CBSE Class 12 Mathematics

Sample Paper – 5

Duration: 180 Minutes **Maximum Marks: 80**

General Instructions

- This question paper contains **38 questions**. **All questions are compulsory**.
- The paper is divided into **five sections** – A, B, C, D and E.
- **Section A** (Q1–Q20) carries **1 mark** each: Q1–Q18 are multiple choice questions and Q19–Q20 are Assertion–Reason questions.
- **Section B** (Q21–Q25) carries **2 marks** each (Very Short Answer).
- **Section C** (Q26–Q31) carries **3 marks** each (Short Answer).
- **Section D** (Q32–Q35) carries **5 marks** each (Long Answer).
- **Section E** (Q36–Q38) carries **4 marks** each (case study based, with sub-parts).
- There is **no overall choice**, but an **internal choice** has been provided in some questions. Attempt only one of the alternatives in such questions.
- There is **no negative marking**. Use of a **calculator is not permitted**.

Section A (Q1–Q20) – 1 Mark Each

Q1. A matrix has 12 elements. The number of possible orders it can have is:

(A) 3

(B) 6

(C) 12

(D) 4

Q2. The value of the determinant $\begin{vmatrix} 5 & 3 \\ 2 & 4 \end{vmatrix}$ is:

(A) 26

(B) -14

(C) 6

(D) 14

Q3. Let $A = \{1, 2, 3\}$ and $R = \{(1,1), (2,2), (3,3), (1,2), (2,1), (2,3), (3,2)\}$ be a relation on A. Then R is:

(A) reflexive and symmetric but not transitive

(B) transitive but not symmetric

(C) an equivalence relation

(D) symmetric and transitive but not reflexive

Q4. The principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is:

(A) $\frac{\pi}{3}$

(B) $-\frac{\pi}{3}$

(C) $\frac{2\pi}{3}$

(D) $-\frac{2\pi}{3}$

Q5. If $y = e^{\tan x}$, then $\frac{dy}{dx}$ equals:

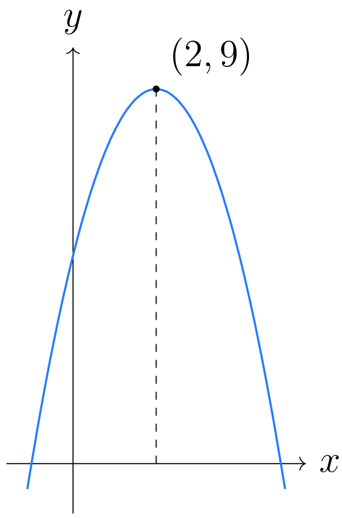
(A) $e^{\tan x}$

(B) $\sec x \, e^{\tan x}$

(C) $e^{\sec^2 x}$

(D) $\sec^2 x \, e^{\tan x}$

Q6. The graph of $f(x) = -x^2 + 4x + 5$ is shown below. The function is *increasing* on the interval:



(A) $(-\infty, 2)$

(B) $(2, \infty)$

(C) $(-\infty, \infty)$

(D) $(0, 2)$

Q7. $\int e^{2x}\, dx$ equals:

(A) $e^{2x} + C$

(B) $\dfrac{e^{2x}}{2} + C$

(C) $2e^{2x} + C$

(D) $\dfrac{e^{x}}{2} + C$

Q8. The order and degree of the differential equation $\left(\dfrac{d^2y}{dx^2}\right)^2 + 3\dfrac{dy}{dx} + y = 0$ are respectively:

(A) 2 and 1

(B) 1 and 2

(C) 2 and 3

(D) 2 and 2

Q9. The magnitude of the vector $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ is:

(A) 5

(B) $\sqrt{13}$

(C) 7

(D) 49

Q10. The direction cosines of a line whose direction ratios are $2, 3, -6$ are:

(A) $2, 3, -6$

(B) $\frac{2}{7}, \frac{3}{7}, -\frac{6}{7}$

(C) $\frac{2}{49}, \frac{3}{49}, -\frac{6}{49}$

(D) $\frac{2}{\sqrt{7}}, \frac{3}{\sqrt{7}}, -\frac{6}{\sqrt{7}}$

Q11. $\int_0^1 e^x \, dx$ equals:

(A) $e - 1$

(B) $1 - e$

(C) e

(D) 1

Q12. If $P(A \cap B) = 0.12$ and $P(A) = 0.4$, then $P(B \mid A)$ equals:

(A) 0.12

(B) 0.48

(C) 0.4

(D) 0.3

Q13. $\int \frac{3x^2}{1 + x^3} \, dx$ equals:

(A) $3 \ln(1 + x^3) + C$

(B) $\ln(1 + x^3) + C$

(C) $\ln(1 + x^2) + C$

(D) $\frac{1}{1 + x^3} + C$

Q14. $\int_{-2}^{2} x^2\, dx$ equals:

(A) 0

(B) $\frac{8}{3}$

(C) $\frac{16}{3}$

(D) 8

Q15. The local maximum value of $f(x) = x^3 - 12x$ is:

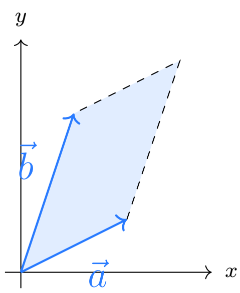
(A) 16

(B) -16

(C) 12

(D) -12

Q16. The area of the parallelogram whose two adjacent sides are the vectors $\vec{a} = 2\hat{i} + \hat{j}$ and $\vec{b} = \hat{i} + 3\hat{j}$ (shown below) is:



(A) 10

(B) 7

(C) 6

(D) 5

Q17. The angle between two lines whose direction ratios are $1, 0, 1$ and $1, 1, 0$ is:

(A) 90°

(B) 60°

(C) 45°

(D) 30°

Q18. If A and B are independent events with $P(A) = \frac{1}{4}$ and $P(B) = \frac{1}{3}$, then $P(A \cup B)$ equals:

(A) $\frac{7}{12}$

(B) $\frac{1}{12}$

(C) $\frac{1}{2}$

(D) $\frac{1}{3}$

Q19. Assertion (A): The determinant of a skew-symmetric matrix of order 3 is 0.

Reason (R): For a skew-symmetric matrix of odd order, the determinant is always 0.

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is *not* the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

Q20. Assertion (A): $\tan^{-1}\left(\tan \frac{3\pi}{4}\right) = -\frac{\pi}{4}$.

