CBSE Class 12 Mathematics

Sample Paper – 4

Duration: 180 Minutes **Maximum Marks: 80**

General Instructions

- This question paper contains **38 questions**. **All questions are compulsory**.
- The paper is divided into **five sections** – A, B, C, D and E.
- **Section A** (Q1–Q20) carries **1 mark** each: Q1–Q18 are multiple choice questions and Q19–Q20 are Assertion–Reason questions.
- **Section B** (Q21–Q25) carries **2 marks** each (Very Short Answer).
- **Section C** (Q26–Q31) carries **3 marks** each (Short Answer).
- **Section D** (Q32–Q35) carries **5 marks** each (Long Answer).
- **Section E** (Q36–Q38) carries **4 marks** each (case study based, with sub-parts).
- There is **no overall choice**, but an **internal choice** has been provided in some questions. Attempt only one of the alternatives in such questions.
- There is **no negative marking**. Use of a **calculator is not permitted**.

Section A (Q1–Q20) – 1 Mark Each

Q1. If $A = [a_{ij}]$ is a matrix of order 3×3 whose elements are given by $a_{ij} = 2i - j$, then the element a_{32} is:

(A) 3

(B) 4

(C) 5

(D) 1

Q2. The value of the determinant $\begin{vmatrix} 5 & 3 \\ 2 & 4 \end{vmatrix}$ is:

(A) 26

(B) 8

(C) -14

(D) 14

Q3. Let $A = \{1, 2, 3\}$ and $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2)\}$ be a relation on A. Then R is:

(A) reflexive and symmetric but not transitive

(B) an equivalence relation

(C) transitive but not reflexive

(D) symmetric and transitive but not reflexive

Q4. The principal value of $\sec^{-1}(2)$ is:

(A) $\dfrac{\pi}{6}$

(B) $\dfrac{\pi}{4}$

(C) $\dfrac{\pi}{3}$

(D) $\dfrac{2\pi}{3}$

Q5. If $y = \ln(\sec x)$, then $\dfrac{dy}{dx}$ equals:

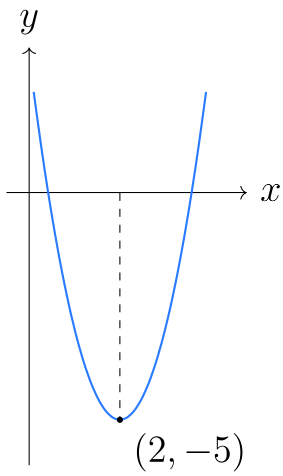
(A) $\cot x$

(B) $\tan x$

(C) $\sec x \tan x$

(D) $-\tan x$

Q6. The graph of $f(x) = 2x^2 - 8x + 3$ is shown below. The function is *increasing* on the interval:



(A) $(-\infty, 2)$

(B) $(2, \infty)$

(C) $(-\infty, \infty)$

(D) $(0, 2)$

Q7. $\int e^{2x}\,dx$ equals:

(A) $2e^{2x} + C$

(B) $e^{2x} + C$

(C) $\dfrac{e^{x}}{2} + C$

(D) $\dfrac{e^{2x}}{2} + C$

Q8. The order and degree of the differential equation $\left(\dfrac{d^2y}{dx^2}\right)^2 + \left(\dfrac{dy}{dx}\right)^3 + y = 0$ are respectively:

(A) 2 and 3

(B) 2 and 2

(C) 1 and 2

(D) 2 and 1

Q9. The magnitude of the vector $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ is:

(A) 5

(B) 11

(C) 7

(D) $\sqrt{11}$

Q10. The direction cosines of a line whose direction ratios are $2, -1, 2$ are:

(A) $\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}$

(B) $2, -1, 2$

(C) $\frac{2}{9}, -\frac{1}{9}, \frac{2}{9}$

(D) $\frac{1}{3}, -\frac{1}{3}, \frac{1}{3}$

Q11. $\int_0^1 x^2\,dx$ equals:

(A) 1

(B) 3

(C) $\frac{1}{2}$

(D) $\frac{1}{3}$

Q12. If $P(A \cap B) = 0.15$ and $P(B) = 0.6$, then $P(A \mid B)$ equals:

(A) 0.4

(B) 0.25

(C) 0.09

(D) 0.75

Q13. $\int \frac{3x^2}{1+x^3}\,dx$ equals:

(A) $3\ln(1+x^3) + C$

(B) $\frac{1}{1+x^3} + C$

(C) $\ln(1+x^3) + C$

(D) $\ln(1+x^2) + C$

Q14. $\int_{-2}^{2} x^2\,dx$ equals:

(A) $\frac{16}{3}$

(B) 0

(C) $\frac{8}{3}$

(D) 16

Q15. The local minimum value of $f(x) = x^3 - 12x$ is:

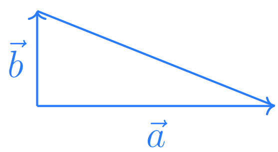
(A) 16

(B) -8

(C) 8

(D) -16

Q16. The area of the triangle whose two sides are represented by the vectors $\vec{a} = 6\hat{i}$ and $\vec{b} = 2\hat{j}$ (shown below) is:



(A) 6

(B) 12

(C) 24

(D) 8

Q17. The angle between two lines whose direction ratios are $2, 4, 6$ and $1, 2, 3$ is:

(A) 90°

(B) 45°

(C) 0°

(D) 60°

Q18. If A and B are independent events with $P(A) = 0.3$ and $P(B) = 0.4$, then $P(A \cup B)$ equals:

(A) 0.7

(B) 0.42

(C) 0.12

(D) 0.58

Q19. **Assertion (A):** For any two square matrices A and B of the same order, $(AB)^T = B^T A^T$.

Reason (R): Matrix multiplication is commutative, that is, $AB = BA$ for all square matrices A, B.

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is *not* the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

Q20. **Assertion (A):** The determinant of every 3×3 skew-symmetric matrix is 0.

Reason (R): For a skew-symmetric matrix of odd order, the determinant is always 0.

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is *not* the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

Section B (Q21–Q25) – 2 Marks Each

Q21. Find the principal value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{1}{2}\right)$. **[2]**

Q22. Differentiate $y = x^{\ln x}$ with respect to x. **[2]**

Q23. Evaluate $\int x \cos x \, dx$. **[2]**

OR

Evaluate $\int \ln x \, dx$.

