CBSE Class 12 Mathematics(Set 65/1/3) Question Paper

Time Allowed :3 Hour	Maximum Marks :70	Total Questions :38

General Instructions

Read the following instructions very carefully and strictly follow them:

(i) This Question paper contains 38 questions. All questions are compulsory.

(ii) Question paper is divided into FIVE Sections – Section A, B, C, D and E.

(iii) In Section A – Question Number 1 to 18 are Multiple Choice Questions (MCQs) and Question Number 19 & 20 are Assertion-Reason based questions of 1 mark each.

(iv) In Section B – Question Number 21 to 25 are Very Short Answer (VSA) type questions, carrying 2 marks each.

(v) In Section C – Question Number 26 to 31 are Short Answer (SA) type questions, carrying 3 marks each.

(vi) In Section D – Question Number 32 to 35 are Long Answer (LA) type questions, carrying 5 marks each.

(vii) In Section E – Question Number 36 to 38 are case study based questions, carrying 4 marks each.

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.

(ix) Use of calculator is NOT allowed.

1. If the feasible region of a linear programming problem with objective function $Z = ax + by$ is bounded, then which of the following is correct?

(A) It will only have a maximum value.
(B) It will only have a minimum value.
(C) It will have both maximum and minimum values.
(D) It will have neither maximum nor minimum value.

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2. The unit vector perpendicular to the vectors $\hat{i} - \hat{j}$ and $\hat{i} + \hat{j}$ is

(A) $\hat{k}$
(B) $-\hat{k}$
(C) $\frac{\hat{i} - \hat{j}}{\sqrt{2}}$
(D) $\frac{\hat{i} + \hat{j}}{\sqrt{2}}$

3. If $\int_0^1 \frac{e^x}{1+x}\,dx = \alpha$, then $\int_0^1 \frac{e^x}{(1+x)^2}\,dx$ is equal to

(A) $\alpha - 1 + \frac{e}{2}$

(B) $\alpha + 1 - \frac{e}{2}$

(C) $\alpha - 1 - \frac{e}{2}$

(D) $\alpha + 1 + \frac{e}{2}$

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4. If $\int \frac{2^{\frac{1}{x}}}{x^2}\,dx = k\,2^{\frac{1}{x}} + C$, then k is equal to

(A) $-\frac{1}{\log 2}$

(B) $-\log 2$

(C) -1

(D) $\frac{1}{2}$

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5. If $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then A^{-1} is

(A) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(D) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

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6. If

$$\begin{bmatrix} x+y & 3y \\ 3x & x+3 \end{bmatrix} = \begin{bmatrix} 9 & 4x+y \\ x+6 & y \end{bmatrix}$$

then $x - y =$?

(A) -7

(B) -3

(C) 3
(D) 7

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7. Let M and N be two events such that $P(M) = 0.6$, $P(N) = 0.2$ and $P(M \cap N) = 0.15$. Then $P(M|N)$ is

(A) $\frac{7}{8}$
(B) $\frac{2}{5}$
(C) $\frac{1}{2}$
(D) $\frac{2}{3}$

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8. Which of the following is not a homogeneous function of x and y?

(A) $y^2 - xy$
(B) $7x - 3y$
(C) $\sin^2\left(\frac{y}{x}\right) + \frac{y}{x}$
(D) $\tan x - \sec y$

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9. If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, $|\vec{a}| = \sqrt{37}$, $|\vec{b}| = 3$ and $|\vec{c}| = 4$, then the angle between $\vec{b}$ and $\vec{c}$ is

(A) $\frac{\pi}{6}$
(B) $\frac{\pi}{4}$
(C) $\frac{\pi}{3}$
(D) $\frac{\pi}{2}$

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10. If $f(x) = |x| + |x - 1|$, then which of the following is correct?

(A) $f(x)$ is both continuous and differentiable at $x = 0$ and $x = 1$.
(B) $f(x)$ is differentiable but not continuous at $x = 0$ and $x = 1$.
(C) $f(x)$ is continuous but not differentiable at $x = 0$ and $x = 1$.
(D) $f(x)$ is neither continuous nor differentiable at $x = 0$ and $x = 1$.

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11. A system of linear equations is represented as $AX = B$, where A is the coefficient matrix, X is the variable matrix and B is the constant matrix. Then

(A) Consistent, if $|A| \neq 0$, solution is given by $X = A^{-1}B$.
(B) Inconsistent if $|A| = 0$ and $\text{adj}(A)B = 0$.
(C) Inconsistent if $|A| = 0$.
(D) May or may not be consistent if $|A| = 0$ and $\text{adj}(A)B = 0$.

