CBSE Class 12 Mathematics

Sample Paper – 8

Duration: 180 Minutes **Maximum Marks: 80**

General Instructions

- This question paper contains **38 questions**. **All questions are compulsory**.
- The paper is divided into **five sections** – A, B, C, D and E.
- **Section A** (Q1–Q20) carries **1 mark** each: Q1–Q18 are multiple choice questions and Q19–Q20 are Assertion–Reason questions.
- **Section B** (Q21–Q25) carries **2 marks** each (Very Short Answer).
- **Section C** (Q26–Q31) carries **3 marks** each (Short Answer).
- **Section D** (Q32–Q35) carries **5 marks** each (Long Answer).
- **Section E** (Q36–Q38) carries **4 marks** each (case study based, with sub-parts).
- There is **no overall choice**, but an **internal choice** has been provided in some questions. Attempt only one of the alternatives in such questions.
- There is **no negative marking**. Use of a **calculator is not permitted**.

Section A (Q1–Q20) – 1 Mark Each

Q1. The trace of the matrix $A = \begin{bmatrix} 3 & 1 & 0 \\ 2 & 5 & 4 \\ -1 & 0 & 2 \end{bmatrix}$ is:

(A) 8

(B) 10

(C) 5

(D) 12

Q2. If A is a square matrix of order 3 with $|A| = 4$, then $|2A|$ equals:

(A) 8

(B) 12

(C) 16

(D) 32

Q3. The number of reflexive relations that can be defined on a set A having 3 elements is:

(A) 64

(B) 512

(C) 32

(D) 8

Q4. The principal value of $\tan^{-1}\left(\sqrt{3}\right)$ is:

(A) $\dfrac{\pi}{6}$

(B) $\dfrac{\pi}{4}$

(C) $\dfrac{\pi}{3}$

(D) $\dfrac{2\pi}{3}$

Q5. If $y = e^{\tan x}$, then $\dfrac{dy}{dx}$ equals:

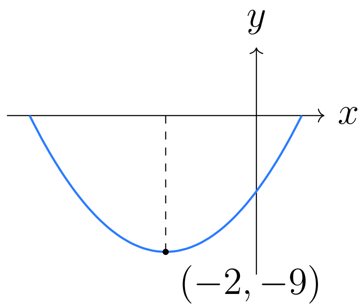
(A) $\sec^2 x\, e^{\tan x}$

(B) $e^{\tan x}$

(C) $\sec^2 x$

(D) $\tan x\, e^{\tan x}$

Q6. The graph of $f(x) = x^2 + 4x - 5$ is shown below. The function is *increasing* on the interval:



(A) $(-\infty, -2)$

(B) $(-\infty, \infty)$

(C) $(-2, \infty)$

(D) $(-2, 0)$

Q7. $\int \frac{1}{\sqrt{1-x^2}}\,dx$ equals:

(A) $\cos^{-1} x + C$

(B) $\tan^{-1} x + C$

(C) $\sec^{-1} x + C$

(D) $\sin^{-1} x + C$

Q8. The order and degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{dy}{dx}\right)^4 + y = 0$ are respectively:

(A) 3 and 4

(B) 2 and 3

(C) 3 and 2

(D) 4 and 2

Q9. The magnitude of the vector $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ is:

(A) 7

(B) 5

(C) 9

(D) 49

Q10. The direction cosines of a line whose direction ratios are $2, 3, -6$ are:

(A) $2, 3, -6$

(B) $\frac{2}{49}, \frac{3}{49}, -\frac{6}{49}$

(C) $\frac{1}{7}, \frac{3}{7}, -\frac{6}{7}$

(D) $\frac{2}{7}, \frac{3}{7}, -\frac{6}{7}$

Q11. $\int_0^1 x^2\,dx$ equals:

(A) $\frac{1}{3}$

(B) 1

(C) $\frac{1}{2}$

(D) 0

Q12. If $P(A \cap B) = 0.15$ and $P(B) = 0.6$, then $P(A \mid B)$ equals:

(A) 0.15

(B) 0.6

(C) 0.25

(D) 0.4

Q13. $\int \frac{3x^2}{1+x^3}\,dx$ equals:

(A) $3\ln(1+x^3) + C$

(B) $\ln(1+x^3) + C$

(C) $\ln(1+x^2) + C$

(D) $\frac{1}{1+x^3} + C$

Q14. $\int_{-2}^{2} x^2\,dx$ equals:

(A) 0

(B) $\frac{8}{3}$

(C) 8

(D) $\frac{16}{3}$

Q15. The local maximum value of $f(x) = x^3 - 12x$ is:

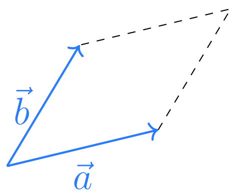
(A) -16

(B) 16

(C) 8

(D) -8

Q16. The area of the parallelogram whose adjacent sides are the vectors $\vec{a} = \hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$ (shown below) is:



(A) 18

(B) 9

(C) $\frac{9}{2}$

(D) 81

Q17. The angle between two lines whose direction ratios are $1, 2, 3$ and $2, 4, 6$ is:

(A) $90°$

(B) $45°$

(C) $60°$

(D) $0°$

Q18. If A and B are independent events with $P(A) = \frac{1}{4}$ and $P(B) = \frac{1}{3}$, then $P(A \cup B)$ equals:

(A) $\frac{7}{12}$

(B) $\frac{1}{2}$

(C) $\frac{1}{12}$

(D) $\frac{5}{12}$

Q19. Assertion (A): The function $f(x) = x^3$ is increasing on $\mathbb{R}$.
Reason (R): If $f'(x) \geq 0$ for all x in an interval, then f is increasing on that interval.

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is *not* the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

Q20. Assertion (A): $\tan^{-1}\left(\tan \frac{3\pi}{4}\right) = -\frac{\pi}{4}$.
Reason (R): $\tan^{-1}(\tan x) = x$ for every real number x.

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is *not* the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

Section B (Q21–Q25) – 2 Marks Each

Q21. Find the principal value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{1}{2}\right)$. **[2]**

Q22. Differentiate $y = (\cos x)^{\sin x}$ with respect to x. **[2]**

Q23. Evaluate $\int x \sin x \, dx$. **[2]**

OR

Evaluate $\int \ln x \, dx$.

