# CBSE Class 12 Mathematics

## Sample Paper – 2

**Duration: 180 Minutes** **Maximum Marks: 80**

### General Instructions

- This question paper contains **38 questions**. **All questions are compulsory**.
- The paper is divided into **five sections** – A, B, C, D and E.
- **Section A** (Q1–Q20) carries **1 mark** each: Q1–Q18 are multiple choice questions and Q19–Q20 are Assertion–Reason questions.
- **Section B** (Q21–Q25) carries **2 marks** each (Very Short Answer).
- **Section C** (Q26–Q31) carries **3 marks** each (Short Answer).
- **Section D** (Q32–Q35) carries **5 marks** each (Long Answer).
- **Section E** (Q36–Q38) carries **4 marks** each (case study based, with sub-parts).
- There is **no overall choice**, but an **internal choice** has been provided in some questions. Attempt only one of the alternatives in such questions.
- There is **no negative marking**. Use of a **calculator is not permitted**.

## Section A (Q1–Q20) – 1 Mark Each

**Q1.** If $A$ is a skew-symmetric matrix, then $A'$ (the transpose of $A$) equals:

(A) $A$

(B) $-A$

(C) $A^2$

(D) $I$

**Q2.** The value of the determinant $\begin{vmatrix} 5 & 2 \\ -3 & 4 \end{vmatrix}$ is:

(A) $26$

(B) $14$

(C) $-26$

(D) $20$

**Q3.** Let $A = \{1, 2, 3\}$ and $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2)\}$ be a relation on $A$. Then $R$ is:

(A) transitive

(B) an equivalence relation

(C) reflexive and symmetric but not transitive

(D) symmetric but not reflexive

**Q4.** The principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is:

(A) $\frac{\pi}{3}$

(B) $-\frac{\pi}{3}$

(C) $\frac{\pi}{6}$

(D) $\frac{2\pi}{3}$

**Q5.** If $y = e^{\tan x}$, then $\frac{dy}{dx}$ equals:

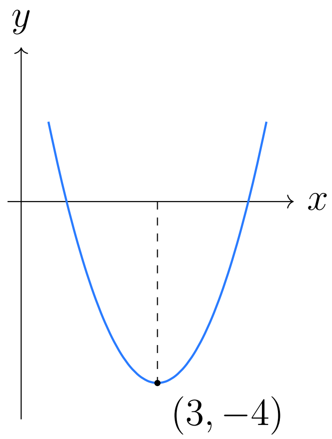
(A) $e^{\tan x}\sec^2 x$

(B) $e^{\tan x}$

(C) $e^{\sec x}\tan x$

(D) $\sec^2 x$

**Q6.** The graph of $f(x) = x^2 - 6x + 5$ is shown below. The function is *increasing* on the interval:




(A) $(-\infty, 3)$

(B) $(-\infty, \infty)$

(C) $(3, \infty)$

(D) $(0, 3)$

**Q7.** $\int \csc^2 x \, dx$ equals:

(A) $\cot x + C$

(B) $\tan x + C$

(C) $-\tan x + C$

(D) $-\cot x + C$

**Q8.** The order and degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{d^2y}{dx^2}\right)^3 + y = 0$ are respectively:

(A) 2 and 3

(B) 3 and 2

(C) 3 and 3

(D) 2 and 2

**Q9.** The magnitude of the vector $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ is:

(A) 7

(B) 49

(C) $\sqrt{7}$

(D) $11$

**Q10.** The direction cosines of a line whose direction ratios are $2, -1, 2$ are:

(A) $2, -1, 2$

(B) $\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}$

(C) $\frac{2}{9}, -\frac{1}{9}, \frac{2}{9}$

(D) $\frac{2}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

**Q11.** $\int_0^{\pi/2} \sin x \, dx$ equals:

(A) $0$

(B) $-1$

(C) $1$

(D) $\frac{\pi}{2}$

**Q12.** If $P(A \cap B) = 0.18$ and $P(B) = 0.6$, then $P(A \mid B)$ equals:

(A) $0.108$

(B) $0.78$

(C) $0.5$

(D) $0.3$

**Q13.** $\int \frac{3x^2}{1+x^3} \, dx$ equals:

(A) $\ln|1+x^3| + C$

(B) $3\ln|1+x^3| + C$

(C) $\frac{1}{1+x^3} + C$

(D) $\tan^{-1}(x^3) + C$

**Q14.** $\int_{-3}^{3} x^3\, dx$ equals:

(A) $81$

(B) $0$

(C) $162$

(D) $40.5$

**Q15.** The local minimum value of $f(x) = x^3 - 12x$ is:

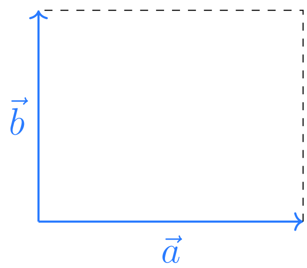
(A) $16$

(B) $12$

(C) $-12$

(D) $-16$

**Q16.** The area of the parallelogram whose adjacent sides are the vectors $\vec{a} = 5\hat{i}$ and $\vec{b} = 4\hat{j}$ (shown below) is:




(A) $9$

(B) $40$

(C) $20$

(D) $10$

**Q17.** The angle between two lines whose direction ratios are $1, 2, 2$ and $2, 4, 4$ is:

(A) $90^\circ$

(B) $0^\circ$

(C) $45^\circ$

(D) $60^\circ$

**Q18.** If $A$ and $B$ are independent events with $P(A) = \frac{1}{4}$ and $P(B) = \frac{2}{5}$, then $P(A \cup B)$ equals:

(A) $\frac{11}{20}$

(B) $\frac{13}{20}$

(C) $\frac{1}{10}$

(D) $\frac{3}{5}$

**Q19.** **Assertion (A):** Every continuous function is differentiable.

**Reason (R):** The function $f(x) = |x|$ is continuous at $x = 0$ but not differentiable at $x = 0$.

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is *not* the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

**Q20.** **Assertion (A):** $\sec^{-1}(2) = \frac{\pi}{3}$.

**Reason (R):** The principal value branch (range) of $\sec^{-1} x$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is *not* the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

## Section B (Q21–Q25) – 2 Marks Each

**Q21.** Find the principal value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{1}{2}\right)$. **[2]**

**Q22.** Differentiate $y = x^x$ with respect to $x$. **[2]**

**Q23.** Evaluate $\int x \sin x \, dx$. **[2]**

**OR**

Evaluate $\int \sin^2 x \, dx$.

