CBSE Class 12 Mathematics

Sample Paper – 10

Duration: 180 Minutes **Maximum Marks: 80**

General Instructions

- This question paper contains **38 questions**. **All questions are compulsory**.
- The paper is divided into **five sections** – A, B, C, D and E.
- **Section A** (Q1–Q20) carries **1 mark** each: Q1–Q18 are multiple choice questions and Q19–Q20 are Assertion–Reason questions.
- **Section B** (Q21–Q25) carries **2 marks** each (Very Short Answer).
- **Section C** (Q26–Q31) carries **3 marks** each (Short Answer).
- **Section D** (Q32–Q35) carries **5 marks** each (Long Answer).
- **Section E** (Q36–Q38) carries **4 marks** each (case study based, with sub-parts).
- There is **no overall choice**, but an **internal choice** has been provided in some questions. Attempt only one of the alternatives in such questions.
- There is **no negative marking**. Use of a **calculator is not permitted**.

Section A (Q1–Q20) – 1 Mark Each

Q1. If $A = \begin{bmatrix} 0 & a \\ -3 & 0 \end{bmatrix}$ is a skew-symmetric matrix, then the value of a is:

(A) -3

(B) 3

(C) 0

(D) 9

Q2. If A is a square matrix of order 3 with $|A| = 4$, then the value of $|2A|$ is:

(A) 8

(B) 12

(C) 16

(D) 32

Q3. Let $A = \{1, 2, 3\}$ and $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2)\}$ be a relation on A. Then R is:

(A) reflexive and symmetric but not transitive

(B) an equivalence relation

(C) transitive but not symmetric

(D) reflexive only

Q4. The principal value of $\tan^{-1}(\sqrt{3})$ is:

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$

(D) $\frac{2\pi}{3}$

Q5. If $y = \sqrt{1 + x^2}$, then $\frac{dy}{dx}$ equals:

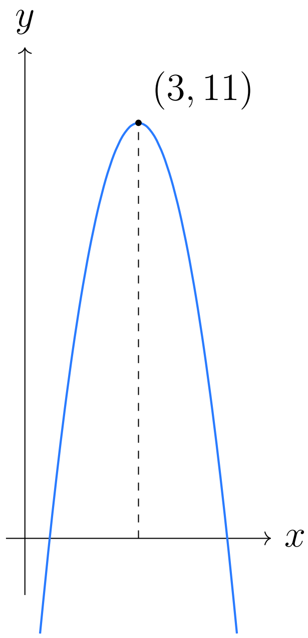
(A) $\frac{1}{2\sqrt{1 + x^2}}$

(B) $\frac{2x}{\sqrt{1 + x^2}}$

(C) $\frac{x}{\sqrt{1 + x^2}}$

(D) $\frac{x}{2\sqrt{1 + x^2}}$

Q6. The graph of $f(x) = -2x^2 + 12x - 7$ is shown below. The function is *increasing* on the interval:



(A) $(3, \infty)$

(B) $(-\infty, \infty)$

(C) $(-\infty, 3)$

(D) $(0, 3)$

Q7. $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,dx$ equals:

(A) $\cos^{-1} x + C$

(B) $\tan^{-1} x + C$

(C) $-\dfrac{1}{\sqrt{1-x^2}} + C$

(D) $\sin^{-1} x + C$

Q8. The order and degree of the differential equation $\left(\dfrac{d^3y}{dx^3}\right)^2 + \left(\dfrac{dy}{dx}\right)^4 + y = 0$ are respectively:

(A) 3 and 4

(B) 3 and 2

(C) 2 and 3

(D) 4 and 2

Q9. The magnitude of the vector $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ is:

(A) 6

(B) 7

(C) 49

(D) $\sqrt{41}$

Q10. The direction cosines of a line whose direction ratios are $3, 4, 0$ are:

(A) $\frac{3}{5}, \frac{4}{5}, 0$

(B) $3, 4, 0$

(C) $\frac{3}{25}, \frac{4}{25}, 0$

(D) $\frac{3}{5}, \frac{4}{5}, 1$

Q11. $\int_0^1 x^2\,dx$ equals:

(A) 1

(B) $\frac{1}{2}$

(C) $\frac{1}{4}$

(D) $\frac{1}{3}$

Q12. If $P(A \cap B) = 0.18$ and $P(B) = 0.6$, then $P(A \mid B)$ equals:

(A) 0.6

(B) 0.108

(C) 0.3

(D) 0.42

Q13. $\int \frac{3x^2}{1+x^3}\,dx$ equals:

(A) $3\ln|1+x^3| + C$

(B) $\ln|1+x^3| + C$

(C) $\dfrac{1}{1+x^3} + C$

(D) $\dfrac{x^3}{1+x^3} + C$

Q14. $\displaystyle\int_{-1}^{1} x\cos x\,dx$ equals:

(A) 0

(B) $2\sin 1$

(C) 1

(D) 2

Q15. The local minimum value of $f(x) = x^3 - 12x$ is:

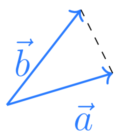
(A) 16

(B) -8

(C) 8

(D) -16

Q16. The area of the triangle whose adjacent sides are the vectors $\vec{a} = \hat{i}+\hat{j}+\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$ (shown below) is:



(A) $\sqrt{2}$

(B) $2\sqrt{2}$

(C) 4

(D) 2

Q17. The angle between two lines whose direction ratios are $2, -1, 3$ and $4, -2, 6$ is:

(A) 90°

(B) 0°

(C) 45°

(D) 30°

Q18. If A and B are independent events with $P(A) = \frac{1}{4}$ and $P(B) = \frac{1}{3}$, then $P(A \cup B)$ equals:

(A) $\frac{1}{2}$

(B) $\frac{7}{12}$

(C) $\frac{1}{12}$

(D) $\frac{5}{12}$

Q19. Assertion (A): The derivative of the even function $f(x) = \cos x$ is an even function.

Reason (R): The derivative of every even differentiable function is an odd function.

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is *not* the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

Q20. Assertion (A): $\tan^{-1}\left(\tan \frac{3\pi}{4}\right) = -\frac{\pi}{4}$.

Reason (R): $\tan^{-1}(\tan x) = x$ for every real number x.

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is *not* the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

Section B (Q21–Q25) – 2 Marks Each

Q21. Find the principal value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{1}{2}\right)$. **[2]**

Q22. Differentiate $y = x^x$ with respect to x. **[2]**

Q23. Evaluate $\int x \cos x \, dx$. **[2]**

OR

Evaluate $\int \ln x \, dx$.

