CBSE Class 12 Mathematics

Sample Paper – 7

Duration: 180 Minutes **Maximum Marks: 80**

General Instructions

- This question paper contains **38 questions**. **All questions are compulsory**.
- The paper is divided into **five sections** – A, B, C, D and E.
- **Section A** (Q1–Q20) carries **1 mark** each: Q1–Q18 are multiple choice questions and Q19–Q20 are Assertion–Reason questions.
- **Section B** (Q21–Q25) carries **2 marks** each (Very Short Answer).
- **Section C** (Q26–Q31) carries **3 marks** each (Short Answer).
- **Section D** (Q32–Q35) carries **5 marks** each (Long Answer).
- **Section E** (Q36–Q38) carries **4 marks** each (case study based, with sub-parts).
- There is **no overall choice**, but an **internal choice** has been provided in some questions. Attempt only one of the alternatives in such questions.
- There is **no negative marking**. Use of a **calculator is not permitted**.

Section A (Q1–Q20) – 1 Mark Each

Q1. In a skew-symmetric matrix, every element on the main diagonal is:

(A) 0

(B) 1

(C) equal to one another and non-zero

(D) not defined

Q2. The value of the determinant $\begin{vmatrix} 4 & -1 \\ 2 & 3 \end{vmatrix}$ is:

(A) 10

(B) 14

(C) 12

(D) -14

Q3. Let $A = \{1, 2, 3\}$ and $R = \{(1,1), (2,2), (3,3), (1,2), (2,1), (2,3), (3,2)\}$ be a relation on A. Then R is:

(A) reflexive and transitive but not symmetric

(B) an equivalence relation

(C) reflexive and symmetric but not transitive

(D) symmetric and transitive but not reflexive

Q4. The principal value of $\sec^{-1}(2)$ is:

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{4}$

(C) $\frac{2\pi}{3}$

(D) $\frac{\pi}{3}$

Q5. If $y = \ln(\sec x)$, then $\frac{dy}{dx}$ equals:

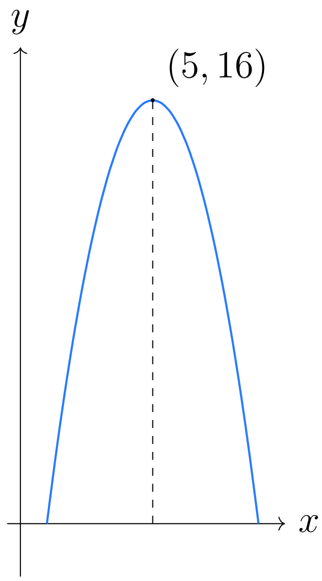
(A) $\sec x$

(B) $\tan x$

(C) $\cot x$

(D) $\sec x \tan x$

Q6. The graph of $f(x) = -x^2 + 10x - 9$ is shown below. The function is *increasing* on the interval:



(A) $(-\infty, 5)$

(B) $(5, \infty)$

(C) $(-\infty, \infty)$

(D) $(0, 5)$

Q7. $\int \frac{1}{\sqrt{1-x^2}}\, dx$ equals:

(A) $\cos^{-1} x + C$

(B) $\tan^{-1} x + C$

(C) $\sec^{-1} x + C$

(D) $\sin^{-1} x + C$

Q8. The order and degree of the differential equation $\frac{d^3y}{dx^3} + x\left(\frac{d^2y}{dx^2}\right)^2 + y = 0$ are respectively:

(A) 2 and 2

(B) 2 and 1

(C) 3 and 1

(D) 3 and 2

Q9. The magnitude of the vector $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ is:

(A) 5

(B) 7

(C) 11

(D) $\sqrt{41}$

Q10. The direction cosines of a line whose direction ratios are $2, 3, -6$ are:

(A) $2, 3, -6$

(B) $\frac{2}{49}, \frac{3}{49}, -\frac{6}{49}$

(C) $\frac{2}{\sqrt{7}}, \frac{3}{\sqrt{7}}, -\frac{6}{\sqrt{7}}$

(D) $\frac{2}{7}, \frac{3}{7}, -\frac{6}{7}$

Q11. $\int_1^e \frac{1}{x}\,dx$ equals:

(A) 1

(B) 0

(C) e

(D) $e-1$

Q12. If $P(A \cap B) = 0.18$ and $P(A) = 0.6$, then $P(B \mid A)$ equals:

(A) 0.108

(B) 0.3

(C) 0.5

(D) 0.4

Q13. $\int \frac{e^x}{1+e^x}\,dx$ equals:

(A) $\frac{e^x}{1+e^x} + C$

(B) $\ln(e^x) + C$

(C) $\ln(1+e^x) + C$

(D) $\tan^{-1}(e^x) + C$

Q14. $\int_{-2}^{2} x^2\,dx$ equals:

(A) 0

(B) $\frac{8}{3}$

(C) $\frac{32}{3}$

(D) $\frac{16}{3}$

Q15. The local minimum value of $f(x) = x^3 - 12x$ is:

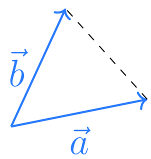
(A) 16

(B) -16

(C) -8

(D) 8

Q16. The area of the triangle whose two sides (from a common vertex) are represented by the vectors $\vec{a} = 2\hat{i} + 2\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 2\hat{k}$ (shown below) is:



(A) $\frac{9}{2}$

(B) 9

(C) $\frac{81}{2}$

(D) $\frac{9}{4}$

Q17. The angle between two lines whose direction ratios are $1, 0, 1$ and $1, 1, 0$ is:

(A) 90°

(B) $30°$

(C) $60°$

(D) $45°$

Q18. If A and B are independent events with $P(A) = 0.3$ and $P(B) = 0.4$, then $P(A \cup B)$ equals:

(A) 0.7

(B) 0.12

(C) 0.42

(D) 0.58

Q19. **Assertion (A):** The function $f(x) = x^3$ is increasing on $\mathbb{R}$.
Reason (R): If $f'(x) \geq 0$ for every x in an interval, then f is increasing on that interval.

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is *not* the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

Q20. **Assertion (A):** For any two matrices A and B for which AB is defined, $(AB)^T = B^T A^T$.
Reason (R): For any two matrices A and B for which AB is defined, $(AB)^T = A^T B^T$.

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is *not* the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

Section B (Q21–Q25) – 2 Marks Each

Q21. Find the principal value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{1}{2}\right)$. **[2]**

Q22. If $x = a\cos t$ and $y = b\sin t$, find $\dfrac{dy}{dx}$. **[2]**

Q23. Evaluate $\displaystyle\int x\cos x\,dx$. **[2]**

OR

Evaluate $\displaystyle\int \sin^2 x\,dx$.