Q24. Find the value of λ for which the vectors $\vec{a} = \lambda\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b} = 3\hat{i}-3\hat{j}-\hat{k}$ are perpendicular. **[2]**

Q25. A fair die is thrown twice. Find the probability of getting a sum of 7. **[2]**

OR

If $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cap B) = 0.2$, find $P(A \mid B)$.

Section C (Q26–Q31) – 3 Marks Each

Q26. Evaluate $\int \frac{2x+3}{(x+1)(x+3)} dx$. **[3]**

Q27. Solve the differential equation $\frac{dy}{dx} + \frac{1}{x} y = x^2$. **[3]**

OR

Solve the differential equation $\frac{dy}{dx} = e^{x-y}$.

Q28. Find the inverse of the matrix $A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ using the adjoint method. **[3]**

Q29. Find the intervals in which $f(x) = x^3 - 3x^2 - 9x + 5$ is increasing or decreasing. **[3]**

OR

Find the equation of the tangent to the curve $y = x^3$ at the point $(1, 1)$.

Q30. Find the vector equation of the line passing through the point $(2, -1, 3)$ and parallel to the vector $\hat{i} + 2\hat{j} - \hat{k}$. Hence check whether the point $(4, 3, 1)$ lies on this line. **[3]**

Q31. Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 3x - 2$ is one-one and onto. **[3]**

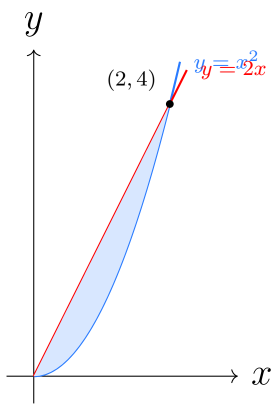
Section D (Q32–Q35) – 5 Marks Each

Q32. Using the matrix method, solve the following system of linear equations:

$$x + 2y + z = 7, \qquad 2x - y + z = 6, \qquad x + y - z = 0.$$

[5]

Q33. Using integration, find the area of the region bounded by the curve $y = x^2$ and the line $y = 2x$.



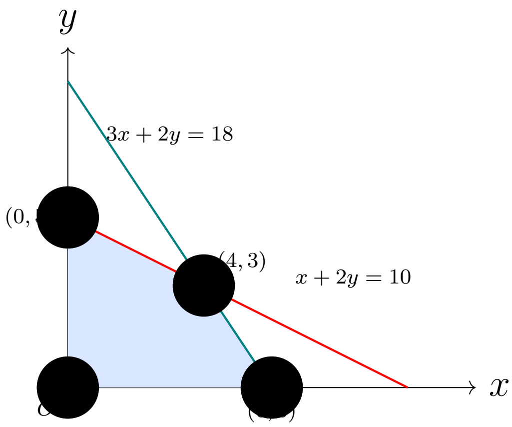
[5]

OR

Using integration, find the area of the region bounded by the parabola $y = 4 - x^2$ and the x-axis.

Q34. Solve the following linear programming problem graphically. Maximise $Z = 4x + 3y$ subject to the constraints

$$3x + 2y \leq 18, \qquad x + 2y \leq 10, \qquad x \geq 0,\ y \geq 0.$$



[5]

Q35. Find the shortest distance between the lines

$$\vec{r} = (\hat{i}+2\hat{j}+3\hat{k})+\lambda(2\hat{i}+3\hat{j}+4\hat{k}) \quad \text{and} \quad \vec{r} = (2\hat{i}+4\hat{j}+5\hat{k})+\mu(3\hat{i}+4\hat{j}+5\hat{k}).$$

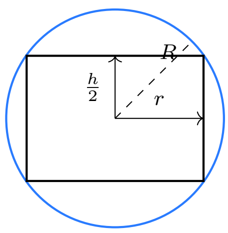
[5]

OR

Find the angle between the line $\vec{r} = (\hat{i} + \hat{j} + \hat{k}) + \lambda(2\hat{i} - \hat{j} + 2\hat{k})$ and the plane $2x + y - 2z = 5$.

Section E (Q36–Q38) – 4 Marks Each (Case Study)

Q36. Case Study – Cylinder Inside a Sphere.
A designer wants to cut the largest possible right circular cylinder from a solid spherical ball of radius R. The cylinder of radius r and height h is inscribed in the sphere so that the rims of its two circular ends lie on the sphere, as shown.



Based on the above, answer the following:

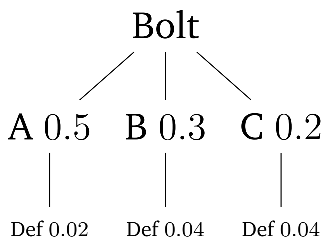
(i) Using the sphere, express r^2 in terms of R and h, and hence write the volume $V = \pi r^2 h$ as a function of h. (1)

(ii) Find $\dfrac{dV}{dh}$. (1)

(iii) Find the height h for which the volume is maximum, and the corresponding radius r. (2)

Q37. Case Study – Three Production Plants.

A company manufactures bolts at three plants A, B and C. Plant A makes 50% of the total output, plant B makes 30% and plant C makes the remaining 20%. The proportion of defective bolts is 2% from plant A, 4% from plant B and 4% from plant C. A bolt is drawn at random from the combined stock and is found to be defective.



Based on the above, answer the following:

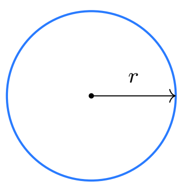
(i) Write the prior probabilities $P(A)$, $P(B)$ and $P(C)$. (1)

(ii) Write $P(\text{Def} \mid A)$, $P(\text{Def} \mid B)$, $P(\text{Def} \mid C)$ and find the total probability that a bolt is defective. (1)

(iii) Using Bayes' theorem, find the probability that the defective bolt was produced by plant A. (2)

Q38. Case Study – The Inflating Balloon.

A spherical balloon is being inflated so that its radius r (in cm) increases at the constant rate of $\dfrac{dr}{dt} = 0.5$ cm/s. Recall that the volume of a sphere of radius r is $V = \dfrac{4}{3}\pi r^3$.