Q24. Find the value of λ for which the vectors $\vec{a} = 3\hat{i} + \lambda\hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ are perpendicular. **[2]**

Q25. A fair die is thrown twice. Find the probability of getting a sum of 7. **[2]**

OR

If $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cap B) = 0.2$, find $P(B \mid A)$.

Section C (Q26–Q31) – 3 Marks Each

Q26. Evaluate $\int \frac{1}{(x-1)(x+2)} \, dx$. **[3]**

Q27. Solve the differential equation $\frac{dy}{dx} + 2y = e^{-x}$. **[3]**

OR

Solve the differential equation $\frac{dy}{dx} = e^{x-y}$.

Q28. Find the inverse of the matrix $A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ using the adjoint method. **[3]**

Q29. Find the intervals in which $f(x) = x^3 - 3x^2 - 9x + 11$ is increasing or decreasing. **[3]**

OR

Find the equation of the tangent to the curve $y = \sqrt{x}$ at the point $(4, 2)$.

Q30. Find the vector equation of the line passing through the point $(2, -1, 4)$ and parallel to the vector $\hat{i} + 3\hat{j} - \hat{k}$. Hence check whether the point $(4, 5, 2)$ lies on this line. **[3]**

Q31. Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 3x - 2$ is one-one and onto. **[3]**

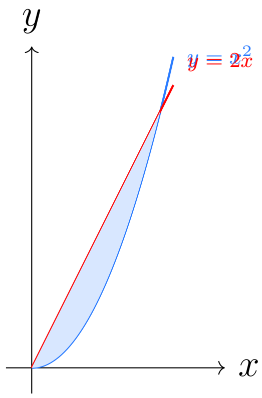
Section D (Q32–Q35) – 5 Marks Each

Q32. Using the matrix method, solve the following system of linear equations:

$$x + 2y + z = 8, \qquad 2x - y + z = 5, \qquad x + y - z = 4.$$

[5]

Q33. Using integration, find the area of the region bounded by the curves $y = x^2$ and $y = 2x$.



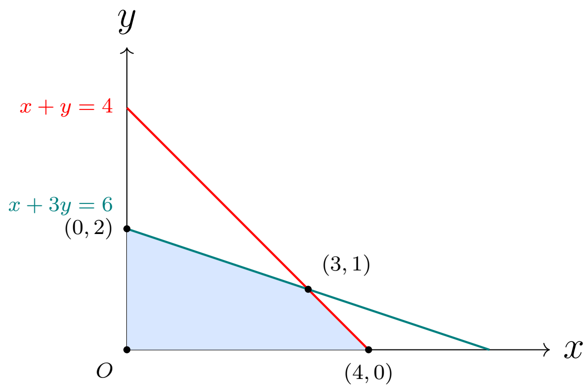
[5]

OR

Using integration, find the area of the region bounded by the parabola $y = 4 - x^2$ and the x-axis.

Q34. A manufacturer produces two products, earning a profit of Rs. 4 per unit of the first and Rs. 5 per unit of the second. Solve the following linear programming problem graphically. Maximise the profit $Z = 4x + 5y$ subject to the constraints

$$x + y \leq 4, \qquad x + 3y \leq 6, \qquad x \geq 0,\ y \geq 0.$$



[5]

Q35. Find the shortest distance between the lines

$$\vec{r} = (\hat{i}+2\hat{j}+3\hat{k})+\lambda(\hat{i}-\hat{j}+2\hat{k}) \quad \text{and} \quad \vec{r} = (2\hat{i}+4\hat{j}+5\hat{k})+\mu(2\hat{i}+\hat{j}+3\hat{k}).$$

[5]

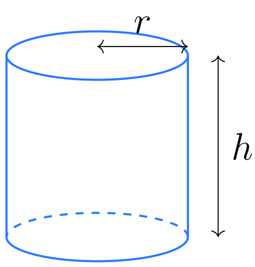
OR

Find the angle between the line $\vec{r} = (2\hat{i}-\hat{j}+\hat{k})+\lambda(2\hat{i}+2\hat{j}+\hat{k})$ and the plane $2x-y+z=5$.

Section E (Q36–Q38) – 4 Marks Each (Case Study)

Q36. Case Study – The Open Cylindrical Tank.

An open-top cylindrical tank of base radius r cm and height h cm is to be built so that it holds a fixed volume of 64π cm^3. The material used is proportional to the total surface area $S = \pi r^2 + 2\pi rh$ (base plus curved surface).



Based on the above, answer the following:

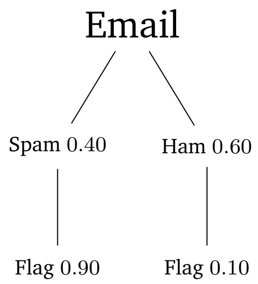
(i) Express the surface area S as a function of r alone. (1)

(ii) Find $\dfrac{dS}{dr}$. (1)

(iii) Find the value of r for which the material used is minimum, and the corresponding height h. (2)

Q37. Case Study – The Spam-Email Filter.

An email service classifies every incoming message as either *spam* or *ham* (legitimate). Records show that 40% of all incoming emails are spam and the remaining 60% are ham. An automatic filter flags suspicious mail: it flags a genuine spam email 90% of the time (true positive), but it also wrongly flags a ham email 10% of the time (false positive). One incoming email is flagged by the filter.



Based on the above, answer the following:

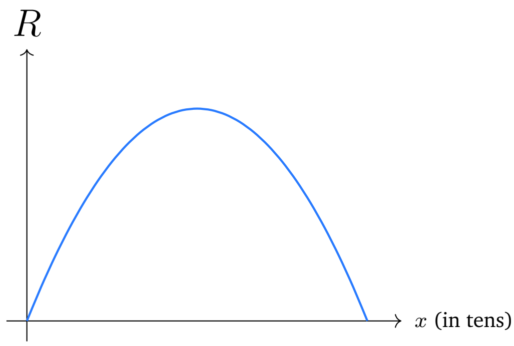
(i) Write $P(\text{Spam})$ and $P(\text{Ham})$. (1)

(ii) Write $P(\text{Flag} \mid \text{Spam})$ and $P(\text{Flag} \mid \text{Ham})$. (1)

(iii) Using Bayes' theorem, find the probability that a flagged email is actually spam. (2)

Q38. Case Study – Revenue of a Company.

A company finds that the total revenue (in rupees) obtained from selling x units of a product is given by $R(x) = 100x - 2x^2$, for $x \geq 0$.