Q24. Find the value of λ for which the vectors $\vec{a} = 2\hat{i}+3\hat{j}+\lambda\hat{k}$ and $\vec{b} = \hat{i}-2\hat{j}+\hat{k}$ are perpendicular. **[2]**

Q25. A fair die is thrown twice. Find the probability of getting a sum of 7. **[2]**

OR

If $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cap B) = 0.2$, find $P(A \mid B)$.

Section C (Q26–Q31) – 3 Marks Each

Q26. Evaluate $\displaystyle\int \frac{1}{(x-1)(x+2)}\,dx$. **[3]**

Q27. Solve the differential equation $\dfrac{dy}{dx} + 2y = e^{-x}$. **[3]**

OR

Solve the differential equation $\dfrac{dy}{dx} = e^{x-y}$.

Q28. Find the inverse of the matrix $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$ using the adjoint method. **[3]**

Q29. Find the intervals in which $f(x) = x^3 - 3x^2 - 9x + 11$ is increasing or decreasing. **[3]**

OR

Find the equation of the tangent to the curve $y = x^3$ at the point $(1, 1)$.

Q30. Find the vector equation of the line passing through the point $(2, -1, 3)$ and parallel to the vector $\hat{i} + 2\hat{j} - \hat{k}$. Hence check whether the point $(4, 3, 1)$ lies on this line. **[3]**

Q31. Show that the function $f : \mathbb{R} \to (-1, 1)$ defined by $f(x) = \dfrac{x}{1+|x|}$ is one-one and onto. **[3]**

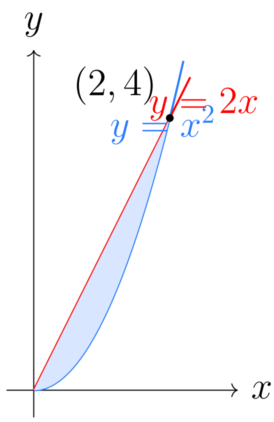
Section D (Q32–Q35) – 5 Marks Each

Q32. Using the matrix method, solve the following system of linear equations:

$$2x + y - z = -1, \qquad x - y + z = 4, \qquad 3x + 2y + z = 3.$$

[5]

Q33. Using integration, find the area of the region bounded by the parabola $y = x^2$ and the line $y = 2x$.



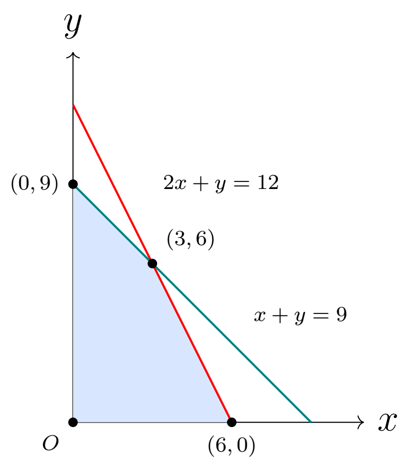
[5]

OR

Using integration, find the area of the region bounded by the curve $y = 4 - x^2$ and the x-axis.

Q34. Solve the following linear programming problem graphically. Maximise $Z = 6x + 5y$ subject to the constraints

$$x + y \le 9, \qquad 2x + y \le 12, \qquad x \ge 0,\ y \ge 0.$$



[5]

Q35. Find the shortest distance between the lines

$$\vec{r} = (\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k}) \quad \text{and} \quad \vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(2\hat{i} + 3\hat{j} + 4\hat{k}).$$

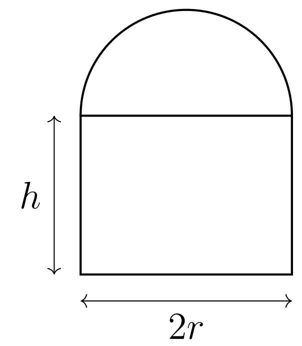
[5]

OR

Find the angle between the line $\vec{r} = (\hat{i} + \hat{j} + \hat{k}) + \lambda(2\hat{i} - \hat{j} + 2\hat{k})$ and the plane $2x + y - 2z = 5$.

Section E (Q36–Q38) – 4 Marks Each (Case Study)

Q36. Case Study – The Norman Window.
A window is in the shape of a rectangle surmounted by a semicircle (a Norman window). The width of the rectangle equals the diameter of the semicircle, so if the radius of the semicircle is r metres, the rectangle is $2r$ wide and h metres tall. The total perimeter of the window is 20 m.



Based on the above, answer the following:

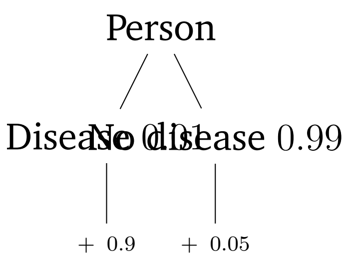
(i) Using the perimeter condition, express the total area A of the window as a function of r. (1)

(ii) Find $\dfrac{dA}{dr}$. (1)

(iii) Find the value of r for which the area of the window is maximum, and the maximum area. (2)

Q37. Case Study – A Diagnostic Test.

A certain disease is present in 1% of a population. A diagnostic test detects the disease correctly in 90% of the people who actually have it (true positive), but it also returns a positive result for 5% of the people who do not have the disease (false positive). A randomly chosen person is tested and the result is positive.



Based on the above, answer the following:

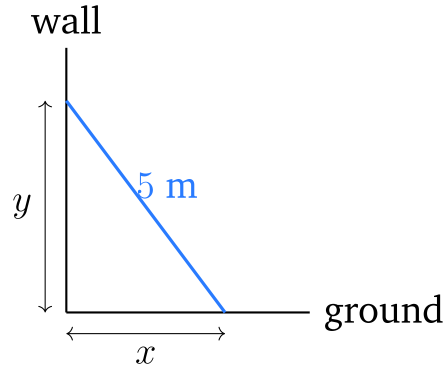
(i) Write the prior probabilities $P(\text{Disease})$ and $P(\text{No disease})$. (1)

(ii) Write $P(+ \mid \text{Disease})$ and $P(+ \mid \text{No disease})$. (1)

(iii) Using Bayes' theorem, find the probability that the person actually has the disease, given the positive result. (2)

Q38. Case Study – The Sliding Ladder.

A ladder 5 m long rests against a vertical wall. The foot of the ladder is pulled away from the wall along the ground at a constant rate of 2 m/s. Let x be the distance of the foot of the ladder from the wall and y the height of the top of the ladder above the ground.



Based on the above, answer the following:

(i) Write the relation connecting x and y. (1)

(ii) Differentiate this relation with respect to time t to relate $\frac{dx}{dt}$ and $\frac{dy}{dt}$. (1)

(iii) Find the rate at which the top of the ladder is sliding down the wall at the instant when the foot is 3 m from the wall. (2)

Detailed Solutions

Q1.