**Q24.** Find the value of $\lambda$ for which the vectors $\vec{a} = 3\hat{i}+\lambda\hat{j}+2\hat{k}$ and $\vec{b} = 2\hat{i}+4\hat{j}-\hat{k}$ are perpendicular. **[2]**

**Q25.** A fair die is thrown twice. Find the probability of getting a sum of 7. **[2]**

**OR**

If $P(A) = 0.5$, $P(B) = 0.45$ and $P(A \cap B) = 0.15$, find $P(A \mid B)$.

## Section C (Q26–Q31) – 3 Marks Each

**Q26.** Evaluate $\int \frac{1}{(x-1)(x+2)} \, dx$. **[3]**

**Q27.** Solve the differential equation $\frac{dy}{dx} + 2y = e^{-x}$. **[3]**

**OR**

Solve the differential equation $\frac{dy}{dx} = e^{x-y}$.

**Q28.** Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ using the adjoint method. **[3]**

**Q29.** Find the intervals in which $f(x) = x^3 - 3x^2 - 9x + 1$ is increasing or decreasing. **[3]**

**OR**

Find the equation of the tangent to the curve $y = x^3$ at the point $(1, 1)$.

**Q30.** Find the vector equation of the line passing through the point $(2, -1, 3)$ and parallel to the vector $\hat{i} + 2\hat{j} - \hat{k}$. Hence check whether the point $(4, 3, 1)$ lies on this line. **[3]**

**Q31.** Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 3x - 2$ is one-one and onto. **[3]**

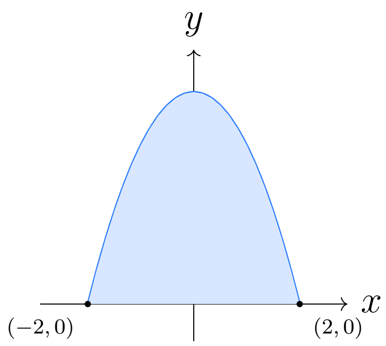
## Section D (Q32–Q35) – 5 Marks Each

**Q32.** Using the matrix method, solve the following system of linear equations:

$$x + 2y + z = 3, \qquad 2x - y + z = 8, \qquad x + y - z = -2.$$

**[5]**

**Q33.** Using integration, find the area of the region bounded by the parabola $y = 4 - x^2$ and the $x$-axis.



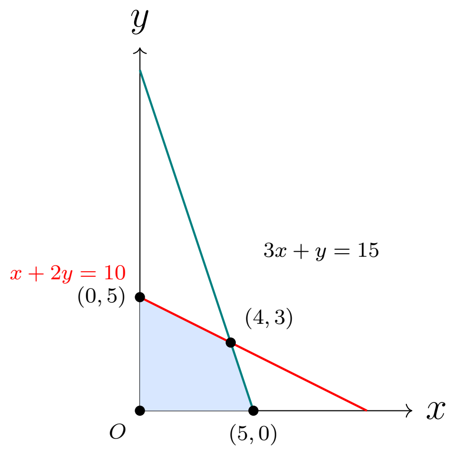

**[5]**

**OR**

Using integration, find the area of the region bounded by the curves $y = x^2$ and $y = 2x$ between $x = 0$ and $x = 2$.

**Q34.** Solve the following linear programming problem graphically. Maximise $Z = 3x + 2y$ subject to the constraints

$$x + 2y \leq 10, \qquad 3x + y \leq 15, \qquad x \geq 0,\ y \geq 0.$$




**[5]**

**Q35.** Find the shortest distance between the lines

$$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} + 2\hat{j} - \hat{k}) \quad \text{and} \quad \vec{r} = (2\hat{i} - \hat{j} + \hat{k}) + \mu(2\hat{i} - \hat{j} + \hat{k}).$$

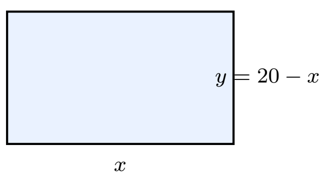
**[5]**

**OR**

Find the distance of the point $(1, 2, -1)$ from the plane $2x - y + 2z + 3 = 0$.

## Section E (Q36–Q38) – 4 Marks Each (Case Study)

**Q36. Case Study – Fencing a Garden.**
A farmer has $40$ metres of fencing wire and wishes to enclose a rectangular garden of length $x$ metres and breadth $y$ metres using all the wire, as shown.




Based on the above, answer the following:

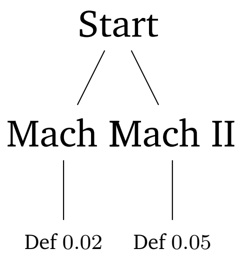
(i) Express the area $A$ of the garden as a function of $x$. (1)

(ii) Find $\dfrac{dA}{dx}$. (1)

(iii) Find the value of $x$ for which the area is maximum, and the maximum area. (2)

**Q37. Case Study – Factory Machines.**

In a factory, Machine I manufactures $60\%$ of the items and Machine II manufactures the remaining $40\%$. Of their output, $2\%$ of the items from Machine I and $5\%$ of the items from Machine II are defective. An item is picked at random and is found to be defective.




Based on the above, answer the following:

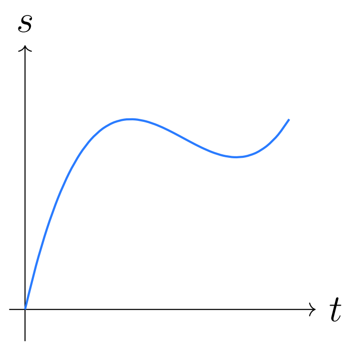
(i) Write $P(\text{Machine I})$ and $P(\text{Machine II})$. (1)

(ii) Write $P(\text{Defective} \mid \text{Machine I})$ and $P(\text{Defective} \mid \text{Machine II})$. (1)

(iii) Using Bayes' theorem, find the probability that the defective item was produced by Machine I. (2)

**Q38. Case Study – Motion of a Particle.**

A particle moves along a straight line so that its position (in metres) at time $t$ seconds is given by $s(t) = t^3 - 9t^2 + 24t$, for $t \geq 0$.




Based on the above, answer the following:

(i) Find the velocity $v(t)$ of the particle. (1)

(ii) Find the time(s) at which the particle is momentarily at rest. (1)

(iii) Find the acceleration of the particle at $t = 5$ s, and state whether it is speeding up or slowing down at that instant. (2)

# Detailed Solutions

## Q1.

**Solution**

**Concept — Skew-symmetric matrix:** A square matrix $A$ is skew-symmetric if its transpose equals its own negative, that is $A' = -A$.