Reason (R): The range (principal value branch) of $\tan^{-1} x$ is $[0, \pi]$.

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is *not* the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

Section B (Q21–Q25) – 2 Marks Each

Q21. Find the principal value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{1}{2}\right)$. **[2]**

Q22. Differentiate $y = x^x$ with respect to x. **[2]**

Q23. Evaluate $\int x \sin x \, dx$. **[2]**

OR

Evaluate $\int \ln x \, dx$.

Q24. Find the value of λ for which the vectors $\vec{a} = \lambda\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = 2\hat{i} + \hat{j} + 3\hat{k}$ are perpendicular. **[2]**

Q25. A fair die is thrown twice. Find the probability of getting a sum of 7. **[2]**

OR

If $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cap B) = 0.2$, find $P(B \mid A)$.

Section C (Q26–Q31) – 3 Marks Each

Q26. Evaluate $\int \frac{1}{(x-1)(x+2)} \, dx$. **[3]**

Q27. Solve the differential equation $\frac{dy}{dx} + 2y = e^{-x}$. **[3]**

OR

Solve the differential equation $\frac{dy}{dx} = e^{x-y}$.

Q28. Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ using the adjoint method. **[3]**

Q29. Find the intervals in which $f(x) = 2x^3 - 3x^2 - 12x + 5$ is increasing or decreasing. **[3]**

OR

Find the equation of the tangent to the curve $y = x^3$ at the point $(1, 1)$.

Q30. Find the vector equation of the line passing through the point $(2, 1, -1)$ and parallel to the vector $2\hat{i} - \hat{j} + 3\hat{k}$. Hence check whether the point $(4, 0, 2)$ lies on this line. **[3]**

Q31. Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 3x - 2$ is one-one and onto. **[3]**

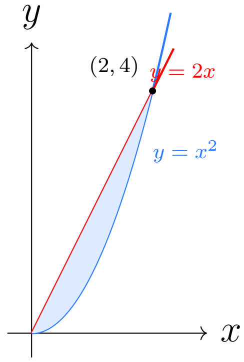
Section D (Q32–Q35) – 5 Marks Each

Q32. Using the matrix method, solve the following system of linear equations:

$$x + y + z = 2, \qquad x + 2y + 3z = 1, \qquad x - y + z = 0.$$

[5]

Q33. Using integration, find the area of the region bounded by the parabola $y = x^2$ and the line $y = 2x$.



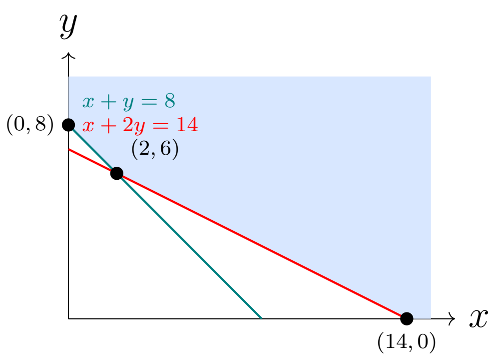
[5]

OR

Using integration, find the area of the region bounded by the curve $y = \sin x$ and the x-axis between $x = 0$ and $x = \pi$.

Q34. Solve the following linear programming problem graphically. Minimise the cost $Z = 50x + 70y$ subject to the constraints

$$x + 2y \geq 14, \qquad x + y \geq 8, \qquad x \geq 0,\ y \geq 0.$$



[5]

Q35. Find the shortest distance between the lines

$$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k}) \quad \text{and} \quad \vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} - \hat{k}).$$

[5]

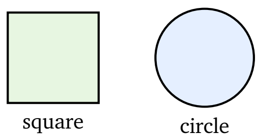
OR

Find the angle between the line $\vec{r} = (\hat{i} + \hat{j} + \hat{k}) + \lambda(2\hat{i} + 2\hat{j} + \hat{k})$ and the plane $x + 2y + 2z = 7$.

Section E (Q36–Q38) – 4 Marks Each (Case Study)

Q36. Case Study – Wire, a Square and a Circle.

A piece of wire of length 28 cm is cut into two parts. One part of length x cm is bent into a *square*, and the remaining part is bent into a *circle*. An engineer wants the *total* area enclosed by the two shapes to be as small as possible.



Based on the above, answer the following:

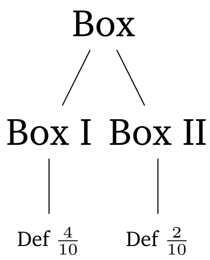
(i) Express the total area A enclosed as a function of x. (1)

(ii) Find $\dfrac{dA}{dx}$. (1)

(iii) Find the value of x for which the total area is minimum. (2)

Q37. Case Study – Two Boxes of Bulbs.

A shop has two identical-looking boxes of light bulbs. Box I contains 6 good and 4 defective bulbs, while Box II contains 8 good and 2 defective bulbs. A box is chosen at random and one bulb is drawn from it; the bulb turns out to be *defective*.



Based on the above, answer the following:

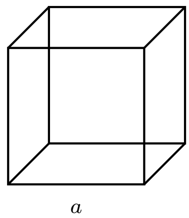
(i) Write $P(\text{Box I})$ and $P(\text{Box II})$. (1)

(ii) Write $P(\text{Defective} \mid \text{Box I})$ and $P(\text{Defective} \mid \text{Box II})$. (1)

(iii) Using Bayes' theorem, find the probability that the defective bulb came from Box I. (2)

Q38. Case Study – An Expanding Cube.

A metal cube is heated so that each edge expands. At a certain instant the length of an edge is increasing at the rate $\dfrac{da}{dt} = 3$ cm/s, where a cm is the edge length at time t seconds.



Based on the above, answer the following:

(i) Express $\dfrac{dV}{dt}$ (rate of change of volume) in terms of a. (1)

(ii) Find the rate of increase of the volume when the edge is 10 cm. (1)

(iii) Find the rate of increase of the total surface area when the edge is 10 cm. (2)

Detailed Solutions

Q1.

Solution

Concept — Order of a matrix: A matrix of order $m \times n$ has mn elements. So the possible orders correspond to the ways of writing the number of elements as a product $m \times n$.

Step 1 — Factor the number of elements: We need ordered pairs (m, n) with $m \times n = 12$.

Step 2 — List the factor pairs:

$$1 \times 12, \quad 2 \times 6, \quad 3 \times 4, \quad 4 \times 3, \quad 6 \times 2, \quad 12 \times 1.$$

There are 6 such ordered pairs.