Solution

Concept — Skew-symmetric matrix: A matrix A is skew-symmetric if $A^T = -A$, i.e. $a_{ji} = -a_{ij}$ for all i, j.

Step 1 — Apply the condition to a diagonal entry: Put $i = j$. Then $a_{ii} = -a_{ii}$.

Step 2 — Solve for the diagonal entry:

$$a_{ii} = -a_{ii}.$$

$$2a_{ii} = 0.$$

$$a_{ii} = 0.$$

So every main-diagonal element is 0.

Why other options are wrong: (B) 1 and (C) a non-zero constant would violate $a_{ii} = -a_{ii}$; (D) the entries are perfectly well defined.

Final Answer: Each diagonal element is $0 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q1

Q2.

Solution

Concept — Value of a 2×2 determinant: For $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$, the value is $ad - bc$.

Step 1 — Identify entries: Here $a = 4$, $b = -1$, $c = 2$, $d = 3$.

Step 2 — Apply the formula:

$$ad - bc = (4)(3) - (-1)(2).$$

$$= 12 - (-2).$$

$$= 12 + 2.$$

$$= 14.$$

Why other options are wrong: (A) 10 comes from $12 - 2$ (wrong sign on bc); (C) 12 ignores the bc term; (D) -14 flips the overall sign.

Final Answer: The determinant is $14 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q2

Q3.

Solution

Concept — Properties of a relation: *Reflexive*: $(a,a) \in R$ for all a. *Symmetric*: $(a,b) \in R \Rightarrow (b,a) \in R$. *Transitive*: $(a,b),(b,c) \in R \Rightarrow (a,c) \in R$.

Step 1 — Reflexive? $(1,1),(2,2),(3,3) \in R$, so R is reflexive.

Step 2 — Symmetric? The off-diagonal pairs are $(1,2)\&(2,1)$ and $(2,3)\&(3,2)$; each pair and its reverse are present. So R is symmetric.

Step 3 — Transitive? $(1,2) \in R$ and $(2,3) \in R$, but $(1,3) \notin R$.

Hence R is not transitive.

Why other options are wrong: (A) fails since R is symmetric; (B) an equivalence relation must also be transitive, which fails; (D) R is reflexive, so "not reflexive" is false.

Final Answer: R is reflexive and symmetric but not transitive $\Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q3

Q4.

Solution

Concept — Principal value of $\sec^{-1}$: The principal value branch of $\sec^{-1} x$ is $[0,\pi] \setminus \left\{\frac{\pi}{2}\right\}$.

Step 1 — Set up the equation: Let $\sec^{-1}(2) = \theta$, so $\sec\theta = 2$.

Step 2 — Rewrite using cosine: $\sec\theta = 2 \Rightarrow \cos\theta = \frac{1}{2}$.

Step 3 — Find θ in the branch: We know $\cos\frac{\pi}{3} = \frac{1}{2}$, and $\frac{\pi}{3} \in [0,\pi]$.

So $\theta = \frac{\pi}{3}$.

Why other options are wrong: (A) $\frac{\pi}{6}$ and (B) $\frac{\pi}{4}$ do not give $\cos\theta = \frac{1}{2}$; (C) $\frac{2\pi}{3}$ gives $\cos\theta = -\frac{1}{2}$, i.e. $\sec\theta = -2$.

Final Answer: $\sec^{-1}(2) = \frac{\pi}{3} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q4

Q5.

Solution

Concept — Chain rule with logarithm: $\dfrac{d}{dx}\ln(u) = \dfrac{1}{u}\cdot\dfrac{du}{dx}$.

Step 1 — Identify the inner function: Let $u = \sec x$, so $\dfrac{du}{dx} = \sec x \tan x$.

Step 2 — Apply the chain rule:

$$\frac{dy}{dx} = \frac{1}{\sec x}\cdot \sec x \tan x.$$

Step 3 — Simplify:

$$= \tan x.$$

Why other options are wrong: (A) $\sec x$ and (D) $\sec x \tan x$ forget to divide by $\sec x$; (C) $\cot x$ is the derivative of $\ln(\sin x)$.

Final Answer: $\dfrac{dy}{dx} = \tan x \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q5

Q6.

Solution

Concept — Increasing function: f is increasing where $f'(x) > 0$.

Step 1 — Differentiate:

$$f(x) = -x^2 + 10x - 9.$$

$$f'(x) = -2x + 10.$$

Step 2 — Solve $f'(x) > 0$:

$$-2x + 10 > 0.$$

$$10 > 2x.$$

$$x < 5.$$

So f is increasing on $(-\infty, 5)$, which matches the left, rising branch of this downward parabola (vertex at $(5, 16)$).

Why other options are wrong: (B) $(5, \infty)$ is where $f'(x) < 0$ (decreasing); (C) the whole line is false since the parabola rises then falls; (D) $(0, 5)$ misses the part $x < 0$ where f is still increasing.

Final Answer: f increases on $(-\infty, 5) \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q6

Q7.

Solution

Concept — Standard integral: $\frac{d}{dx} \sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$, hence $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$.

Step 1 — Recall the antiderivative: The derivative of $\sin^{-1} x$ is $\frac{1}{\sqrt{1-x^2}}$.

Step 2 — Write the integral:

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C.$$

Why other options are wrong: (A) $\cos^{-1} x$ has derivative $-\frac{1}{\sqrt{1-x^2}}$ (wrong sign); (B) is $\int \frac{1}{1+x^2} dx$; (C) is $\int \frac{1}{|x|\sqrt{x^2-1}} dx$.

Final Answer: $\sin^{-1} x + C \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q7

Q8.

Solution

Concept — Order and degree: The *order* is the highest derivative present; the *degree* is the power of that highest derivative when the equation is polynomial in derivatives.

Step 1 — Highest derivative: The highest derivative is $\frac{d^3y}{dx^3}$, so the order is 3.

Step 2 — Its power: The term $\frac{d^3y}{dx^3}$ appears to the first power (the square is on $\frac{d^2y}{dx^2}$, a lower derivative).
So the degree is 1.

Why other options are wrong: (A),(B) misread the highest derivative as second order; (D) wrongly takes the power of the highest derivative as 2.

Final Answer: Order 3, degree 1 $\Rightarrow$ $\boxed{\text{C}}$

Answer: (C) Go Back to Q8

Q9.

Solution

Concept — Magnitude of a vector: For $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

Step 1 — Square the components:

$$2^2 + (-3)^2 + 6^2 = 4 + 9 + 36.$$

$$= 49.$$

Step 2 — Take the square root:

$$|\vec{a}| = \sqrt{49} = 7.$$

Why other options are wrong: (A) 5 ignores one term; (C) 11 adds the components' absolute values; (D) $\sqrt{41}$ drops the $\hat{j}$ term.

