CBSE Class 12 Mathematics

Sample Paper – 9

Duration: 180 Minutes **Maximum Marks: 80**

General Instructions

- This question paper contains **38 questions**. **All questions are compulsory**.
- The paper is divided into **five sections** – A, B, C, D and E.
- **Section A** (Q1–Q20) carries **1 mark** each: Q1–Q18 are multiple choice questions and Q19–Q20 are Assertion–Reason questions.
- **Section B** (Q21–Q25) carries **2 marks** each (Very Short Answer).
- **Section C** (Q26–Q31) carries **3 marks** each (Short Answer).
- **Section D** (Q32–Q35) carries **5 marks** each (Long Answer).
- **Section E** (Q36–Q38) carries **4 marks** each (case study based, with sub-parts).
- There is **no overall choice**, but an **internal choice** has been provided in some questions. Attempt only one of the alternatives in such questions.
- There is **no negative marking**. Use of a **calculator is not permitted**.

Section A (Q1–Q20) – 1 Mark Each

Q1. If $A = [a_{ij}]$ is a skew-symmetric matrix, then each diagonal element a_{ii} is equal to:

(A) 0

(B) 1

(C) -1

(D) the trace of A

Q2. If A is a square matrix of order 3 with $|A| = 4$, then $|2A|$ equals:

(A) 8

(B) 32

(C) 16

(D) 64

Q3. Let $A = \{1, 2, 3\}$ and $R = \{(1,1), (2,2), (3,3), (1,2), (2,1), (2,3), (3,2)\}$ be a relation on A. Then R is:

(A) an equivalence relation

(B) transitive but not symmetric

(C) reflexive and symmetric but not transitive

(D) neither reflexive nor symmetric

Q4. The principal value of $\cot^{-1}(-1)$ is:

(A) $\frac{\pi}{4}$

(B) $-\frac{\pi}{4}$

(C) $\frac{\pi}{2}$

(D) $\frac{3\pi}{4}$

Q5. If $y = e^{\tan x}$, then $\frac{dy}{dx}$ equals:

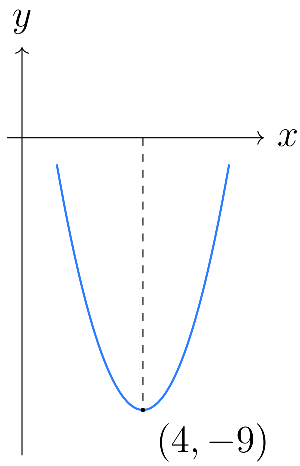
(A) $e^{\tan x} \sec^2 x$

(B) $e^{\tan x}$

(C) $\sec^2 x$

(D) $e^{\tan x} \tan x$

Q6. The graph of $f(x) = x^2 - 8x + 7$ is shown below. The function is *decreasing* on the interval:



(A) $(4, \infty)$

(B) $(-\infty, \infty)$

(C) $(-\infty, 4)$

(D) $(0, 4)$

Q7. $\int \frac{1}{\sqrt{1-x^2}}\,dx$ equals:

(A) $\tan^{-1} x + C$

(B) $-\sin^{-1} x + C$

(C) $\sec^{-1} x + C$

(D) $\sin^{-1} x + C$

Q8. The order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^4 + y = 0$ are respectively:

(A) 2 and 1

(B) 2 and 4

(C) 4 and 2

(D) 2 and 2

Q9. The magnitude of the vector $\vec{a} = 2\hat{i} - 3\hat{j} + 6\hat{k}$ is:

(A) 7

(B) 5

(C) 49

(D) $\sqrt{7}$

Q10. The direction cosines of a line whose direction ratios are $2, 3, -6$ are:

(A) $2,\ 3,\ -6$

(B) $\frac{2}{7},\ \frac{3}{7},\ -\frac{6}{7}$

(C) $\frac{2}{49},\ \frac{3}{49},\ -\frac{6}{49}$

(D) $\frac{2}{\sqrt{7}},\ \frac{3}{\sqrt{7}},\ -\frac{6}{\sqrt{7}}$

Q11. $\int_0^1 x^2\,dx$ equals:

(A) 1

(B) $\frac{1}{2}$

(C) $\frac{1}{3}$

(D) 0

Q12. If $P(A \cap B) = 0.12$ and $P(A) = 0.3$, then $P(B \mid A)$ equals:

(A) 0.12

(B) 0.3

(C) 0.75

(D) 0.4

Q13. $\int \frac{e^x}{1+e^x}\,dx$ equals:

(A) $\ln(1+e^x) + C$

(B) $\frac{e^x}{1+e^x} + C$

(C) $\ln(e^x) + C$

(D) $\tan^{-1}(e^x) + C$

Q14. $\int_{-2}^{2} x^2\,dx$ equals:

(A) $\frac{8}{3}$

(B) $\frac{16}{3}$

(C) 0

(D) 8

Q15. The local minimum value of $f(x) = x^3 - 12x$ is:

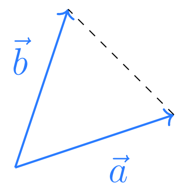
(A) 16

(B) 0

(C) -16

(D) -8

Q16. The area of the triangle whose two adjacent sides are the vectors $\vec{a} = 3\hat{i} + \hat{j}$ and $\vec{b} = \hat{i} + 3\hat{j}$ (shown below) is:



(A) 8

(B) 4

(C) 2

(D) 16

Q17. The angle between two lines whose direction ratios are $1, 2, 2$ and $2, 4, 4$ is:

(A) 90°

(B) 0°

(C) 45°

(D) 60°

Q18. If A and B are independent events with $P(A) = 0.3$ and $P(B) = 0.4$, then $P(A' \cap B')$ equals:

(A) 0.12

(B) 0.7

(C) 0.58

(D) 0.42

Q19. **Assertion (A):** If $f(x) = x^2$, then $f'(x) = 2x$ is an odd function.
Reason (R): The derivative of an even differentiable function is always an odd function.

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is *not* the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

Q20. **Assertion (A):** $\sin^{-1}\left(\sin\frac{\pi}{3}\right) = \frac{\pi}{3}$.
Reason (R): $\sin^{-1}(\sin x) = x$ for every real number x.

(A) Both A and R are true and R is the correct explanation of A.

(B) Both A and R are true but R is *not* the correct explanation of A.

(C) A is true but R is false.

(D) A is false but R is true.