Q24. Find the value of λ for which the vectors $\vec{a} = 3\hat{i}+\lambda\hat{j}-2\hat{k}$ and $\vec{b} = \hat{i}+2\hat{j}+\hat{k}$ are perpendicular. **[2]**

Q25. A fair die is thrown twice. Find the probability of getting a sum of 7. **[2]**

OR

If $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cap B) = 0.2$, find $P(B \mid A)$.

Section C (Q26–Q31) – 3 Marks Each

Q26. Evaluate $\displaystyle\int \frac{x}{(x-1)(x-2)}\,dx$. **[3]**

Q27. Solve the differential equation $\dfrac{dy}{dx} + 2y = e^{-x}$. **[3]**

OR

Solve the differential equation $\dfrac{dy}{dx} = e^{x-y}$.

Q28. Find the inverse of the matrix $A = \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}$ using the adjoint method. **[3]**

Q29. Find the intervals in which $f(x) = 2x^3 - 3x^2 - 12x + 5$ is increasing or decreasing. **[3]**

OR

Find the equation of the tangent to the curve $y = x^3$ at the point $(1, 1)$.

Q30. Find the vector equation of the line passing through the point $(1, 0, 2)$ and parallel to the vector $2\hat{i} + 3\hat{j} + \hat{k}$. Hence check whether the point $(5, 6, 4)$ lies on this line. **[3]**

Q31. Show that the function $f : \mathbb{N} \to \mathbb{N}$ defined by $f(x) = 2x$ is one-one but not onto. **[3]**

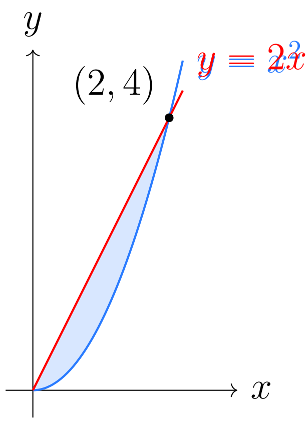
Section D (Q32–Q35) – 5 Marks Each

Q32. Using the matrix method, solve the following system of linear equations:

$$x + y + z = 2, \qquad 2x - y + z = 6, \qquad x + 2y + 3z = 3.$$

[5]

Q33. Using integration, find the area of the region bounded by the parabola $y = x^2$ and the line $y = 2x$.



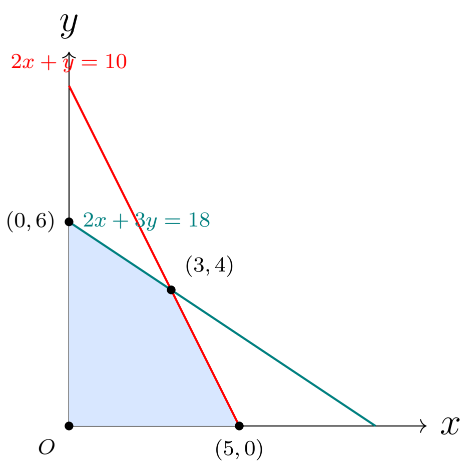
[5]

OR

Using integration, find the area of the region bounded by the curve $y = \sin x$, the x-axis and the ordinates $x = 0$ and $x = \pi$.

Q34. Solve the following linear programming problem graphically. Maximise $Z = 5x + 4y$ subject to the constraints

$$2x + 3y \leq 18, \qquad 2x + y \leq 10, \qquad x \geq 0,\ y \geq 0.$$



[5]

Q35. Find the shortest distance between the lines

$$\vec{r} = (2\hat{i} - \hat{j} + \hat{k}) + \lambda(\hat{i} + 2\hat{j} - \hat{k}) \quad \text{and} \quad \vec{r} = (\hat{i} + \hat{j} + 2\hat{k}) + \mu(2\hat{i} - \hat{j} + 3\hat{k}).$$

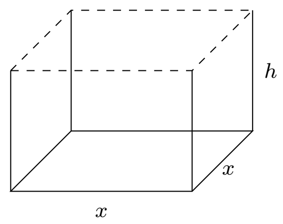
[5]

OR

Find the angle between the line $\vec{r} = (\hat{i} + \hat{j} + \hat{k}) + \lambda(2\hat{i} + 2\hat{j} + \hat{k})$ and the plane $2x - y + 2z = 5$.

Section E (Q36–Q38) – 4 Marks Each (Case Study)

Q36. Case Study – The Water Tank.
An open-top water tank with a square base of side x metres and height h metres is to be constructed to hold a fixed volume of $32\ \text{m}^3$. The cost of the sheet used is proportional to the total surface area (base + four walls), so the engineer wishes to *minimise* the surface area.



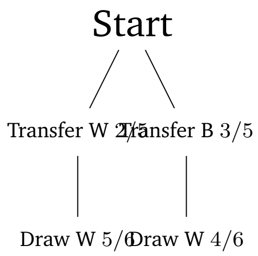
Based on the above, answer the following:

(i) Using $x^2h = 32$, express the surface area S as a function of x alone. (1)

(ii) Find $\dfrac{dS}{dx}$. (1)

(iii) Find the value of x that minimises S, the corresponding height h, and the minimum surface area. (2)

Q37. Case Study – Transferring a Ball.
Bag A contains 2 white and 3 black balls; Bag B contains 4 white and 1 black ball. One ball is transferred at random from Bag A to Bag B, and then a ball is drawn at random from Bag B. The ball drawn from Bag B is found to be white.



Based on the above, answer the following:

(i) Write the probabilities that the transferred ball is white and that it is black. (1)

(ii) Write $P(\text{draw white} \mid \text{transferred white})$ and $P(\text{draw white} \mid \text{transferred black})$. (1)

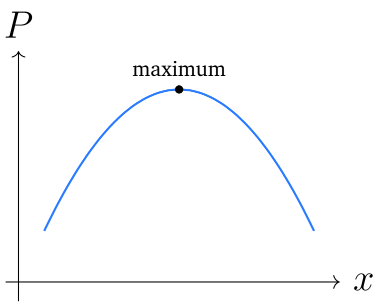
(iii) Using Bayes' theorem, find the probability that the transferred ball was white, given that the ball drawn is white. (2)

Q38. Case Study – Maximising Profit.

For a certain product, when x units are produced and sold, the total revenue and total cost (in hundreds of rupees) are

$$R(x) = 70x - 2x^2, \qquad C(x) = x^2 + 10x + 30.$$

The profit is $P(x) = R(x) - C(x)$, and profit is maximum where marginal revenue equals marginal cost.



