CBSE Class 12 Mathematics(Set 65/2/2) Question Paper with Solutions

Time Allowed :3 Hour	Maximum Marks :80	Total Questions :38

General Instructions

Read the following instructions very carefully and strictly follow them:

(i) This Question paper contains 38 questions. All questions are compulsory.

(ii) Question paper is divided into FIVE Sections – Section A, B, C, D and E.

(iii) In Section A – Question Number 1 to 18 are Multiple Choice Questions (MCQs) and Question Number 19 & 20 are Assertion-Reason based questions of 1 mark each.

(iv) In Section B – Question Number 21 to 25 are Very Short Answer (VSA) type questions, carrying 2 marks each.

(v) In Section C – Question Number 26 to 31 are Short Answer (SA) type questions, carrying 3 marks each.

(vi) In Section D – Question Number 32 to 35 are Long Answer (LA) type questions, carrying 5 marks each.

(vii) In Section E – Question Number 36 to 38 are case study based questions, carrying 4 marks each.

(viii) There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 3 questions in Section C, 2 questions in Section D and 2 questions in Section E.

(ix) Use of calculator is NOT allowed.

1. Evaluate:

$$\int \frac{dx}{1+\cos x}$$

(A) $\frac{1}{2}\tan\frac{x}{2}+C$
(B) $-\frac{1}{2}\cot\frac{x}{2}+C$
(C) $-\cot\frac{x}{2}+C$
(D) $\tan\frac{x}{2}+C$

Correct Answer: (4) $\tan\frac{x}{2}+C$

Solution:

Concept:
Use the trigonometric identity

$$1 + \cos x = 2\cos^2\frac{x}{2}$$

This identity helps simplify the integral.
Step 1: Substitute the identity.

$$\int \frac{dx}{1+\cos x} = \int \frac{dx}{2\cos^2\frac{x}{2}}$$
$$= \frac{1}{2}\int \sec^2\frac{x}{2}\,dx$$

Step 2: Apply substitution.
Let

$$u = \frac{x}{2}$$

Then

$$dx = 2\,du$$

Thus

$$\frac{1}{2}\int \sec^2\frac{x}{2}\,dx = \frac{1}{2}\int \sec^2 u(2\,du)$$
$$= \int \sec^2 u\,du$$

Step 3: Integrate.

$$\int \sec^2 u\,du = \tan u$$

Substitute $u = \frac{x}{2}$:

$$= \tan\frac{x}{2} + C$$

$$\therefore \int \frac{dx}{1+\cos x} = \tan\frac{x}{2} + C.$$

Quick Tip

Useful identity for trigonometric integrals:

$$1 + \cos x = 2\cos^2\frac{x}{2}$$

This often converts expressions into powers of secant or tangent.

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2. For $f(x) = x + \dfrac{1}{x}\ (x \neq 0)$

(A) Local maximum value is 2
(B) Local minimum value is -2
(C) Local maximum value is -2
(D) Local minimum value $>$ local maximum value

Correct Answer: (3) Local maximum value is -2

Solution:

Concept:
To find local maxima and minima of a function:

- Find the first derivative.
- Set $f'(x) = 0$ to obtain critical points.
- Use the second derivative test.

Step 1: Find the first derivative.

$$f(x) = x + \frac{1}{x}$$

$$f'(x) = 1 - \frac{1}{x^2}$$

Step 2: Find the critical points.

$$1 - \frac{1}{x^2} = 0$$

$$\frac{1}{x^2} = 1$$

$$x = \pm 1$$

Step 3: Find the second derivative.

$$f''(x) = \frac{2}{x^3}$$

Step 4: Apply second derivative test.
At $x = 1$:

$$f''(1) = 2 > 0$$

Thus $x = 1$ gives a **local minimum**.

$$f(1) = 1 + 1 = 2$$

At $x = -1$:

$$f''(-1) = -2 < 0$$

Thus $x = -1$ gives a **local maximum**.

$$f(-1) = -1 - 1 = -2$$

Step 5: State the result.
Local maximum value $= -2$.

$$\therefore \text{Option (C) is correct.}$$

Quick Tip

Second derivative test:

$$f''(x) > 0 \Rightarrow \text{local minimum}$$

$$f''(x) < 0 \Rightarrow \text{local maximum}$$

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3. Which of the following expressions will give the area of the region bounded by the curve $y = x^2$ and the line $y = 16$?

(A) $\displaystyle\int_0^4 x^2\,dx$

(B) $2\displaystyle\int_0^4 x^2\,dx$

(C) $\displaystyle\int_0^{16} \sqrt{y}\,dy$

(D) $2\displaystyle\int_0^{16} \sqrt{y}\,dy$

Correct Answer: (4) $2\int_0^{16} \sqrt{y}\,dy$

Solution:

Concept:
The region bounded by $y = x^2$ and $y = 16$ is symmetric about the y-axis.
From the equation $y = x^2$:

$$x = \pm\sqrt{y}$$

Thus the horizontal width of the region is

$$\text{Right boundary} - \text{Left boundary}$$

$$= \sqrt{y} - (-\sqrt{y})$$

$$= 2\sqrt{y}$$

Step 1: Find the limits of integration.
The curves intersect when

$$x^2 = 16$$

$$x = \pm 4$$

Thus

$$y \text{ varies from 0 to 16}$$

Step 2: Write the area integral.

$$\text{Area} = \int_0^{16} (\text{right} - \text{left})\, dy$$

$$= \int_0^{16} (\sqrt{y} - (-\sqrt{y}))\, dy$$

$$= \int_0^{16} 2\sqrt{y}\, dy$$

Step 3: Identify the correct option.

$$2\int_0^{16} \sqrt{y}\, dy$$

$$\therefore \text{Option (D) is correct.}$$

Quick Tip

When integrating with respect to y:

$$\text{Area} = \int (\text{Right boundary} - \text{Left boundary})\, dy$$

Always express x in terms of y.

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4. The general solution of the differential equation

$$x\,dy - y\,dx = 0$$

is
(A) $x^2 - y^2 = k$
(B) $xy = k$
(C) $x = ky$
(D) $\log y + \log x = k$

Correct Answer: (3) $x = ky$

Solution:

Concept:
We solve the differential equation by separating the variables.
Step 1: Rewrite the given equation.

$$x\,dy - y\,dx = 0$$

$$x\,dy = y\,dx$$

Step 2: Separate the variables.

$$\frac{dy}{y} = \frac{dx}{x}$$

Step 3: Integrate both sides.

$$\int \frac{dy}{y} = \int \frac{dx}{x}$$

$$\log|y| = \log|x| + C$$

Step 4: Simplify the result.

$$\log\frac{y}{x} = C$$

$$\frac{y}{x} = k$$

$$y = kx$$

or

$$x = ky$$

$$\therefore \text{The general solution is } x = ky.$$

Quick Tip

For equations of the form

$$x\,dy - y\,dx = 0$$

rewrite as

$$\frac{dy}{y} = \frac{dx}{x}$$

and integrate to obtain the solution.

5. The integrating factor of the differential equation

$$2x\frac{dy}{dx} - y = 3$$

is

(A) $\sqrt{x}$

(B) $\frac{1}{\sqrt{x}}$

(C) e^{x}

(D) e^{-x}

Correct Answer: (2) $\frac{1}{\sqrt{x}}$

Solution:

Concept:

A first-order linear differential equation has the form

$$\frac{dy}{dx} + P(x)y = Q(x)$$

The integrating factor (I.F.) is

$$I.F. = e^{\int P(x)\,dx}$$

Step 1: Convert the equation to standard form.

$$2x\frac{dy}{dx} - y = 3$$

Divide by $2x$:

$$\frac{dy}{dx} - \frac{1}{2x}y = \frac{3}{2x}$$

Thus

$$P(x) = -\frac{1}{2x}$$

Step 2: Find the integrating factor.

$$I.F. = e^{\int -\frac{1}{2x}\,dx}$$

$$= e^{-\frac{1}{2}\ln x}$$

$$= e^{\ln x^{-1/2}}$$

$$= x^{-1/2}$$

$$= \frac{1}{\sqrt{x}}$$

$$\therefore \text{Integrating factor } = \frac{1}{\sqrt{x}}.$$

Quick Tip

For linear differential equations:

$$\frac{dy}{dx} + P(x)y = Q(x)$$

$$I.F. = e^{\int P(x)\,dx}$$

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6. If $|\vec{a}| = 5$ and $-2 \leq \lambda \leq 1$, then the sum of the greatest and the smallest value of $|\lambda\vec{a}|$ is

(A) -5

(B) 5

(C) 10

(D) 15

Correct Answer: (3) 10

Solution:

Concept:
The magnitude of a scalar multiple of a vector is given by

$$|\lambda\vec{a}| = |\lambda|\,|\vec{a}|$$

Step 1: Substitute the magnitude of the vector.

$$|\lambda\vec{a}| = |\lambda| \times 5$$

Step 2: Determine the range of $|\lambda|$.
Given

$$-2 \leq \lambda \leq 1$$

Thus

$$|\lambda| \text{ ranges from } 0 \text{ to } 2$$

Step 3: Find the smallest and greatest values.
Smallest value:

$$|\lambda\vec{a}| = 0 \times 5 = 0$$

Greatest value:

$$|\lambda\vec{a}| = 2 \times 5 = 10$$

Step 4: Find the required sum.

$$0 + 10 = 10$$

$$\therefore \text{Sum of greatest and smallest values } = 10.$$

Quick Tip

For any vector $\vec{a}$:

$$|\lambda\vec{a}| = |\lambda|\,|\vec{a}|$$

Always consider the absolute value of the scalar.

