CBSE Class 12 Mathematics

Sample Paper – 6

Duration: 180 Minutes **Maximum Marks: 80**

General Instructions

- This question paper contains **38 questions**. **All questions are compulsory**.
- The paper is divided into **five sections** – A, B, C, D and E.
- **Section A** (Q1–Q20) carries **1 mark** each: Q1–Q18 are multiple choice questions and Q19–Q20 are Assertion–Reason questions.
- **Section B** (Q21–Q25) carries **2 marks** each (Very Short Answer).
- **Section C** (Q26–Q31) carries **3 marks** each (Short Answer).
- **Section D** (Q32–Q35) carries **5 marks** each (Long Answer).
- **Section E** (Q36–Q38) carries **4 marks** each (case study based, with sub-parts).
- There is **no overall choice**, but an **internal choice** has been provided in some questions. Attempt only one of the alternatives in such questions.
- There is **no negative marking**. Use of a **calculator is not permitted**.

Section A (Q1–Q20) – 1 Mark Each

Q1. The matrix $A = \begin{bmatrix} x & 3 \\ 3 & x \end{bmatrix}$ is singular for:

(A) $x = 3$ only

(B) $x = \pm 3$

(C) $x = 9$

(D) $x = \pm 9$

Q2. The area of the triangle with vertices $(2,0)$, $(0,3)$ and $(4,4)$ is:

(A) 14

(B) 5

(C) 7

(D) $\frac{7}{2}$

Q3. On the set $A = \{1,2,3\}$, the relation $R = \{(1,1),(2,2),(3,3),(1,2),(2,1),(2,3),(3,2$
is:

(A) reflexive and symmetric but not transitive

(B) an equivalence relation

(C) symmetric and transitive but not reflexive

(D) transitive only

Q4. The principal value of $\cot^{-1}(-1)$ is:

(A) $\frac{\pi}{4}$

(B) $-\frac{\pi}{4}$

(C) $-\frac{3\pi}{4}$

(D) $\frac{3\pi}{4}$

Q5. If $y = \tan(2x)$, then $\frac{dy}{dx}$ equals:

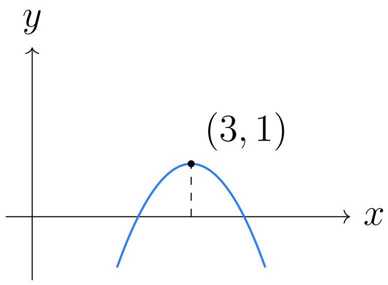
(A) $\sec^2(2x)$

(B) $2\sec(2x)\tan(2x)$

(C) $-2\sec^2(2x)$

(D) $2\sec^2(2x)$

Q6. The graph of $f(x) = -x^2 + 6x - 8$ is shown below. The function is *decreasing* on the interval:



(A) $(-\infty, 3)$

(B) $(-\infty, \infty)$

(C) $(0, 3)$

(D) $(3, \infty)$

Q7. $\int \frac{1}{\sqrt{1-x^2}}\,dx$ equals:

(A) $\cos^{-1} x + C$

(B) $\tan^{-1} x + C$

(C) $\sin^{-1} x + C$

(D) $\sec^{-1} x + C$

Q8. The order and degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{dy}{dx}\right)^4 + y = 0$ are respectively:

(A) 3 and 2

(B) 3 and 4

(C) 3 and 1

(D) 2 and 3

Q9. The magnitude of the vector $\vec{a} = 4\hat{i} - 3\hat{j}$ is:

(A) 7

(B) 5

(C) 25

(D) 1

Q10. The direction cosines of a line whose direction ratios are $3, 4, 0$ are:

(A) $3, 4, 0$

(B) $\frac{3}{25}, \frac{4}{25}, 0$

(C) $\frac{3}{5}, \frac{4}{5}, 0$

(D) $\frac{3}{7}, \frac{4}{7}, 0$

Q11. $\int_0^1 x^2\,dx$ equals:

(A) $\frac{1}{3}$

(B) $\frac{1}{2}$

(C) 1

(D) $\frac{2}{3}$

Q12. If $P(A \cap B) = 0.18$ and $P(A) = 0.6$, then $P(B \mid A)$ equals:

(A) 0.108

(B) 0.42

(C) 0.5

(D) 0.3

Q13. $\int \frac{1}{x \ln x}\,dx$ equals:

(A) $\ln x + C$

(B) $\ln|\ln x| + C$

(C) $\frac{(\ln x)^2}{2} + C$

(D) $\frac{1}{\ln x} + C$

Q14. $\int_{-2}^{2} x^2\,dx$ equals:

(A) 0

(B) $\frac{8}{3}$

(C) $\frac{16}{3}$

(D) 8

Q15. The local maximum value of $f(x) = x + \frac{1}{x}$ is:

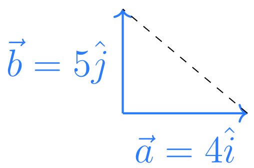
(A) -2

(B) 2

(C) 0

(D) -1

Q16. The area of the triangle whose two adjacent sides are the vectors $\vec{a} = 4\hat{i}$ and $\vec{b} = 5\hat{j}$ (shown below) is:



(A) 20

(B) 10

(C) 40

(D) 5

Q17. The angle between two lines whose direction ratios are $1, 2, -1$ and $2, 4, -2$ is:

(A) 90°

(B) 0°

(C) 45°

(D) 60°

Q18. If A and B are independent events with $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$, then $P(A' \cap B')$ equals:

(A) $\frac{1}{12}$

(B) $\frac{7}{12}$

(C) $\frac{5}{12}$

(D) $\frac{1}{2}$

Q19. Assertion (A): The derivative of an even function is an odd function.
Reason (R): If $f(-x) = f(x)$, then differentiating both sides gives $f'(-x) = -f'(x)$.

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is *not* the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

Q20. Assertion (A): The determinant of a 3×3 skew-symmetric matrix is 0.
Reason (R): The determinant of a skew-symmetric matrix of *any* order is 0.

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is *not* the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

Section B (Q21–Q25) – 2 Marks Each

Q21. Find the value of $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right)$. **[2]**

Q22. Differentiate $y = x^{\tan x}$ with respect to x. **[2]**

Q23. Evaluate $\int x \cos x \, dx$. **[2]**

OR

Evaluate $\int \sin^2 x \, dx$.

Q24. Find the value of λ for which the vectors $\vec{a} = 2\hat{i} + 3\hat{j} + \lambda\hat{k}$ and $\vec{b} = 4\hat{i} + 6\hat{j} - 4\hat{k}$ are parallel. **[2]**

Q25. A card is drawn at random from a well-shuffled pack of 52 playing cards. Find the probability that the card drawn is a king or a heart. **[2]**

OR

If $P(B) = 0.4$ and $P(A \cap B) = 0.25$, find $P(A \mid B)$.