Based on the above, answer the following:

(i) Find the marginal revenue function $\frac{dR}{dx}$. (1)

(ii) Find the marginal revenue when $x = 10$ units. (1)

(iii) Find the number of units for which the revenue is maximum, and the maximum revenue. (2)

Detailed Solutions

Q1.

Solution

Concept — Skew-symmetric matrix: A matrix A is skew-symmetric if $A^T = -A$; equivalently every entry satisfies $a_{ij} = -a_{ji}$ (and the diagonal entries are 0).

Step 1 — Apply the condition to the off-diagonal entries: Here $a_{12} = a$ and $a_{21} = -3$.

The condition $a_{12} = -a_{21}$ gives

$$a = -(-3).$$

$$a = 3.$$

Step 2 — Check: With $a = 3$, $A = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$ and $A^T = \begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix} = -A.$ ✓

Why other options are wrong: (A) -3 makes A symmetric, not skew; (C) 0 and (D) 9 do not satisfy $a = -a_{21}$.

Final Answer: $a = 3 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q1

Q2.

Solution

Concept — Scalar multiple of a determinant: For a square matrix of order n, $|kA| = k^n\,|A|$.

Step 1 — Identify n and k: Here the order is $n = 3$ and $k = 2$.

Step 2 — Apply the property:

$$|2A| = 2^3\,|A|.$$

$$= 8 \times 4.$$

$$= 32.$$

Why other options are wrong: (A) 8 multiplies only by k; (B) 12 and (C) 16 use wrong powers of 2.

Final Answer: $|2A| = 32 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q2

Q3.

Solution

Concept — Properties of a relation: *Reflexive*: $(a, a) \in R$ for all a. *Symmetric*: $(a, b) \in R \Rightarrow (b, a) \in R$. *Transitive*: $(a, b), (b, c) \in R \Rightarrow (a, c) \in R$.

Step 1 — Reflexive? $(1, 1), (2, 2), (3, 3) \in R$, so R is reflexive.

Step 2 — Symmetric? The pairs present are $(1, 2)\&(2, 1)$ and $(2, 3)\&(3, 2)$; each has its reverse. So R is symmetric.

Step 3 — Transitive? $(1, 2) \in R$ and $(2, 3) \in R$, but $(1, 3) \notin R$.

Hence R is not transitive.

Why other options are wrong: (B) an equivalence relation must also be transitive, which fails; (C) R is symmetric; (D) R is symmetric as well as reflexive.

Final Answer: R is reflexive and symmetric but not transitive $\Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q3

Q4.

Solution

Concept — Principal value of $\tan^{-1}$: The principal value branch of $\tan^{-1} x$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Step 1 — Set up the equation: Let $\tan^{-1}(\sqrt{3}) = \theta$, so $\tan\theta = \sqrt{3}$.

Step 2 — Find θ in the branch: We know $\tan\frac{\pi}{3} = \sqrt{3}$, and $\frac{\pi}{3} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

So $\theta = \frac{\pi}{3}$.

Why other options are wrong: (A) $\frac{\pi}{6}$ gives $\tan = \frac{1}{\sqrt{3}}$; (B) $\frac{\pi}{4}$ gives $\tan = 1$; (D) $\frac{2\pi}{3}$ lies outside the principal branch.

Final Answer: $\tan^{-1}(\sqrt{3}) = \frac{\pi}{3} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q4

Q5.

Solution

Concept — Chain rule on a square root: $\frac{d}{dx}\sqrt{u} = \frac{1}{2\sqrt{u}} \cdot \frac{du}{dx}$.

Step 1 — Identify the inner function: Let $u = 1 + x^2$, so $\frac{du}{dx} = 2x$.

Step 2 — Apply the chain rule:

$$\begin{aligned} \frac{dy}{dx} &= \frac{1}{2\sqrt{1+x^2}} \cdot 2x. \\ &= \frac{2x}{2\sqrt{1+x^2}}. \\ &= \frac{x}{\sqrt{1+x^2}}. \end{aligned}$$

Why other options are wrong: (A) drops the factor $2x$; (B) forgets to cancel the 2; (D) keeps an extra 2 in the denominator.

Final Answer: $\frac{dy}{dx} = \frac{x}{\sqrt{1+x^2}} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q5

Q6.

Solution

Concept — Increasing function: f is increasing where $f'(x) > 0$.

Step 1 — Differentiate:

$$f(x) = -2x^2 + 12x - 7.$$

$$f'(x) = -4x + 12.$$

Step 2 — Solve $f'(x) > 0$:

$$-4x + 12 > 0.$$

$$12 > 4x.$$

$$x < 3.$$

So f is increasing on $(-\infty, 3)$, matching the rising left branch up to the vertex $(3, 11)$.

Why other options are wrong: (A) $(3, \infty)$ is where $f'(x) < 0$ (decreasing); (B)

$(-\infty, \infty)$ is false as the parabola turns; (D) $(0, 3)$ is only part of the increasing interval.

Final Answer: f increases on $(-\infty, 3) \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q6

Q7.

Solution

Concept — Standard integral: $\dfrac{d}{dx}\sin^{-1}x = \dfrac{1}{\sqrt{1-x^2}}$, hence $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + C$.

Step 1 — Recall the antiderivative: The derivative of $\sin^{-1}x$ is $\dfrac{1}{\sqrt{1-x^2}}$.

Step 2 — Write the integral:

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + C.$$

Why other options are wrong: (A) $\cos^{-1}x$ has derivative $-\dfrac{1}{\sqrt{1-x^2}}$ (wrong sign); (B) is $\int \dfrac{1}{1+x^2}dx$; (C) is not an antiderivative at all.

Final Answer: $\int \dfrac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + C \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q7

Q8.

Solution

Concept — Order and degree: The *order* is the highest derivative present; the *degree* is the power of that highest derivative when the equation is polynomial in derivatives.

Step 1 — Highest derivative: The highest derivative is $\dfrac{d^3y}{dx^3}$, so the order is 3.

Step 2 — Its power: The term $\left(\dfrac{d^3y}{dx^3}\right)^2$ shows this highest derivative appears to the power 2 (the fourth power is on $\dfrac{dy}{dx}$, a lower derivative).

So the degree is 2.

Why other options are wrong: (A),(C),(D) misread either the highest derivative or its power.

Final Answer: Order 3, degree 2 $\Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q8

Q9.