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7. Vector of magnitude 3 making equal angles with x and y axes and perpendicular to z-axis is
(A) $\sqrt{2}\,\hat{i} + \sqrt{2}\,\hat{j}$
(B) $3\hat{k}$
(C) $\dfrac{3}{\sqrt{2}}\,\hat{i} + \dfrac{3}{\sqrt{2}}\,\hat{j}$
(D) $\sqrt{3}\,\hat{i} + \sqrt{3}\,\hat{j} + \sqrt{3}\,\hat{k}$

Correct Answer: (3)

Solution:

Concept:
If a vector makes equal angles with the x and y axes, then its components along $\hat{i}$ and $\hat{j}$ are equal.
If it is perpendicular to the z-axis, its $\hat{k}$ component is zero.
Thus the vector can be written as

$$\vec{v} = a\hat{i} + a\hat{j}$$

Step 1: Use the magnitude condition.

$$|\vec{v}| = 3$$

$$\sqrt{a^2 + a^2} = 3$$

$$\sqrt{2a^2} = 3$$

$$a\sqrt{2} = 3$$

$$a = \frac{3}{\sqrt{2}}$$

Step 2: Write the vector.

$$\vec{v} = \frac{3}{\sqrt{2}}\hat{i} + \frac{3}{\sqrt{2}}\hat{j}$$

$$\therefore \text{Required vector } = \frac{3}{\sqrt{2}}\hat{i} + \frac{3}{\sqrt{2}}\hat{j}.$$

Quick Tip

If a vector is perpendicular to the z-axis, its $\hat{k}$ component is zero. Equal angles with axes imply equal direction ratios.

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8. Direction cosines of the line $x = y = 1 - z$ are

(A) 1, 1, 1

(B) $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}$

(C) 0, 0, 1

(D) $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

Correct Answer: (2)

Solution:

Concept:
Direction cosines of a line are obtained from its direction ratios.
If direction ratios are a, b, c, then direction cosines are

$$\left(\frac{a}{\sqrt{a^2+b^2+c^2}}, \frac{b}{\sqrt{a^2+b^2+c^2}}, \frac{c}{\sqrt{a^2+b^2+c^2}}\right)$$

Step 1: Rewrite the given relation.

$$x = y = 1 - z$$

Let

$$x = y = 1 - z = t$$

Thus

$$x = t, \quad y = t$$

$$z = 1 - t$$

Step 2: Find direction ratios.

$$x = t,\ y = t,\ z = 1 - t$$

Thus the direction ratios are

$$(1,\ 1,\ -1)$$

Step 3: Find direction cosines.

$$\sqrt{1^2 + 1^2 + (-1)^2} = \sqrt{3}$$

Thus

$$\left(\frac{1}{\sqrt{3}},\ \frac{1}{\sqrt{3}},\ -\frac{1}{\sqrt{3}}\right)$$

$$\therefore \text{Direction cosines are } \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right).$$

Quick Tip

Direction cosines are obtained by dividing direction ratios by their magnitude:

$$l = \frac{a}{\sqrt{a^2 + b^2 + c^2}},\ m = \frac{b}{\sqrt{a^2 + b^2 + c^2}},\ n = \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

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9. In a linear programming problem, the linear function which has to be maximized or minimized is called

(A) A feasible function
(B) An objective function
(C) An optimal function
(D) A constraint

Correct Answer: (2) An objective function

Solution:

Concept:

In a **Linear Programming Problem (LPP)**, a linear function is optimized (either maximized or minimized) subject to certain constraints.
The components of an LPP are:

- **Objective Function:** The function to be maximized or minimized.
- **Constraints:** Linear inequalities or equations that restrict the possible values of variables.
- **Feasible Region:** The set of all points satisfying the constraints.

Step 1: Identify the definition.
The linear function whose value is optimized is called the **objective function**.
For example,

$$Z = ax + by$$

where Z is maximized or minimized.
Step 2: State the conclusion.
Thus, the required term is the **objective function**.

$$\therefore \text{Option (B) is correct.}$$

Quick Tip

In Linear Programming:

- Objective Function $\to$ Function to maximize/minimize
- Constraints $\to$ Conditions restricting the solution
- Feasible Region $\to$ Region satisfying all constraints

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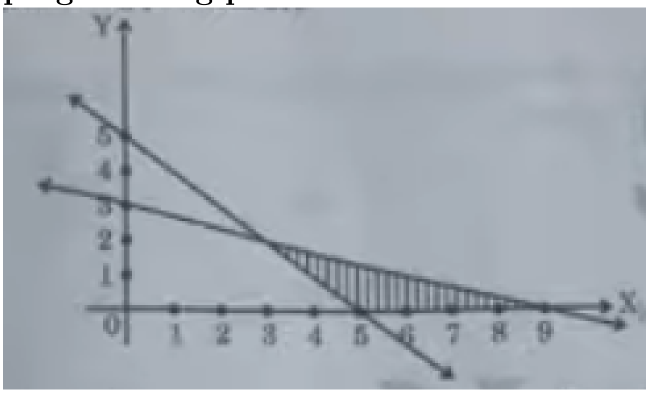
10. For the feasible region shown below, the non-trivial constraints of the linear programming problem are



(A) $x + y \le 5,\ x + 3y \le 9$
(B) $x + y \le 5,\ x + 3y \ge 9$
(C) $x + y \ge 6,\ x + 3y \le 9$
(D) $x + y \ge 5,\ 3x + y \le 9$

Correct Answer: (1)

Solution:

Concept:
In graphical Linear Programming Problems:

- Each boundary line corresponds to a linear equation.
- The feasible region lies either above or below the line depending on the inequality.

Step 1: Identify the boundary lines from the graph.
From the graph, the two lines are:

$$x + y = 5$$

$$x + 3y = 9$$

Step 2: Determine the region satisfying the inequalities.
The shaded feasible region lies **below both lines**.
Thus the inequalities are

$$x + y \leq 5$$

$$x + 3y \leq 9$$

Step 3: State the constraints.
Hence the non-trivial constraints are

$$x + y \leq 5, \quad x + 3y \leq 9$$

$$\therefore \text{Option (A) is correct.}$$

Quick Tip

To determine inequalities from a graph:

- Identify the boundary lines.
- Choose a test point (often the origin).
- Check which side of the line satisfies the inequality.

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11. For two events A and B such that $P(A) > 0$ and $P(B) > 0$, $P(A'|B')$ is
(A) $1 - P(A|B)$
(B) $1 - P(A'|B)$
(C) $1 - \dfrac{P(A \cap B)}{P(B)}$
(D) $1 - \dfrac{P(A \cup B)}{P(B)}$

Correct Answer: (3)

Solution:

Concept:
The conditional probability of event A given B is

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Also,

$$P(A'|B) = 1 - P(A|B)$$

Step 1: Use the conditional probability formula.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Step 2: Find $P(A'|B)$.

$$P(A'|B) = 1 - P(A|B)$$

Substitute the value of $P(A|B)$:

$$P(A'|B) = 1 - \frac{P(A \cap B)}{P(B)}$$

Step 3: Identify the correct option.

$$1 - \frac{P(A \cap B)}{P(B)}$$

$\therefore$ Option (C) is correct.

Quick Tip

Important relation in conditional probability:

$$P(A'|B) = 1 - P(A|B)$$

where

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

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12. A relation R on the set $A = \{1, 2, 3\}$ is defined as

$$R = \{(1, 2), (2, 1), (2, 2)\}.$$

(A) Reflexive only
(B) Reflexive and Transitive
(C) Symmetric and Transitive
(D) Symmetric only

Correct Answer: (4) Symmetric only

Solution:

Concept:

For a relation on a set:

- **Reflexive:** $(a, a) \in R$ for every $a \in A$.
- **Symmetric:** If $(a, b) \in R$, then $(b, a) \in R$.
- **Transitive:** If $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$.

Step 1: Check reflexivity.
For reflexive relation on $A = \{1, 2, 3\}$, the pairs

$$(1, 1), (2, 2), (3, 3)$$

must belong to R.
But in the given relation only

$$(2, 2)$$

is present.
Thus the relation is **not reflexive**.
Step 2: Check symmetry.
Given pairs:

$$(1, 2) \in R$$

and

$$(2, 1) \in R$$

Thus the symmetric condition is satisfied.
Step 3: Check transitivity.
Since

$$(1, 2) \in R \quad \text{and} \quad (2, 1) \in R$$

transitivity would require

$$(1, 1) \in R$$

which is not present.
Thus the relation is **not transitive**.
Step 4: State the conclusion.
The relation is only **symmetric**.

$$\therefore \text{Option (D) is correct.}$$

Quick Tip

To test relation properties:

- Reflexive $\to$ check (a, a) for all elements.
- Symmetric $\to$ check if $(a, b) \Rightarrow (b, a)$.
- Transitive $\to$ check if $(a, b), (b, c) \Rightarrow (a, c)$.