Section C (Q26–Q31) – 3 Marks Each

Q26. Evaluate $\displaystyle\int \frac{2x+3}{(x+1)(x+3)}\,dx$. **[3]**

Q27. Solve the differential equation $\dfrac{dy}{dx} + 2y = e^{-x}$. **[3]**

OR

Solve the differential equation $\dfrac{dy}{dx} = e^{x-y}$.

Q28. Find the inverse of the matrix $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ using the adjoint method. **[3]**

Q29. Find the intervals in which $f(x) = x^3 - 3x^2 - 9x + 7$ is increasing or decreasing. **[3]**

OR

Find the equation of the tangent to the curve $y = x^3$ at the point $(1, 1)$.

Q30. Find the vector equation of the line passing through the point $(2, -1, 3)$ and parallel to the vector $2\hat{i} + 3\hat{j} - \hat{k}$. Hence check whether the point $(4, 2, 4)$ lies on this line. **[3]**

Q31. Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 3x - 2$ is one-one and onto. **[3]**

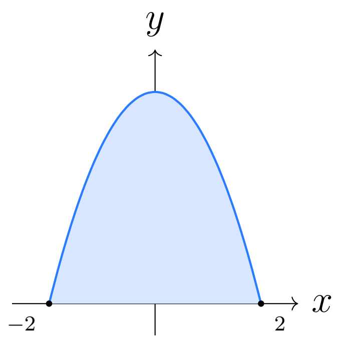
Section D (Q32–Q35) – 5 Marks Each

Q32. Using the matrix method, solve the following system of linear equations:

$$x + y + z = 2, \qquad 2x - y + z = -1, \qquad x + 2y - z = 6.$$

[5]

Q33. Using integration, find the area of the region bounded by the parabola $y = 4 - x^2$ and the x-axis.



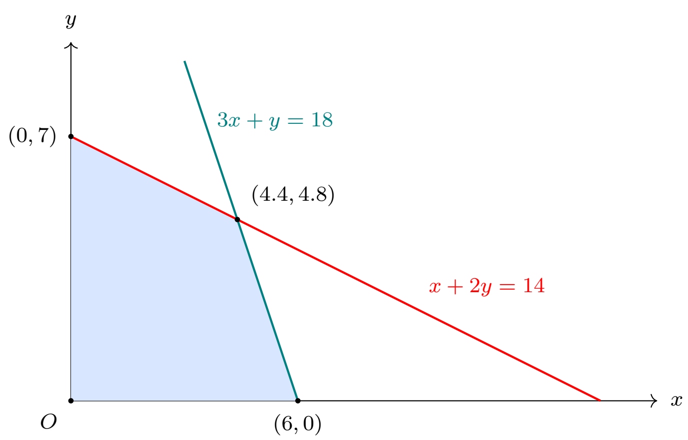
[5]

OR

Using integration, find the area of the region bounded by the curves $y = x^2$ and $y = 2x$.

Q34. Solve the following linear programming problem graphically. Maximise $Z = x + 4y$ subject to the constraints

$$x + 2y \leq 14, \qquad 3x + y \leq 18, \qquad x \geq 0, \; y \geq 0.$$



[5]

Q35. Find the shortest distance between the lines

$$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k}) \quad \text{and} \quad \vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k}).$$

[5]

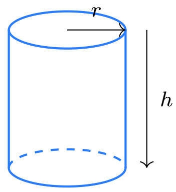
OR

Find the angle between the planes $2x - y + 2z = 5$ and $x + y + z = 3$.

Section E (Q36–Q38) – 4 Marks Each (Case Study)

Q36. Case Study – The Cylindrical Can.

A closed right circular cylinder (a sealed tin can) of base radius r and height h is to be manufactured to hold a fixed volume V. To keep costs low, the maker wants to use the least possible sheet metal, that is, to *minimise the total surface area* S.



Based on the above, answer the following:

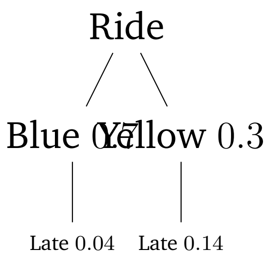
(i) Using $V = \pi r^2 h$, express the total surface area S as a function of r. (1)

(ii) Find $\dfrac{dS}{dr}$. (1)

(iii) Find the value of r for which S is minimum, and show that then $h = 2r$. (2)

Q37. Case Study – Two Cab Companies.

In a city, two app-based cab companies operate. Blue Cabs run 70% of all rides while Yellow Cabs run the remaining 30%. From past records, 4% of Blue Cab rides receive a late-arrival complaint, whereas 14% of Yellow Cab rides receive one. A ride is chosen at random and it received a late-arrival complaint.



Based on the above, answer the following:

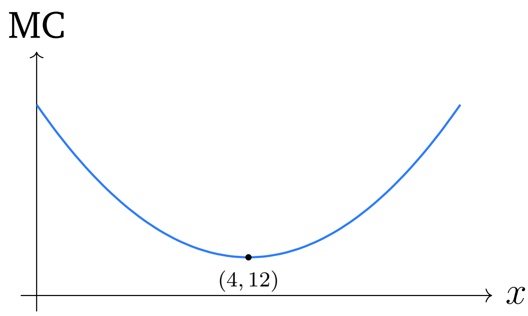
(i) Write $P(\text{Blue})$ and $P(\text{Yellow})$. (1)

(ii) Write $P(\text{Late} \mid \text{Blue})$ and $P(\text{Late} \mid \text{Yellow})$. (1)

(iii) Using Bayes' theorem, find the probability that the ride was a Yellow Cab ride. (2)

Q38. Case Study – Marginal Cost.

The total cost (in rupees) of producing x units of a commodity is given by $C(x) = x^3 - 12x^2 + 60x + 100$. The marginal cost is the rate of change of total cost with respect to output, that is MC $= \dfrac{dC}{dx}$.



Based on the above, answer the following:

(i) Find the marginal cost function MC. (1)

(ii) Find the marginal cost when $x = 5$ units. (1)

(iii) Find the value of x at which the marginal cost is minimum, and the minimum marginal cost. (2)

Detailed Solutions

Q1.

Solution

Concept — Singular matrix: A square matrix A is singular when its determinant is 0.