Why other options are wrong: (A) 3 and (D) 4 count too few; (C) 12 counts the number of elements, not the number of orders.

Final Answer: The number of possible orders is $6 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q1

Q2.

Solution

Concept — Value of a 2×2 determinant: For $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$, the value is $ad - bc$.

Step 1 — Identify entries: Here $a = 5$, $b = 3$, $c = 2$, $d = 4$.

Step 2 — Apply the formula:

$$ad - bc = (5)(4) - (3)(2).$$

$$= 20 - 6.$$

$$= 14.$$

Why other options are wrong: (A) 26 adds instead of subtracting; (B) -14 flips the sign; (C) 6 keeps only the bc term.

Final Answer: The determinant is $14 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q2

Q3.

Solution

Concept — Properties of a relation: *Reflexive*: $(a, a) \in R$ for all a. *Symmetric*: $(a, b) \in R \Rightarrow (b, a) \in R$. *Transitive*: $(a, b), (b, c) \in R \Rightarrow (a, c) \in R$.

Step 1 — Reflexive? $(1, 1), (2, 2), (3, 3) \in R$, so R is reflexive.

Step 2 — Symmetric? Every off-diagonal pair appears with its reverse: $(1, 2)$ and $(2, 1)$; $(2, 3)$ and $(3, 2)$.

Hence R is symmetric.

Step 3 — Transitive? $(1, 2) \in R$ and $(2, 3) \in R$, but $(1, 3) \notin R$.

Hence R is not transitive.

Why other options are wrong: (B) R is symmetric, so "not symmetric" is false; (C) an equivalence relation must also be transitive, which fails; (D) R is reflexive, so "not reflexive" is false.

Final Answer: R is reflexive and symmetric but not transitive $\Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q3

Q4.

Solution

Concept — Principal value of $\cos^{-1}$: The principal value branch of $\cos^{-1} x$ is $[0, \pi]$.

Step 1 — Set up the equation: Let $\cos^{-1}\left(-\frac{1}{2}\right) = \theta$, so $\cos\theta = -\frac{1}{2}$.

Step 2 — Find θ in the branch: We know $\cos\frac{\pi}{3} = \frac{1}{2}$, so $\cos\left(\pi - \frac{\pi}{3}\right) = -\frac{1}{2}$.

$$\pi - \frac{\pi}{3} = \frac{2\pi}{3}.$$

Since $\frac{2\pi}{3} \in [0, \pi]$, it is the principal value.

Why other options are wrong: (A) $\frac{\pi}{3}$ gives $+\frac{1}{2}$; (B) $-\frac{\pi}{3}$ and (D) $-\frac{2\pi}{3}$ lie outside $[0, \pi]$.

Final Answer: $\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q4

Q5.

Solution

Concept — Chain rule: $\dfrac{d}{dx}e^u = e^u \cdot \dfrac{du}{dx}$.

Step 1 — Identify the inner function: Let $u = \tan x$, so $\dfrac{du}{dx} = \sec^2 x$.

Step 2 — Apply the chain rule:

$$\frac{dy}{dx} = e^{\tan x} \cdot \sec^2 x.$$

$$= \sec^2 x\, e^{\tan x}.$$

Why other options are wrong: (A) omits the factor $\sec^2 x$; (B) uses $\sec x$ instead of $\sec^2 x$; (C) wrongly puts $\sec^2 x$ in the exponent.

Final Answer: $\dfrac{dy}{dx} = \sec^2 x\, e^{\tan x} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q5

Q6.

Solution

Concept — Increasing function: f is increasing where $f'(x) > 0$.

Step 1 — Differentiate:

$$f(x) = -x^2 + 4x + 5.$$

$$f'(x) = -2x + 4.$$

Step 2 — Solve $f'(x) > 0$:

$$-2x + 4 > 0.$$

$$4 > 2x.$$

$$x < 2.$$

So f is increasing on $(-\infty, 2)$, matching the left (rising) branch of the downward parabola with vertex $(2, 9)$.

Why other options are wrong: (B) $(2, \infty)$ is where $f'(x) < 0$ (decreasing); (C) is false since the parabola rises then falls; (D) is only part of the increasing region.

Final Answer: f increases on $(-\infty, 2) \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q6

Q7.

Solution

Concept — Exponential integral: $\int e^{ax}\,dx = \dfrac{e^{ax}}{a} + C.$

Step 1 — Identify a: Here $a = 2$.

Step 2 — Write the integral:

$$\int e^{2x}\,dx = \frac{e^{2x}}{2} + C.$$

Why other options are wrong: (A) forgets to divide by 2; (C) multiplies by 2 instead of dividing; (D) wrongly halves the exponent.

Final Answer: $\int e^{2x}\,dx = \dfrac{e^{2x}}{2} + C \Rightarrow \boxed{\text{B}}$

$\boxed{\textbf{Answer: (B)}}$ **Go Back to Q7**

Q8.

Solution

Concept — Order and degree: The *order* is the highest derivative present; the *degree* is the power of that highest derivative when the equation is polynomial in derivatives.

Step 1 — Highest derivative: The highest derivative is $\dfrac{d^2y}{dx^2}$, so the order is 2.

Step 2 — Its power: The term $\left(\dfrac{d^2y}{dx^2}\right)^2$ shows the highest derivative raised to the power 2.

So the degree is 2.

Why other options are wrong: (A),(B),(C) misread either the highest derivative or its power.

Final Answer: Order 2, degree 2 $\Rightarrow \boxed{\text{D}}$

$\boxed{\textbf{Answer: (D)}}$ **Go Back to Q8**

Q9.

Solution

Concept — Magnitude of a vector: For $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

Step 1 — Square the components:

$$2^2 + (-3)^2 + 6^2 = 4 + 9 + 36.$$

$$= 49.$$

Step 2 — Take the square root:

$$|\vec{a}| = \sqrt{49} = 7.$$

Why other options are wrong: (A) 5 and (B) $\sqrt{13}$ use wrong components; (D) 49 skips the square root.

Final Answer: $|\vec{a}| = 7 \Rightarrow \boxed{\text{C}}$

$\boxed{\text{Answer: (C)}}$ Go Back to Q9

Q10.

Solution

Concept — Direction cosines: Direction cosines are the direction ratios divided by their magnitude $\sqrt{a^2 + b^2 + c^2}$.