Section B (Q21–Q25) – 2 Marks Each

Q21. Find the value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{1}{2}\right)$. **[2]**

Q22. Differentiate $y = x^{\cos x}$ with respect to x. **[2]**

Q23. Evaluate $\int x\cos x\,dx$. **[2]**

OR

Evaluate $\int \ln x\,dx$.

Q24. Find the value of λ for which the vectors $\vec{a} = 3\hat{i} - \hat{j} + \lambda\hat{k}$ and $\vec{b} = 2\hat{i} + 3\hat{j} - \hat{k}$ are perpendicular. **[2]**

Q25. A fair die is thrown twice. Find the probability of getting a sum of 7. **[2]**

OR

If $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cap B) = 0.25$, find $P(A \mid B)$.

Section C (Q26–Q31) – 3 Marks Each

Q26. Evaluate $\int \frac{1}{(x-1)(x+2)}\,dx$. **[3]**

Q27. Solve the differential equation $\frac{dy}{dx} + 2y = e^{-x}$. **[3]**

OR

Solve the differential equation $\frac{dy}{dx} = e^{x-y}$.

Q28. Find the inverse of the matrix $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ using the adjoint method. **[3]**

Q29. Find the intervals in which $f(x) = 2x^3 - 3x^2 - 12x + 5$ is increasing or decreasing. **[3]**

OR

Find the equation of the tangent to the curve $y = \sqrt{x}$ at the point $(4, 2)$.

Q30. Find the vector equation of the line passing through the points $(1, -1, 2)$ and $(3, 2, 4)$. Hence check whether the point $(5, 5, 6)$ lies on this line. **[3]**

Q31. Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 3x - 2$ is one-one and onto. **[3]**

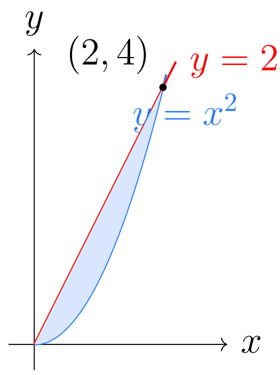
Section D (Q32–Q35) – 5 Marks Each

Q32. Using the matrix method, solve the following system of linear equations:

$$x + 2y + z = 7, \qquad 2x - y + 3z = 12, \qquad x + y - z = 0.$$

[5]

Q33. Using integration, find the area of the region bounded by the parabola $y = x^2$ and the line $y = 2x$.



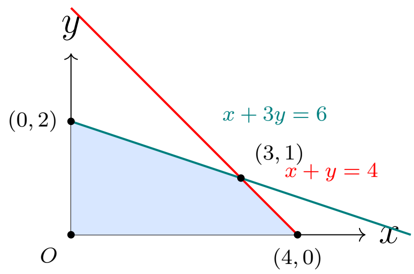
[5]

OR

Using integration, find the area of the region bounded by the curve $y = \sin x$ and the x-axis between $x = 0$ and $x = \pi$.

Q34. Solve the following linear programming problem graphically. Maximise $Z = 5x + 3y$ subject to the constraints

$$x + y \leq 4, \qquad x + 3y \leq 6, \qquad x \geq 0,\ y \geq 0.$$



[5]

Q35. Find the shortest distance between the lines

$$\vec{r} = (\hat{i}+\hat{j}+\hat{k})+\lambda(2\hat{i}+3\hat{j}+4\hat{k}) \quad \text{and} \quad \vec{r} = (2\hat{i}+4\hat{j}+5\hat{k})+\mu(3\hat{i}+4\hat{j}+5\hat{k}).$$

[5]

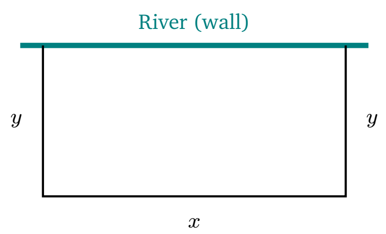
OR

Find the angle between the line $\vec{r} = (\hat{i}+\hat{j}+\hat{k})+\lambda(2\hat{i}-\hat{j}+2\hat{k})$ and the plane $2x+y-2z=5$.

Section E (Q36–Q38) – 4 Marks Each (Case Study)

Q36. Case Study – The Garden Fence.

A farmer wants to fence a rectangular garden along a straight river. The side along the river needs no fence. He has 60 metres of fencing for the other three sides, as shown. Let x metres be the side parallel to the river and y metres be each side perpendicular to it.



Based on the above, answer the following:

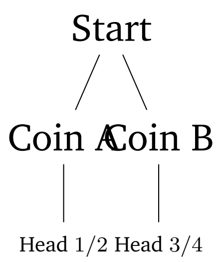
(i) Express the area A of the garden as a function of y. (1)

(ii) Find $\dfrac{dA}{dy}$. (1)

(iii) Find the value of y for which the area is maximum, and the maximum area. (2)

Q37. Case Study – Two Coins.

A box contains two coins. Coin A is a fair coin with $P(\text{Head}) = \dfrac{1}{2}$; Coin B is biased with $P(\text{Head}) = \dfrac{3}{4}$. One coin is chosen at random and tossed once. The toss shows a head.



Based on the above, answer the following:

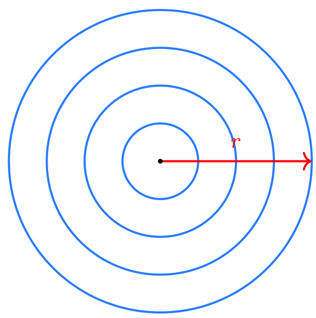
(i) Write $P(\text{Coin A})$ and $P(\text{Coin B})$. (1)

(ii) Write $P(\text{Head} \mid \text{Coin A})$ and $P(\text{Head} \mid \text{Coin B})$. (1)

(iii) Using Bayes' theorem, find the probability that the biased coin (Coin B) was tossed. (2)

Q38. Case Study – Spreading Ripples.

A stone is dropped into a still pond, creating circular ripples that move outwards. The radius r (in cm) of the outermost ripple increases at the constant rate of 3 cm/s.



Based on the above, answer the following:

(i) Express the area A enclosed by the outermost ripple in terms of r, and write $\dfrac{dA}{dr}$. (1)

(ii) Write $\dfrac{dA}{dt}$ in terms of r. (1)

(iii) Find the rate at which the enclosed area is increasing when $r = 8$ cm. (2)

Detailed Solutions

Q1.

Solution

Concept — Skew-symmetric matrix: A matrix A is skew-symmetric if $A^T = -A$, i.e. $a_{ji} = -a_{ij}$ for all i, j.