**Step 1 — Recall the definition:** By definition of a skew-symmetric matrix, $a_{ji} = -a_{ij}$ for all entries.

**Step 2 — Write in matrix form:** Collecting all entries, this is exactly the statement

$$A' = -A.$$

**Why other options are wrong:**

- (A) $A' = A$ is the condition for a *symmetric* matrix, not skew-symmetric.
- (C) $A^2$ is a product, unrelated to the transpose condition.
- (D) $I$ would force $A$ to be a fixed matrix, which is false in general.

**Final Answer:** $A' = -A \Rightarrow \boxed{\text{B}}$

**Answer: (B)** Go Back to Q1

## Q2.

**Solution**

**Concept — Value of a $2 \times 2$ determinant:** For $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$, the value is $ad - bc$.

**Step 1 — Identify entries:** Here $a = 5$, $b = 2$, $c = -3$, $d = 4$.

**Step 2 — Apply the formula:**

$$ad - bc = (5)(4) - (2)(-3).$$

$$= 20 - (-6).$$

$$= 20 + 6.$$

$$= 26.$$

**Why other options are wrong:** (B) $14$ comes from $20 - 6$ (wrong sign on $bc$); (C) $-26$ flips the overall sign; (D) $20$ keeps only the product $ad$.

**Final Answer:** The determinant is $26 \Rightarrow \boxed{\text{A}}$

**Answer: (A)** Go Back to Q2

## Q3.

**Solution**

**Concept — Properties of a relation:** *Reflexive*: $(a, a) \in R$ for all $a$. *Symmetric*: $(a, b) \in R \Rightarrow (b, a) \in R$. *Transitive*: $(a, b), (b, c) \in R \Rightarrow (a, c) \in R$.

**Step 1 — Reflexive?** $(1, 1), (2, 2), (3, 3) \in R$, so $R$ is reflexive.

**Step 2 — Symmetric?** The off-diagonal pairs occur in both orders: $(1, 2)$ and $(2, 1)$, $(2, 3)$ and $(3, 2)$. So $R$ is symmetric.

**Step 3 — Transitive?** $(1, 2) \in R$ and $(2, 3) \in R$, but $(1, 3) \notin R$.

Hence $R$ is not transitive.

**Why other options are wrong:** (A) fails by Step 3; (B) an equivalence relation needs transitivity, which fails; (D) is false since $R$ is reflexive.

**Final Answer:** $R$ is reflexive and symmetric but not transitive $\Rightarrow \boxed{\text{C}}$

**Answer: (C)** Go Back to Q3

## Q4.

**Solution**

**Concept — Principal value of $\cos^{-1}$:** The principal value branch of $\cos^{-1} x$ is $[0, \pi]$.

**Step 1 — Set up the equation:** Let $\cos^{-1}\left(-\frac{1}{2}\right) = \theta$, so $\cos\theta = -\frac{1}{2}$ with $\theta \in [0, \pi]$.

**Step 2 — Find $\theta$ in the branch:** We know $\cos\frac{\pi}{3} = \frac{1}{2}$, so the angle with cosine $-\frac{1}{2}$ in $[0, \pi]$ is

$$\theta = \pi - \frac{\pi}{3} = \frac{2\pi}{3}.$$

Since $\frac{2\pi}{3} \in [0, \pi]$, it is the principal value.

**Why other options are wrong:** (A) $\frac{\pi}{3}$ gives $+\frac{1}{2}$; (B) $-\frac{\pi}{3}$ lies outside $[0, \pi]$; (C) $\frac{\pi}{6}$ gives $\frac{\sqrt{3}}{2}$.

**Final Answer:** $\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3} \Rightarrow \boxed{\text{D}}$

$\boxed{\text{Answer: (D)}}$ Go Back to Q4

## Q5.

### Solution

**Concept — Chain rule:** $\frac{d}{dx}e^u = e^u \cdot \frac{du}{dx}$.

**Step 1 — Identify the inner function:** Let $u = \tan x$, so $\frac{du}{dx} = \sec^2 x$.

**Step 2 — Apply the chain rule:**

$$\frac{dy}{dx} = e^{\tan x} \cdot \sec^2 x.$$

$$= e^{\tan x} \sec^2 x.$$

**Why other options are wrong:** (B) omits the factor $\sec^2 x$; (C) has the wrong exponent and factor; (D) drops the exponential factor.

**Final Answer:** $\frac{dy}{dx} = e^{\tan x} \sec^2 x \Rightarrow \boxed{\text{A}}$

$\boxed{\text{Answer: (A)}}$ Go Back to Q5

## Q6.

### Solution

**Concept — Increasing function:** $f$ is increasing where $f'(x) > 0$.

**Step 1 — Differentiate:**

$$f(x) = x^2 - 6x + 5.$$

$$f'(x) = 2x - 6.$$

**Step 2 — Solve $f'(x) > 0$:**

$$2x - 6 > 0.$$

$$2x > 6.$$

$$x > 3.$$

So $f$ is increasing on $(3, \infty)$, which matches the right branch of the parabola.

**Why other options are wrong:** (A) and (D) describe where $f'(x) < 0$ (decreasing); (B) is false since the parabola first falls then rises.

**Final Answer:** $f$ increases on $(3, \infty) \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q6

## Q7.

### Solution

**Concept — Standard integral:** $\frac{d}{dx}(-\cot x) = \csc^2 x$, hence $\int \csc^2 x\,dx = -\cot x + C$.

**Step 1 — Recall the antiderivative:** The derivative of $-\cot x$ is $\csc^2 x$.

**Step 2 — Write the integral:**

$$\int \csc^2 x\,dx = -\cot x + C.$$

**Why other options are wrong:** (A) $\cot x$ has the wrong sign; (B) is $\int \sec^2 x\,dx$; (C) is not an antiderivative of $\csc^2 x$.

**Final Answer:** $\int \csc^2 x\,dx = -\cot x + C \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q7

## Q8.

### Solution

**Concept — Order and degree:** The *order* is the highest derivative present; the *degree* is the power of that highest derivative when the equation is polynomial in derivatives.

**Step 1 — Highest derivative:** The highest derivative is $\frac{d^3y}{dx^3}$, so the order is $3$.

**Step 2 — Its power:** The term $\frac{d^3y}{dx^3}$ appears to the power $2$ (the cube is on the lower derivative $\frac{d^2y}{dx^2}$).
So the degree is $2$.

**Why other options are wrong:** (A),(C),(D) misread either the highest derivative or its power.

**Final Answer:** Order $3$, degree $2 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q8

## Q9.

**Solution**

**Concept — Magnitude of a vector:** For $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

**Step 1 — Square the components:**

$$2^2 + (-3)^2 + 6^2 = 4 + 9 + 36.$$

$$= 49.$$

**Step 2 — Take the square root:**

$$|\vec{a}| = \sqrt{49} = 7.$$

**Why other options are wrong:** (B) $49$ skips the square root; (C) $\sqrt{7}$ mis-adds; (D) $11$ adds the components' absolute values.