Based on the above, answer the following:

(i) Find $\dfrac{dV}{dr}$. (1)

(ii) Using the chain rule, write $\dfrac{dV}{dt}$ in terms of r. (1)

(iii) Find the rate at which the volume is increasing when $r = 7$ cm. (2)

Detailed Solutions

Q1.

Solution

Concept — Element from a defining rule: The entry a_{ij} of a matrix is obtained by substituting the row number i and column number j into the given rule.

Step 1 — Identify i and j: For a_{32} we take $i = 3$ and $j = 2$.

Step 2 — Substitute in $a_{ij} = 2i - j$:

$$a_{32} = 2(3) - 2.$$

$$= 6 - 2.$$

$$= 4.$$

Why other options are wrong: (A) 3 uses $2i - j$ with wrong indices; (C) 5 comes from $2(3) - 1$; (D) 1 comes from $2(1) - 1$.

Final Answer: $a_{32} = 4 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q1

Q2.

Solution

Concept — Value of a 2×2 determinant: For $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$, the value is $ad - bc$.

Step 1 — Identify entries: Here $a = 5$, $b = 3$, $c = 2$, $d = 4$.

Step 2 — Apply the formula:

$$ad - bc = (5)(4) - (3)(2).$$

$$= 20 - 6.$$

$$= 14.$$

Why other options are wrong: (A) 26 adds instead of subtracting; (B) 8 subtracts wrongly; (C) -14 flips the overall sign.

Final Answer: The determinant is $14 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q2

Q3.

Solution

Concept — Properties of a relation: *Reflexive*: $(a, a) \in R$ for all a. *Symmetric*: $(a, b) \in R \Rightarrow (b, a) \in R$. *Transitive*: $(a, b), (b, c) \in R \Rightarrow (a, c) \in R$.

Step 1 — Reflexive? $(1, 1), (2, 2), (3, 3) \in R$, so R is reflexive.

Step 2 — Symmetric? The off-diagonal pairs are $(1, 2)$ with $(2, 1)$, and $(2, 3)$ with $(3, 2)$; both reverses are present.

Hence R is symmetric.

Step 3 — Transitive? $(1, 2) \in R$ and $(2, 3) \in R$, but $(1, 3) \notin R$.

Hence R is not transitive.

Why other options are wrong: (B) an equivalence relation needs transitivity, which fails; (C) R is reflexive, so "not reflexive" is false; (D) likewise fails as R is reflexive.

Final Answer: R is reflexive and symmetric but not transitive $\Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q3

Q4.

Solution

Concept — Principal value of $\sec^{-1}$: The principal value branch of $\sec^{-1} x$ is $[0, \pi] \setminus \left\{\frac{\pi}{2}\right\}$.

Step 1 — Set up the equation: Let $\sec^{-1}(2) = \theta$, so $\sec\theta = 2$.

Step 2 — Convert to cosine: $\sec\theta = 2 \Rightarrow \cos\theta = \frac{1}{2}$.

Step 3 — Find θ in the branch: We know $\cos\frac{\pi}{3} = \frac{1}{2}$, and $\frac{\pi}{3} \in [0, \pi]$.

So $\theta = \frac{\pi}{3}$.

Why other options are wrong: (A) $\frac{\pi}{6}$ gives $\cos\theta = \frac{\sqrt{3}}{2}$; (B) $\frac{\pi}{4}$ gives $\frac{1}{\sqrt{2}}$; (D) $\frac{2\pi}{3}$ gives $\cos\theta = -\frac{1}{2}$.

Final Answer: $\sec^{-1}(2) = \frac{\pi}{3} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q4

Q5.

Solution

Concept — Derivative of a logarithm: $\frac{d}{dx}\ln(u) = \frac{1}{u} \cdot \frac{du}{dx}$.

Step 1 — Identify the inner function: Let $u = \sec x$, so $\frac{du}{dx} = \sec x \tan x$.

Step 2 — Apply the rule:

$$\frac{dy}{dx} = \frac{1}{\sec x} \cdot \sec x \tan x.$$

Step 3 — Simplify:

$$= \tan x.$$

Why other options are wrong: (A) $\cot x$ inverts the ratio; (C) $\sec x \tan x$ forgets to divide by $\sec x$; (D) $-\tan x$ has a wrong sign.

Final Answer: $\frac{dy}{dx} = \tan x \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q5

Q6.

Solution

Concept — Increasing function: f is increasing where $f'(x) > 0$.

Step 1 — Differentiate:

$$f(x) = 2x^2 - 8x + 3.$$

$$f'(x) = 4x - 8.$$

Step 2 — Solve $f'(x) > 0$:

$$4x - 8 > 0.$$

$$4x > 8.$$

$$x > 2.$$

So f is increasing on $(2, \infty)$, which matches the right, rising part of the upward parabola beyond the vertex $(2, -5)$.

Why other options are wrong: (A) $(-\infty, 2)$ is where the parabola falls; (C) is false since the curve falls then rises; (D) $(0, 2)$ lies inside the decreasing part.

Final Answer: f increases on $(2, \infty) \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q6

Q7.

Solution

Concept — Integral of an exponential: $\int e^{ax}\,dx = \frac{e^{ax}}{a} + C.$

Step 1 — Identify a**:** Here $a = 2$.

Step 2 — Apply the formula:

$$\int e^{2x}\,dx = \frac{e^{2x}}{2} + C.$$

Why other options are wrong: (A) $2e^{2x} + C$ differentiates instead of integrating; (B) $e^{2x} + C$ forgets to divide by 2; (C) $\frac{e^{x}}{2} + C$ drops the factor 2 in the exponent.

Final Answer: $\int e^{2x}\,dx = \frac{e^{2x}}{2} + C \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q7

Q8.

Solution

Concept — Order and degree: The *order* is the highest derivative present; the *degree* is the power of that highest derivative when the equation is polynomial in derivatives.

Step 1 — Highest derivative: The highest derivative is $\frac{d^2y}{dx^2}$, so the order is 2.

Step 2 — Its power: The term $\left(\frac{d^2y}{dx^2}\right)^2$ shows the highest derivative appears to the power 2.

So the degree is 2.

Why other options are wrong: (A) 2 and 3 reads the power of $\frac{dy}{dx}$; (C),(D) misread the order or its power.