Solution

Concept — Magnitude of a vector: For $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

Step 1 — Square the components:

$$2^2 + (-3)^2 + 6^2 = 4 + 9 + 36.$$

$$= 49.$$

Step 2 — Take the square root:

$$|\vec{a}| = \sqrt{49} = 7.$$

Why other options are wrong: (A) 6 uses one component; (C) 49 skips the square root; (D) $\sqrt{41}$ uses wrong components.

Final Answer: $|\vec{a}| = 7 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q9

Q10.

Solution

Concept — Direction cosines: Direction cosines are the direction ratios divided by their magnitude $\sqrt{a^2 + b^2 + c^2}$.

Step 1 — Magnitude of ratios $3, 4, 0$:

$$\sqrt{3^2 + 4^2 + 0^2} = \sqrt{9 + 16 + 0} = \sqrt{25} = 5.$$

Step 2 — Divide each ratio by 5:

$$\left(\frac{3}{5}, \frac{4}{5}, 0\right).$$

Why other options are wrong: (B) are the ratios themselves; (C) divides by 25;

(D) has a wrong third cosine of 1.

Final Answer: Direction cosines $\left(\frac{3}{5}, \frac{4}{5}, 0\right) \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q10

Q11.

Solution

Concept — Fundamental theorem of calculus: $\int_a^b f(x)\,dx = F(b) - F(a)$, where $F' = f$.

Step 1 — Antiderivative: $\int x^2\,dx = \frac{x^3}{3}$.

Step 2 — Apply limits 0 **to** 1**:**

$$\left[\frac{x^3}{3}\right]_0^1 = \frac{1^3}{3} - \frac{0^3}{3}.$$

$$= \frac{1}{3} - 0.$$

$$= \frac{1}{3}.$$

Why other options are wrong: (A) 1, (B) $\frac{1}{2}$ and (C) $\frac{1}{4}$ misapply the power rule or the limits.

Final Answer: The integral equals $\frac{1}{3} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q11

Q12.

Solution

Concept — Conditional probability: $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$, provided $P(B) \neq 0$.

Step 1 — Substitute the values:

$$P(A \mid B) = \frac{0.18}{0.6}.$$

Step 2 — Simplify:

$$= \frac{18}{60} = \frac{3}{10} = 0.3.$$

Why other options are wrong: (A) 0.6 ignores the numerator; (B) 0.108 multiplies instead of dividing; (D) 0.42 subtracts.

Final Answer: $P(A \mid B) = 0.3 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q12

Q13.

Solution

Concept — Integration by recognising a derivative: If the numerator is the derivative of the denominator, $\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$.

Step 1 — Spot the pattern: Here $f(x) = 1 + x^3$, so $f'(x) = 3x^2$, which is exactly the numerator.

Step 2 — Write the result:

$$\int \frac{3x^2}{1+x^3}\,dx = \ln|1+x^3| + C.$$

Why other options are wrong: (A) has an extra factor 3; (C) is a wrong antiderivative; (D) is not an antiderivative of the integrand.

Final Answer: $\ln|1+x^3| + C \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q13

Q14.

Solution

Concept — Integral of an odd function over a symmetric interval: If $g(-x) = -g(x)$, then $\int_{-a}^{a} g(x)\,dx = 0$.

Step 1 — Test the integrand: Let $g(x) = x\cos x$.

$$g(-x) = (-x)\cos(-x) = -x\cos x = -g(x).$$

So $x\cos x$ is an odd function.

Step 2 — Apply the property: The interval $[-1, 1]$ is symmetric about 0, hence

$$\int_{-1}^{1} x\cos x\,dx = 0.$$

Why other options are wrong: (B) $2\sin 1$ would be the answer for an even integrand; (C),(D) ignore the odd symmetry.

Final Answer: The integral equals $0 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q14

Q15.

Solution

Concept — Local minimum via second derivative: At a critical point where $f'(x) = 0$, a local minimum occurs if $f''(x) > 0$.

Step 1 — Critical points:

$$f(x) = x^3 - 12x.$$

$$f'(x) = 3x^2 - 12 = 0.$$

$$x^2 = 4 \;\Rightarrow\; x = \pm 2.$$

Step 2 — Second-derivative test:

$$f''(x) = 6x.$$

At $x = 2$, $f''(2) = 12 > 0$, so $x = 2$ gives a local minimum.

Step 3 — Minimum value:

$$f(2) = 2^3 - 12(2) = 8 - 24 = -16.$$

Why other options are wrong: (A) 16 is the local maximum value at $x = -2$; (B) -8 and (C) 8 are not extreme values.

Final Answer: Local minimum value is $-16 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q15

Q16.

Solution

Concept — Area of a triangle from two sides: Area $= \dfrac{1}{2}\,|\vec{a} \times \vec{b}|$ for adjacent side vectors $\vec{a}, \vec{b}$.

Step 1 — Compute the cross product:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix}.$$

$$= \hat{i}\big[(1)(1) - (1)(-1)\big] - \hat{j}\big[(1)(1) - (1)(1)\big] + \hat{k}\big[(1)(-1) - (1)(1)\big].$$
$$= \hat{i}(1+1) - \hat{j}(1-1) + \hat{k}(-1-1) = 2\hat{i} + 0\hat{j} - 2\hat{k}.$$

Step 2 — Magnitude of the cross product:

$$|\vec{a} \times \vec{b}| = \sqrt{2^2 + 0^2 + (-2)^2} = \sqrt{8} = 2\sqrt{2}.$$

Step 3 — Take half the magnitude:

$$\frac{1}{2}|\vec{a} \times \vec{b}| = \frac{1}{2}(2\sqrt{2}) = \sqrt{2}.$$

Why other options are wrong: (B) $2\sqrt{2}$ is $|\vec{a} \times \vec{b}|$ (the parallelogram area, forgetting the factor $\frac{1}{2}$); (C) 4 and (D) 2 come from arithmetic slips in the cross product.

Final Answer: Area $= \sqrt{2} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q16

Q17.

Solution

Concept — Parallel lines: Two lines are parallel when their direction ratios are proportional; the angle between them is 0°.

Step 1 — Check proportionality: Compare $2, -1, 3$ with $4, -2, 6$:

$$\frac{4}{2} = 2, \qquad \frac{-2}{-1} = 2, \qquad \frac{6}{3} = 2.$$

All ratios equal 2, so the direction ratios are proportional.

Step 2 — Interpret: Proportional direction ratios mean the lines are parallel, so the angle between them is 0°.

Why other options are wrong: (A) 90° needs a zero dot product; (C),(D) would need a non-proportional, non-perpendicular pair.