**Final Answer:** $|\vec{a}| = 7 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q9

## Q10.

**Solution**

**Concept — Direction cosines:** Direction cosines are the direction ratios divided by their magnitude $\sqrt{a^2 + b^2 + c^2}$.

**Step 1 — Magnitude of ratios $2, -1, 2$:**

$$\sqrt{2^2 + (-1)^2 + 2^2} = \sqrt{4 + 1 + 4} = \sqrt{9} = 3.$$

**Step 2 — Divide each ratio by $3$:**

$$\left(\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}\right).$$

**Why other options are wrong:** (A) are the ratios themselves; (C) divides by $9$; (D) divides by $\sqrt{3}$.

**Final Answer:** Direction cosines $\left(\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}\right) \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q10

## Q11.

**Solution**

**Concept — Fundamental theorem of calculus:** $\int_a^b f(x)\,dx = F(b) - F(a)$, where $F' = f$.

**Step 1 — Antiderivative:** $\int \sin x\,dx = -\cos x$.

**Step 2 — Apply limits** $0$ **to** $\frac{\pi}{2}$**:**

$$\big[-\cos x\big]_0^{\pi/2} = -\cos\frac{\pi}{2} - (-\cos 0).$$

$$= -0 + 1.$$

$$= 1.$$

**Why other options are wrong:** (A) $0$ and (B) $-1$ misuse the limits or sign; (D) confuses the value with the interval length.

**Final Answer:** The integral equals $1 \Rightarrow \boxed{\text{C}}$

**Answer: (C)** Go Back to Q11

## Q12.

**Solution**

**Concept — Conditional probability:** $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$, provided $P(B) \neq 0$.

**Step 1 — Substitute the values:**

$$P(A \mid B) = \frac{0.18}{0.6}.$$

**Step 2 — Simplify:**

$$= \frac{18}{60} = \frac{3}{10} = 0.3.$$

**Why other options are wrong:** (A) $0.108$ multiplies instead of dividing; (B) $0.78$ adds; (C) $0.5$ uses wrong arithmetic.

**Final Answer:** $P(A \mid B) = 0.3 \Rightarrow \boxed{\text{D}}$

**Answer: (D)** Go Back to Q12

## Q13.

**Solution**

**Concept — Integration by recognising a derivative:** If the numerator is the derivative of the denominator, $\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$.

**Step 1 — Spot the pattern:** Here $f(x) = 1 + x^3$, so $f'(x) = 3x^2$, which is exactly the numerator.

**Step 2 — Write the result:**

$$\int \frac{3x^2}{1+x^3}\,dx = \ln|1+x^3| + C.$$

**Why other options are wrong:** (B) has an extra factor $3$; (C) is a wrong antiderivative; (D) is $\int \frac{3x^2}{1+x^6}\,dx$ form, not this.

**Final Answer:** $\ln|1+x^3| + C \Rightarrow \boxed{\text{A}}$

**Answer: (A)** Go Back to Q13

## Q14.

**Solution**

**Concept — Integral of an odd function over a symmetric interval:** If $g(-x) = -g(x)$, then $\int_{-a}^{a} g(x)\,dx = 0$.

**Step 1 — Test the integrand:** $g(x) = x^3$ gives $g(-x) = (-x)^3 = -x^3 = -g(x)$. So $x^3$ is an odd function.

**Step 2 — Apply the property:** The interval $[-3, 3]$ is symmetric about $0$, hence

$$\int_{-3}^{3} x^3\,dx = 0.$$

**Why other options are wrong:** (A),(C),(D) ignore the symmetry; direct integration also gives $\left[\frac{x^4}{4}\right]_{-3}^{3} = \frac{81}{4} - \frac{81}{4} = 0$.

**Final Answer:** The integral equals $0 \Rightarrow \boxed{\text{B}}$

**Answer: (B)** Go Back to Q14

## Q15.

**Solution**

**Concept — Local minimum via second derivative:** At a critical point where $f'(x) = 0$, a local minimum occurs if $f''(x) > 0$.

**Step 1 — Critical points:**

$$f(x) = x^3 - 12x.$$

$$f'(x) = 3x^2 - 12 = 0.$$

$$x^2 = 4 \;\Rightarrow\; x = \pm 2.$$

**Step 2 — Second-derivative test:**

$$f''(x) = 6x.$$

At $x = 2$, $f''(2) = 12 > 0$, so $x = 2$ gives a local minimum.

**Step 3 — Minimum value:**

$$f(2) = (2)^3 - 12(2) = 8 - 24 = -16.$$

**Why other options are wrong:** (A) $16$ is the local maximum value at $x = -2$; (B),(C) are not extreme values.

**Final Answer:** Local minimum value is $-16 \Rightarrow \boxed{\text{D}}$

**Answer: (D)** Go Back to Q15

## Q16.

**Solution**

**Concept — Area of a parallelogram:** Area $= |\vec{a} \times \vec{b}|$ for adjacent side vectors $\vec{a}, \vec{b}$.

**Step 1 — Compute the cross product:**

$$\vec{a} \times \vec{b} = (5\hat{\imath}) \times (4\hat{\jmath}) = 20\,(\hat{\imath} \times \hat{\jmath}) = 20\hat{k}.$$

**Step 2 — Take the magnitude:**

$$|\vec{a} \times \vec{b}| = |20\hat{k}| = 20.$$

(Equivalently, the rectangle has sides $5$ and $4$, so area $= 5 \times 4 = 20$.)

**Why other options are wrong:** (A) $9$ adds the sides; (B) $40$ doubles the area; (D) $10$ halves the area.

**Final Answer:** Area $= 20 \Rightarrow \boxed{\text{C}}$

**Answer: (C)** Go Back to Q16

## Q17.

### Solution

**Concept — Angle between two lines:** If the direction ratios of the two lines are proportional, the lines are parallel and the angle between them is $0^\circ$.

**Step 1 — Check proportionality:** Compare $1, 2, 2$ with $2, 4, 4$:

$$\frac{2}{1} = \frac{4}{2} = \frac{4}{2} = 2.$$

All three ratios are equal, so the direction ratios are proportional.

**Step 2 — Interpret:** Proportional direction ratios mean the lines are parallel, so the angle is $0^\circ$.

**Why other options are wrong:** (A) $90^\circ$ needs a zero dot product; (C),(D) would need a non-proportional pair giving those cosines.

