

# CBSE Class 12 Mathematics

## Sample Paper – 1

Duration: 180 Minutes

Maximum Marks: 80

### General Instructions

- This question paper contains **38 questions**. All questions are **compulsory**.
- The paper is divided into **five sections** – A, B, C, D and E.
- **Section A** (Q1–Q20) carries **1 mark** each: Q1–Q18 are multiple choice questions and Q19–Q20 are Assertion–Reason questions.
- **Section B** (Q21–Q25) carries **2 marks** each (Very Short Answer).
- **Section C** (Q26–Q31) carries **3 marks** each (Short Answer).
- **Section D** (Q32–Q35) carries **5 marks** each (Long Answer).
- **Section E** (Q36–Q38) carries **4 marks** each (case study based, with sub-parts).
- There is **no overall choice**, but an **internal choice** has been provided in some questions. Attempt only one of the alternatives in such questions.
- There is **no negative marking**. Use of a **calculator is not permitted**.

### Section A (Q1–Q20) – 1 Mark Each

**Q1.** If  $A$  is a matrix of order  $2 \times 3$  and  $B$  is a matrix of order  $3 \times 2$ , then the order of the product  $AB$  is:

- (A)  $2 \times 2$
- (B)  $3 \times 3$
- (C)  $3 \times 2$
- (D)  $2 \times 3$



**Q2.** The value of the determinant  $\begin{vmatrix} 3 & -2 \\ 4 & 5 \end{vmatrix}$  is:

- (A) 7
- (B) 17
- (C) 23
- (D) -23

**Q3.** Let  $A = \{1, 2, 3\}$  and  $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3)\}$  be a relation on  $A$ . Then  $R$  is:

- (A) symmetric
- (B) reflexive but neither symmetric nor transitive
- (C) transitive
- (D) an equivalence relation

**Q4.** The principal value of  $\sin^{-1}\left(-\frac{1}{2}\right)$  is:

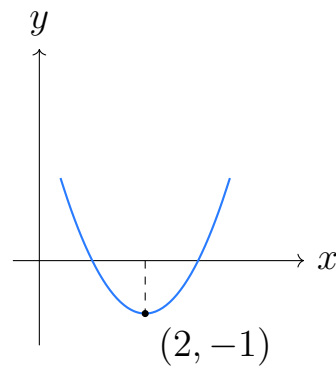
- (A)  $\frac{\pi}{6}$
- (B)  $\frac{5\pi}{6}$
- (C)  $-\frac{5\pi}{6}$
- (D)  $-\frac{\pi}{6}$

**Q5.** If  $y = \sin(x^2)$ , then  $\frac{dy}{dx}$  equals:

- (A)  $2x \cos(x^2)$
- (B)  $\cos(x^2)$
- (C)  $2x \cos x$
- (D)  $-2x \cos(x^2)$

**Q6.** The graph of  $f(x) = x^2 - 4x + 3$  is shown below. The function is *increasing* on the interval:





- (A)  $(-\infty, 2)$
- (B)  $(-\infty, \infty)$
- (C)  $(2, \infty)$
- (D)  $(0, 2)$

**Q7.**  $\int \sec^2 x \, dx$  equals:

- (A)  $-\cot x + C$
- (B)  $\tan x + C$
- (C)  $\sec x \tan x + C$
- (D)  $\ln |\sec x| + C$

**Q8.** The order and degree of the differential equation  $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 + y = 0$  are respectively:

- (A) 1 and 2
- (B) 2 and 3
- (C) 3 and 1
- (D) 2 and 1

**Q9.** The magnitude of the vector  $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}$  is:

- (A) 3
- (B) 5
- (C) 9



(D)  $\sqrt{5}$

**Q10.** The direction cosines of a line whose direction ratios are 1, -2, 2 are:

(A) 1, -2, 2

(B)  $\frac{1}{9}, -\frac{2}{9}, \frac{2}{9}$

(C)  $\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}$

(D)  $\frac{1}{\sqrt{3}}, -\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

**Q11.**  $\int_0^{\pi/2} \cos x \, dx$  equals:

(A) 0

(B) 1

(C) -1

(D)  $\frac{\pi}{2}$

**Q12.** If  $P(A \cap B) = 0.2$  and  $P(B) = 0.5$ , then  $P(A | B)$  equals:

(A) 0.1

(B) 0.7

(C) 0.25

(D) 0.4

**Q13.**  $\int \frac{2x}{1+x^2} \, dx$  equals:

(A)  $\ln(1+x^2) + C$

(B)  $2 \ln(1+x^2) + C$

(C)  $\tan^{-1} x + C$

(D)  $\frac{1}{1+x^2} + C$

**Q14.**  $\int_{-2}^2 x^3 \, dx$  equals:

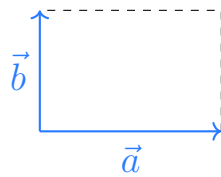


- (A) 16
- (B) 8
- (C) 0
- (D) 4

**Q15.** The local maximum value of  $f(x) = x^3 - 3x$  is:

- (A)  $-2$
- (B)  $2$
- (C)  $0$
- (D)  $-1$

**Q16.** The area of the parallelogram whose adjacent sides are the vectors  $\vec{a} = 3\hat{i}$  and  $\vec{b} = 2\hat{j}$  (shown below) is:



- (A) 5
- (B) 1
- (C) 12
- (D) 6

**Q17.** The angle between two lines whose direction ratios are  $1, 2, 3$  and  $2, -1, 0$  is:

- (A)  $90^\circ$
- (B)  $0^\circ$
- (C)  $45^\circ$
- (D)  $60^\circ$

**Q18.** If  $A$  and  $B$  are independent events with  $P(A) = \frac{1}{3}$  and  $P(B) = \frac{1}{2}$ , then  $P(A \cup B)$  equals:



- (A)  $\frac{5}{6}$   
 (B)  $\frac{1}{6}$   
 (C)  $\frac{2}{3}$   
 (D)  $\frac{1}{2}$

**Q19. Assertion (A):** The function  $f(x) = |x|$  is continuous at  $x = 0$ .

**Reason (R):** Every differentiable function is continuous.

- (A) Both A and R are true and R is the correct explanation of A.  
 (B) Both A and R are true but R is *not* the correct explanation of A.  
 (C) A is true but R is false.  
 (D) A is false but R is true.

**Q20. Assertion (A):**  $\cos^{-1}\left(\cos \frac{7\pi}{6}\right) = \frac{7\pi}{6}$ .

**Reason (R):** The range (principal value branch) of  $\cos^{-1} x$  is  $[0, \pi]$ .

- (A) Both A and R are true and R is the correct explanation of A.  
 (B) Both A and R are true but R is *not* the correct explanation of A.  
 (C) A is true but R is false.  
 (D) A is false but R is true.