Final Answer: The angle is $0^\circ \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q17

Q18.

Solution

Concept — Union with independent events: $P(A \cup B) = P(A) + P(B) - P(A)P(B)$ when A, B are independent.

Step 1 — Product term:

$$P(A)P(B) = \frac{1}{4} \cdot \frac{1}{3} = \frac{1}{12}.$$

Step 2 — Substitute:

$$\begin{aligned} P(A \cup B) &= \frac{1}{4} + \frac{1}{3} - \frac{1}{12}. \\ &= \frac{3}{12} + \frac{4}{12} - \frac{1}{12}. \\ &= \frac{6}{12} = \frac{1}{2}. \end{aligned}$$

Why other options are wrong: (B) $\frac{7}{12}$ forgets to subtract the intersection; (C) $\frac{1}{12}$ is only $P(A \cap B)$; (D) $\frac{5}{12}$ uses wrong arithmetic.

Final Answer: $P(A \cup B) = \frac{1}{2} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q18

Q19.

Solution

Concept — Assertion–Reason analysis: Judge each statement's truth, then decide whether R correctly explains A.

Step 1 — Assertion: $f(x) = \cos x$ is even, but its derivative is $f'(x) = -\sin x$, which is an *odd* function, not even. So A is **false**.

Step 2 — Reason: It is a standard true result that the derivative of an even differentiable function is odd (and the derivative of an odd function is even). So R is **true**.

Step 3 — Decide the option: Since A is false and R is true, the correct choice is

(D). (Indeed R is exactly the fact that makes A false.)

Why other options are wrong: (A),(B) require A to be true; (C) requires R to be false.

Final Answer: A false, R true $\Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q19

Q20.

Solution

Concept — Inverse tangent of a tangent: $\tan^{-1}(\tan x) = x$ only when $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$; otherwise reduce to the principal branch.

Step 1 — Test the Assertion: $\frac{3\pi}{4} \notin \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, so we reduce it.

$$\tan \frac{3\pi}{4} = -1.$$

$$\tan^{-1}(-1) = -\frac{\pi}{4}.$$

Since $-\frac{\pi}{4} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, we get $\tan^{-1}\left(\tan \frac{3\pi}{4}\right) = -\frac{\pi}{4}$. So A is **true**.

Step 2 — Test the Reason: The identity $\tan^{-1}(\tan x) = x$ holds only on the principal branch, *not* for every real x (e.g. it fails at $x = \frac{3\pi}{4}$). So R is **false**.

Step 3 — Decide the option: A true and R false gives (C).

Why other options are wrong: (A),(B) require R true; (D) requires A false.

Final Answer: A true, R false $\Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q20

Q21.

Solution

Concept — Principal values: $\sin^{-1}$ lies in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $\cos^{-1}$ lies in $[0, \pi]$.

Step 1 — Evaluate $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$:

$$\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2} \Rightarrow \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}.$$

Step 2 — Evaluate $\cos^{-1}\left(\frac{1}{2}\right)$:

$$\cos\frac{\pi}{3} = \frac{1}{2} \Rightarrow \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}.$$

Step 3 — Add:

$$\frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}.$$

Final Answer: $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{1}{2}\right) = \frac{2\pi}{3}$. Go Back to Q21

Q22.

Solution

Concept — Logarithmic differentiation: Used when the variable appears in both base and exponent.

Step 1 — Take natural log of both sides:

$$y = x^x.$$

$$\ln y = x \ln x.$$

Step 2 — Differentiate both sides w.r.t. x: (product rule on the right)

$$\frac{1}{y}\frac{dy}{dx} = 1 \cdot \ln x + x \cdot \frac{1}{x}.$$

$$\frac{1}{y}\frac{dy}{dx} = \ln x + 1.$$

Step 3 — Solve for $\frac{dy}{dx}$:

$$\frac{dy}{dx} = y\big(\ln x + 1\big).$$

$$= x^x\big(\ln x + 1\big).$$

Final Answer: $\frac{dy}{dx} = x^x\big(1 + \ln x\big)$. Go Back to Q22

Q23.

Solution

Concept — Integration by parts: $\int u\,dv = uv - \int v\,du$, choosing u by the ILATE rule.

Step 1 — Choose parts: Let $u = x$ and $dv = \cos x\,dx$.

Then $du = dx$ and $v = \sin x$.

Step 2 — Apply the formula:

$$\int x\cos x\,dx = x\sin x - \int \sin x\,dx.$$

$$= x\sin x - (-\cos x) + C.$$

$$= x\sin x + \cos x + C.$$

Final Answer: $\int x\cos x\,dx = x\sin x + \cos x + C$.

OR — $\int \ln x\,dx$**:**

Step 1 — Parts with $u = \ln x$, $dv = dx$: then $du = \frac{1}{x}dx$ and $v = x$.

$$\int \ln x\,dx = x\ln x - \int x\cdot\frac{1}{x}\,dx.$$

Step 2 — Simplify and integrate:

$$= x\ln x - \int 1\,dx.$$

$$= x\ln x - x + C.$$

Final Answer (OR): $\int \ln x\,dx = x\ln x - x + C$. Go Back to Q23

Q24.

Solution

Concept — Perpendicular vectors: $\vec{a}\perp\vec{b} \iff \vec{a}\cdot\vec{b} = 0$.

Step 1 — Compute the dot product:

$$\vec{a}\cdot\vec{b} = (3)(1) + (\lambda)(2) + (-2)(1).$$

$$= 3 + 2\lambda - 2.$$

$$= 1 + 2\lambda.$$

Step 2 — Set equal to zero and solve:

$$1 + 2\lambda = 0.$$

$$2\lambda = -1.$$

$$\lambda = -\frac{1}{2}.$$

Final Answer: $\lambda = -\frac{1}{2}$. **Go Back to Q24**

Q25.

Solution

Concept — Equally likely outcomes: Probability $= \dfrac{\text{favourable outcomes}}{\text{total outcomes}}$.

Step 1 — Total outcomes: Two throws of a die give $6 \times 6 = 36$ equally likely outcomes.

Step 2 — Favourable outcomes (sum $= 7$):

$$(1,6),\ (6,1),\ (2,5),\ (5,2),\ (3,4),\ (4,3).$$

There are 6 such outcomes.

Step 3 — Probability:

$$P(\text{sum} = 7) = \frac{6}{36} = \frac{1}{6}.$$

Final Answer: $P(\text{sum} = 7) = \frac{1}{6}$.

OR — $P(B \mid A)$:

Step 1 — Formula: $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)}$.