Step 1 — Apply the condition to a diagonal entry: Put $i = j$ in $a_{ji} = -a_{ij}$.

$$a_{ii} = -a_{ii}.$$

Step 2 — Solve:

$$a_{ii} + a_{ii} = 0.$$

$$2a_{ii} = 0.$$

$$a_{ii} = 0.$$

So every diagonal element of a skew-symmetric matrix is 0.

Why other options are wrong: (B) 1 and (C) -1 contradict $a_{ii} = -a_{ii}$; (D) the trace is the *sum* of the diagonal entries, which is 0, not the individual entry description asked.

Final Answer: Each diagonal element is $0 \Rightarrow \boxed{\text{A}}$

$\boxed{\text{Answer: (A)}}$ Go Back to Q1

Q2.

Solution

Concept — Scalar multiple of a determinant: For a square matrix of order n, $|kA| = k^n\,|A|$.

Step 1 — Identify n and k: Here $n = 3$ and $k = 2$.

Step 2 — Apply the rule:

$$|2A| = 2^3\,|A|.$$

$$= 8 \times 4.$$

$$= 32.$$

Why other options are wrong: (A) 8 is only 2^3; (C) 16 uses k^2; (D) 64 uses $|A| = 8$.

Final Answer: $|2A| = 32 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q2

Q3.

Solution

Concept — Properties of a relation: *Reflexive*: $(a, a) \in R$ for all a. *Symmetric*: $(a, b) \in R \Rightarrow (b, a) \in R$. *Transitive*: $(a, b), (b, c) \in R \Rightarrow (a, c) \in R$.

Step 1 — Reflexive? $(1, 1), (2, 2), (3, 3) \in R$, so R is reflexive.

Step 2 — Symmetric? $(1, 2)$ pairs with $(2, 1)$, and $(2, 3)$ pairs with $(3, 2)$; all reverse pairs are present, so R is symmetric.

Step 3 — Transitive? $(1, 2) \in R$ and $(2, 3) \in R$, but $(1, 3) \notin R$.

Hence R is not transitive.

Why other options are wrong: (A) an equivalence relation must also be transitive, which fails; (B) R *is* symmetric; (D) R is both reflexive and symmetric.

Final Answer: R is reflexive and symmetric but not transitive $\Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q3

Q4.

Solution

Concept — Principal value of $\cot^{-1}$: The principal value branch of $\cot^{-1} x$ is $(0, \pi)$.

Step 1 — Set up the equation: Let $\cot^{-1}(-1) = \theta$, so $\cot\theta = -1$ with $\theta \in (0, \pi)$.

Step 2 — Locate θ: $\cot \dfrac{\pi}{4} = 1$, and cotangent is negative in the second quadrant, so

$$\cot\left(\pi - \frac{\pi}{4}\right) = -1.$$

$$\theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4}.$$

Since $\dfrac{3\pi}{4} \in (0, \pi)$, it is the principal value.

Why other options are wrong: (A) $\dfrac{\pi}{4}$ gives $\cot = +1$; (B) $-\dfrac{\pi}{4}$ lies outside $(0, \pi)$; (C) $\dfrac{\pi}{2}$ gives $\cot = 0$.

Final Answer: $\cot^{-1}(-1) = \dfrac{3\pi}{4} \Rightarrow \boxed{\text{D}}$

$\boxed{\textbf{Answer: (D)}}$ **Go Back to Q4**

Q5.

Solution

Concept — Chain rule for exponentials: $\dfrac{d}{dx}e^u = e^u \cdot \dfrac{du}{dx}$.

Step 1 — Identify the inner function: Let $u = \tan x$, so $\dfrac{du}{dx} = \sec^2 x$.

Step 2 — Apply the chain rule:

$$\frac{dy}{dx} = e^{\tan x} \cdot \sec^2 x.$$

$$= e^{\tan x} \sec^2 x.$$

Why other options are wrong: (B) omits the derivative of the exponent; (C) drops the exponential factor; (D) uses the wrong derivative $\tan x$ instead of $\sec^2 x$.

Final Answer: $\dfrac{dy}{dx} = e^{\tan x} \sec^2 x \Rightarrow \boxed{\text{A}}$

$\boxed{\textbf{Answer: (A)}}$ **Go Back to Q5**

Q6.

Solution

Concept — Decreasing function: f is decreasing where $f'(x) < 0$.

Step 1 — Differentiate:

$$f(x) = x^2 - 8x + 7.$$

$$f'(x) = 2x - 8.$$

Step 2 — Solve $f'(x) < 0$:

$$2x - 8 < 0.$$

$$2x < 8.$$

$$x < 4.$$

So f is decreasing on $(-\infty, 4)$, which matches the falling left branch of the upward parabola down to the vertex $(4, -9)$.

Why other options are wrong: (A) $(4, \infty)$ is where f increases; (B) $(-\infty, \infty)$ is false since the curve falls then rises; (D) $(0, 4)$ is contained in the decreasing region but is not the complete interval on which f decreases.

Final Answer: f decreases on $(-\infty, 4) \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q6

Q7.

Solution

Concept — Standard integral: $\frac{d}{dx}\sin^{-1}x = \frac{1}{\sqrt{1-x^2}}$, hence $\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + C$.

Step 1 — Recall the antiderivative: The derivative of $\sin^{-1}x$ is $\frac{1}{\sqrt{1-x^2}}$.

Step 2 — Write the integral:

$$\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + C.$$

Why other options are wrong: (A) is $\int \frac{1}{1+x^2}\,dx$; (B) has an incorrect negative sign (that is $\cos^{-1}x$ up to constant); (C) is a different standard integral.

Final Answer: $\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + C \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q7

Q8.

Solution

Concept — Order and degree: The *order* is the highest derivative present; the *degree* is the power of that highest derivative when the equation is polynomial in derivatives.

Step 1 — Highest derivative: The highest derivative is $\frac{d^2y}{dx^2}$, so the order is 2.

Step 2 — Its power: The term $\frac{d^2y}{dx^2}$ appears squared, so its power is 2.

Hence the degree is 2 (the fourth power is on the lower derivative $\frac{dy}{dx}$ and does not affect the degree).

Why other options are wrong: (A) $2, 1$ misreads the power; (B) $2, 4$ takes the power of the wrong term; (C) $4, 2$ swaps order and degree.

Final Answer: Order 2, degree $2 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q8

Q9.