Step 1 — Write the determinant:

$$|A| = \begin{vmatrix} x & 3 \\ 3 & x \end{vmatrix} = (x)(x) - (3)(3).$$

$$= x^2 - 9.$$

Step 2 — Set the determinant to zero:

$$x^2 - 9 = 0.$$

$$x^2 = 9.$$

$$x = \pm 3.$$

Why other options are wrong: (A) $x = 3$ is only one of the two roots; (C) $x = 9$ solves $x = 9$, not $x^2 = 9$; (D) $x = \pm 9$ comes from misreading $x^2 = 81$.

Final Answer: A is singular for $x = \pm 3 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q1

Q2.

Solution

Concept — Area of a triangle by determinant: For vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$,

$$\text{Area} = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|.$$

Step 1 — Substitute the vertices $(2, 0), (0, 3), (4, 4)$:

$$\text{Area} = \frac{1}{2}\left|2(3 - 4) + 0(4 - 0) + 4(0 - 3)\right|.$$

Step 2 — Simplify inside:

$$= \frac{1}{2}\left|2(-1) + 0 + 4(-3)\right|.$$

$$= \frac{1}{2}\left| -2 - 12\right|.$$

$$= \frac{1}{2} \times 14.$$

$$= 7.$$

Why other options are wrong: (A) 14 forgets the factor $\frac{1}{2}$; (B) 5 and (D) $\frac{7}{2}$ come from arithmetic slips.

Final Answer: Area $= 7$ square units $\Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q2

Q3.

Solution

Concept — Properties of a relation: *Reflexive*: $(a, a) \in R$ for all a. *Symmetric*: $(a, b) \in R \Rightarrow (b, a) \in R$. *Transitive*: $(a, b), (b, c) \in R \Rightarrow (a, c) \in R$.

Step 1 — Reflexive? $(1, 1), (2, 2), (3, 3) \in R$, so R is reflexive.

Step 2 — Symmetric? $(1, 2)$ and $(2, 1)$ are both in R; $(2, 3)$ and $(3, 2)$ are both in R.

Hence R is symmetric.

Step 3 — Transitive? $(1, 2) \in R$ and $(2, 3) \in R$, but $(1, 3) \notin R$.

Hence R is not transitive.

Why other options are wrong: (B) an equivalence relation must also be transitive, which fails; (C) R is reflexive, so "not reflexive" is false; (D) R is not transitive.

Final Answer: R is reflexive and symmetric but not transitive $\Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q3

Q4.

Solution

Concept — Principal value of $\cot^{-1}$: The principal value branch of $\cot^{-1} x$ is $(0, \pi)$.

Step 1 — Set up the equation: Let $\cot^{-1}(-1) = \theta$, so $\cot\theta = -1$ with $\theta \in (0, \pi)$.

Step 2 — Find θ in the branch: $\cot\frac{3\pi}{4} = \frac{\cos(3\pi/4)}{\sin(3\pi/4)} = \frac{-1/\sqrt{2}}{1/\sqrt{2}} = -1$.

Since $\frac{3\pi}{4} \in (0, \pi)$, it is the principal value.

Why other options are wrong: (A) $\frac{\pi}{4}$ gives $\cot = +1$; (B) $-\frac{\pi}{4}$ and (C) $-\frac{3\pi}{4}$ lie outside $(0, \pi)$.

Final Answer: $\cot^{-1}(-1) = \frac{3\pi}{4} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q4

Q5.

Solution

Concept — Chain rule: $\frac{d}{dx}\tan(u) = \sec^2(u) \cdot \frac{du}{dx}$.

Step 1 — Identify the inner function: Let $u = 2x$, so $\frac{du}{dx} = 2$.

Step 2 — Apply the chain rule:

$$\frac{dy}{dx} = \sec^2(2x) \cdot 2.$$

$$= 2\sec^2(2x).$$

Why other options are wrong: (A) omits the factor 2; (B) is the derivative of $\sec(2x)$; (C) has a wrong sign.

Final Answer: $\frac{dy}{dx} = 2\sec^2(2x) \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q5

Q6.

Solution

Concept — Decreasing function: f is decreasing where $f'(x) < 0$.

Step 1 — Differentiate:

$$f(x) = -x^2 + 6x - 8.$$

$$f'(x) = -2x + 6.$$

Step 2 — Solve $f'(x) < 0$:

$$-2x + 6 < 0.$$

$$-2x < -6.$$

$$x > 3.$$

So f is decreasing on $(3, \infty)$, which matches the falling right branch of the downward parabola.

Why other options are wrong: (A) $(-\infty, 3)$ is where $f'(x) > 0$ (increasing); (B) is false since the parabola first rises then falls; (C) $(0, 3)$ is only part of the increasing region.

Final Answer: f decreases on $(3, \infty) \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q6

Q7.

Solution

Concept — Standard integral: $\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$, hence $\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + C$.

Step 1 — Recall the antiderivative: The derivative of $\sin^{-1}x$ is $\frac{1}{\sqrt{1-x^2}}$.

Step 2 — Write the integral:

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + C.$$

Why other options are wrong: (A) $\cos^{-1}x$ differentiates to $-\frac{1}{\sqrt{1-x^2}}$; (B) $\tan^{-1}x$ is $\int \frac{1}{1+x^2}\,dx$; (D) $\sec^{-1}x$ is $\int \frac{1}{x\sqrt{x^2-1}}\,dx$.

Final Answer: $\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C \Rightarrow \boxed{C}$

Answer: (C) Go Back to Q7

Q8.

Solution

Concept — Order and degree: The *order* is the highest derivative present; the *degree* is the power of that highest derivative when the equation is polynomial in derivatives.

Step 1 — Highest derivative: The highest derivative is $\frac{d^3y}{dx^3}$, so the order is 3.

Step 2 — Its power: The term $\left(\frac{d^3y}{dx^3}\right)^2$ appears to the second power (the fourth power is on $\frac{dy}{dx}$, a lower derivative).
So the degree is 2.

Why other options are wrong: (B),(C),(D) misread either the highest derivative or its power.

Final Answer: Order 3, degree $2 \Rightarrow \boxed{A}$

Answer: (A) Go Back to Q8

Q9.

Solution

Concept — Magnitude of a vector: For $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

Step 1 — Square the components:

$$4^2 + (-3)^2 = 16 + 9.$$

$$= 25.$$

Step 2 — Take the square root:

$$|\vec{a}| = \sqrt{25} = 5.$$

Why other options are wrong: (A) 7 adds the components $4+3$; (C) 25 skips the square root; (D) 1 subtracts the components.

Final Answer: $|\vec{a}| = 5 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q9

Q10.

Solution

Concept — Direction cosines: Direction cosines are the direction ratios divided by their magnitude $\sqrt{a^2 + b^2 + c^2}$.

