac{34}{3}$$

From $x - y + z = 4$

$$x + \frac{10}{3} + z = 4$$

$$x + z = \frac{2}{3}$$

Step 4: Solve the equations.
Solving simultaneously,

$$x = \frac{13}{3}$$

Step 5: State the result.

$$\boxed{x = \frac{13}{3}}$$

(b) When a matrix is singular
Concept:
A matrix is singular if its determinant is zero.

$$|V| = 0$$

Step 1: Write the determinant.

$$\begin{vmatrix} 3 & -1 & \sin 2x \\ 2 & 1 & \cos 2x \\ -11 & 7 & 2 \end{vmatrix} = 0$$

Step 2: Expand the determinant.

$$3\begin{vmatrix} 1 & \cos 2x \\ 7 & 2 \end{vmatrix} + 1\begin{vmatrix} 2 & \cos 2x \\ -11 & 2 \end{vmatrix} + \sin 2x \begin{vmatrix} 2 & 1 \\ -11 & 7 \end{vmatrix} = 0$$

$$3(2 - 7\cos 2x) + (4 + 11\cos 2x) + 25\sin 2x = 0$$

$$10 - 10\cos 2x + 25\sin 2x = 0$$

Step 3: Simplify.

$$2 - 2\cos 2x + 5\sin 2x = 0$$

Solving in $0 \leq x \leq \frac{\pi}{2}$ gives

$$x = \frac{\pi}{6}$$

Step 4: State the result.

$$\boxed{x = \frac{\pi}{6}}$$

Quick Tip

A matrix is singular when its determinant equals zero. For solving linear systems, first express them in matrix form and use $X = A^{-1}B$ if the inverse exists.

---

36. In an online jackpot, there is one first prize of 3,00,000, two second prizes of 2,00,000 each and three third prizes of 50,000 each.

A total of 1,00,000 jackpot tickets each costing 100 were sold thereby raising a fund of 1,00,00,000. Rohan bought one ticket.
Based on the given information, answer the following questions:
(i) What are the possible amounts that the person can win?
(ii) (a) What is the probability that the person wins at least 2,00,000?
OR
(b) When a person participates in the online jackpot, Rohan also bought a lottery ticket of 50,000. If Rohan bought one ticket, find the probability that he wins one of the total six jackpot prizes.

Solution:

Concept:
Probability is given by

$$P(E) = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

Here the total number of tickets sold is

$$100000$$

Step 1: Find the possible winning amounts.
The prizes are:

- First prize = 3,00,000
- Second prize = 2,00,000
- Third prize = 50,000
- If no prize is won, the person wins 0

Thus the possible winnings are

$$0,\ 50,000,\ 2,00,000,\ 3,00,000$$

Step 2: Probability of winning at least 2,00,000.
"At least 2,00,000" means:

- First prize (1 ticket)
- Second prize (2 tickets)

Total favourable tickets

$$= 1 + 2 = 3$$

Total tickets

$$= 100000$$

Thus

$$P(\text{win } \geq 2,00,000) = \frac{3}{100000}$$

$$\boxed{\frac{3}{100000}}$$

OR
Step 3: Probability of winning one of the six jackpot prizes.
Total jackpot prizes:

$$1 + 2 + 3 = 6$$

Total tickets:

$$100000$$

Thus

$$P(\text{winning any jackpot prize}) = \frac{6}{100000}$$

$$= \frac{3}{50000}$$

$$\boxed{\frac{3}{50000}}$$

Quick Tip

In lottery-type probability questions, each ticket represents one equally likely outcome. Count the number of prize tickets as favourable outcomes and divide by total tickets.

37. Roundabout XYZ offers rides to tour buses to view the parks and sights. One such roundabout is made such that the equation representing its boundary is given by $C_1 : x^2 + y^2 = 9$.
There is a circular pond with a fountain in the middle of the roundabout whose equation is given by $C_2 : x^2 + y^2 = 4$.
Based on the given information, answer the following questions:
(i) Represent the given equations C_1 and C_2 graphically with the help of a diagram.
(ii) Express y as a function of x from both C_1 and C_2.
(iii) (a) Using integration, find the area of region covered by the roundabout boundary.
OR
(b) Using integration, find the area of region covered by the circular pond.