Step 1 — Magnitude of ratios $2, 3, -6$:

$$\sqrt{2^2 + 3^2 + (-6)^2} = \sqrt{4 + 9 + 36} = \sqrt{49} = 7.$$

Step 2 — Divide each ratio by 7:

$$\left(\frac{2}{7}, \frac{3}{7}, -\frac{6}{7}\right).$$

Why other options are wrong: (A) are the ratios themselves; (C) divides by 49; (D) divides by $\sqrt{7}$.

Final Answer: Direction cosines $\left(\frac{2}{7}, \frac{3}{7}, -\frac{6}{7}\right) \Rightarrow \boxed{\text{B}}$

$\boxed{\text{Answer: (B)}}$ Go Back to Q10

Q11.

Solution

Concept — Fundamental theorem of calculus: $\int_a^b f(x)\,dx = F(b) - F(a)$, where $F' = f$.

Step 1 — Antiderivative: $\int e^x\,dx = e^x$.

Step 2 — Apply limits 0 **to** 1**:**

$$[e^x]_0^1 = e^1 - e^0.$$

$$= e - 1.$$

Why other options are wrong: (B) $1 - e$ reverses the limits; (C) e forgets $e^0 = 1$; (D) 1 ignores the upper limit.

Final Answer: The integral equals $e - 1 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q11

Q12.

Solution

Concept — Conditional probability: $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)}$, provided $P(A) \neq 0$.

Step 1 — Substitute the values:

$$P(B \mid A) = \frac{0.12}{0.4}.$$

Step 2 — Simplify:

$$= \frac{12}{40} = 0.3.$$

Why other options are wrong: (A) 0.12 is $P(A \cap B)$; (B) 0.48 multiplies; (C) 0.4 is $P(A)$.

Final Answer: $P(B \mid A) = 0.3 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q12

Q13.

Solution

Concept — Integration by recognising a derivative: If the numerator is the derivative of the denominator, $\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$.

Step 1 — Spot the pattern: Here $f(x) = 1 + x^3$, so $f'(x) = 3x^2$, which is exactly the numerator.

Step 2 — Write the result:

$$\int \frac{3x^2}{1+x^3}\,dx = \ln|1+x^3| + C.$$

Why other options are wrong: (A) has an extra factor 3; (C) uses the wrong denominator $1 + x^2$; (D) is a wrong antiderivative.

Final Answer: $\ln|1 + x^3| + C \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q13

Q14.

Solution

Concept — Integral of an even function over a symmetric interval: If $g(-x) = g(x)$, then $\int_{-a}^{a} g(x)\,dx = 2\int_0^a g(x)\,dx$.

Step 1 — Test the integrand: $g(x) = x^2$ gives $g(-x) = (-x)^2 = x^2 = g(x)$.

So x^2 is an even function.

Step 2 — Apply the property and integrate:

$$\int_{-2}^{2} x^2\,dx = 2\int_0^2 x^2\,dx.$$

$$= 2\left[\frac{x^3}{3}\right]_0^2.$$

$$= 2 \cdot \frac{8}{3} = \frac{16}{3}.$$

Why other options are wrong: (A) 0 treats it as odd; (B) $\frac{8}{3}$ forgets the factor 2; (D) 8 uses a wrong antiderivative.

Final Answer: The integral equals $\frac{16}{3} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q14

Q15.

Solution

Concept — Local maximum via second derivative: At a critical point where $f'(x) = 0$, a local maximum occurs if $f''(x) < 0$.

Step 1 — Critical points:

$$f(x) = x^3 - 12x.$$

$$f'(x) = 3x^2 - 12 = 0.$$

$$x^2 = 4 \;\Rightarrow\; x = \pm 2.$$

Step 2 — Second-derivative test:

$$f''(x) = 6x.$$

At $x = -2$, $f''(-2) = -12 < 0$, so $x = -2$ gives a local maximum.

Step 3 — Maximum value:

$$f(-2) = (-2)^3 - 12(-2) = -8 + 24 = 16.$$

Why other options are wrong: (B) -16 is the local minimum value at $x = 2$; (C),(D) are not extreme values.

Final Answer: Local maximum value is $16 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q15

Q16.

Solution

Concept — Area of a parallelogram: Area $= |\vec{a} \times \vec{b}|$ for adjacent side vectors $\vec{a}, \vec{b}$.

Step 1 — Compute the cross product: For $\vec{a} = 2\hat{\imath} + \hat{\jmath}$ and $\vec{b} = \hat{\imath} + 3\hat{\jmath}$, the cross product is along $\hat{k}$:

$$\vec{a} \times \vec{b} = (a_x b_y - a_y b_x)\hat{k} = \big((2)(3) - (1)(1)\big)\hat{k} = (6-1)\hat{k} = 5\hat{k}.$$

Step 2 — Take its magnitude:

$$|\vec{a} \times \vec{b}| = |5\hat{k}| = 5.$$

Why other options are wrong: (A) 10 doubles the area; (B) 7 adds the products instead of subtracting; (C) 6 keeps only the first product.

Final Answer: Area $= 5 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q16

Q17.

Solution

Concept — Angle between two lines: $\cos\theta = \dfrac{|\vec{b}_1 \cdot \vec{b}_2|}{|\vec{b}_1||\vec{b}_2|}$.

Step 1 — Dot product of direction ratios:

$$(1)(1) + (0)(1) + (1)(0).$$

$$= 1 + 0 + 0 = 1.$$

Step 2 — Magnitudes:

$$|\vec{b}_1| = \sqrt{1^2 + 0^2 + 1^2} = \sqrt{2}, \qquad |\vec{b}_2| = \sqrt{1^2 + 1^2 + 0^2} = \sqrt{2}.$$

Step 3 — Compute $\cos\theta$:

$$\cos\theta = \frac{1}{\sqrt{2} \cdot \sqrt{2}} = \frac{1}{2}.$$

$$\theta = 60^\circ.$$

Why other options are wrong: (A) 90° needs a zero dot product; (C) 45° and (D) 30° correspond to different cosine values.

Final Answer: The angle is $60^\circ \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q17

Q18.

Solution

Concept — Union with independent events: $P(A \cup B) = P(A) + P(B) - P(A)P(B)$ when A, B are independent.