**Final Answer:** The angle is $0^\circ \Rightarrow \boxed{\text{B}}$

**Answer: (B)** Go Back to Q17

## Q18.

### Solution

**Concept — Union with independent events:** $P(A \cup B) = P(A) + P(B) - P(A)P(B)$ when $A, B$ are independent.

**Step 1 — Product term:**

$$P(A)P(B) = \frac{1}{4} \cdot \frac{2}{5} = \frac{2}{20} = \frac{1}{10}.$$

**Step 2 — Substitute:**

$$P(A \cup B) = \frac{1}{4} + \frac{2}{5} - \frac{1}{10}.$$
$$= \frac{5}{20} + \frac{8}{20} - \frac{2}{20}.$$

$$= \frac{11}{20}.$$

**Why other options are wrong:** (B) $\frac{13}{20}$ forgets to subtract the intersection; (C) $\frac{1}{10}$ is only $P(A \cap B)$; (D) $\frac{3}{5}$ uses wrong arithmetic.

**Final Answer:** $P(A \cup B) = \frac{11}{20} \Rightarrow \boxed{\text{A}}$

**Answer: (A)** Go Back to Q18

## Q19.

### Solution

**Concept — Assertion–Reason analysis:** Judge each statement's truth, then decide whether R correctly explains A.

**Step 1 — Assertion:** "Every continuous function is differentiable" is **false**. Continuity does not guarantee differentiability.

**Step 2 — Reason:** $f(x) = |x|$ is continuous at $x = 0$ (both one-sided limits equal $0 = f(0)$) but is not differentiable there, since the left derivative $-1$ and right derivative $+1$ differ. So R is **true**.

**Step 3 — Relationship:** R is in fact a counterexample that shows A is false. Hence A is false while R is true.

**Why other options are wrong:** (A),(B) require A true; (C) requires R false.

**Final Answer:** A false, R true $\Rightarrow \boxed{\text{D}}$

**Answer: (D)** Go Back to Q19

## Q20.

### Solution

**Concept — Principal value of $\sec^{-1}$:** The principal value branch (range) of $\sec^{-1} x$ is $[0, \pi] \setminus \left\{\frac{\pi}{2}\right\}$.

**Step 1 — Test the Assertion:** $\sec^{-1}(2) = \theta$ means $\sec\theta = 2$, i.e. $\cos\theta = \frac{1}{2}$. In $[0, \pi]$ this gives $\theta = \frac{\pi}{3}$. So A is **true**.

**Step 2 — Test the Reason:** The stated range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is the branch of $\sin^{-1}$, *not* of $\sec^{-1}$ (whose range is $[0, \pi] \setminus \{\pi/2\}$). So R is **false**.

**Why other options are wrong:** (A),(B) require R true; (D) requires A false.

**Final Answer:** A true, R false $\Rightarrow \boxed{\text{C}}$

**Answer: (C)** Go Back to Q20

## Q21.

### Solution

**Concept — Principal values:** $\sin^{-1}$ lies in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $\cos^{-1}$ lies in $[0, \pi]$.

**Step 1 — Evaluate $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$:**

$$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} \Rightarrow \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}.$$

**Step 2 — Evaluate $\cos^{-1}\left(\frac{1}{2}\right)$:**

$$\cos\frac{\pi}{3} = \frac{1}{2} \Rightarrow \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}.$$

**Step 3 — Add:**

$$\frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}.$$

**Final Answer:** $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{1}{2}\right) = \frac{2\pi}{3}$. Go Back to Q21

## Q22.

### Solution

**Concept — Logarithmic differentiation:** Used when the variable appears in both base and exponent.

**Step 1 — Take natural log of both sides:**

$$y = x^x.$$

$$\ln y = x \ln x.$$

**Step 2 — Differentiate both sides w.r.t. $x$:** (product rule on the right)

$$\frac{1}{y}\frac{dy}{dx} = 1 \cdot \ln x + x \cdot \frac{1}{x}.$$

$$\frac{1}{y}\frac{dy}{dx} = \ln x + 1.$$

**Step 3 — Solve for $\frac{dy}{dx}$:**

$$\frac{dy}{dx} = y\,(1 + \ln x).$$

$$= x^x\,(1 + \ln x).$$

**Final Answer:** $\frac{dy}{dx} = x^x\,(1 + \ln x)$. **Go Back to Q22**

## Q23.

**Solution**

**Concept — Integration by parts:** $\int u\,dv = uv - \int v\,du$, choosing $u$ by the ILATE rule.

**Step 1 — Choose parts:** Let $u = x$ and $dv = \sin x\,dx$.

Then $du = dx$ and $v = -\cos x$.

**Step 2 — Apply the formula:**

$$\int x\,\sin x\,dx = x(-\cos x) - \int (-\cos x)\,dx.$$

$$= -x\cos x + \int \cos x\,dx.$$

$$= -x\cos x + \sin x + C.$$

**Final Answer:** $\int x\,\sin x\,dx = \sin x - x\cos x + C$.

**OR —** $\int \sin^2 x\,dx$**:**

**Step 1 — Use the identity** $\sin^2 x = \frac{1 - \cos 2x}{2}$**:**

$$\int \sin^2 x\,dx = \int \frac{1 - \cos 2x}{2}\,dx.$$

**Step 2 — Integrate term by term:**

$$= \frac{1}{2}\int 1\,dx - \frac{1}{2}\int \cos 2x\,dx.$$

$$= \frac{x}{2} - \frac{1}{2}\cdot\frac{\sin 2x}{2} + C.$$

$$= \frac{x}{2} - \frac{\sin 2x}{4} + C.$$

**Final Answer (OR):** $\int \sin^2 x\,dx = \frac{x}{2} - \frac{\sin 2x}{4} + C$. **Go Back to Q23**

## Q24.

**Solution**

**Concept — Perpendicular vectors:** $\vec{a} \perp \vec{b} \iff \vec{a}\cdot\vec{b} = 0$.

**Step 1 — Compute the dot product:**

$$\vec{a}\cdot\vec{b} = (3)(2) + (\lambda)(4) + (2)(-1).$$

$$= 6 + 4\lambda - 2.$$

$$= 4 + 4\lambda.$$

**Step 2 — Set equal to zero and solve:**

$$4 + 4\lambda = 0.$$

$$4\lambda = -4.$$

$$\lambda = -1.$$

**Final Answer:** $\lambda = -1$. **Go Back to Q24**

## Q25.

**Solution**

**Concept — Equally likely outcomes:** Probability $= \dfrac{\text{favourable outcomes}}{\text{total outcomes}}$.

**Step 1 — Total outcomes:** Two throws of a die give $6 \times 6 = 36$ equally likely outcomes.

**Step 2 — Favourable outcomes (sum $= 7$):**

$$(1,6),\ (6,1),\ (2,5),\ (5,2),\ (3,4),\ (4,3).$$

There are $6$ such outcomes.