Final Answer: Order 2, degree $2 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q8

Q9.

Solution

Concept — Magnitude of a vector: For $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

Step 1 — Square the components:

$$2^2 + (-3)^2 + 6^2 = 4 + 9 + 36.$$

$$= 49.$$

Step 2 — Take the square root:

$$|\vec{a}| = \sqrt{49} = 7.$$

Why other options are wrong: (A) 5 and (B) 11 misuse the components; (D) $\sqrt{11}$ forgets the squares.

Final Answer: $|\vec{a}| = 7 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q9

Q10.

Solution

Concept — Direction cosines: Direction cosines are the direction ratios divided by their magnitude $\sqrt{a^2 + b^2 + c^2}$.

Step 1 — Magnitude of ratios $2, -1, 2$**:**

$$\sqrt{2^2 + (-1)^2 + 2^2} = \sqrt{4 + 1 + 4} = \sqrt{9} = 3.$$

Step 2 — Divide each ratio by 3**:**

$$\left(\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}\right).$$

Why other options are wrong: (B) are the ratios themselves; (C) divides by 9; (D) uses wrong components.

Final Answer: Direction cosines $\left(\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}\right) \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q10

Q11.

Solution

Concept — Fundamental theorem of calculus: $\int_a^b f(x)\,dx = F(b) - F(a)$, where $F' = f$.

Step 1 — Antiderivative: $\int x^2\,dx = \frac{x^3}{3}$.

Step 2 — Apply limits 0 **to** 1**:**

$$\left[\frac{x^3}{3}\right]_0^1 = \frac{1^3}{3} - \frac{0^3}{3}.$$

$$= \frac{1}{3} - 0.$$

$$= \frac{1}{3}.$$

Why other options are wrong: (A) 1 and (C) $\frac{1}{2}$ misuse the power rule; (B) 3 inverts the fraction.

Final Answer: The integral equals $\frac{1}{3} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q11

Q12.

Solution

Concept — Conditional probability: $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$, provided $P(B) \neq 0$.

Step 1 — Substitute the values:

$$P(A \mid B) = \frac{0.15}{0.6}.$$

Step 2 — Simplify:

$$= \frac{15}{60} = \frac{1}{4} = 0.25.$$

Why other options are wrong: (A) 0.4 inverts a different ratio; (C) 0.09 multiplies the two probabilities; (D) 0.75 divides in the wrong order.

Final Answer: $P(A \mid B) = 0.25 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q12

Q13.

Solution

Concept — Integration by recognising a derivative: If the numerator is the derivative of the denominator, $\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$.

Step 1 — Spot the pattern: Here $f(x) = 1 + x^3$, so $f'(x) = 3x^2$, which is exactly the numerator.

Step 2 — Write the result:

$$\int \frac{3x^2}{1+x^3}\,dx = \ln|1+x^3| + C.$$

Why other options are wrong: (A) has an extra factor 3; (B) is a wrong antiderivative; (D) uses the wrong denominator $1 + x^2$.

Final Answer: $\ln(1 + x^3) + C \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q13

Q14.

Solution

Concept — Integral of an even function over a symmetric interval: If $g(-x) = g(x)$, then $\int_{-a}^{a} g(x)\,dx = 2\int_0^a g(x)\,dx$.

Step 1 — Test the integrand: $g(x) = x^2$ gives $g(-x) = (-x)^2 = x^2 = g(x)$.

So x^2 is an even function.

Step 2 — Apply the property:

$$\int_{-2}^{2} x^2\,dx = 2\int_0^2 x^2\,dx.$$

Step 3 — Integrate:

$$= 2\left[\frac{x^3}{3}\right]_0^2.$$

$$= 2 \cdot \frac{8}{3}.$$

$$= \frac{16}{3}.$$

Why other options are wrong: (B) 0 would hold for an odd function; (C) $\frac{8}{3}$

forgets the factor 2; (D) 16 ignores the division by 3.

Final Answer: The integral equals $\frac{16}{3} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q14

Q15.

Solution

Concept — Local minimum via second derivative: At a critical point where $f'(x) = 0$, a local minimum occurs if $f''(x) > 0$.

Step 1 — Critical points:

$$f(x) = x^3 - 12x.$$

$$f'(x) = 3x^2 - 12 = 0.$$

$$x^2 = 4 \;\Rightarrow\; x = \pm 2.$$

Step 2 — Second-derivative test:

$$f''(x) = 6x.$$

At $x = 2$, $f''(2) = 12 > 0$, so $x = 2$ gives a local minimum.

Step 3 — Minimum value:

$$f(2) = (2)^3 - 12(2) = 8 - 24 = -16.$$

Why other options are wrong: (A) 16 is the local maximum value at $x = -2$; (B),(C) are not extreme values.

Final Answer: Local minimum value is $-16 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q15

Q16.

Solution

Concept — Area of a triangle from two side vectors: Area $= \frac{1}{2}|\vec{a} \times \vec{b}|$.

Step 1 — Compute the cross product:

$$\vec{a} \times \vec{b} = (6\hat{i}) \times (2\hat{j}) = 12\,(\hat{i} \times \hat{j}) = 12\hat{k}.$$

Step 2 — Take the magnitude:

$$|\vec{a} \times \vec{b}| = |12\hat{k}| = 12.$$

Step 3 — Halve it:

$$\text{Area} = \frac{1}{2} \times 12 = 6.$$

Why other options are wrong: (B) 12 is the parallelogram area (forgets the $\frac{1}{2}$); (C) 24 doubles it; (D) 8 adds the sides.

Final Answer: Area $= 6 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q16

Q17.

Solution

Concept — Angle between two lines: If the direction ratios are proportional, the lines are parallel and the angle between them is 0°.

Step 1 — Compare the ratios:

$$\frac{2}{1} = \frac{4}{2} = \frac{6}{3} = 2.$$

Step 2 — Interpret: Since all ratios are equal, the direction ratios $2, 4, 6$ and $1, 2, 3$ are proportional, so the lines are parallel.

Hence the angle between them is 0°.

Why other options are wrong: (A) 90° needs a zero dot product; (B),(D) would need non-proportional ratios.

Final Answer: The angle is $0^\circ \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q17

Q18.