Step 2 — Substitute:

$$P(B \mid A) = \frac{0.2}{0.5} = \frac{2}{5}.$$

Final Answer (OR): $P(B \mid A) = \frac{2}{5} = 0.4$. **Go Back to Q25**

Q26.

Solution

Concept — Partial fractions: Split a proper rational function into simpler fractions, then integrate each.

Step 1 — Set up the decomposition:

$$\frac{1}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}.$$

$$1 = A(x+2) + B(x-1).$$

Step 2 — Find A and B:

Put $x = 1$: $1 = A(3) + B(0) \Rightarrow A = \frac{1}{3}$.

Put $x = -2$: $1 = A(0) + B(-3) \Rightarrow B = -\frac{1}{3}$.

Step 3 — Integrate:

$$\int \frac{1}{(x-1)(x+2)}\,dx = \int \left(\frac{1/3}{x-1} - \frac{1/3}{x+2}\right) dx.$$

$$= \frac{1}{3}\ln|x-1| - \frac{1}{3}\ln|x+2| + C.$$

$$= \frac{1}{3}\ln\left|\frac{x-1}{x+2}\right| + C.$$

Final Answer: $\frac{1}{3}\ln\left|\frac{x-1}{x+2}\right| + C$. **Go Back to Q26**

Q27.

Solution

Concept — Linear differential equation: For $\frac{dy}{dx} + Py = Q$, the integrating factor is I.F. $= e^{\int P\,dx}$ and the solution is $y \cdot \text{I.F.} = \int Q \cdot \text{I.F.}\,dx$.

Step 1 — Identify P, Q: Here $P = 2$, $Q = e^{-x}$.

Step 2 — Integrating factor:

$$\text{I.F.} = e^{\int 2\,dx} = e^{2x}.$$

Step 3 — Solve:

$$y\,e^{2x} = \int e^{-x}\cdot e^{2x}\,dx = \int e^{x}\,dx.$$

$$y\,e^{2x} = e^{x} + C.$$

$$y = e^{-x} + C\,e^{-2x}.$$

Final Answer: $y = e^{-x} + C\,e^{-2x}$.

OR — Variables separable $\frac{dy}{dx} = e^{x-y}$:

Step 1 — Rewrite and separate:

$$\frac{dy}{dx} = e^{x}\,e^{-y}.$$

$$e^{y}\,dy = e^{x}\,dx.$$

Step 2 — Integrate both sides:

$$\int e^{y}\,dy = \int e^{x}\,dx.$$

$$e^{y} = e^{x} + C.$$

Final Answer (OR): $e^{y} = e^{x} + C$. **Go Back to Q27**

Q28.

Solution

Concept — Inverse by adjoint: $A^{-1} = \frac{1}{|A|}\,\mathrm{adj}(A)$, valid when $|A| \neq 0$.

Step 1 — Determinant:

$$|A| = \begin{vmatrix} 3 & 1 \\ 2 & 1 \end{vmatrix} = (3)(1) - (1)(2) = 3 - 2 = 1.$$

Step 2 — Adjoint of a 2×2 matrix: Swap the leading diagonal entries and negate the off-diagonal entries:

$$\mathrm{adj}(A) = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}.$$

Step 3 — Inverse:

$$A^{-1} = \frac{1}{1}\begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}.$$

Step 4 — Verify: $A\,A^{-1} = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. ✓

Final Answer: $A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$. **Go Back to Q28**

Q29.

Solution

Concept — Monotonicity: f increases where $f'(x) > 0$ and decreases where $f'(x) < 0$.

Step 1 — Differentiate:

$$f(x) = x^3 - 3x^2 - 9x + 11.$$

$$f'(x) = 3x^2 - 6x - 9.$$

$$= 3(x^2 - 2x - 3) = 3(x - 3)(x + 1).$$

Step 2 — Critical points: $f'(x) = 0$ at $x = -1$ and $x = 3$.

Step 3 — Sign analysis:

For $x < -1$: $f' > 0$; for $-1 < x < 3$: $f' < 0$; for $x > 3$: $f' > 0$.

Conclusion: Increasing on $(-\infty, -1) \cup (3, \infty)$; decreasing on $(-1, 3)$.

Final Answer: Increasing on $(-\infty, -1) \cup (3, \infty)$, decreasing on $(-1, 3)$.

OR — Tangent to $y = \sqrt{x}$ at $(4, 2)$:

Step 1 — Slope: $\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{x}}$, so at $x = 4$ slope $= \dfrac{1}{2\sqrt{4}} = \dfrac{1}{4}$.

Step 2 — Equation: $y - 2 = \dfrac{1}{4}(x - 4)$.

$$y = \frac{1}{4}x - 1 + 2 = \frac{1}{4}x + 1.$$

Final Answer (OR): $y = \dfrac{1}{4}x + 1$, i.e. $x - 4y + 4 = 0$. **Go Back to Q29**

Q30.

Solution

Concept — Vector equation of a line: A line through point $\vec{a}$ parallel to $\vec{b}$ is $\vec{r} = \vec{a} + \lambda\vec{b}$.

Step 1 — Write the equation: With $\vec{a} = 2\hat{\imath} - \hat{\jmath} + 4\hat{k}$ and $\vec{b} = \hat{\imath} + 3\hat{\jmath} - \hat{k}$,

$$\vec{r} = (2\hat{\imath} - \hat{\jmath} + 4\hat{k}) + \lambda(\hat{\imath} + 3\hat{\jmath} - \hat{k}).$$

Step 2 — Test the point $(4, 5, 2)$: The point lies on the line if some λ satisfies all three component equations.

$$2 + \lambda = 4 \;\Rightarrow\; \lambda = 2.$$

$$-1 + 3\lambda = 5 \;\Rightarrow\; 3\lambda = 6 \;\Rightarrow\; \lambda = 2.$$

$$4 - \lambda = 2 \;\Rightarrow\; \lambda = 2.$$

Step 3 — Conclusion: All three give the same value $\lambda = 2$, so the point $(4, 5, 2)$ lies on the line.

Final Answer: $\vec{r} = (2\hat{\imath} - \hat{\jmath} + 4\hat{k}) + \lambda(\hat{\imath} + 3\hat{\jmath} - \hat{k})$; the point $(4, 5, 2)$ lies on it (at $\lambda = 2$). **Go Back to Q30**

Q31.

Solution

Concept — One-one (injective) and onto (surjective): *One-one*: $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$. *Onto*: every y in the codomain has a pre-image.

