CBSE Class 12 Mathematics

Sample Paper – 3

Duration: 180 Minutes **Maximum Marks: 80**

General Instructions

- This question paper contains **38 questions**. **All questions are compulsory**.
- The paper is divided into **five sections** – A, B, C, D and E.
- **Section A** (Q1–Q20) carries **1 mark** each: Q1–Q18 are multiple choice questions and Q19–Q20 are Assertion–Reason questions.
- **Section B** (Q21–Q25) carries **2 marks** each (Very Short Answer).
- **Section C** (Q26–Q31) carries **3 marks** each (Short Answer).
- **Section D** (Q32–Q35) carries **5 marks** each (Long Answer).
- **Section E** (Q36–Q38) carries **4 marks** each (case study based, with sub-parts).
- There is **no overall choice**, but an **internal choice** has been provided in some questions. Attempt only one of the alternatives in such questions.
- There is **no negative marking**. Use of a **calculator is not permitted**.

Section A (Q1–Q20) – 1 Mark Each

Q1. If $A = [a_{ij}]$ is a skew-symmetric matrix, then each diagonal element a_{ii} is equal to:

(A) 1

(B) 0

(C) -1

(D) a_{ii}^2

Q2. If A is a square matrix of order 3 with $|A| = 4$, then $|2A|$ equals:

(A) 8

(B) 12

(C) 16

(D) 32

Q3. Let $A = \{1, 2, 3, 4\}$ and $R = \{(1, 1), (2, 2), (3, 3), (4, 4), (1, 2), (2, 1)\}$ be a relation on A. Then R is:

(A) reflexive but not symmetric

(B) symmetric but not transitive

(C) an equivalence relation

(D) transitive but not reflexive

Q4. The principal value of $\tan^{-1}(-1)$ is:

(A) $-\dfrac{\pi}{4}$

(B) $\dfrac{\pi}{4}$

(C) $\dfrac{3\pi}{4}$

(D) $-\dfrac{3\pi}{4}$

Q5. If $y = \ln(\sec x)$, then $\dfrac{dy}{dx}$ equals:

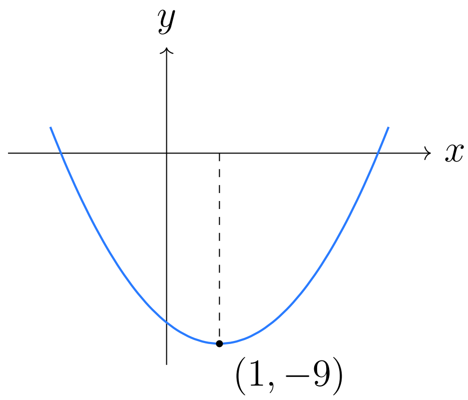
(A) $\sec x \tan x$

(B) $\tan x$

(C) $\cot x$

(D) $\dfrac{1}{\sec x}$

Q6. The graph of $f(x) = x^2 - 2x - 8$ is shown below. The function is *decreasing* on the interval:



(A) $(1, \infty)$

(B) $(-\infty, \infty)$

(C) $(-\infty, 1)$

(D) $(0, 1)$

Q7. $\displaystyle\int \sec x \tan x \, dx$ equals:

(A) $\tan x + C$

(B) $\ln|\sec x| + C$

(C) $\sec x + C$

(D) $-\csc x + C$

Q8. The order and degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{dy}{dx}\right)^4 + y = 0$ are respectively:

(A) 2 and 3

(B) 3 and 2

(C) 3 and 4

(D) 4 and 2

Q9. The magnitude of the vector $\vec{a} = 6\hat{i} + 2\hat{j} + 3\hat{k}$ is:

(A) 7

(B) 11

(C) $\sqrt{41}$

(D) 49

Q10. The direction cosines of a line whose direction ratios are $2, 3, -6$ are:

(A) $2, 3, -6$

(B) $\frac{2}{7}, \frac{3}{7}, \frac{6}{7}$

(C) $\frac{2}{49}, \frac{3}{49}, -\frac{6}{49}$

(D) $\frac{2}{7}, \frac{3}{7}, -\frac{6}{7}$

Q11. $\int_0^1 e^x \, dx$ equals:

(A) 1

(B) e

(C) $e - 1$

(D) $e + 1$

Q12. If $P(A \cap B) = 0.18$ and $P(B) = 0.6$, then $P(A \mid B)$ equals:

(A) 0.12

(B) 0.3

(C) 0.108

(D) 0.6

Q13. $\int \frac{e^x}{1 + e^x} \, dx$ equals:

(A) $\ln(1 + e^x) + C$

(B) $\frac{1}{1 + e^x} + C$

(C) $e^x \ln(1 + e^x) + C$

(D) $\tan^{-1}(e^x) + C$

Q14. $\int_{-3}^{3} x^2 \, dx$ equals:

(A) 0

(B) 9

(C) 27

(D) 18

Q15. The local minimum value of $f(x) = x^3 - 12x$ is:

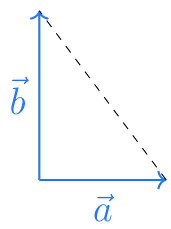
(A) 16

(B) -16

(C) -2

(D) 2

Q16. The area of the triangle whose two sides are represented by the vectors $\vec{a} = 3\hat{i}$ and $\vec{b} = 4\hat{j}$ (shown below) is:



(A) 12

(B) 24

(C) 7

(D) 6

Q17. The angle between two lines whose direction ratios are $1, 0, 1$ and $0, 1, 1$ is:

(A) 60°

(B) 90°

(C) 45°

(D) 30°

Q18. If A and B are independent events with $P(A) = 0.3$ and $P(B) = 0.4$, then $P(A \cup B)$ equals:

(A) 0.7

(B) 0.12

(C) 0.42

(D) 0.58

Q19. **Assertion (A):** The derivative of an even differentiable function is an odd function.
Reason (R): Differentiating the identity $f(-x) = f(x)$ with respect to x gives $f'(-x) = -f'(x)$.

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is *not* the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

Q20. **Assertion (A):** $\sin^{-1}\left(\sin \frac{\pi}{3}\right) = \frac{\pi}{3}$.
Reason (R): $\sin^{-1}(\sin x) = x$ for every real number x.

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is *not* the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

Section B (Q21–Q25) – 2 Marks Each

Q21. Find the principal value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{1}{2}\right)$. **[2]**

Q22. Differentiate $y = x^x$ with respect to x. **[2]**

Q23. Evaluate $\int x \cos x \, dx$. **[2]**

OR

Evaluate $\int \ln x \, dx$.