Final Answer: $|\vec{a}| = 7 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q9

Q10.

Solution

Concept — Direction cosines: Direction cosines are the direction ratios divided by their magnitude $\sqrt{a^2 + b^2 + c^2}$.

Step 1 — Magnitude of ratios $2, 3, -6$:

$$\sqrt{2^2 + 3^2 + (-6)^2} = \sqrt{4 + 9 + 36} = \sqrt{49} = 7.$$

Step 2 — Divide each ratio by 7:

$$\left(\frac{2}{7}, \frac{3}{7}, -\frac{6}{7}\right).$$

Why other options are wrong: (A) are the ratios themselves; (B) divides by 49; (C) divides by $\sqrt{7}$.

Final Answer: Direction cosines $\left(\frac{2}{7}, \frac{3}{7}, -\frac{6}{7}\right) \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q10

Q11.

Solution

Concept — Fundamental theorem of calculus: $\int_a^b f(x)\,dx = F(b) - F(a)$, where $F' = f$.

Step 1 — Antiderivative: $\int \frac{1}{x}\,dx = \ln|x|$.

Step 2 — Apply limits 1 **to** e**:**

$$\left[\ln x\right]_1^e = \ln e - \ln 1.$$

$$= 1 - 0.$$

$$= 1.$$

Why other options are wrong: (B) 0 uses $\ln 1$ at both ends; (C) e confuses the value with the upper limit; (D) $e - 1$ integrates e^x instead of $\frac{1}{x}$.

Final Answer: The integral equals $1 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q11

Q12.

Solution

Concept — Conditional probability: $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)}$, provided $P(A) \neq 0$.

Step 1 — Substitute the values:

$$P(B \mid A) = \frac{0.18}{0.6}.$$

Step 2 — Simplify:

$$= \frac{18}{60} = \frac{3}{10} = 0.3.$$

Why other options are wrong: (A) 0.108 multiplies instead of dividing; (C) 0.5 and (D) 0.4 use wrong arithmetic.

Final Answer: $P(B \mid A) = 0.3 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q12

Q13.

Solution

Concept — Integration by recognising a derivative: If the numerator is the derivative of the denominator, $\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$.

Step 1 — Spot the pattern: Here $f(x) = 1 + e^x$, so $f'(x) = e^x$, which is exactly the numerator.

Step 2 — Write the result:

$$\int \frac{e^x}{1+e^x}\,dx = \ln(1+e^x) + C.$$

(The absolute value is dropped since $1 + e^x > 0$.)

Why other options are wrong: (A) is not an antiderivative; (B) $\ln(e^x) = x$; (D) is a wrong antiderivative.

Final Answer: $\ln(1+e^x) + C \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q13

Q14.

Solution

Concept — Integral of an even function over a symmetric interval: If $g(-x) = g(x)$, then $\int_{-a}^{a} g(x)\,dx = 2\int_0^a g(x)\,dx$.

Step 1 — Test the integrand: $g(x) = x^2$ gives $g(-x) = (-x)^2 = x^2 = g(x)$.

So x^2 is an even function.

Step 2 — Use the property:

$$\int_{-2}^{2} x^2\,dx = 2\int_0^2 x^2\,dx.$$

Step 3 — Integrate:

$$= 2\left[\frac{x^3}{3}\right]_0^2.$$

$$= 2 \cdot \frac{8}{3}.$$
$$= \frac{16}{3}.$$

Why other options are wrong: (A) 0 would hold for an odd function; (B) $\frac{8}{3}$ forgets the factor 2; (C) $\frac{32}{3}$ doubles too much.

Final Answer: The integral equals $\frac{16}{3} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q14

Q15.

Solution

Concept — Local minimum via second derivative: At a critical point where $f'(x) = 0$, a local minimum occurs if $f''(x) > 0$.

Step 1 — Critical points:

$$f(x) = x^3 - 12x.$$
$$f'(x) = 3x^2 - 12 = 0.$$
$$x^2 = 4 \;\Rightarrow\; x = \pm 2.$$

Step 2 — Second-derivative test:

$$f''(x) = 6x.$$

At $x = 2$, $f''(2) = 12 > 0$, so $x = 2$ gives a local minimum.

Step 3 — Minimum value:

$$f(2) = (2)^3 - 12(2) = 8 - 24 = -16.$$

Why other options are wrong: (A) 16 is the local maximum value at $x = -2$; (C) -8 and (D) 8 are not extreme values.

Final Answer: Local minimum value is $-16 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q15

Q16.

Solution

Concept — Area of a triangle: Area $= \frac{1}{2}|\vec{a}\times\vec{b}|$ for side vectors $\vec{a}, \vec{b}$ from a common vertex.

Step 1 — Compute the cross product:

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & 1 \\ 1 & -2 & 2 \end{vmatrix}.$$

$$= \hat{i}\big[(2)(2)-(1)(-2)\big] - \hat{j}\big[(2)(2)-(1)(1)\big] + \hat{k}\big[(2)(-2)-(2)(1)\big].$$

$$= \hat{i}(4+2) - \hat{j}(4-1) + \hat{k}(-4-2).$$

$$= 6\hat{i} - 3\hat{j} - 6\hat{k}.$$

Step 2 — Take the magnitude:

$$|\vec{a}\times\vec{b}| = \sqrt{6^2+(-3)^2+(-6)^2} = \sqrt{36+9+36} = \sqrt{81} = 9.$$

Step 3 — Halve it:

$$\text{Area} = \frac{1}{2}\times 9 = \frac{9}{2}.$$

Why other options are wrong: (B) 9 is $|\vec{a}\times\vec{b}|$ (the parallelogram area); (C) $\frac{81}{2}$ halves $|\vec{a}\times\vec{b}|^2$ (forgets the square root); (D) $\frac{9}{4}$ halves twice.

Final Answer: Area $= \frac{9}{2} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q16

Q17.

Solution

Concept — Angle between two lines: $\cos\theta = \frac{|\vec{b}_1\cdot\vec{b}_2|}{|\vec{b}_1||\vec{b}_2|}$, using the direction ratios.

Step 1 — Dot product of direction ratios:

$$(1)(1)+(0)(1)+(1)(0) = 1+0+0 = 1.$$

Step 2 — Magnitudes:

$$|\vec{b}_1| = \sqrt{1^2 + 0^2 + 1^2} = \sqrt{2}, \qquad |\vec{b}_2| = \sqrt{1^2 + 1^2 + 0^2} = \sqrt{2}.$$

Step 3 — Compute $\cos\theta$:

$$\cos\theta = \frac{1}{\sqrt{2}\cdot\sqrt{2}} = \frac{1}{2}.$$

$$\theta = 60^\circ.$$

Why other options are wrong: (A) 90° needs a zero dot product; (B) 30° and (D) 45° do not give $\cos\theta = \frac{1}{2}$.