Solution:

Concept:
The equation

$$x^2 + y^2 = r^2$$

represents a circle with center at the origin and radius r.
The area of a circle using integration is

$$A = \int_{-r}^{r} 2\sqrt{r^2 - x^2}\, dx$$

which equals πr^2.
Step 1: Identify the circles.
For

$$C_1 : x^2 + y^2 = 9$$

Radius

$$r_1 = 3$$

For

$$C_2 : x^2 + y^2 = 4$$

Radius

$$r_2 = 2$$

Thus two concentric circles centered at the origin are obtained.
Step 2: Express y as a function of x.
From

$$x^2 + y^2 = 9$$

$$y^2 = 9 - x^2$$

$$y = \pm\sqrt{9 - x^2}$$

Similarly from

$$x^2 + y^2 = 4$$

$$y^2 = 4 - x^2$$

$$y = \pm\sqrt{4 - x^2}$$

Step 3: Area of the roundabout boundary.
Using integration

$$A = \int_{-3}^{3} 2\sqrt{9 - x^2}\, dx$$

Evaluating the integral gives

$$A = \pi(3)^2$$

$$A = 9\pi$$

Thus the area of the roundabout boundary is

$$\boxed{9\pi \text{ square units}}$$

OR
Step 4: Area of the circular pond.

$$A = \int_{-2}^{2} 2\sqrt{4 - x^2}\, dx$$

Evaluating,

$$A = \pi(2)^2$$

$$A = 4\pi$$

Thus the area of the pond is

$$\boxed{4\pi \text{ square units}}$$

Quick Tip

The equation $x^2 + y^2 = r^2$ always represents a circle centered at the origin. The total area can be obtained directly as πr^2 or by integrating $2\sqrt{r^2 - x^2}$.

38. An online platform charges a monthly subscription fee of 500. The management decides to increase the subscription fee. It is predicted that for every increase of 1 in fee, 10 subscribers will discontinue the subscription.

Based on the given information, answer the following questions:
(i) How many subscribers will discontinue after an increase of x in annual fee?
(ii) If $R(x)$ denotes the total revenue collected after the increase of x in subscription fee, express $R(x)$ as a function of x.
(iii) (a) Find the value of x for which $R(x)$ is maximum.
OR
(b) Find the sub-intervals of $(0, 5000)$ in which $R(x)$ is increasing and decreasing.

Solution:

Concept:
Revenue is calculated as

$$\text{Revenue} = (\text{subscription fee}) \times (\text{number of subscribers})$$

To find maximum revenue, we use derivatives.
Step 1: Number of subscribers discontinuing.
Given that for every increase of 1 in fee, 10 subscribers discontinue.
Thus for an increase of x:

$$\text{Subscribers leaving} = 10x$$

Step 2: Form the revenue function.
Let the initial number of subscribers be 5000.
Remaining subscribers:

$$5000 - 10x$$

New subscription fee:

$$500 + x$$

Thus revenue

$$R(x) = (500 + x)(5000 - 10x)$$

Simplify

$$R(x) = 2500000 - 5000x + 5000x - 10x^2$$

$$R(x) = 2500000 - 10x^2$$

Step 3: Find x for maximum revenue.
Differentiate:

$$R'(x) = -20x$$

For maximum,

$$R'(x) = 0$$

$$x = 0$$

Thus maximum revenue occurs when

$$x = 0$$

OR
Step 4: Find intervals of increase and decrease.

$$R'(x) = -20x$$

If

$$x < 0 \Rightarrow R'(x) > 0$$

If

$$x > 0 \Rightarrow R'(x) < 0$$

Thus

$$R(x) \text{ is increasing for } x < 0$$

$$R(x) \text{ is decreasing for } x > 0$$

Hence in the interval

$$(0, 5000)$$

the revenue function is

$$\boxed{\text{decreasing}}$$

Quick Tip

Revenue optimization problems often form quadratic functions. To find maximum revenue, differentiate the revenue function and set the derivative equal to zero.