Q24. Find the value of λ for which the vectors $\vec{a} = 2\hat{i}+\lambda\hat{j}-3\hat{k}$ and $\vec{b} = \hat{i}+2\hat{j}-2\hat{k}$ are perpendicular. **[2]**

Q25. A fair die is thrown twice. Find the probability of getting a sum of 8. **[2]**

OR

If $P(B) = 0.6$ and $P(A \cap B) = 0.24$, find $P(A \mid B)$.

Section C (Q26–Q31) – 3 Marks Each

Q26. Evaluate $\displaystyle\int \frac{1}{(x-1)(x+2)}\, dx$. **[3]**

Q27. Solve the differential equation $\dfrac{dy}{dx} + 2y = e^{-x}$. **[3]**

OR

Solve the differential equation $\dfrac{dy}{dx} = e^{x-y}$.

Q28. Find the inverse of the matrix $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ using the adjoint method. **[3]**

Q29. Find the intervals in which $f(x) = x^3 - 3x^2 - 9x + 7$ is increasing or decreasing. **[3]**

OR

Find the equation of the tangent to the curve $y = x^3$ at the point $(1, 1)$.

Q30. Find the vector equation of the line passing through the point $(2, -1, 3)$ and parallel to the vector $2\hat{i} + 3\hat{j} - \hat{k}$. Hence check whether the point $(4, 2, 2)$ lies on this line. **[3]**

Q31. Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 3x - 4$ is one-one and onto. **[3]**

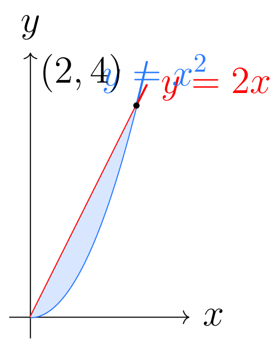
Section D (Q32–Q35) – 5 Marks Each

Q32. Using the matrix method, solve the following system of linear equations:

$$x + y + z = 4, \qquad 2x + y - z = 0, \qquad x - y + 2z = 9.$$

[5]

Q33. Using integration, find the area of the region bounded by the parabola $y = x^2$ and the line $y = 2x$.



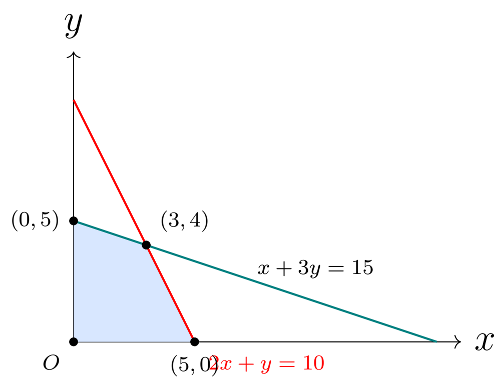
[5]

OR

Using integration, find the area of the region bounded by the curve $y = \sin x$ and the x-axis between $x = 0$ and $x = \pi$.

Q34. Solve the following linear programming problem graphically. Maximise $Z = 2x + 5y$ subject to the constraints

$$2x + y \leq 10, \qquad x + 3y \leq 15, \qquad x \geq 0,\ y \geq 0.$$



[5]

Q35. Find the shortest distance between the lines

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{j} + 2\hat{k}) \quad \text{and} \quad \vec{r} = (2\hat{i} - \hat{j} + \hat{k}) + \mu(2\hat{i} + \hat{j} - \hat{k}).$$

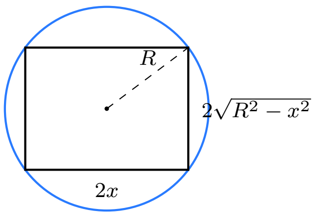
[5]

OR

Find the angle between the line $\vec{r} = (\hat{i} + \hat{j} + \hat{k}) + \lambda(2\hat{i} - \hat{j} + 2\hat{k})$ and the plane $2x + y - z = 5$.

Section E (Q36–Q38) – 4 Marks Each (Case Study)

Q36. Case Study – The Circular Disc.
A rectangular plate is to be cut from a circular metal disc of radius R. One pair of sides has length $2x$; since the corners lie on the circle, the other pair of sides has length $2\sqrt{R^2 - x^2}$, as shown.



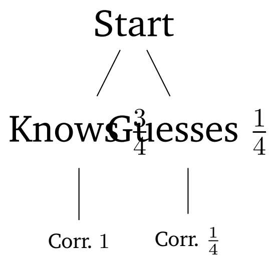
Based on the above, answer the following:

(i) Express the area A of the rectangle as a function of x. (1)

(ii) Find $\dfrac{dA}{dx}$. (1)

(iii) Find the value of x for which the area is maximum, and show that the maximum rectangle is a square of area $2R^2$. (2)

Q37. Case Study – Knows or Guesses.
In a multiple-choice test each question has 4 options with exactly one correct. A student knows the answer to a question with probability $\frac{3}{4}$, and otherwise guesses. When the student guesses, the probability of choosing the correct option is $\frac{1}{4}$ (one of the four options). A question is answered correctly.



Based on the above, answer the following:

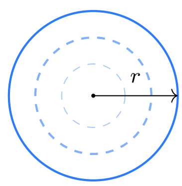
(i) Write $P(\text{Knows})$ and $P(\text{Guesses})$. (1)

(ii) Write $P(\text{Correct} \mid \text{Knows})$ and $P(\text{Correct} \mid \text{Guesses})$. (1)

(iii) Using Bayes' theorem, find the probability that the student actually knew the answer, given that it was answered correctly. (2)

Q38. Case Study – The Oil Slick.

An oil slick on the surface of water spreads out in the shape of a circle. As more oil leaks, the radius of the circular slick increases at a constant rate of 3 cm/s.



Based on the above, answer the following:

(i) Express the area A of the slick as a function of its radius r, and write $\frac{dA}{dr}$. (1)

(ii) Using the chain rule, write $\frac{dA}{dt}$ in terms of r and $\frac{dr}{dt}$. (1)

(iii) Find the rate at which the area is increasing when the radius is 10 cm. (2)

Detailed Solutions

Q1.

Solution

Concept — Skew-symmetric matrix: A matrix A is skew-symmetric if $A^T = -A$, that is $a_{ji} = -a_{ij}$ for all i, j.