12. The absolute maximum value of the function $f(x) = x^3 - 3x + 2$ in $[0, 2]$ is
(A) 0
(B) 2
(C) 4
(D) 5

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13. The order and degree of the differential equation

$$1 + \left(\frac{dy}{dx}\right)^2 = \frac{d^2y}{dx^2}$$

are
(A) Order 1, degree 1
(B) Order 1, degree 2
(C) Order 2, degree 1
(D) Order 2, degree 2

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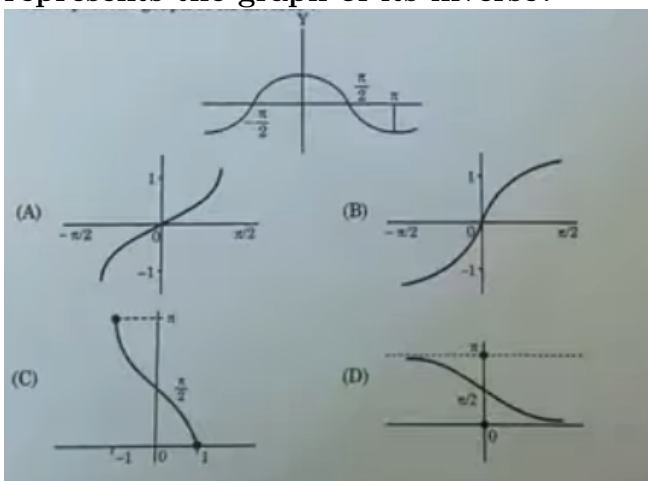
14. The graph of a trigonometric function is as shown. Which of the following represents the graph of its inverse?



(A) Graph (A)
(B) Graph (B)
(C) Graph (C)
(D) Graph (D)

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15. The corner points of the feasible region in graphical representation of a L.P.P. are $(72, 15)$, $(40, 15)$ and $(40, 10)$. If $Z = 18x + 19y$ is the objective function, then
(A) Z is maximum at $(72, 15)$, minimum at $(40, 10)$
(B) Z is maximum at $(15, 20)$, minimum at $(40, 15)$
(C) Z is maximum at $(40, 15)$, minimum at $(15, 20)$
(D) Z is maximum at $(40, 15)$, minimum at $(72, 15)$

16. Let

$$A = \begin{bmatrix} 1 & -2 & -1 \\ 3 & 4 & -1 \\ -3 & 2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} -2 \\ -5 \\ -7 \end{bmatrix}, \quad C = [9\ 8\ 7]$$

Which of the following is defined?

(A) Only AB
(B) Only AC
(C) Only BA
(D) All AB, AC and BA

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17. If A and B are invertible matrices, then which of the following is not correct?

(A) $(A+B)^{-1} = B^{-1} + A^{-1}$
(B) $(AB)^{-1} = B^{-1}A^{-1}$
(C) $\text{adj}(A) = |A|A^{-1}$
(D) $I^{-1} = I$

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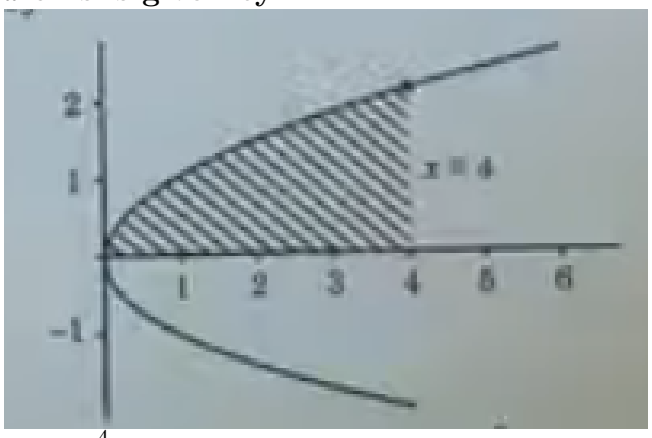
18. The area of the shaded region bounded by the curves $y^2 = x$, $x = 4$ and the x-axis is given by



(A) $\displaystyle\int_0^4 x\,dx$
(B) $\displaystyle\int_0^2 y^2\,dy$
(C) $\displaystyle 2\int_0^4 \sqrt{x}\,dx$
(D) $\displaystyle\int_0^4 \sqrt{x}\,dx$

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19. Assertion (A):

$$f(x) = \begin{cases} 3x - 8, & x \leq 5 \\ 2x, & x > 5 \end{cases}$$

is continuous at $x = 5$ for $k = \dfrac{5}{2}$.
Reason (R): For a function f to be continuous at $x = a$,

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$$

Choose the correct answer from the options below.
(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

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20. Assertion (A): Let $\mathbb{Z}$ be the set of integers. A function $f : \mathbb{Z} \to \mathbb{Z}$ defined by

$$f(x) = 3x - 5, \quad x \in \mathbb{Z}$$

is a bijective function.
Reason (R): A function is bijective if it is both injective and surjective.
Choose the correct answer from the options below.
(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

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21. The diagonals of a parallelogram are given by

$$\vec{a} = 2\hat{i} - \hat{j} + \hat{k} \quad \textbf{and} \quad \vec{b} = \hat{i} + 3\hat{j} - \hat{k}.$$

Find the area of the parallelogram.