Solution

Concept — Union with independent events: $P(A \cup B) = P(A) + P(B) - P(A)P(B)$ when A, B are independent.

Step 1 — Product term:

$$P(A)P(B) = (0.3)(0.4) = 0.12.$$

Step 2 — Substitute:

$$P(A \cup B) = 0.3 + 0.4 - 0.12.$$
$$= 0.7 - 0.12.$$
$$= 0.58.$$

Why other options are wrong: (A) 0.7 forgets to subtract the intersection; (B) 0.42 uses wrong arithmetic; (C) 0.12 is only $P(A \cap B)$.

Final Answer: $P(A \cup B) = 0.58 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q18

Q19.

Solution

Concept — Assertion–Reason analysis: Judge each statement's truth, then decide whether R correctly explains A.

Step 1 — Assertion: The reversal law of transpose states $(AB)^T = B^T A^T$ for conformable matrices. So A is **true**.

Step 2 — Reason: Matrix multiplication is *not* commutative in general; usually $AB \neq BA$. So R is **false**.

Step 3 — Conclusion: A is true and R is false.

Why other options are wrong: (A),(B) require R true; (D) requires A false.

Final Answer: A true, R false $\Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q19

Q20.

Solution

Concept — Determinant of a skew-symmetric matrix: A matrix A is skew-symmetric if $A^T = -A$. For odd order n, $|A^T| = |A|$ but also $|-A| = (-1)^n|A|$.

Step 1 — Test the Reason: For skew-symmetric A of order n,

$$|A| = |A^T| = |-A| = (-1)^n|A|.$$

When n is odd, $(-1)^n = -1$, so $|A| = -|A| \Rightarrow 2|A| = 0 \Rightarrow |A| = 0$. Hence R is **true**.

Step 2 — Test the Assertion: A 3×3 matrix has odd order $n = 3$, so by the same argument its determinant is 0. Hence A is **true**.

Step 3 — Does R explain A? The general odd-order result (R) is exactly the reason the specific 3×3 case (A) holds. So R correctly explains A.

Why other options are wrong: (B) denies the explanation; (C),(D) misjudge a truth value.

Final Answer: Both true, R explains A $\Rightarrow \boxed{\text{A}}$

$\boxed{\text{Answer: (A)}}$ Go Back to Q20

Q21.

Solution

Concept — Principal values: $\sin^{-1}$ lies in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $\cos^{-1}$ lies in $[0, \pi]$.

Step 1 — Evaluate $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$:

$$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} \Rightarrow \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}.$$

Step 2 — Evaluate $\cos^{-1}\left(\frac{1}{2}\right)$:

$$\cos\frac{\pi}{3} = \frac{1}{2} \Rightarrow \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}.$$

Step 3 — Add:

$$\frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}.$$

Final Answer: $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{1}{2}\right) = \frac{2\pi}{3}$. Go Back to Q21

Q22.

Solution

Concept — Logarithmic differentiation: Used when the variable appears in both base and exponent.

Step 1 — Take natural log of both sides:

$$y = x^{\ln x}.$$

$$\ln y = (\ln x)(\ln x) = (\ln x)^2.$$

Step 2 — Differentiate both sides w.r.t. x: (chain rule on the right)

$$\frac{1}{y}\frac{dy}{dx} = 2(\ln x) \cdot \frac{1}{x}.$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2\ln x}{x}.$$

Step 3 — Solve for $\dfrac{dy}{dx}$:

$$\frac{dy}{dx} = y \cdot \frac{2\ln x}{x}.$$

$$= x^{\ln x} \cdot \frac{2\ln x}{x}.$$

Final Answer: $\dfrac{dy}{dx} = \dfrac{2\ln x}{x}\, x^{\ln x}$. Go Back to Q22

Q23.

Solution

Concept — Integration by parts: $\displaystyle\int u\,dv = uv - \int v\,du$, choosing u by the ILATE rule.

Step 1 — Choose parts: Let $u = x$ and $dv = \cos x\,dx$.

Then $du = dx$ and $v = \sin x$.

Step 2 — Apply the formula:

$$\int x\cos x\,dx = x\sin x - \int \sin x\,dx.$$

$$= x\sin x - (-\cos x) + C.$$

$$= x\sin x + \cos x + C.$$

Final Answer: $\int x \cos x \, dx = x \sin x + \cos x + C$.

OR — $\int \ln x \, dx$:

Step 1 — Choose parts: Let $u = \ln x$ and $dv = dx$.

Then $du = \frac{1}{x}\, dx$ and $v = x$.

Step 2 — Apply the formula:

$$\int \ln x \, dx = x \ln x - \int x \cdot \frac{1}{x}\, dx.$$

$$= x \ln x - \int 1 \, dx.$$

$$= x \ln x - x + C.$$

Final Answer (OR): $\int \ln x \, dx = x \ln x - x + C$. Go Back to Q23

Q24.

Solution

Concept — Perpendicular vectors: $\vec{a} \perp \vec{b} \iff \vec{a} \cdot \vec{b} = 0$.

Step 1 — Compute the dot product:

$$\vec{a} \cdot \vec{b} = (\lambda)(3) + (2)(-3) + (-3)(-1).$$

$$= 3\lambda - 6 + 3.$$

$$= 3\lambda - 3.$$

Step 2 — Set equal to zero and solve:

$$3\lambda - 3 = 0.$$

$$3\lambda = 3.$$

$$\lambda = 1.$$

Final Answer: $\lambda = 1$. Go Back to Q24

Q25.

Solution

Concept — Equally likely outcomes: Probability $= \dfrac{\text{favourable outcomes}}{\text{total outcomes}}$.

Step 1 — Total outcomes: Two throws of a die give $6 \times 6 = 36$ equally likely outcomes.

Step 2 — Favourable outcomes (sum $= 7$):

$$(1,6),\ (6,1),\ (2,5),\ (5,2),\ (3,4),\ (4,3).$$

There are 6 such outcomes.

Step 3 — Probability:

$$P(\text{sum} = 7) = \frac{6}{36} = \frac{1}{6}.$$

Final Answer: $P(\text{sum} = 7) = \dfrac{1}{6}$.

OR — $P(A \mid B)$**:**

Step 1 — Formula: $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$.