Solution

Concept — Magnitude of a vector: For $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

Step 1 — Square the components:

$$2^2 + (-3)^2 + 6^2 = 4 + 9 + 36.$$

$$= 49.$$

Step 2 — Take the square root:

$$|\vec{a}| = \sqrt{49} = 7.$$

Why other options are wrong: (B) 5 uses wrong components; (C) 49 forgets the square root; (D) $\sqrt{7}$ mis-adds the squares.

Final Answer: $|\vec{a}| = 7 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q9

Q10.

Solution

Concept — Direction cosines: Direction cosines are the direction ratios divided by their magnitude $\sqrt{a^2 + b^2 + c^2}$.

Step 1 — Magnitude of ratios $2, 3, -6$:

$$\sqrt{2^2 + 3^2 + (-6)^2} = \sqrt{4 + 9 + 36} = \sqrt{49} = 7.$$

Step 2 — Divide each ratio by 7:

$$\left(\frac{2}{7}, \frac{3}{7}, -\frac{6}{7}\right).$$

Why other options are wrong: (A) are the ratios themselves; (C) divides by 49;

(D) divides by $\sqrt{7}$.

Final Answer: Direction cosines $\left(\frac{2}{7}, \frac{3}{7}, -\frac{6}{7}\right) \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q10

Q11.

Solution

Concept — Fundamental theorem of calculus: $\int_a^b f(x)\,dx = F(b) - F(a)$, where $F' = f$.

Step 1 — Antiderivative: $\int x^2\,dx = \frac{x^3}{3}$.

Step 2 — Apply limits 0 **to** 1**:**

$$\left[\frac{x^3}{3}\right]_0^1 = \frac{1^3}{3} - \frac{0^3}{3}.$$

$$= \frac{1}{3} - 0.$$

$$= \frac{1}{3}.$$

Why other options are wrong: (A) 1 forgets to divide by 3; (B) $\frac{1}{2}$ integrates x; (D) 0 ignores the upper limit.

Final Answer: The integral equals $\frac{1}{3} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q11

Q12.

Solution

Concept — Conditional probability: $P(B \mid A) = \frac{P(A \cap B)}{P(A)}$, provided $P(A) \neq 0$.

Step 1 — Substitute the values:

$$P(B \mid A) = \frac{0.12}{0.3}.$$

Step 2 — Simplify:

$$= \frac{12}{30} = \frac{2}{5} = 0.4.$$

Why other options are wrong: (A) 0.12 is $P(A \cap B)$; (B) 0.3 is $P(A)$; (C) 0.75 inverts the ratio.

Final Answer: $P(B \mid A) = 0.4 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q12

Q13.

Solution

Concept — Integration by recognising a derivative: If the numerator is the derivative of the denominator, $\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$.

Step 1 — Spot the pattern: Here $f(x) = 1 + e^x$, so $f'(x) = e^x$, which is exactly the numerator.

Step 2 — Write the result:

$$\int \frac{e^x}{1+e^x}\,dx = \ln(1+e^x) + C.$$

(The absolute value is dropped since $1 + e^x > 0$.)

Why other options are wrong: (B) is the integrand, not its integral; (C) $\ln(e^x) = x$ ignores the "1+"; (D) is a wrong antiderivative.

Final Answer: $\ln(1+e^x) + C \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q13

Q14.

Solution

Concept — Integral of an even function over a symmetric interval: If $g(-x) = g(x)$, then $\int_{-a}^{a} g(x)\,dx = 2\int_0^a g(x)\,dx$.

Step 1 — Test the integrand: $g(x) = x^2$ gives $g(-x) = (-x)^2 = x^2 = g(x)$, so x^2 is even.

Step 2 — Apply the property:

$$\int_{-2}^{2} x^2\,dx = 2\int_0^2 x^2\,dx.$$

$$= 2\left[\frac{x^3}{3}\right]_0^2.$$

$$= 2 \cdot \frac{8}{3}.$$
$$= \frac{16}{3}.$$

Why other options are wrong: (A) $\frac{8}{3}$ forgets the factor 2 from symmetry; (C) 0 wrongly treats x^2 as odd; (D) 8 drops the division by 3.

Final Answer: The integral equals $\frac{16}{3} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q14

Q15.

Solution

Concept — Local minimum via second derivative: At a critical point where $f'(x) = 0$, a local minimum occurs if $f''(x) > 0$.

Step 1 — Critical points:

$$f(x) = x^3 - 12x.$$
$$f'(x) = 3x^2 - 12 = 0.$$
$$x^2 = 4 \;\Rightarrow\; x = \pm 2.$$

Step 2 — Second-derivative test:

$$f''(x) = 6x.$$

At $x = 2$, $f''(2) = 12 > 0$, so $x = 2$ gives a local minimum.

Step 3 — Minimum value:

$$f(2) = 2^3 - 12(2) = 8 - 24 = -16.$$

Why other options are wrong: (A) 16 is the local maximum value at $x = -2$; (B) 0 and (D) -8 are not extreme values.

Final Answer: Local minimum value is $-16 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q15

Q16.

Solution

Concept — Area of a triangle: Area $= \frac{1}{2}|\vec{a}\times\vec{b}|$ for two adjacent side vectors $\vec{a},\vec{b}$.

Step 1 — Compute the cross product:

$$\vec{a}\times\vec{b} = (3\hat{\imath}+\hat{\jmath})\times(\hat{\imath}+3\hat{\jmath}).$$

$$= 3\cdot 3(\hat{\imath}\times\hat{\jmath}) + 1\cdot 1(\hat{\jmath}\times\hat{\imath}) = 9\hat{k} - \hat{k} = 8\hat{k}.$$

Step 2 — Take half the magnitude:

$$\frac{1}{2}|\vec{a}\times\vec{b}| = \frac{1}{2}|8\hat{k}| = \frac{1}{2}\times 8 = 4.$$

Why other options are wrong: (A) 8 is $|\vec{a}\times\vec{b}|$, the parallelogram area; (C) 2 halves the answer again; (D) 16 doubles the parallelogram area.

Final Answer: Area $= 4 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q16

Q17.

Solution

Concept — Parallel lines: Two lines are parallel when their direction ratios are proportional, and then the angle between them is 0°.

Step 1 — Check proportionality: Compare $1,2,2$ and $2,4,4$:

$$\frac{2}{1} = \frac{4}{2} = \frac{4}{2} = 2.$$

All ratios equal 2, so the direction ratios are proportional.