Based on the above, answer the following:

(i) Find the marginal revenue $\text{MR} = \dfrac{dR}{dx}$. (1)

(ii) Find the marginal cost $\text{MC} = \dfrac{dC}{dx}$. (1)

(iii) Find the output x at which profit is maximum, and the maximum profit. (2)

Detailed Solutions

Q1.

Solution

Concept — Trace of a matrix: The trace is the sum of the entries on the leading (top-left to bottom-right) diagonal.

Step 1 — Identify the diagonal entries: They are 3 (row 1), 5 (row 2) and 2 (row 3).

Step 2 — Add them:

$$\text{trace}(A) = 3 + 5 + 2.$$
$$= 10.$$

Why other options are wrong: (A) 8 and (C) 5 drop a diagonal entry; (D) 12 wrongly includes an off-diagonal entry.

Final Answer: $\text{trace}(A) = 10 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q1

Q2.

Solution

Concept — Scalar multiple of a determinant: For an $n \times n$ matrix, $|kA| = k^n\,|A|$.

Step 1 — Identify n and k: Here $n = 3$ and $k = 2$.

Step 2 — Apply the rule:

$$|2A| = 2^3\,|A|.$$
$$= 8 \times 4.$$
$$= 32.$$

Why other options are wrong: (A) 8 is only 2^3; (B) 12 uses $2 \times |A| + |A|$; (C) 16 multiplies $|A|$ by 2^2 (treating it as order 2).

Final Answer: $|2A| = 32 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q2

Q3.

Solution

Concept — Counting reflexive relations: A relation on a set of n elements is a subset of the n^2 ordered pairs. For a reflexive relation, all n diagonal pairs must be present; the remaining $n^2 - n$ pairs are free, giving 2^{n^2-n} reflexive relations.

Step 1 — Count the free pairs: With $n = 3$, total pairs $= 3^2 = 9$; the 3 diagonal pairs are forced, leaving $9 - 3 = 6$ free pairs.

Step 2 — Count the relations:

$$\text{Number} = 2^6.$$

$$= 64.$$

Why other options are wrong: (B) $512 = 2^9$ counts all relations; (C) $32 = 2^5$ miscounts free pairs; (D) $8 = 2^3$ counts only the diagonal choices.

Final Answer: There are 64 reflexive relations $\Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q3

Q4.

Solution

Concept — Principal value of $\tan^{-1}$: The principal value branch of $\tan^{-1} x$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Step 1 — Set up the equation: Let $\tan^{-1}(\sqrt{3}) = \theta$, so $\tan\theta = \sqrt{3}$.

Step 2 — Find θ in the branch: We know $\tan\frac{\pi}{3} = \sqrt{3}$, and $\frac{\pi}{3} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Hence $\theta = \frac{\pi}{3}$.

Why other options are wrong: (A) $\frac{\pi}{6}$ gives $\tan = \frac{1}{\sqrt{3}}$; (B) $\frac{\pi}{4}$ gives $\tan = 1$; (D) $\frac{2\pi}{3}$ lies outside the principal branch.

Final Answer: $\tan^{-1}(\sqrt{3}) = \frac{\pi}{3} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q4

Q5.

Solution

Concept — Chain rule for the exponential: $\frac{d}{dx}e^u = e^u \cdot \frac{du}{dx}$.

Step 1 — Identify the inner function: Let $u = \tan x$, so $\frac{du}{dx} = \sec^2 x$.

Step 2 — Apply the chain rule:

$$\frac{dy}{dx} = e^{\tan x} \cdot \sec^2 x.$$

$$= \sec^2 x\, e^{\tan x}.$$

Why other options are wrong: (B) omits the factor $\sec^2 x$; (C) drops the exponential; (D) wrongly differentiates $\tan x$ as $\tan x$.

Final Answer: $\frac{dy}{dx} = \sec^2 x\, e^{\tan x} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q5

Q6.

Solution

Concept — Increasing function: f is increasing where $f'(x) > 0$.

Step 1 — Differentiate:

$$f(x) = x^2 + 4x - 5.$$

$$f'(x) = 2x + 4.$$

Step 2 — Solve $f'(x) > 0$:

$$2x + 4 > 0.$$

$$2x > -4.$$

$$x > -2.$$

So f is increasing on $(-2, \infty)$, which matches the right (rising) branch of the upward parabola with vertex $(-2, -9)$.

Why other options are wrong: (A) $(-\infty, -2)$ is where $f'(x) < 0$ (decreasing); (B) $\mathbb{R}$ is false since the parabola falls then rises; (D) $(-2, 0)$ is only part of the increasing interval.

Final Answer: f increases on $(-2, \infty) \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q6

Q7.

Solution

Concept — Standard integral: $\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$, hence $\int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1}x + C$.

Step 1 — Recall the antiderivative: The derivative of $\sin^{-1}x$ is $\frac{1}{\sqrt{1-x^2}}$.

Step 2 — Write the integral:

$$\int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1}x + C.$$

Why other options are wrong: (A) $\cos^{-1}x$ has derivative $-\frac{1}{\sqrt{1-x^2}}$ (wrong sign); (B) $\tan^{-1}x$ is $\int \frac{1}{1+x^2}dx$; (C) $\sec^{-1}x$ is a different antiderivative.

Final Answer: $\int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1}x + C \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q7

Q8.

Solution

Concept — Order and degree: The *order* is the highest derivative present; the *degree* is the power of that highest derivative when the equation is polynomial in the derivatives.

Step 1 — Highest derivative: The highest derivative is $\frac{d^3y}{dx^3}$, so the order is 3.

Step 2 — Its power: The term $\left(\frac{d^3y}{dx^3}\right)^2$ shows the highest derivative appears to the power 2.

So the degree is 2. (The power 4 is on $\frac{dy}{dx}$, a lower derivative, so it does not decide the degree.)

Why other options are wrong: (A),(B),(D) misread either the highest derivative or its power.

Final Answer: Order 3, degree $2 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q8

Q9.

Solution

Concept — Magnitude of a vector: For $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

Step 1 — Square the components:

$$2^2 + (-3)^2 + 6^2 = 4 + 9 + 36.$$

$$= 49.$$

Step 2 — Take the square root:

$$|\vec{a}| = \sqrt{49} = 7.$$

Why other options are wrong: (B) 5 and (C) 9 use wrong components; (D) 49 forgets to take the square root.

Final Answer: $|\vec{a}| = 7 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q9

Q10.

Solution

Concept — Direction cosines: Direction cosines are the direction ratios divided by their magnitude $\sqrt{a^2 + b^2 + c^2}$.