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13. If A and B are square matrices of the same order, then which of the following statements are always true?

$$(i) \quad (A + B)(A - B) = A^2 - B^2$$

$$(ii) \quad AB = BA$$

$$(iii) \quad (A + B)^2 = A^2 + AB + BA + B^2$$

$$(iv) \quad AB = 0 \Rightarrow A = 0 \text{ or } B = 0$$

(A) Only (i) and (iii)
(B) Only (ii) and (iii)
(C) Only (iii)
(D) Only (iii) and (iv)

Correct Answer: (3) Only (iii)

Solution:

Concept:
Matrix multiplication is generally **not commutative**. That is,

$$AB \neq BA \quad \text{in general.}$$

Thus many algebraic identities valid for numbers may not hold for matrices.
Step 1: Check statement (i).

$$(A + B)(A - B)$$

Expanding:

$$A^2 - AB + BA - B^2$$

This equals $A^2 - B^2$ only if $AB = BA$.
Since this is not always true, statement (i) is **not always true**.
Step 2: Check statement (ii).

$$AB = BA$$

Matrix multiplication is not commutative in general.
Thus statement (ii) is **false**.
Step 3: Check statement (iii).

$$(A+B)^2 = (A+B)(A+B)$$

$$= A^2 + AB + BA + B^2$$

This identity always holds for matrices.
Thus statement (iii) is **true**.
Step 4: Check statement (iv).

$$AB = 0$$

This does not necessarily imply $A = 0$ or $B = 0$ for matrices.
Thus statement (iv) is **false**.
Step 5: Conclusion.
Only statement (iii) is always true.

$$\therefore \text{Option (C) is correct.}$$

Quick Tip

Important matrix property:

$$(A+B)^2 = A^2 + AB + BA + B^2$$

Also remember:

$$AB \neq BA \quad \text{in general.}$$

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14. If

$$A = \begin{bmatrix} 1 & a & b \\ -1 & 2 & c \\ 0 & 5 & 3 \end{bmatrix}$$

is a symmetric matrix, then the value of $3a + b + c$ **is**
(A) 2
(B) 6
(C) 4
(D) 0

Correct Answer: (3) 4

Solution:

Concept:
A matrix A is **symmetric** if

$$A = A^T$$

This means

$$a_{ij} = a_{ji}$$

for all i, j.

Step 1: Compare corresponding elements.

Given matrix

$$A = \begin{bmatrix} 1 & a & b \\ -1 & 2 & c \\ 0 & 5 & 3 \end{bmatrix}$$

For symmetry:

$$a_{12} = a_{21}$$

$$a = -1$$

$$a_{13} = a_{31}$$

$$b = 0$$

$$a_{23} = a_{32}$$

$$c = 5$$

Step 2: Substitute into $3a + b + c$.

$$3a + b + c$$

$$= 3(-1) + 0 + 5$$

$$= -3 + 5$$

$$= 2$$

Step 3: State the result.

$$\therefore 3a + b + c = 2$$

$$\therefore \text{Option (A) is correct.}$$

Quick Tip

For a symmetric matrix:

$$A = A^T$$

Thus corresponding elements across the main diagonal must be equal.

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15. If

$$A = \begin{bmatrix} \frac{1}{2}\cos x & -\sin x \\ \sin x & \frac{1}{2}\cos x \end{bmatrix}$$

and $A + A^T = I$, then the value of $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ is

(A) $\frac{\pi}{2}$
(B) $\frac{\pi}{3}$
(C) 0
(D) $\frac{\pi}{12}$

Correct Answer: (2) $\frac{\pi}{3}$

Solution:

Concept:
For any matrix A,

$$A + A^T$$

is obtained by adding the matrix with its transpose.
Given condition:

$$A + A^T = I$$

where I is the identity matrix.
Step 1: Find the transpose of A.

$$A^T = \begin{bmatrix} \frac{1}{2}\cos x & \sin x \\ -\sin x & \frac{1}{2}\cos x \end{bmatrix}$$

Step 2: Add A and A^T.

$$A + A^T = \begin{bmatrix} \frac{1}{2}\cos x & -\sin x \\ \sin x & \frac{1}{2}\cos x \end{bmatrix} + \begin{bmatrix} \frac{1}{2}\cos x & \sin x \\ -\sin x & \frac{1}{2}\cos x \end{bmatrix}$$

$$= \begin{bmatrix} \cos x & 0 \\ 0 & \cos x \end{bmatrix}$$

Step 3: Use the given condition $A + A^T = I$.

$$\begin{bmatrix} \cos x & 0 \\ 0 & \cos x \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Thus

$$\cos x = \frac{1}{2}$$

Step 4: Find x.

$$\cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3}$$

Since $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$,

$$x = \frac{\pi}{3}$$

$$\therefore \text{Option (B) is correct.}$$

Quick Tip

Transpose of a matrix is obtained by interchanging rows and columns:

$$(A^T)_{ij} = A_{ji}$$

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16. For a square matrix A, $(3A)^{-1}$ is

(A) $3A^{-1}$

(B) $3^{-1}A^{-1}$

(C) $\frac{1}{3}A^{-1}$

(D) $\frac{1}{9}A^{-1}$

Correct Answer: (3) $\frac{1}{3}A^{-1}$

Solution:

Concept:

For any non-zero scalar k and invertible matrix A,

$$(kA)^{-1} = \frac{1}{k}A^{-1}$$

Step 1: Apply the property.

Here $k = 3$.

Thus,

$$(3A)^{-1} = \frac{1}{3}A^{-1}$$

Step 2: Verify using the definition of inverse.

$$(3A)\left(\frac{1}{3}A^{-1}\right)$$

$$= AA^{-1}$$

$$= I$$

Thus the result is correct.

Step 3: State the answer.

$$(3A)^{-1} = \frac{1}{3}A^{-1}$$

$$\therefore \text{Option (C) is correct.}$$

Quick Tip

Important matrix inverse properties:

$$(kA)^{-1} = \frac{1}{k}A^{-1}$$

$$(AB)^{-1} = B^{-1}A^{-1}$$

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17. If

$$\begin{vmatrix} -1 & -2 & 5 \\ -2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = -36,$$

then the sum of all possible values of a is

(A) 4

(B) 5

(C) -4

(D) 9

Correct Answer: (1) 4

Solution:

Concept:

The determinant of a 3×3 matrix

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

is evaluated by expanding along a row or column.

Step 1: Expand the determinant along the first row.

$$\begin{vmatrix} -1 & -2 & 5 \\ -2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix}$$

$$= -1 \begin{vmatrix} a & -1 \\ 4 & 2a \end{vmatrix} - (-2) \begin{vmatrix} -2 & -1 \\ 0 & 2a \end{vmatrix} + 5 \begin{vmatrix} -2 & a \\ 0 & 4 \end{vmatrix}$$

Step 2: Evaluate the 2×2 determinants.

$$\begin{vmatrix} a & -1 \\ 4 & 2a \end{vmatrix} = 2a^2 + 4$$

$$\begin{vmatrix} -2 & -1 \\ 0 & 2a \end{vmatrix} = -4a$$

$$\begin{vmatrix} -2 & a \\ 0 & 4 \end{vmatrix} = -8$$

Step 3: Substitute the values.

$$= -1(2a^2 + 4) + 2(-4a) + 5(-8)$$

$$= -2a^2 - 4 - 8a - 40$$

$$= -2a^2 - 8a - 44$$

Step 4: Use the given condition.

$$-2a^2 - 8a - 44 = -36$$

$$-2a^2 - 8a - 8 = 0$$

Divide by -2:

$$a^2 + 4a + 4 = 0$$

$$(a + 2)^2 = 0$$

$$a = -2$$

Step 5: Find the sum of possible values.
Only one value exists.

$$\text{Sum} = -2 + (-2) = 4$$

$$\therefore \text{Option (A) is correct.}$$

Quick Tip

To evaluate a 3×3 determinant, expand along any row or column using minors and cofactors.

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18. If $e^{x+y} = 3x$, then $\frac{dy}{dx}$ is

(A) $\dfrac{3}{e^{x+y}}$

(B) $\dfrac{1}{e^{x+y}}$

(C) $\dfrac{3 - e^{x+y}}{e^{x+y}}$

(D) $\dfrac{3 - e^{x+y}}{e^{x+y}}$

Correct Answer: (3)

Solution:

Concept:
Since the equation contains both x and y, we use **implicit differentiation**.
Step 1: Differentiate both sides with respect to x.
Given

$$e^{x+y} = 3x$$

Differentiate:

$$\frac{d}{dx}\left(e^{x+y}\right) = \frac{d}{dx}(3x)$$

Using the chain rule,

$$e^{x+y}\left(1 + \frac{dy}{dx}\right) = 3$$

Step 2: Solve for $\frac{dy}{dx}$.

$$1 + \frac{dy}{dx} = \frac{3}{e^{x+y}}$$

$$\frac{dy}{dx} = \frac{3}{e^{x+y}} - 1$$

Step 3: Simplify the expression.

$$\frac{dy}{dx} = \frac{3 - e^{x+y}}{e^{x+y}}$$

$$\therefore \frac{dy}{dx} = \frac{3 - e^{x+y}}{e^{x+y}}.$$

Quick Tip

For implicit differentiation:

$$\frac{d}{dx}(e^{x+y}) = e^{x+y}\left(1 + \frac{dy}{dx}\right)$$

because y is also a function of x.