Step 2 — Substitute:

$$P(A \mid B) = \frac{0.2}{0.4} = \frac{1}{2}.$$

Final Answer (OR): $P(A \mid B) = \dfrac{1}{2}$. **Go Back to Q25**

Q26.

Solution

Concept — Partial fractions: Split a proper rational function into simpler fractions, then integrate each.

Step 1 — Set up the decomposition:

$$\frac{2x+3}{(x+1)(x+3)} = \frac{A}{x+1} + \frac{B}{x+3}.$$

$$2x + 3 = A(x+3) + B(x+1).$$

Step 2 — Find A and B:

Put $x = -1$: $2(-1) + 3 = A(2) \Rightarrow 1 = 2A \Rightarrow A = \dfrac{1}{2}$.

Put $x=-3$: $2(-3)+3=B(-2) \Rightarrow -3=-2B \Rightarrow B=\frac{3}{2}$.

Step 3 — Integrate:

$$\int \frac{2x+3}{(x+1)(x+3)}\,dx = \int \left(\frac{1/2}{x+1}+\frac{3/2}{x+3}\right) dx.$$

$$= \frac{1}{2}\ln|x+1| + \frac{3}{2}\ln|x+3| + C.$$

Final Answer: $\frac{1}{2}\ln|x+1| + \frac{3}{2}\ln|x+3| + C$. **Go Back to Q26**

Q27.

Solution

Concept — Linear differential equation: For $\frac{dy}{dx}+Py=Q$, the integrating factor is I.F. $= e^{\int P\,dx}$ and the solution is $y \cdot \text{I.F.} = \int Q \cdot \text{I.F.}\,dx$.

Step 1 — Identify P, Q: Here $P=\frac{1}{x}$, $Q=x^2$.

Step 2 — Integrating factor:

$$\text{I.F.} = e^{\int (1/x)\,dx} = e^{\ln x} = x.$$

Step 3 — Solve:

$$y \cdot x = \int x^2 \cdot x\,dx = \int x^3\,dx.$$

$$y\,x = \frac{x^4}{4} + C.$$

$$y = \frac{x^3}{4} + \frac{C}{x}.$$

Final Answer: $y = \frac{x^3}{4} + \frac{C}{x}$.

OR — Variables separable $\frac{dy}{dx} = e^{x-y}$:

Step 1 — Rewrite and separate:

$$\frac{dy}{dx} = e^x \cdot e^{-y}.$$

$$e^y\,dy = e^x\,dx.$$

Step 2 — Integrate both sides:

$$\int e^y\,dy = \int e^x\,dx.$$

$$e^y = e^x + C.$$

Final Answer (OR): $e^y = e^x + C$. **Go Back to Q27**

Q28.

Solution

Concept — Inverse by adjoint: $A^{-1} = \frac{1}{|A|}\text{adj}(A)$, valid when $|A| \neq 0$.

Step 1 — Determinant:

$$|A| = \begin{vmatrix} 3 & 1 \\ 2 & 1 \end{vmatrix} = (3)(1) - (1)(2) = 3 - 2 = 1.$$

Step 2 — Adjoint of a 2×2 matrix: Swap the leading diagonal entries and negate the off-diagonal entries:

$$\text{adj}(A) = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}.$$

Step 3 — Inverse:

$$A^{-1} = \frac{1}{1}\begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}.$$

Final Answer: $A^{-1} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$. **Go Back to Q28**

Q29.

Solution

Concept — Monotonicity: f increases where $f'(x) > 0$ and decreases where $f'(x) < 0$.

Step 1 — Differentiate:

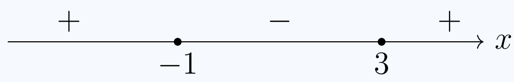
$$f(x) = x^3 - 3x^2 - 9x + 5.$$

$$f'(x) = 3x^2 - 6x - 9.$$

$$= 3(x^2 - 2x - 3) = 3(x-3)(x+1).$$

Step 2 — Critical points: $f'(x) = 0$ at $x = -1$ and $x = 3$.

Step 3 — Sign analysis:



For $x < -1$: $f' > 0$; for $-1 < x < 3$: $f' < 0$; for $x > 3$: $f' > 0$.

Conclusion: Increasing on $(-\infty, -1) \cup (3, \infty)$; decreasing on $(-1, 3)$.

Final Answer: Increasing on $(-\infty, -1) \cup (3, \infty)$, decreasing on $(-1, 3)$.

OR — Tangent to $y = x^3$ at $(1, 1)$:

Step 1 — Slope: $\dfrac{dy}{dx} = 3x^2$, so at $x = 1$ slope $= 3$.

Step 2 — Equation: $y - 1 = 3(x - 1) \Rightarrow y = 3x - 2$.

Final Answer (OR): $y = 3x - 2$. **Go Back to Q29**

Q30.

Solution

Concept — Vector equation of a line: A line through point $\vec{a}$ parallel to $\vec{b}$ is $\vec{r} = \vec{a} + \lambda\vec{b}$.

Step 1 — Write the equation: With $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$,

$$\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} - \hat{k}).$$

Step 2 — Test the point $(4, 3, 1)$: The point lies on the line if some λ satisfies all three component equations.

$$2 + \lambda = 4 \;\Rightarrow\; \lambda = 2.$$

$$-1 + 2\lambda = 3 \;\Rightarrow\; 2\lambda = 4 \;\Rightarrow\; \lambda = 2.$$

$$3 - \lambda = 1 \;\Rightarrow\; \lambda = 2.$$

Step 3 — Conclusion: All three give the same value $\lambda = 2$, so the point $(4, 3, 1)$ lies on the line.

Final Answer: $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})$; the point $(4, 3, 1)$ lies on it (at $\lambda = 2$). **Go Back to Q30**

Q31.

Solution

Concept — One-one (injective) and onto (surjective): *One-one*: $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$. *Onto*: every y in the codomain has a pre-image.

Step 1 — One-one: Suppose $f(x_1) = f(x_2)$.

$$3x_1 - 2 = 3x_2 - 2.$$

$$3x_1 = 3x_2.$$

$$x_1 = x_2.$$

Hence f is one-one.