Step 1 — Magnitude of ratios $2, 3, -6$**:**

$$\sqrt{2^2 + 3^2 + (-6)^2} = \sqrt{4 + 9 + 36} = \sqrt{49} = 7.$$

Step 2 — Divide each ratio by 7**:**

$$\left(\frac{2}{7}, \frac{3}{7}, -\frac{6}{7}\right).$$

Why other options are wrong: (A) are the ratios themselves; (B) divides by 49; (C) has a wrong first component.

Final Answer: Direction cosines $\left(\frac{2}{7}, \frac{3}{7}, -\frac{6}{7}\right) \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q10

Q11.

Solution

Concept — Fundamental theorem of calculus: $\int_a^b f(x)\,dx = F(b) - F(a)$, where $F' = f$.

Step 1 — Antiderivative: $\int x^2\,dx = \frac{x^3}{3}$.

Step 2 — Apply limits 0 **to** 1**:**

$$\left[\frac{x^3}{3}\right]_0^1 = \frac{1^3}{3} - \frac{0^3}{3}.$$

$$= \frac{1}{3} - 0.$$

$$= \frac{1}{3}.$$

Why other options are wrong: (B) 1 forgets to divide by 3; (C) $\frac{1}{2}$ integrates x; (D) 0 ignores the antiderivative.

Final Answer: The integral equals $\frac{1}{3} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q11

Q12.

Solution

Concept — Conditional probability: $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$, provided $P(B) \neq 0$.

Step 1 — Substitute the values:

$$P(A \mid B) = \frac{0.15}{0.6}.$$

Step 2 — Simplify:

$$= \frac{15}{60} = \frac{1}{4} = 0.25.$$

Why other options are wrong: (A) 0.15 is only $P(A \cap B)$; (B) 0.6 is $P(B)$; (D) 0.4 inverts or misdivides the ratio.

Final Answer: $P(A \mid B) = 0.25 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q12

Q13.

Solution

Concept — Integration by recognising a derivative: If the numerator is the derivative of the denominator, $\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$.

Step 1 — Spot the pattern: Here $f(x) = 1 + x^3$, so $f'(x) = 3x^2$, which is exactly the numerator.

Step 2 — Write the result:

$$\int \frac{3x^2}{1+x^3}\,dx = \ln|1+x^3| + C.$$

Why other options are wrong: (A) has an extra factor 3; (C) uses the wrong denominator $1 + x^2$; (D) is a wrong antiderivative.

Final Answer: $\ln|1+x^3| + C \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q13

Q14.

Solution

Concept — Integral of an even function over a symmetric interval: If $g(-x) = g(x)$, then $\int_{-a}^{a} g(x)\,dx = 2\int_0^a g(x)\,dx$.

Step 1 — Test the integrand: $g(x) = x^2$ gives $g(-x) = (-x)^2 = x^2 = g(x)$, so x^2 is even.

Step 2 — Use the property:

$$\int_{-2}^{2} x^2\,dx = 2\int_0^2 x^2\,dx.$$

$$= 2\left[\frac{x^3}{3}\right]_0^2.$$

$$= 2 \cdot \frac{8}{3}.$$

$$= \frac{16}{3}.$$

Why other options are wrong: (A) 0 treats x^2 as odd; (B) $\frac{8}{3}$ forgets the factor 2; (C) 8 drops the division by 3.

Final Answer: The integral equals $\frac{16}{3} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q14

Q15.

Solution

Concept — Local maximum via second derivative: At a critical point where $f'(x) = 0$, a local maximum occurs if $f''(x) < 0$.

Step 1 — Critical points:

$$f(x) = x^3 - 12x.$$

$$f'(x) = 3x^2 - 12 = 0.$$

$$x^2 = 4 \;\Rightarrow\; x = \pm 2.$$

Step 2 — Second-derivative test:

$$f''(x) = 6x.$$

At $x = -2$, $f''(-2) = -12 < 0$, so $x = -2$ gives a local maximum.

Step 3 — Maximum value:

$$f(-2) = (-2)^3 - 12(-2) = -8 + 24 = 16.$$

Why other options are wrong: (A) -16 is the local minimum value at $x = 2$; (C),(D) are not extreme values.

Final Answer: Local maximum value is $16 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q15

Q16.

Solution

Concept — Area of a parallelogram: Area $= |\vec{a}\times\vec{b}|$ for adjacent side vectors $\vec{a},\vec{b}$.

Step 1 — Compute the cross product:

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 2 & -2 & 1 \end{vmatrix}.$$

$$= \hat{i}\big[(2)(1)-(2)(-2)\big] - \hat{j}\big[(1)(1)-(2)(2)\big] + \hat{k}\big[(1)(-2)-(2)(2)\big].$$

$$= \hat{i}(2+4) - \hat{j}(1-4) + \hat{k}(-2-4).$$

$$= 6\hat{i} + 3\hat{j} - 6\hat{k}.$$

Step 2 — Take the magnitude:

$$|\vec{a}\times\vec{b}| = \sqrt{6^2+3^2+(-6)^2} = \sqrt{36+9+36} = \sqrt{81} = 9.$$

Why other options are wrong: (A) 18 doubles the area; (C) $\frac{9}{2}$ halves it (that would be the triangle); (D) 81 forgets to take the square root.

Final Answer: Area $= 9 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q16

Q17.

Solution

Concept — Angle between two lines: If the direction ratios are proportional, the lines are parallel and the angle between them is 0°.

Step 1 — Test proportionality: Compare $1, 2, 3$ with $2, 4, 6$:

$$\frac{2}{1} = \frac{4}{2} = \frac{6}{3} = 2.$$

Step 2 — Interpret: All three ratios equal 2, so the second set is 2 times the first; the lines are parallel and the angle is 0°.

Why other options are wrong: (A) 90° needs a zero dot product; (B),(C) would need non-proportional ratios giving those cosines.

Final Answer: The angle is $0^\circ \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q17

Q18.

Solution

Concept — Union with independent events: $P(A \cup B) = P(A) + P(B) - P(A)P(B)$ when A, B are independent.

Step 1 — Product term:

$$P(A)P(B) = \frac{1}{4} \cdot \frac{1}{3} = \frac{1}{12}.$$

Step 2 — Substitute:

$$\begin{aligned} P(A \cup B) &= \frac{1}{4} + \frac{1}{3} - \frac{1}{12}. \\ &= \frac{3}{12} + \frac{4}{12} - \frac{1}{12}. \\ &= \frac{6}{12} = \frac{1}{2}. \end{aligned}$$

Why other options are wrong: (A) $\frac{7}{12}$ forgets to subtract the intersection; (C) $\frac{1}{12}$ is only $P(A \cap B)$; (D) $\frac{5}{12}$ uses wrong arithmetic.