**Section B (Q21–Q25) – 2 Marks Each**

**Q21.** Find the principal value of  $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right)$ . [2]

**Q22.** Differentiate  $y = (\sin x)^x$  with respect to  $x$ . [2]

**Q23.** Evaluate  $\int x e^x dx$ . [2]

**OR**

Evaluate  $\int \cos^2 x dx$ .



**Q24.** Find the value of  $\lambda$  for which the vectors  $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$  and  $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$  are perpendicular. [2]

**Q25.** A fair die is thrown twice. Find the probability of getting a sum of 9. [2]

**OR**

If  $P(A) = 0.6$ ,  $P(B) = 0.3$  and  $P(A \cap B) = 0.2$ , find  $P(A | B)$ .

**Section C (Q26–Q31) – 3 Marks Each**

**Q26.** Evaluate  $\int \frac{x}{(x+1)(x+2)} dx$ . [3]

**Q27.** Solve the differential equation  $\frac{dy}{dx} + y = e^x$ . [3]

**OR**

Solve the differential equation  $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ .

**Q28.** Find the inverse of the matrix  $A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$  using the adjoint method. [3]

**Q29.** Find the intervals in which  $f(x) = x^3 - 6x^2 + 9x + 5$  is increasing or decreasing. [3]

**OR**

Find the equation of the tangent to the curve  $y = x^2$  at the point  $(1, 1)$ .

**Q30.** Find the vector equation of the line passing through the point  $(1, 2, 3)$  and parallel to the vector  $3\hat{i} + 2\hat{j} - 2\hat{k}$ . Hence check whether the point  $(4, 4, 1)$  lies on this line. [3]

**Q31.** Show that the function  $f : \mathbb{R} \rightarrow \mathbb{R}$  defined by  $f(x) = 2x + 3$  is one-one and onto. [3]

**Section D (Q32–Q35) – 5 Marks Each**

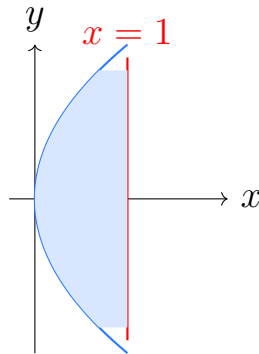


**Q32.** Using the matrix method, solve the following system of linear equations:

$$x + y + z = 6, \quad y + 3z = 11, \quad x - 2y + z = 0.$$

[5]

**Q33.** Using integration, find the area of the region bounded by the parabola  $y^2 = 4x$  and the line  $x = 1$ .



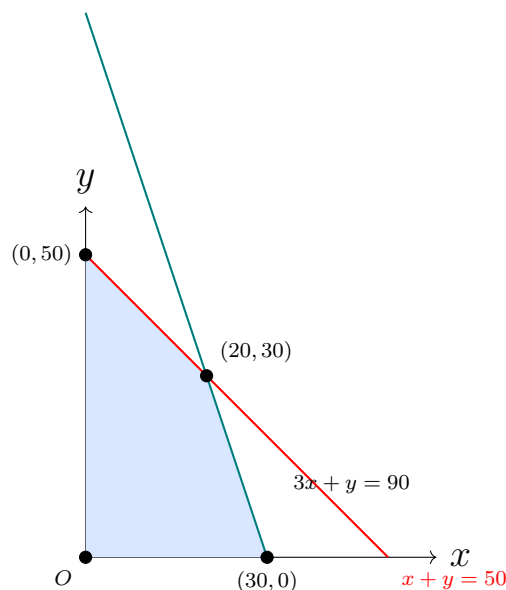
[5]

**OR**

Using integration, find the area of the region bounded by the curves  $y = x^2$  and  $y = x$  between  $x = 0$  and  $x = 1$ .

**Q34.** Solve the following linear programming problem graphically. Maximise  $Z = 4x + y$  subject to the constraints

$$x + y \leq 50, \quad 3x + y \leq 90, \quad x \geq 0, \quad y \geq 0.$$



[5]

**Q35.** Find the shortest distance between the lines

$$\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k}) \quad \text{and} \quad \vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(3\hat{i} - 5\hat{j} + 2\hat{k}).$$

[5]

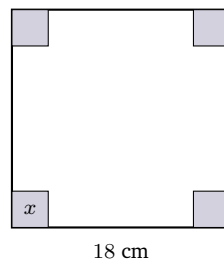
**OR**

Find the angle between the line  $\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$  and the plane  $x + y + z = 5$ .

**Section E (Q36–Q38) – 4 Marks Each (Case Study)**

**Q36. Case Study – The Open Box.**

A square sheet of tin of side 18 cm is to be made into an open-top box by cutting a small square of side  $x$  cm from each corner and folding up the flaps, as shown.



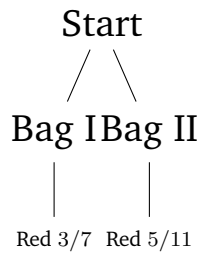
Based on the above, answer the following:

- (i) Express the volume  $V$  of the box as a function of  $x$ . (1)
- (ii) Find  $\frac{dV}{dx}$ . (1)
- (iii) Find the value of  $x$  for which the volume is maximum, and the maximum volume. (2)

**Q37. Case Study – Choosing a Bag.**

Bag I contains 3 red and 4 black balls; Bag II contains 5 red and 6 black balls. One bag is selected at random and a ball is drawn from it. The ball drawn is found to be red.



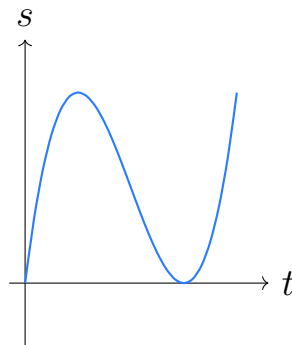


Based on the above, answer the following:

- (i) Write  $P(\text{Bag I})$  and  $P(\text{Bag II})$ . (1)
- (ii) Write  $P(\text{Red} \mid \text{Bag I})$  and  $P(\text{Red} \mid \text{Bag II})$ . (1)
- (iii) Using Bayes' theorem, find the probability that the red ball was drawn from Bag I. (2)

**Q38. Case Study – Motion of a Particle.**

A particle moves along a straight line so that its position (in metres) at time  $t$  seconds is given by  $s(t) = t^3 - 6t^2 + 9t$ , for  $t \geq 0$ .