Step 1 — Magnitude of ratios $3, 4, 0$:

$$\sqrt{3^2 + 4^2 + 0^2} = \sqrt{9 + 16 + 0} = \sqrt{25} = 5.$$

Step 2 — Divide each ratio by 5:

$$\left(\frac{3}{5}, \frac{4}{5}, 0\right).$$

Why other options are wrong: (A) are the ratios themselves; (B) divides by 25; (D) divides by 7.

Final Answer: Direction cosines $\left(\frac{3}{5}, \frac{4}{5}, 0\right) \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q10

Q11.

Solution

Concept — Fundamental theorem of calculus: $\int_a^b f(x)\,dx = F(b) - F(a)$, where $F' = f$.

Step 1 — Antiderivative: $\int x^2\,dx = \frac{x^3}{3}$.

Step 2 — Apply limits 0 to 1:

$$\left[\frac{x^3}{3}\right]_0^1 = \frac{1^3}{3} - \frac{0^3}{3}.$$

$$= \frac{1}{3} - 0.$$

$$= \frac{1}{3}.$$

Why other options are wrong: (B) $\frac{1}{2}$ is $\int_0^1 x\,dx$; (C) 1 ignores the power; (D) $\frac{2}{3}$ misplaces the constant.

Final Answer: The integral equals $\dfrac{1}{3} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q11

Q12.

Solution

Concept — Conditional probability: $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)}$, provided $P(A) \neq 0$.

Step 1 — Identify the values: $P(A \cap B) = 0.18$ and $P(A) = 0.6$.

Step 2 — Substitute:

$$P(B \mid A) = \frac{0.18}{0.6}.$$

Step 3 — Simplify:

$$= \frac{18}{60} = 0.3.$$

Why other options are wrong: (A) 0.108 multiplies instead of dividing; (B) 0.42 subtracts; (C) 0.5 inverts the ratio.

Final Answer: $P(B \mid A) = 0.3 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q12

Q13.

Solution

Concept — Integration by substitution: If a factor is the derivative of another expression present, substitute to simplify.

Step 1 — Substitute: Let $u = \ln x$, so $du = \dfrac{1}{x}\,dx$.

Step 2 — Rewrite the integral:

$$\int \frac{1}{x \ln x}\,dx = \int \frac{1}{\ln x} \cdot \frac{1}{x}\,dx = \int \frac{1}{u}\,du.$$

Step 3 — Integrate:

$$= \ln|u| + C = \ln|\ln x| + C.$$

Why other options are wrong: (A) $\ln x$ ignores the extra factor; (C) is $\int \frac{\ln x}{x}\,dx$; (D) is a wrong antiderivative.

Final Answer: $\ln|\ln x| + C \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q13

Q14.

Solution

Concept — Integral of an even function over a symmetric interval: If $g(-x) = g(x)$, then $\int_{-a}^{a} g(x)\,dx = 2\int_{0}^{a} g(x)\,dx$.

Step 1 — Test the integrand: $g(x) = x^2$ gives $g(-x) = (-x)^2 = x^2 = g(x)$.

So x^2 is an even function.

Step 2 — Apply the property:

$$\int_{-2}^{2} x^2\,dx = 2\int_{0}^{2} x^2\,dx.$$

Step 3 — Integrate:

$$= 2\left[\frac{x^3}{3}\right]_0^2.$$

$$= 2 \cdot \frac{8}{3}.$$

$$= \frac{16}{3}.$$

Why other options are wrong: (A) 0 is the value for an odd function; (B) $\frac{8}{3}$ is only the half-integral; (D) 8 drops the factor $\frac{1}{3}$.

Final Answer: The integral equals $\frac{16}{3} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q14

Q15.

Solution

Concept — Local maximum via second derivative: At a critical point where $f'(x) = 0$, a local maximum occurs if $f''(x) < 0$.

Step 1 — Critical points:

$$f(x) = x + \frac{1}{x}.$$

$$f'(x) = 1 - \frac{1}{x^2} = 0.$$

$$x^2 = 1 \;\Rightarrow\; x = \pm 1.$$

Step 2 — Second-derivative test:

$$f''(x) = \frac{2}{x^3}.$$

At $x = -1$, $f''(-1) = \frac{2}{-1} = -2 < 0$, so $x = -1$ gives a local maximum.

Step 3 — Maximum value:

$$f(-1) = -1 + \frac{1}{-1} = -1 - 1 = -2.$$

Why other options are wrong: (B) 2 is the local minimum value at $x = 1$; (C),(D) are not extreme values.

Final Answer: Local maximum value is $-2 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q15

Q16.

Solution

Concept — Area of a triangle from two side vectors: Area $= \frac{1}{2}|\vec{a} \times \vec{b}|$.

Step 1 — Compute the cross product:

$$\vec{a} \times \vec{b} = (4\hat{\imath}) \times (5\hat{\jmath}) = 20\,(\hat{\imath} \times \hat{\jmath}) = 20\hat{k}.$$

Step 2 — Take half the magnitude:

$$\frac{1}{2}|\vec{a} \times \vec{b}| = \frac{1}{2}|20\hat{k}| = \frac{1}{2} \times 20 = 10.$$

Why other options are wrong: (A) 20 is $|\vec{a} \times \vec{b}|$, the parallelogram area, not the triangle; (C) 40 doubles the parallelogram area; (D) 5 halves the wrong quantity.

Final Answer: Area of triangle $= 10 \Rightarrow \boxed{\text{B}}$

$\boxed{\text{Answer: (B)}}$ Go Back to Q16

Q17.

Solution

Concept — Parallel lines: Two lines are parallel when their direction ratios are proportional; the angle between them is $0°$.

Step 1 — Check proportionality: Compare $1, 2, -1$ with $2, 4, -2$.

$$\frac{2}{1} = 2, \qquad \frac{4}{2} = 2, \qquad \frac{-2}{-1} = 2.$$

Step 2 — Interpret: All ratios equal 2, so the second set of direction ratios is exactly 2 times the first; the lines are parallel, giving an angle of $0°$.

Why other options are wrong: (A) $90°$ needs a zero dot product; (C),(D) would need non-proportional ratios.

Final Answer: The angle is $0° \Rightarrow \boxed{\text{B}}$

$\boxed{\text{Answer: (B)}}$ Go Back to Q17

Q18.

Solution

Concept — Complements of independent events: If A, B are independent, so are A', B', and $P(A' \cap B') = P(A')P(B')$.