Final Answer: $P(A \cup B) = \frac{1}{2} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q18

Q19.

Solution

Concept — Assertion–Reason analysis: Judge each statement's truth, then decide whether R correctly explains A.

Step 1 — Assertion: For $f(x) = x^3$, $f'(x) = 3x^2 \geq 0$ for all x (equal to 0 only at the single point $x = 0$), so f is increasing on all of $\mathbb{R}$. So A is **true**.

Step 2 — Reason: "If $f'(x) \geq 0$ on an interval then f is increasing there" is a standard true theorem. So R is **true**.

Step 3 — Does R explain A? It is precisely because $f'(x) = 3x^2 \geq 0$ that x^3 is increasing; R gives the exact reason for A. Hence R *is* the correct explanation of A.

Why other options are wrong: (B) denies the explanation (but R does explain

A); (C),(D) misjudge a truth value.

Final Answer: Both true, R the correct explanation ⇒ $\boxed{\text{A}}$

Answer: (A) Go Back to Q19

Q20.

Solution

Concept — Inverse tangent of a tangent: $\tan^{-1}(\tan\theta) = \theta$ only when $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$; otherwise reduce to the principal branch.

Step 1 — Test the Assertion: $\frac{3\pi}{4} \notin \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, so evaluate the tangent first.

$$\tan\frac{3\pi}{4} = -1.$$

$$\tan^{-1}(-1) = -\frac{\pi}{4} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right).$$

So the Assertion value $-\frac{\pi}{4}$ is **true**.

Step 2 — Test the Reason: The claim "$\tan^{-1}(\tan x) = x$ for *every* real x" is **false**; it holds only for $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (indeed it fails at $x = \frac{3\pi}{4}$).

Step 3 — Combine: A is true and R is false.

Why other options are wrong: (A),(B) require R true; (D) requires A false.

Final Answer: A true, R false ⇒ $\boxed{\text{C}}$

Answer: (C) Go Back to Q20

Q21.

Solution

Concept — Principal values: $\sin^{-1}$ lies in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $\cos^{-1}$ lies in $[0, \pi]$.

Step 1 — Evaluate $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$:

$$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} \Rightarrow \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}.$$

Step 2 — Evaluate $\cos^{-1}\left(\frac{1}{2}\right)$:

$$\cos\frac{\pi}{3} = \frac{1}{2} \Rightarrow \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}.$$

Step 3 — Add:

$$\frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}.$$

Final Answer: $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{1}{2}\right) = \frac{2\pi}{3}$. Go Back to Q21

Q22.

Solution

Concept — Logarithmic differentiation: Used when the variable appears in both base and exponent.

Step 1 — Take natural log of both sides:

$$y = (\cos x)^{\sin x}.$$

$$\ln y = \sin x\, \ln(\cos x).$$

Step 2 — Differentiate both sides w.r.t. x: (product rule on the right)

$$\frac{1}{y}\frac{dy}{dx} = \cos x \cdot \ln(\cos x) + \sin x \cdot \frac{-\sin x}{\cos x}.$$

$$\frac{1}{y}\frac{dy}{dx} = \cos x\, \ln(\cos x) - \frac{\sin^2 x}{\cos x}.$$

Step 3 — Solve for $\frac{dy}{dx}$:

$$\frac{dy}{dx} = y\left[\cos x\, \ln(\cos x) - \frac{\sin^2 x}{\cos x}\right].$$

$$= (\cos x)^{\sin x}\left[\cos x\, \ln(\cos x) - \frac{\sin^2 x}{\cos x}\right].$$

Final Answer: $\frac{dy}{dx} = (\cos x)^{\sin x}\left[\cos x\, \ln(\cos x) - \frac{\sin^2 x}{\cos x}\right]$. Go Back to Q22

Q23.

Solution

Concept — Integration by parts: $\int u\,dv = uv - \int v\,du$, choosing u by the ILATE rule.

Step 1 — Choose parts: Let $u = x$ and $dv = \sin x\,dx$.

Then $du = dx$ and $v = -\cos x$.

Step 2 — Apply the formula:

$$\int x\,\sin x\,dx = x(-\cos x) - \int (-\cos x)\,dx.$$

$$= -x\cos x + \int \cos x\,dx.$$

$$= -x\cos x + \sin x + C.$$

Final Answer: $\int x\,\sin x\,dx = -x\cos x + \sin x + C.$

OR — $\int \ln x\,dx$**:**

Step 1 — Integration by parts with $u = \ln x$, $dv = dx$:

Then $du = \frac{1}{x}\,dx$ and $v = x$.

Step 2 — Apply the formula:

$$\int \ln x\,dx = x\ln x - \int x \cdot \frac{1}{x}\,dx.$$

$$= x\ln x - \int 1\,dx.$$

$$= x\ln x - x + C.$$

Final Answer (OR): $\int \ln x\,dx = x\ln x - x + C.$ Go Back to Q23

Q24.

Solution

Concept — Perpendicular vectors: $\vec{a} \perp \vec{b} \iff \vec{a}\cdot\vec{b} = 0.$

Step 1 — Compute the dot product:

$$\vec{a}\cdot\vec{b} = (3)(1) + (\lambda)(2) + (-2)(1).$$

$$= 3 + 2\lambda - 2.$$

$$= 1 + 2\lambda.$$

Step 2 — Set equal to zero and solve:

$$1 + 2\lambda = 0.$$

$$2\lambda = -1.$$

$$\lambda = -\frac{1}{2}.$$

Final Answer: $\lambda = -\frac{1}{2}$. Go Back to Q24

Q25.

Solution

Concept — Equally likely outcomes: Probability $= \dfrac{\text{favourable outcomes}}{\text{total outcomes}}$.

Step 1 — Total outcomes: Two throws of a die give $6 \times 6 = 36$ equally likely outcomes.

Step 2 — Favourable outcomes (sum $= 7$):

$$(1, 6),\ (6, 1),\ (2, 5),\ (5, 2),\ (3, 4),\ (4, 3).$$

There are 6 such outcomes.

Step 3 — Probability:

$$P(\text{sum} = 7) = \frac{6}{36} = \frac{1}{6}.$$

Final Answer: $P(\text{sum} = 7) = \frac{1}{6}$.

OR — $P(B \mid A)$:

Step 1 — Formula: $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)}$.