Based on the above, answer the following:

- (i) Find the velocity  $v(t)$  of the particle. (1)
- (ii) Find the time(s) at which the particle is momentarily at rest. (1)
- (iii) Find the acceleration of the particle at  $t = 2$  s, and state whether it is speeding up or slowing down at that instant. (2)



## Detailed Solutions

Q1.

## Solution

**Concept — Order of a matrix product:** The product  $AB$  is defined only when the number of columns of  $A$  equals the number of rows of  $B$ , and the resulting order is (rows of  $A$ )  $\times$  (columns of  $B$ ).

**Step 1 — Check compatibility:**  $A$  is  $2 \times 3$  and  $B$  is  $3 \times 2$ .

The inner numbers are 3 and 3; they match, so  $AB$  exists.

**Step 2 — Read off the order:** The outer numbers give the order of  $AB$ .

Order of  $AB = 2 \times 2$ .

**Why other options are wrong:**

- (B)  $3 \times 3$  is the order of the other product  $BA$ , not of  $AB$ .
- (C)  $3 \times 2$  is the order of  $B$ , not of  $AB$ .
- (D)  $2 \times 3$  is the order of  $A$ , not of  $AB$ .

**Final Answer:** Order of  $AB = 2 \times 2 \Rightarrow$  A

Answer: (A) [Go Back to Q1](#)

Q2.

## Solution

**Concept — Value of a  $2 \times 2$  determinant:** For  $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ , the value is  $ad - bc$ .

**Step 1 — Identify entries:** Here  $a = 3$ ,  $b = -2$ ,  $c = 4$ ,  $d = 5$ .

**Step 2 — Apply the formula:**

$$\begin{aligned} ad - bc &= (3)(5) - (-2)(4). \\ &= 15 - (-8). \\ &= 15 + 8. \\ &= 23. \end{aligned}$$

**Why other options are wrong:** (A) 7 comes from  $15 - 8$  (wrong sign on  $bc$ ); (B) 17 from  $9 + 8$ ; (D)  $-23$  flips the overall sign.



**Final Answer:** The determinant is 23  $\Rightarrow$  C

**Answer: (C)** [Go Back to Q2](#)

**Q3.**

### Solution

**Concept — Properties of a relation:** *Reflexive:*  $(a, a) \in R$  for all  $a$ . *Symmetric:*  $(a, b) \in R \Rightarrow (b, a) \in R$ . *Transitive:*  $(a, b), (b, c) \in R \Rightarrow (a, c) \in R$ .

**Step 1 — Reflexive?**  $(1, 1), (2, 2), (3, 3) \in R$ , so  $R$  is reflexive.

**Step 2 — Symmetric?**  $(1, 2) \in R$  but  $(2, 1) \notin R$ .

Hence  $R$  is not symmetric.

**Step 3 — Transitive?**  $(1, 2) \in R$  and  $(2, 3) \in R$ , but  $(1, 3) \notin R$ .

Hence  $R$  is not transitive.

**Why other options are wrong:** (A) fails by Step 2; (C) fails by Step 3; (D) an equivalence relation must be reflexive, symmetric and transitive, but two of these fail.

**Final Answer:**  $R$  is reflexive but neither symmetric nor transitive  $\Rightarrow$  B

**Answer: (B)** [Go Back to Q3](#)

**Q4.**

### Solution

**Concept — Principal value of  $\sin^{-1}$ :** The principal value branch of  $\sin^{-1} x$  is  $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ .

**Step 1 — Set up the equation:** Let  $\sin^{-1}\left(-\frac{1}{2}\right) = \theta$ , so  $\sin \theta = -\frac{1}{2}$ .

**Step 2 — Find  $\theta$  in the branch:** We know  $\sin \frac{\pi}{6} = \frac{1}{2}$ , so  $\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$ .

Since  $-\frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ , it is the principal value.

**Why other options are wrong:** (A)  $\frac{\pi}{6}$  gives  $+\frac{1}{2}$ ; (B)  $\frac{5\pi}{6}$  and (C)  $-\frac{5\pi}{6}$  lie outside the principal branch.

**Final Answer:**  $\sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6} \Rightarrow$  D

**Answer: (D)** [Go Back to Q4](#)



Q5.

**Solution**

**Concept — Chain rule:**  $\frac{d}{dx} \sin(u) = \cos(u) \cdot \frac{du}{dx}$ .

**Step 1 — Identify the inner function:** Let  $u = x^2$ , so  $\frac{du}{dx} = 2x$ .

**Step 2 — Apply the chain rule:**

$$\begin{aligned} \frac{dy}{dx} &= \cos(x^2) \cdot 2x. \\ &= 2x \cos(x^2). \end{aligned}$$

**Why other options are wrong:** (B) omits the factor  $2x$ ; (C) wrongly differentiates the inside as  $\cos x$ ; (D) has an incorrect negative sign.

**Final Answer:**  $\frac{dy}{dx} = 2x \cos(x^2) \Rightarrow \boxed{\text{A}}$

**Answer: (A)** [Go Back to Q5](#)

Q6.

**Solution**

**Concept — Increasing function:**  $f$  is increasing where  $f'(x) > 0$ .

**Step 1 — Differentiate:**

$$\begin{aligned} f(x) &= x^2 - 4x + 3. \\ f'(x) &= 2x - 4. \end{aligned}$$

**Step 2 — Solve  $f'(x) > 0$ :**

$$\begin{aligned} 2x - 4 &> 0. \\ 2x &> 4. \\ x &> 2. \end{aligned}$$

So  $f$  is increasing on  $(2, \infty)$ , which matches the right branch of the parabola.

**Why other options are wrong:** (A) and (D) describe where  $f'(x) < 0$  (decreasing); (B) is false since the parabola first falls then rises.

**Final Answer:**  $f$  increases on  $(2, \infty) \Rightarrow \boxed{\text{C}}$

**Answer: (C)** [Go Back to Q6](#)



Q7.

**Solution**

**Concept — Standard integral:**  $\frac{d}{dx} \tan x = \sec^2 x$ , hence  $\int \sec^2 x dx = \tan x + C$ .

**Step 1 — Recall the antiderivative:** The derivative of  $\tan x$  is  $\sec^2 x$ .

**Step 2 — Write the integral:**

$$\int \sec^2 x dx = \tan x + C.$$

**Why other options are wrong:** (A) is  $\int \csc^2 x dx$ ; (C) is  $\int \sec x \tan x dx$ ; (D) is  $\int \tan x dx$ .

**Final Answer:**  $\int \sec^2 x dx = \tan x + C \Rightarrow \boxed{\text{B}}$

**Answer: (B)** [Go Back to Q7](#)

Q8.

**Solution**

**Concept — Order and degree:** The *order* is the highest derivative present; the *degree* is the power of that highest derivative when the equation is polynomial in derivatives.