Step 2 — Onto: Let $y \in \mathbb{R}$ be arbitrary. Solve $y = 3x - 2$ for x:

$$x = \frac{y+2}{3} \in \mathbb{R}.$$

Then $f(x) = 3\left(\frac{y+2}{3}\right) - 2 = y$.

So every y has a pre-image; f is onto.

Step 3 — Conclusion: f is both one-one and onto, hence bijective.

Final Answer: $f(x) = 3x - 2$ is one-one and onto (a bijection). **Go Back to Q31**

Q32.

Solution

Concept — Matrix method: Write $AX = B$; then $X = A^{-1}B$ when $|A| \neq 0$.

Step 1 — Form the matrices:

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 7 \\ 6 \\ 0 \end{bmatrix}.$$

Step 2 — Determinant of A:

$$|A| = 1\big[(-1)(-1) - (1)(1)\big] - 2\big[(2)(-1) - (1)(1)\big] + 1\big[(2)(1) - (-1)(1)\big].$$

$$= 1(1-1) - 2(-2-1) + 1(2+1).$$

$$= 0 + 6 + 3 = 9 \ (\neq 0).$$

Step 3 — Cofactors and adjoint: Computing the cofactor matrix and transposing gives

$$\text{adj}(A) = \begin{bmatrix} 0 & 3 & 3 \\ 3 & -2 & 1 \\ 3 & 1 & -5 \end{bmatrix}.$$

Step 4 — Solve $X = \frac{1}{|A|}$ adj$(A)\,B$:

$$X = \frac{1}{9}\begin{bmatrix} 0 & 3 & 3 \\ 3 & -2 & 1 \\ 3 & 1 & -5 \end{bmatrix}\begin{bmatrix} 7 \\ 6 \\ 0 \end{bmatrix}.$$

$$= \frac{1}{9}\begin{bmatrix} 0 \cdot 7 + 3 \cdot 6 + 3 \cdot 0 \\ 3 \cdot 7 - 2 \cdot 6 + 1 \cdot 0 \\ 3 \cdot 7 + 1 \cdot 6 - 5 \cdot 0 \end{bmatrix}.$$

$$= \frac{1}{9}\begin{bmatrix} 18 \\ 21 - 12 \\ 21 + 6 \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 18 \\ 9 \\ 27 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}.$$

Step 5 — Verify: $x + 2y + z = 2 + 2 + 3 = 7$ ✓; $2x - y + z = 4 - 1 + 3 = 6$ ✓; $x + y - z = 2 + 1 - 3 = 0$ ✓.

Final Answer: $x = 2,\ y = 1,\ z = 3$. **Go Back to Q32**

Q33.

Solution

Concept — Area between two curves: Area $= \int_a^b (\text{upper} - \text{lower})\, dx$ over the interval where they intersect.

Step 1 — Points of intersection: Set $x^2 = 2x$:

$$x^2 - 2x = 0.$$

$$x(x-2) = 0 \ \Rightarrow\ x = 0,\ x = 2.$$

Step 2 — Identify upper curve: On $[0, 2]$, the line $y = 2x$ lies above $y = x^2$ (e.g. at $x = 1$, $2 > 1$).

Step 3 — Set up the integral:

$$\text{Area} = \int_0^2 (2x - x^2)\,dx.$$

Step 4 — Integrate:

$$= \left[x^2 - \frac{x^3}{3}\right]_0^2.$$

$$= \left(4 - \frac{8}{3}\right) - 0.$$

$$= \frac{12-8}{3} = \frac{4}{3}.$$

Final Answer: Area $= \frac{4}{3}$ square units.

OR — Region bounded by $y = 4 - x^2$ and the x-axis:

Step 1 — Limits: $y = 0$ gives $4 - x^2 = 0 \Rightarrow x = \pm 2$.

Step 2 — Use symmetry: The curve is even, so

$$\text{Area} = 2\int_0^2 (4 - x^2)\,dx.$$

Step 3 — Integrate:

$$= 2\left[4x - \frac{x^3}{3}\right]_0^2.$$

$$= 2\left(8 - \frac{8}{3}\right) = 2 \cdot \frac{16}{3} = \frac{32}{3}.$$

Final Answer (OR): Area $= \frac{32}{3}$ square units. **Go Back to Q33**

Q34.

Solution

Concept — Corner-point method: The optimum of a linear objective over a bounded feasible region occurs at a corner (vertex).

Step 1 — Find the corner points: The constraints $3x + 2y \le 18$, $x + 2y \le 10$, $x \ge 0$, $y \ge 0$ bound the region with vertices $O(0,0)$, $(6,0)$ and $(0,5)$.

The lines $3x + 2y = 18$ and $x + 2y = 10$ intersect: subtracting the second from the first,

$$(3x + 2y) - (x + 2y) = 18 - 10.$$

$$2x = 8 \Rightarrow x = 4.$$

Substituting into $x + 2y = 10$: $4 + 2y = 10 \Rightarrow y = 3$. So the fourth vertex is $(4, 3)$.

Step 2 — Evaluate $Z = 4x + 3y$ at each corner:

$$Z(0,0) = 0.$$

$$Z(6,0) = 4(6) + 3(0) = 24.$$

$$Z(4,3) = 4(4) + 3(3) = 16 + 9 = 25.$$

$$Z(0,5) = 4(0) + 3(5) = 15.$$

Step 3 — Pick the maximum: The largest value is 25 at $(4, 3)$.

Final Answer: Z is maximum $= 25$ at $(x, y) = (4, 3)$. **Go Back to Q34**

Q35.

Solution

Concept — Shortest distance between skew lines: For $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$,

$$d = \frac{\left|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)\right|}{|\vec{b}_1 \times \vec{b}_2|}.$$

Step 1 — Identify vectors: $\vec{a}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b}_1 = 2\hat{i} + 3\hat{j} + 4\hat{k}$; $\vec{a}_2 = 2\hat{i} + 4\hat{j} + 5\hat{k}$, $\vec{b}_2 = 3\hat{i} + 4\hat{j} + 5\hat{k}$.