Step 2 — Interpret: Proportional direction ratios mean the lines are parallel, so the angle between them is 0°.

Why other options are wrong: (A) 90° needs a zero dot product; (C) 45° and (D) 60° would need non-proportional ratios giving those cosines.

Final Answer: The angle is $0^\circ \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q17

Q18.

Solution

Concept — Independence of complements: If A, B are independent, so are A' and B', and $P(A' \cap B') = P(A')\,P(B')$.

Step 1 — Complement probabilities:

$$P(A') = 1 - 0.3 = 0.7.$$

$$P(B') = 1 - 0.4 = 0.6.$$

Step 2 — Multiply:

$$P(A' \cap B') = 0.7 \times 0.6.$$
$$= 0.42.$$

Why other options are wrong: (A) 0.12 is $P(A)P(B) = P(A \cap B)$; (B) 0.7 is only $P(A')$; (C) 0.58 subtracts instead of multiplying.

Final Answer: $P(A' \cap B') = 0.42 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q18

Q19.

Solution

Concept — Assertion–Reason analysis: Judge each statement's truth, then decide whether R correctly explains A.

Step 1 — Assertion: For $f(x) = x^2$, $f'(x) = 2x$. Testing: $f'(-x) = 2(-x) = -2x = -f'(x)$, so f' is odd. Hence A is **true**.

Step 2 — Reason: If g is even and differentiable, then $g(-x) = g(x)$; differentiating both sides gives $-g'(-x) = g'(x)$, i.e. $g'(-x) = -g'(x)$, so g' is odd. Hence R is **true**.

Step 3 — Does R explain A? The general result in R (derivative of an even function is odd) is exactly why the specific $2x$ in A is odd. So R correctly explains A.

Why other options are wrong: (B) denies the explanation, which is valid here; (C),(D) misjudge a truth value.

Final Answer: Both true, R the correct explanation $\Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q19

Q20.

Solution

Concept — Inverse sine of a sine: $\sin^{-1}(\sin x) = x$ holds only when $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$; otherwise it must be reduced to the principal branch.

Step 1 — Test the Assertion: Here $x = \frac{\pi}{3}$, and $\frac{\pi}{3} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

So $\sin^{-1}\left(\sin\frac{\pi}{3}\right) = \frac{\pi}{3}$, and A is **true**.

Step 2 — Test the Reason: The claim "$\sin^{-1}(\sin x) = x$ for every real x" is **false**; for example, $\sin^{-1}\left(\sin\frac{5\pi}{6}\right) = \frac{\pi}{6} \neq \frac{5\pi}{6}$. So R is false.

Step 3 — Conclusion: A is true but R is false.

Why other options are wrong: (A),(B) require R true; (D) requires A false.

Final Answer: A true, R false $\Rightarrow \boxed{\text{C}}$

$\boxed{\textbf{Answer: (C)}}$ Go Back to Q20

Q21.

Solution

Concept — Principal values: $\sin^{-1}$ lies in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $\cos^{-1}$ lies in $[0, \pi]$.

Step 1 — Evaluate $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$:

$$\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} \Rightarrow \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}.$$

Step 2 — Evaluate $\cos^{-1}\left(\frac{1}{2}\right)$:

$$\cos\frac{\pi}{3} = \frac{1}{2} \Rightarrow \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}.$$

Step 3 — Add:

$$\frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}.$$

Final Answer: $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \cos^{-1}\left(\frac{1}{2}\right) = \frac{2\pi}{3}$. Go Back to Q21

Q22.

Solution

Concept — Logarithmic differentiation: Used when the variable appears in both base and exponent.

Step 1 — Take natural log of both sides:

$$y = x^{\cos x}.$$

$$\ln y = \cos x \cdot \ln x.$$

Step 2 — Differentiate both sides w.r.t. x: (product rule on the right)

$$\frac{1}{y}\frac{dy}{dx} = (-\sin x) \cdot \ln x + \cos x \cdot \frac{1}{x}.$$

$$\frac{1}{y}\frac{dy}{dx} = -\sin x\ \ln x + \frac{\cos x}{x}.$$

Step 3 — Solve for $\frac{dy}{dx}$:

$$\frac{dy}{dx} = y\left[\frac{\cos x}{x} - \sin x\ \ln x\right].$$

$$= x^{\cos x}\left[\frac{\cos x}{x} - \sin x\ \ln x\right].$$

Final Answer: $\frac{dy}{dx} = x^{\cos x}\left[\frac{\cos x}{x} - \sin x\ \ln x\right]$. **Go Back to Q22**

Q23.

Solution

Concept — Integration by parts: $\int u\,dv = uv - \int v\,du$, choosing u by the ILATE rule.

Step 1 — Choose parts: Let $u = x$ and $dv = \cos x\,dx$.

Then $du = dx$ and $v = \sin x$.

Step 2 — Apply the formula:

$$\int x\cos x\,dx = x\sin x - \int \sin x\,dx.$$

$$= x\sin x - (-\cos x) + C.$$

$$= x\sin x + \cos x + C.$$

Final Answer: $\int x\cos x\,dx = x\sin x + \cos x + C.$

OR — $\int \ln x\,dx$**:**

Step 1 — Parts with $u = \ln x$, $dv = dx$: then $du = \frac{1}{x}\,dx$ and $v = x$.

Step 2 — Apply the formula:

$$\int \ln x\,dx = x\ln x - \int x \cdot \frac{1}{x}\,dx.$$

$$= x\ln x - \int 1\,dx.$$

$$= x\ln x - x + C.$$

Final Answer (OR): $\int \ln x\,dx = x\ln x - x + C.$ **Go Back to Q23**

Q24.

Solution

Concept — Perpendicular vectors: $\vec{a} \perp \vec{b} \iff \vec{a}\cdot\vec{b} = 0.$

Step 1 — Compute the dot product:

$$\vec{a}\cdot\vec{b} = (3)(2) + (-1)(3) + (\lambda)(-1).$$

$$= 6 - 3 - \lambda.$$

$$= 3 - \lambda.$$

Step 2 — Set equal to zero and solve:

$$3 - \lambda = 0.$$

$$\lambda = 3.$$

Final Answer: $\lambda = 3.$ **Go Back to Q24**

Q25.

Solution

Concept — Equally likely outcomes: Probability $= \dfrac{\text{favourable outcomes}}{\text{total outcomes}}$.

Step 1 — Total outcomes: Two throws of a die give $6 \times 6 = 36$ equally likely outcomes.