**Step 1 — Highest derivative:** The highest derivative is  $\frac{d^2 y}{dx^2}$ , so the order is 2.

**Step 2 — Its power:** The term  $\frac{d^2 y}{dx^2}$  appears to the first power (the cube is on  $\frac{dy}{dx}$ , a lower derivative).

So the degree is 1.

**Why other options are wrong:** (A),(B),(C) misread either the highest derivative or its power.

**Final Answer:** Order 2, degree 1  $\Rightarrow \boxed{\text{D}}$

**Answer: (D)** [Go Back to Q8](#)



Q9.

**Solution**

**Concept — Magnitude of a vector:** For  $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ ,  $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$ .

**Step 1 — Square the components:**

$$\begin{aligned} 1^2 + 2^2 + 2^2 &= 1 + 4 + 4. \\ &= 9. \end{aligned}$$

**Step 2 — Take the square root:**

$$|\vec{a}| = \sqrt{9} = 3.$$

**Why other options are wrong:** (B) 5 and (C) 9 skip or misuse the square root; (D)  $\sqrt{5}$  uses wrong components.

**Final Answer:**  $|\vec{a}| = 3 \Rightarrow \boxed{\text{A}}$

**Answer: (A)** [Go Back to Q9](#)

Q10.

**Solution**

**Concept — Direction cosines:** Direction cosines are the direction ratios divided by their magnitude  $\sqrt{a^2 + b^2 + c^2}$ .

**Step 1 — Magnitude of ratios 1, -2, 2:**

$$\sqrt{1^2 + (-2)^2 + 2^2} = \sqrt{1 + 4 + 4} = \sqrt{9} = 3.$$

**Step 2 — Divide each ratio by 3:**

$$\left(\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}\right).$$

**Why other options are wrong:** (A) are the ratios themselves; (B) divides by 9; (D) divides by  $\sqrt{3}$ .

**Final Answer:** Direction cosines  $\left(\frac{1}{3}, -\frac{2}{3}, \frac{2}{3}\right) \Rightarrow \boxed{\text{C}}$

**Answer: (C)** [Go Back to Q10](#)



Q11.

**Solution**

**Concept — Fundamental theorem of calculus:**  $\int_a^b f(x) dx = F(b) - F(a)$ , where  $F' = f$ .

**Step 1 — Antiderivative:**  $\int \cos x dx = \sin x$ .

**Step 2 — Apply limits 0 to  $\frac{\pi}{2}$ :**

$$\begin{aligned} [\sin x]_0^{\pi/2} &= \sin \frac{\pi}{2} - \sin 0 \\ &= 1 - 0 \\ &= 1. \end{aligned}$$

**Why other options are wrong:** (A) 0 and (C)  $-1$  misuse the limits; (D) confuses the value with the interval length.

**Final Answer:** The integral equals 1  $\Rightarrow$  **B**

**Answer: (B)** [Go Back to Q11](#)

Q12.

**Solution**

**Concept — Conditional probability:**  $P(A | B) = \frac{P(A \cap B)}{P(B)}$ , provided  $P(B) \neq 0$ .

**Step 1 — Substitute the values:**

$$P(A | B) = \frac{0.2}{0.5}.$$

**Step 2 — Simplify:**

$$= \frac{2}{5} = 0.4.$$

**Why other options are wrong:** (A) 0.1 multiplies instead of dividing; (B) 0.7 adds; (C) 0.25 inverts the ratio.

**Final Answer:**  $P(A | B) = 0.4 \Rightarrow$  **D**

**Answer: (D)** [Go Back to Q12](#)



Q13.

**Solution**

**Concept — Integration by recognising a derivative:** If the numerator is the derivative of the denominator,  $\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + C$ .

**Step 1 — Spot the pattern:** Here  $f(x) = 1 + x^2$ , so  $f'(x) = 2x$ , which is exactly the numerator.

**Step 2 — Write the result:**

$$\int \frac{2x}{1+x^2} dx = \ln(1+x^2) + C.$$

(The absolute value is dropped since  $1+x^2 > 0$ .)

**Why other options are wrong:** (B) has an extra factor 2; (C) is  $\int \frac{1}{1+x^2} dx$ ; (D) is a wrong antiderivative.

**Final Answer:**  $\ln(1+x^2) + C \Rightarrow \boxed{\text{A}}$

**Answer: (A)** [Go Back to Q13](#)

Q14.

**Solution**

**Concept — Integral of an odd function over a symmetric interval:** If  $g(-x) = -g(x)$ , then  $\int_{-a}^a g(x) dx = 0$ .

**Step 1 — Test the integrand:**  $g(x) = x^3$  gives  $g(-x) = (-x)^3 = -x^3 = -g(x)$ .

So  $x^3$  is an odd function.

**Step 2 — Apply the property:** The interval  $[-2, 2]$  is symmetric about 0, hence

$$\int_{-2}^2 x^3 dx = 0.$$

**Why other options are wrong:** (A),(B),(D) ignore the symmetry; direct integration also gives  $\left[ \frac{x^4}{4} \right]_{-2}^2 = 4 - 4 = 0$ .

**Final Answer:** The integral equals 0  $\Rightarrow \boxed{\text{C}}$

**Answer: (C)** [Go Back to Q14](#)



Q15.

**Solution**

**Concept — Local maximum via second derivative:** At a critical point where  $f'(x) = 0$ , a local maximum occurs if  $f''(x) < 0$ .

**Step 1 — Critical points:**

$$f(x) = x^3 - 3x.$$

$$f'(x) = 3x^2 - 3 = 0.$$

$$x^2 = 1 \Rightarrow x = \pm 1.$$

**Step 2 — Second-derivative test:**

$$f''(x) = 6x.$$

At  $x = -1$ ,  $f''(-1) = -6 < 0$ , so  $x = -1$  gives a local maximum.

**Step 3 — Maximum value:**

$$f(-1) = (-1)^3 - 3(-1) = -1 + 3 = 2.$$

**Why other options are wrong:** (A)  $-2$  is the local minimum value at  $x = 1$ ; (C),(D) are not extreme values.

**Final Answer:** Local maximum value is 2  $\Rightarrow$  **B**

**Answer: (B)** [Go Back to Q15](#)

Q16.

**Solution**

**Concept — Area of a parallelogram:** Area =  $|\vec{a} \times \vec{b}|$  for adjacent side vectors  $\vec{a}, \vec{b}$ .