Final Answer: The angle is $60^\circ \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q17

Q18.

Solution

Concept — Union with independent events: $P(A \cup B) = P(A) + P(B) - P(A)P(B)$ when A, B are independent.

Step 1 — Product term:

$$P(A)P(B) = (0.3)(0.4) = 0.12.$$

Step 2 — Substitute:

$$P(A \cup B) = 0.3 + 0.4 - 0.12.$$
$$= 0.7 - 0.12.$$
$$= 0.58.$$

Why other options are wrong: (A) 0.7 forgets to subtract $P(A \cap B)$; (B) 0.12 is only $P(A \cap B)$; (C) $0.42 = P(A' \cap B')$, the complement.

Final Answer: $P(A \cup B) = 0.58 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q18

Q19.

Solution

Concept — Assertion–Reason analysis: Judge each statement's truth, then decide whether R correctly explains A.

Step 1 — Reason: "If $f'(x) \geq 0$ throughout an interval then f is increasing there" is the standard monotonicity theorem. So R is **true**.

Step 2 — Assertion: For $f(x) = x^3$, $f'(x) = 3x^2 \geq 0$ for all x (and $f' = 0$ only at the isolated point $x = 0$). Hence f is increasing on $\mathbb{R}$. So A is **true**.

Step 3 — Does R explain A? The conclusion in A follows directly from R by taking $f'(x) = 3x^2 \geq 0$. So R is the correct explanation of A.

Why other options are wrong: (B) denies the explanation, which is valid here; (C),(D) misjudge a truth value.

Final Answer: Both true, R is the correct explanation $\Rightarrow$ $\boxed{\text{A}}$

Answer: (A) Go Back to Q19

Q20.

Solution

Concept — Transpose of a product: The reversal law states $(AB)^T = B^T A^T$; the order of the factors is reversed.

Step 1 — Test the Assertion: $(AB)^T = B^T A^T$ is the correct, standard property. So A is **true**.

Step 2 — Test the Reason: The Reason claims $(AB)^T = A^T B^T$, which drops the reversal. In general $A^T B^T$ may not even be defined, and when it is it usually differs from $(AB)^T$. So R is **false**.

Step 3 — Combine: A is true and R is false.

Why other options are wrong: (A),(B) require R to be true; (D) requires A to be false.

Final Answer: A true, R false $\Rightarrow$ $\boxed{\text{C}}$

Answer: (C) Go Back to Q20

Q21.

Solution

Concept — Principal values: $\sin^{-1}$ lies in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $\cos^{-1}$ lies in $[0, \pi]$.

Step 1 — Evaluate $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$:

$$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} \Rightarrow \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}.$$

Step 2 — Evaluate $\cos^{-1}\left(\frac{1}{2}\right)$:

$$\cos\frac{\pi}{3} = \frac{1}{2} \Rightarrow \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}.$$

Step 3 — Add:

$$\frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}.$$

Final Answer: $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{1}{2}\right) = \frac{2\pi}{3}$. Go Back to Q21

Q22.

Solution

Concept — Parametric differentiation: If x and y are given in terms of a parameter t, then $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$.

Step 1 — Differentiate x with respect to t:

$$x = a\cos t.$$

$$\frac{dx}{dt} = -a\sin t.$$

Step 2 — Differentiate y with respect to t:

$$y = b\sin t.$$

$$\frac{dy}{dt} = b\cos t.$$

Step 3 — Divide:

$$\frac{dy}{dx} = \frac{b\cos t}{-a\sin t}.$$

$$= -\frac{b}{a}\cot t.$$

Final Answer: $\frac{dy}{dx} = -\frac{b}{a}\cot t$. **Go Back to Q22**

Q23.

Solution

Concept — Integration by parts: $\int u\,dv = uv - \int v\,du$, choosing u by the ILATE rule.

Step 1 — Choose parts: Let $u = x$ and $dv = \cos x\,dx$.

Then $du = dx$ and $v = \sin x$.

Step 2 — Apply the formula:

$$\int x\cos x\,dx = x\sin x - \int \sin x\,dx.$$

$$= x\sin x - (-\cos x) + C.$$

$$= x\sin x + \cos x + C.$$

Final Answer: $\int x\cos x\,dx = x\sin x + \cos x + C$.

OR — $\int \sin^2 x\,dx$**:**

Step 1 — Use the identity $\sin^2 x = \frac{1-\cos 2x}{2}$**:**

$$\int \sin^2 x\,dx = \int \frac{1-\cos 2x}{2}\,dx.$$

Step 2 — Integrate term by term:

$$= \frac{1}{2}\int 1\,dx - \frac{1}{2}\int \cos 2x\,dx.$$

$$= \frac{x}{2} - \frac{1}{2}\cdot\frac{\sin 2x}{2} + C.$$

$$= \frac{x}{2} - \frac{\sin 2x}{4} + C.$$

Final Answer (OR): $\int \sin^2 x\,dx = \frac{x}{2} - \frac{\sin 2x}{4} + C$. Go Back to Q23

Q24.

Solution

Concept — Perpendicular vectors: $\vec{a} \perp \vec{b} \iff \vec{a}\cdot\vec{b} = 0$.

Step 1 — Compute the dot product:

$$\vec{a}\cdot\vec{b} = (2)(1) + (3)(-2) + (\lambda)(1).$$

$$= 2 - 6 + \lambda.$$

$$= \lambda - 4.$$

Step 2 — Set equal to zero and solve:

$$\lambda - 4 = 0.$$

$$\lambda = 4.$$

Final Answer: $\lambda = 4$. Go Back to Q24

Q25.

Solution

Concept — Equally likely outcomes: Probability $= \frac{\text{favourable outcomes}}{\text{total outcomes}}$.

Step 1 — Total outcomes: Two throws of a die give $6 \times 6 = 36$ equally likely outcomes.

Step 2 — Favourable outcomes (sum $= 7$):

$$(1,6),\ (6,1),\ (2,5),\ (5,2),\ (3,4),\ (4,3).$$

There are 6 such outcomes.

Step 3 — Probability:

$$P(\text{sum} = 7) = \frac{6}{36} = \frac{1}{6}.$$

Final Answer: $P(\text{sum} = 7) = \frac{1}{6}$.

OR — $P(A \mid B)$**:**

Step 1 — Formula: $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$.

Step 2 — Substitute:

$$P(A \mid B) = \frac{0.2}{0.4} = \frac{1}{2}.$$

Final Answer (OR): $P(A \mid B) = \dfrac{1}{2}$. Go Back to Q25

Q26.