Step 1 — Complement probabilities:

$$P(A') = 1 - \frac{1}{3} = \frac{2}{3}.$$

$$P(B') = 1 - \frac{1}{4} = \frac{3}{4}.$$

Step 2 — Multiply:

$$P(A' \cap B') = \frac{2}{3} \cdot \frac{3}{4}.$$
$$= \frac{6}{12} = \frac{1}{2}.$$

Why other options are wrong: (A) $\frac{1}{12}$ is $P(A \cap B)$; (B) $\frac{7}{12}$ is $P(A' \cap B')$ computed with an error; (C) $\frac{5}{12}$ mixes complements.

Final Answer: $P(A' \cap B') = \frac{1}{2} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q18

Q19.

Solution

Concept — Assertion–Reason analysis: Judge each statement's truth, then decide whether R correctly explains A.

Step 1 — Assertion: If f is even and differentiable, its derivative f' satisfies $f'(-x) = -f'(x)$, i.e. f' is odd. So A is **true**.

Step 2 — Reason: Start from $f(-x) = f(x)$ and differentiate both sides with respect to x. The chain rule on the left gives $\frac{d}{dx}f(-x) = -f'(-x)$, while the right gives $f'(x)$.

$$-f'(-x) = f'(x) \;\Rightarrow\; f'(-x) = -f'(x).$$

So R is a **true** statement.

Step 3 — Does R explain A? The relation $f'(-x) = -f'(x)$ is precisely the definition of f' being odd, so R directly justifies A. Hence R is the correct explanation.

Why other options are wrong: (B) denies the explanation, which is valid; (C),(D) misjudge a truth value.

Final Answer: Both true and R correctly explains A $\Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q19

Q20.

Solution

Concept — Determinant of a skew-symmetric matrix: For a skew-symmetric matrix, $A^T = -A$, so $|A| = |A^T| = |-A| = (-1)^n|A|$ for order n.

Step 1 — Test the Assertion: For order $n = 3$ (odd), $(-1)^3 = -1$, so $|A| = -|A|$, giving $2|A| = 0$, i.e. $|A| = 0$. So A is **true**.

Step 2 — Test the Reason: The result $|A| = 0$ holds only for *odd* order. For even order it can fail; for example $\begin{vmatrix} 0 & 1 \\ -1 & 0 \end{vmatrix} = 1 \neq 0$. So the claim "any order" is **false**.

Why other options are wrong: (A),(B) require R true; (D) requires A false.

Final Answer: A true, R false $\Rightarrow \boxed{\text{C}}$

$\boxed{\text{Answer: (C)}}$ **Go Back to Q20**

Q21.

Solution

Concept — Principal value branches: $\cos^{-1}$ has range $[0, \pi]$ and $\sin^{-1}$ has range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$; reduce the angle to the correct branch.

Step 1 — Evaluate $\cos^{-1}\left(\cos\frac{2\pi}{3}\right)$: Since $\frac{2\pi}{3} \in [0, \pi]$,

$$\cos^{-1}\left(\cos\frac{2\pi}{3}\right) = \frac{2\pi}{3}.$$

Step 2 — Evaluate $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$: Here $\frac{2\pi}{3} \notin \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, so reduce it.

$$\sin\frac{2\pi}{3} = \sin\left(\pi - \frac{\pi}{3}\right) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}.$$

$$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}.$$

Step 3 — Add:

$$\frac{2\pi}{3} + \frac{\pi}{3} = \frac{3\pi}{3} = \pi.$$

Final Answer: $\cos^{-1}\left(\cos\frac{2\pi}{3}\right) + \sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \pi$. **Go Back to Q21**

Q22.

Solution

Concept — Logarithmic differentiation: Used when the variable appears in both base and exponent.

Step 1 — Take natural log of both sides:

$$y = x^{\tan x}.$$

$$\ln y = \tan x \cdot \ln x.$$

Step 2 — Differentiate both sides w.r.t. x: (product rule on the right)

$$\frac{1}{y}\frac{dy}{dx} = \sec^2 x \cdot \ln x + \tan x \cdot \frac{1}{x}.$$

Step 3 — Solve for $\frac{dy}{dx}$:

$$\frac{dy}{dx} = y\left[\sec^2 x\ \ln x + \frac{\tan x}{x}\right].$$

$$= x^{\tan x}\left[\sec^2 x\ \ln x + \frac{\tan x}{x}\right].$$

Final Answer: $\frac{dy}{dx} = x^{\tan x}\left[\sec^2 x\ \ln x + \frac{\tan x}{x}\right]$. Go Back to Q22

Q23.

Solution

Concept — Integration by parts: $\int u\,dv = uv - \int v\,du$, choosing u by the ILATE rule.

Step 1 — Choose parts: Let $u = x$ and $dv = \cos x\,dx$.

Then $du = dx$ and $v = \sin x$.

Step 2 — Apply the formula:

$$\int x\cos x\,dx = x\sin x - \int \sin x\,dx.$$

$$= x\sin x - (-\cos x) + C.$$

$$= x\sin x + \cos x + C.$$

Final Answer: $\int x\cos x\,dx = x\sin x + \cos x + C.$

OR — $\int \sin^2 x\,dx$**:**

Step 1 — Use the identity $\sin^2 x = \frac{1-\cos 2x}{2}$**:**

$$\int \sin^2 x\,dx = \int \frac{1-\cos 2x}{2}\,dx.$$

Step 2 — Integrate term by term:

$$= \frac{1}{2}\int 1\,dx - \frac{1}{2}\int \cos 2x\,dx.$$

$$= \frac{x}{2} - \frac{1}{2}\cdot\frac{\sin 2x}{2} + C.$$

$$= \frac{x}{2} - \frac{\sin 2x}{4} + C.$$

Final Answer (OR): $\int \sin^2 x\,dx = \frac{x}{2} - \frac{\sin 2x}{4} + C.$ **Go Back to Q23**

Q24.

Solution

Concept — Parallel vectors: $\vec{a} \parallel \vec{b}$ when their corresponding components are proportional, i.e. $\vec{b} = k\vec{a}$ for some scalar k.

Step 1 — Write the proportionality:

$$\frac{4}{2} = \frac{6}{3} = \frac{-4}{\lambda}.$$

Step 2 — Evaluate the known ratios:

$$\frac{4}{2} = 2, \qquad \frac{6}{3} = 2.$$

So the common ratio is $k = 2$.

Step 3 — Solve for λ:

$$\frac{-4}{\lambda} = 2.$$

$$-4 = 2\lambda.$$

$$\lambda = -2.$$

Final Answer: $\lambda = -2$. **Go Back to Q24**

Q25.

Solution

Concept — Addition rule of probability: $P(E \cup F) = P(E) + P(F) - P(E \cap F)$.