**Step 1 — Compute the cross product:**

$$\vec{a} \times \vec{b} = (3\hat{i}) \times (2\hat{j}) = 6(\hat{i} \times \hat{j}) = 6\hat{k}.$$

**Step 2 — Take the magnitude:**

$$|\vec{a} \times \vec{b}| = |6\hat{k}| = 6.$$

(Equivalently, the rectangle has sides 3 and 2, so area =  $3 \times 2 = 6$ .)



**Why other options are wrong:** (A) 5 adds the sides; (B) 1 ignores magnitudes; (C) 12 doubles the area.

**Final Answer:** Area = 6  $\Rightarrow$   D

**Answer:** (D) [Go Back to Q16](#)

Q17.

### Solution

**Concept — Angle between two lines:** If  $\vec{b}_1 \cdot \vec{b}_2 = 0$ , the lines are perpendicular ( $90^\circ$ ).

**Step 1 — Dot product of direction ratios:**

$$\begin{aligned}(1)(2) + (2)(-1) + (3)(0) \\ = 2 - 2 + 0 \\ = 0.\end{aligned}$$

**Step 2 — Interpret:** A zero dot product means the lines are perpendicular, so the angle is  $90^\circ$ .

**Why other options are wrong:** (B)  $0^\circ$  needs parallel ratios; (C),(D) would need a non-zero dot product giving those cosines.

**Final Answer:** The angle is  $90^\circ \Rightarrow$   A

**Answer:** (A) [Go Back to Q17](#)

Q18.

### Solution

**Concept — Union with independent events:**  $P(A \cup B) = P(A) + P(B) - P(A)P(B)$  when  $A, B$  are independent.

**Step 1 — Product term:**

$$P(A)P(B) = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}.$$

**Step 2 — Substitute:**

$$P(A \cup B) = \frac{1}{3} + \frac{1}{2} - \frac{1}{6}.$$



$$= \frac{2}{6} + \frac{3}{6} - \frac{1}{6}$$

$$= \frac{4}{6} = \frac{2}{3}$$

**Why other options are wrong:** (A)  $\frac{5}{6}$  forgets to subtract the intersection; (B)  $\frac{1}{6}$  is only  $P(A \cap B)$ ; (D)  $\frac{1}{2}$  uses wrong arithmetic.

**Final Answer:**  $P(A \cup B) = \frac{2}{3} \Rightarrow$   C

Answer: (C) [Go Back to Q18](#)

Q19.

### Solution

**Concept — Assertion–Reason analysis:** Judge each statement's truth, then decide whether R correctly explains A.

**Step 1 — Assertion:**  $f(x) = |x|$  is continuous everywhere, including at  $x = 0$  (left and right limits both equal  $0 = f(0)$ ). So A is **true**.

**Step 2 — Reason:** "Every differentiable function is continuous" is a standard true theorem. So R is **true**.

**Step 3 — Does R explain A?**  $|x|$  is *not* differentiable at  $x = 0$ , so its continuity there cannot be justified by differentiability. Hence R is a true statement but *not* the correct explanation of A.

**Why other options are wrong:** (A) needs R to explain A (it does not); (C),(D) misjudge a truth value.

**Final Answer:** Both true, R not the correct explanation  $\Rightarrow$   B

Answer: (B) [Go Back to Q19](#)

Q20.

### Solution

**Concept — Inverse cosine of a cosine:**  $\cos^{-1}(\cos \theta) = \theta$  only when  $\theta \in [0, \pi]$ ; otherwise reduce to the principal branch.

**Step 1 — Test the Assertion:**  $\frac{7\pi}{6} \notin [0, \pi]$ , so we cannot simply write the answer as  $\frac{7\pi}{6}$ .



Now  $\cos \frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$ , and  $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5\pi}{6}$ .

Hence the correct value is  $\frac{5\pi}{6}$ , not  $\frac{7\pi}{6}$ . So A is **false**.

**Step 2 — Test the Reason:** The principal value branch of  $\cos^{-1}$  is indeed  $[0, \pi]$ . So R is **true**.

**Why other options are wrong:** (A),(B) require A true; (C) requires R false.

**Final Answer:** A false, R true  $\Rightarrow$  **D**

**Answer: (D)** [Go Back to Q20](#)

**Q21.**

### Solution

**Concept — Principal values:**  $\tan^{-1} 1$  lies in  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$  and  $\cos^{-1}$  lies in  $[0, \pi]$ .

**Step 1 — Evaluate  $\tan^{-1} 1$ :**

$$\tan \frac{\pi}{4} = 1 \Rightarrow \tan^{-1} 1 = \frac{\pi}{4}.$$

**Step 2 — Evaluate  $\cos^{-1}\left(-\frac{1}{2}\right)$ :**

$$\cos \frac{2\pi}{3} = -\frac{1}{2} \Rightarrow \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}.$$

**Step 3 — Add:**

$$\frac{\pi}{4} + \frac{2\pi}{3} = \frac{3\pi}{12} + \frac{8\pi}{12} = \frac{11\pi}{12}.$$

**Final Answer:**  $\tan^{-1} 1 + \cos^{-1}\left(-\frac{1}{2}\right) = \frac{11\pi}{12}$ . [Go Back to Q21](#)

**Q22.**

### Solution

**Concept — Logarithmic differentiation:** Used when the variable appears in both base and exponent.

**Step 1 — Take natural log of both sides:**

$$y = (\sin x)^x.$$



$$\ln y = x \ln(\sin x).$$

**Step 2 — Differentiate both sides w.r.t.  $x$ :** (product rule on the right)

$$\frac{1}{y} \frac{dy}{dx} = 1 \cdot \ln(\sin x) + x \cdot \frac{\cos x}{\sin x}.$$

$$\frac{1}{y} \frac{dy}{dx} = \ln(\sin x) + x \cot x.$$

**Step 3 — Solve for  $\frac{dy}{dx}$ :**

$$\frac{dy}{dx} = y [\ln(\sin x) + x \cot x].$$

$$= (\sin x)^x [\ln(\sin x) + x \cot x].$$

**Final Answer:**  $\frac{dy}{dx} = (\sin x)^x [\ln(\sin x) + x \cot x]$ . [Go Back to Q22](#)

**Q23.**

### Solution

**Concept — Integration by parts:**  $\int u dv = uv - \int v du$ , choosing  $u$  by the ILATE rule.

**Step 1 — Choose parts:** Let  $u = x$  and  $dv = e^x dx$ .

Then  $du = dx$  and  $v = e^x$ .