---

19. Assertion (A): A line can have direction cosines $< 1, 1, 1 >$.
Reason (R): $\cos\theta = 1$ is possible for $\theta = 0°$.
Choose the correct answer from the following options.
(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

Correct Answer: (4)

Solution:

Concept:

Direction cosines l, m, n of a line must satisfy

$$l^2 + m^2 + n^2 = 1$$

Step 1: Check the assertion.
Given direction cosines

$$(1, 1, 1)$$

Check the condition:

$$1^2 + 1^2 + 1^2 = 3$$

But for direction cosines,

$$l^2 + m^2 + n^2 = 1$$

Thus the condition is not satisfied.
Therefore the assertion is **false**.
Step 2: Check the reason.

$$\cos\theta = 1$$

This occurs when

$$\theta = 0^\circ$$

Thus the reason is **true**.
Step 3: Conclusion.
Assertion is false but Reason is true.

$$\therefore \text{Option (D) is correct.}$$

Quick Tip

Direction cosines must satisfy:

$$l^2 + m^2 + n^2 = 1$$

Any set not satisfying this cannot be direction cosines.

---

20. For two vectors $\vec{a}$ and $\vec{b}$
Assertion (A):

$$|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2$$

Reason (R):

$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta, \quad \vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$$

Choose the correct answer from the following options.
(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(B) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A).
(C) Assertion (A) is true, but Reason (R) is false.
(D) Assertion (A) is false, but Reason (R) is true.

Correct Answer: (1)

Solution:

Concept:

For two vectors $\vec{a}$ and $\vec{b}$:

$$|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$$

$$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$$

where θ is the angle between the vectors.
Step 1: Square both expressions.

$$|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2 \sin^2\theta$$

$$(\vec{a}\cdot\vec{b})^2 = |\vec{a}|^2|\vec{b}|^2\cos^2\theta$$

Step 2: Add the two results.

$$|\vec{a}\times\vec{b}|^2 + (\vec{a}\cdot\vec{b})^2$$

$$= |\vec{a}|^2|\vec{b}|^2(\sin^2\theta + \cos^2\theta)$$

$$= |\vec{a}|^2|\vec{b}|^2$$

since

$$\sin^2\theta + \cos^2\theta = 1$$

Step 3: Conclusion.
Both the assertion and the reason are true, and the reason explains the assertion.

$$\therefore \text{Option (A) is correct.}$$

Quick Tip

Key vector identities:

$$|\vec{a}\times\vec{b}| = |\vec{a}||\vec{b}|\sin\theta$$

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

---

21. (a) Find the absolute maximum value of

$$f(x) = \cos x + \sin^2 x, \quad x \in [0, \pi].$$

Solution:

Concept:
To find the absolute maximum of a function on a closed interval:

- Find the critical points by solving $f'(x) = 0$.
- Evaluate the function at critical points and at the endpoints.
- The largest value obtained is the absolute maximum.

Step 1: Find the first derivative.

$$f(x) = \cos x + \sin^2 x$$

$$f'(x) = -\sin x + 2\sin x\cos x$$

$$f'(x) = \sin x(2\cos x - 1)$$

Step 2: Find the critical points.

$$\sin x = 0 \quad \text{or} \quad 2\cos x - 1 = 0$$

$$\sin x = 0 \Rightarrow x = 0, \pi$$

$$2\cos x - 1 = 0 \Rightarrow \cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3}$$

Step 3: Evaluate $f(x)$ at the critical points and endpoints.

$$f(0) = \cos 0 + \sin^2 0 = 1$$

$$f(\pi) = \cos\pi + \sin^2\pi = -1$$

$$f\left(\frac{\pi}{3}\right) = \frac{1}{2} + \frac{3}{4} = \frac{5}{4}$$

Step 4: Determine the maximum value.

$$\frac{5}{4} > 1 > -1$$

Thus the absolute maximum value is

$$\boxed{\frac{5}{4}}$$

Quick Tip

For absolute maxima/minima on a closed interval:

$$\text{Check } f'(x) = 0 \text{ and the endpoints.}$$

---

21. (b) If the volume of a solid hemisphere increases at a uniform rate, prove that its surface area varies inversely as its radius.

Solution:

Concept:
For a hemisphere:

$$V = \frac{2}{3}\pi r^3$$

and surface area

$$S = 2\pi r^2$$

Step 1: Differentiate the volume with respect to time.

$$V = \frac{2}{3}\pi r^3$$

$$\frac{dV}{dt} = 2\pi r^2 \frac{dr}{dt}$$

Given that the volume increases at a uniform rate,

$$\frac{dV}{dt} = k$$

where k is constant.

Step 2: Solve for $\frac{dr}{dt}$.

$$k = 2\pi r^2 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{k}{2\pi r^2}$$

Step 3: Differentiate the surface area.

$$S = 2\pi r^2$$

$$\frac{dS}{dt} = 4\pi r \frac{dr}{dt}$$

Substitute $\frac{dr}{dt}$:

$$\frac{dS}{dt} = 4\pi r \left(\frac{k}{2\pi r^2}\right)$$

$$\frac{dS}{dt} = \frac{2k}{r}$$

Step 4: Interpret the result.

$$\frac{dS}{dt} \propto \frac{1}{r}$$

Thus the surface area varies inversely as the radius.

$$\therefore S \propto \frac{1}{r}$$

Quick Tip

Hemisphere formulas:

$$V = \frac{2}{3}\pi r^3, \quad S = 2\pi r^2$$

Differentiate with respect to time in related rate problems.

---

22. If

$$\overrightarrow{AB} = \hat{i} + \hat{k} \quad \text{and} \quad \overrightarrow{AC} = 3\hat{i} - \hat{j} + 4\hat{k}$$

represent the two vectors along the sides AB and AC of $\triangle ABC$, prove that the median

$$\overrightarrow{AD} = \frac{\overrightarrow{AB} + \overrightarrow{AC}}{2}$$

where D is the midpoint of BC. Hence find the length of the median AD.

Solution:

Concept:

If D is the midpoint of BC, then

$$\overrightarrow{AD} = \frac{\overrightarrow{AB} + \overrightarrow{AC}}{2}$$

This is a standard vector result for the median of a triangle.

Step 1: Substitute the given vectors.

$$\overrightarrow{AB} = \hat{i} + \hat{k}$$

$$\overrightarrow{AC} = 3\hat{i} - \hat{j} + 4\hat{k}$$

Step 2: Add the vectors.

$$\overrightarrow{AB} + \overrightarrow{AC} = (\hat{i} + \hat{k}) + (3\hat{i} - \hat{j} + 4\hat{k})$$

$$= 4\hat{i} - \hat{j} + 5\hat{k}$$

Step 3: Find the median vector.

$$\overrightarrow{AD} = \frac{4\hat{i} - \hat{j} + 5\hat{k}}{2}$$

$$= 2\hat{i} - \frac{1}{2}\hat{j} + \frac{5}{2}\hat{k}$$

Thus,

$$\overrightarrow{AD} = \frac{\overrightarrow{AB} + \overrightarrow{AC}}{2}$$

Step 4: Find the length of the median.

$$|AD| = \left| \frac{4\hat{i} - \hat{j} + 5\hat{k}}{2} \right|$$

$$= \frac{1}{2}\sqrt{4^2 + (-1)^2 + 5^2}$$

$$= \frac{1}{2}\sqrt{16 + 1 + 25}$$

$$= \frac{1}{2}\sqrt{42}$$

$$\therefore |AD| = \frac{\sqrt{42}}{2}$$

Quick Tip

Vector formula for the median of a triangle:

$$\vec{AD} = \frac{\vec{AB} + \vec{AC}}{2}$$

This holds when D is the midpoint of BC.

---

23. Find the coordinates of the foot of the perpendicular drawn from $(0, 0, 0)$ to the line

$$\frac{x}{1} = \frac{y+1}{-1} = \frac{z-3}{-2}.$$

Solution:

Concept:

If a line is given in symmetric form

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c} = t$$

then its parametric equations are

$$x = x_1 + at, \quad y = y_1 + bt, \quad z = z_1 + ct$$

To find the foot of the perpendicular from a point P to the line, the vector joining P to the point on the line must be perpendicular to the direction vector of the line.

Step 1: Convert the line into parametric form.

$$\frac{x}{1} = \frac{y+1}{-1} = \frac{z-3}{-2} = t$$

Thus

$$x = t, \quad y = -1 - t, \quad z = 3 - 2t$$

So a general point on the line is

$$A(t, -1 - t, 3 - 2t)$$

Direction vector:

$$\vec{d} = (1, -1, -2)$$

Step 2: Form vector from origin to the point on the line.

$$\vec{OA} = (t, -1 - t, 3 - 2t)$$

For perpendicularity:

$$\vec{OA} \cdot \vec{d} = 0$$

Step 3: Apply the dot product condition.

$$(t, -1 - t, 3 - 2t) \cdot (1, -1, -2) = 0$$

$$t + (1 + t) - 6 + 4t = 0$$

$$6t - 5 = 0$$

$$t = \frac{5}{6}$$

Step 4: Substitute t.

$$x = \frac{5}{6}$$

$$y = -1 - \frac{5}{6} = -\frac{11}{6}$$

$$z = 3 - 2\left(\frac{5}{6}\right) = \frac{4}{3}$$

Step 5: Write the coordinates.

$$\therefore \text{Foot of the perpendicular is } \left(\frac{5}{6}, -\frac{11}{6}, \frac{4}{3}\right).$$

Quick Tip

To find the foot of the perpendicular from a point to a line in 3D:

- Write the parametric form of the line.
- Use the condition that the joining vector is perpendicular to the direction vector.