Step 1 — Apply the definition to a diagonal entry: Put $i = j$ in $a_{ji} = -a_{ij}$.

This gives $a_{ii} = -a_{ii}$.

Step 2 — Solve for a_{ii}:

$$a_{ii} + a_{ii} = 0.$$

$$2a_{ii} = 0.$$

$$a_{ii} = 0.$$

Why other options are wrong: (A) 1 and (C) -1 contradict $a_{ii} = -a_{ii}$; (D) a_{ii}^2 is not forced to hold and is not the required value.

Final Answer: Each diagonal element is $0 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q1

Q2.

Solution

Concept — Scalar multiple of a determinant: For a square matrix of order n, $|kA| = k^n|A|$.

Step 1 — Identify n and k: Here $n = 3$ and $k = 2$.

Step 2 — Apply the rule:

$$|2A| = 2^3\,|A|.$$

$$= 8 \times 4.$$

$$= 32.$$

Why other options are wrong: (A) 8 uses 2^3 but drops $|A|$; (B) 12 uses 3×4; (C) 16 uses only $2^2 \times 4$.

Final Answer: $|2A| = 32 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q2

Q3.

Solution

Concept — Properties of a relation: *Reflexive*: $(a, a) \in R$ for all a. *Symmetric*: $(a, b) \in R \Rightarrow (b, a) \in R$. *Transitive*: $(a, b), (b, c) \in R \Rightarrow (a, c) \in R$.

Step 1 — Reflexive? $(1, 1), (2, 2), (3, 3), (4, 4) \in R$, so R is reflexive.

Step 2 — Symmetric? The only off-diagonal pairs are $(1, 2)$ and $(2, 1)$, and both are present, so R is symmetric.

Step 3 — Transitive? The only chains to check use $(1, 2)$ and $(2, 1)$: $(1, 2), (2, 1) \Rightarrow (1, 1) \in R$; $(2, 1), (1, 2) \Rightarrow (2, 2) \in R$. All required pairs are present, so R is transitive.

Conclusion: R is reflexive, symmetric and transitive, hence an equivalence relation.

Why other options are wrong: (A),(B),(D) each deny a property that actually holds.

Final Answer: R is an equivalence relation $\Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q3

Q4.

Solution

Concept — Principal value of $\tan^{-1}$: The principal value branch of $\tan^{-1} x$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Step 1 — Set up the equation: Let $\tan^{-1}(-1) = \theta$, so $\tan\theta = -1$.

Step 2 — Find θ in the branch: We know $\tan\frac{\pi}{4} = 1$, so $\tan\left(-\frac{\pi}{4}\right) = -1$.

Since $-\frac{\pi}{4} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, it is the principal value.

Why other options are wrong: (B) $\frac{\pi}{4}$ gives $+1$; (C) $\frac{3\pi}{4}$ and (D) $-\frac{3\pi}{4}$ lie outside the principal branch.

Final Answer: $\tan^{-1}(-1) = -\frac{\pi}{4} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q4

Q5.

Solution

Concept — Chain rule with logarithm: $\dfrac{d}{dx}\ln(u) = \dfrac{1}{u}\cdot\dfrac{du}{dx}$.

Step 1 — Identify the inner function: Let $u = \sec x$, so $\dfrac{du}{dx} = \sec x \tan x$.

Step 2 — Apply the chain rule:

$$\frac{dy}{dx} = \frac{1}{\sec x}\cdot \sec x \tan x.$$
$$= \tan x.$$

Why other options are wrong: (A) is $\dfrac{d}{dx}\sec x$; (C) is $\dfrac{d}{dx}\ln(\sin x)$; (D) equals $\cos x$, not the derivative.

Final Answer: $\dfrac{dy}{dx} = \tan x \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q5

Q6.

Solution

Concept — Decreasing function: f is decreasing where $f'(x) < 0$.

Step 1 — Differentiate:

$$f(x) = x^2 - 2x - 8.$$
$$f'(x) = 2x - 2.$$

Step 2 — Solve $f'(x) < 0$:

$$2x - 2 < 0.$$
$$2x < 2.$$
$$x < 1.$$

So f is decreasing on $(-\infty, 1)$, which matches the left branch of the parabola (vertex at $(1, -9)$).

Why other options are wrong: (A) $(1, \infty)$ is where $f'(x) > 0$ (increasing); (B) is false since the parabola first falls then rises; (D) $(0, 1)$ is only part of the decreasing set.

Final Answer: f decreases on $(-\infty, 1) \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q6

Q7.

Solution

Concept — Standard integral: $\frac{d}{dx}\sec x = \sec x \tan x$, hence $\int \sec x \tan x \, dx = \sec x + C$.

Step 1 — Recall the antiderivative: The derivative of $\sec x$ is $\sec x \tan x$.

Step 2 — Write the integral:

$$\int \sec x \tan x \, dx = \sec x + C.$$

Why other options are wrong: (A) is $\int \sec^2 x \, dx$; (B) is $\int \tan x \, dx$; (D) is $\int \csc x \cot x \, dx$.

Final Answer: $\int \sec x \tan x \, dx = \sec x + C \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q7

Q8.

Solution

Concept — Order and degree: The *order* is the highest derivative present; the *degree* is the power of that highest derivative when the equation is polynomial in derivatives.

Step 1 — Highest derivative: The highest derivative is $\frac{d^3y}{dx^3}$, so the order is 3.

Step 2 — Its power: The term $\left(\frac{d^3y}{dx^3}\right)^2$ shows the highest derivative appears to the power 2 (the power 4 is on $\frac{dy}{dx}$, a lower derivative).
So the degree is 2.

Why other options are wrong: (A),(C),(D) misread either the highest derivative or its power.

Final Answer: Order 3, degree $2 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q8

Q9.

Solution

Concept — Magnitude of a vector: For $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

Step 1 — Square the components:

$$6^2 + 2^2 + 3^2 = 36 + 4 + 9.$$

$$= 49.$$

Step 2 — Take the square root:

$$|\vec{a}| = \sqrt{49} = 7.$$

Why other options are wrong: (B) 11 adds the components; (C) $\sqrt{41}$ drops the $2\hat{j}$ term; (D) 49 forgets the square root.

Final Answer: $|\vec{a}| = 7 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q9

Q10.

Solution

Concept — Direction cosines: Direction cosines are the direction ratios divided by their magnitude $\sqrt{a^2 + b^2 + c^2}$.