Solution

Concept — Partial fractions: Split a proper rational function into simpler fractions, then integrate each.

Step 1 — Set up the decomposition:

$$\frac{1}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}.$$

$$1 = A(x+2) + B(x-1).$$

Step 2 — Find A and B:

Put $x = 1$: $1 = A(3) + B(0) \Rightarrow A = \dfrac{1}{3}$.

Put $x = -2$: $1 = A(0) + B(-3) \Rightarrow B = -\dfrac{1}{3}$.

Step 3 — Integrate:

$$\int \frac{1}{(x-1)(x+2)}\,dx = \int \left(\frac{1/3}{x-1} - \frac{1/3}{x+2} \right) dx.$$

$$= \frac{1}{3}\ln|x-1| - \frac{1}{3}\ln|x+2| + C.$$

$$= \frac{1}{3}\ln\left|\frac{x-1}{x+2}\right| + C.$$

Final Answer: $\dfrac{1}{3}\ln\left|\dfrac{x-1}{x+2}\right| + C$. Go Back to Q26

Q27.

Solution

Concept — Linear differential equation: For $\frac{dy}{dx} + Py = Q$, the integrating factor is I.F. $= e^{\int P\,dx}$ and the solution is $y \cdot \text{I.F.} = \int Q \cdot \text{I.F.}\, dx$.

Step 1 — Identify P, Q: Here $P = 2$, $Q = e^{-x}$.

Step 2 — Integrating factor:

$$\text{I.F.} = e^{\int 2\,dx} = e^{2x}.$$

Step 3 — Solve:

$$y\,e^{2x} = \int e^{-x} \cdot e^{2x}\,dx = \int e^{x}\,dx.$$

$$y\,e^{2x} = e^{x} + C.$$

$$y = e^{-x} + C\,e^{-2x}.$$

Final Answer: $y = e^{-x} + C\,e^{-2x}$.

OR — Variables separable $\frac{dy}{dx} = e^{x-y}$:

Step 1 — Rewrite and separate:

$$\frac{dy}{dx} = e^{x}\,e^{-y}.$$

$$e^{y}\,dy = e^{x}\,dx.$$

Step 2 — Integrate both sides:

$$\int e^{y}\,dy = \int e^{x}\,dx.$$

$$e^{y} = e^{x} + C.$$

Final Answer (OR): $e^{y} = e^{x} + C$. **Go Back to Q27**

Q28.

Solution

Concept — Inverse by adjoint: $A^{-1} = \frac{1}{|A|}\,\text{adj}(A)$, valid when $|A| \neq 0$.

Step 1 — Determinant:

$$|A| = \begin{vmatrix} 2 & 5 \\ 1 & 3 \end{vmatrix} = (2)(3) - (5)(1) = 6 - 5 = 1.$$

Step 2 — Adjoint of a 2×2 matrix: Swap the leading diagonal entries and negate the off-diagonal entries:

$$\text{adj}(A) = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}.$$

Step 3 — Inverse:

$$A^{-1} = \frac{1}{1}\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}.$$

Step 4 — Verify:

$$A\,A^{-1} = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Final Answer: $A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$. **Go Back to Q28**

Q29.

Solution

Concept — Monotonicity: f increases where $f'(x) > 0$ and decreases where $f'(x) < 0$.

Step 1 — Differentiate:

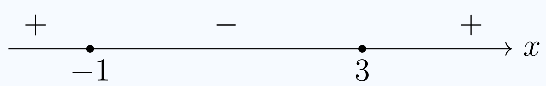
$$f(x) = x^3 - 3x^2 - 9x + 11.$$

$$f'(x) = 3x^2 - 6x - 9.$$

$$= 3(x^2 - 2x - 3) = 3(x-3)(x+1).$$

Step 2 — Critical points: $f'(x) = 0$ at $x = -1$ and $x = 3$.

Step 3 — Sign analysis:



For $x < -1$: $f' > 0$; for $-1 < x < 3$: $f' < 0$; for $x > 3$: $f' > 0$.

Conclusion: Increasing on $(-\infty, -1) \cup (3, \infty)$; decreasing on $(-1, 3)$.

Final Answer: Increasing on $(-\infty, -1) \cup (3, \infty)$, decreasing on $(-1, 3)$.

OR — Tangent to $y = x^3$ at $(1, 1)$:

Step 1 — Slope: $\frac{dy}{dx} = 3x^2$, so at $x = 1$ slope $= 3$.

Step 2 — Equation: $y - 1 = 3(x - 1) \Rightarrow y = 3x - 2$.

Final Answer (OR): $y = 3x - 2$. Go Back to Q29

Q30.

Solution

Concept — Vector equation of a line: A line through point $\vec{a}$ parallel to $\vec{b}$ is $\vec{r} = \vec{a} + \lambda\vec{b}$.

Step 1 — Write the equation: With $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$,

$$\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} - \hat{k}).$$

Step 2 — Test the point $(4, 3, 1)$: The point lies on the line if some λ satisfies all three component equations.

$$2 + \lambda = 4 \;\Rightarrow\; \lambda = 2.$$

$$-1 + 2\lambda = 3 \;\Rightarrow\; \lambda = 2.$$

$$3 - \lambda = 1 \;\Rightarrow\; \lambda = 2.$$

Step 3 — Conclusion: All three give the same value $\lambda = 2$, so the point $(4, 3, 1)$ lies on the line.

Final Answer: $\vec{r} = (2\hat{i} - \hat{j} + 3\hat{k}) + \lambda(\hat{i} + 2\hat{j} - \hat{k})$; the point $(4, 3, 1)$ lies on it (at $\lambda = 2$). Go Back to Q30

Q31.

Solution

Concept — One-one (injective) and onto (surjective): *One-one*: $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$. *Onto*: every y in the codomain has a pre-image.

Step 1 — Split by sign of x: By definition of $|x|$,

$$f(x) = \frac{x}{1+|x|} = \begin{cases} \dfrac{x}{1+x}, & x \geq 0, \\ \dfrac{x}{1-x}, & x < 0. \end{cases}$$

For $x \geq 0$, $f(x) \in [0, 1)$; for $x < 0$, $f(x) \in (-1, 0)$.

Step 2 — One-one: On $x \geq 0$, $f(x) = 1 - \dfrac{1}{1+x}$ increases strictly with x; on $x < 0$, $f(x) = -1 + \dfrac{1}{1-x}$ also increases strictly. Since the two pieces cover disjoint output ranges and each is strictly increasing, f is strictly increasing on all of $\mathbb{R}$. Hence $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$; f is one-one.