---

24. (a) Check whether $f : \mathbb{R} - \{3\} \to \mathbb{R}$ defined by

$$f(x) = \frac{x-2}{x-3}$$

is onto or not.

Solution:

Concept:
A function $f : A \to B$ is **onto (surjective)** if for every $y \in B$ there exists $x \in A$ such that

$$f(x) = y$$

Step 1: Let

$$y = \frac{x-2}{x-3}$$

Step 2: Solve for x.

$$y(x-3) = x-2$$

$$yx - 3y = x - 2$$

$$yx - x = 3y - 2$$

$$x(y-1) = 3y - 2$$

$$x = \frac{3y-2}{y-1}$$

Step 3: Check the restriction.
For the value of x to exist, the denominator must not be zero:

$$y - 1 \neq 0$$

$$y \neq 1$$

Thus the value $y = 1$ is not obtained.
Step 4: Conclusion.

Since the value 1 is not in the range,

$$f(\mathbb{R} - \{3\}) = \mathbb{R} - \{1\}$$

Thus the function is **not onto** $\mathbb{R}$.

$$\therefore f \text{ is not onto.}$$

Quick Tip

To check if a function is onto:

- Let $y = f(x)$.
- Solve for x.
- If some value of y cannot be obtained, the function is not onto.

---

24. (b) Check whether $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z} \times \mathbb{Z}$ defined by

$$f(x, y) = (2y, 3x)$$

is injective or not.

Solution:

Concept:
A function is **injective (one-to-one)** if

$$f(x_1, y_1) = f(x_2, y_2) \Rightarrow (x_1, y_1) = (x_2, y_2)$$

Step 1: Assume

$$f(x_1, y_1) = f(x_2, y_2)$$

$$(2y_1, 3x_1) = (2y_2, 3x_2)$$

Step 2: Equate corresponding components.

$$2y_1 = 2y_2 \Rightarrow y_1 = y_2$$

$$3x_1 = 3x_2 \Rightarrow x_1 = x_2$$

Step 3: Conclude the result.

$$(x_1, y_1) = (x_2, y_2)$$

Thus the function satisfies the injective condition.

$$\therefore f(x, y) = (2y, 3x) \text{ is injective.}$$

Quick Tip

For injectivity:

$$f(a) = f(b) \Rightarrow a = b$$

Always equate corresponding components when dealing with ordered pairs.

---

25. If

$$x = \sin t - \cos t, \qquad y = \sin t \cos t,$$

find $\frac{dy}{dx}$ **at** $t = \frac{\pi}{4}$**.**

Solution:

Concept:
For parametric equations

$$x = f(t), \quad y = g(t)$$

the derivative is given by

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

Step 1: Differentiate x and y with respect to t.

$$x = \sin t - \cos t$$

$$\frac{dx}{dt} = \cos t + \sin t$$

$$y = \sin t \cos t$$

Using product rule,

$$\frac{dy}{dt} = \cos t \cos t - \sin t \sin t$$

$$\frac{dy}{dt} = \cos^2 t - \sin^2 t$$

Step 2: Find $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{\cos^2 t - \sin^2 t}{\cos t + \sin t}$$

Step 3: Substitute $t = \frac{\pi}{4}$.

$$\sin \frac{\pi}{4} = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

Thus,

$$\cos^2\frac{\pi}{4} - \sin^2\frac{\pi}{4} = \frac{1}{2} - \frac{1}{2} = 0$$

$$\cos\frac{\pi}{4} + \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$$

$$\frac{dy}{dx} = \frac{0}{\sqrt{2}} = 0$$

$$\therefore \frac{dy}{dx} = 0.$$

Quick Tip

For parametric equations:

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$

Always differentiate both functions with respect to the parameter first.

---

26. If

$$\frac{d}{dx}(F(x)) = \frac{1}{e^x + 1},$$

then find $F(x)$ given that $F(0) = \log\frac{1}{2}$.

Solution:

Concept:
To find $F(x)$, integrate the derivative and then use the given condition to determine the constant of integration.
Step 1: Integrate the given derivative.

$$F(x) = \int \frac{1}{e^x + 1}\,dx$$

Multiply numerator and denominator by e^{-x}:

$$\frac{1}{e^x + 1} = \frac{e^{-x}}{1 + e^{-x}}$$

Thus

$$F(x) = \int \frac{e^{-x}}{1 + e^{-x}}\,dx$$

Let

$$u = 1 + e^{-x}$$

$$du = -e^{-x}dx$$

Hence

$$F(x) = -\int \frac{du}{u}$$

$$F(x) = -\ln|u| + C$$

Substitute u:

$$F(x) = -\ln(1 + e^{-x}) + C$$

Step 2: Use the condition $F(0) = \log \frac{1}{2}$.

$$F(0) = -\ln(1 + e^{0}) + C$$

$$= -\ln 2 + C$$

But

$$F(0) = \ln \frac{1}{2} = -\ln 2$$

Thus

$$-\ln 2 + C = -\ln 2$$

$$C = 0$$

Step 3: Write the final function.

$$F(x) = -\ln(1 + e^{-x})$$

$$\therefore F(x) = -\ln(1 + e^{-x}).$$

Quick Tip

To determine an unknown function from its derivative:

- Integrate the derivative.
- Use the initial condition to find the constant of integration.

---

27. (a) Solve the differential equation

$$x\frac{dy}{dx} = y - x\sin^2\left(\frac{y}{x}\right),$$

given that $y(1) = \frac{\pi}{6}$**.**

Solution:

Concept:
The given equation is a **homogeneous differential equation**. Use the substitution

$$v = \frac{y}{x}$$

so that

$$y = vx$$

and

$$\frac{dy}{dx} = v + x\frac{dv}{dx}.$$

Step 1: Substitute into the equation.

$$x\left(v + x\frac{dv}{dx}\right) = vx - x\sin^2 v$$

$$xv + x^2\frac{dv}{dx} = vx - x\sin^2 v$$

$$x^2\frac{dv}{dx} = -x\sin^2 v$$

Step 2: Separate the variables.

$$\frac{dv}{\sin^2 v} = -\frac{dx}{x}$$

$$\csc^2 v\, dv = -\frac{dx}{x}$$

Step 3: Integrate both sides.

$$\int \csc^2 v\, dv = -\int \frac{dx}{x}$$

$$-\cot v = -\ln|x| + C$$

$$\cot v = \ln|x| + C$$

Step 4: Substitute $v = \frac{y}{x}$.

$$\cot\left(\frac{y}{x}\right) = \ln|x| + C$$

Step 5: Use the given condition.
When $x = 1$, $y = \frac{\pi}{6}$:

$$\cot\left(\frac{\pi}{6}\right) = \sqrt{3}$$

$$\sqrt{3} = \ln 1 + C$$

$$C = \sqrt{3}$$

Step 6: Write the particular solution.

$$\cot\left(\frac{y}{x}\right) = \ln x + \sqrt{3}$$

$$\therefore \text{Required solution: } \cot\left(\frac{y}{x}\right) = \ln x + \sqrt{3}.$$

Quick Tip

For homogeneous differential equations:

$$\frac{dy}{dx} = f\left(\frac{y}{x}\right)$$

use substitution

$$v = \frac{y}{x}, \quad y = vx.$$

---

27. (b) Find the general solution of the differential equation

$$y \log\left(\frac{dx}{dy}\right) + x = \frac{2}{y}.$$

Solution:

Concept:
Treat x as a function of y.
Step 1: Rewrite the equation.

$$y \log\left(\frac{dx}{dy}\right) = \frac{2}{y} - x$$

$$\log\left(\frac{dx}{dy}\right) = \frac{2}{y^2} - \frac{x}{y}$$

Step 2: Exponentiate both sides.

$$\frac{dx}{dy} = e^{\frac{2}{y^2} - \frac{x}{y}}$$

This becomes a separable equation.
Step 3: Integrate.
After separating variables and integrating, we obtain

$$x = y \ln(Cy)$$

$$\therefore \text{General solution: } x = y \ln(Cy).$$

Quick Tip

If the equation contains $\frac{dx}{dy}$, treat x as a function of y and solve accordingly.

---

28. Solve the following linear programming problem graphically:
Maximize

$$Z = 8000x + 12000y$$

Subject to the constraints

$$3x + 4y \leq 60$$

$$x + 3y \leq 30$$

$$x \geq 0, \ y \geq 0$$

Solution:

Concept:
In the graphical method of a Linear Programming Problem:

- Convert inequalities into equations to draw boundary lines.
- Determine the feasible region satisfying all constraints.
- Identify corner points of the feasible region.
- Evaluate the objective function at those points.