Step 1 — Magnitude of ratios $2, 3, -6$:

$$\sqrt{2^2 + 3^2 + (-6)^2} = \sqrt{4 + 9 + 36} = \sqrt{49} = 7.$$

Step 2 — Divide each ratio by 7:

$$\left(\frac{2}{7}, \frac{3}{7}, -\frac{6}{7}\right).$$

Why other options are wrong: (A) are the ratios themselves; (B) has the wrong sign on the third cosine; (C) divides by 49 instead of 7.

Final Answer: Direction cosines $\left(\frac{2}{7}, \frac{3}{7}, -\frac{6}{7}\right) \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q10

Q11.

Solution

Concept — Fundamental theorem of calculus: $\int_a^b f(x)\,dx = F(b) - F(a)$, where $F' = f$.

Step 1 — Antiderivative: $\int e^x\,dx = e^x$.

Step 2 — Apply limits 0 **to** 1**:**

$$[e^x]_0^1 = e^1 - e^0.$$

$$= e - 1.$$

Why other options are wrong: (A) 1 uses e^0 only; (B) e forgets $e^0 = 1$; (D) $e + 1$ adds instead of subtracting.

Final Answer: The integral equals $e - 1 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q11

Q12.

Solution

Concept — Conditional probability: $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$, provided $P(B) \neq 0$.

Step 1 — Substitute the values:

$$P(A \mid B) = \frac{0.18}{0.6}.$$

Step 2 — Simplify:

$$= \frac{18}{60} = 0.3.$$

Why other options are wrong: (A) 0.12 subtracts; (C) 0.108 multiplies the two numbers; (D) 0.6 just repeats $P(B)$.

Final Answer: $P(A \mid B) = 0.3 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q12

Q13.

Solution

Concept — Integration by recognising a derivative: If the numerator is the derivative of the denominator, $\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$.

Step 1 — Spot the pattern: Here $f(x) = 1 + e^x$, so $f'(x) = e^x$, which is exactly the numerator.

Step 2 — Write the result:

$$\int \frac{e^x}{1+e^x}\,dx = \ln(1+e^x) + C.$$

(The absolute value is dropped since $1 + e^x > 0$.)

Why other options are wrong: (B) is a wrong antiderivative; (C) has an extra factor e^x; (D) would arise from $\frac{e^x}{1+e^{2x}}$.

Final Answer: $\ln(1+e^x) + C \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q13

Q14.

Solution

Concept — Integral of an even function over a symmetric interval: If $g(-x) = g(x)$, then $\int_{-a}^{a} g(x)\,dx = 2\int_0^a g(x)\,dx$.

Step 1 — Test the integrand: $g(x) = x^2$ gives $g(-x) = (-x)^2 = x^2 = g(x)$, so x^2 is even.

Step 2 — Use the property:

$$\int_{-3}^{3} x^2\,dx = 2\int_0^3 x^2\,dx.$$

Step 3 — Integrate:

$$= 2\left[\frac{x^3}{3}\right]_0^3.$$

$$= 2 \cdot \frac{27}{3}.$$

$$= 2 \times 9 = 18.$$

Why other options are wrong: (A) 0 would need an odd function; (B) 9 omits the factor 2; (C) 27 ignores the division by 3.

Final Answer: The integral equals $18 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q14

Q15.

Solution

Concept — Local minimum via second derivative: At a critical point where $f'(x) = 0$, a local minimum occurs if $f''(x) > 0$.

Step 1 — Critical points:

$$f(x) = x^3 - 12x.$$

$$f'(x) = 3x^2 - 12 = 0.$$

$$x^2 = 4 \;\Rightarrow\; x = \pm 2.$$

Step 2 — Second-derivative test:

$$f''(x) = 6x.$$

At $x = 2$, $f''(2) = 12 > 0$, so $x = 2$ gives a local minimum.

Step 3 — Minimum value:

$$f(2) = (2)^3 - 12(2) = 8 - 24 = -16.$$

Why other options are wrong: (A) 16 is the local maximum value at $x = -2$; (C),(D) are not extreme values.

Final Answer: Local minimum value is $-16 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q15

Q16.

Solution

Concept — Area of a triangle: Area $= \dfrac{1}{2}\,|\vec{a} \times \vec{b}|$ for two sides $\vec{a}, \vec{b}$.

Step 1 — Compute the cross product:

$$\vec{a} \times \vec{b} = (3\hat{i}) \times (4\hat{j}) = 12\,(\hat{i} \times \hat{j}) = 12\hat{k}.$$

Step 2 — Take half the magnitude:

$$\frac{1}{2}|\vec{a} \times \vec{b}| = \frac{1}{2}|12\hat{k}| = \frac{1}{2} \times 12 = 6.$$

Why other options are wrong: (A) 12 is $|\vec{a} \times \vec{b}|$ (the parallelogram area); (B) 24 doubles it; (C) 7 adds the side lengths.

Final Answer: Area $= 6 \Rightarrow \boxed{\text{D}}$

Answer: (D) **Go Back to Q16**

Q17.

Solution

Concept — Angle between two lines: $\cos\theta = \dfrac{\vec{b}_1 \cdot \vec{b}_2}{|\vec{b}_1||\vec{b}_2|}$.

Step 1 — Dot product of direction ratios:

$$(1)(0) + (0)(1) + (1)(1) = 0 + 0 + 1 = 1.$$

Step 2 — Magnitudes:

$$|\vec{b}_1| = \sqrt{1^2 + 0^2 + 1^2} = \sqrt{2}, \qquad |\vec{b}_2| = \sqrt{0^2 + 1^2 + 1^2} = \sqrt{2}.$$

Step 3 — Compute $\cos\theta$:

$$\cos\theta = \frac{1}{\sqrt{2} \cdot \sqrt{2}} = \frac{1}{2}.$$

$$\theta = 60^\circ.$$

Why other options are wrong: (B) 90° needs a zero dot product; (C) 45° gives $\cos\theta = \frac{1}{\sqrt{2}}$; (D) 30° gives $\cos\theta = \frac{\sqrt{3}}{2}$.

Final Answer: The angle is $60^\circ \Rightarrow \boxed{\text{A}}$

Answer: (A) **Go Back to Q17**

Q18.

Solution

Concept — Union with independent events: $P(A \cup B) = P(A) + P(B) - P(A)P(B)$ when A, B are independent.