**Step 3 — Probability:**

$$P(\text{sum} = 7) = \frac{6}{36} = \frac{1}{6}.$$

**Final Answer:** $P(\text{sum} = 7) = \frac{1}{6}$.

**OR —** $P(A \mid B)$**:**

**Step 1 — Formula:** $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$.

**Step 2 — Substitute:**

$$P(A \mid B) = \frac{0.15}{0.45} = \frac{1}{3}.$$

**Final Answer (OR):** $P(A \mid B) = \frac{1}{3}$. **Go Back to Q25**

## Q26.

**Solution**

**Concept — Partial fractions:** Split a proper rational function into simpler fractions, then integrate each.

**Step 1 — Set up the decomposition:**

$$\frac{1}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}.$$

$$1 = A(x+2) + B(x-1).$$

**Step 2 — Find $A$ and $B$:**

Put $x = 1$: $1 = A(3) + B(0) \Rightarrow A = \frac{1}{3}$.

Put $x = -2$: $1 = A(0) + B(-3) \Rightarrow B = -\frac{1}{3}$.

**Step 3 — Integrate:**

$$\int \frac{1}{(x-1)(x+2)}\,dx = \int \left( \frac{1/3}{x-1} - \frac{1/3}{x+2} \right) dx.$$

$$= \frac{1}{3}\ln|x-1| - \frac{1}{3}\ln|x+2| + C.$$

$$= \frac{1}{3}\ln\left|\frac{x-1}{x+2}\right| + C.$$

**Final Answer:** $\frac{1}{3}\ln\left|\frac{x-1}{x+2}\right| + C$. Go Back to Q26

## Q27.

**Solution**

**Concept — Linear differential equation:** For $\frac{dy}{dx} + Py = Q$, the integrating factor is I.F. $= e^{\int P\,dx}$ and the solution is $y \cdot \text{I.F.} = \int Q \cdot \text{I.F.}\,dx$.

**Step 1 — Identify $P, Q$:** Here $P = 2$, $Q = e^{-x}$.

**Step 2 — Integrating factor:**

$$\text{I.F.} = e^{\int 2\,dx} = e^{2x}.$$

**Step 3 — Solve:**

$$y\,e^{2x} = \int e^{-x} \cdot e^{2x}\,dx = \int e^{x}\,dx.$$
$$y\,e^{2x} = e^{x} + C.$$
$$y = e^{-x} + C\,e^{-2x}.$$

**Final Answer:** $y = e^{-x} + C\,e^{-2x}$.

**OR — Variables separable $\frac{dy}{dx} = e^{x-y}$:**

**Step 1 — Separate variables:** Write $e^{x-y} = e^{x}e^{-y}$, so

$$e^{y}\,dy = e^{x}\,dx.$$

**Step 2 — Integrate both sides:**

$$e^{y} = e^{x} + C.$$

**Final Answer (OR):** $e^{y} = e^{x} + C$. Go Back to Q27

## Q28.

### Solution

**Concept — Inverse by adjoint:** $A^{-1} = \dfrac{1}{|A|}\,\text{adj}(A)$, valid when $|A| \neq 0$.

**Step 1 — Determinant:**

$$|A| = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = (1)(4) - (2)(3) = 4 - 6 = -2.$$

**Step 2 — Adjoint of a $2\times 2$ matrix:** Swap the leading diagonal entries and negate the off-diagonal entries:

$$\text{adj}(A) = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}.$$

**Step 3 — Inverse:**

$$A^{-1} = \frac{1}{-2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}.$$

**Final Answer:** $A^{-1} = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$. **Go Back to Q28**

## Q29.

### Solution

**Concept — Monotonicity:** $f$ increases where $f'(x) > 0$ and decreases where $f'(x) < 0$.

**Step 1 — Differentiate:**

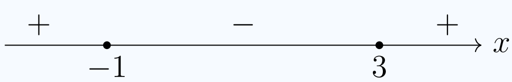
$$f(x) = x^3 - 3x^2 - 9x + 1.$$

$$f'(x) = 3x^2 - 6x - 9.$$

$$= 3(x^2 - 2x - 3) = 3(x-3)(x+1).$$

**Step 2 — Critical points:** $f'(x) = 0$ at $x = -1$ and $x = 3$.

**Step 3 — Sign analysis:**



For $x < -1$: $f' > 0$; for $-1 < x < 3$: $f' < 0$; for $x > 3$: $f' > 0$.

**Conclusion:** Increasing on $(-\infty, -1) \cup (3, \infty)$; decreasing on $(-1, 3)$.

**Final Answer:** Increasing on $(-\infty, -1) \cup (3, \infty)$, decreasing on $(-1, 3)$.

**OR — Tangent to $y = x^3$ at $(1, 1)$:**

**Step 1 — Slope:** $\frac{dy}{dx} = 3x^2$, so at $x = 1$ slope $= 3$.

**Step 2 — Equation:** $y - 1 = 3(x - 1) \Rightarrow y = 3x - 2$.

**Final Answer (OR):** $y = 3x - 2$. **Go Back to Q29**

## Q30.

**Solution**

**Concept — Vector equation of a line:** A line through point $\vec{a}$ parallel to $\vec{b}$ is $\vec{r} = \vec{a} + \lambda\vec{b}$.

**Step 1 — Write the equation:** With $\vec{a} = 2\hat{\imath} - \hat{\jmath} + 3\hat{k}$ and $\vec{b} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$,

$$\vec{r} = (2\hat{\imath} - \hat{\jmath} + 3\hat{k}) + \lambda(\hat{\imath} + 2\hat{\jmath} - \hat{k}).$$

**Step 2 — Test the point $(4, 3, 1)$:** The point lies on the line if some $\lambda$ satisfies all three component equations.

$$2 + \lambda = 4 \;\Rightarrow\; \lambda = 2.$$

$$-1 + 2\lambda = 3 \;\Rightarrow\; \lambda = 2.$$

$$3 - \lambda = 1 \;\Rightarrow\; \lambda = 2.$$

**Step 3 — Conclusion:** All three give the same value $\lambda = 2$, so the point $(4, 3, 1)$ lies on the line.

**Final Answer:** $\vec{r} = (2\hat{\imath} - \hat{\jmath} + 3\hat{k}) + \lambda(\hat{\imath} + 2\hat{\jmath} - \hat{k})$; the point $(4, 3, 1)$ lies on it (at $\lambda = 2$). **Go Back to Q30**

## Q31.

**Solution**

**Concept — One-one (injective) and onto (surjective):** *One-one*: $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$. *Onto*: every $y$ in the codomain has a pre-image.