Step 2 — Favourable outcomes (sum $= 7$):

$$(1,6),\ (6,1),\ (2,5),\ (5,2),\ (3,4),\ (4,3).$$

There are 6 such outcomes.

Step 3 — Probability:

$$P(\text{sum} = 7) = \frac{6}{36} = \frac{1}{6}.$$

Final Answer: $P(\text{sum} = 7) = \frac{1}{6}$.

OR — $P(A \mid B)$**:**

Step 1 — Formula: $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$.

Step 2 — Substitute:

$$P(A \mid B) = \frac{0.25}{0.4} = \frac{25}{40} = \frac{5}{8}.$$

Final Answer (OR): $P(A \mid B) = \frac{5}{8}$. **Go Back to Q25**

Q26.

Solution

Concept — Partial fractions: Split a proper rational function into simpler fractions, then integrate each.

Step 1 — Set up the decomposition:

$$\frac{1}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}.$$

$$1 = A(x+2) + B(x-1).$$

Step 2 — Find A and B:

Put $x = 1$: $1 = A(3) + B(0) \Rightarrow A = \frac{1}{3}$.

Put $x = -2$: $1 = A(0) + B(-3) \Rightarrow B = -\frac{1}{3}$.

Step 3 — Integrate:

$$\int \frac{1}{(x-1)(x+2)}\,dx = \int \left(\frac{1/3}{x-1} - \frac{1/3}{x+2}\right) dx.$$

$$= \frac{1}{3}\ln|x-1| - \frac{1}{3}\ln|x+2| + C.$$

$$= \frac{1}{3}\ln\left|\frac{x-1}{x+2}\right| + C.$$

Final Answer: $\frac{1}{3}\ln\left|\frac{x-1}{x+2}\right| + C$. Go Back to Q26

Q27.

Solution

Concept — Linear differential equation: For $\frac{dy}{dx} + Py = Q$, the integrating factor is I.F. $= e^{\int P\,dx}$ and the solution is $y \cdot \text{I.F.} = \int Q \cdot \text{I.F.}\,dx$.

Step 1 — Identify P, Q: Here $P = 2$, $Q = e^{-x}$.

Step 2 — Integrating factor:

$$\text{I.F.} = e^{\int 2\,dx} = e^{2x}.$$

Step 3 — Solve:

$$y\,e^{2x} = \int e^{-x} \cdot e^{2x}\,dx = \int e^{x}\,dx.$$

$$y\,e^{2x} = e^{x} + C.$$

$$y = e^{-x} + C\,e^{-2x}.$$

Final Answer: $y = e^{-x} + C\,e^{-2x}$.

OR — Variables separable $\frac{dy}{dx} = e^{x-y}$**:**

Step 1 — Separate variables: Write $e^{x-y} = e^{x}e^{-y}$, so

$$\frac{dy}{dx} = e^{x}e^{-y}.$$

$$e^{y}\,dy = e^{x}\,dx.$$

Step 2 — Integrate both sides:

$$\int e^y \, dy = \int e^x \, dx.$$

$$e^y = e^x + C.$$

Final Answer (OR): $e^y = e^x + C$. **Go Back to Q27**

Q28.

Solution

Concept — Inverse by adjoint: $A^{-1} = \frac{1}{|A|}\text{adj}(A)$, valid when $|A| \neq 0$.

Step 1 — Determinant:

$$|A| = \begin{vmatrix} 2 & 1 \\ 5 & 3 \end{vmatrix} = (2)(3) - (1)(5) = 6 - 5 = 1.$$

Step 2 — Adjoint of a 2×2 matrix: Swap the leading diagonal entries and negate the off-diagonal entries:

$$\text{adj}(A) = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}.$$

Step 3 — Inverse:

$$A^{-1} = \frac{1}{1}\begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}.$$

Final Answer: $A^{-1} = \begin{bmatrix} 3 & -1 \\ -5 & 2 \end{bmatrix}$. **Go Back to Q28**

Q29.

Solution

Concept — Monotonicity: f increases where $f'(x) > 0$ and decreases where $f'(x) < 0$.

Step 1 — Differentiate:

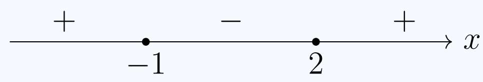
$$f(x) = 2x^3 - 3x^2 - 12x + 5.$$

$$f'(x) = 6x^2 - 6x - 12.$$

$$= 6(x^2 - x - 2) = 6(x-2)(x+1).$$

Step 2 — Critical points: $f'(x) = 0$ at $x = -1$ and $x = 2$.

Step 3 — Sign analysis:



For $x < -1$: $f' > 0$; for $-1 < x < 2$: $f' < 0$; for $x > 2$: $f' > 0$.

Conclusion: Increasing on $(-\infty, -1) \cup (2, \infty)$; decreasing on $(-1, 2)$.

Final Answer: Increasing on $(-\infty, -1) \cup (2, \infty)$, decreasing on $(-1, 2)$.

OR — Tangent to $y = \sqrt{x}$ at $(4, 2)$:

Step 1 — Slope: $\dfrac{dy}{dx} = \dfrac{1}{2\sqrt{x}}$, so at $x = 4$ slope $= \dfrac{1}{2\sqrt{4}} = \dfrac{1}{4}$.

Step 2 — Equation: $y - 2 = \dfrac{1}{4}(x - 4) \Rightarrow y = \dfrac{x}{4} + 1$.

Final Answer (OR): $y = \dfrac{x}{4} + 1$. Go Back to Q29

Q30.

Solution

Concept — Line through two points: A line through $\vec{a}$ and $\vec{b}$ has direction $\vec{b} - \vec{a}$, so $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$.

Step 1 — Direction vector: With $A(1, -1, 2)$ and $B(3, 2, 4)$,

$$\vec{b} - \vec{a} = (3 - 1)\hat{i} + (2 - (-1))\hat{j} + (4 - 2)\hat{k} = 2\hat{i} + 3\hat{j} + 2\hat{k}.$$

Step 2 — Write the equation:

$$\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 2\hat{k}).$$

Step 3 — Test the point $(5, 5, 6)$: The point lies on the line if some λ satisfies all three component equations.

$$1 + 2\lambda = 5 \;\Rightarrow\; \lambda = 2.$$

$$-1 + 3\lambda = 5 \;\Rightarrow\; \lambda = 2.$$

$$2 + 2\lambda = 6 \;\Rightarrow\; \lambda = 2.$$

Step 4 — Conclusion: All three give the same value $\lambda = 2$, so the point $(5, 5, 6)$

lies on the line.