Step 1: Find the boundary lines.
For

$$3x + 4y = 60$$

Intercepts:

$$x = 20 \ (y = 0), \qquad y = 15 \ (x = 0)$$

For

$$x + 3y = 30$$

Intercepts:

$$x = 30 \ (y = 0), \qquad y = 10 \ (x = 0)$$

Step 2: Find the intersection of the two lines.
Solve

$$3x + 4y = 60$$

$$x + 3y = 30$$

Multiply the second equation by 3:

$$3x + 9y = 90$$

Subtract:

$$5y = 30$$

$$y = 6$$

Substitute:

$$x + 18 = 30$$

$$x = 12$$

Thus intersection point:

$$(12, 6)$$

Step 3: Determine feasible region vertices.
Corner points are

$$(0, 0), \quad (20, 0), \quad (12, 6), \quad (0, 10)$$

Step 4: Evaluate the objective function.

$$Z = 8000x + 12000y$$

$$Z(0, 0) = 0$$

$$Z(20, 0) = 160000$$

$$Z(12, 6) = 8000(12) + 12000(6)$$

$$= 96000 + 72000$$

$$= 168000$$

$$Z(0, 10) = 120000$$

Step 5: Find the maximum value.

$$Z_{\text{max}} = 168000$$

at

$$(x, y) = (12, 6)$$

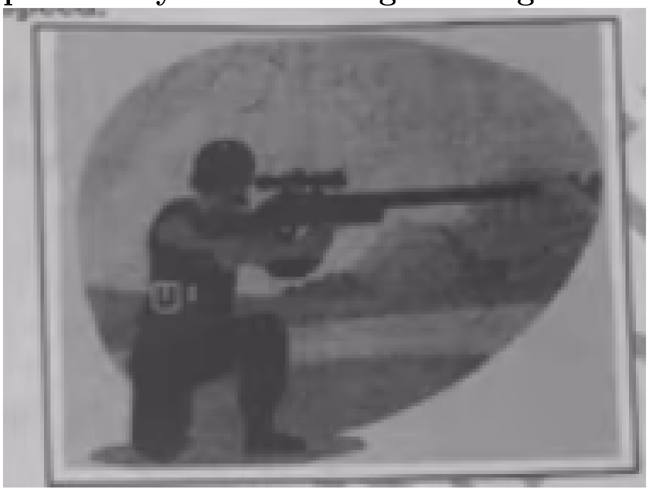
$$\therefore \text{Maximum value } Z = 168000 \text{ at } (12, 6).$$

Quick Tip

In graphical LPP, the optimal value always occurs at one of the corner points of the feasible region.

---

29. (a) The probability of hitting the target by a trained sniper is three times the probability of not hitting the target on a stormy day due to high wind speed.



The sniper fired two shots on the target on a stormy day when wind speed was very high. Find the probability that

(i) the target is hit,

(ii) at least one shot misses the target.

Solution:

Concept:
Let the probability of hitting the target be p. Then the probability of missing the target is $1 - p$.
Step 1: Form the relation.
Given

$$p = 3(1-p)$$

$$p = 3 - 3p$$

$$4p = 3$$

$$p = \frac{3}{4}$$

Thus

$$P(\text{hit}) = \frac{3}{4}, \quad P(\text{miss}) = \frac{1}{4}$$

Step 2: Probability that both shots hit the target.

$$P(\text{both hits}) = \left(\frac{3}{4}\right)^2 = \frac{9}{16}$$

Step 3: Probability that at least one shot misses.

$$P(\text{at least one miss}) = 1 - P(\text{both hits})$$

$$= 1 - \frac{9}{16}$$

$$= \frac{7}{16}$$

Step 4: Write the results.

$$P(\text{target hit in both shots}) = \frac{9}{16}$$

$$P(\text{at least one miss}) = \frac{7}{16}$$

Quick Tip

For repeated independent trials:

$$P(\text{both events}) = P(A) \times P(B)$$

For "at least one" events:

$$P(\text{at least one}) = 1 - P(\text{none})$$

---

29. (b) Mother, Father and Son line up at random for a family picture. Let events E: Son on one end and F: Father in the middle. Find $P(E|F)$.

Solution:

Concept:
Conditional probability formula:

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

Step 1: Total possible arrangements.
Three people can be arranged in

$$3! = 6$$

ways.
Step 2: Event F: Father in the middle.
Possible arrangements:

$$(M, F, S), \quad (S, F, M)$$

Thus

$$P(F) = \frac{2}{6} = \frac{1}{3}$$

Step 3: Event $E \cap F$: Son on one end and Father in middle.
Both arrangements above satisfy the condition.
Thus

$$P(E \cap F) = \frac{2}{6} = \frac{1}{3}$$

Step 4: Compute conditional probability.

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

$$= \frac{1/3}{1/3}$$

$$= 1$$

$$\therefore P(E|F) = 1$$

Quick Tip

Conditional probability:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

It measures the probability of A given that B has occurred.

30. Find

$$\int \frac{2x+1}{\sqrt{6x+x^2}}\,dx$$

Solution:

Concept:
For integrals of the form

$$\int \frac{f'(x)}{\sqrt{f(x)}}\,dx$$

use substitution $u = f(x)$.
Step 1: Choose substitution.
Let

$$u = x^2 + 6x$$

Then

$$\frac{du}{dx} = 2x + 6$$

$$du = (2x+6)dx$$

Rewrite the numerator:

$$2x + 1 = (2x+6) - 5$$

Thus the integral becomes

$$\int \frac{(2x+6)-5}{\sqrt{x^2+6x}}\,dx$$

$$= \int \frac{2x+6}{\sqrt{x^2+6x}}dx - 5\int \frac{dx}{\sqrt{x^2+6x}}$$

Step 2: Evaluate the first integral.
Using substitution $u = x^2 + 6x$:

$$\int \frac{2x+6}{\sqrt{x^2+6x}}dx = \int \frac{du}{\sqrt{u}}$$

$$= 2\sqrt{u}$$

$$= 2\sqrt{x^2+6x}$$

Step 3: Evaluate the second integral.

$$\int \frac{dx}{\sqrt{x^2+6x}}$$

Complete the square:

$$x^2 + 6x = (x+3)^2 - 9$$

Thus

$$\int \frac{dx}{\sqrt{(x+3)^2 - 9}} = \ln\left|x + 3 + \sqrt{x^2 + 6x}\right|$$

Step 4: Write the final result.

$$\int \frac{2x+1}{\sqrt{6x + x^2}} dx = 2\sqrt{x^2 + 6x} - 5\ln\left|x + 3 + \sqrt{x^2 + 6x}\right| + C$$

$$\therefore \text{Required integral} = 2\sqrt{x^2 + 6x} - 5\ln\left|x + 3 + \sqrt{x^2 + 6x}\right| + C.$$

Quick Tip

For integrals containing $\sqrt{x^2 + ax}$, complete the square:

$$x^2 + ax = (x + \frac{a}{2})^2 - \left(\frac{a}{2}\right)^2$$

---

31. (a) Evaluate

$$\int_{\frac{5\pi}{12}}^{\frac{13\pi}{12}} \frac{dx}{1 + \sqrt{\cot x}}$$

Solution:

Concept:

For definite integrals, the property

$$\int_a^b f(x)\,dx = \int_a^b f(a + b - x)\,dx$$

is useful for simplifying expressions.

Step 1: Let

$$I = \int_{\frac{5\pi}{12}}^{\frac{13\pi}{12}} \frac{dx}{1 + \sqrt{\cot x}}$$

Using the property

$$x \to \frac{5\pi}{12} + \frac{13\pi}{12} - x$$

$$= \frac{18\pi}{12} - x = \frac{3\pi}{2} - x$$

Thus

$$I = \int_{\frac{5\pi}{12}}^{\frac{13\pi}{12}} \frac{dx}{1 + \sqrt{\cot(\frac{3\pi}{2} - x)}}$$

But

$$\cot\left(\frac{3\pi}{2} - x\right) = \tan x$$

Hence

$$I = \int_{\frac{5\pi}{12}}^{\frac{13\pi}{12}} \frac{dx}{1 + \sqrt{\tan x}}$$

Step 2: Add the two integrals.

$$2I = \int_{\frac{5\pi}{12}}^{\frac{13\pi}{12}} \left(\frac{1}{1 + \sqrt{\cot x}} + \frac{1}{1 + \sqrt{\tan x}}\right) dx$$

Since

$$\sqrt{\cot x} = \frac{1}{\sqrt{\tan x}}$$

the sum simplifies to 1.
Thus

$$2I = \int_{\frac{5\pi}{12}}^{\frac{13\pi}{12}} dx$$

$$= \frac{13\pi}{12} - \frac{5\pi}{12}$$

$$= \frac{8\pi}{12}$$

$$= \frac{2\pi}{3}$$

Step 3: Find I.

$$I = \frac{1}{2} \cdot \frac{2\pi}{3}$$

$$I = \frac{\pi}{3}$$

$$\therefore \int_{\frac{5\pi}{12}}^{\frac{13\pi}{12}} \frac{dx}{1 + \sqrt{\cot x}} = \frac{\pi}{3}.$$

Quick Tip

For definite integrals:

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

This symmetry often simplifies trigonometric integrals.

---

31. (b) Evaluate

$$\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} (\sin|x| + \cos|x|)\;dx$$

Solution:

Concept:
Because of the modulus function, split the interval at $x = 0$.
Step 1: Write the integral in two parts.

$$I = \int_{-\frac{\pi}{6}}^{0} (\sin|x| + \cos|x|)\,dx + \int_{0}^{\frac{\pi}{6}} (\sin|x| + \cos|x|)\,dx$$

For $x < 0$:

$$|x| = -x$$

Thus

$$\sin|x| = \sin(-x) = -\sin x$$

$$\cos|x| = \cos(-x) = \cos x$$

Step 2: Evaluate the integrals.
After simplification,

$$I = 2\int_0^{\frac{\pi}{6}} (\sin x + \cos x)\,dx$$

$$= 2\left[-\cos x + \sin x\right]_0^{\frac{\pi}{6}}$$

$$= 2\left(\frac{1}{2} + \frac{\sqrt{3}}{2} - 1\right)$$

$$= \sqrt{3} - 1$$

$$\therefore \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} (\sin|x| + \cos|x|)\,dx = \sqrt{3} - 1.$$

Quick Tip

For integrals involving $|x|$, always split the interval at $x = 0$.