Step 1 — Individual probabilities: In a pack of 52 cards there are 4 kings and 13 hearts.

$$P(\text{King}) = \frac{4}{52}, \qquad P(\text{Heart}) = \frac{13}{52}.$$

Step 2 — Overlap: The king of hearts is counted in both, so

$$P(\text{King} \cap \text{Heart}) = \frac{1}{52}.$$

Step 3 — Apply the addition rule:

$$P(\text{King} \cup \text{Heart}) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52}.$$

$$= \frac{16}{52} = \frac{4}{13}.$$

Final Answer: $P(\text{King or Heart}) = \frac{4}{13}$.

OR — $P(A \mid B)$**:**

Step 1 — Formula: $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$.

Step 2 — Substitute:

$$P(A \mid B) = \frac{0.25}{0.4} = \frac{25}{40} = \frac{5}{8}.$$

Final Answer (OR): $P(A \mid B) = \frac{5}{8}$. **Go Back to Q25**

Q26.

Solution

Concept — Partial fractions: Split a proper rational function into simpler fractions, then integrate each.

Step 1 — Set up the decomposition:

$$\frac{2x+3}{(x+1)(x+3)} = \frac{A}{x+1} + \frac{B}{x+3}.$$

$$2x + 3 = A(x+3) + B(x+1).$$

Step 2 — Find A and B:

Put $x = -1$: $2(-1) + 3 = A(2) \Rightarrow 1 = 2A \Rightarrow A = \frac{1}{2}$.

Put $x = -3$: $2(-3) + 3 = B(-2) \Rightarrow -3 = -2B \Rightarrow B = \frac{3}{2}$.

Step 3 — Integrate:

$$\int \frac{2x+3}{(x+1)(x+3)}\,dx = \int \left(\frac{1/2}{x+1} + \frac{3/2}{x+3}\right) dx.$$

$$= \frac{1}{2}\ln|x+1| + \frac{3}{2}\ln|x+3| + C.$$

Final Answer: $\frac{1}{2}\ln|x+1| + \frac{3}{2}\ln|x+3| + C$. **Go Back to Q26**

Q27.

Solution

Concept — Linear differential equation: For $\frac{dy}{dx} + Py = Q$, the integrating factor is I.F. $= e^{\int P\,dx}$ and the solution is $y \cdot \text{I.F.} = \int Q \cdot \text{I.F.}\,dx$.

Step 1 — Identify P, Q: Here $P = 2$, $Q = e^{-x}$.

Step 2 — Integrating factor:

$$\text{I.F.} = e^{\int 2\,dx} = e^{2x}.$$

Step 3 — Solve:

$$y\,e^{2x} = \int e^{-x} \cdot e^{2x}\,dx = \int e^{x}\,dx.$$

$$y\,e^{2x} = e^{x} + C.$$

$$y = e^{-x} + C\,e^{-2x}.$$

Final Answer: $y = e^{-x} + C\,e^{-2x}$.

OR — Variables separable $\frac{dy}{dx} = e^{x-y}$:

Step 1 — Rewrite and separate:

$$\frac{dy}{dx} = e^{x}\,e^{-y}.$$

$$e^{y}\,dy = e^{x}\,dx.$$

Step 2 — Integrate both sides:

$$e^{y} = e^{x} + C.$$

Final Answer (OR): $e^{y} = e^{x} + C$. **Go Back to Q27**

Q28.

Solution

Concept — Inverse by adjoint: $A^{-1} = \frac{1}{|A|}\text{adj}(A)$, valid when $|A| \neq 0$.

Step 1 — Determinant:

$$|A| = \begin{vmatrix} 2 & 1 \\ 5 & 3 \end{vmatrix} = (2)(3) - (1)(5) = 6 - 5 = 1.$$

Step 2 — Adjoint of a 2×2 matrix: Swap the leading diagonal entries and negate the off-diagonal entries:

$$\text{adj}(A) = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}.$$

Step 3 — Inverse:

$$A^{-1} = \frac{1}{1}\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}.$$

Final Answer: $A^{-1} = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$. **Go Back to Q28**

Q29.

Solution

Concept — Monotonicity: f increases where $f'(x) > 0$ and decreases where $f'(x) < 0$.

Step 1 — Differentiate:

$$f(x) = x^3 - 3x^2 - 9x + 7.$$

$$f'(x) = 3x^2 - 6x - 9.$$

$$= 3(x^2 - 2x - 3) = 3(x - 3)(x + 1).$$

Step 2 — Critical points: $f'(x) = 0$ at $x = -1$ and $x = 3$.

Step 3 — Sign analysis:

For $x < -1$: $f' > 0$; for $-1 < x < 3$: $f' < 0$; for $x > 3$: $f' > 0$.

Conclusion: Increasing on $(-\infty, -1) \cup (3, \infty)$; decreasing on $(-1, 3)$.

Final Answer: Increasing on $(-\infty, -1) \cup (3, \infty)$, decreasing on $(-1, 3)$.

OR — Tangent to $y = x^3$ at $(1, 1)$:

Step 1 — Slope: $\frac{dy}{dx} = 3x^2$, so at $x = 1$ slope $= 3(1)^2 = 3$.

Step 2 — Equation: $y - 1 = 3(x - 1) \Rightarrow y = 3x - 2$.

Final Answer (OR): $y = 3x - 2$. **Go Back to Q29**

Q30.

Solution

Concept — Vector equation of a line: A line through point $\vec{a}$ parallel to $\vec{b}$ is $\vec{r} = \vec{a} + \lambda\vec{b}$.

Step 1 — Write the equation: With $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} - \hat{k}$,

$$\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} - \hat{k}).$$

Step 2 — Test the point $(4, 2, 4)$: The point lies on the line only if the same λ satisfies all three component equations.

$$2 + 2\lambda = 4 \;\Rightarrow\; \lambda = 1.$$

$$-1 + 3\lambda = 2 \;\Rightarrow\; \lambda = 1.$$

$$3 - \lambda = 4 \;\Rightarrow\; \lambda = -1.$$

Step 3 — Conclusion: The first two equations give $\lambda = 1$, but the third gives $\lambda = -1$. Since the values disagree, the point $(4, 2, 4)$ does *not* lie on the line.

Final Answer: $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$; the point $(4, 2, 4)$ does not lie on it. **Go Back to Q30**

Q31.

Solution

Concept — One-one (injective) and onto (surjective): *One-one*: $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$. *Onto*: every y in the codomain has a pre-image.

Step 1 — One-one: Suppose $f(x_1) = f(x_2)$.

$$3x_1 - 2 = 3x_2 - 2.$$

$$3x_1 = 3x_2.$$

$$x_1 = x_2.$$

Hence f is one-one.