Final Answer: $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 2\hat{k})$; the point $(5, 5, 6)$ lies on it (at $\lambda = 2$). **Go Back to Q30**

Q31.

Solution

Concept — One-one (injective) and onto (surjective): *One-one*: $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$. *Onto*: every y in the codomain has a pre-image.

Step 1 — One-one: Suppose $f(x_1) = f(x_2)$.

$$3x_1 - 2 = 3x_2 - 2.$$

$$3x_1 = 3x_2.$$

$$x_1 = x_2.$$

Hence f is one-one.

Step 2 — Onto: Let $y \in \mathbb{R}$ be arbitrary. Solve $y = 3x - 2$ for x:

$$x = \frac{y+2}{3} \in \mathbb{R}.$$

Then $f(x) = 3\left(\frac{y+2}{3}\right) - 2 = (y+2) - 2 = y$.
So every y has a pre-image; f is onto.

Step 3 — Conclusion: f is both one-one and onto, hence bijective.

Final Answer: $f(x) = 3x - 2$ is one-one and onto (a bijection). **Go Back to Q31**

Q32.

Solution

Concept — Matrix method: Write $AX = B$; then $X = A^{-1}B$ when $|A| \neq 0$.

Step 1 — Form the matrices:

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & 3 \\ 1 & 1 & -1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 7 \\ 12 \\ 0 \end{bmatrix}.$$

Step 2 — Determinant of A:

$$|A| = 1\big[(-1)(-1) - (3)(1)\big] - 2\big[(2)(-1) - (3)(1)\big] + 1\big[(2)(1) - (-1)(1)\big].$$

$$= 1(1-3) - 2(-2-3) + 1(2+1).$$

$$= 1(-2) - 2(-5) + 1(3).$$

$$= -2 + 10 + 3 = 11 \ (\neq 0).$$

Step 3 — Cofactors and adjoint: Computing the cofactor matrix and transposing gives

$$\text{adj}(A) = \begin{bmatrix} -2 & 3 & 7 \\ 5 & -2 & -1 \\ 3 & 1 & -5 \end{bmatrix}.$$

Step 4 — Solve $X = \dfrac{1}{|A|}\,\text{adj}(A)\,B$:

$$X = \frac{1}{11}\begin{bmatrix} -2 & 3 & 7 \\ 5 & -2 & -1 \\ 3 & 1 & -5 \end{bmatrix}\begin{bmatrix} 7 \\ 12 \\ 0 \end{bmatrix}.$$

$$= \frac{1}{11}\begin{bmatrix} -2\cdot 7 + 3\cdot 12 + 7\cdot 0 \\ 5\cdot 7 - 2\cdot 12 - 1\cdot 0 \\ 3\cdot 7 + 1\cdot 12 - 5\cdot 0 \end{bmatrix}.$$

$$= \frac{1}{11}\begin{bmatrix} -14+36 \\ 35-24 \\ 21+12 \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 22 \\ 11 \\ 33 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}.$$

Step 5 — Verify: $x + 2y + z = 2 + 2 + 3 = 7$ ✓; $2x - y + 3z = 4 - 1 + 9 = 12$ ✓; $x + y - z = 2 + 1 - 3 = 0$ ✓.

Final Answer: $x = 2,\ y = 1,\ z = 3$. **Go Back to Q32**

Q33.

Solution

Concept — Area between two curves: On $[a, b]$ where the line is above the parabola, area $= \displaystyle\int_a^b (\text{upper} - \text{lower})\,dx$.

Step 1 — Points of intersection: Set $x^2 = 2x$:

$$x^2 - 2x = 0.$$

$$x(x-2) = 0 \;\Rightarrow\; x = 0 \text{ or } x = 2.$$

Step 2 — Decide upper curve: For $0 < x < 2$, $2x > x^2$, so the line $y = 2x$ is above the parabola $y = x^2$.

Step 3 — Set up the integral:

$$\text{Area} = \int_0^2 (2x - x^2)\,dx.$$

Step 4 — Integrate:

$$= \left[x^2 - \frac{x^3}{3}\right]_0^2.$$

$$= \left(2^2 - \frac{2^3}{3}\right) - (0).$$

$$= 4 - \frac{8}{3}.$$

$$= \frac{12-8}{3} = \frac{4}{3}.$$

Final Answer: Area $= \dfrac{4}{3}$ square units.

OR — Area under $y = \sin x$, $0 \le x \le \pi$:

Step 1 — Set up the integral (since $\sin x \ge 0$ on $[0, \pi]$):

$$\text{Area} = \int_0^{\pi} \sin x\,dx.$$

Step 2 — Integrate:

$$= \big[-\cos x\big]_0^{\pi}.$$

$$= (-\cos\pi) - (-\cos 0).$$

$$= -(-1) - (-1) = 1 + 1 = 2.$$

Final Answer (OR): Area $= 2$ square units. Go Back to Q33

Q34.

Solution

Concept — Corner-point method: The optimum of a linear objective over a bounded feasible region occurs at a corner (vertex).

Step 1 — Find the corner points: The constraints $x + y \leq 4$, $x + 3y \leq 6$, $x \geq 0$, $y \geq 0$ bound the region with vertices:

$$O(0,0), \quad (4,0), \quad (0,2).$$

The lines $x + y = 4$ and $x + 3y = 6$ intersect where

$$(x + 3y) - (x + y) = 6 - 4 \Rightarrow 2y = 2 \Rightarrow y = 1, \; x = 3.$$

So the fourth vertex is $(3,1)$.

Step 2 — Evaluate $Z = 5x + 3y$ at each corner:

$$Z(0,0) = 0.$$

$$Z(4,0) = 5(4) + 0 = 20.$$

$$Z(3,1) = 5(3) + 3(1) = 18.$$

$$Z(0,2) = 5(0) + 3(2) = 6.$$

Step 3 — Pick the maximum: The largest value is 20 at $(4,0)$.

Final Answer: Z is maximum $= 20$ at $(x,y) = (4,0)$. **Go Back to Q34**

Q35.