Step 1 — Product term:

$$P(A)P(B) = \frac{1}{4} \cdot \frac{1}{3} = \frac{1}{12}.$$

Step 2 — Substitute:

$$\begin{aligned} P(A \cup B) &= \frac{1}{4} + \frac{1}{3} - \frac{1}{12}. \\ &= \frac{3}{12} + \frac{4}{12} - \frac{1}{12}. \\ &= \frac{6}{12} = \frac{1}{2}. \end{aligned}$$

Why other options are wrong: (A) $\frac{7}{12}$ forgets to subtract the intersection; (B) $\frac{1}{12}$ is only $P(A \cap B)$; (D) $\frac{1}{3}$ uses wrong arithmetic.

Final Answer: $P(A \cup B) = \frac{1}{2} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q18

Q19.

Solution

Concept — Assertion–Reason analysis: Judge each statement's truth, then decide whether R correctly explains A.

Step 1 — Reason: For any skew-symmetric matrix, $A^T = -A$, so $|A^T| = |-A| = (-1)^n|A|$. Since $|A^T| = |A|$, we get $|A| = (-1)^n|A|$. For odd n, $(-1)^n = -1$, giving $|A| = -|A|$, hence $2|A| = 0$ and $|A| = 0$. So R is **true**.

Step 2 — Assertion: Order 3 is odd, so by the result in Step 1 the determinant of a 3×3 skew-symmetric matrix is 0. So A is **true**.

Step 3 — Does R explain A? The general odd-order result in R is exactly the reason the order-3 case in A holds. So R is the correct explanation of A.

Why other options are wrong: (B) denies the explanation link (which does hold); (C),(D) misjudge a truth value.

Final Answer: Both true, R the correct explanation $\Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q19

Q20.

Solution

Concept — Inverse tangent of a tangent: $\tan^{-1}(\tan\theta) = \theta$ only when $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$; otherwise reduce to the principal branch.

Step 1 — Test the Assertion: $\frac{3\pi}{4} \notin \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, so reduce first.

$$\tan\frac{3\pi}{4} = -1.$$

$$\tan^{-1}(-1) = -\frac{\pi}{4}.$$

Since $-\frac{\pi}{4} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, the value is $-\frac{\pi}{4}$. So A is **true**.

Step 2 — Test the Reason: The principal value branch of $\tan^{-1} x$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, *not* $[0, \pi]$. So R is **false**.

Why other options are wrong: (A),(B) require R true; (D) requires A false.

Final Answer: A true, R false $\Rightarrow \boxed{C}$

Answer: (C) Go Back to Q20

Q21.

Solution

Concept — Principal values: $\sin^{-1}$ lies in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $\cos^{-1}$ lies in $[0, \pi]$.

Step 1 — Evaluate $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$:

$$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} \Rightarrow \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}.$$

Step 2 — Evaluate $\cos^{-1}\left(\frac{1}{2}\right)$:

$$\cos\frac{\pi}{3} = \frac{1}{2} \Rightarrow \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}.$$

Step 3 — Add:

$$\frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}.$$

Final Answer: $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{1}{2}\right) = \frac{2\pi}{3}$. Go Back to Q21

Q22.

Solution

Concept — Logarithmic differentiation: Used when the variable appears in both base and exponent.

Step 1 — Take natural log of both sides:

$$y = x^x.$$

$$\ln y = x \ln x.$$

Step 2 — Differentiate both sides w.r.t. x: (product rule on the right)

$$\frac{1}{y}\frac{dy}{dx} = 1 \cdot \ln x + x \cdot \frac{1}{x}.$$

$$\frac{1}{y}\frac{dy}{dx} = \ln x + 1.$$

Step 3 — Solve for $\frac{dy}{dx}$:

$$\frac{dy}{dx} = y\big(\ln x + 1\big).$$

$$= x^x\big(1 + \ln x\big).$$

Final Answer: $\frac{dy}{dx} = x^x\big(1 + \ln x\big)$. Go Back to Q22

Q23.

Solution

Concept — Integration by parts: $\int u\,dv = uv - \int v\,du$, choosing u by the ILATE rule.

Step 1 — Choose parts: Let $u = x$ and $dv = \sin x\,dx$.

Then $du = dx$ and $v = -\cos x$.

Step 2 — Apply the formula:

$$\int x \,\sin x\, dx = x(-\cos x) - \int (-\cos x)\, dx.$$

$$= -x\cos x + \int \cos x\, dx.$$

$$= -x\cos x + \sin x + C.$$

Final Answer: $\int x \,\sin x\, dx = -x\cos x + \sin x + C.$

OR — $\int \ln x\, dx$**:**

Step 1 — Choose parts: Let $u = \ln x$ and $dv = dx$.

Then $du = \frac{1}{x}\, dx$ and $v = x$.

Step 2 — Apply the formula:

$$\int \ln x\, dx = x\ln x - \int x \cdot \frac{1}{x}\, dx.$$

$$= x\ln x - \int 1\, dx.$$

$$= x\ln x - x + C.$$

Final Answer (OR): $\int \ln x\, dx = x\ln x - x + C.$ **Go Back to Q23**

Q24.

Solution

Concept — Perpendicular vectors: $\vec{a} \perp \vec{b} \iff \vec{a}\cdot\vec{b} = 0.$

Step 1 — Compute the dot product:

$$\vec{a}\cdot\vec{b} = (\lambda)(2) + (-2)(1) + (1)(3).$$

$$= 2\lambda - 2 + 3.$$

$$= 2\lambda + 1.$$

Step 2 — Set equal to zero and solve:

$$2\lambda + 1 = 0.$$

$$2\lambda = -1.$$

$$\lambda = -\frac{1}{2}.$$

Final Answer: $\lambda = -\frac{1}{2}$. Go Back to Q24

Q25.

Solution

Concept — Equally likely outcomes: Probability $= \dfrac{\text{favourable outcomes}}{\text{total outcomes}}$.

Step 1 — Total outcomes: Two throws of a die give $6 \times 6 = 36$ equally likely outcomes.

Step 2 — Favourable outcomes (sum $= 7$):

$$(1,6),\ (6,1),\ (2,5),\ (5,2),\ (3,4),\ (4,3).$$

There are 6 such outcomes.

Step 3 — Probability:

$$P(\text{sum} = 7) = \frac{6}{36} = \frac{1}{6}.$$

Final Answer: $P(\text{sum} = 7) = \frac{1}{6}$.

OR — $P(B \mid A)$:

Step 1 — Formula: $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)}$.