Step 2 — Substitute:

$$P(B \mid A) = \frac{0.2}{0.5} = \frac{2}{5}.$$

Final Answer (OR): $P(B \mid A) = \frac{2}{5}$. Go Back to Q25

Q26.

Solution

Concept — Partial fractions: Split a proper rational function into simpler fractions, then integrate each.

Step 1 — Set up the decomposition:

$$\frac{x}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}.$$

$$x = A(x-2) + B(x-1).$$

Step 2 — Find A and B:

Put $x = 1$: $1 = A(-1) + B(0) \Rightarrow A = -1$.

Put $x = 2$: $2 = A(0) + B(1) \Rightarrow B = 2$.

Step 3 — Integrate:

$$\int \frac{x}{(x-1)(x-2)}\,dx = \int \left(\frac{-1}{x-1} + \frac{2}{x-2}\right) dx.$$

$$= -\ln|x-1| + 2\ln|x-2| + C.$$

Final Answer: $-\ln|x-1| + 2\ln|x-2| + C$. **Go Back to Q26**

Q27.

Solution

Concept — Linear differential equation: For $\frac{dy}{dx} + Py = Q$, the integrating factor is I.F. $= e^{\int P\,dx}$ and the solution is $y \cdot \text{I.F.} = \int Q \cdot \text{I.F.}\,dx$.

Step 1 — Identify P, Q: Here $P = 2$, $Q = e^{-x}$.

Step 2 — Integrating factor:

$$\text{I.F.} = e^{\int 2\,dx} = e^{2x}.$$

Step 3 — Solve:

$$y\,e^{2x} = \int e^{-x} \cdot e^{2x}\,dx = \int e^{x}\,dx.$$

$$y\,e^{2x} = e^{x} + C.$$

$$y = e^{-x} + C\,e^{-2x}.$$

Final Answer: $y = e^{-x} + C\,e^{-2x}$.

OR — Variables separable $\dfrac{dy}{dx} = e^{x-y}$:

Step 1 — Rewrite and separate:

$$\frac{dy}{dx} = e^{x}\,e^{-y}.$$

$$e^{y}\,dy = e^{x}\,dx.$$

Step 2 — Integrate both sides:

$$\int e^{y}\,dy = \int e^{x}\,dx.$$

$$e^{y} = e^{x} + C.$$

Final Answer (OR): $e^{y} = e^{x} + C$. Go Back to Q27

Q28.

Solution

Concept — Inverse by adjoint: $A^{-1} = \dfrac{1}{|A|}\,\text{adj}(A)$, valid when $|A| \neq 0$.

Step 1 — Determinant:

$$|A| = \begin{vmatrix} 4 & 3 \\ 3 & 2 \end{vmatrix} = (4)(2) - (3)(3) = 8 - 9 = -1.$$

Step 2 — Adjoint of a 2×2 matrix: Swap the leading diagonal entries and negate the off-diagonal entries:

$$\text{adj}(A) = \begin{bmatrix} 2 & -3 \\ -3 & 4 \end{bmatrix}.$$

Step 3 — Inverse:

$$A^{-1} = \frac{1}{-1}\begin{bmatrix} 2 & -3 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix}.$$

Step 4 — Verify:

$$A\,A^{-1} = \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} -8+9 & 12-12 \\ -6+6 & 9-8 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Final Answer: $A^{-1} = \begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix}$. Go Back to Q28

Q29.

Solution

Concept — Monotonicity: f increases where $f'(x) > 0$ and decreases where $f'(x) < 0$.

Step 1 — Differentiate:

$$f(x) = 2x^3 - 3x^2 - 12x + 5.$$

$$f'(x) = 6x^2 - 6x - 12.$$

$$= 6(x^2 - x - 2) = 6(x-2)(x+1).$$

Step 2 — Critical points: $f'(x) = 0$ at $x = -1$ and $x = 2$.

Step 3 — Sign analysis:

For $x < -1$: $f' > 0$; for $-1 < x < 2$: $f' < 0$; for $x > 2$: $f' > 0$.

Conclusion: Increasing on $(-\infty, -1) \cup (2, \infty)$; decreasing on $(-1, 2)$.

Final Answer: Increasing on $(-\infty, -1) \cup (2, \infty)$, decreasing on $(-1, 2)$.

OR — Tangent to $y = x^3$ at $(1, 1)$:

Step 1 — Slope: $\dfrac{dy}{dx} = 3x^2$, so at $x = 1$ slope $= 3$.

Step 2 — Equation: $y - 1 = 3(x - 1) \Rightarrow y = 3x - 2$.

Final Answer (OR): $y = 3x - 2$. Go Back to Q29

Q30.

Solution

Concept — Vector equation of a line: A line through point $\vec{a}$ parallel to $\vec{b}$ is $\vec{r} = \vec{a} + \lambda\vec{b}$.

Step 1 — Write the equation: With $\vec{a} = \hat{i} + 2\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} + \hat{k}$,

$$\vec{r} = (\hat{i} + 2\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + \hat{k}).$$

Step 2 — Test the point $(5, 6, 4)$**:** The point lies on the line if some λ satisfies all three component equations.

$$1 + 2\lambda = 5 \;\Rightarrow\; \lambda = 2.$$

$$0 + 3\lambda = 6 \;\Rightarrow\; \lambda = 2.$$

$$2 + \lambda = 4 \;\Rightarrow\; \lambda = 2.$$

Step 3 — Conclusion: All three give the same value $\lambda = 2$, so the point $(5, 6, 4)$ lies on the line.

Final Answer: $\vec{r} = (\hat{i} + 2\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + \hat{k})$; the point $(5, 6, 4)$ lies on it (at $\lambda = 2$).
Go Back to Q30

Q31.

Solution

Concept — One-one (injective) and onto (surjective): *One-one*: $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$. *Onto*: every element of the codomain has a pre-image.

Step 1 — One-one: Suppose $f(x_1) = f(x_2)$.

$$2x_1 = 2x_2.$$

$$x_1 = x_2.$$

Hence f is one-one.

Step 2 — Onto? The codomain is $\mathbb{N}$. Take $y = 3 \in \mathbb{N}$. A pre-image would need $2x = 3$, i.e. $x = \frac{3}{2}$, which is *not* a natural number.
So 3 has no pre-image in $\mathbb{N}$.

Step 3 — Conclusion: The range of f is the set of even natural numbers $\{2, 4, 6, \dots\}$, which is a proper subset of $\mathbb{N}$. Therefore f is not onto.