**Step 1 — One-one:** Suppose $f(x_1) = f(x_2)$.

$$3x_1 - 2 = 3x_2 - 2.$$

$$3x_1 = 3x_2.$$

$$x_1 = x_2.$$

Hence $f$ is one-one.

**Step 2 — Onto:** Let $y \in \mathbb{R}$ be arbitrary. Solve $y = 3x - 2$ for $x$:

$$x = \frac{y+2}{3} \in \mathbb{R}.$$

Then $f(x) = 3\left(\frac{y+2}{3}\right) - 2 = y$.
So every $y$ has a pre-image; $f$ is onto.

**Step 3 — Conclusion:** $f$ is both one-one and onto, hence bijective.

**Final Answer:** $f(x) = 3x - 2$ is one-one and onto (a bijection). **Go Back to Q31**

## Q32.

**Solution**

**Concept — Matrix method:** Write $AX = B$; then $X = A^{-1}B$ when $|A| \neq 0$.

**Step 1 — Form the matrices:**

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 3 \\ 8 \\ -2 \end{bmatrix}.$$

**Step 2 — Determinant of $A$:**

$$|A| = 1\big[(-1)(-1) - (1)(1)\big] - 2\big[(2)(-1) - (1)(1)\big] + 1\big[(2)(1) - (-1)(1)\big].$$

$$= 1(1-1) - 2(-2-1) + 1(2+1).$$

$$= 0 + 6 + 3 = 9 \; (\neq 0).$$

**Step 3 — Cofactors and adjoint:** Computing the cofactor matrix and transposing gives

$$\text{adj}(A) = \begin{bmatrix} 0 & 3 & 3 \\ 3 & -2 & 1 \\ 3 & 1 & -5 \end{bmatrix}.$$

**Step 4 — Solve $X = \frac{1}{|A|}$ adj$(A)\,B$:**

$$X = \frac{1}{9}\begin{bmatrix} 0 & 3 & 3 \\ 3 & -2 & 1 \\ 3 & 1 & -5 \end{bmatrix}\begin{bmatrix} 3 \\ 8 \\ -2 \end{bmatrix}.$$

$$= \frac{1}{9}\begin{bmatrix} 0 \cdot 3 + 3 \cdot 8 + 3 \cdot (-2) \\ 3 \cdot 3 + (-2) \cdot 8 + 1 \cdot (-2) \\ 3 \cdot 3 + 1 \cdot 8 + (-5) \cdot (-2) \end{bmatrix}.$$

$$= \frac{1}{9}\begin{bmatrix} 0 + 24 - 6 \\ 9 - 16 - 2 \\ 9 + 8 + 10 \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 18 \\ -9 \\ 27 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}.$$

**Step 5 — Verify:** $x + 2y + z = 2 - 2 + 3 = 3 \checkmark$; $2x - y + z = 4 + 1 + 3 = 8 \checkmark$; $x + y - z = 2 - 1 - 3 = -2 \checkmark$.

**Final Answer:** $x = 2,\ y = -1,\ z = 3$. **Go Back to Q32**

## Q33.

**Solution**

**Concept — Area by integration:** The area between a curve $y = f(x)$ lying above the $x$-axis and the axis is $\int_a^b f(x)\,dx$.

**Step 1 — Find where the curve meets the axis:** Set $y = 0$:

$$4 - x^2 = 0 \;\Rightarrow\; x = \pm 2.$$

So the region runs from $x = -2$ to $x = 2$.

**Step 2 — Set up the integral** (use symmetry about the $y$-axis):

$$\text{Area} = \int_{-2}^{2} (4 - x^2)\,dx = 2\int_0^2 (4 - x^2)\,dx.$$

**Step 3 — Integrate:**

$$= 2\left[4x - \frac{x^3}{3}\right]_0^2.$$

$$= 2\left[\left(8 - \frac{8}{3}\right) - 0\right].$$

$$= 2\left[\frac{24-8}{3}\right] = 2 \cdot \frac{16}{3} = \frac{32}{3}.$$

**Final Answer:** Area $= \frac{32}{3}$ square units.

**OR — Region between $y = 2x$ and $y = x^2$, $0 \le x \le 2$:**

**Step 1 — Top minus bottom:** On $[0, 2]$, $2x \ge x^2$, so

$$\text{Area} = \int_0^2 (2x - x^2)\,dx.$$

**Step 2 — Integrate:**

$$= \left[x^2 - \frac{x^3}{3}\right]_0^2 = 4 - \frac{8}{3} = \frac{4}{3}.$$

**Final Answer (OR):** Area $= \frac{4}{3}$ square units. **Go Back to Q33**

## Q34.

**Solution**

**Concept — Corner-point method:** The optimum of a linear objective over a bounded feasible region occurs at a corner (vertex).

**Step 1 — Find the corner points:** The constraints $x+2y \le 10$, $3x+y \le 15$, $x \ge 0$, $y \ge 0$ bound the region with vertices:

$$O(0,0), \quad (5,0), \quad (0,5).$$

The lines $x + 2y = 10$ and $3x + y = 15$ intersect: from $3x + y = 15$, $y = 15 - 3x$; substitute,

$$x + 2(15 - 3x) = 10 \;\Rightarrow\; x + 30 - 6x = 10 \;\Rightarrow\; -5x = -20 \;\Rightarrow\; x = 4,\; y = 3.$$

So the fourth vertex is $(4, 3)$.

**Step 2 — Evaluate $Z = 3x + 2y$ at each corner:**

$$Z(0,0) = 0.$$

$$Z(5,0) = 3(5) + 0 = 15.$$

$$Z(4,3) = 3(4) + 2(3) = 12 + 6 = 18.$$

$$Z(0,5) = 3(0) + 2(5) = 10.$$

**Step 3 — Pick the maximum:** The largest value is 18 at $(4,3)$.

**Final Answer:** $Z$ is maximum $= 18$ at $(x,y) = (4,3)$. **Go Back to Q34**

## Q35.

**Solution**

**Concept — Shortest distance between skew lines:** For $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$,

$$d = \frac{\left|(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)\right|}{|\vec{b}_1 \times \vec{b}_2|}.$$

**Step 1 — Identify vectors:** $\vec{a}_1 = \hat{\imath} + 2\hat{\jmath} + \hat{k}$, $\vec{b}_1 = \hat{\imath} + 2\hat{\jmath} - \hat{k}$; $\vec{a}_2 = 2\hat{\imath} - \hat{\jmath} + \hat{k}$, $\vec{b}_2 = 2\hat{\imath} - \hat{\jmath} + \hat{k}$.