**Step 2 — Apply the formula:**

$$\int x e^x dx = x e^x - \int e^x dx.$$

$$= x e^x - e^x + C.$$

$$= e^x(x - 1) + C.$$

**Final Answer:**  $\int x e^x dx = e^x(x - 1) + C$ .

**OR —  $\int \cos^2 x dx$ :**



**Step 1 — Use the identity**  $\cos^2 x = \frac{1 + \cos 2x}{2}$ :

$$\int \cos^2 x \, dx = \int \frac{1 + \cos 2x}{2} \, dx.$$

**Step 2 — Integrate term by term:**

$$= \frac{1}{2} \int 1 \, dx + \frac{1}{2} \int \cos 2x \, dx.$$

$$= \frac{x}{2} + \frac{1}{2} \cdot \frac{\sin 2x}{2} + C.$$

$$= \frac{x}{2} + \frac{\sin 2x}{4} + C.$$

**Final Answer (OR):**  $\int \cos^2 x \, dx = \frac{x}{2} + \frac{\sin 2x}{4} + C$ . **Go Back to Q23**

**Q24.**

### Solution

**Concept — Perpendicular vectors:**  $\vec{a} \perp \vec{b} \iff \vec{a} \cdot \vec{b} = 0$ .

**Step 1 — Compute the dot product:**

$$\vec{a} \cdot \vec{b} = (2)(1) + (\lambda)(-2) + (1)(3).$$

$$= 2 - 2\lambda + 3.$$

$$= 5 - 2\lambda.$$

**Step 2 — Set equal to zero and solve:**

$$5 - 2\lambda = 0.$$

$$2\lambda = 5.$$

$$\lambda = \frac{5}{2}.$$

**Final Answer:**  $\lambda = \frac{5}{2}$ . **Go Back to Q24**



Q25.

**Solution**

**Concept — Equally likely outcomes:** Probability =  $\frac{\text{favourable outcomes}}{\text{total outcomes}}$ .

**Step 1 — Total outcomes:** Two throws of a die give  $6 \times 6 = 36$  equally likely outcomes.

**Step 2 — Favourable outcomes (sum = 9):**

$$(3, 6), (6, 3), (4, 5), (5, 4).$$

There are 4 such outcomes.

**Step 3 — Probability:**

$$P(\text{sum} = 9) = \frac{4}{36} = \frac{1}{9}.$$

**Final Answer:**  $P(\text{sum} = 9) = \frac{1}{9}$ .

**OR —  $P(A | B)$ :**

**Step 1 — Formula:**  $P(A | B) = \frac{P(A \cap B)}{P(B)}$ .

**Step 2 — Substitute:**

$$P(A | B) = \frac{0.2}{0.3} = \frac{2}{3}.$$

**Final Answer (OR):**  $P(A | B) = \frac{2}{3}$ . **Go Back to Q25**

Q26.

**Solution**

**Concept — Partial fractions:** Split a proper rational function into simpler fractions, then integrate each.

**Step 1 — Set up the decomposition:**

$$\frac{x}{(x+1)(x+2)} = \frac{A}{x+1} + \frac{B}{x+2}.$$

$$x = A(x+2) + B(x+1).$$

**Step 2 — Find A and B:**

Put  $x = -1$ :  $-1 = A(1) + B(0) \Rightarrow A = -1$ .

Put  $x = -2$ :  $-2 = A(0) + B(-1) \Rightarrow B = 2$ .



**Step 3 — Integrate:**

$$\int \frac{x}{(x+1)(x+2)} dx = \int \left( \frac{-1}{x+1} + \frac{2}{x+2} \right) dx.$$

$$= -\ln|x+1| + 2\ln|x+2| + C.$$

**Final Answer:**  $-\ln|x+1| + 2\ln|x+2| + C$ . [Go Back to Q26](#)

**Q27.**

### Solution

**Concept — Linear differential equation:** For  $\frac{dy}{dx} + Py = Q$ , the integrating factor is I.F. =  $e^{\int P dx}$  and the solution is  $y \cdot \text{I.F.} = \int Q \cdot \text{I.F.} dx$ .

**Step 1 — Identify  $P, Q$ :** Here  $P = 1, Q = e^x$ .

**Step 2 — Integrating factor:**

$$\text{I.F.} = e^{\int 1 dx} = e^x.$$

**Step 3 — Solve:**

$$y e^x = \int e^x \cdot e^x dx = \int e^{2x} dx.$$

$$y e^x = \frac{e^{2x}}{2} + C.$$

$$y = \frac{e^x}{2} + C e^{-x}.$$

**Final Answer:**  $y = \frac{e^x}{2} + C e^{-x}$ .

**OR — Variables separable**  $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ :

**Step 1 — Separate variables:**

$$\frac{dy}{1+y^2} = \frac{dx}{1+x^2}.$$

**Step 2 — Integrate both sides:**

$$\tan^{-1} y = \tan^{-1} x + C.$$

**Final Answer (OR):**  $\tan^{-1} y = \tan^{-1} x + C$ . [Go Back to Q27](#)



Q28.

**Solution**

**Concept — Inverse by adjoint:**  $A^{-1} = \frac{1}{|A|} \text{adj}(A)$ , valid when  $|A| \neq 0$ .

**Step 1 — Determinant:**

$$|A| = \begin{vmatrix} 2 & 3 \\ 1 & 2 \end{vmatrix} = (2)(2) - (3)(1) = 4 - 3 = 1.$$

**Step 2 — Adjoint of a  $2 \times 2$  matrix:** Swap the leading diagonal entries and negate the off-diagonal entries:

$$\text{adj}(A) = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}.$$

**Step 3 — Inverse:**

$$A^{-1} = \frac{1}{1} \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}.$$

**Final Answer:**  $A^{-1} = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$ . [Go Back to Q28](#)

Q29.

**Solution**

**Concept — Monotonicity:**  $f$  increases where  $f'(x) > 0$  and decreases where  $f'(x) < 0$ .

**Step 1 — Differentiate:**

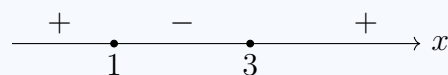
$$f(x) = x^3 - 6x^2 + 9x + 5.$$

$$f'(x) = 3x^2 - 12x + 9.$$

$$= 3(x^2 - 4x + 3) = 3(x - 1)(x - 3).$$

**Step 2 — Critical points:**  $f'(x) = 0$  at  $x = 1$  and  $x = 3$ .

**Step 3 — Sign analysis:**



For  $x < 1$ :  $f' > 0$ ; for  $1 < x < 3$ :  $f' < 0$ ; for  $x > 3$ :  $f' > 0$ .