Final Answer: $f(x) = 2x$ is one-one but not onto (no odd natural number has a pre-image). **Go Back to Q31**

Q32.

Solution

Concept — Matrix method: Write $AX = B$; then $X = A^{-1}B$ when $|A| \neq 0$.

Step 1 — Form the matrices:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 1 \\ 1 & 2 & 3 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ 6 \\ 3 \end{bmatrix}.$$

Step 2 — Determinant of A:

$$|A| = 1\big[(-1)(3) - (1)(2)\big] - 1\big[(2)(3) - (1)(1)\big] + 1\big[(2)(2) - (-1)(1)\big].$$

$$= 1(-3-2) - 1(6-1) + 1(4+1).$$

$$= -5 - 5 + 5 = -5\ (\neq 0).$$

Step 3 — Cofactors and adjoint: Computing the cofactor matrix and transposing gives

$$\text{adj}(A) = \begin{bmatrix} -5 & -1 & 2 \\ -5 & 2 & 1 \\ 5 & -1 & -3 \end{bmatrix}.$$

Step 4 — Solve $X = \dfrac{1}{|A|}$ adj$(A)\,B$:

$$X = \frac{1}{-5} \begin{bmatrix} -5 & -1 & 2 \\ -5 & 2 & 1 \\ 5 & -1 & -3 \end{bmatrix} \begin{bmatrix} 2 \\ 6 \\ 3 \end{bmatrix}.$$

$$= \frac{1}{-5} \begin{bmatrix} -5\cdot 2 - 1\cdot 6 + 2\cdot 3 \\ -5\cdot 2 + 2\cdot 6 + 1\cdot 3 \\ 5\cdot 2 - 1\cdot 6 - 3\cdot 3 \end{bmatrix}.$$

$$= \frac{1}{-5} \begin{bmatrix} -10 - 6 + 6 \\ -10 + 12 + 3 \\ 10 - 6 - 9 \end{bmatrix} = \frac{1}{-5} \begin{bmatrix} -10 \\ 5 \\ -5 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}.$$

Step 5 — Verify: $x + y + z = 2 - 1 + 1 = 2$ ✓; $2x - y + z = 4 + 1 + 1 = 6$ ✓; $x + 2y + 3z = 2 - 2 + 3 = 3$ ✓.

Final Answer: $x = 2,\ y = -1,\ z = 1$. **Go Back to Q32**

Q33.

Solution

Concept — Area between two curves: Area $= \int_a^b (y_{\text{top}} - y_{\text{bottom}})\, dx$ over the interval where the curves meet.

Step 1 — Points of intersection: Set $x^2 = 2x$:

$$x^2 - 2x = 0.$$

$$x(x-2) = 0 \;\Rightarrow\; x = 0,\; x = 2.$$

Step 2 — Identify top and bottom: On $[0,2]$, the line $y = 2x$ lies above the parabola $y = x^2$, so

$$\text{Area} = \int_0^2 (2x - x^2)\, dx.$$

Step 3 — Integrate:

$$= \left[x^2 - \frac{x^3}{3}\right]_0^2.$$

$$= \left(4 - \frac{8}{3}\right) - 0.$$

$$= \frac{12-8}{3} = \frac{4}{3}.$$

Final Answer: Area $= \frac{4}{3}$ square units.

OR — Area under $y = \sin x$ on $[0, \pi]$:

Step 1 — Set up the integral: Since $\sin x \geq 0$ on $[0, \pi]$,

$$\text{Area} = \int_0^\pi \sin x\, dx.$$

Step 2 — Integrate:

$$= \big[-\cos x\big]_0^\pi.$$

$$= (-\cos\pi) - (-\cos 0).$$

$$= -(-1) - (-1) = 1 + 1 = 2.$$

Final Answer (OR): Area $= 2$ square units. **Go Back to Q33**

Q34.

Solution

Concept — Corner-point method: The optimum of a linear objective over a bounded feasible region occurs at a corner (vertex).

Step 1 — Find the corner points: The constraints $2x + 3y \le 18$, $2x + y \le 10$, $x \ge 0$, $y \ge 0$ bound the region with vertices:

$$O(0,0), \quad (5,0), \quad (0,6).$$

The lines $2x + 3y = 18$ and $2x + y = 10$ intersect where, subtracting the second from the first,

$$(2x+3y) - (2x+y) = 18 - 10 \ \Rightarrow\ 2y = 8 \ \Rightarrow\ y = 4,$$

and then $2x + 4 = 10 \Rightarrow x = 3$. So the fourth vertex is $(3,4)$.

Step 2 — Evaluate $Z = 5x + 4y$ at each corner:

$$Z(0,0) = 0.$$

$$Z(5,0) = 5(5) + 0 = 25.$$

$$Z(3,4) = 5(3) + 4(4) = 15 + 16 = 31.$$

$$Z(0,6) = 5(0) + 4(6) = 24.$$

Step 3 — Pick the maximum: The largest value is 31 at $(3,4)$.

Final Answer: Z is maximum $= 31$ at $(x,y) = (3,4)$. **Go Back to Q34**

Q35.

Solution

Concept — Shortest distance between skew lines: For $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$,

$$d = \frac{\left|(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)\right|}{|\vec{b}_1 \times \vec{b}_2|}.$$

Step 1 — Identify vectors: $\vec{a}_1 = 2\hat{i} - \hat{j} + \hat{k}$, $\vec{b}_1 = \hat{i} + 2\hat{j} - \hat{k}$; $\vec{a}_2 = \hat{i} + \hat{j} + 2\hat{k}$, $\vec{b}_2 = 2\hat{i} - \hat{j} + 3\hat{k}$.

Step 2 — Cross product $\vec{b}_1 \times \vec{b}_2$:

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ 2 & -1 & 3 \end{vmatrix}.$$

$$= \hat{i}\big[(2)(3) - (-1)(-1)\big] - \hat{j}\big[(1)(3) - (-1)(2)\big] + \hat{k}\big[(1)(-1) - (2)(2)\big].$$

$$= \hat{i}(6-1) - \hat{j}(3+2) + \hat{k}(-1-4).$$

$$= 5\hat{i} - 5\hat{j} - 5\hat{k}.$$

Step 3 — Its magnitude:

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{5^2 + (-5)^2 + (-5)^2} = \sqrt{25+25+25} = \sqrt{75} = 5\sqrt{3}.$$

Step 4 — $(\vec{a}_2 - \vec{a}_1)$ and the dot product:

$$\vec{a}_2 - \vec{a}_1 = (1-2)\hat{i} + (1+1)\hat{j} + (2-1)\hat{k} = -\hat{i} + 2\hat{j} + \hat{k}.$$

$$(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2) = (-1)(5) + (2)(-5) + (1)(-5) = -5 - 10 - 5 = -20.$$

Step 5 — Distance:

$$d = \frac{|-20|}{5\sqrt{3}} = \frac{20}{5\sqrt{3}} = \frac{4}{\sqrt{3}} = \frac{4\sqrt{3}}{3} \text{ units.}$$

Final Answer: Shortest distance $= \dfrac{4\sqrt{3}}{3}$ units.