**Step 2 — Cross product $\vec{b}_1 \times \vec{b}_2$:**

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 2 & -1 \\ 2 & -1 & 1 \end{vmatrix}.$$

$$= \hat{\imath}\big[(2)(1) - (-1)(-1)\big] - \hat{\jmath}\big[(1)(1) - (-1)(2)\big] + \hat{k}\big[(1)(-1) - (2)(2)\big].$$

$$= \hat{\imath}(2-1) - \hat{\jmath}(1+2) + \hat{k}(-1-4).$$

$$= \hat{\imath} - 3\hat{\jmath} - 5\hat{k}.$$

**Step 3 — Its magnitude:**

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{1^2 + (-3)^2 + (-5)^2} = \sqrt{1 + 9 + 25} = \sqrt{35}.$$

**Step 4 — $(\vec{a}_2 - \vec{a}_1)$ and the dot product:**

$$\vec{a}_2 - \vec{a}_1 = (2-1)\hat{\imath} + (-1-2)\hat{\jmath} + (1-1)\hat{k} = \hat{\imath} - 3\hat{\jmath}.$$

$$(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2) = (1)(1) + (-3)(-3) + (0)(-5) = 1 + 9 + 0 = 10.$$

**Step 5 — Distance:**

$$d = \frac{|10|}{\sqrt{35}} = \frac{10}{\sqrt{35}} \text{ units.}$$

**Final Answer:** Shortest distance $= \dfrac{10}{\sqrt{35}}$ units.

**OR — Distance of $(1, 2, -1)$ from the plane $2x - y + 2z + 3 = 0$:**

**Step 1 — Formula:** $d = \dfrac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}$.

**Step 2 — Substitute** $a = 2, b = -1, c = 2, d = 3$ and $(x_1, y_1, z_1) = (1, 2, -1)$:

$$d = \frac{|2(1) - 1(2) + 2(-1) + 3|}{\sqrt{2^2 + (-1)^2 + 2^2}} = \frac{|2 - 2 - 2 + 3|}{\sqrt{9}} = \frac{1}{3}.$$

**Final Answer (OR):** Distance $= \dfrac{1}{3}$ units. **Go Back to Q35**

## Q36.

### Solution

**Concept — Maxima by derivatives:** Build the area function, set $\dfrac{dA}{dx} = 0$, and select the value giving a genuine maximum.

**(i) Area function:** The perimeter uses all $40$ m of wire, so

$$2(x + y) = 40 \;\Rightarrow\; x + y = 20 \;\Rightarrow\; y = 20 - x.$$

Hence the area is

$$A = x\,y = x(20 - x), \qquad 0 < x < 20.$$

**(ii) Differentiate:**

$$A = 20x - x^2.$$
$$\frac{dA}{dx} = 20 - 2x.$$

**(iii) Critical point and maximum:** Set $\dfrac{dA}{dx} = 0$:

$$20 - 2x = 0 \;\Rightarrow\; x = 10.$$

Then $\dfrac{d^2A}{dx^2} = -2 < 0$, so $x = 10$ gives a maximum.

Maximum area:

$$A(10) = 10(20 - 10) = 10 \times 10 = 100.$$

(The garden is a $10$ m $\times$ $10$ m square.)

**Final Answer:** $A = x(20 - x)$; $\dfrac{dA}{dx} = 20 - 2x$; maximum at $x = 10$ m with $A_{\max} = 100$ m$^2$. **Go Back to Q36**

## Q37.

**Solution**

**Concept — Bayes' theorem:** $P(E_i \mid A) = \dfrac{P(E_i)P(A \mid E_i)}{\sum_j P(E_j)P(A \mid E_j)}$.

**(i) Prior probabilities:** The item comes from Machine I with probability $60\%$ and Machine II with $40\%$:

$$P(\text{Machine I}) = 0.6, \qquad P(\text{Machine II}) = 0.4.$$

**(ii) Conditional (defective) probabilities:**

$$P(\text{Def} \mid \text{Machine I}) = 0.02, \qquad P(\text{Def} \mid \text{Machine II}) = 0.05.$$

**(iii) Apply Bayes' theorem:**

$$P(\text{Machine I} \mid \text{Def}) = \frac{(0.6)(0.02)}{(0.6)(0.02) + (0.4)(0.05)}.$$

Compute each product:

$$(0.6)(0.02) = 0.012, \qquad (0.4)(0.05) = 0.020.$$

$$= \frac{0.012}{0.012 + 0.020} = \frac{0.012}{0.032}.$$

Multiply numerator and denominator by $1000$:

$$= \frac{12}{32} = \frac{3}{8}.$$

**Final Answer:** $P(\text{Machine I} \mid \text{Def}) = \dfrac{3}{8}$. **Go Back to Q37**

## Q38.

### Solution

**Concept — Velocity and acceleration:** Velocity $v = \frac{ds}{dt}$, acceleration $a = \frac{dv}{dt}$. The body is at rest when $v = 0$.

**(i) Velocity:**

$$s(t) = t^3 - 9t^2 + 24t.$$

$$v(t) = \frac{ds}{dt} = 3t^2 - 18t + 24.$$

**(ii) Times at rest:** Set $v(t) = 0$:

$$3t^2 - 18t + 24 = 0.$$

$$t^2 - 6t + 8 = 0.$$

$$(t-2)(t-4) = 0 \;\Rightarrow\; t = 2 \text{ s and } t = 4 \text{ s}.$$

**(iii) Acceleration at $t = 5$:**

$$a(t) = \frac{dv}{dt} = 6t - 18.$$

$$a(5) = 6(5) - 18 = 30 - 18 = 12 \text{ m/s}^2.$$

At $t = 5$, $v(5) = 3(25) - 18(5) + 24 = 75 - 90 + 24 = 9$ m/s $> 0$. Since $v$ and $a$ have the same sign (both positive), the particle is **speeding up** at $t = 5$ s.

**Final Answer:** $v(t) = 3t^2 - 18t + 24$; at rest at $t = 2$ s and $t = 4$ s; $a(5) = 12$ m/s$^2$ and the particle is speeding up. **Go Back to Q38**

## Answer Key – Section A (Q1–Q20)

| Q | Ans | Q | Ans | Q | Ans | Q | Ans | Q | Ans |
|---|---|---|---|---|---|---|---|---|---|
| 1 | B | 2 | A | 3 | C | 4 | D | 5 | A |
| 6 | C | 7 | D | 8 | B | 9 | A | 10 | B |
| 11 | C | 12 | D | 13 | A | 14 | B | 15 | D |
| 16 | C | 17 | B | 18 | A | 19 | D | 20 | C |

*Sections B–E are descriptive; refer to the Detailed Solutions above for full model answers and step marking.*