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23. (a) Two friends while flying kites from different locations find the strings of their kites crossing each other. The strings can be represented by the vectors

$$\vec{a} = 3\hat{i} + \hat{j} + 2\hat{k} \quad \text{and} \quad \vec{b} = 2\hat{i} - 2\hat{j} + 4\hat{k}.$$

Determine the angle formed between the kite strings. Assume there is no slack in the strings.

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23. (b) Find a vector of magnitude 21 units in the direction opposite to that of $\overrightarrow{AB}$, where $A(2, 1, 3)$ and $B(6, -1, 0)$.

24. Solve for x:

$$2\tan^{-1}x + \sin^{-1}\left(\frac{2x}{1+x^2}\right) = 4\sqrt{3}$$

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25. (a) Differentiate $2^{\cos^2 x}$ with respect to $\cos^2 x$.

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25. (b) If $\tan^{-1}(x^2+y^2) = a^2$, then find $\frac{dy}{dx}$.

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26. Solve the following Linear Programming Problem graphically:
Maximize

$$Z = 8x + 9y$$

Subject to the constraints

$$2x + 3y \leq 6$$

$$3x - 2y \leq 6$$

$$y \leq 1$$

$$x \geq 0, \quad y \geq 0$$

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27. (a) Find:

$$\int \frac{2x-1}{(x-1)(x+2)(x-3)}\,dx$$

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27. (b) Evaluate:

$$\int_0^5 (|x-1| + |x-2| + |x-5|)\,dx$$

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28. A spherical medicine ball when dropped in water dissolves in such a way that the rate of decrease of volume at any instant is proportional to its surface area. Calculate the rate of decrease of its radius.

29. Sketch the graph of $y = |x + 3|$ and find the area of the region enclosed by the curve and the x-axis between $x = -6$ and $x = 0$, using integration.

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30. (a) Verify that the lines given by

$$\vec{r} = (1 - \lambda)\hat{i} + (0 - 2\lambda)\hat{j} + (3 - 2\lambda)\hat{k}$$

$$\vec{r} = (\mu + 1)\hat{i} + (2\mu - 1)\hat{j} - (2\mu + 1)\hat{k}$$

are skew lines. Hence find the shortest distance between them.

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30. (b) During a cricket match, the position of the bowler, the wicket keeper and the leg slip fielder are given by

$$\vec{B} = 2\hat{i} + 8\hat{j}, \quad \vec{W} = 6\hat{i} + 12\hat{j}, \quad \vec{F} = 12\hat{i} + 18\hat{j}$$

Calculate the ratio in which the wicket keeper divides the line segment joining the bowler and the leg slip fielder.

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31. (a) The probability distribution for the number of students being absent in a class on a Saturday is as follows:

X	0	2	4	5
$P(X)$	p	$2p$	$3p$	p

Where X is the number of students absent.

(i) Calculate p.

(ii) Calculate the mean number of absent students on Saturday.

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31. (b) For the vacancy advertised in the newspaper, 3000 candidates submitted applications. Two-thirds of the applicants were females and the rest were males. The selection was done through a written test. The probability that a male gets distinction in the written test is 0.4 and that a female gets distinction is 0.35. Find the probability that the candidate chosen at random will have a distinction in the written test.

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32. A school wants to allocate students into three clubs: Sports, Music and Drama, under the following conditions:

- The number of students in Sports club should be equal to the sum of the number of students in Music and Drama club.

- The number of students in Music club should be 20 more than half the number of students in Sports club.
- The total number of students to be allocated in all three clubs are 180.

Find the number of students allocated to different clubs, using matrix method.

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33. Find:

$$\int \sin^{-1}\sqrt{\frac{x}{a+x}}\,dx$$

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34. (a) If

$$\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y),$$

then prove that

$$\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}.$$

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34. (b) If

$$x=a\left(\cos\theta+\log\tan\frac{\theta}{2}\right), \qquad y=a\sin\theta,$$

find

$$\frac{d^2y}{dx^2} \quad \text{at} \quad \theta=\frac{\pi}{4}.$$

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35. (a) Find the image A' of the point $A(1,6,3)$ in the line

$$\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{3}.$$

Also find the equation of the line joining A and A'.

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35. (b) Find a point P on the line

$$\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}$$

such that its distance from the point $Q(2,4,-1)$ is 7 units. Also find the equation of the line joining P and Q.