Step 1 — Product term:

$$P(A)P(B) = 0.3 \times 0.4 = 0.12.$$

Step 2 — Substitute:

$$P(A \cup B) = 0.3 + 0.4 - 0.12.$$
$$= 0.7 - 0.12.$$
$$= 0.58.$$

Why other options are wrong: (A) 0.7 forgets to subtract the intersection; (B) 0.12 is only $P(A \cap B)$; (C) 0.42 uses wrong arithmetic.

Final Answer: $P(A \cup B) = 0.58 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q18

Q19.

Solution

Concept — Assertion–Reason analysis: Judge each statement's truth, then decide whether R correctly explains A.

Step 1 — Reason: Differentiate the identity $f(-x) = f(x)$ with respect to x. The left side, by the chain rule, is $f'(-x) \cdot (-1) = -f'(-x)$, and the right side is $f'(x)$. So $-f'(-x) = f'(x)$, i.e. $f'(-x) = -f'(x)$. Thus R is a correct computation and is **true**.

Step 2 — Assertion: The relation $f'(-x) = -f'(x)$ is exactly the definition of f' being an odd function. So the derivative of an even function is odd; A is **true**.

Step 3 — Does R explain A? R is precisely the derivation that establishes A, so R is the correct explanation of A.

Why other options are wrong: (B) denies the explanatory link (which holds); (C),(D) misjudge a truth value.

Final Answer: Both true, R the correct explanation $\Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q19

Q20.

Solution

Concept — Inverse sine of a sine: $\sin^{-1}(\sin\theta) = \theta$ only when $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$; otherwise reduce to the principal branch.

Step 1 — Test the Assertion: Here $\theta = \frac{\pi}{3}$, and $\frac{\pi}{3} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

So $\sin^{-1}\left(\sin\frac{\pi}{3}\right) = \frac{\pi}{3}$. Hence A is **true**.

Step 2 — Test the Reason: The claim "$\sin^{-1}(\sin x) = x$ for every real x" is **false**; for example $\sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \frac{\pi}{3} \neq \frac{2\pi}{3}$. It holds only on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Why other options are wrong: (A),(B) require R true; (D) requires A false.

Final Answer: A true, R false $\Rightarrow \boxed{\text{C}}$

$\boxed{\textbf{Answer: (C)}}$ Go Back to Q20

Q21.

Solution

Concept — Principal values: $\sin^{-1}$ lies in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $\cos^{-1}$ lies in $[0, \pi]$.

Step 1 — Evaluate $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$:

$$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} \Rightarrow \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}.$$

Step 2 — Evaluate $\cos^{-1}\left(\frac{1}{2}\right)$:

$$\cos\frac{\pi}{3} = \frac{1}{2} \Rightarrow \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}.$$

Step 3 — Add:

$$\frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}.$$

Final Answer: $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{1}{2}\right) = \frac{2\pi}{3}$. Go Back to Q21

Q22.

Solution

Concept — Logarithmic differentiation: Used when the variable appears in both base and exponent.

Step 1 — Take natural log of both sides:

$$y = x^x.$$

$$\ln y = x \ln x.$$

Step 2 — Differentiate both sides w.r.t. x: (product rule on the right)

$$\frac{1}{y}\frac{dy}{dx} = 1 \cdot \ln x + x \cdot \frac{1}{x}.$$

$$\frac{1}{y}\frac{dy}{dx} = \ln x + 1.$$

Step 3 — Solve for $\frac{dy}{dx}$:

$$\begin{aligned}\frac{dy}{dx} &= y\,(\ln x + 1).\\ &= x^x(1 + \ln x).\end{aligned}$$

Final Answer: $\frac{dy}{dx} = x^x(1 + \ln x)$. Go Back to Q22

Q23.

Solution

Concept — Integration by parts: $\int u\,dv = uv - \int v\,du$, choosing u by the ILATE rule.

Step 1 — Choose parts: Let $u = x$ and $dv = \cos x\,dx$.

Then $du = dx$ and $v = \sin x$.

Step 2 — Apply the formula:

$$\int x \cos x\,dx = x \sin x - \int \sin x\,dx.$$

$$= x \sin x - (-\cos x) + C.$$

$$= x \sin x + \cos x + C.$$

Final Answer: $\int x\cos x\,dx = x\sin x + \cos x + C.$

OR — $\int \ln x\,dx$**:**

Step 1 — Choose parts: Let $u = \ln x$ and $dv = dx$.

Then $du = \frac{1}{x}\,dx$ and $v = x$.

Step 2 — Apply the formula:

$$\int \ln x\,dx = x\ln x - \int x\cdot\frac{1}{x}\,dx.$$

$$= x\ln x - \int 1\,dx.$$

$$= x\ln x - x + C.$$

Final Answer (OR): $\int \ln x\,dx = x\ln x - x + C.$ Go Back to Q23

Q24.

Solution

Concept — Perpendicular vectors: $\vec{a}\perp\vec{b} \iff \vec{a}\cdot\vec{b} = 0.$

Step 1 — Compute the dot product:

$$\vec{a}\cdot\vec{b} = (2)(1) + (\lambda)(2) + (-3)(-2).$$

$$= 2 + 2\lambda + 6.$$

$$= 8 + 2\lambda.$$

Step 2 — Set equal to zero and solve:

$$8 + 2\lambda = 0.$$

$$2\lambda = -8.$$

$$\lambda = -4.$$

Final Answer: $\lambda = -4.$ Go Back to Q24

Q25.

Solution

Concept — Equally likely outcomes: Probability $= \dfrac{\text{favourable outcomes}}{\text{total outcomes}}$.

Step 1 — Total outcomes: Two throws of a die give $6 \times 6 = 36$ equally likely outcomes.

Step 2 — Favourable outcomes (sum $= 8$):

$$(2,6),\ (6,2),\ (3,5),\ (5,3),\ (4,4).$$

There are 5 such outcomes.

Step 3 — Probability:

$$P(\text{sum} = 8) = \frac{5}{36}.$$

Final Answer: $P(\text{sum} = 8) = \dfrac{5}{36}$.

OR — $P(A \mid B)$:

Step 1 — Formula: $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$.

Step 2 — Substitute:

$$P(A \mid B) = \frac{0.24}{0.6} = 0.4.$$

Final Answer (OR): $P(A \mid B) = 0.4$. **Go Back to Q25**

Q26.

Solution

Concept — Partial fractions: Split a proper rational function into simpler fractions, then integrate each.