Step 2 — Onto: Let $y \in \mathbb{R}$ be arbitrary. Solve $y = 3x - 2$ for x:

$$x = \frac{y+2}{3} \in \mathbb{R}.$$

Then $f(x) = 3\left(\frac{y+2}{3}\right) - 2 = y + 2 - 2 = y.$
So every y has a pre-image; f is onto.

Step 3 — Conclusion: f is both one-one and onto, hence bijective.

Final Answer: $f(x) = 3x - 2$ is one-one and onto (a bijection). Go Back to Q31

Q32.

Solution

Concept — Matrix method: Write $AX = B$; then $X = A^{-1}B$ when $|A| \neq 0$.

Step 1 — Form the matrices:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 1 & 2 & -1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}.$$

Step 2 — Determinant of A:

$$|A| = 1\big[(-1)(-1) - (1)(2)\big] - 1\big[(2)(-1) - (1)(1)\big] + 1\big[(2)(2) - (-1)(1)\big].$$

$$= 1(1-2) - 1(-2-1) + 1(4+1).$$

$$= -1 + 3 + 5 = 7 \ (\neq 0).$$

Step 3 — Cofactors and adjoint: Computing the cofactor matrix and transposing gives

$$\text{adj}(A) = \begin{bmatrix} -1 & 3 & 2 \\ 3 & -2 & 1 \\ 5 & -1 & -3 \end{bmatrix}.$$

Step 4 — Solve $X = \dfrac{1}{|A|}\,\text{adj}(A)\,B$:

$$X = \frac{1}{7}\begin{bmatrix} -1 & 3 & 2 \\ 3 & -2 & 1 \\ 5 & -1 & -3 \end{bmatrix}\begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}.$$

$$= \frac{1}{7}\begin{bmatrix} (-1)(2) + 3(-1) + 2(6) \\ 3(2) + (-2)(-1) + 1(6) \\ 5(2) + (-1)(-1) + (-3)(6) \end{bmatrix}.$$

$$= \frac{1}{7}\begin{bmatrix} -2 - 3 + 12 \\ 6 + 2 + 6 \\ 10 + 1 - 18 \end{bmatrix} = \frac{1}{7}\begin{bmatrix} 7 \\ 14 \\ -7 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}.$$

Step 5 — Verify: $x + y + z = 1 + 2 - 1 = 2\ \checkmark$; $2x - y + z = 2 - 2 - 1 = -1\ \checkmark$; $x + 2y - z = 1 + 4 + 1 = 6\ \checkmark$.

Final Answer: $x = 1,\ y = 2,\ z = -1$. **Go Back to Q32**

Q33.

Solution

Concept — Area by integration: Area between a curve above the x-axis and the axis is $\displaystyle\int_a^b y\,dx$ between the intercepts.

Step 1 — Find the x-intercepts: Put $y = 0$ in $y = 4 - x^2$.

$$4 - x^2 = 0 \ \Rightarrow\ x = \pm 2.$$

Step 2 — Set up the integral (the curve is even, so double the right half):

$$\text{Area} = \int_{-2}^{2} (4 - x^2)\,dx = 2\int_0^2 (4 - x^2)\,dx.$$

Step 3 — Integrate:

$$= 2\left[4x - \frac{x^3}{3}\right]_0^2.$$

$$= 2\left[4(2) - \frac{2^3}{3}\right].$$

$$= 2\left[8 - \frac{8}{3}\right].$$

$$= 2 \cdot \frac{24-8}{3} = 2 \cdot \frac{16}{3} = \frac{32}{3}.$$

Final Answer: Area $= \dfrac{32}{3}$ square units.

OR — Region between $y = x^2$ and $y = 2x$:

Step 1 — Points of intersection: $x^2 = 2x \Rightarrow x^2 - 2x = 0 \Rightarrow x(x-2) = 0 \Rightarrow x = 0,\ 2.$

Step 2 — Top minus bottom: On $[0, 2]$, $2x \geq x^2$, so

$$\text{Area} = \int_0^2 (2x - x^2)\, dx.$$

Step 3 — Integrate:

$$= \left[x^2 - \frac{x^3}{3}\right]_0^2 = 4 - \frac{8}{3} = \frac{12-8}{3} = \frac{4}{3}.$$

Final Answer (OR): Area $= \dfrac{4}{3}$ square units. **Go Back to Q33**

Q34.

Solution

Concept — Corner-point method: The optimum of a linear objective over a bounded feasible region occurs at a corner (vertex).

Step 1 — Find the corner points: The constraints $x+2y \leq 14$, $3x+y \leq 18$, $x \geq 0$, $y \geq 0$ bound a region with vertices $O(0,0)$, $(6,0)$ and $(0,7)$. The lines $x + 2y = 14$ and $3x + y = 18$ intersect: from the first, $x = 14 - 2y$; substitute into the second,

$$3(14-2y) + y = 18 \ \Rightarrow\ 42 - 6y + y = 18 \ \Rightarrow\ -5y = -24 \ \Rightarrow\ y = \frac{24}{5} = 4.8.$$

Then $x = 14 - 2(4.8) = 4.4$, so the fourth vertex is $(4.4, 4.8)$.

Step 2 — Evaluate $Z = x + 4y$ at each corner:

$$Z(0,0) = 0.$$

$$Z(6,0) = 6 + 4(0) = 6.$$

$$Z(4.4, 4.8) = 4.4 + 4(4.8) = 4.4 + 19.2 = 23.6.$$

$$Z(0,7) = 0 + 4(7) = 28.$$

Step 3 — Pick the maximum: The largest value is 28 at $(0, 7)$.

Final Answer: Z is maximum $= 28$ at $(x, y) = (0, 7)$. **Go Back to Q34**

Q35.

Solution

Concept — Shortest distance between skew lines: For $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$,

$$d = \frac{\left|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)\right|}{|\vec{b}_1 \times \vec{b}_2|}.$$

Step 1 — Identify vectors: $\vec{a}_1 = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{b}_1 = \hat{i} - \hat{j} + \hat{k}$; $\vec{a}_2 = 2\hat{i} - \hat{j} - \hat{k}$, $\vec{b}_2 = 2\hat{i} + \hat{j} + 2\hat{k}$.