Step 2 — Substitute:

$$P(B \mid A) = \frac{0.2}{0.5} = \frac{2}{5}.$$

Final Answer (OR): $P(B \mid A) = \frac{2}{5} = 0.4$. Go Back to Q25

Q26.

Solution

Concept — Partial fractions: Split a proper rational function into simpler fractions, then integrate each.

Step 1 — Set up the decomposition:

$$\frac{1}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}.$$

$$1 = A(x+2) + B(x-1).$$

Step 2 — Find A and B:

Put $x = 1$: $1 = A(3) + B(0) \Rightarrow A = \frac{1}{3}$.

Put $x = -2$: $1 = A(0) + B(-3) \Rightarrow B = -\frac{1}{3}$.

Step 3 — Integrate:

$$\int \frac{1}{(x-1)(x+2)}\,dx = \int \left(\frac{1/3}{x-1} - \frac{1/3}{x+2}\right) dx.$$

$$= \frac{1}{3}\ln|x-1| - \frac{1}{3}\ln|x+2| + C.$$

$$= \frac{1}{3}\ln\left|\frac{x-1}{x+2}\right| + C.$$

Final Answer: $\frac{1}{3}\ln\left|\frac{x-1}{x+2}\right| + C$. **Go Back to Q26**

Q27.

Solution

Concept — Linear differential equation: For $\frac{dy}{dx} + Py = Q$, the integrating factor is I.F. $= e^{\int P\,dx}$ and the solution is $y \cdot \text{I.F.} = \int Q \cdot \text{I.F.}\,dx$.

Step 1 — Identify P, Q: Here $P = 2$, $Q = e^{-x}$.

Step 2 — Integrating factor:

$$\text{I.F.} = e^{\int 2\,dx} = e^{2x}.$$

Step 3 — Solve:

$$y\,e^{2x} = \int e^{-x} \cdot e^{2x}\,dx = \int e^{x}\,dx.$$

$$y\,e^{2x} = e^{x} + C.$$

$$y = e^{-x} + C\,e^{-2x}.$$

Final Answer: $y = e^{-x} + C\,e^{-2x}$.

OR — Variables separable $\frac{dy}{dx} = e^{x-y}$**:**

Step 1 — Rewrite and separate variables:

$$\frac{dy}{dx} = e^x e^{-y}.$$

$$e^y \, dy = e^x \, dx.$$

Step 2 — Integrate both sides:

$$\int e^y \, dy = \int e^x \, dx.$$

$$e^y = e^x + C.$$

Final Answer (OR): $e^y = e^x + C$. **Go Back to Q27**

Q28.

Solution

Concept — Inverse by adjoint: $A^{-1} = \frac{1}{|A|}\,\text{adj}(A)$, valid when $|A| \neq 0$.

Step 1 — Determinant:

$$|A| = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = (1)(4) - (2)(3) = 4 - 6 = -2.$$

Step 2 — Adjoint of a 2×2 matrix: Swap the leading diagonal entries and negate the off-diagonal entries:

$$\text{adj}(A) = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}.$$

Step 3 — Inverse:

$$A^{-1} = \frac{1}{-2}\begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}.$$

$$= \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}.$$

Final Answer: $A^{-1} = \begin{bmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{bmatrix}$. **Go Back to Q28**

Q29.

Solution

Concept — Monotonicity: f increases where $f'(x) > 0$ and decreases where $f'(x) < 0$.

Step 1 — Differentiate:

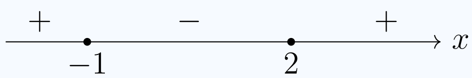
$$f(x) = 2x^3 - 3x^2 - 12x + 5.$$

$$f'(x) = 6x^2 - 6x - 12.$$

$$= 6(x^2 - x - 2) = 6(x-2)(x+1).$$

Step 2 — Critical points: $f'(x) = 0$ at $x = -1$ and $x = 2$.

Step 3 — Sign analysis:



For $x < -1$: $f' > 0$; for $-1 < x < 2$: $f' < 0$; for $x > 2$: $f' > 0$.

Conclusion: Increasing on $(-\infty, -1) \cup (2, \infty)$; decreasing on $(-1, 2)$.

Final Answer: Increasing on $(-\infty, -1) \cup (2, \infty)$, decreasing on $(-1, 2)$.

OR — Tangent to $y = x^3$ at $(1, 1)$:

Step 1 — Slope: $\dfrac{dy}{dx} = 3x^2$, so at $x = 1$ slope $= 3$.

Step 2 — Equation: $y - 1 = 3(x - 1) \Rightarrow y = 3x - 2$.

Final Answer (OR): $y = 3x - 2$. Go Back to Q29

Q30.

Solution

Concept — Vector equation of a line: A line through point $\vec{a}$ parallel to $\vec{b}$ is $\vec{r} = \vec{a} + \lambda\vec{b}$.

Step 1 — Write the equation: With $\vec{a} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$ and $\vec{b} = 2\hat{\imath} - \hat{\jmath} + 3\hat{k}$,

$$\vec{r} = (2\hat{\imath} + \hat{\jmath} - \hat{k}) + \lambda(2\hat{\imath} - \hat{\jmath} + 3\hat{k}).$$

Step 2 — Test the point $(4, 0, 2)$: The point lies on the line if some λ satisfies all three component equations.

$$2 + 2\lambda = 4 \;\Rightarrow\; \lambda = 1.$$

$$1 - \lambda = 0 \;\Rightarrow\; \lambda = 1.$$

$$-1 + 3\lambda = 2 \;\Rightarrow\; \lambda = 1.$$

Step 3 — Conclusion: All three give the same value $\lambda = 1$, so the point $(4, 0, 2)$ lies on the line.

Final Answer: $\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \lambda(2\hat{i} - \hat{j} + 3\hat{k})$; the point $(4, 0, 2)$ lies on it (at $\lambda = 1$). **Go Back to Q30**

Q31.

Solution

Concept — One-one (injective) and onto (surjective): *One-one*: $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$. *Onto*: every y in the codomain has a pre-image.

Step 1 — One-one: Suppose $f(x_1) = f(x_2)$.

$$3x_1 - 2 = 3x_2 - 2.$$

$$3x_1 = 3x_2.$$

$$x_1 = x_2.$$

Hence f is one-one.

Step 2 — Onto: Let $y \in \mathbb{R}$ be arbitrary. Solve $y = 3x - 2$ for x:

$$x = \frac{y+2}{3} \in \mathbb{R}.$$

Then $f(x) = 3\left(\frac{y+2}{3}\right) - 2 = y$.