Step 1 — Set up the decomposition:

$$\frac{1}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}.$$

$$1 = A(x+2) + B(x-1).$$

Step 2 — Find A and B:

Put $x = 1$: $1 = A(3) + B(0) \Rightarrow A = \dfrac{1}{3}$.

Put $x = -2$: $1 = A(0) + B(-3) \Rightarrow B = -\dfrac{1}{3}$.

Step 3 — Integrate:

$$\int \frac{1}{(x-1)(x+2)}\,dx = \int \left(\frac{1/3}{x-1} - \frac{1/3}{x+2}\right) dx.$$

$$= \frac{1}{3}\ln|x-1| - \frac{1}{3}\ln|x+2| + C.$$

Final Answer: $\frac{1}{3}\ln\left|\frac{x-1}{x+2}\right| + C$. **Go Back to Q26**

Q27.

Solution

Concept — Linear differential equation: For $\frac{dy}{dx} + Py = Q$, the integrating factor is I.F. $= e^{\int P\,dx}$ and the solution is $y \cdot \text{I.F.} = \int Q \cdot \text{I.F.}\,dx$.

Step 1 — Identify P, Q: Here $P = 2$, $Q = e^{-x}$.

Step 2 — Integrating factor:

$$\text{I.F.} = e^{\int 2\,dx} = e^{2x}.$$

Step 3 — Solve:

$$y\,e^{2x} = \int e^{-x} \cdot e^{2x}\,dx = \int e^{x}\,dx.$$

$$y\,e^{2x} = e^{x} + C.$$

$$y = e^{-x} + C\,e^{-2x}.$$

Final Answer: $y = e^{-x} + C\,e^{-2x}$.

OR — Variables separable $\frac{dy}{dx} = e^{x-y}$:

Step 1 — Rewrite and separate:

$$\frac{dy}{dx} = e^{x}\,e^{-y}.$$

$$e^{y}\,dy = e^{x}\,dx.$$

Step 2 — Integrate both sides:

$$e^{y} = e^{x} + C.$$

Final Answer (OR): $e^{y} = e^{x} + C$. **Go Back to Q27**

Q28.

Solution

Concept — Inverse by adjoint: $A^{-1} = \frac{1}{|A|}\,\text{adj}(A)$, valid when $|A| \neq 0$.

Step 1 — Determinant:

$$|A| = \begin{vmatrix} 2 & 1 \\ 5 & 3 \end{vmatrix} = (2)(3) - (1)(5) = 6 - 5 = 1.$$

Step 2 — Adjoint of a 2×2 matrix: Swap the leading diagonal entries and negate the off-diagonal entries:

$$\text{adj}(A) = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}.$$

Step 3 — Inverse:

$$A^{-1} = \frac{1}{1}\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}.$$

Final Answer: $A^{-1} = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$. **Go Back to Q28**

Q29.

Solution

Concept — Monotonicity: f increases where $f'(x) > 0$ and decreases where $f'(x) < 0$.

Step 1 — Differentiate:

$$f(x) = x^3 - 3x^2 - 9x + 7.$$

$$f'(x) = 3x^2 - 6x - 9.$$

$$= 3(x^2 - 2x - 3) = 3(x - 3)(x + 1).$$

Step 2 — Critical points: $f'(x) = 0$ at $x = -1$ and $x = 3$.

Step 3 — Sign analysis:

For $x < -1$: $f' > 0$; for $-1 < x < 3$: $f' < 0$; for $x > 3$: $f' > 0$.

Conclusion: Increasing on $(-\infty, -1) \cup (3, \infty)$; decreasing on $(-1, 3)$.

Final Answer: Increasing on $(-\infty, -1) \cup (3, \infty)$, decreasing on $(-1, 3)$.

OR — Tangent to $y = x^3$ at $(1, 1)$:

Step 1 — Slope: $\frac{dy}{dx} = 3x^2$, so at $x = 1$ slope $= 3$.

Step 2 — Equation: $y - 1 = 3(x - 1) \Rightarrow y = 3x - 2$.

Final Answer (OR): $y = 3x - 2$. Go Back to Q29

Q30.

Solution

Concept — Vector equation of a line: A line through point $\vec{a}$ parallel to $\vec{b}$ is $\vec{r} = \vec{a} + \lambda\vec{b}$.

Step 1 — Write the equation: With $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} - \hat{k}$,

$$\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} - \hat{k}).$$

Step 2 — Test the point $(4, 2, 2)$: The point lies on the line if some λ satisfies all three component equations.

$$2 + 2\lambda = 4 \;\Rightarrow\; \lambda = 1.$$

$$-1 + 3\lambda = 2 \;\Rightarrow\; \lambda = 1.$$

$$3 - \lambda = 2 \;\Rightarrow\; \lambda = 1.$$

Step 3 — Conclusion: All three give the same value $\lambda = 1$, so the point $(4, 2, 2)$ lies on the line.

Final Answer: $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$; the point $(4, 2, 2)$ lies on it (at $\lambda = 1$). Go Back to Q30

Q31.

Solution

Concept — One-one (injective) and onto (surjective): *One-one*: $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$. *Onto*: every y in the codomain has a pre-image.

Step 1 — One-one: Suppose $f(x_1) = f(x_2)$.

$$3x_1 - 4 = 3x_2 - 4.$$

$$3x_1 = 3x_2.$$

$$x_1 = x_2.$$

Hence f is one-one.

Step 2 — Onto: Let $y \in \mathbb{R}$ be arbitrary. Solve $y = 3x - 4$ for x:

$$x = \frac{y+4}{3} \in \mathbb{R}.$$

Then $f(x) = 3\left(\frac{y+4}{3}\right) - 4 = y$.

So every y has a pre-image; f is onto.

Step 3 — Conclusion: f is both one-one and onto, hence bijective.

Final Answer: $f(x) = 3x - 4$ is one-one and onto (a bijection). **Go Back to Q31**

Q32.

Solution

Concept — Matrix method: Write $AX = B$; then $X = A^{-1}B$ when $|A| \neq 0$.