Step 2 — Cross product $\vec{b}_1 \times \vec{b}_2$:

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix}.$$

$$= \hat{i}\big[(-1)(2) - (1)(1)\big] - \hat{j}\big[(1)(2) - (1)(2)\big] + \hat{k}\big[(1)(1) - (-1)(2)\big].$$

$$= \hat{i}(-2 - 1) - \hat{j}(2 - 2) + \hat{k}(1 + 2).$$

$$= -3\hat{i} - 0\hat{j} + 3\hat{k}.$$

Step 3 — Its magnitude:

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{(-3)^2 + 0^2 + 3^2} = \sqrt{9 + 0 + 9} = \sqrt{18} = 3\sqrt{2}.$$

Step 4 — $(\vec{a}_2 - \vec{a}_1)$ and the dot product:

$$\vec{a}_2 - \vec{a}_1 = (2 - 1)\hat{i} + (-1 - 2)\hat{j} + (-1 - 1)\hat{k} = \hat{i} - 3\hat{j} - 2\hat{k}.$$

$$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = (1)(-3) + (-3)(0) + (-2)(3) = -3 + 0 - 6 = -9.$$

Step 5 — Distance:

$$d = \frac{|-9|}{3\sqrt{2}} = \frac{9}{3\sqrt{2}} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2} \text{ units.}$$

Final Answer: Shortest distance $= \dfrac{3\sqrt{2}}{2}$ units.

OR — Angle between two planes:

Step 1 — Normals: For $2x - y + 2z = 5$, $\vec{n}_1 = 2\hat{i} - \hat{j} + 2\hat{k}$; for $x + y + z = 3$, $\vec{n}_2 = \hat{i} + \hat{j} + \hat{k}$.

Step 2 — Use $\cos\theta = \dfrac{|\vec{n}_1 \cdot \vec{n}_2|}{|\vec{n}_1||\vec{n}_2|}$:

$$\vec{n}_1 \cdot \vec{n}_2 = (2)(1) + (-1)(1) + (2)(1) = 2 - 1 + 2 = 3.$$

$$|\vec{n}_1| = \sqrt{4+1+4} = 3, \qquad |\vec{n}_2| = \sqrt{3}.$$

$$\cos\theta = \frac{3}{3\sqrt{3}} = \frac{1}{\sqrt{3}}.$$

Final Answer (OR): $\theta = \cos^{-1}\left(\dfrac{1}{\sqrt{3}}\right)$. **Go Back to Q35**

Q36.

Solution

Concept — Minima by derivatives: Build the surface-area function $S(r)$, set $\dfrac{dS}{dr} = 0$, and confirm a genuine minimum with the second derivative.

(i) Surface-area function: A closed cylinder has two circular ends and a curved side, so

$$S = 2\pi r^2 + 2\pi r h.$$

From $V = \pi r^2 h$ we get $h = \dfrac{V}{\pi r^2}$. Substitute:

$$S = 2\pi r^2 + 2\pi r \cdot \frac{V}{\pi r^2} = 2\pi r^2 + \frac{2V}{r}.$$

(ii) Differentiate:

$$\frac{dS}{dr} = 4\pi r - \frac{2V}{r^2}.$$

(iii) Critical point and minimum: Set $\dfrac{dS}{dr} = 0$:

$$4\pi r = \frac{2V}{r^2} \;\Rightarrow\; 4\pi r^3 = 2V \;\Rightarrow\; r^3 = \frac{V}{2\pi} \;\Rightarrow\; r = \sqrt[3]{\frac{V}{2\pi}}.$$

Second derivative: $\dfrac{d^2S}{dr^2} = 4\pi + \dfrac{4V}{r^3} > 0$, so this r gives a minimum.

For the height, $h = \dfrac{V}{\pi r^2}$ and $V = 2\pi r^3$, so

$$h = \frac{2\pi r^3}{\pi r^2} = 2r.$$

Final Answer: $S = 2\pi r^2 + \dfrac{2V}{r}$; $\dfrac{dS}{dr} = 4\pi r - \dfrac{2V}{r^2}$; minimum at $r = \sqrt[3]{\dfrac{V}{2\pi}}$, where $h = 2r$. **Go Back to Q36**

Q37.

Solution

Concept — Bayes' theorem: $P(E_i \mid D) = \dfrac{P(E_i)P(D \mid E_i)}{\sum_j P(E_j)P(D \mid E_j)}$.

(i) Prior probabilities: Blue Cabs run 70% and Yellow Cabs run 30% of the rides, so

$$P(\text{Blue}) = 0.7, \qquad P(\text{Yellow}) = 0.3.$$

(ii) Conditional (complaint) probabilities:

$$P(\text{Late} \mid \text{Blue}) = 0.04, \qquad P(\text{Late} \mid \text{Yellow}) = 0.14.$$

(iii) Apply Bayes' theorem:

$$P(\text{Yellow} \mid \text{Late}) = \frac{P(\text{Yellow})P(\text{Late} \mid \text{Yellow})}{P(\text{Blue})P(\text{Late} \mid \text{Blue}) + P(\text{Yellow})P(\text{Late} \mid \text{Yellow})}.$$

$$= \frac{(0.3)(0.14)}{(0.7)(0.04) + (0.3)(0.14)}.$$

$$= \frac{0.042}{0.028 + 0.042}.$$

$$= \frac{0.042}{0.070} = \frac{42}{70} = \frac{3}{5}.$$

Final Answer: $P(\text{Yellow} \mid \text{Late}) = \dfrac{3}{5}$. **Go Back to Q37**

Q38.

Solution

Concept — Marginal cost: Marginal cost is the derivative of total cost, $\text{MC} = \dfrac{dC}{dx}$; its minimum is found by setting $\dfrac{d(\text{MC})}{dx} = 0$.

(i) Marginal cost function:

$$C(x) = x^3 - 12x^2 + 60x + 100.$$

$$\text{MC} = \frac{dC}{dx} = 3x^2 - 24x + 60.$$

(ii) Marginal cost at $x = 5$:

$$\text{MC}(5) = 3(5)^2 - 24(5) + 60.$$

$$= 75 - 120 + 60.$$

$$= 15.$$

(iii) Minimum marginal cost: Differentiate MC and set it to zero.

$$\frac{d(\text{MC})}{dx} = 6x - 24 = 0.$$

$$x = 4.$$

Since $\dfrac{d^2(\text{MC})}{dx^2} = 6 > 0$, this gives a minimum. Then

$$\text{MC}(4) = 3(4)^2 - 24(4) + 60 = 48 - 96 + 60 = 12.$$

Final Answer: $\text{MC} = 3x^2 - 24x + 60$; $\text{MC}(5) = 15$ rupees; minimum marginal cost is 12 rupees at $x = 4$ units. **Go Back to Q38**

Answer Key – Section A (Q1–Q20)

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	C	3	A	4	D	5	D
6	D	7	C	8	A	9	B	10	C
11	A	12	D	13	B	14	C	15	A
16	B	17	B	18	D	19	A	20	C

Sections B–E are descriptive; refer to the Detailed Solutions above for full model answers and step marking.