Step 1 — One-one: Suppose $f(x_1) = f(x_2)$.

$$3x_1 - 2 = 3x_2 - 2.$$

$$3x_1 = 3x_2.$$

$$x_1 = x_2.$$

Hence f is one-one.

Step 2 — Onto: Let $y \in \mathbb{R}$ be arbitrary. Solve $y = 3x - 2$ for x:

$$x = \frac{y+2}{3} \in \mathbb{R}.$$

Then $f(x) = 3\left(\dfrac{y+2}{3}\right) - 2 = y$.

So every y has a pre-image; f is onto.

Step 3 — Conclusion: f is both one-one and onto, hence bijective.

Final Answer: $f(x) = 3x - 2$ is one-one and onto (a bijection). **Go Back to Q31**

Q32.

Solution

Concept — Matrix method: Write $AX = B$; then $X = A^{-1}B$ when $|A| \neq 0$.

Step 1 — Form the matrices:

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 8 \\ 5 \\ 4 \end{bmatrix}.$$

Step 2 — Determinant of A:

$$|A| = 1\big[(-1)(-1) - (1)(1)\big] - 2\big[(2)(-1) - (1)(1)\big] + 1\big[(2)(1) - (-1)(1)\big].$$

$$= 1(1-1) - 2(-2-1) + 1(2+1).$$

$$= 0 + 6 + 3 = 9\ (\neq 0).$$

Step 3 — Cofactors and adjoint: Computing the cofactor matrix and transposing gives

$$\text{adj}(A) = \begin{bmatrix} 0 & 3 & 3 \\ 3 & -2 & 1 \\ 3 & 1 & -5 \end{bmatrix}.$$

Step 4 — Solve $X = \dfrac{1}{|A|}\,\text{adj}(A)\,B$:

$$X = \frac{1}{9}\begin{bmatrix} 0 & 3 & 3 \\ 3 & -2 & 1 \\ 3 & 1 & -5 \end{bmatrix}\begin{bmatrix} 8 \\ 5 \\ 4 \end{bmatrix}.$$

$$= \frac{1}{9}\begin{bmatrix} 0\cdot 8 + 3\cdot 5 + 3\cdot 4 \\ 3\cdot 8 - 2\cdot 5 + 1\cdot 4 \\ 3\cdot 8 + 1\cdot 5 - 5\cdot 4 \end{bmatrix}.$$

$$= \frac{1}{9}\begin{bmatrix} 0+15+12 \\ 24-10+4 \\ 24+5-20 \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 27 \\ 18 \\ 9 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}.$$

Step 5 — Verify: $x + 2y + z = 3 + 4 + 1 = 8$ ✓; $2x - y + z = 6 - 2 + 1 = 5$ ✓; $x + y - z = 3 + 2 - 1 = 4$ ✓.

Final Answer: $x = 3,\ y = 2,\ z = 1$. **Go Back to Q32**

Q33.

Solution

Concept — Area between two curves: Area $= \int_a^b \left[y_{\text{top}} - y_{\text{bottom}}\right] dx$ between the points of intersection.

Step 1 — Points of intersection: Set $x^2 = 2x$:

$$x^2 - 2x = 0.$$

$$x(x-2) = 0 \ \Rightarrow \ x = 0 \text{ and } x = 2.$$

Step 2 — Decide top and bottom: On $(0, 2)$, the line $y = 2x$ lies above the parabola $y = x^2$ (e.g. at $x = 1$, $2 > 1$).

Step 3 — Set up the integral:

$$\text{Area} = \int_0^2 (2x - x^2)\, dx.$$

Step 4 — Integrate:

$$= \left[x^2 - \frac{x^3}{3}\right]_0^2.$$

$$= \left(4 - \frac{8}{3}\right) - (0).$$

$$= \frac{12-8}{3} = \frac{4}{3}.$$

Final Answer: Area $= \frac{4}{3}$ square units.

OR — Region bounded by $y = 4 - x^2$ and the x-axis:

Step 1 — x-intercepts: $4 - x^2 = 0 \Rightarrow x = \pm 2$. The curve is symmetric about the y-axis.

Step 2 — Integrate (double the right half):

$$\text{Area} = 2\int_0^2 (4 - x^2)\,dx.$$

$$= 2\left[4x - \frac{x^3}{3}\right]_0^2.$$

$$= 2\left(8 - \frac{8}{3}\right) = 2 \cdot \frac{16}{3} = \frac{32}{3}.$$

Final Answer (OR): Area $= \dfrac{32}{3}$ square units. **Go Back to Q33**

Q34.

Solution

Concept — Corner-point method: The optimum of a linear objective over a bounded feasible region occurs at a corner (vertex).

Step 1 — Find the corner points: The constraints $x + y \le 4$, $x + 3y \le 6$, $x \ge 0$, $y \ge 0$ bound the region with vertices:

$$O(0,0), \quad (4,0), \quad (0,2).$$

The lines $x + y = 4$ and $x + 3y = 6$ intersect where

$$(x + 3y) - (x + y) = 6 - 4 \;\Rightarrow\; 2y = 2 \;\Rightarrow\; y = 1,\; x = 3.$$

So the fourth vertex is $(3,1)$.

Step 2 — Evaluate $Z = 4x + 5y$ at each corner:

$$Z(0,0) = 0.$$

$$Z(4,0) = 4(4) + 5(0) = 16.$$

$$Z(3,1) = 4(3) + 5(1) = 12 + 5 = 17.$$

$$Z(0,2) = 4(0) + 5(2) = 10.$$

Step 3 — Pick the maximum: The largest value is 17 at $(3,1)$.

Final Answer: The profit Z is maximum $= 17$ (i.e. Rs. 17) at $(x,y) = (3,1)$. **Go Back to Q34**

Q35.

Solution

Concept — Shortest distance between skew lines: For $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$,

$$d = \frac{\left|(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)\right|}{|\vec{b}_1 \times \vec{b}_2|}.$$

Step 1 — Identify vectors: $\vec{a}_1 = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$, $\vec{b}_1 = \hat{\imath} - \hat{\jmath} + 2\hat{k}$; $\vec{a}_2 = 2\hat{\imath} + 4\hat{\jmath} + 5\hat{k}$, $\vec{b}_2 = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$.