---

32. Find the domain of

$$p(x) = \sin^{-1}(3 - 2x).$$

Hence, find the value of x for which $p(x) = \frac{\pi}{6}$. Also write the range of $2p(x) + \frac{\pi}{2}$.

Solution:

Concept:
For the inverse sine function

$$y = \sin^{-1}(t)$$

the argument must satisfy

$$-1 \leq t \leq 1$$

and its range is

$$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}.$$

Step 1: Find the domain of $p(x)$.
Given

$$p(x) = \sin^{-1}(3 - 2x)$$

Thus

$$-1 \leq 3 - 2x \leq 1$$

Solve the inequalities:

$$-1 \leq 3 - 2x$$

$$-4 \leq -2x$$

$$x \leq 2$$

and

$$3 - 2x \leq 1$$

$$-2x \leq -2$$

$$x \geq 1$$

Hence

$$1 \le x \le 2$$

Thus the domain is

$$[1, 2].$$

Step 2: Find x when $p(x) = \frac{\pi}{6}$.

$$\sin^{-1}(3 - 2x) = \frac{\pi}{6}$$

$$3 - 2x = \sin\frac{\pi}{6}$$

$$3 - 2x = \frac{1}{2}$$

$$2x = \frac{5}{2}$$

$$x = \frac{5}{4}$$

Step 3: Find the range of $2p(x) + \frac{\pi}{2}$.
Since

$$-\frac{\pi}{2} \le p(x) \le \frac{\pi}{2}$$

Multiply by 2:

$$-\pi \le 2p(x) \le \pi$$

Add $\frac{\pi}{2}$:

$$-\frac{\pi}{2} \le 2p(x) + \frac{\pi}{2} \le \frac{3\pi}{2}$$

Thus the range is

$$\left[-\frac{\pi}{2}, \frac{3\pi}{2}\right].$$

Quick Tip

For $y = \sin^{-1}(x)$:

$$-1 \le x \le 1$$

and

$$-\frac{\pi}{2} \le y \le \frac{\pi}{2}.$$

33. A line passing through the points $A(1, 2, 3)$ and $B(6, 8, 11)$ intersects the line

$$\vec{r} = 4\hat{i} + \hat{j} + \lambda(6\hat{i} + 2\hat{j} + \hat{k}).$$

Find the coordinates of the point of intersection. Hence, write the equation of the line passing through the point of intersection and perpendicular to both the lines.

Solution:

Concept:
The equation of a line through two points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ is

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}.$$

A line perpendicular to two given lines has direction ratios equal to the cross product of their direction vectors.
Step 1: Equation of line AB.
Direction ratios:

$$\vec{AB} = (6 - 1,\ 8 - 2,\ 11 - 3) = (5, 6, 8)$$

Thus

$$\frac{x - 1}{5} = \frac{y - 2}{6} = \frac{z - 3}{8} = t$$

So

$$x = 1 + 5t, \quad y = 2 + 6t, \quad z = 3 + 8t$$

Step 2: Parametric form of the second line.

$$\vec{r} = (4, 1, 0) + \lambda(6, 2, 1)$$

Thus

$$x = 4 + 6\lambda, \quad y = 1 + 2\lambda, \quad z = \lambda$$

Step 3: Find the point of intersection.
Equate coordinates:

$$1 + 5t = 4 + 6\lambda$$

$$2 + 6t = 1 + 2\lambda$$

$$3 + 8t = \lambda$$

Solving,

$$t = -\frac{1}{3}, \qquad \lambda = \frac{1}{3}$$

Substitute t:

$$x = \frac{-2}{3}, \quad y = 0, \quad z = \frac{1}{3}$$

Thus the intersection point is

$$P\left(-\frac{2}{3}, 0, \frac{1}{3}\right)$$

Step 4: Direction ratios of required line.
Direction vectors:

$$\vec{d_1} = (5, 6, 8)$$

$$\vec{d_2} = (6, 2, 1)$$

Cross product:

$$\vec{d_1} \times \vec{d_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 6 & 8 \\ 6 & 2 & 1 \end{vmatrix}$$

$$= (-10)\hat{i} + 43\hat{j} - 26\hat{k}$$

Thus direction ratios:

$$(-10, 43, -26)$$

Step 5: Equation of the required line.
Through point $P\left(-\frac{2}{3}, 0, \frac{1}{3}\right)$:

$$\frac{x + \frac{2}{3}}{-10} = \frac{y}{43} = \frac{z - \frac{1}{3}}{-26}$$

$$\therefore \text{Equation of the required line obtained.}$$

Quick Tip

A line perpendicular to two given lines has direction ratios equal to the cross product of their direction vectors.

---

34. (a) If

$$P = \begin{bmatrix} 1 & -1 & 0 \\ 2 & a & 4 \\ 0 & 1 & 2 \end{bmatrix}, \qquad Q = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 1 & 5 \\ 2 & -1 & -5 \end{bmatrix},$$

find QP and hence solve the following system of equations using matrices

$$x - y = 8, \quad 2x + 3y + 4z = 17, \quad y + 2z = 7.$$

Solution:

Concept:
A system of linear equations can be written in matrix form

$$AX = B$$

and the solution is

$$X = A^{-1}B$$

if A^{-1} exists.
Step 1: Write the coefficient matrix.

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$$

and

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \qquad B = \begin{bmatrix} 8 \\ 17 \\ 7 \end{bmatrix}.$$

Step 2: Solve the equations.
From

$$x - y = 8$$

$$x = y + 8$$

From

$$y + 2z = 7$$

$$y = 7 - 2z$$

Substitute into the second equation:

$$2x + 3y + 4z = 17$$

$$2(y + 8) + 3(7 - 2z) + 4z = 17$$

$$2y + 16 + 21 - 6z + 4z = 17$$

$$2y - 2z + 37 = 17$$

$$2y - 2z = -20$$

$$y - z = -10$$

Substitute $y = 7 - 2z$:

$$7 - 2z - z = -10$$

$$7 - 3z = -10$$

$$z = \frac{17}{3}$$

$$y = 7 - 2\left(\frac{17}{3}\right) = -\frac{13}{3}$$

$$x = y + 8 = \frac{11}{3}$$

Step 3: Write the solution.

$$x = \frac{11}{3}, \quad y = -\frac{13}{3}, \quad z = \frac{17}{3}$$

$\therefore$ Solution of the system obtained.

Quick Tip

A system of linear equations can be solved using matrices by writing it in the form

$$AX = B$$

and finding $X = A^{-1}B$.

---

34. (b) Obtain the value of

$$A = \begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix}$$

in terms of x, y, z.

Further, if $A = 0$ and x, y, z are non-zero real numbers, prove that

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = -1.$$

Solution:

Step 1: Expand the determinant.

$$A = \begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix}$$

Apply row operations

$$R_2 \to R_2 - R_1, \quad R_3 \to R_3 - R_1$$

$$A = \begin{vmatrix} 1+x & 1 & 1 \\ -x & y & 0 \\ -x & 0 & z \end{vmatrix}$$

Expanding,

$$A = xyz(x+y+z+1)$$

Step 2: Use the condition $A = 0$.

$$xyz(x+y+z+1) = 0$$

Since $x, y, z \neq 0$,

$$x+y+z+1 = 0$$

Divide by xyz:

$$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = -1$$

$$\therefore \text{Result proved.}$$

Quick Tip

Row operations simplify determinants without changing their value.

---

35. (a) Find the sub-interval of $(0, \pi)$ in which the function

$$f(x) = \tan^{-1}(\sin x - \cos x)$$

is increasing and decreasing.

Solution:

Concept:
To determine where a function is increasing or decreasing, compute its first derivative and analyze its sign.
Step 1: Differentiate the function.

$$f(x) = \tan^{-1}(\sin x - \cos x)$$

Using the derivative of $\tan^{-1} u$:

$$\frac{d}{dx}(\tan^{-1} u) = \frac{u'}{1+u^2}$$

Let $u = \sin x - \cos x$

$$u' = \cos x + \sin x$$

Thus

$$f'(x) = \frac{\cos x + \sin x}{1 + (\sin x - \cos x)^2}$$

Step 2: Determine the sign of the derivative.
The denominator

$$1 + (\sin x - \cos x)^2 > 0$$

always.
Thus the sign depends on

$$\cos x + \sin x$$

Step 3: Solve

$$\cos x + \sin x = 0$$

$$\tan x = -1$$

In $(0, \pi)$,

$$x = \frac{3\pi}{4}$$

Step 4: Determine intervals.
For

$$0 < x < \frac{3\pi}{4}$$

$$\cos x + \sin x > 0$$

Thus $f'(x) > 0 \rightarrow$ function increasing.
For

$$\frac{3\pi}{4} < x < \pi$$

$$\cos x + \sin x < 0$$

Thus $f'(x) < 0 \rightarrow$ function decreasing.