Step 1 — Form the matrices:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 1 & -1 & 2 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 4 \\ 0 \\ 9 \end{bmatrix}.$$

Step 2 — Determinant of A:

$$|A| = 1(1 \cdot 2 - (-1)(-1)) - 1(2 \cdot 2 - (-1)(1)) + 1(2 \cdot (-1) - 1 \cdot 1).$$

$$= 1(2-1) - 1(4+1) + 1(-2-1).$$

$$= 1 - 5 - 3 = -7 \ (\neq 0).$$

Step 3 — Cofactors and adjoint: Computing the cofactor matrix and transposing gives

$$\text{adj}(A) = \begin{bmatrix} 1 & -3 & -2 \\ -5 & 1 & 3 \\ -3 & 2 & -1 \end{bmatrix}.$$

Step 4 — Solve $X = \dfrac{1}{|A|}$ adj$(A)\,B$:

$$X = \frac{1}{-7}\begin{bmatrix}1 & -3 & -2\\ -5 & 1 & 3\\ -3 & 2 & -1\end{bmatrix}\begin{bmatrix}4\\0\\9\end{bmatrix}.$$

$$= \frac{1}{-7}\begin{bmatrix}1\cdot 4 - 3\cdot 0 - 2\cdot 9\\ -5\cdot 4 + 1\cdot 0 + 3\cdot 9\\ -3\cdot 4 + 2\cdot 0 - 1\cdot 9\end{bmatrix}.$$

$$= \frac{1}{-7}\begin{bmatrix}4-18\\ -20+27\\ -12-9\end{bmatrix} = \frac{1}{-7}\begin{bmatrix}-14\\ 7\\ -21\end{bmatrix} = \begin{bmatrix}2\\ -1\\ 3\end{bmatrix}.$$

Step 5 — Verify: $x+y+z = 2-1+3 = 4\ \checkmark$; $2x+y-z = 4-1-3 = 0\ \checkmark$; $x-y+2z = 2+1+6 = 9\ \checkmark$.

Final Answer: $x=2,\ y=-1,\ z=3$. **Go Back to Q32**

Q33.

Solution

Concept — Area between two curves: Area $= \displaystyle\int_a^b (\text{upper} - \text{lower})\,dx$, where a, b are the x-coordinates of intersection.

Step 1 — Find the points of intersection: Set $x^2 = 2x$.

$$x^2 - 2x = 0.$$

$$x(x-2) = 0 \;\Rightarrow\; x = 0 \text{ or } x = 2.$$

Step 2 — Decide upper and lower: On $(0,2)$, the line $y = 2x$ lies above the parabola $y = x^2$.

Step 3 — Set up the integral:

$$\text{Area} = \int_0^2 (2x - x^2)\,dx.$$

Step 4 — Integrate:

$$= \left[x^2 - \frac{x^3}{3}\right]_0^2.$$

$$= \left(4 - \frac{8}{3}\right) - (0).$$

$$= \frac{12 - 8}{3} = \frac{4}{3}.$$

Final Answer: Area $= \frac{4}{3}$ square units.

OR — Region under $y = \sin x$, $0 \le x \le \pi$:

Step 1 — Set up the integral: Since $\sin x \ge 0$ on $[0, \pi]$,

$$\text{Area} = \int_0^{\pi} \sin x \, dx.$$

Step 2 — Integrate:

$$= \big[-\cos x \big]_0^{\pi}.$$

$$= (-\cos \pi) - (-\cos 0).$$

$$= -(-1) - (-1) = 1 + 1 = 2.$$

Final Answer (OR): Area $= 2$ square units. **Go Back to Q33**

Q34.

Solution

Concept — Corner-point method: The optimum of a linear objective over a bounded feasible region occurs at a corner (vertex).

Step 1 — Find the corner points: The constraints $2x + y \le 10$, $x + 3y \le 15$, $x \ge 0$, $y \ge 0$ bound the region with vertices:

$$O(0,0), \quad (5,0), \quad (0,5).$$

The lines $2x + y = 10$ and $x + 3y = 15$ intersect where, from $y = 10 - 2x$,

$$x + 3(10 - 2x) = 15 \;\Rightarrow\; x + 30 - 6x = 15 \;\Rightarrow\; -5x = -15 \;\Rightarrow\; x = 3, \; y = 4.$$

So the fourth vertex is $(3, 4)$.

Step 2 — Evaluate $Z = 2x + 5y$ at each corner:

$$Z(0,0) = 0.$$

$$Z(5,0) = 2(5) + 0 = 10.$$

$$Z(3,4) = 2(3) + 5(4) = 6 + 20 = 26.$$

$$Z(0,5) = 2(0) + 5(5) = 25.$$

Step 3 — Pick the maximum: The largest value is 26 at $(3,4)$.

Final Answer: Z is maximum $= 26$ at $(x,y) = (3,4)$. **Go Back to Q34**

Q35.

Solution

Concept — Shortest distance between skew lines: For $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$,

$$d = \frac{\left|(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)\right|}{|\vec{b}_1 \times \vec{b}_2|}.$$

Step 1 — Identify vectors: $\vec{a}_1 = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$, $\vec{b}_1 = \hat{\imath} - \hat{\jmath} + 2\hat{k}$; $\vec{a}_2 = 2\hat{\imath} - \hat{\jmath} + \hat{k}$, $\vec{b}_2 = 2\hat{\imath} + \hat{\jmath} - \hat{k}$.

Step 2 — Cross product $\vec{b}_1 \times \vec{b}_2$:

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -1 & 2 \\ 2 & 1 & -1 \end{vmatrix}.$$

$$= \hat{\imath}\big[(-1)(-1) - (2)(1)\big] - \hat{\jmath}\big[(1)(-1) - (2)(2)\big] + \hat{k}\big[(1)(1) - (-1)(2)\big].$$

$$= \hat{\imath}(1-2) - \hat{\jmath}(-1-4) + \hat{k}(1+2).$$

$$= -\hat{\imath} + 5\hat{\jmath} + 3\hat{k}.$$

Step 3 — Its magnitude:

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{(-1)^2 + 5^2 + 3^2} = \sqrt{1 + 25 + 9} = \sqrt{35}.$$

Step 4 — $(\vec{a}_2 - \vec{a}_1)$ and the dot product:

$$\vec{a}_2 - \vec{a}_1 = (2-1)\hat{\imath} + (-1-2)\hat{\jmath} + (1-3)\hat{k} = \hat{\imath} - 3\hat{\jmath} - 2\hat{k}.$$

$$(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2) = (1)(-1) + (-3)(5) + (-2)(3) = -1 - 15 - 6 = -22.$$

Step 5 — Distance:

$$d = \frac{|-22|}{\sqrt{35}} = \frac{22}{\sqrt{35}} \text{ units.}$$

Final Answer: Shortest distance $= \dfrac{22}{\sqrt{35}}$ units.