**Conclusion:** Increasing on  $(-\infty, 1) \cup (3, \infty)$ ; decreasing on  $(1, 3)$ .

**Final Answer:** Increasing on  $(-\infty, 1) \cup (3, \infty)$ , decreasing on  $(1, 3)$ .

**OR — Tangent to  $y = x^2$  at  $(1, 1)$ :**

**Step 1 — Slope:**  $\frac{dy}{dx} = 2x$ , so at  $x = 1$  slope = 2.

**Step 2 — Equation:**  $y - 1 = 2(x - 1) \Rightarrow y = 2x - 1$ .

**Final Answer (OR):**  $y = 2x - 1$ . [Go Back to Q29](#)

**Q30.**

### Solution

**Concept — Vector equation of a line:** A line through point  $\vec{a}$  parallel to  $\vec{b}$  is  $\vec{r} = \vec{a} + \lambda\vec{b}$ .

**Step 1 — Write the equation:** With  $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$  and  $\vec{b} = 3\hat{i} + 2\hat{j} - 2\hat{k}$ ,

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(3\hat{i} + 2\hat{j} - 2\hat{k}).$$

**Step 2 — Test the point  $(4, 4, 1)$ :** The point lies on the line if some  $\lambda$  satisfies all three component equations.

$$1 + 3\lambda = 4 \Rightarrow \lambda = 1.$$

$$2 + 2\lambda = 4 \Rightarrow \lambda = 1.$$

$$3 - 2\lambda = 1 \Rightarrow \lambda = 1.$$

**Step 3 — Conclusion:** All three give the same value  $\lambda = 1$ , so the point  $(4, 4, 1)$  lies on the line.

**Final Answer:**  $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(3\hat{i} + 2\hat{j} - 2\hat{k})$ ; the point  $(4, 4, 1)$  lies on it (at  $\lambda = 1$ ). [Go Back to Q30](#)

**Q31.**

### Solution

**Concept — One-one (injective) and onto (surjective):** *One-one:*  $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ . *Onto:* every  $y$  in the codomain has a pre-image.

**Step 1 — One-one:** Suppose  $f(x_1) = f(x_2)$ .

$$2x_1 + 3 = 2x_2 + 3.$$



$$2x_1 = 2x_2.$$

$$x_1 = x_2.$$

Hence  $f$  is one-one.

**Step 2 — Onto:** Let  $y \in \mathbb{R}$  be arbitrary. Solve  $y = 2x + 3$  for  $x$ :

$$x = \frac{y - 3}{2} \in \mathbb{R}.$$

Then  $f(x) = 2 \left( \frac{y - 3}{2} \right) + 3 = y$ .

So every  $y$  has a pre-image;  $f$  is onto.

**Step 3 — Conclusion:**  $f$  is both one-one and onto, hence bijective.

**Final Answer:**  $f(x) = 2x + 3$  is one-one and onto (a bijection). [Go Back to Q31](#)

**Q32.**

### Solution

**Concept — Matrix method:** Write  $AX = B$ ; then  $X = A^{-1}B$  when  $|A| \neq 0$ .

**Step 1 — Form the matrices:**

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}.$$

**Step 2 — Determinant of  $A$ :**

$$\begin{aligned} |A| &= 1(1 \cdot 1 - 3 \cdot (-2)) - 1(0 \cdot 1 - 3 \cdot 1) + 1(0 \cdot (-2) - 1 \cdot 1). \\ &= 1(1 + 6) - 1(0 - 3) + 1(0 - 1). \\ &= 7 + 3 - 1 = 9 (\neq 0). \end{aligned}$$

**Step 3 — Cofactors and adjoint:** Computing the cofactor matrix and transposing gives

$$\text{adj}(A) = \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}.$$



**Step 4 — Solve**  $X = \frac{1}{|A|} \text{adj}(A) B$ :

$$\begin{aligned} X &= \frac{1}{9} \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix} \\ &= \frac{1}{9} \begin{bmatrix} 7 \cdot 6 - 3 \cdot 11 + 2 \cdot 0 \\ 3 \cdot 6 + 0 \cdot 11 - 3 \cdot 0 \\ -1 \cdot 6 + 3 \cdot 11 + 1 \cdot 0 \end{bmatrix} \\ &= \frac{1}{9} \begin{bmatrix} 42 - 33 \\ 18 \\ -6 + 33 \end{bmatrix} = \frac{1}{9} \begin{bmatrix} 9 \\ 18 \\ 27 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} . \end{aligned}$$

**Step 5 — Verify:**  $x + y + z = 1 + 2 + 3 = 6 \checkmark$ ;  $y + 3z = 2 + 9 = 11 \checkmark$ ;  
 $x - 2y + z = 1 - 4 + 3 = 0 \checkmark$ .

**Final Answer:**  $x = 1, y = 2, z = 3$ . [Go Back to Q32](#)

**Q33.**

### Solution

**Concept — Area by integration:** For a curve symmetric about the  $x$ -axis, total area =  $2 \times$  area above the axis.

**Step 1 — Express  $y$ :** From  $y^2 = 4x$ , the upper branch is  $y = 2\sqrt{x}$ .

**Step 2 — Set up the integral** (from  $x = 0$  to  $x = 1$ , doubled for symmetry):

$$\begin{aligned} \text{Area} &= 2 \int_0^1 2\sqrt{x} \, dx \\ &= 4 \int_0^1 x^{1/2} \, dx. \end{aligned}$$

**Step 3 — Integrate:**

$$\begin{aligned} &= 4 \left[ \frac{x^{3/2}}{3/2} \right]_0^1 \\ &= 4 \cdot \frac{2}{3} [x^{3/2}]_0^1 \\ &= \frac{8}{3} (1 - 0) = \frac{8}{3}. \end{aligned}$$

**Final Answer:** Area =  $\frac{8}{3}$  square units.



**OR — Region between  $y = x$  and  $y = x^2$ ,  $0 \leq x \leq 1$ :**

**Step 1 — Top minus bottom:** On  $[0, 1]$ ,  $x \geq x^2$ , so

$$\text{Area} = \int_0^1 (x - x^2) dx.$$

**Step 2 — Integrate:**

$$= \left[ \frac{x^2}{2} - \frac{x^3}{3} \right]_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}.$$

**Final Answer (OR):** Area =  $\frac{1}{6}$  square units. [Go Back to Q33](#)

**Q34.**

### Solution

**Concept — Corner-point method:** The optimum of a linear objective over a bounded feasible region occurs at a corner (vertex).

**Step 1 — Find the corner points:** The constraints  $x + y \leq 50$ ,  $3x + y \leq 90$ ,  $x \geq 0$ ,  $y \geq 0$  bound the region with vertices:

$$O(0, 0), \quad (30, 0), \quad (0, 50).$$

The lines  $x + y = 50$  and  $3x + y = 90$  intersect where

$$(3x + y) - (x + y) = 90 - 50 \Rightarrow 2x = 40 \Rightarrow x = 20, y = 30.$$

So the fourth vertex is  $(20, 30)$ .