OR — Angle between line and plane:

Step 1 — Direction $\vec{b} = 2\hat{i} + 2\hat{j} + \hat{k}$, normal $\vec{n} = 2\hat{i} - \hat{j} + 2\hat{k}$.

Step 2 — Use $\sin\theta = \dfrac{|\vec{b}\cdot\vec{n}|}{|\vec{b}||\vec{n}|}$:

$$\vec{b}\cdot\vec{n} = (2)(2) + (2)(-1) + (1)(2) = 4 - 2 + 2 = 4.$$

$$|\vec{b}| = \sqrt{4+4+1} = 3, \qquad |\vec{n}| = \sqrt{4+1+4} = 3.$$

$$\sin\theta = \frac{4}{3\cdot 3} = \frac{4}{9}.$$

Final Answer (OR): $\theta = \sin^{-1}\left(\dfrac{4}{9}\right)$. **Go Back to Q35**

Q36.

Solution

Concept — Minima by derivatives: Build the surface-area function, set $\dfrac{dS}{dx} = 0$, and confirm a minimum with the second derivative.

(i) Surface-area function: The tank has a square base of area x^2 and four walls each of area xh, with no top:

$$S = x^2 + 4xh.$$

Using the volume condition $x^2h = 32$, we get $h = \dfrac{32}{x^2}$, so

$$S = x^2 + 4x \cdot \frac{32}{x^2} = x^2 + \frac{128}{x}, \qquad x > 0.$$

(ii) Differentiate:

$$\frac{dS}{dx} = 2x - \frac{128}{x^2}.$$

(iii) Critical point and minimum: Set $\dfrac{dS}{dx} = 0$:

$$2x - \frac{128}{x^2} = 0.$$

$$2x^3 = 128.$$

$$x^3 = 64 \;\Rightarrow\; x = 4.$$

Second derivative:

$$\frac{d^2S}{dx^2} = 2 + \frac{256}{x^3}.$$

At $x = 4$, $\dfrac{d^2S}{dx^2} = 2 + \dfrac{256}{64} = 2 + 4 = 6 > 0$, so $x = 4$ gives a minimum.
Corresponding height:

$$h = \frac{32}{4^2} = \frac{32}{16} = 2.$$

Minimum surface area:

$$S = 4^2 + \frac{128}{4} = 16 + 32 = 48.$$

Final Answer: $S = x^2 + \dfrac{128}{x}$; $\dfrac{dS}{dx} = 2x - \dfrac{128}{x^2}$; minimum at $x = 4$ m with $h = 2$ m and $S_{\min} = 48$ m^2. **Go Back to Q36**

Q37.

Solution

Concept — Bayes' theorem: $P(E_i \mid A) = \dfrac{P(E_i)P(A \mid E_i)}{\sum_j P(E_j)P(A \mid E_j)}$.

(i) Prior probabilities: The transferred ball comes from Bag A (2 white, 3 black), so

$$P(\text{transferred white}) = \frac{2}{5}, \qquad P(\text{transferred black}) = \frac{3}{5}.$$

(ii) Conditional (draw-white) probabilities: After the transfer, Bag B has 6 balls.

$$P(\text{draw W} \mid \text{transferred W}) = \frac{5}{6} \quad (5 \text{ white out of } 6),$$

$$P(\text{draw W} \mid \text{transferred B}) = \frac{4}{6} \quad (4 \text{ white out of } 6).$$

(iii) Apply Bayes' theorem:

$$P(\text{transferred W} \mid \text{draw W}) = \frac{\frac{2}{5}\cdot\frac{5}{6}}{\frac{2}{5}\cdot\frac{5}{6} + \frac{3}{5}\cdot\frac{4}{6}}.$$

Compute each product:

$$\frac{2}{5}\cdot\frac{5}{6} = \frac{10}{30}, \qquad \frac{3}{5}\cdot\frac{4}{6} = \frac{12}{30}.$$

$$= \frac{10/30}{(10+12)/30} = \frac{10}{22} = \frac{5}{11}.$$

Final Answer: $P(\text{transferred white} \mid \text{drew white}) = \dfrac{5}{11}$. **Go Back to Q37**

Q38.

Solution

Concept — Marginal analysis: Marginal revenue MR $= \dfrac{dR}{dx}$, marginal cost MC $= \dfrac{dC}{dx}$; profit is maximum where MR $=$ MC (equivalently $P'(x) = 0$ with $P''(x) < 0$).

(i) Marginal revenue:

$$R(x) = 70x - 2x^2.$$

$$\text{MR} = \frac{dR}{dx} = 70 - 4x.$$

(ii) Marginal cost:

$$C(x) = x^2 + 10x + 30.$$

$$\text{MC} = \frac{dC}{dx} = 2x + 10.$$

(iii) Maximum profit: Set MR = MC:

$$70 - 4x = 2x + 10.$$

$$60 = 6x.$$

$$x = 10.$$

The profit function is

$$P(x) = R(x) - C(x) = 70x - 2x^2 - (x^2 + 10x + 30) = 60x - 3x^2 - 30.$$

Check it is a maximum: $P'(x) = 60 - 6x$, and $P''(x) = -6 < 0$, so $x = 10$ maximises profit. Maximum profit:

$$P(10) = 60(10) - 3(10)^2 - 30 = 600 - 300 - 30 = 270.$$

Final Answer: MR $= 70 - 4x$; MC $= 2x + 10$; profit is maximum at $x = 10$ units, with maximum profit $P(10) = 270$ (hundred rupees). **Go Back to Q38**

Answer Key – Section A (Q1–Q20)

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	D	3	A	4	C	5	A
6	C	7	D	8	C	9	A	10	D
11	A	12	C	13	B	14	D	15	B
16	B	17	D	18	B	19	A	20	C

Sections B–E are descriptive; refer to the Detailed Solutions above for full model answers and step marking.