Solution

Concept — Shortest distance between skew lines: For $\vec{r} = \vec{a}_1 + \lambda\vec{b}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{b}_2$,

$$d = \frac{\left|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)\right|}{|\vec{b}_1 \times \vec{b}_2|}.$$

Step 1 — Identify vectors: $\vec{a}_1 = \hat{i} + \hat{j} + \hat{k}$, $\vec{b}_1 = 2\hat{i} + 3\hat{j} + 4\hat{k}$; $\vec{a}_2 = 2\hat{i} + 4\hat{j} + 5\hat{k}$, $\vec{b}_2 = 3\hat{i} + 4\hat{j} + 5\hat{k}$.

Step 2 — Cross product $\vec{b}_1 \times \vec{b}_2$:

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}.$$

$$= \hat{i}\big[(3)(5) - (4)(4)\big] - \hat{j}\big[(2)(5) - (4)(3)\big] + \hat{k}\big[(2)(4) - (3)(3)\big].$$

$$= \hat{i}(15 - 16) - \hat{j}(10 - 12) + \hat{k}(8 - 9).$$

$$= -\hat{i} + 2\hat{j} - \hat{k}.$$

Step 3 — Its magnitude:

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{(-1)^2 + 2^2 + (-1)^2} = \sqrt{1 + 4 + 1} = \sqrt{6}.$$

Step 4 — $(\vec{a}_2 - \vec{a}_1)$ and the dot product:

$$\vec{a}_2 - \vec{a}_1 = (2-1)\hat{i} + (4-1)\hat{j} + (5-1)\hat{k} = \hat{i} + 3\hat{j} + 4\hat{k}.$$

$$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = (1)(-1) + (3)(2) + (4)(-1) = -1 + 6 - 4 = 1.$$

Step 5 — Distance:

$$d = \frac{|1|}{\sqrt{6}} = \frac{1}{\sqrt{6}} \text{ units.}$$

Final Answer: Shortest distance $= \dfrac{1}{\sqrt{6}}$ units.

OR — Angle between line and plane:

Step 1 — Direction $\vec{b} = 2\hat{i} - \hat{j} + 2\hat{k}$, normal $\vec{n} = 2\hat{i} + \hat{j} - 2\hat{k}$.

Step 2 — Use $\sin\theta = \dfrac{|\vec{b} \cdot \vec{n}|}{|\vec{b}||\vec{n}|}$:

$$\vec{b} \cdot \vec{n} = (2)(2) + (-1)(1) + (2)(-2) = 4 - 1 - 4 = -1.$$

$$|\vec{b}| = \sqrt{4 + 1 + 4} = 3, \qquad |\vec{n}| = \sqrt{4 + 1 + 4} = 3.$$

$$\sin\theta = \frac{|-1|}{3 \times 3} = \frac{1}{9}.$$

Final Answer (OR): $\theta = \sin^{-1}\left(\dfrac{1}{9}\right)$. **Go Back to Q35**

Q36.

Solution

Concept — Maxima by derivatives: Build the area function of one variable, set $\frac{dA}{dy} = 0$, and select the value giving a genuine maximum.

(i) Area function: The three fenced sides use $x + 2y = 60$, so $x = 60 - 2y$. The area is

$$A = x \cdot y = (60 - 2y)\, y = 60y - 2y^2, \qquad 0 < y < 30.$$

(ii) Differentiate:

$$\frac{dA}{dy} = 60 - 4y.$$

(iii) Critical point and maximum: Set $\frac{dA}{dy} = 0$:

$$60 - 4y = 0.$$

$$4y = 60.$$

$$y = 15.$$

Second-derivative check: $\frac{d^2A}{dy^2} = -4 < 0$, so $y = 15$ gives a maximum.

Then $x = 60 - 2(15) = 30$, and the maximum area is

$$A = 30 \times 15 = 450.$$

Final Answer: $A = 60y - 2y^2$; $\frac{dA}{dy} = 60 - 4y$; maximum at $y = 15$ m (with $x = 30$ m) giving $A_{\max} = 450$ m^2. **Go Back to Q36**

Q37.

Solution

Concept — Bayes' theorem: $P(E_i \mid A) = \dfrac{P(E_i)P(A \mid E_i)}{\sum_j P(E_j)P(A \mid E_j)}$.

(i) Prior probabilities: A coin is chosen at random, so

$$P(\text{Coin A}) = P(\text{Coin B}) = \frac{1}{2}.$$

(ii) Conditional (head) probabilities:

$$P(\text{Head} \mid \text{Coin A}) = \frac{1}{2}, \qquad P(\text{Head} \mid \text{Coin B}) = \frac{3}{4}.$$

(iii) Apply Bayes' theorem:

$$P(\text{Coin B} \mid \text{Head}) = \frac{\frac{1}{2} \cdot \frac{3}{4}}{\frac{1}{2} \cdot \frac{1}{2} + \frac{1}{2} \cdot \frac{3}{4}}.$$

Cancel the common $\frac{1}{2}$:

$$= \frac{\frac{3}{4}}{\frac{1}{2} + \frac{3}{4}}.$$

Take LCM 4 in the denominator:

$$\frac{1}{2} = \frac{2}{4}, \qquad \frac{3}{4} = \frac{3}{4}.$$

$$= \frac{3/4}{(2+3)/4} = \frac{3}{5}.$$

Final Answer: $P(\text{Coin B} \mid \text{Head}) = \frac{3}{5}$. **Go Back to Q37**

Q38.

Solution

Concept — Related rates: Differentiate the area formula with respect to time and use the chain rule, $\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt}$.

(i) Area and its derivative: The area enclosed by a circle of radius r is

$$A = \pi r^2.$$

$$\frac{dA}{dr} = 2\pi r.$$

(ii) Rate of change with time: Given $\frac{dr}{dt} = 3$ cm/s,

$$\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt} = 2\pi r \cdot 3 = 6\pi r.$$

(iii) Evaluate at $r = 8$ cm:

$$\left.\frac{dA}{dt}\right|_{r=8} = 6\pi(8).$$

$$= 48\pi.$$

So the area is increasing at 48π cm^2/s.

Final Answer: $A = \pi r^2$, $\dfrac{dA}{dr} = 2\pi r$; $\dfrac{dA}{dt} = 6\pi r$; at $r = 8$ cm the area increases at 48π cm^2/s. **Go Back to Q38**

Answer Key – Section A (Q1–Q20)

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	B	3	C	4	D	5	A
6	C	7	D	8	D	9	A	10	B
11	C	12	D	13	A	14	B	15	C
16	B	17	B	18	D	19	A	20	C

Sections B–E are descriptive; refer to the Detailed Solutions above for full model answers and step marking.