**Step 2 — Evaluate  $Z = 4x + y$  at each corner:**

$$Z(0, 0) = 0.$$

$$Z(30, 0) = 4(30) + 0 = 120.$$

$$Z(20, 30) = 4(20) + 30 = 110.$$

$$Z(0, 50) = 4(0) + 50 = 50.$$

**Step 3 — Pick the maximum:** The largest value is 120 at  $(30, 0)$ .

**Final Answer:**  $Z$  is maximum = 120 at  $(x, y) = (30, 0)$ . [Go Back to Q34](#)



Q35.

**Solution**

**Concept — Shortest distance between skew lines:** For  $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$  and  $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$ ,

$$d = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$$

**Step 1 — Identify vectors:**  $\vec{a}_1 = \hat{i} + \hat{j}$ ,  $\vec{b}_1 = 2\hat{i} - \hat{j} + \hat{k}$ ;  $\vec{a}_2 = 2\hat{i} + \hat{j} - \hat{k}$ ,  $\vec{b}_2 = 3\hat{i} - 5\hat{j} + 2\hat{k}$ .

**Step 2 — Cross product  $\vec{b}_1 \times \vec{b}_2$ :**

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix}$$

$$\begin{aligned} &= \hat{i}[(-1)(2) - (1)(-5)] - \hat{j}[(2)(2) - (1)(3)] + \hat{k}[(2)(-5) - (-1)(3)] \\ &= \hat{i}(-2 + 5) - \hat{j}(4 - 3) + \hat{k}(-10 + 3) \\ &= 3\hat{i} - \hat{j} - 7\hat{k}. \end{aligned}$$

**Step 3 — Its magnitude:**

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{3^2 + (-1)^2 + (-7)^2} = \sqrt{9 + 1 + 49} = \sqrt{59}.$$

**Step 4 —  $(\vec{a}_2 - \vec{a}_1)$  and the dot product:**

$$\vec{a}_2 - \vec{a}_1 = (2 - 1)\hat{i} + (1 - 1)\hat{j} + (-1 - 0)\hat{k} = \hat{i} - \hat{k}.$$

$$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = (1)(3) + (0)(-1) + (-1)(-7) = 3 + 0 + 7 = 10.$$

**Step 5 — Distance:**

$$d = \frac{|10|}{\sqrt{59}} = \frac{10}{\sqrt{59}} \text{ units.}$$

**Final Answer:** Shortest distance =  $\frac{10}{\sqrt{59}}$  units.

**OR — Angle between line and plane:**

**Step 1 — Direction  $\vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}$ , normal  $\vec{n} = \hat{i} + \hat{j} + \hat{k}$ .**

**Step 2 — Use  $\sin \theta = \frac{|\vec{b} \cdot \vec{n}|}{|\vec{b}||\vec{n}|}$ :**

$$\vec{b} \cdot \vec{n} = 1 + 2 + 2 = 5, \quad |\vec{b}| = 3, \quad |\vec{n}| = \sqrt{3}.$$



$$\sin \theta = \frac{5}{3\sqrt{3}}.$$

**Final Answer (OR):**  $\theta = \sin^{-1}\left(\frac{5}{3\sqrt{3}}\right)$ . **Go Back to Q35**

**Q36.**

### Solution

**Concept — Maxima by derivatives:** Build the volume function, set  $\frac{dV}{dx} = 0$ , and select the value giving a genuine maximum.

**(i) Volume function:** After cutting squares of side  $x$ , the base is  $(18 - 2x)$  by  $(18 - 2x)$  and the height is  $x$ :

$$V = x(18 - 2x)^2, \quad 0 < x < 9.$$

**(ii) Differentiate:**

$$\begin{aligned} \frac{dV}{dx} &= (18 - 2x)^2 + x \cdot 2(18 - 2x)(-2). \\ &= (18 - 2x)[(18 - 2x) - 4x]. \\ &= (18 - 2x)(18 - 6x). \end{aligned}$$

**(iii) Critical points and maximum:** Set  $\frac{dV}{dx} = 0$ :

$$18 - 2x = 0 \Rightarrow x = 9 \quad (\text{rejected; gives } V = 0).$$

$$18 - 6x = 0 \Rightarrow x = 3.$$

For  $x < 3$ ,  $\frac{dV}{dx} > 0$  and for  $3 < x < 9$ ,  $\frac{dV}{dx} < 0$ , so  $x = 3$  gives a maximum.

Maximum volume:

$$V(3) = 3(18 - 6)^2 = 3(12)^2 = 3 \times 144 = 432.$$

**Final Answer:**  $V = x(18 - 2x)^2$ ;  $\frac{dV}{dx} = (18 - 2x)(18 - 6x)$ ; maximum at  $x = 3$  cm with  $V_{\max} = 432 \text{ cm}^3$ . **Go Back to Q36**





(ii) Times at rest: Set  $v(t) = 0$ :

$$3t^2 - 12t + 9 = 0.$$

$$t^2 - 4t + 3 = 0.$$

$$(t - 1)(t - 3) = 0 \Rightarrow t = 1 \text{ s and } t = 3 \text{ s}.$$

(iii) Acceleration at  $t = 2$ :

$$a(t) = \frac{dv}{dt} = 6t - 12.$$

$$a(2) = 6(2) - 12 = 0.$$

At  $t = 2$ ,  $v(2) = 3(4) - 24 + 9 = -3 \text{ m/s}$  ( $\neq 0$ ) and  $a = 0$ , so the speed is momentarily neither increasing nor decreasing (the velocity is at its minimum).

**Final Answer:**  $v(t) = 3t^2 - 12t + 9$ ; at rest at  $t = 1 \text{ s}$  and  $t = 3 \text{ s}$ ;  $a(2) = 0 \text{ m/s}^2$  (instantaneously neither speeding up nor slowing down). [Go Back to Q38](#)



**Answer Key – Section A (Q1–Q20)**

| Q  | Ans | Q  | Ans | Q  | Ans | Q  | Ans | Q  | Ans |
|----|-----|----|-----|----|-----|----|-----|----|-----|
| 1  | A   | 2  | C   | 3  | B   | 4  | D   | 5  | A   |
| 6  | C   | 7  | B   | 8  | D   | 9  | A   | 10 | C   |
| 11 | B   | 12 | D   | 13 | A   | 14 | C   | 15 | B   |
| 16 | D   | 17 | A   | 18 | C   | 19 | B   | 20 | D   |

*Sections B–E are descriptive; refer to the Detailed Solutions above for full model answers and step marking.*