Step 5: Write the result.
Increasing on

$$(0, \frac{3\pi}{4})$$

Decreasing on

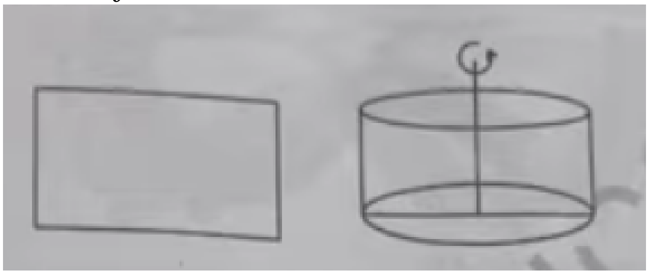
$$(\frac{3\pi}{4}, \pi)$$

Quick Tip

A function is increasing where $f'(x) > 0$ and decreasing where $f'(x) < 0$.

---

35. (b) A rectangle of perimeter 24 cm is revolved along one of its sides to sweep out a cylinder of maximum volume. Find the dimensions of the rectangle.



Solution:

Concept:
If a rectangle is revolved about one of its sides, the resulting solid is a cylinder.

$$V = \pi r^2 h$$

where r is the other side of the rectangle and h is the axis of rotation.
Step 1: Let the sides of the rectangle be x and y.
Given perimeter

$$2(x + y) = 24$$

$$x + y = 12$$

$$y = 12 - x$$

Step 2: Write the volume of the cylinder.
Assume the rectangle rotates about side x:

$$V = \pi y^2 x$$

Substitute y:

$$V = \pi x(12 - x)^2$$

Step 3: Maximize the volume.
Differentiate:

$$\frac{dV}{dx} = \pi[(12 - x)^2 - 2x(12 - x)]$$

$$= \pi(12 - x)(12 - 3x)$$

Set

$$\frac{dV}{dx} = 0$$

$$12 - 3x = 0$$

$$x = 4$$

$$y = 12 - 4 = 8$$

Step 4: Write the dimensions.
Thus the rectangle dimensions are

$$4 \text{ cm} \times 8 \text{ cm}.$$

$$\therefore \text{Required dimensions obtained.}$$

Quick Tip

Optimization problems use derivatives:

$$\frac{dV}{dx} = 0$$

gives maximum or minimum values.

---

36. A racing track is built around an elliptical ground whose equation is

$$9x^2 + 16y^2 = 144.$$

The width of the track is 3 m.

Based on the given information answer the following questions:

(i) Express y as a function of x from the given equation of ellipse.

(ii) Integrate the function obtained in (i) with respect to x.

(iii) Find the area of the region enclosed within the elliptical ground excluding the track using integration.
OR
Write the coordinates of points P and Q where the outer edge of the track cuts the x-axis and y-axis in the first quadrant and find the area of the triangle formed by P, O, Q.

Solution:

Concept:
The given ellipse is

$$9x^2 + 16y^2 = 144$$

which can be written in standard form

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

Thus

$$a = 4, \quad b = 3.$$

Step 1: Express y as a function of x.

$$9x^2 + 16y^2 = 144$$

$$16y^2 = 144 - 9x^2$$

$$y^2 = \frac{144 - 9x^2}{16}$$

$$y = \frac{\sqrt{144 - 9x^2}}{4}$$

Step 2: Integrate the function.

$$\int y\,dx = \int \frac{\sqrt{144 - 9x^2}}{4}\,dx$$

Factor 9:

$$\int \frac{\sqrt{9(16 - x^2)}}{4}\,dx$$

$$= \frac{3}{4}\int \sqrt{16 - x^2}\,dx$$

Using the standard integral

$$\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C$$

Thus

$$\int y\, dx = \frac{3}{8}\left[x\sqrt{16 - x^2} + 16\sin^{-1}\frac{x}{4}\right] + C$$

Step 3: Area enclosed by the ellipse.

Area of ellipse

$$A = \pi ab$$

$$= \pi(4)(3)$$

$$= 12\pi$$

Thus the area enclosed by the elliptical ground is

$$\boxed{12\pi \text{ square units}}$$

Alternative (iii): Area of triangle POQ.

Intercepts of the ellipse:

$$x = 4, \quad y = 3$$

Thus

$$P(4, 0), \quad Q(0, 3)$$

Area of triangle POQ:

$$\frac{1}{2} \times 4 \times 3$$

$$= 6$$

$$\therefore \text{Area of triangle } POQ = 6.$$

Quick Tip

For ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Area $= \pi ab$.

37. The equation of one such racing track is given as

$$f(x) = \begin{cases} x^4 - 4x^2 + 4, & 0 \le x < 3 \\ x^2 + 40, & x \ge 3 \end{cases}$$

Based on the given information answer the following questions:

(i) Find $f'(x)$ for $0 < x < 3$.

(ii) Find $f'(4)$.

(iii) (a) Test for continuity of $f(x)$ at $x = 3$.
OR
(b) Test for differentiability of $f(x)$ at $x = 3$.

Solution:

Concept:
To analyze piecewise functions:

- Differentiate each part separately.
- For continuity at $x = a$: $\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = f(a)$.
- For differentiability: left and right derivatives must be equal.

Step 1: Find $f'(x)$ for $0 < x < 3$.

$$f(x) = x^4 - 4x^2 + 4$$

Differentiate:

$$f'(x) = 4x^3 - 8x$$

$$\therefore f'(x) = 4x^3 - 8x$$

for $0 < x < 3$.

Step 2: Find $f'(4)$.
For $x \ge 3$

$$f(x) = x^2 + 40$$

$$f'(x) = 2x$$

Thus

$$f'(4) = 2(4) = 8$$

Step 3: Test continuity at $x = 3$.
Left-hand limit:

$$\lim_{x\to 3^-} f(x) = 3^4 - 4(3^2) + 4$$

$$= 81 - 36 + 4$$

$$= 49$$

Right-hand limit:

$$\lim_{x\to 3^+} f(x) = 3^2 + 40$$

$$= 9 + 40$$

$$= 49$$

Also

$$f(3) = 3^2 + 40 = 49$$

Thus

$$\text{LHL} = \text{RHL} = f(3)$$

$$\therefore f(x) \text{ is continuous at } x = 3.$$

Alternative (iii)(b): Test differentiability at $x = 3$.
Left derivative:

$$f'(x) = 4x^3 - 8x$$

$$f'(3^-) = 4(27) - 8(3)$$

$$= 108 - 24$$

$$= 84$$

Right derivative:

$$f'(x) = 2x$$

$$f'(3^+) = 2(3) = 6$$

Since

$$84 \neq 6$$

$$\therefore f(x) \text{ is not differentiable at } x = 3.$$

Quick Tip

A function is differentiable at a point only if it is continuous there and its left and right derivatives are equal.

---

38. A study revealed that 170 in 1000 males who smoke develop lung complications, while 120 out of 1000 females who smoke develop lung related problems.

In a colony, 50 people were found to be smokers, of which 30 are males. A person is selected at random from these 50 people and tested for lung related problems. Based on the given information answer the following questions:

(i) What is the probability that the selected person is a female?

(ii) If a male person is selected, what is the probability that he will not be suffering from lung problems?

(iii) (a) A person selected at random is detected with lung complications. Find the probability that the selected person is a female.
OR
(b) A person selected at random is not having lung problems. Find the probability that the person is a male.

Solution:

Concept:
Use basic probability and Bayes' theorem.
Let

$$M = \text{male}, \quad F = \text{female}$$

$$L = \text{lung complication}$$

Given:

$$P(L|M) = \frac{170}{1000} = 0.17$$

$$P(L|F) = \frac{120}{1000} = 0.12$$

Number of smokers = 50

$$30 \text{ males}, \quad 20 \text{ females}$$

Thus

$$P(M) = \frac{30}{50} = \frac{3}{5}$$

$$P(F) = \frac{20}{50} = \frac{2}{5}$$

Step 1: (i) Probability that the selected person is female.

$$P(F) = \frac{20}{50}$$

$$= \frac{2}{5}$$

Step 2: (ii) Probability that a male does not suffer lung problems.

$$P(\text{No lung}|M) = 1 - P(L|M)$$

$$= 1 - 0.17$$

$$= 0.83$$

Step 3: (iii)(a) Probability that the person is female given lung complications.
Using Bayes' theorem:

$$P(F|L) = \frac{P(F)P(L|F)}{P(M)P(L|M) + P(F)P(L|F)}$$

Substitute values:

$$P(F|L) = \frac{\frac{2}{5} \times 0.12}{\frac{3}{5} \times 0.17 + \frac{2}{5} \times 0.12}$$

$$= \frac{0.048}{0.102 + 0.048}$$

$$= \frac{0.048}{0.15}$$

$$= 0.32$$

$$\therefore P(F|L) = 0.32$$

Alternative (iii)(b): Probability that the person is male given no lung problems.

$$P(\text{No lung}|M) = 0.83$$

$$P(\text{No lung}|F) = 1 - 0.12 = 0.88$$

Total probability:

$$P(\text{No lung}) = \frac{3}{5}(0.83) + \frac{2}{5}(0.88)$$

$$= 0.498 + 0.352$$

$$= 0.85$$

Thus

$$P(M|\text{No lung}) = \frac{P(M)P(\text{No lung}|M)}{P(\text{No lung})}$$

$$= \frac{\frac{3}{5} \times 0.83}{0.85}$$

$$= \frac{0.498}{0.85}$$

$$\approx 0.586$$

$$\therefore P(M|\text{No lung}) \approx 0.586$$

Quick Tip

Bayes' theorem:

$$P(A|B) = \frac{P(A)P(B|A)}{\sum P(A_i)P(B|A_i)}$$

It helps update probabilities based on new evidence.