So every y has a pre-image; f is onto.

Step 3 — Conclusion: f is both one-one and onto, hence bijective.

Final Answer: $f(x) = 3x - 2$ is one-one and onto (a bijection). **Go Back to Q31**

Q32.

Solution

Concept — Matrix method: Write $AX = B$; then $X = A^{-1}B$ when $|A| \neq 0$.

Step 1 — Form the matrices:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & -1 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}.$$

Step 2 — Determinant of A:

$$|A| = 1(2 \cdot 1 - 3 \cdot (-1)) - 1(1 \cdot 1 - 3 \cdot 1) + 1(1 \cdot (-1) - 2 \cdot 1).$$

$$= 1(2 + 3) - 1(1 - 3) + 1(-1 - 2).$$

$$= 5 + 2 - 3 = 4 \ (\neq 0).$$

Step 3 — Cofactors and adjoint: Computing the cofactor matrix and transposing gives

$$\text{adj}(A) = \begin{bmatrix} 5 & -2 & 1 \\ 2 & 0 & -2 \\ -3 & 2 & 1 \end{bmatrix}.$$

Step 4 — Solve $X = \frac{1}{|A|}\,\text{adj}(A)\,B$:

$$X = \frac{1}{4}\begin{bmatrix} 5 & -2 & 1 \\ 2 & 0 & -2 \\ -3 & 2 & 1 \end{bmatrix}\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}.$$

$$= \frac{1}{4}\begin{bmatrix} 5 \cdot 2 - 2 \cdot 1 + 1 \cdot 0 \\ 2 \cdot 2 + 0 \cdot 1 - 2 \cdot 0 \\ -3 \cdot 2 + 2 \cdot 1 + 1 \cdot 0 \end{bmatrix}.$$

$$= \frac{1}{4}\begin{bmatrix} 10 - 2 \\ 4 \\ -6 + 2 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 8 \\ 4 \\ -4 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}.$$

Step 5 — Verify: $x + y + z = 2 + 1 - 1 = 2$ ✓; $x + 2y + 3z = 2 + 2 - 3 = 1$ ✓; $x - y + z = 2 - 1 - 1 = 0$ ✓.

Final Answer: $x = 2,\ y = 1,\ z = -1$. Go Back to Q32

Q33.

Solution

Concept — Area between two curves: Area $= \int_a^b \left(y_{\text{top}} - y_{\text{bottom}}\right) dx$ over the interval where they overlap.

Step 1 — Find the points of intersection: Set $x^2 = 2x$:

$$x^2 - 2x = 0.$$

$$x(x-2) = 0 \;\Rightarrow\; x = 0 \text{ and } x = 2.$$

Step 2 — Identify top and bottom: On $[0, 2]$, the line $y = 2x$ lies above the parabola $y = x^2$.

Step 3 — Set up the integral:

$$\text{Area} = \int_0^2 (2x - x^2)\, dx.$$

Step 4 — Integrate:

$$= \left[x^2 - \frac{x^3}{3}\right]_0^2.$$

$$= \left(4 - \frac{8}{3}\right) - 0.$$

$$= \frac{12-8}{3} = \frac{4}{3}.$$

Final Answer: Area $= \frac{4}{3}$ square units.

OR — Area under $y = \sin x$, $0 \le x \le \pi$:

Step 1 — Set up the integral: Since $\sin x \ge 0$ on $[0, \pi]$,

$$\text{Area} = \int_0^\pi \sin x\, dx.$$

Step 2 — Integrate:

$$= \big[-\cos x\big]_0^\pi.$$

$$= -\cos\pi - (-\cos 0).$$

$$= -(-1) + 1 = 2.$$

Final Answer (OR): Area $= 2$ square units. Go Back to Q33

Q34.

Solution

Concept — Corner-point method (minimisation): For an unbounded feasible region, the minimum of a linear objective (when it exists) occurs at a corner; confirm it by checking that the objective cannot be made smaller anywhere in the region.

Step 1 — Find the corner points: The constraints $x + 2y \geq 14$, $x + y \geq 8$, $x \geq 0$, $y \geq 0$ give an unbounded region lying above the two lines. Its corners come from the boundary lines. The lines $x + 2y = 14$ and $x + y = 8$ intersect: subtracting the second from the first gives $y = 6$, and then $x = 8 - 6 = 2$. So one corner is $(2, 6)$. On the axes, $(0, 8)$ satisfies $x + 2y = 16 \geq 14$, and $(14, 0)$ satisfies $x + y = 14 \geq 8$. Hence the corners are

$$(0, 8), \quad (2, 6), \quad (14, 0).$$

Step 2 — Evaluate $Z = 50x + 70y$ at each corner:

$$Z(0, 8) = 50(0) + 70(8) = 560.$$

$$Z(2, 6) = 50(2) + 70(6) = 100 + 420 = 520.$$

$$Z(14, 0) = 50(14) + 70(0) = 700.$$

Step 3 — Pick the minimum: The smallest value is 520 at $(2, 6)$. Since the open half-plane $50x + 70y < 520$ contains no point of the feasible region, this minimum is genuine.

Final Answer: Z is minimum $= 520$ at $(x, y) = (2, 6)$. **Go Back to Q34**

Q35.

Solution

Concept — Shortest distance between skew lines: For $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$,

$$d = \frac{\left|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)\right|}{|\vec{b}_1 \times \vec{b}_2|}.$$

Step 1 — Identify vectors: $\vec{a}_1 = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b}_1 = \hat{i} - \hat{j} + \hat{k}$; $\vec{a}_2 = 2\hat{i} - \hat{j} - \hat{k}$, $\vec{b}_2 = 2\hat{i} + \hat{j} - \hat{k}$.