Step 3 — Onto $(-1, 1)$: Take any $y \in (-1, 1)$.

$$\text{If } 0 \leq y < 1: \ y = \frac{x}{1+x} \Rightarrow x = \frac{y}{1-y} \geq 0.$$

$$\text{If } -1 < y < 0: \ y = \frac{x}{1-x} \Rightarrow x = \frac{y}{1+y} < 0.$$

In each case $x \in \mathbb{R}$ and $f(x) = y$, so every $y \in (-1, 1)$ has a pre-image.

Step 4 — Conclusion: f is one-one and onto, hence a bijection from $\mathbb{R}$ to $(-1, 1)$.

Final Answer: $f(x) = \dfrac{x}{1+|x|}$ is one-one and onto $(-1, 1)$. **Go Back to Q31**

Q32.

Solution

Concept — Matrix method: Write $AX = B$; then $X = A^{-1}B$ when $|A| \neq 0$.

Step 1 — Form the matrices:

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & -1 & 1 \\ 3 & 2 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix}.$$

Step 2 — Determinant of A:

$$|A| = 2\big[(-1)(1) - (1)(2)\big] - 1\big[(1)(1) - (1)(3)\big] + (-1)\big[(1)(2) - (-1)(3)\big].$$

$$= 2(-1-2) - 1(1-3) + (-1)(2+3).$$

$$= 2(-3) - 1(-2) + (-1)(5).$$

$$= -6 + 2 - 5 = -9 \ (\neq 0).$$

Step 3 — Cofactors and adjoint: Computing the cofactor matrix and transposing gives

$$\text{adj}(A) = \begin{bmatrix} -3 & -3 & 0 \\ 2 & 5 & -3 \\ 5 & -1 & -3 \end{bmatrix}.$$

Step 4 — Solve $X = \frac{1}{|A|}$ adj$(A)\,B$:

$$X = \frac{1}{-9} \begin{bmatrix} -3 & -3 & 0 \\ 2 & 5 & -3 \\ 5 & -1 & -3 \end{bmatrix} \begin{bmatrix} -1 \\ 4 \\ 3 \end{bmatrix}.$$

$$= \frac{1}{-9} \begin{bmatrix} (-3)(-1) + (-3)(4) + 0(3) \\ 2(-1) + 5(4) + (-3)(3) \\ 5(-1) + (-1)(4) + (-3)(3) \end{bmatrix}.$$

$$= \frac{1}{-9} \begin{bmatrix} 3 - 12 \\ -2 + 20 - 9 \\ -5 - 4 - 9 \end{bmatrix} = \frac{1}{-9} \begin{bmatrix} -9 \\ 9 \\ -18 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}.$$

Step 5 — Verify: $2x + y - z = 2 - 1 - 2 = -1$ ✓; $x - y + z = 1 + 1 + 2 = 4$ ✓; $3x + 2y + z = 3 - 2 + 2 = 3$ ✓.

Final Answer: $x = 1, \ y = -1, \ z = 2$. **Go Back to Q32**

Q33.

Solution

Concept — Area between two curves: Area $= \int_a^b (\text{upper} - \text{lower})\,dx$ between the intersection abscissae.

Step 1 — Find intersection points: Set $x^2 = 2x$:

$$x^2 - 2x = 0.$$

$$x(x - 2) = 0 \ \Rightarrow \ x = 0, \ x = 2.$$

Step 2 — Decide upper/lower: On $(0, 2)$, the line $y = 2x$ lies above the parabola $y = x^2$ (e.g. at $x = 1$: $2 > 1$).

Step 3 — Set up the integral:

$$\text{Area} = \int_0^2 (2x - x^2)\, dx.$$

Step 4 — Integrate:

$$= \left[x^2 - \frac{x^3}{3}\right]_0^2.$$
$$= \left(4 - \frac{8}{3}\right) - (0).$$
$$= \frac{12-8}{3} = \frac{4}{3}.$$

Final Answer: Area $= \frac{4}{3}$ square units.

OR — Region bounded by $y = 4 - x^2$ and the x-axis:

Step 1 — Limits: The curve meets $y = 0$ where $4 - x^2 = 0 \Rightarrow x = \pm 2$.

Step 2 — Integrate (even function, so double the half):

$$\text{Area} = \int_{-2}^2 (4 - x^2)\, dx = 2\int_0^2 (4 - x^2)\, dx.$$

$$= 2\left[4x - \frac{x^3}{3}\right]_0^2.$$
$$= 2\left(8 - \frac{8}{3}\right) = 2 \cdot \frac{16}{3} = \frac{32}{3}.$$

Final Answer (OR): Area $= \frac{32}{3}$ square units. **Go Back to Q33**

Q34.

Solution

Concept — Corner-point method: The optimum of a linear objective over a bounded feasible region occurs at a corner (vertex).

Step 1 — Find the corner points: The constraints $x + y \leq 9$, $2x + y \leq 12$, $x \geq 0$, $y \geq 0$ bound the region with vertices:

$$O(0,0), \quad (6,0), \quad (0,9).$$

The lines $2x + y = 12$ and $x + y = 9$ intersect: subtract the second from the first,

$$(2x + y) - (x + y) = 12 - 9 \;\Rightarrow\; x = 3, \text{ then } y = 9 - 3 = 6.$$

So the fourth vertex is $(3, 6)$.

Step 2 — Evaluate $Z = 6x + 5y$ at each corner:

$$Z(0, 0) = 0.$$

$$Z(6, 0) = 6(6) + 0 = 36.$$

$$Z(3, 6) = 6(3) + 5(6) = 18 + 30 = 48.$$

$$Z(0, 9) = 0 + 5(9) = 45.$$

Step 3 — Pick the maximum: The largest value is 48 at $(3, 6)$.

Final Answer: Z is maximum $= 48$ at $(x, y) = (3, 6)$. **Go Back to Q34**

Q35.

Solution

Concept — Shortest distance between skew lines: For $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$,

$$d = \frac{\left|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)\right|}{|\vec{b}_1 \times \vec{b}_2|}.$$

Step 1 — Identify vectors: $\vec{a}_1 = \hat{i} - \hat{j}$, $\vec{b}_1 = \hat{i} + 2\hat{j} + 3\hat{k}$; $\vec{a}_2 = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{b}_2 = 2\hat{i} + 3\hat{j} + 4\hat{k}$.