Step 2 — Cross product $\vec{b}_1 \times \vec{b}_2$:

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -1 & 2 \\ 2 & 1 & 3 \end{vmatrix}.$$

$$= \hat{\imath}\big[(-1)(3) - (2)(1)\big] - \hat{\jmath}\big[(1)(3) - (2)(2)\big] + \hat{k}\big[(1)(1) - (-1)(2)\big].$$

$$= \hat{\imath}(-3-2) - \hat{\jmath}(3-4) + \hat{k}(1+2).$$

$$= -5\hat{\imath} + \hat{\jmath} + 3\hat{k}.$$

Step 3 — Its magnitude:

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{(-5)^2 + 1^2 + 3^2} = \sqrt{25+1+9} = \sqrt{35}.$$

Step 4 — $(\vec{a}_2 - \vec{a}_1)$ and the dot product:

$$\vec{a}_2 - \vec{a}_1 = (2-1)\hat{\imath} + (4-2)\hat{\jmath} + (5-3)\hat{k} = \hat{\imath} + 2\hat{\jmath} + 2\hat{k}.$$

$$(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2) = (1)(-5) + (2)(1) + (2)(3) = -5 + 2 + 6 = 3.$$

Step 5 — Distance:

$$d = \frac{|3|}{\sqrt{35}} = \frac{3}{\sqrt{35}} \text{ units.}$$

Final Answer: Shortest distance $= \dfrac{3}{\sqrt{35}}$ units.

OR — Angle between line and plane:

Step 1 — Direction $\vec{b} = 2\hat{\imath} + 2\hat{\jmath} + \hat{k}$, normal $\vec{n} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$.

Step 2 — Use $\sin\theta = \dfrac{|\vec{b}\cdot\vec{n}|}{|\vec{b}||\vec{n}|}$:

$$\vec{b}\cdot\vec{n} = (2)(2) + (2)(-1) + (1)(1) = 4 - 2 + 1 = 3.$$

$$|\vec{b}| = \sqrt{4+4+1} = 3, \qquad |\vec{n}| = \sqrt{4+1+1} = \sqrt{6}.$$

$$\sin\theta = \frac{3}{3\sqrt{6}} = \frac{1}{\sqrt{6}}.$$

Final Answer (OR): $\theta = \sin^{-1}\left(\dfrac{1}{\sqrt{6}}\right)$. **Go Back to Q35**

Q36.

Solution

Concept — Minima by derivatives: Build the surface-area function, set $\dfrac{dS}{dr} = 0$, and select the value giving a genuine minimum.

(i) Surface area function: The fixed volume gives $\pi r^2 h = 64\pi$, so

$$h = \frac{64}{r^2}.$$

Substitute into $S = \pi r^2 + 2\pi r h$:

$$S = \pi r^2 + 2\pi r \cdot \frac{64}{r^2}.$$

$$S = \pi r^2 + \frac{128\pi}{r}, \qquad r > 0.$$

(ii) Differentiate:

$$\frac{dS}{dr} = 2\pi r - \frac{128\pi}{r^2}.$$

(iii) Minimise: Set $\dfrac{dS}{dr} = 0$:

$$2\pi r = \frac{128\pi}{r^2}.$$

$$2r^3 = 128.$$

$$r^3 = 64 \;\Rightarrow\; r = 4.$$

Second-derivative check: $\dfrac{d^2S}{dr^2} = 2\pi + \dfrac{256\pi}{r^3} > 0$, so $r = 4$ gives a minimum.

Corresponding height:

$$h = \frac{64}{4^2} = \frac{64}{16} = 4 \text{ cm}.$$

Final Answer: $S = \pi r^2 + \dfrac{128\pi}{r}$; $\dfrac{dS}{dr} = 2\pi r - \dfrac{128\pi}{r^2}$; material is minimum at $r = 4$ cm with $h = 4$ cm (minimum $S = 48\pi$ cm^2). Go Back to Q36

Q37.

Solution

Concept — Bayes' theorem: $P(E_i \mid A) = \dfrac{P(E_i)P(A \mid E_i)}{\sum_j P(E_j)P(A \mid E_j)}$.

(i) Prior probabilities: Since 40% of emails are spam,

$$P(\text{Spam}) = 0.40, \qquad P(\text{Ham}) = 0.60.$$

(ii) Conditional (flag) probabilities:

$$P(\text{Flag} \mid \text{Spam}) = 0.90, \qquad P(\text{Flag} \mid \text{Ham}) = 0.10.$$

(iii) Apply Bayes' theorem:

$$P(\text{Spam} \mid \text{Flag}) = \frac{(0.40)(0.90)}{(0.40)(0.90) + (0.60)(0.10)}.$$

Compute each product:

$$(0.40)(0.90) = 0.36, \qquad (0.60)(0.10) = 0.06.$$

Denominator:

$$0.36 + 0.06 = 0.42.$$

So

$$P(\text{Spam} \mid \text{Flag}) = \frac{0.36}{0.42} = \frac{36}{42} = \frac{6}{7}.$$

Final Answer: $P(\text{Spam} \mid \text{Flag}) = \dfrac{6}{7} \approx 0.857$. Go Back to Q37

Q38.

Solution

Concept — Marginal revenue: Marginal revenue is the rate of change of total revenue, MR $= \dfrac{dR}{dx}$; revenue is maximum where $\dfrac{dR}{dx} = 0$ and $\dfrac{d^2R}{dx^2} < 0$.

(i) Marginal revenue function:

$$R(x) = 100x - 2x^2.$$

$$\frac{dR}{dx} = 100 - 4x.$$

(ii) Marginal revenue at $x = 10$:

$$\left.\frac{dR}{dx}\right|_{x=10} = 100 - 4(10) = 100 - 40 = 60.$$

So the marginal revenue is 60 (rupees per unit).

(iii) Maximum revenue: Set $\frac{dR}{dx} = 0$:

$$100 - 4x = 0 \;\Rightarrow\; x = 25.$$

Since $\frac{d^2R}{dx^2} = -4 < 0$, $x = 25$ gives a maximum. Maximum revenue:

$$R(25) = 100(25) - 2(25)^2 = 2500 - 2(625) = 2500 - 1250 = 1250.$$

Final Answer: $\frac{dR}{dx} = 100 - 4x$; marginal revenue at $x = 10$ is 60; revenue is maximum at $x = 25$ units, with maximum revenue $= 1250$ (Rs. 1250). **Go Back to Q38**

Answer Key – Section A (Q1–Q20)

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	D	3	A	4	C	5	C
6	C	7	D	8	B	9	B	10	A
11	D	12	C	13	B	14	A	15	D
16	A	17	B	18	A	19	D	20	C

Sections B–E are descriptive; refer to the Detailed Solutions above for full model answers and step marking.