Step 2 — Cross product $\vec{b}_1 \times \vec{b}_2$:

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}.$$

$$= \hat{i}\left[(3)(5) - (4)(4)\right] - \hat{j}\left[(2)(5) - (4)(3)\right] + \hat{k}\left[(2)(4) - (3)(3)\right].$$

$$= \hat{i}(15 - 16) - \hat{j}(10 - 12) + \hat{k}(8 - 9).$$

$$= -\hat{i} + 2\hat{j} - \hat{k}.$$

Step 3 — Its magnitude:

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{(-1)^2 + 2^2 + (-1)^2} = \sqrt{1 + 4 + 1} = \sqrt{6}.$$

Step 4 — $(\vec{a}_2 - \vec{a}_1)$ and the dot product:

$$\vec{a}_2 - \vec{a}_1 = (2-1)\hat{i} + (4-2)\hat{j} + (5-3)\hat{k} = \hat{i} + 2\hat{j} + 2\hat{k}.$$

$$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = (1)(-1) + (2)(2) + (2)(-1) = -1 + 4 - 2 = 1.$$

Step 5 — Distance:

$$d = \frac{|1|}{\sqrt{6}} = \frac{1}{\sqrt{6}} \text{ units.}$$

Final Answer: Shortest distance $= \dfrac{1}{\sqrt{6}}$ units.

OR — Angle between line and plane:

Step 1 — Direction $\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$, normal $\vec{n} = 2\hat{i} + \hat{j} - 2\hat{k}$.

Step 2 — Use $\sin\theta = \dfrac{|\vec{b} \cdot \vec{n}|}{|\vec{b}||\vec{n}|}$**:**

$$\vec{b} \cdot \vec{n} = (2)(2) + (-1)(1) + (2)(-2) = 4 - 1 - 4 = -1.$$

$$|\vec{b}| = \sqrt{4+1+4} = 3, \quad |\vec{n}| = \sqrt{4+1+4} = 3.$$

$$\sin\theta = \frac{|-1|}{3 \cdot 3} = \frac{1}{9}.$$

Final Answer (OR): $\theta = \sin^{-1}\left(\dfrac{1}{9}\right)$. **Go Back to Q35**

Q36.

Solution

Concept — Maxima by derivatives: Express the volume as a function of one variable using the sphere constraint, set $\dfrac{dV}{dh} = 0$, and confirm a maximum.

(i) Volume as a function of h: If the cylinder has height h, each rim lies on the sphere, so a radius drawn to a rim gives

$$r^2 + \left(\frac{h}{2}\right)^2 = R^2 \Rightarrow r^2 = R^2 - \frac{h^2}{4}.$$

Hence

$$V = \pi r^2 h = \pi\left(R^2 - \frac{h^2}{4}\right)h = \pi\left(R^2 h - \frac{h^3}{4}\right).$$

(ii) Differentiate:

$$\frac{dV}{dh} = \pi\left(R^2 - \frac{3h^2}{4}\right).$$

(iii) Critical point and maximum: Set $\frac{dV}{dh} = 0$:

$$R^2 - \frac{3h^2}{4} = 0.$$

$$h^2 = \frac{4R^2}{3} \Rightarrow h = \frac{2R}{\sqrt{3}}.$$

The second derivative $\frac{d^2V}{dh^2} = -\frac{3\pi h}{2} < 0$, so this h gives a maximum.
The corresponding radius satisfies

$$r^2 = R^2 - \frac{1}{4}\cdot\frac{4R^2}{3} = R^2 - \frac{R^2}{3} = \frac{2R^2}{3}.$$

$$r = R\sqrt{\frac{2}{3}}.$$

Final Answer: $V = \pi\left(R^2 h - \frac{h^3}{4}\right)$; $\frac{dV}{dh} = \pi\left(R^2 - \frac{3h^2}{4}\right)$; the volume is maximum when $h = \frac{2R}{\sqrt{3}}$, giving $r = R\sqrt{\frac{2}{3}}$. **Go Back to Q36**

Q37.

Solution

Concept — Bayes' theorem: $P(E_i \mid D) = \dfrac{P(E_i)P(D \mid E_i)}{\sum_j P(E_j)P(D \mid E_j)}$.

(i) Prior probabilities:

$$P(A) = 0.5, \qquad P(B) = 0.3, \qquad P(C) = 0.2.$$

(ii) Conditional probabilities and total defective rate:

$$P(\text{Def} \mid A) = 0.02, \quad P(\text{Def} \mid B) = 0.04, \quad P(\text{Def} \mid C) = 0.04.$$

By the law of total probability,

$$P(\text{Def}) = (0.5)(0.02) + (0.3)(0.04) + (0.2)(0.04).$$

$$= 0.010 + 0.012 + 0.008.$$

$$= 0.030.$$

(iii) Apply Bayes' theorem:

$$P(A \mid \text{Def}) = \frac{P(A)P(\text{Def} \mid A)}{P(\text{Def})} = \frac{0.010}{0.030}.$$

Simplify:

$$= \frac{10}{30} = \frac{1}{3}.$$

Final Answer: $P(\text{plant A} \mid \text{defective}) = \frac{1}{3} \approx 0.333.$ **Go Back to Q37**

Q38.

Solution

Concept — Related rates: Differentiate the volume with respect to r, then use the chain rule $\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt}$.

(i) Differentiate V with respect to r:

$$V = \frac{4}{3}\pi r^3.$$

$$\frac{dV}{dr} = \frac{4}{3}\pi \cdot 3r^2 = 4\pi r^2.$$

(ii) Chain rule for $\frac{dV}{dt}$:

$$\frac{dV}{dt} = \frac{dV}{dr} \cdot \frac{dr}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}.$$

(iii) Rate when $r = 7$ cm: Given $\frac{dr}{dt} = 0.5$ cm/s,

$$\frac{dV}{dt} = 4\pi(7)^2(0.5).$$

$$= 4\pi(49)(0.5).$$

$$= 98\pi \text{ cm}^3/\text{s}.$$

Final Answer: $\frac{dV}{dr} = 4\pi r^2$; $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$; at $r = 7$ cm the volume increases at 98π cm^3/s. **Go Back to Q38**

Answer Key – Section A (Q1–Q20)

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	D	3	A	4	C	5	B
6	B	7	D	8	B	9	C	10	A
11	D	12	B	13	C	14	A	15	D
16	A	17	C	18	D	19	C	20	A

Sections B–E are descriptive; refer to the Detailed Solutions above for full model answers and step marking.