Step 2 — Cross product $\vec{b}_1 \times \vec{b}_2$:

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 3 & 4 \end{vmatrix}.$$

$$= \hat{i}\big[(2)(4) - (3)(3)\big] - \hat{j}\big[(1)(4) - (3)(2)\big] + \hat{k}\big[(1)(3) - (2)(2)\big].$$

$$= \hat{i}(8 - 9) - \hat{j}(4 - 6) + \hat{k}(3 - 4).$$

$$= -\hat{i} + 2\hat{j} - \hat{k}.$$

Step 3 — Its magnitude:

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{(-1)^2 + 2^2 + (-1)^2} = \sqrt{1 + 4 + 1} = \sqrt{6}.$$

Step 4 — $(\vec{a}_2 - \vec{a}_1)$ and the dot product:

$$\vec{a}_2 - \vec{a}_1 = (2-1)\hat{\imath} + (1-(-1))\hat{\jmath} + (-1-0)\hat{k} = \hat{\imath} + 2\hat{\jmath} - \hat{k}.$$

$$(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2) = (1)(-1) + (2)(2) + (-1)(-1) = -1 + 4 + 1 = 4.$$

Step 5 — Distance:

$$d = \frac{|4|}{\sqrt{6}} = \frac{4}{\sqrt{6}} = \frac{4\sqrt{6}}{6} = \frac{2\sqrt{6}}{3} \text{ units.}$$

Final Answer: Shortest distance $= \dfrac{2\sqrt{6}}{3}$ units.

OR — Angle between line and plane:

Step 1 — Direction $\vec{b} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$, normal $\vec{n} = 2\hat{\imath} + \hat{\jmath} - 2\hat{k}$.

Step 2 — Use $\sin\theta = \dfrac{|\vec{b}\cdot\vec{n}|}{|\vec{b}||\vec{n}|}$:

$$\vec{b}\cdot\vec{n} = (2)(2) + (-1)(1) + (2)(-2) = 4 - 1 - 4 = -1.$$

$$|\vec{b}| = \sqrt{4+1+4} = 3, \qquad |\vec{n}| = \sqrt{4+1+4} = 3.$$

$$\sin\theta = \frac{|-1|}{3\cdot 3} = \frac{1}{9}.$$

Final Answer (OR): $\theta = \sin^{-1}\left(\dfrac{1}{9}\right)$. Go Back to Q35

Q36.

Solution

Concept — Maxima by derivatives: Express the area in one variable using the perimeter constraint, set $\dfrac{dA}{dr} = 0$, and select the value giving a maximum.

(i) Area function: The perimeter is the base $2r$, the two vertical sides $2h$, and the semicircular arc πr:

$$2r + 2h + \pi r = 20.$$

Solve for h:

$$h = \frac{20 - 2r - \pi r}{2}.$$

The area is the rectangle plus the semicircle:

$$A = 2r\,h + \frac{1}{2}\pi r^2.$$

Substitute h:

$$A = 2r \cdot \frac{20 - 2r - \pi r}{2} + \frac{1}{2}\pi r^2 = r(20 - 2r - \pi r) + \frac{1}{2}\pi r^2.$$

$$A = 20r - 2r^2 - \pi r^2 + \frac{1}{2}\pi r^2 = 20r - 2r^2 - \frac{1}{2}\pi r^2.$$

(ii) Differentiate:

$$\frac{dA}{dr} = 20 - 4r - \pi r = 20 - (4+\pi)r.$$

(iii) Maximise: Set $\frac{dA}{dr} = 0$:

$$20 - (4+\pi)r = 0 \Rightarrow r = \frac{20}{4+\pi}.$$

Since $\frac{d^2A}{dr^2} = -(4+\pi) < 0$, this r gives a maximum.
Maximum area (using $2 + \frac{\pi}{2} = \frac{4+\pi}{2}$):

$$A = 20r - \frac{4+\pi}{2}r^2.$$

$$= 20 \cdot \frac{20}{4+\pi} - \frac{4+\pi}{2} \cdot \frac{400}{(4+\pi)^2}.$$
$$= \frac{400}{4+\pi} - \frac{200}{4+\pi} = \frac{200}{4+\pi}.$$

Final Answer: $A = 20r - 2r^2 - \frac{1}{2}\pi r^2$; $\frac{dA}{dr} = 20 - (4+\pi)r$; maximum at $r = \frac{20}{4+\pi}$ m with $A_{\max} = \frac{200}{4+\pi}$ m^2. Go Back to Q36

Q37.

Solution

Concept — Bayes' theorem: $P(E_i \mid A) = \dfrac{P(E_i)P(A \mid E_i)}{\sum_j P(E_j)P(A \mid E_j)}$.

(i) Prior probabilities: The disease affects 1% of people, so

$$P(\text{Disease}) = 0.01, \qquad P(\text{No disease}) = 0.99.$$

(ii) Conditional (positive) probabilities:

$$P(+ \mid \text{Disease}) = 0.9, \qquad P(+ \mid \text{No disease}) = 0.05.$$

(iii) Apply Bayes' theorem:

$$P(\text{Disease} \mid +) = \frac{(0.01)(0.9)}{(0.01)(0.9) + (0.99)(0.05)}.$$

Compute each product:

$$(0.01)(0.9) = 0.009, \qquad (0.99)(0.05) = 0.0495.$$

Add the denominator:

$$0.009 + 0.0495 = 0.0585.$$

Divide:

$$P(\text{Disease} \mid +) = \frac{0.009}{0.0585} = \frac{90}{585} = \frac{2}{13}.$$

Final Answer: $P(\text{Disease} \mid +) = \dfrac{2}{13} \approx 0.154$. **Go Back to Q37**

Q38.

Solution

Concept — Related rates: Differentiate a geometric relation with respect to time t; a decreasing quantity has a negative rate.

(i) Relation between x and y: The ladder, wall and ground form a right triangle with hypotenuse 5 m:

$$x^2 + y^2 = 25.$$

(ii) Differentiate with respect to t:

$$\frac{d}{dt}(x^2 + y^2) = \frac{d}{dt}(25).$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0.$$

$$x\frac{dx}{dt} + y\frac{dy}{dt} = 0.$$

(iii) Evaluate at $x = 3$: When $x = 3$, from $x^2 + y^2 = 25$,

$$y = \sqrt{25 - 9} = \sqrt{16} = 4.$$

Given $\dfrac{dx}{dt} = 2$ m/s, substitute:

$$(3)(2) + (4)\frac{dy}{dt} = 0.$$

$$6 + 4\frac{dy}{dt} = 0.$$

$$\frac{dy}{dt} = -\frac{6}{4} = -\frac{3}{2}.$$

The negative sign shows y is decreasing.

Final Answer: $x^2 + y^2 = 25$; $x\frac{dx}{dt} + y\frac{dy}{dt} = 0$; the top of the ladder slides *down* at $\frac{3}{2} = 1.5$ m/s when the foot is 3 m from the wall. **Go Back to Q38**

Answer Key – Section A (Q1–Q20)

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	B	3	C	4	D	5	B
6	A	7	D	8	C	9	B	10	D
11	A	12	B	13	C	14	D	15	B
16	A	17	C	18	D	19	A	20	C

Sections B–E are descriptive; refer to the Detailed Solutions above for full model answers and step marking.