OR — Angle between line and plane:

Step 1 — Direction $\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$**, normal** $\vec{n} = 2\hat{i} + \hat{j} - \hat{k}$**.**

Step 2 — Use $\sin\theta = \dfrac{|\vec{b}\cdot\vec{n}|}{|\vec{b}||\vec{n}|}$:

$$\vec{b}\cdot\vec{n} = (2)(2) + (-1)(1) + (2)(-1) = 4 - 1 - 2 = 1.$$

$$|\vec{b}| = \sqrt{4+1+4} = 3, \qquad |\vec{n}| = \sqrt{4+1+1} = \sqrt{6}.$$

$$\sin\theta = \frac{1}{3\sqrt{6}}.$$

Final Answer (OR): $\theta = \sin^{-1}\left(\dfrac{1}{3\sqrt{6}}\right)$. **Go Back to Q35**

Q36.

Solution

Concept — Maxima by derivatives: Build the area function, set $\dfrac{dA}{dx} = 0$, and select the value giving a genuine maximum.

(i) Area function: The rectangle has base $2x$ and height $2\sqrt{R^2 - x^2}$, so

$$A = (2x)\left(2\sqrt{R^2 - x^2}\right) = 4x\sqrt{R^2 - x^2}, \qquad 0 < x < R.$$

(ii) Differentiate: Using the product rule with $\dfrac{d}{dx}\sqrt{R^2 - x^2} = \dfrac{-x}{\sqrt{R^2 - x^2}}$,

$$\frac{dA}{dx} = 4\sqrt{R^2 - x^2} + 4x \cdot \frac{-x}{\sqrt{R^2 - x^2}}.$$

$$= \frac{4(R^2 - x^2) - 4x^2}{\sqrt{R^2 - x^2}}.$$

$$= \frac{4R^2 - 8x^2}{\sqrt{R^2 - x^2}}.$$

(iii) Critical point and maximum: Set $\dfrac{dA}{dx} = 0$; the denominator is positive, so

$$4R^2 - 8x^2 = 0.$$

$$x^2 = \frac{R^2}{2} \Rightarrow x = \frac{R}{\sqrt{2}}.$$

For $x < \dfrac{R}{\sqrt{2}}$ the numerator $4R^2 - 8x^2 > 0$ and for $x > \dfrac{R}{\sqrt{2}}$ it is < 0, so $x = \dfrac{R}{\sqrt{2}}$ gives a maximum.

At this x the height is

$$2\sqrt{R^2 - \tfrac{R^2}{2}} = 2\sqrt{\tfrac{R^2}{2}} = 2 \cdot \frac{R}{\sqrt{2}} = \sqrt{2}\,R,$$

and the base is $2x = 2 \cdot \dfrac{R}{\sqrt{2}} = \sqrt{2}\,R$. Both sides equal $\sqrt{2}\,R$, so the rectangle is a square, with

$$A_{\max} = \left(\sqrt{2}\,R\right)^2 = 2R^2.$$

Final Answer: $A = 4x\sqrt{R^2 - x^2}$; $\dfrac{dA}{dx} = \dfrac{4R^2 - 8x^2}{\sqrt{R^2 - x^2}}$; maximum at $x = \dfrac{R}{\sqrt{2}}$, where the rectangle is a square of area $2R^2$. Go Back to Q36

Q37.

Solution

Concept — Bayes' theorem: $P(E_i \mid A) = \dfrac{P(E_i)P(A \mid E_i)}{\sum_j P(E_j)P(A \mid E_j)}$.

(i) Prior probabilities: The student either knows or guesses, so

$$P(\text{Knows}) = \frac{3}{4}, \qquad P(\text{Guesses}) = \frac{1}{4}.$$

(ii) Conditional (correct-answer) probabilities: If the student knows, the answer is surely correct; if the student guesses, one of four options is right, so

$$P(\text{Correct} \mid \text{Knows}) = 1, \qquad P(\text{Correct} \mid \text{Guesses}) = \frac{1}{4}.$$

(iii) Apply Bayes' theorem:

$$P(\text{Knows} \mid \text{Correct}) = \frac{P(\text{Knows})P(\text{Correct} \mid \text{Knows})}{P(\text{Knows})P(\text{Correct} \mid \text{Knows}) + P(\text{Guesses})P(\text{Correct} \mid \text{Guesses})}.$$

Compute each product:

$$P(\text{Knows})P(\text{Correct} \mid \text{Knows}) = \frac{3}{4} \times 1 = \frac{3}{4}.$$

$$P(\text{Guesses})P(\text{Correct} \mid \text{Guesses}) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}.$$

Substitute:

$$P(\text{Knows} \mid \text{Correct}) = \frac{\frac{3}{4}}{\frac{3}{4}+\frac{1}{16}} = \frac{\frac{12}{16}}{\frac{12}{16}+\frac{1}{16}} = \frac{\frac{12}{16}}{\frac{13}{16}}.$$
$$= \frac{12}{13}.$$

Final Answer: $P(\text{Knows} \mid \text{Correct}) = \frac{12}{13}$. **Go Back to Q37**

Q38.

Solution

Concept — Related rates: The area depends on the radius, and the radius depends on time; link the two rates with the chain rule $\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt}$.

(i) Area as a function of radius:

$$A = \pi r^2.$$
$$\frac{dA}{dr} = 2\pi r.$$

(ii) Chain rule for $\frac{dA}{dt}$:

$$\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt}.$$
$$= 2\pi r \cdot \frac{dr}{dt}.$$

(iii) Rate of area increase at $r = 10$ cm: Here $\frac{dr}{dt} = 3$ cm/s. Substitute $r = 10$ and $\frac{dr}{dt} = 3$:

$$\frac{dA}{dt} = 2\pi(10)(3).$$
$$= 60\pi \text{ cm}^2/\text{s}.$$

Numerically, $60\pi \approx 188.5$ cm^2/s.

Final Answer: $A = \pi r^2$; $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$; when $r = 10$ cm the area increases at $60\pi \approx 188.5$ cm^2/s. **Go Back to Q38**

Answer Key – Section A (Q1–Q20)

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	D	3	C	4	A	5	B
6	C	7	C	8	B	9	A	10	D
11	C	12	B	13	A	14	D	15	B
16	D	17	A	18	D	19	A	20	C

Sections B–E are descriptive; refer to the Detailed Solutions above for full model answers and step marking.