Step 2 — Cross product $\vec{b}_1 \times \vec{b}_2$:

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & -1 \end{vmatrix}.$$

$$= \hat{i}\big[(-1)(-1) - (1)(1)\big] - \hat{j}\big[(1)(-1) - (1)(2)\big] + \hat{k}\big[(1)(1) - (-1)(2)\big].$$

$$= \hat{i}(1-1) - \hat{j}(-1-2) + \hat{k}(1+2).$$

$$= 0\hat{i} + 3\hat{j} + 3\hat{k}.$$

Step 3 — Its magnitude:

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{0^2 + 3^2 + 3^2} = \sqrt{18} = 3\sqrt{2}.$$

Step 4 — $(\vec{a}_2 - \vec{a}_1)$ and the dot product:

$$\vec{a}_2 - \vec{a}_1 = (2-1)\hat{i} + (-1-2)\hat{j} + (-1-1)\hat{k} = \hat{i} - 3\hat{j} - 2\hat{k}.$$

$$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = (1)(0) + (-3)(3) + (-2)(3) = 0 - 9 - 6 = -15.$$

Step 5 — Distance:

$$d = \frac{|-15|}{3\sqrt{2}} = \frac{15}{3\sqrt{2}} = \frac{5}{\sqrt{2}} \text{ units.}$$

Final Answer: Shortest distance $= \dfrac{5}{\sqrt{2}}$ units.

OR — Angle between line and plane:

Step 1 — Direction $\vec{b} = 2\hat{i} + 2\hat{j} + \hat{k}$, normal $\vec{n} = \hat{i} + 2\hat{j} + 2\hat{k}$.

Step 2 — Use $\sin\theta = \dfrac{|\vec{b}\cdot\vec{n}|}{|\vec{b}||\vec{n}|}$:

$$\vec{b}\cdot\vec{n} = (2)(1) + (2)(2) + (1)(2) = 2 + 4 + 2 = 8.$$

$$|\vec{b}| = \sqrt{4+4+1} = 3, \qquad |\vec{n}| = \sqrt{1+4+4} = 3.$$

$$\sin\theta = \frac{8}{3\cdot 3} = \frac{8}{9}.$$

Final Answer (OR): $\theta = \sin^{-1}\left(\dfrac{8}{9}\right)$. **Go Back to Q35**

Q36.

Solution

Concept — Minima by derivatives: Build the total-area function, set $\frac{dA}{dx} = 0$, and confirm a minimum with the second derivative.

(i) Area function: The square is bent from a length x, so its side is $\frac{x}{4}$ and its area is $\left(\frac{x}{4}\right)^2 = \frac{x^2}{16}$. The circle is bent from the remaining length $28 - x$, which is its circumference, so $2\pi r = 28 - x$ giving $r = \frac{28-x}{2\pi}$ and area $\pi r^2 = \frac{(28-x)^2}{4\pi}$. Hence

$$A(x) = \frac{x^2}{16} + \frac{(28-x)^2}{4\pi}, \qquad 0 < x < 28.$$

(ii) Differentiate:

$$\frac{dA}{dx} = \frac{2x}{16} - \frac{2(28-x)}{4\pi} = \frac{x}{8} - \frac{28-x}{2\pi}.$$

(iii) Critical point and minimum: Set $\frac{dA}{dx} = 0$:

$$\frac{x}{8} = \frac{28-x}{2\pi}.$$

$$2\pi x = 8(28-x).$$

$$2\pi x + 8x = 224.$$

$$x(2\pi + 8) = 224 \;\Rightarrow\; x = \frac{224}{2\pi+8} = \frac{112}{\pi+4}.$$

Second derivative: $\frac{d^2A}{dx^2} = \frac{1}{8} + \frac{1}{2\pi} > 0$, so this value of x gives a minimum.

Final Answer: $A(x) = \frac{x^2}{16} + \frac{(28-x)^2}{4\pi}$; $\frac{dA}{dx} = \frac{x}{8} - \frac{28-x}{2\pi}$; the total area is minimum at $x = \frac{112}{\pi+4} \approx 15.7$ cm. **Go Back to Q36**

Q37.

Solution

Concept — Bayes' theorem: $P(E_i \mid D) = \frac{P(E_i)P(D \mid E_i)}{\sum_j P(E_j)P(D \mid E_j)}$.

(i) Prior probabilities: A box is chosen at random, so

$$P(\text{Box I}) = \frac{1}{2}, \qquad P(\text{Box II}) = \frac{1}{2}.$$

(ii) Conditional (defective) probabilities:

$$P(\text{Def} \mid \text{Box I}) = \frac{4}{10} = \frac{2}{5}, \qquad P(\text{Def} \mid \text{Box II}) = \frac{2}{10} = \frac{1}{5}.$$

(iii) Apply Bayes' theorem:

$$P(\text{Box I} \mid \text{Def}) = \frac{P(\text{Box I})P(\text{Def} \mid \text{Box I})}{P(\text{Box I})P(\text{Def} \mid \text{Box I}) + P(\text{Box II})P(\text{Def} \mid \text{Box II})}.$$

$$= \frac{\frac{1}{2} \cdot \frac{2}{5}}{\frac{1}{2} \cdot \frac{2}{5} + \frac{1}{2} \cdot \frac{1}{5}}.$$

$$= \frac{\frac{1}{5}}{\frac{1}{5} + \frac{1}{10}} = \frac{\frac{2}{10}}{\frac{2}{10} + \frac{1}{10}} = \frac{2}{3}.$$

Final Answer: $P(\text{Box I} \mid \text{Defective}) = \frac{2}{3}$. **Go Back to Q37**

Q38.

Solution

Concept — Related rates: Differentiate the formula for the quantity with respect to time and substitute the known rate.

(i) Rate of change of volume: For a cube of edge a, the volume is $V = a^3$. Differentiating with respect to t,

$$\frac{dV}{dt} = 3a^2\frac{da}{dt}.$$

With $\frac{da}{dt} = 3$,

$$\frac{dV}{dt} = 3a^2(3) = 9a^2.$$

(ii) Volume rate at $a = 10$:

$$\frac{dV}{dt} = 9(10)^2 = 9 \times 100 = 900 \text{ cm}^3/\text{s}.$$

(iii) Surface-area rate at $a = 10$: The total surface area is $S = 6a^2$, so

$$\frac{dS}{dt} = 12a\frac{da}{dt} = 12a(3) = 36a.$$

$$\left.\frac{dS}{dt}\right|_{a=10} = 36(10) = 360\ \text{cm}^2/\text{s}.$$

Final Answer: $\dfrac{dV}{dt} = 9a^2$; at $a = 10$ cm the volume increases at 900 cm^3/s and the total surface area at 360 cm^2/s. **Go Back to Q38**

Answer Key – Section A (Q1–Q20)

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	D	3	A	4	C	5	D
6	A	7	B	8	D	9	C	10	B
11	A	12	D	13	B	14	C	15	A
16	D	17	B	18	C	19	A	20	C

Sections B–E are descriptive; refer to the Detailed Solutions above for full model answers and step marking.